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Design of Crystal Filters

Charles E. Schmidt

DESIGN OF CRYSTAL FILTERS - SCHMIDT

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By

Charles E. Schmidt

A Thesis

Submitted in Partial Fulfillment of the

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Master of Science in Electrical Engineering

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This thesis, directed and approved by the candidate's committee, has been accepted by the Graduate Committee of the University of New Mexico in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

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INTRODUCTION

In some applications, the performance required of frequency selective networks is such that the ordinary types of lumped impedance elements are found to be insufficiently close to the ideal. For instance, in bandpass filters, the squareness of the corners of the attenuation curve depends on the Q of the elements in the filter. Since the maximum Q obtainable from conventional coils is approximately 800, an improvement in the corresponding performance of the filter can be obtained by use of high-Q electromechanical resonators. One important type of resonator is the piezoelectric crystal. Filters in which electromechanical resonators are used are included in the term "crystal filters" whether the resonators are crystals or not.

Part I is a discussion of electromechanical resonators and contains a simple mathematical derivation of the equivalent circuit of a longitudinal piezoelectric crystal resonator which leads to an expression for the Q of such a resonator which I believe to be unique.

Part II is a discussion of crystal lattice filters. One of the difficulties which arises in the design

of crystal filters is the "packaged" form of the resonator impedance; that is, the equivalent circuit consists of several elements which cannot be separated, among which certain relationships apply. This obstacle of impedance packages is usually circumvented by designing crystal filters as symmetrical lattices, since lattice filters afford the designer the most flexibility in design. When the resonators used are piezoelectric crystals, the relationships between the elements in the equivalent circuit will be shown to become limitations on the bandwidth and impedance obtainable. A number of transformations between lattice filters and equivalent unbalanced filters are given, several of which do not appear elsewhere.

The various methods of obtaining unbalanced crystal filters are discussed in Part III. The design formulas developed by Mason (2) are used in conjunction with a technique given in Appendix B for replacing a given crystal by a different crystal in series with a capacitor to obtain realizability conditions for a broadband unbalanced crystal bandpass filter in the form of a twin-T network.

I. Electromechanical Transducers

This thesis concerns the application of electromechanical impedance elements to electric wave filters, so it is appropriate to begin with some discussion of such elements. The most important type is the piezoelectric resonator, most commonly made of quartz.

The piezoelectric effect.--The phenomenon which became known as piezoelectricity first came to the attention of Europeans about 1700, when tourmaline crystals were imported by Dutch traders (1). These crystals, when placed in hot ashes, were observed first to attract and then to repel them. This effect was given the name "pyroelectricity" in 1824 after it had been noticed in several other kinds of crystals. It remained for the brothers Curie to discover that certain crystals, when compressed or extended in certain directions show positive and negative charges on parts of their surfaces which are proportional to the applied pressure. This effect and its converse, (deformation produced by an electric field), were given the name "piezoelectricity." Among the known piezoelectric materials are quartz, tourmaline, topaz, Rochelle salt, and cane sugar. The electrical behavior of the tourmaline crystals had actually been caused by piezoelectricity induced by

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thermal stresses. The piezoelectric effect remained a scientific curiosity until the advent of the First World War and submarines. Paul Langevin used resonators made of quartz slabs sandwiched between steel plates in the first sonar apparatus, which he developed for the French government. Although the device was not ready for use until after the war, it was successfully used as a sonic depth finder for exploration of the ocean bottom and for locating submerged objects. Piezoelectric crystals have since found use in a variety of practical applications, among which are their applications as elements in filter networks.

The equivalent circuit.--In order to make use of crystals in electric networks, it is desirable to have at hand an equivalent circuit which describes the electrical impedance seen looking into the terminals of the crystal resonator. Resonant behavior would be expected from a system containing a vibrating mass, so at least one inductance and one capacitance would be expected in the circuit. Since physically the crystal ordinarily consists of two conducting electrodes separated by a slab of dielectric material, a shunt capacitance would be expected across the input. One important class of crystals is that in which the mechanical strain produced in the crystal material is in the same direction as the electric field which causes the strain. When the field varies with time, such a crystal is said

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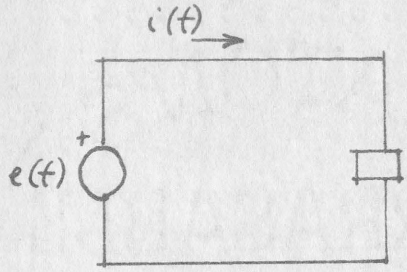
to be in longitudinal motion.

Fig. 1



The relations describing a piezoelectric crystal in longitudinal motion are (2):

$$q(t) = k f(t) + \epsilon A E(t) = k f(t) + \epsilon A \frac{e(t)}{W} \qquad (1)$$

$$\frac{x(t)}{W} = \frac{f(t)}{YA} + k E(t) = \frac{f(t)}{YA} + k \frac{e(t)}{W} \qquad (2)$$

where q(t) charge on the surface of the crystal

f(t) = applied force

ϵ = dielectric constant of crystal material

A = surface area

W = width of crystal between electrodes

k = piezoelectric constant

e(t) = applied voltage

i(t) = crystal current

x(t) = displacement

An intuitive basis for Eq.(1) is that the total charge observed on such a crystal is the sum of the charge due to the piezoelectric effect, with k being the constant of proportionality, and the charge due to the voltage e(t) applied across the capacitor constituted by the crystal material between the electrodes. An intuitive basis for Eq. (2) is that the total strain is the sum of the strain

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due to the piezoelectric effect and the strain given by the stress due to f(t) divided by Young's modulus.

Considering a crystal free of external restraints:

$$f(t) = -D\frac{dx}{dt} - M\frac{d^2x}{dt^2} \tag{3}$$

where D = frictional resistance

M = equivalent mass

Transforming gives:

$$F(s) = -X(s)\left[Ds + Ms^2\right] \tag{4}$$

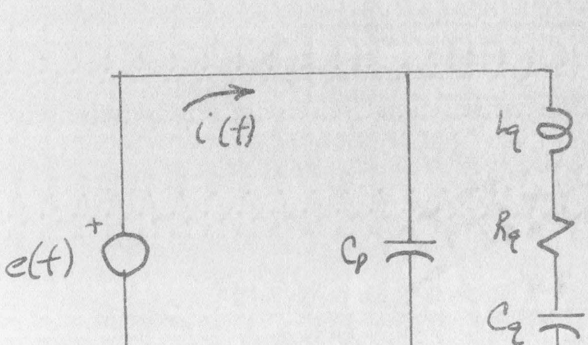
$$Q(s) = kF(s) + C_s E(s) \tag{1a}$$

$$X(s) = \frac{W\,F(s)}{YA} + kE(s) \tag{2a}$$

where $C_s = \epsilon A/W$

In Appendix A these equations are shown to lead to the equivalent circuit of Fig. 2.

Fig. 2



$$C_p = C_s\left(1 - \frac{Yk^2}{\epsilon}\right)$$

$$L_q = M\left(\frac{W}{YAk}\right)^2$$

$$R_q = D\left(\frac{W}{YAk}\right)^2$$

$$C_q = C_s\,\frac{Yk^2}{\epsilon}$$

Since for quartz $Yk^2/\epsilon \approx 0.009$, the ratio $C_p/C_\ell \approx 110$. This ratio is found to be important in the utilization of quartz crystals. In practical crystals, this ratio is usually in the range 200-500, with 140 as the minimum value obtainable (3). The Q of the RLC branch is equal to $\omega L_\ell/R_\ell$ Substituting $\omega = 1/\sqrt{L_\ell C_\ell}$ gives Q $L_\ell/R_\ell\sqrt{C_\ell}$. Substitution of the equivalent mechanical quantities gives:

$$Q = \frac{\sqrt{\dfrac{MW^2}{Y^2A^2k^2}}}{\dfrac{DW^2}{Y^2A^2k^2}\sqrt{C_s\dfrac{Yk^2}{\epsilon}}} = \frac{\sqrt{\dfrac{MAY}{W}}}{D}$$

The equivalent mass M would not be the entire mass of the crystal, but substitution of $M = \rho AW$ as an approximation, where ρ = density, gives:

$$Q = \frac{\sqrt{\rho Y}}{D} A \qquad (13)$$

Q is therefore inversely proportional to the dissipation constant D, as might have been expected, but the rather surprising inference is that Q can be increased indefinitely by increasing the surface area. Considerations of ruggedness and suppression of spurious vibrational modes limit the amount by which Q can be increased in this manner. Also, D is not really wholly independent of A, since dissipation in a vibrating crystal is partly due to friction in minute cracks and

irregularities in the surface (1).

Q of the order of 100,000 is obtainable with careful etching and polishing of the crystal surface and careful mounting in an evacuated container. Special techniques have yielded crystals with Q greater than one million. These high values of Q justify considering the equivalent network to be purely reactive. This does not mean, however, that the value of R_q would be small compared to other resistances in the system. For instance, consider a 100 kilocycle crystal with $Q = 20,000$. A typical value of L_q for such a crystal would be 100 henries, so:

$$R_q = \frac{2\pi f L_q}{Q} = 3.14 \times 10^3 \ \Omega$$

It should be understood than an equivalent circuit for a crystal is only valid when the effects of other resonances can be considered negligible. A given crystal can vibrate in an infinite number of ways, among which are the various compressional, shear, flexural, and torsional modes, and innumerable combinations thereof. Various techniques of proportioning crystals and cutting them along certain crystallographic axes have been developed to minimize coupling between modes, but such coupling can never be entirely eliminated. Fortunately, the coupling effects can often be considered small.

<u>Other transducers</u>.--Magnetostrictive elements

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<u>Other transducers</u>.--Magnetostrictive elements

constitute the principal alternative to crystals for use as electromechanical impedances in filters. Magnetostriction occurs in ferrites and alloys of nickel and iron, and involves a strain induced in the material in the direction of an applied magnetic field. The converse effect of production of a magnetic field by a mechanical stress also exists, which makes possible the use of magnetostrictive resonators in filter circuits. Losses due to hysteresis and eddy currents in the magnetostrictive material limit the Q obtainable with such resonators (2). This limitation has prevented the extensive use of magnetostrictive elements in filters, although some work has been done. Ferroelectric ceramics have found some use as resonators, but they also have the limitation of low Q. Tuning forks with piezoelectric or electromagnetic transducers have been used for audio frequency filters (4).

