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Comparison of Pulse and FM Radar Altimeters Based on Terrain Return Theory

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COMPARISON OF PULSE AND FM RADAR ALTIMETERS • QUINLAN

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By

David L. Quinlan

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INTRODUCTION

In this paper a comparison of pulse and fm radar altimeters is made from the standpoint of the mathematical expressions describing each system's return signal from terrain at near-vertical incidence. The pulse-fm radar relationships resulting from this comparison facilitate application of the extensive information gained in previous pulse radar studies to the investigation of fm radar terrain return and the associated design of fm radar altimeters, and may also assist in the development of new pulse-type radars.

The pulse radar terrain return studies referred to have been performed at the University of New Mexico under contracts with the Sandia Corporation, Albuquerque, New Mexico, and the Naval Ordnance Test Station, China Lake, California. In these studies, actual radar terrain return measurements have verified, to some extent, theoretical expressions predicting radar scatter, and have shown that knowledge of the ground's radar reflection is essential in the design of radar altimeters. The fact that the terrain return sutdies performed have dealt almost exclusively with pulse radar altimeters indicates that an investigation of pulse-fm radar relationships would be useful in the application of pulse radar

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terrain return information to fm radar altimeter systems.

Prior to discussing pulse-fm radar relationships, it is desirable to consider each system separately and examine the effect of a ground target on the return signal. In Chapter II, expressions for the mean return pulse of a pulse radar altimeter, and the mean return spectrum of an fm radar altimeter are obtained after a brief discussion of each system's basic principles of operation. Examination of the return expressions suggests a method for comparing the two systems; for, if properly manipulated mathematically, the expressions for pertinent signals are identical in mathematical form when similar operating conditions are assumed.

With this relationship between the expressions for mean returns as a basis of comparison, additional relationships are obtained in Chapter III by considering similar alterations and Fourier transformation of the related expressions, and the system functions associated with these operations. That is, a pulse radar scanning gate, n(t), and an fm radar scanning filter, M(f), are considered as related system functions. The effect of these devices on the return pulse, p(t), and the fm return spectrum, B(f), is then shown to be

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where $q(t)$ is the pulse through the gate and $C(f)$ is the spectrum in the filter. The functions, $q(t)$ and $c(f)$, are then considered analogous. Fourier transformation of $p(t)$ and $B(f)$ is shown to result in a relationship between the return pulse power spectrum, $|P(f)|$, and the fm beat note voltage, $b(t)$. Alteration of $|P(f)|$ and $b(t)$ by the pulse receiver bandwidth, $D(f)$, and the fm beat note voltage sampling time, $k(t)$, given by

$$H(f) = D(f)\,|P(f)|$$

and

$$g(t) = k(t)\ b(t)$$

is then considered. As before, the functions $g(t)$ and $H(f)$ are considered analogous, and when Fourier transformed, result in the indicator presentations, $h(t)$ and $G(f)$, for the pulse and fm radar systems, respectively. The latter portion of Chapter III suggests an approach to the application of related functions in a comparison of entire pulse and fm systems, and also indicates the advantages of approximations in radar terrain return studies.

In Chapter IV, the general characteristics and statistical properties of the fm return power spectrum and beat note voltage are discussed to permit subsequent approximation of an fm radar's ground return. A discrete spectrum approximation of the continuous return spectrum is considered for an fm radar altimeter employing a cycle-rate counter as a data-converting element. The counter

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range indication is theoretically predicted, and then compared with experimentally obtained data to substantiate the return approximation.

The pulse radar approximation, analogous to the approximation of an fm return power spectrum by a discrete spectrum, is the representation of the return pulse by a series of weighted delta functions. This approximation is considered in Chapter V, and is seen to represent the sampling methods employed in the experimental determination of a pulse radar's mean return. The operations performed to obtain pulse-fm radar relationships, discussed in Chapter III, are applicable to the approximations of both the pulse and the fm radar return, and, as before, result in additional relationships between the two systems.