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II. Crystal Lattice Filters

Crystal resonators have found extensive use as elements in lattice filters. The image impedance of any two-port network is (5):

$$Z_o = \sqrt{Z_{sh}\, Z_{op}} \tag{14}$$

The image propagation constant for such a network terminated in its image impedance is given by:

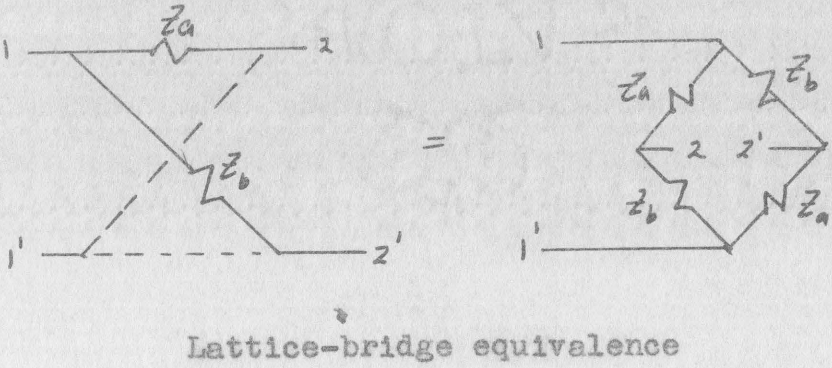
$$\tanh \gamma = \sqrt{\frac{Z_{sh}}{Z_{op}}} \tag{15}$$

where: Z_{sh} = short circuit input impedance

Z_{op} = open circuit input impedance

These quantities can be found for a symmetrical lattice by drawing it in bridge form.

Fig. 3



Lattice-bridge equivalence

The open and short circuit impedances are seen to be:

$$Z_{sh} = 2Z_aZ_b/(Z_a+Z_b) \tag{16}$$

$$Z_{op} = \frac{1}{2}\left(Z_a + Z_b\right) \tag{17}$$

This gives the relations:

$$Z_0 = \sqrt{Z_a Z_b} \tag{18}$$

$$\tanh \gamma = \frac{2\sqrt{Z_a Z_b}}{Z_a + Z_b} \tag{19}$$

The equation for tanh can be written in simpler for by making use of the identity:

$$\tanh \frac{x}{2} = \frac{\cosh x - 1}{\sinh x}$$

and:

$$\cosh x = 1/\sqrt{1-\tanh^2 x} \tag{20}$$

$$\sinh x = \tanh x/\sqrt{1-\tanh^2 x} \tag{21}$$

Therefore,

$$\tanh \frac{\gamma}{2} = \frac{1-\sqrt{1-\tanh^2 \gamma}}{\tanh \gamma} = \frac{2Z_a}{2\sqrt{Z_a Z_b}}$$

$$\tanh \frac{\gamma}{2} = \sqrt{\frac{Z_a}{Z_b}} \tag{22}$$

and

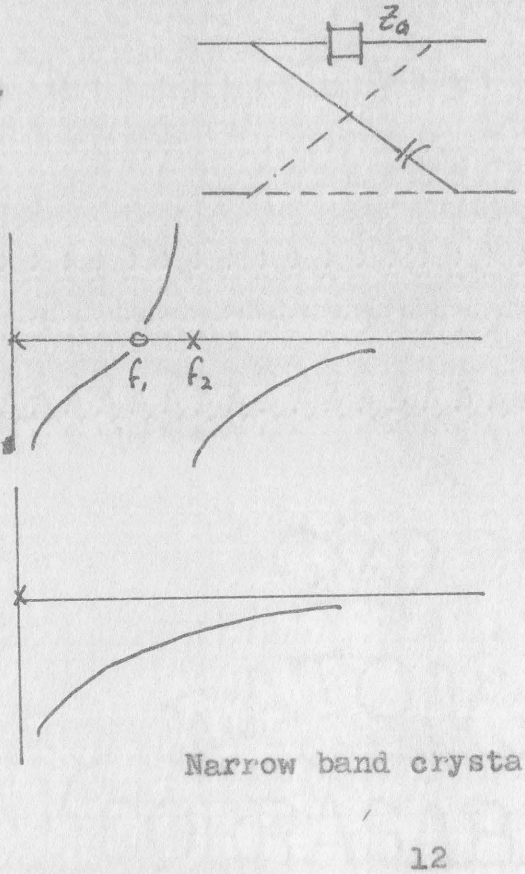
$$\gamma = \alpha + j\beta = 2\tanh^{-1}\sqrt{\frac{Z_a}{Z_b}} \tag{23}$$

where α is the attenuation constant and β is the phase constant.

Assuming purely reactive impedances, $\sqrt{Z_a/Z_b}$ is either purely real when Z_a and Z_b are the same sign, or purely imaginary when Z_a and Z_b are the opposite sign. Hence either α or β is zero, and the filter has a pass band when the reactances are of opposite sign, and a stop band when they have the same sign.

A consequence of Foster's Reactance Theorem is that the poles and zeros of a purely reactive two-terminal network must be simple and lie on the $j\omega$-axis, and alternate with each other. Drawing pole-zero plots for Z_a and Z_b shows how the locations of the critical frequencies determine the performance of the filter.

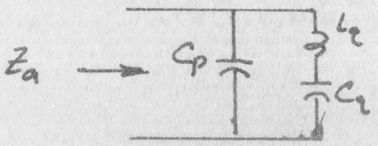
Fig. 4



Narrow band crystal filter

It can be seen that the interval between f_1 and f_2 is the only one in which the criterion for a pass band is met. In general, the pass band will exist where poles are opposite zeros and zeros opposite poles, and the cutoff frequency will occur where either impedance has a pole or zero which is not matched by a zero or pole of the other impedance.

The bandwidth of the simple filter of Fig. 4 is $f_2 - f_1$. The importance of the ratio C_p/C_q is shown by the following analysis, which indicates that the percentage bandwidth of this filter is a function of the ratio.



$$Z_a = \frac{1}{sC_p + \dfrac{C_q}{sL_qC_q + \frac{1}{s}}} = \frac{s^2 + \frac{1}{L_qC_q}}{sC_p\left(s^2 + \dfrac{C_p+C_q}{L_qC_qC_p}\right)}$$

$$\omega_1 = \frac{1}{\sqrt{L_q C_q}}, \qquad \omega_2 = \frac{1}{\sqrt{\dfrac{L_qC_qC_p}{C_q+C_p}}}$$

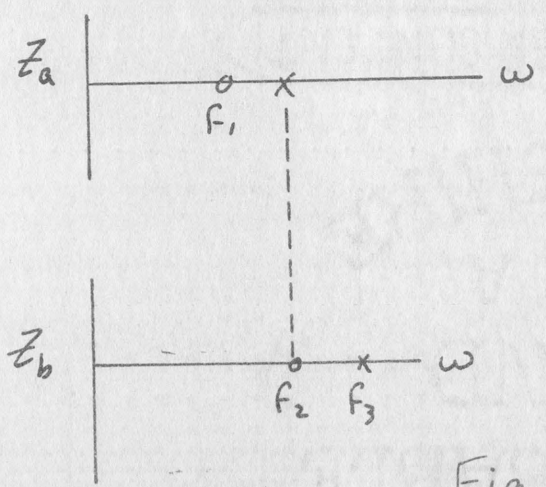
Defining percentage bandwidth as $\%BW = \dfrac{\omega_2 - \omega_1}{\omega_1}$

$$\%BW = \sqrt{1 + C_q/C_p} \; - 1 \qquad (26)$$

For $C_p/C_q = 140$, $\%BW = 0.0036 = 0.36\%$.

Addition of capacitance in parallel with the crystal only serves to increase C_p/C_q and thereby decrease the bandwidth. Capacitance in series with the crystal leaves the pole frequency unchanged while increasing the frequency of the zero, and so also decreases the bandwidth.

The percentage bandwidth can be increased to o.72% by arranging the poles and zeros according to the diagram of Figure 5. Crystals with the minimum value of the capacity ratio (C_p/C_q =140) should be used if maximum bandwidth is desired.



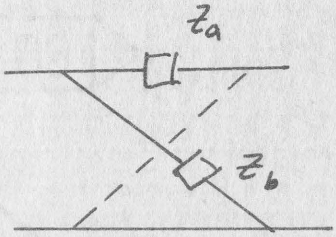


Fig 5

The pass band will extend from f_1 to f_3, giving a bandwidth twice that of the filter of Figure 4. Coils must be used if the bandwidth is to be further increased, and unless special techniques are used, the dissipation inherent in even the best coils will negate the advantage of high Q in the crystal filter. Symmetrical lattice filters with crystals, coils, and capacitors can have percentage bandwidths as high as 13.5%.

The zero frequency of a crystal is readily controllable in manufacture to a very close tolerance, but since the pole frequency depends on C_p, which is the sum of several stray capacitances, a small trimmer capacitor is ordinarily used in parallel with each crystal in order to set the pole frequency exactly where it

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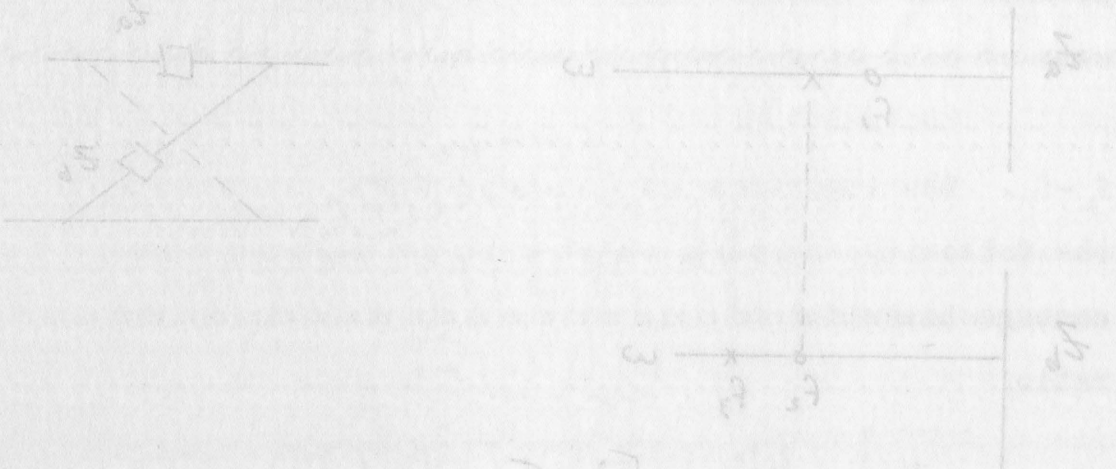

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is supposed to be. This necessarily further reduces the maximum bandwidth of the filter of Figure 5 to about 0.6%.

Since the filter of figure 5 contains two pairs of identical crystals, common practice is to use only two physical crystals, each with two pairs of electrodes. Techniques for doing this are given by Mason (2) and Heising (6).

Bartlett's Bisection Theorem.--This theorem states that for networks having mirror image symmetry, the series arms of the equivalent lattice network are found by bisecting the network along the line of symmetry, short circuiting all the cut wires and using the resulting two terminal networks as the series arms, while the lattice arms are obtained by using the two terminal networks formed by open circuiting all the cut wires.

When the original network contains symmetrical crossed connections, the equivalent series arm will be the input impedance of the network with all the straight through connections short circuited and all the crossed connections open circuited, whilt the equivalent lattice arm will be the input impedance with all the straight through connections open circuited and all the crossed wires short circuited. The second part of the theorem leads to the lattice transformations given in Figure 6 and Figure 7.

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Fig 6

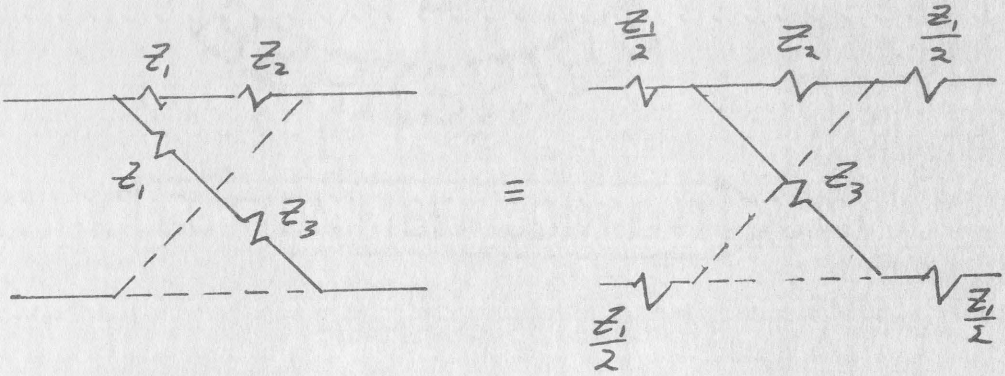
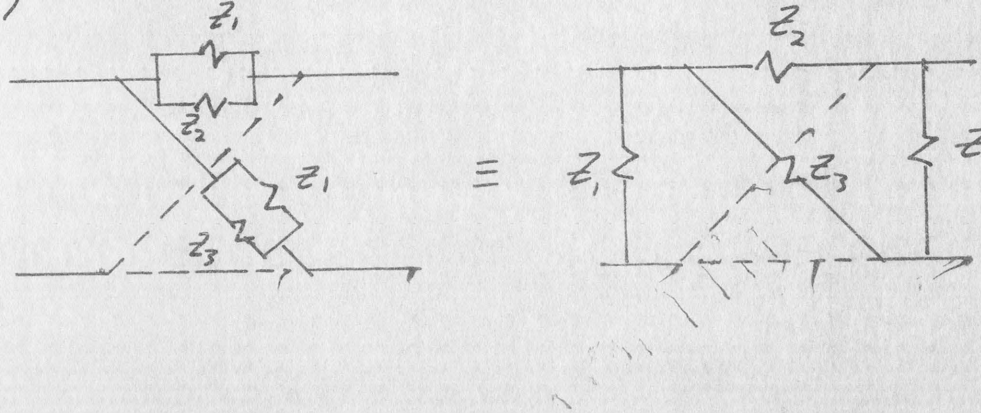


Fig 7



These two transformations are useful for increasing bandwidth by placing coils in series or in parallel with crystals without degrading the performance of the filter.

The equal resistances associated with the inductances (if the resistances are not equal they can be made so by addition of resistance to one pair of coils) can be brought outside the lattice and incorporated with the terminating resistances of the filter. These resistances therefore introduce a constant loss which is independent of frequency, and can therefore be fully compensated by an amplifier. The filter network of Figure 8 has a bandwidth of 13.7% if the elements are propor-

tioned correctly, and the filter retains the sharp cutoff characteristic of crystal filters without coils Fig. 8

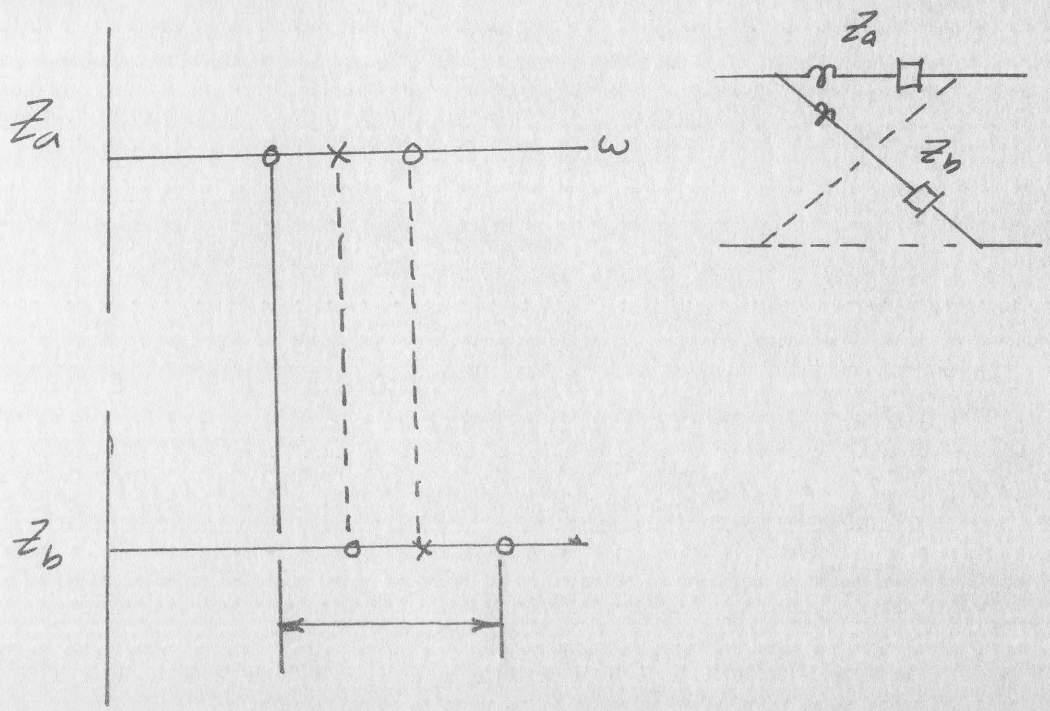


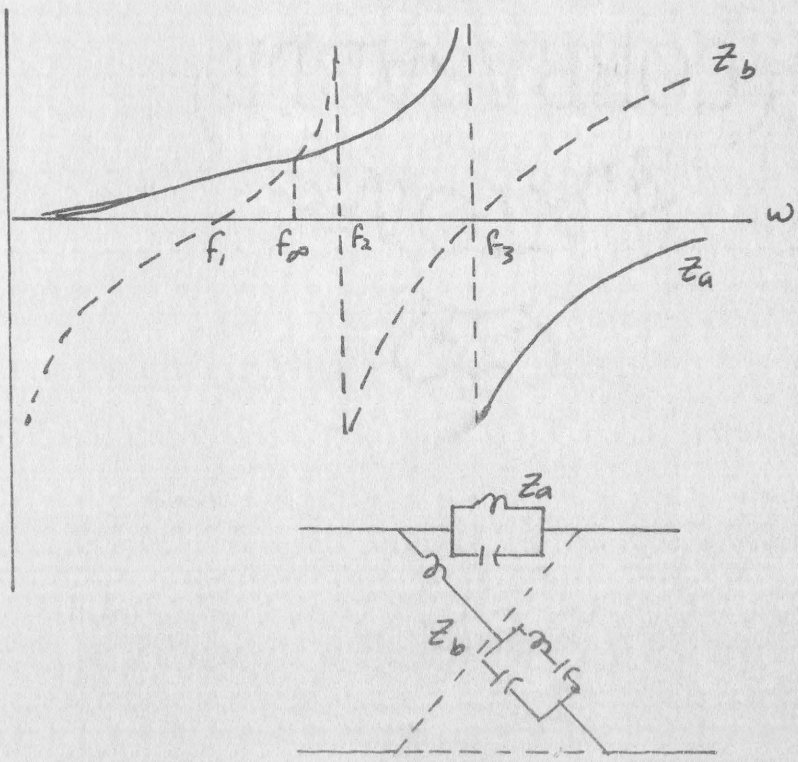
Fig. 8 Broadband crystal filter

Infinite points (frequencies of infinite attenuation) occur when the bridge is balanced; i.e., $Z_a = Z_b$. The filter of Figure 4 will have one infinite point, that of Figure 5 will have two, and that of Figure 8 will have three.

Band - elimination filters. -- Crystals are often used in filters that are designed to reject a narrow band of frequencies and pass all others. Since a purely reactive filter has a stop band where the series and lattice reactances are the same sign and a

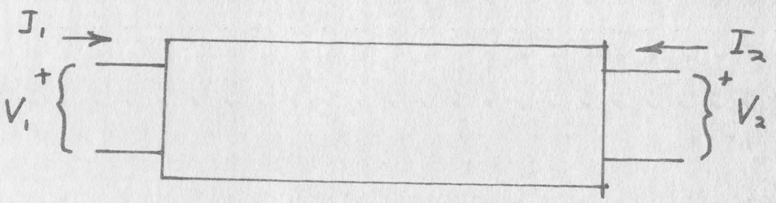
pass band where they are opposite, it is obvious that the filter of Figure 9 has a stop band only between f_1 and f_2, with an infinite point at f_∞.

Fig. 9



Various other configurations are used for band-elimination filters, but that of Figure 9 proves to be the most satisfactory, partly because it transforms easily to a bridged -T equivalent, as will be shown later. Design formulas are given in the reference (7).

Equivalent networks. -- The z and y matrices for a two part network are defined as follows (8).

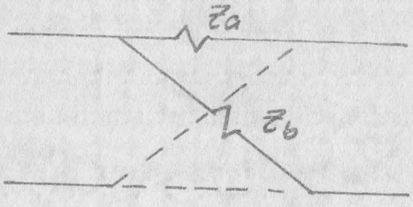


$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

(27)

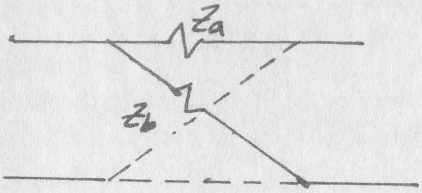
The conditions for the equivalence of two-port networks can be found by equating their z or y matrices. The z matrix for a symmetrical lattice is (8)



$$[z] \quad \begin{bmatrix} \frac{1}{2}(Z_b+Z_a) & \frac{1}{2}(Z_b-Z_a) \\ \frac{1}{2}(Z_b-Z_a) & \frac{1}{2}(Z_b+Z_a) \end{bmatrix}$$

(28)

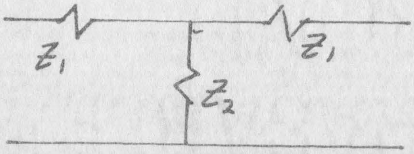
The corresponding y matrix is



$$[Y] = \begin{bmatrix} \dfrac{Z_b + Z_a}{2 Z_a Z_b} & \dfrac{Z_a - Z_b}{2 Z_a Z_b} \\ \dfrac{Z_a - Z_b}{2 Z_a Z_b} & \dfrac{Z_b + Z_a}{2 Z_a Z_b} \end{bmatrix} \tag{29}$$

T - network.

The z matrix for a symmetrical T network is as shown below.



$$[Z] = \begin{bmatrix} Z_1 + Z_2 & Z_2 \\ Z_2 & Z_1 + Z_2 \end{bmatrix} \tag{30}$$

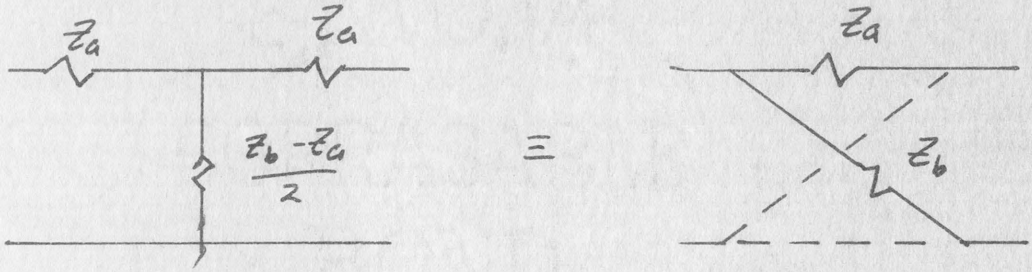
Equating the z matrices gives the relations

$$Z_1 = Z_a \tag{31}$$

$$Z_2 = \frac{Z_b - Z_a}{2} \tag{32}$$

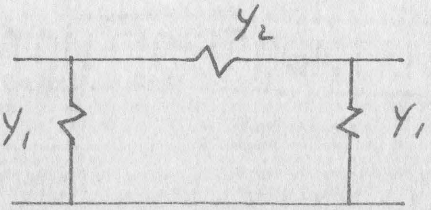
and the equivalent networks

Fig. 10



<u>π - network.</u> -- The y matrix for a symmetrical network is

Fig. 11



$$[Y] = \begin{bmatrix} Y_1 + Y_2 & -Y_2 \\ -Y_2 & Y_1 + Y_2 \end{bmatrix} \tag{33}$$

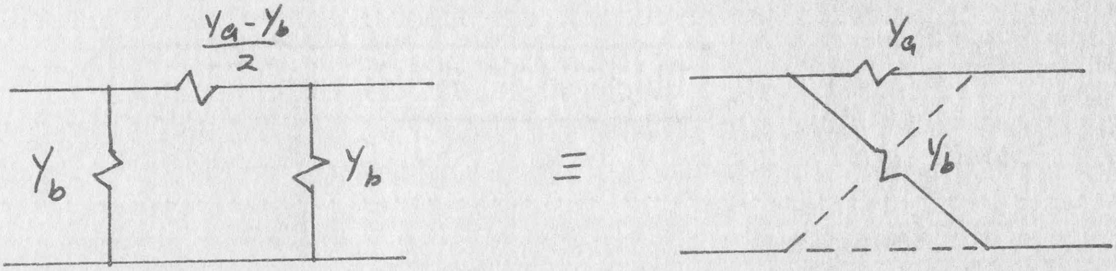
Equating the y matrices gives the relations

$$Y_1 = \frac{1}{Z_b} \tag{34}$$

$$Y_2 = \frac{1}{2}\left(\frac{1}{Z_a} - \frac{1}{Z_b}\right) \tag{35}$$

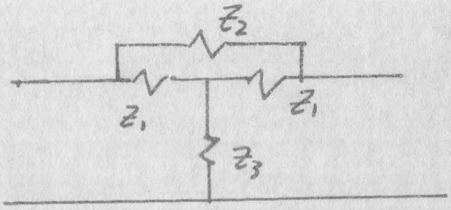
and the equivalent networks

Fig. 12



<u>Bridged - T.</u> The z matrix of a bridged - T network is

Fig. 13



$$[z] = \begin{bmatrix} \frac{Z_1(Z_1+Z_2)}{2Z_1+Z_2} + Z_3 & \frac{Z_1^2}{2Z_1+Z_2} + Z_3 \\ \frac{Z_1^2}{2Z_1+Z_2} + Z_3 & \frac{Z_1(Z_1+Z_2)}{2Z_1+Z_2} + Z_3 \end{bmatrix} \tag{36}$$

Equating the z matrices gives

$$\frac{Z_1(Z_1+Z_2)}{2Z_1+Z_2} + Z_3 = \frac{1}{2}(Z_b + Z_a)$$

$$\frac{Z_1^2}{2Z_1+Z_2} + Z_3 = \frac{1}{2}(Z_b - Z_a)$$

$$\frac{Z_1 Z_2}{2Z_1 + Z_2} = Z_a \tag{37}$$

$$\frac{2Z_1^2 + Z_1 Z_2}{2Z_1 + Z_2} + 2Z_3 = Z_b = Z_1 + 2Z_3 \tag{38}$$

Fig. 14

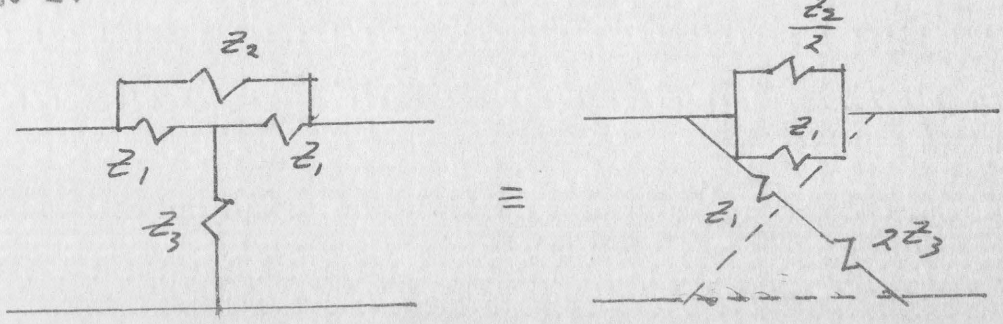


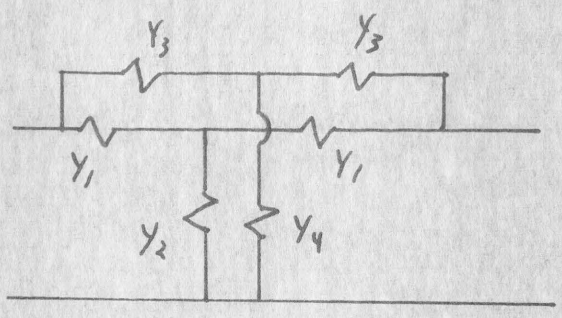
Fig. 14 Bridged-T equivalent.

This transformation is commonly used for the bandstop filter of Figure 9.

<u>Twin-T.</u>

The y matrix for a symmetrical twin - T is

Fig. 15



$$[Y] = \begin{bmatrix} \dfrac{Y_1(Y_1+Y_2)}{2Y_1 Y_2} + \dfrac{Y_3(Y_3+Y_4)}{2Y_3 Y_4} & \dfrac{-Y_1^2}{2Y_1+Y_2} - \dfrac{Y_3^2}{2Y_3+Y_4} \\ \dfrac{-Y_1^2}{2Y_1+Y_2} - \dfrac{Y_3^2}{2Y_3+Y_4} & \dfrac{Y_1(Y_1+Y_2)}{2Y_1 Y_2} + \dfrac{Y_3(Y_3+Y_4)}{2Y_3 Y_4} \end{bmatrix} \quad (39)$$

Equating the y matrices for the twin-T and the lattice gives

$$Y_b = \frac{Y_1 \cdot \frac{Y_2}{2}}{Y_1 + \frac{Y_2}{2}} + \frac{Y_3 \cdot \frac{Y_4}{2}}{Y_3 + \frac{Y_4}{2}} \qquad (40)$$

$$Y_a = Y_1 + Y_3 \qquad (41)$$

Fig. 16

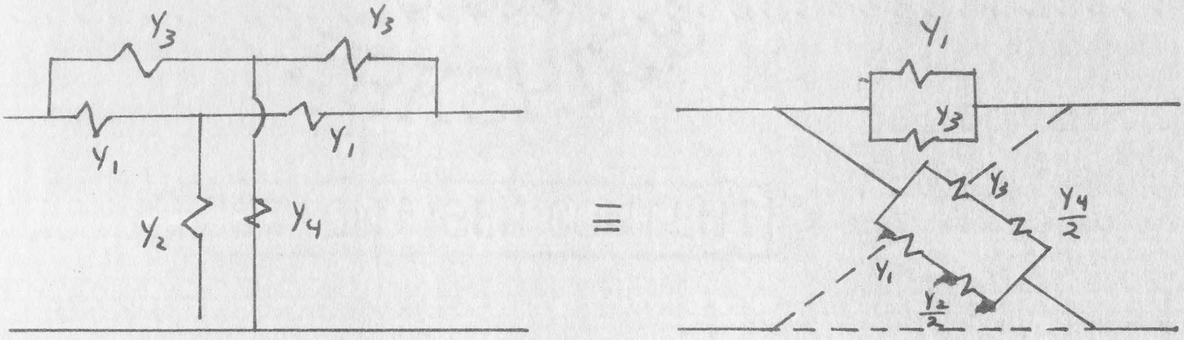


Fig. 16 Twin-T equivalent.

The equivalences shown below follow directly from the bisection theorem.

Fig. 17

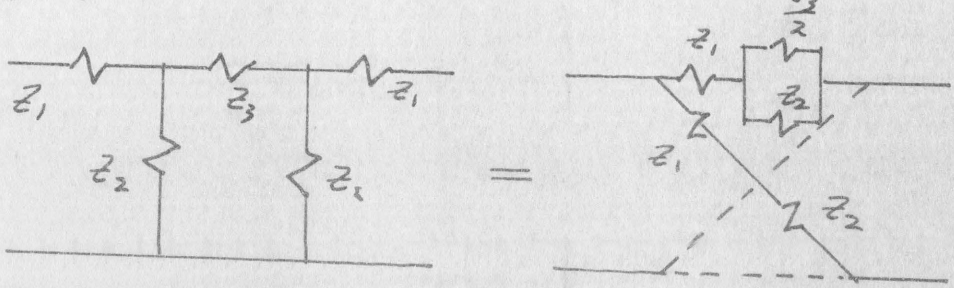


Fig. 17 Ladder equivalent.

Fig. 18

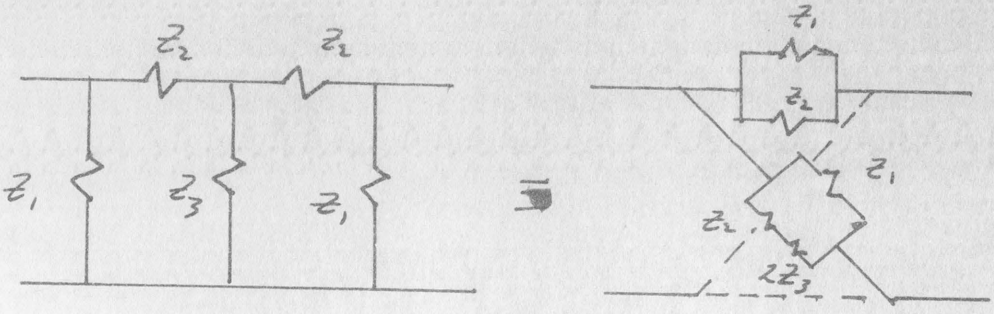


Fig. 18 Ladder equivalent.

When the lattice impedances can be decomposed as indicated in one of the equivalence diagrams, the corresponding unbalanced network can be substituted for the lattice.

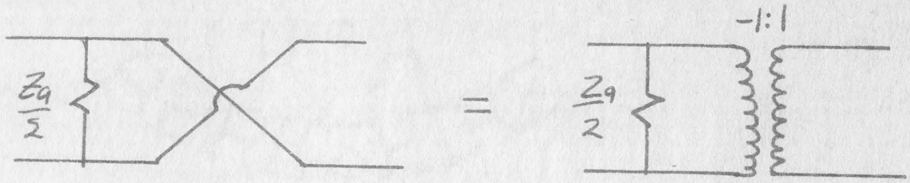
The T and bridged-T transformations were derived by Mason (2) by application of Bartlett's bisection theorem.

Another equivalent network for the symmetrical lattice can be obtained by recognizing that the $\underline{z}$ matrix can be written as the sum of two matrices, corresponding to the connection of two two-port networks in series (9).

$$\begin{bmatrix} \frac{Z_b+Z_a}{2} & \frac{Z_b-Z_a}{2} \\ \frac{Z_b-Z_a}{2} & \frac{Z_b+Z_a}{2} \end{bmatrix} = \begin{bmatrix} \frac{Z_a}{2} & \frac{-Z_a}{2} \\ \frac{-Z_a}{2} & \frac{Z_a}{2} \end{bmatrix} + \begin{bmatrix} \frac{Z_b}{2} & \frac{Z_b}{2} \\ \frac{Z_b}{2} & \frac{Z_b}{2} \end{bmatrix} \tag{42}$$

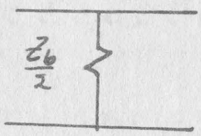
The network corresponding to the first matrix on the right is

Fig. 19



The network corresponding to the second matrix is

Fig. 20



Connecting them in series gives

Fig. 21

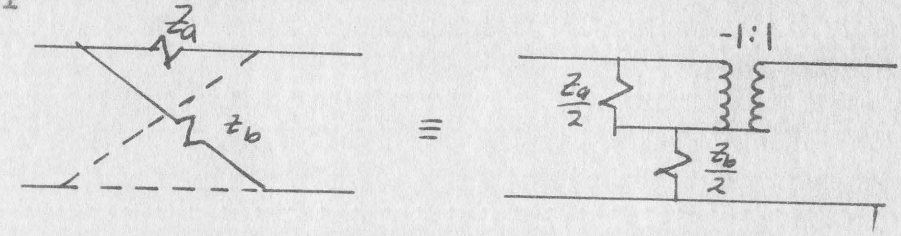


Fig. 21 Narrowband filter

This transformation would be useful to the extent that ideal transformers are possible, and hence would find use only for narrow-band filters.

Application of Bartlett's bisection theorem gives an equivalence between the bridged-T and lattice as shown below (10).

Fig. 22

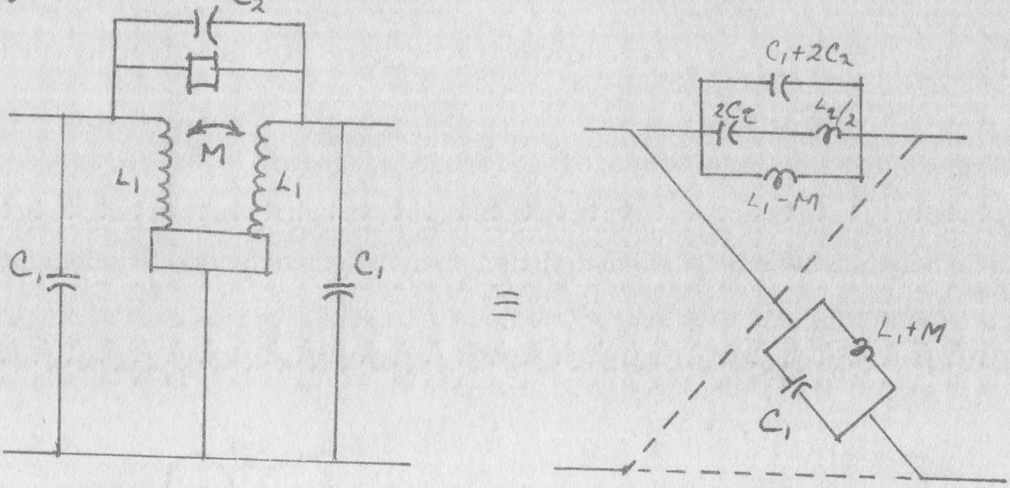


Fig. 22 Transformer bridged-T

Mason (2) gives the following equivalence for the half-lattice or hybrid filter.

Fig. 23

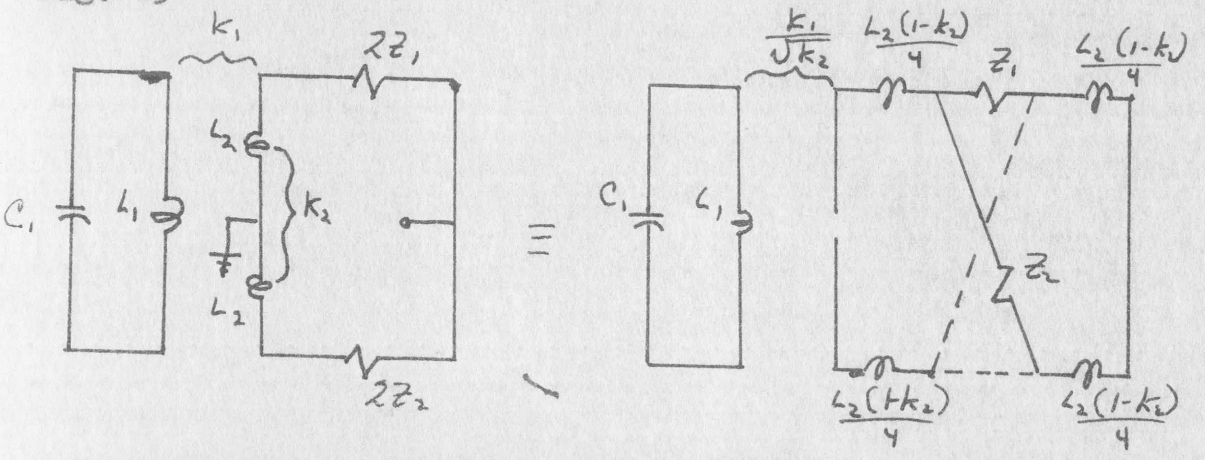


Fig. 23 Half-lattice.

The design considerations which apply to the more complicated broadband filters can be illustrated by the steps in designing the simple filter of Figure 24.

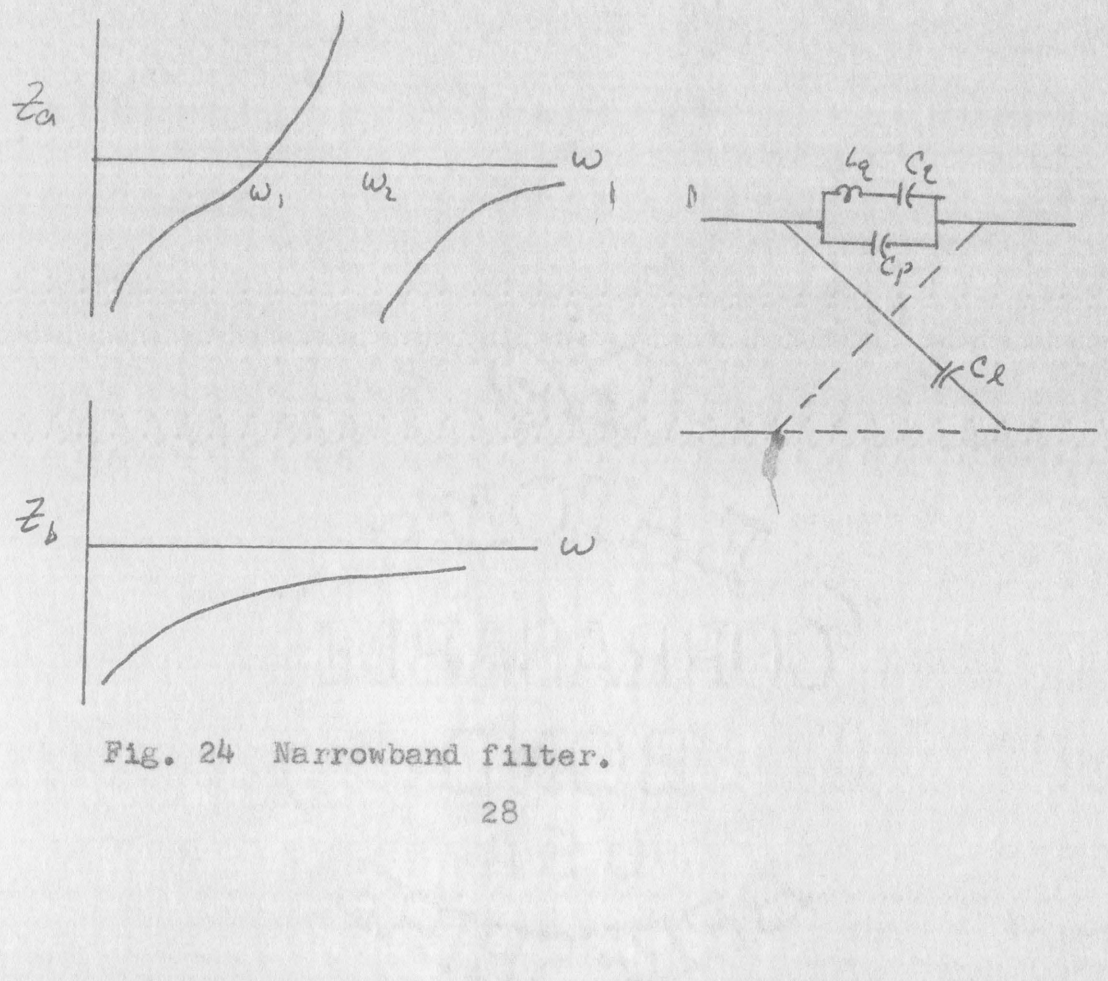


Fig. 24 Narrowband filter.

The desired location of the passband determines the product $L_q C_q$, since $\omega_1 = 1/\sqrt{L_q C_q}$. The ratio $\frac{C_p}{C_q}$ is determined by the desired passband, since %BW= $\frac{W_2 - W_1}{W_1} = \sqrt{1 + C_q/C_p} - 1$

The frequency of infinite attenuation occurs where $Z_a = Z_b$ (i.e., where the bridge is balanced.)

The designer must also obtain the best match of the passband image impedance to the desired terminating resistor. The ratio L_q/C_q and the value of C_l are adjusted to give the desired filter impedance and location of the infinite point. All components are then specified.

The crystal inductance L_q should lie within a certain range for a given frequency if the crystal is to be reasonably rugged, have a good temperature, coefficient, and be free from undesired modes. A curve showing allowable values of L_q is given below (11).

This restriction of the value of L_q becomes a restriction on the impedance level of the filter. Filters of the sort shown above would have impedances in the neighborhood of 50,000 ohms. The ratio L_q/C_q, and thus the impedance of the filter can be modified by replacing the crystal with a different crystal in series with a capacitor. Equivalence formulas for this transformation are developed in Appendix B. In the design of the broad-band crystal lattice filters, in which coils are placed in series or in parallel, with the crystals, the same sorts of considerations apply, although the design is

The desired condition [illegible] constant [illegible]

the product $Z_1 Z_2$, since [illegible] . The [illegible] is

determined by the [illegible] [illegible], since [illegible]

The frequency of infinite [illegible]

where $Z_2 = Z_1$ [illegible], where [illegible]

The [illegible] [illegible] [illegible]

the passband [illegible] [illegible]

resistor. The [illegible] [illegible]

to give the infinite [illegible] [illegible]

infinite point [illegible]

The [illegible]

certain range for a given [illegible] [illegible] to be

be reasonably [illegible] [illegible]

and be free from [illegible] [illegible]

values of [illegible]

This restriction [illegible]

restriction in the [illegible]

of the core [illegible]

neighborhood of [illegible]

the impedance of the [illegible]

the crystal with [illegible]

capacitor. Equivalence [illegible]

are developed in [illegible]

band crystal lattice [illegible]

in series or in [illegible]

sorts of [illegible]

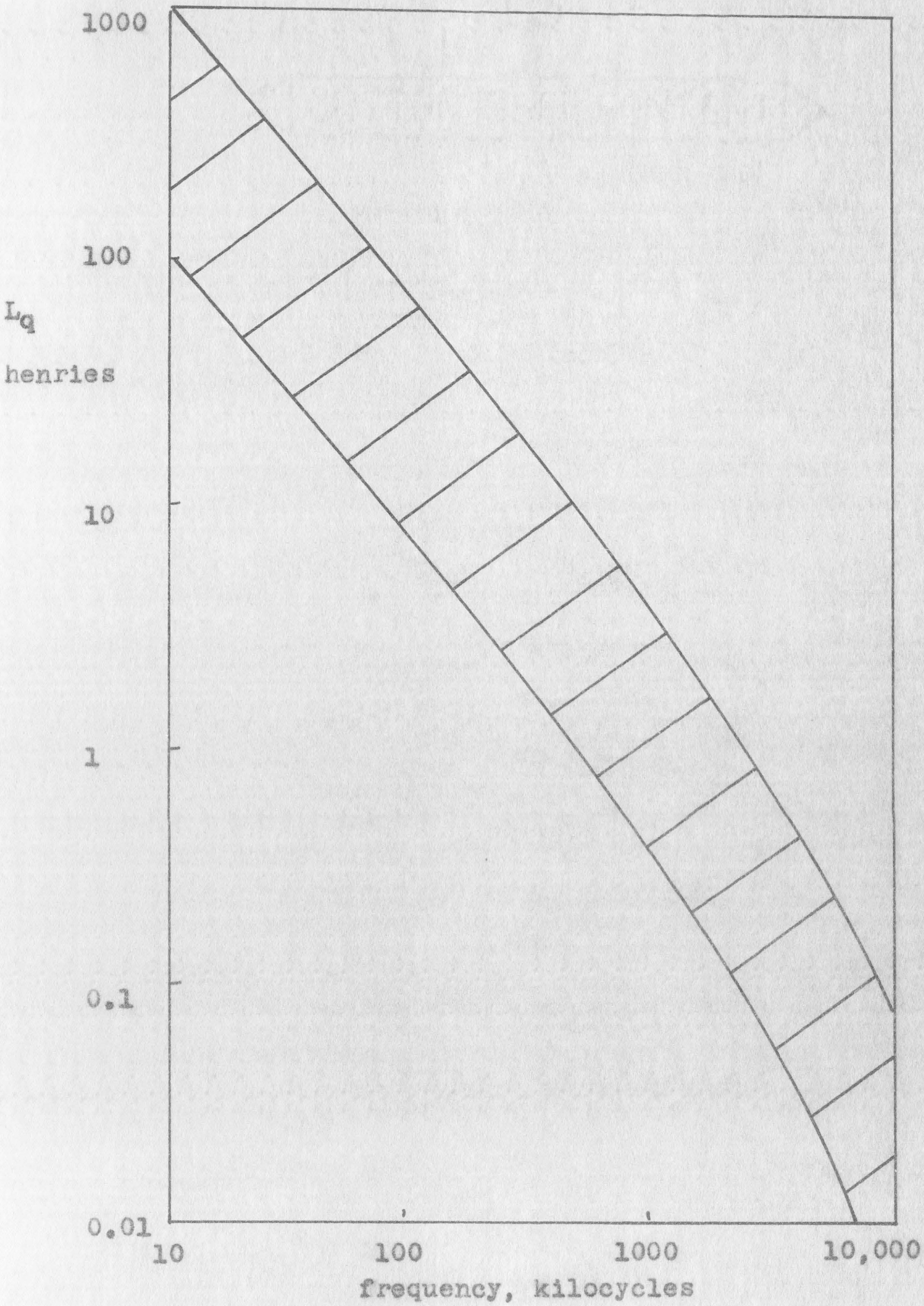


Fig. 25 Range of L_q

considerably more complicated. Design formulas and curves for the design of broadband crystal lattice filters are given by Mason (2) and Kosowsky (12). The general formulas are developed by these authors by making use of frequency normalizations in terms of the cutoff frequencies and the frequencies of the infinite points. Kosowsky (12) gives some approximation techniques which are very useful in preliminary design studies.

considerably [illegible]

curves for (frequency [illegible]

are given by Mason [illegible]

formulas are developed [illegible]

frequency normalized [illegible]

and the frequencies [illegible]

gives some [illegible]

in preliminary design [illegible].

III Ladder Crystal Filters

Very often filters are used in conjunction with three terminal networks such as amplifiers, and so must be realized in unbalanced form. Ladder crystal filters can be abtained either by synthesizing the filter as a lattice and then transforming the lattice to a ladder network by one of the transformations given previously, or by designing the filter as a ladder. Another alternative is to build the filter as a lattice and make use of balance to unbalance transformers. The latter method is usually the least desirable one, since equivalent ladder networks contain fewer elements, and the transformers are expensive and often unsatisfactory.

Poschenrieder (3) has developed designs for ladder half sections of the types shown below.

Fig. 26

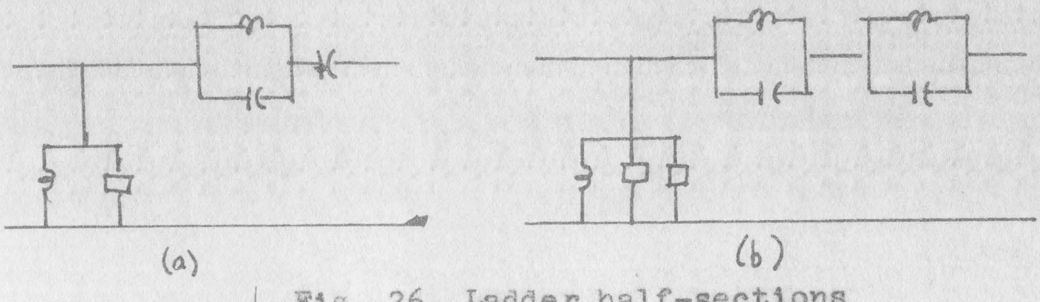


Fig. 26 Ladder half-sections

Design formulas are given in the reference. A computed attenuation curve for a symmetrical filter consisting of two of the type (a) half-sections is shown in Fig. 26.

Very [illegible]

three terminal [illegible]

be realized in [illegible]

can be obtained [illegible]

lattice and then [illegible]

network by one of the [illegible]

or by designing the [illegible]

is to build the filter [illegible]

to unbalance transf[illegible]

the least [illegible]

contain fewer elements, [illegible]

and often unrealizable.

. . . . [illegible]

half sections of the [illegible]

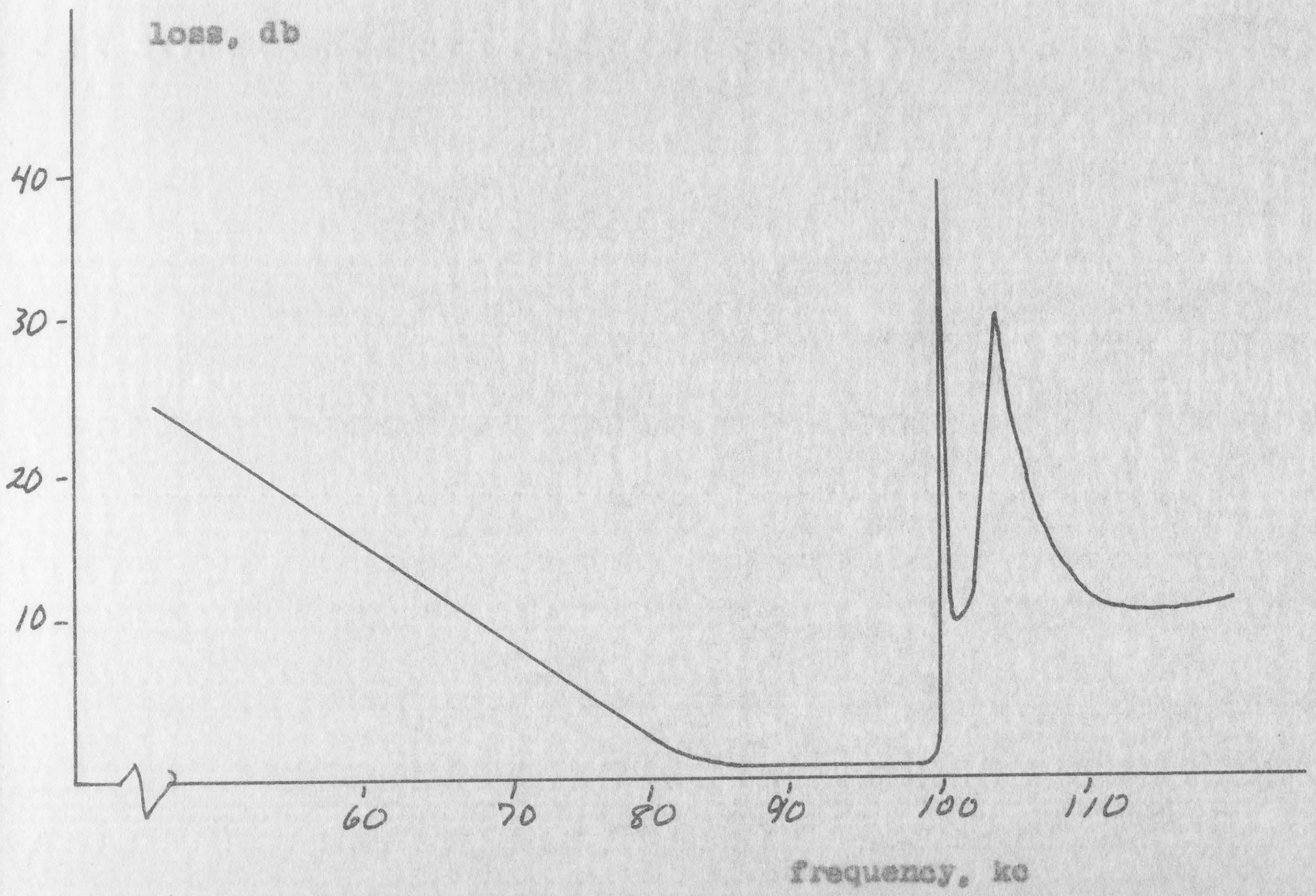
Fig. 26

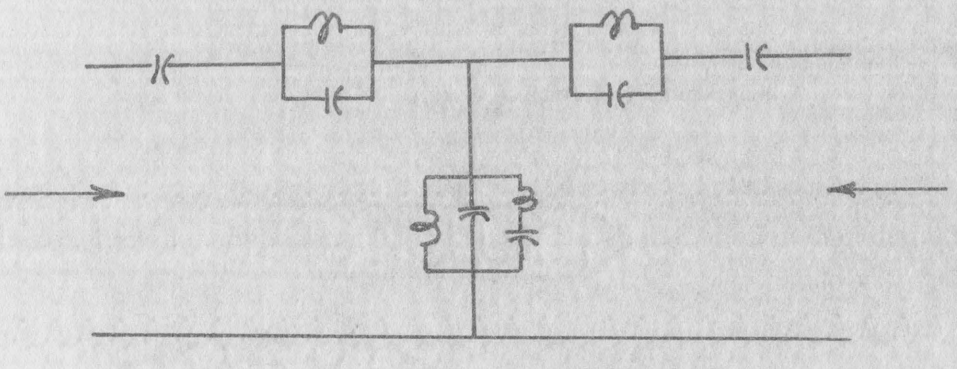
Design [illegible]

computed attenuation [illegible]

consisting of two [illegible]

in Fig. 26.





$C_1 = 20.6 \times 10^{-9}$ farad $L_1 = 28.6 \times 10^{-6}$ henry

$C_2 = 160.4 \times 10^{-9}$ " $L_2 = 14.7 \times 10^{-6}$ "

$C_p = 86.6 \times 10^{-9}$ " $L_q = 8.03 \times 10^{-3}$ "

$C_q = 0.313 \times 10^{-9}$ "

Fig. 27 Ladder crystal filter

The type (b) half-sections are used when sharp cut-off is desired on both sides of the passband. It will be noticed that the value of L_q does not fall in the allowable range. If this filter were to be constructed, the impedance of the crystal would have to be transformed by means of a tap on L_1. The impedance transformation which makes use of a series capacitor would not be applicable since it is only useful for obtaining crystals with decreased values of L_q.

<u>A new unbalanced broadband filter</u>.-- The transformation given earlier between the twin-T and the lattice network can be used to obtain an unbalanced crystal filter with relatively wide bandwidth. The design formulas for the broadband crystal filter shown below are given in the following table (2).

The type (b) all-pass [illegible]

is designed on [illegible]

noticed that the [illegible]

range. If this filter [illegible]

of the crystal [illegible]

top on b_1. [illegible]

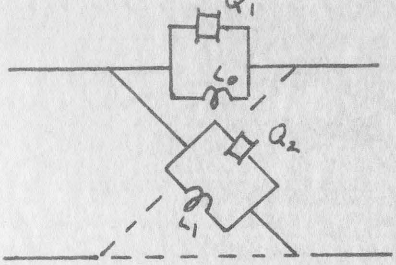
of a series capacitor [illegible]

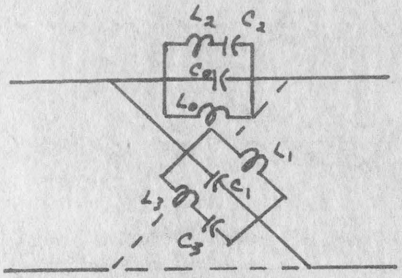
only useful for [illegible]

of b_2.

[illegible] — [illegible] mation given earlier [illegible] network can be used to [illegible] with relatively wide bandwidth [illegible] the broadband crystal filter [illegible] following table (1).

Fig.28





element	formula
L_0	$\dfrac{Z_0(f_B-f_A)(1+B)}{2\pi f_B f_A(A+C)}$
L_1	$\dfrac{Z_0(f_B-f_A)(A+C)}{2\pi f_A f_B(1+B)}$
L_2	$\dfrac{Z_0(1+B)(f_A^2+Bf_B^2)^2}{2\pi f_A f_B(f_B-f_A)(f_B+f_A)^2(AB-C)}$
L_3	$\dfrac{Z_0(A+C)(Af_A^2+Cf_B^2)}{2\pi f_A f_B(f_B-f_A)(f_B+f_A)^2 C(AB-C)}$
C_0	$\dfrac{(Af_A^2+Cf_B^2)f_B}{2\pi Z_0 f_A(f_B-f_A)(f_A^2+Bf_B^2)}$
C_1	$\dfrac{f_A(f_A^2+Bf_B^2)}{2\pi Z_0 f_B(f_B-f_A)(Af_A^2+Cf_B^2)}$
C_2	$\dfrac{(f_B-f_A)(f_B+f_A)^2(AB-C)}{2\pi Z_0 f_A f_B(f_A^2+Bf_B^2)(1+B)^2}$
C_3	$\dfrac{(f_B-f_A)(f_B+f_A)^2 C(AB-C)}{2\pi Z_0 f_A f_B(A+C)^2(Af_A^2+Cf_B^2)}$

Fig. 28 Broadband crystal filter with shunting coils

In these equations, f_B is the upper cutoff frequency and f_A is the lower one. A, B, and C are defined by the following equations.

$$A = \sum_{n=1}^{3} m_n = m_1 + m_2 + m_3 \tag{43}$$

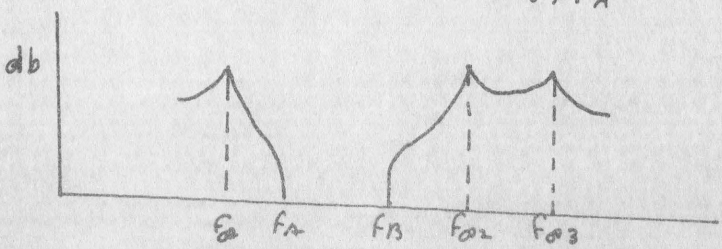
$$B = \sum_{\substack{n=1 \\ (n \neq o)}}^{3} \sum_{o=1}^{3} m_n m_o = m_1(m_2+m_3) + m_2 m_3 \tag{44}$$

$$C = \sum_{\substack{n=1 \\ (n\neq o,\ p\neq n,\ o\neq p)}}^{3} \sum_{o=1}^{3} \sum_{p=1}^{3} m_n m_o m_p = m_1 m_2 m_3 \tag{45}$$

where

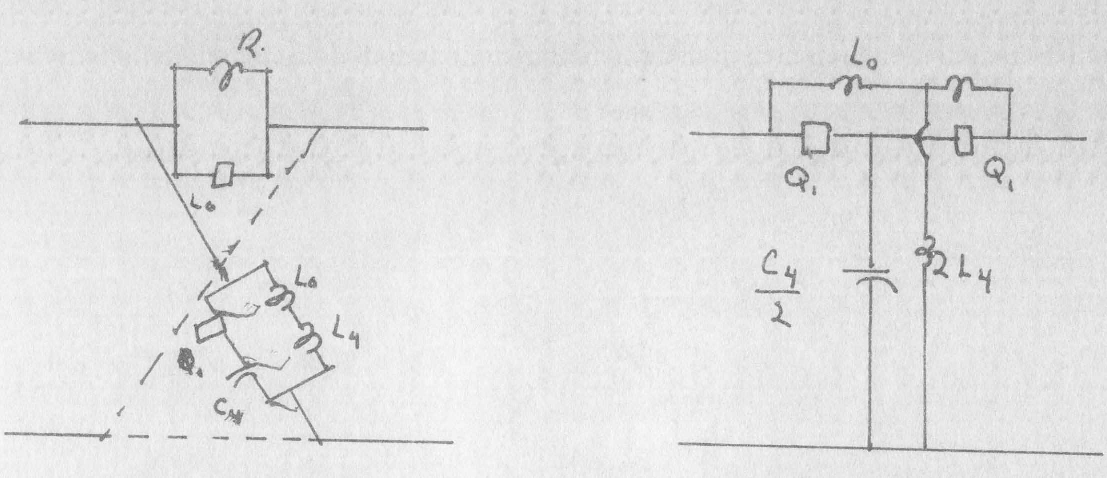
$$m_i = \sqrt{\frac{1 - f_{\infty i}^2/f_B^2}{1 - f_{\infty i}^2/f_A^2}}$$

Fig. 29



The filter desired is the one shown below, with its twin-T equivalent.

Fig. 30



Therefore the crystal Q_1 is found from the design formulas for L_2, C_2, and C_0, and $L_4 = L_1 - L_0$. The crystal which is equivalent to Q_1 and C_4 in series is found from the formulas for L_3, C_3, and C_1.

The conditions for the realizability of this substitution can be found by solving the equivalence formulas given in Appendix B (Eq. 55, 56, 57) for the value of the series capacitor C_4. The result is:

$$\frac{C_4}{C_0} = \frac{1}{\frac{C_0}{C_1} - 1} = \frac{1}{\sqrt{\frac{L_3}{L_2}} - 1} = \frac{C_3 (C_0 + C_2)}{C_1 C_2 - C_0 C_3} \tag{47}$$

The subscripts refer to the elements in the table, except for C_4, which is the series capacitor.

Applying these conditions to the design formulas gives the following

$$k^8(A+C) + k^6(2AB+2BC) + k^4(AB^2+B^2C-A^2C-A^2BC) + k^2(2AC^2+2ABC) + C^3 + BC^2 = 0 \tag{48}$$

and

$$\begin{aligned} &k^{10}(A^2+AB+2AC) + k^8(3A^2B+2AB^2+4ABC+BC^2+2C^2-A^3BC+A^2C^2) \\ &+k^6(2AC^3+3A^2B^2+AB^3+2AB^2C+2B^2C^2+4BC^2+A^3BC-2A^2C^2-A^3C-2A^2BC^2) \\ &+k^4(A^2B^3+B^3C^2+2B^2C^2+2AB^2C-2BC^2-A^3BC-A^2B^2C^2+2A^2BC^2-4AC^3-2A^2C^2-ABC^3+C^4) \\ &+k^2(AB^3C-B^2C^2-2A^2BC^2-2ABC^4+ABC^3-AC^3) \\ &+ABC^3-B^2C^4=0, \qquad \text{where } k = \frac{f_A}{f_B} \end{aligned} \tag{49}$$

Solving the Equations 43, 44, and 45 for C gives

$$C = m_3^{\,3} - m_3^{\,2} A + m_3 B = m_2^{\,3} - m_2^{\,2} A + m_2 B = m_1 m_2 m_3 \quad (50)$$

It follows therefore that the designer may choose the value of only one of the m's, and the values of the other two will be determined by Equations 48 and 49 since these polynomials determine any two of the ABC parameters in terms of the third. The designer is thus at liberty to choose the cutoff frequencies (f_A and f_B), the impedance of the filter (Z_0), and the location of one of the infinite points. Whether or not the location of the other two infinite points will satisfy the attenuation requirements must be determined by trial. The values of A, B, and C to use in the design formulas are obtained by solution of Equations 48, 49, and 50, which will ordinarily be done with a computer.

There are eight design parameters in the design of a filter of the type shown in Fig. 22, namely f_A, f_B, f_{∞}, $f_{\infty 2}$, $f_{\infty 3}$, Z_0, and the two coincident critical frequencies. When the filter is realized in the lattice form of Fig. 28 the designer has correspondingly eight degrees of freedom, since he may specify independently (within limits) the values of eight components. When the filter is realized in the twin-T form of Fig. 30, two degrees of freedom are lost, which results in the restrictions on the locations of two of the infinite points. If it were de-

Solving the equations 4.3, 4.4, and 4.5 for C gives

$$C = m_3^2 - m_3\sqrt{A} - m_1 B \ldots m_1\sqrt{A} + 0.10\, m_1 m_3 \ldots$$ [illegible]

It follows therefore that for a given termination, [illegible] the value of only one of the m's, [illegible] values of the other two will be [illegible] since these parameters are [illegible]. [illegible] parameter m is [illegible] at liberty to choose the cutoff frequencies [illegible] and f_2, the impedance of the filter, Z_0, and [illegible] one of the infinite [illegible] of the other two [illegible] requirements [illegible] A, B, and C so that the filter [illegible] solution of equations 4.3, 4.4, and 4.5 [illegible] be done with a [illegible]

There are eight [illegible] of a filter of the type shown in Fig. [illegible] f_∞, [illegible] When the [illegible] the designer [illegible] since he may specify [illegible] values of eight parameters [illegible] in the [illegible] are lost, which [illegible] cations of two of the [illegible]

sired to realize the filter in the ladder from of Fig. 18 an additional degree of freedom would be lost, resulting in the inability of the designer to specify the location of any of the infinite points.

IV. Conclusion

This paper has presented some information about crystal filters which does not appear elsewhere in a single reference, together with some which does not appear elsewhere at all.

A pa rticularly simple derivation of the equivalent circuit of one class of crystals was presented, and an expression for Q was derived which gives some insight into the manufacture of such crystals. The reasons why crystal filters are usually designed as symmetrical lattices were discussed, and some equivalence relationships between lattices and unbalanced networks were derived, some of which are not available elsewhere. Some equivalence relationships derived in Appendix B were used to obtain design conditions for a new broadband unbalanced crystal filter. Designing such a filter would involve to numerical solution of some complicated algebraic equations, which are given.

A possible extension.--The twin-T filter design could be made more flexible by bridging an additional capacitor across the T. This would give the designer one more degree of freedom, and would therefore allow the selection of two infinite points instead of only one. The analysis becomes considerably more complicated in this case, however.

IV. Conclusions

This paper has presented [illegible]
crystal filters using [illegible]
single reference, Laplace [illegible]
spect elements are [illegible]

A [illegible]
valent circuit of the [illegible]
and an expression for Q was derived which gives some in-
sight into the [illegible]
some way crystal filters [illegible]
metrical lattice [illegible] theorems, and [illegible]
relationships between lattices and [illegible]
were derived, some of which are [illegible]
Some equivalence relations derived [illegible]
were used to obtain design conditions for a [illegible]
band unbalanced crystal filter. [illegible]
would involve the numerical solution of [illegible]
algebraic equations, [illegible]

A possible extension [illegible]
could be made more flexible by [illegible]
capacitor across the [illegible]
one more degree of freedom [illegible]
the selection of [illegible]
one. The analysis [illegible]
in this case, however, [illegible]

Appendix A

Substitution of (4) in (2a) gives:

$$(5)\quad \frac{YAk}{W}E(s) = \left[Ms^2 + Ds + \frac{YA}{W}\right]X(s)$$

Rewriting gives:

$$(6)\quad K\left[Ms^2 + Ds\right]X(s) = \frac{YAK^2}{W}E(s) - \frac{YAk}{W}X(s)$$

Substitution of this relation in (Ia) gives:

$$(7)\quad Q(s) = -\frac{YAk^2}{W}E(s) + \frac{YAk}{W}X(s) + C_s E(s)$$

Rewriting and substituting $C_s = \frac{\epsilon A}{W}$ gives:

$$(8)\quad Q(s) = C_s\left[1 - \frac{Yk^2}{\epsilon}\right]E(s) + \frac{YAk}{W}X(s)$$

Substitution of (5) in (8) gives:

$$(9)\quad Q(s) = C_s\left[1 - \frac{Yk^2}{\epsilon}\right]E(s) + \frac{\left(\frac{YAk}{W}\right)^2}{Ms^2 + Ds + \frac{YA}{W}}E(s)$$

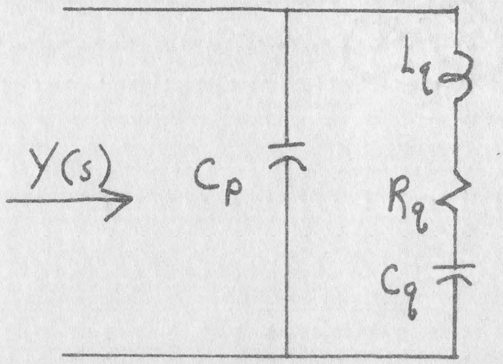
Using the relation I (s) $= s\,Q(s)$ = current gives:

$$(10)\quad I(s) = sC_s\left[1 - \frac{Yk^2}{\epsilon}\right]E(s) + \frac{s\left(\frac{YAk}{W}\right)^2}{Ms^2 + Ds + \frac{YA}{W}}E(s)$$

The admittance, Y(s) I(s)/E(s), can therefore be written:

$$Y(s) = {}_{s}C_s\left[1-\frac{Yk^2}{\epsilon}\right] + \left(\frac{YAk}{W}\right)^2 \frac{}{Ms + D + \frac{YA}{sW}} \tag{11}$$

The circuit corresponding to this admittance equation is:



$$C_p = C_s\left(1-\frac{Yk^2}{\epsilon}\right)$$

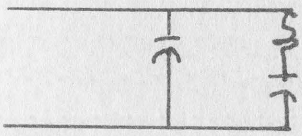
$$L_q = M\left(\frac{W}{YAK}\right)^2$$

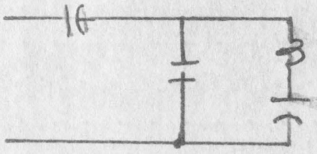
$$R_q = D\left(\frac{W}{YAK}\right)^2$$

$$C_q = C_s\,\frac{Yk^2}{\epsilon}$$

Appendix B

Equivalence relations between a crystal and a crystal with a series capacitor can be found by writing the expressions for impedance and equating the corresponding critical frequencies and the amplitude factor.





$$Z_1 = \frac{1}{C_0} \frac{s^2 + \frac{1}{L_1 C_1}}{s\left(s^2 + \frac{C_0 + C_1}{L_1 C_1 C_0}\right)} \qquad Z_2 = \frac{C_3 + C_4}{C_3 C_4} \; \frac{s^2 + \frac{C_2 C_3 + C_4}{(C_3 + C_4) L_2 C_2}}{s\left(s^2 + \frac{C_2 C_3}{L_2 C_1 C_3}\right)}$$

For equivalence,

$$\frac{1}{C_2} = \frac{C_3 + C_4}{C_3 C_4} \qquad (52)$$

$$\frac{1}{L_1 C_1} \quad \frac{C_2 + C_3 + C_4}{(C_3 + C_4) L_2 C_2} \qquad (53)$$

$$\frac{C_0 + C_1}{L_1 C_1 C_0} = \frac{C_2 + C_3}{L_2 C_2 C_3} \qquad (54)$$

When a given crystal is connected in series with a capacitor, the element values of the equivalent crystal are:

$$C_0 = \frac{C_3 C_4}{C_3 + C_4} \qquad (55)$$

$$C_1 = \frac{C_2 C_4^{\,2}}{(C_3+C_4)(C_2+C_3+C_4)} \tag{56}$$

$$L_1 = \left(\frac{C_3 + C_4}{C_4}\right)^2 L_2 \tag{57}$$

When a given crystal is to be replaced by a different crystal in series with a capacitor, the results are:

$$C_2 = C_0 \frac{L_1/L_2}{1 + C_0/C_1 - \sqrt{L_1/L_2}} \tag{58}$$

$$C_3 = C_0 \sqrt{L_1/L_2} \tag{59}$$

$$C_4 = \frac{C_0}{1 - \sqrt{L_1/L_2}} \tag{60}$$

This is the transformation that is used when the value of L falls outside the acceptable range. It will be noticed that L must be greater than L or C becomes negative. If a crystal is to be replaced by another crystal and a capacitor, there is a restriction on the allowable pole-zero spacing of the crystal to be replaced since

$$\frac{C_3}{C_2} = \frac{1 + \frac{C_0}{C_1} - \sqrt{L_1/L_2}}{\sqrt{L_1/L_2}} \geqslant 140 \tag{61}$$

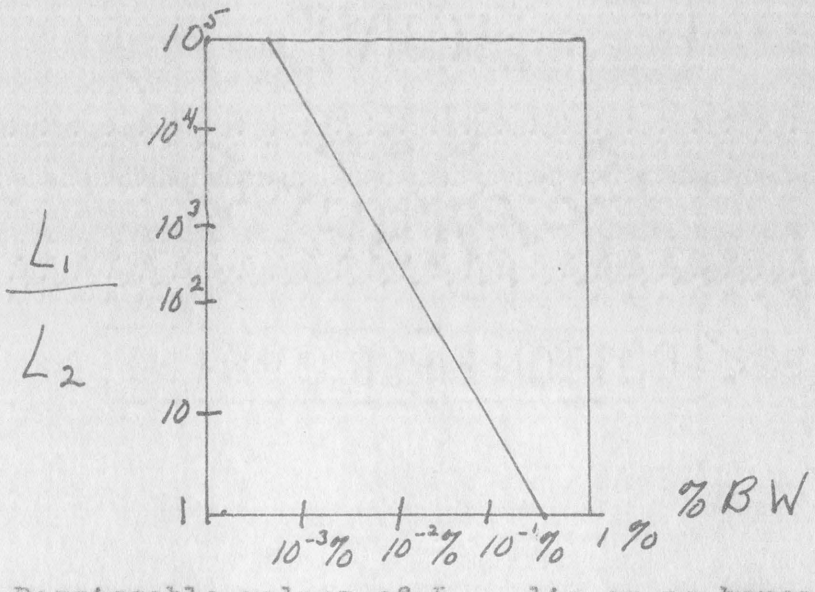
Setting $C_3/C_2 = 140$ gives the greatest range of possibility for the transformation.

$$1 + \frac{C_0}{C_1} = 141\sqrt{L_1/L_2}$$

$$\frac{C_0}{C_1} = 141\sqrt{L_1/L_2} - 1 \geq 140 \qquad (62)$$

Thus if the desired change in crystal inductance is a ratio of 100:1, the crystal to be replaced must have a capacity ratio of at least 1409:1. This capacity ratio is related to the pole-zero spacing as derived previously. Large inductance transformations of this type are therefore only possible in filters of relatively narrow bandwidth. A curve relating inductance ratio to percentage separation of the pole and zero of the crystal is shown below.

Fig. 31



Permissable values of $\frac{L_1}{L_2}$ lie on or beneath the curve and above the line $\frac{L_1}{L_2} = 1$.

[illegible] possibility [illegible]

[illegible]

is a ratio of 10:1, [illegible]

a capacity ratio [illegible]

ratio is [illegible]

previously. [illegible]

type are therefore [illegible]

narrow band[illegible]

percentage [illegible]

is shown below. [illegible]

Fig. 31

[illegible]

curve and shows [illegible]

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