In Chapter VI, fm radar investigation of the ground's scattering properties is considered for a counter-type fm altimeter. The general approach is similar to previous pulse radar terrain return studies in that theoretical expressions for terrain return are compared to actual terrain return measurements in order to determine the validity of the expression describing the ground target's characteristics. Such an investigation is considered necessary if counter-type fm radar altimeters are to be employed as altitude indicating devices.

Considerably more attention is given to fm radar systems throughout this paper, since, as mentioned above, previous investigation of radar terrain return has dealt, for the most part, with pulse radar altimeters. If additional information pertaining to pulse radar terrain return is desired, the references listed provide a thorough discussion of the terrain return studies performed utilizing pulse radar systems.

CHAPTER II

PULSE AND FM RADAR TERRAIN RETURN

Prior to discussing the relationships that exist between pulse and fm radar altimeters, each system must be considered separately to determine how it is affected by terrain return. In the following material, the basic principles of operation for each system are briefly discussed, and then the effect of ground return is considered.

2.1 Range Information as a Time Delay.

The relation between target distance and time delay is unchanged for all radar ranging. If a signal is transmitted with a constant velocity, c, travels to a target at range, r, and an echo returns in time, τ, after transmission, the relationship between target range, velocity of propagation, and propagation time is

$$r = \frac{c\tau}{2} \tag{1}$$

In a pulse radar system, the signal transmitted is a short burst of r-f energy, and the echo is a highly attenuated pulse of r-f energy occuring after the propagation time. In continuous wave radar systems other methods of determining propagation time are

necessary; one method used by frequency modulated radar systems will be discussed in the next section.

2.2 Range Information as a Frequency Difference.

If, in frequency modulated radar, the frequency of the transmitted signal increases linearly with time, a signal transmitted at time, t_1, with frequency, F_1, will be received at $t_1 + \tau$ still with frequency F_1 (for a stationary radar and target). At $t_1 + \tau$ the transmitted signal will have a frequency $F_2 = F_1 + \dot{F}\tau$ where $\dot{F} = dF/dt$ is the rate of change of transmitted signal frequency. (See Figure 1).

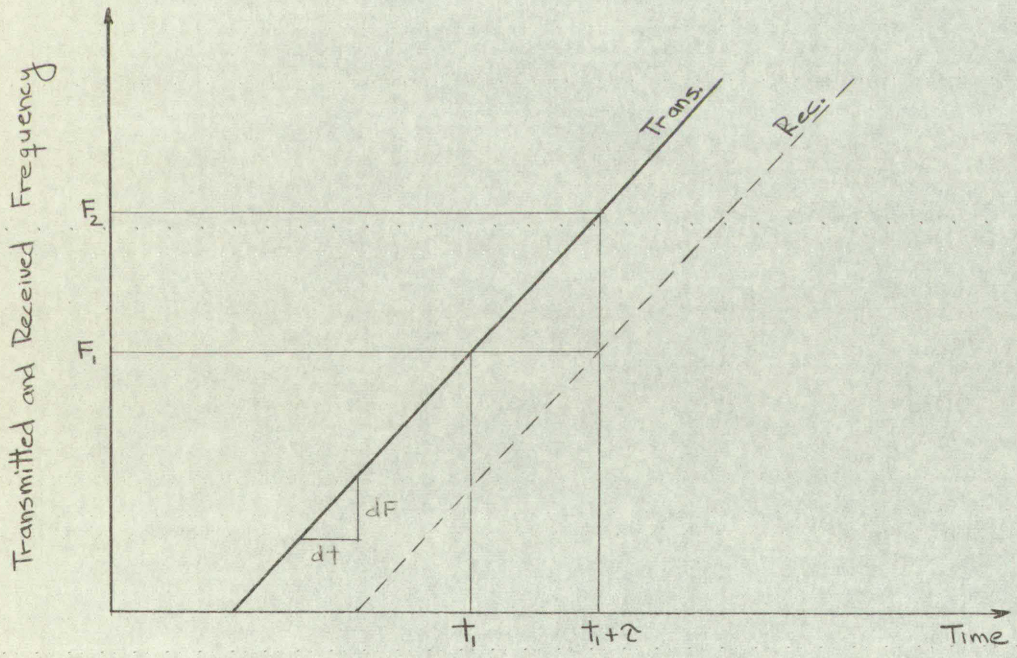

Figure 1. Effect of Time Delay on a Signal with Uniformly Varying Frequency

Since the frequency difference, $\dot{F}t$, between transmitted and received signal is a function of propagation time and a radar parameter, $\dot{F}$, constant for the system considered, its value is directly proportional to target range. Therefore, if a beat note voltage is obtained by mixing the transmitted and received signals and filtering out the high frequency content, its frequency will be given by:

$$f_R = \dot{F}\tau = \frac{2r\dot{F}}{c} \tag{2}$$

where f_R = the beat frequency

$\dot{F}$ = the rate of change of transmitted signal frequency

τ = the propagation time

r = the target range

c = the velocity of propagation

Determining this beat frequency, f_R, will then yield the desired target range information.

2.3 Radar Return from the Ground.

The nature of the ground as a radar target at near-vertical incidence is important, as this paper is primarily restricted to altimeter applications of pulse and fm radar. Recent studies have shown that radar return from the ground can be thought of as being composed of two components, a specular return and a scattered

Since the frequency difference (f_R) between transmitted and received signal is a function of propagation time and a fixed parameter, $\dot{F}$, constant for the system considered, its value is directly proportional to target range. Therefore, if a heterodyne voltage is obtained by mixing the transmitted and received signals and filtering out the high frequency content, its frequency will be given by:

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return.[1] The specular return is a mirror-like reflection occurring directly beneath the radar, and is normally dealt with using image theory from optics. The scattered return results when a large number of individual scatterers, located throughout a radar's illuminated area, reradiate more or less equally the incident radar energy. The studies show that the principal contribution to the radar return is due to the scattering process.

In the following material, the effect of scattering ground on radar range information will be considered, first for a pulse radar altimeter, and second, for an fm radar altimeter.

2.4 The Effect of Scattering Ground on a Pulse Radar Altimeter.

In the following discussion the ground is assumed to be an approximately plane surface. Such an assumption is not valid in some situations, such as mountainous terrain; however, a flat surface serves the purpose of illustrating the effect of scattering ground. The coordinate system used to describe the area illuminated by a radar altimeter[2] is shown in Figure 2.

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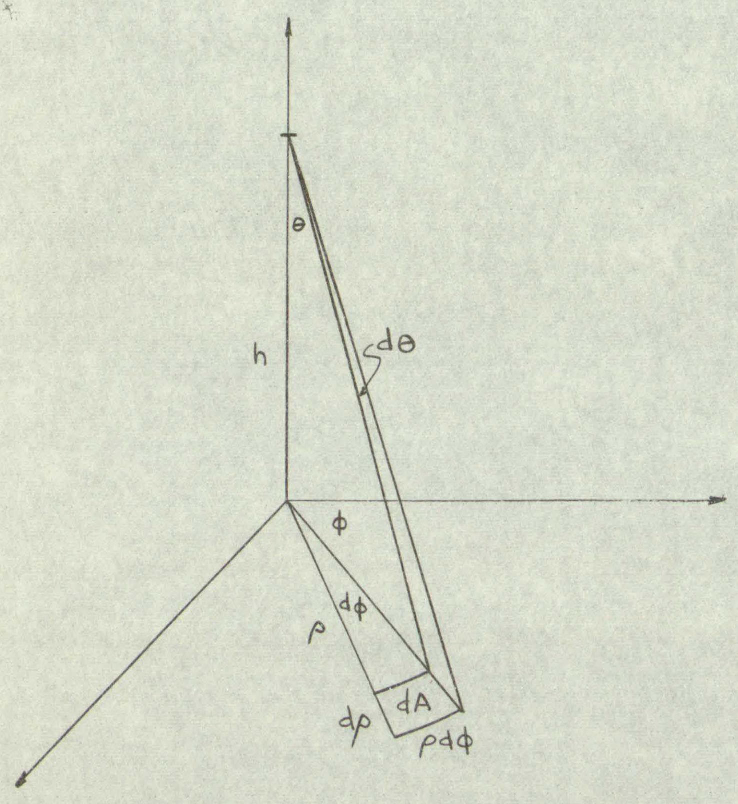
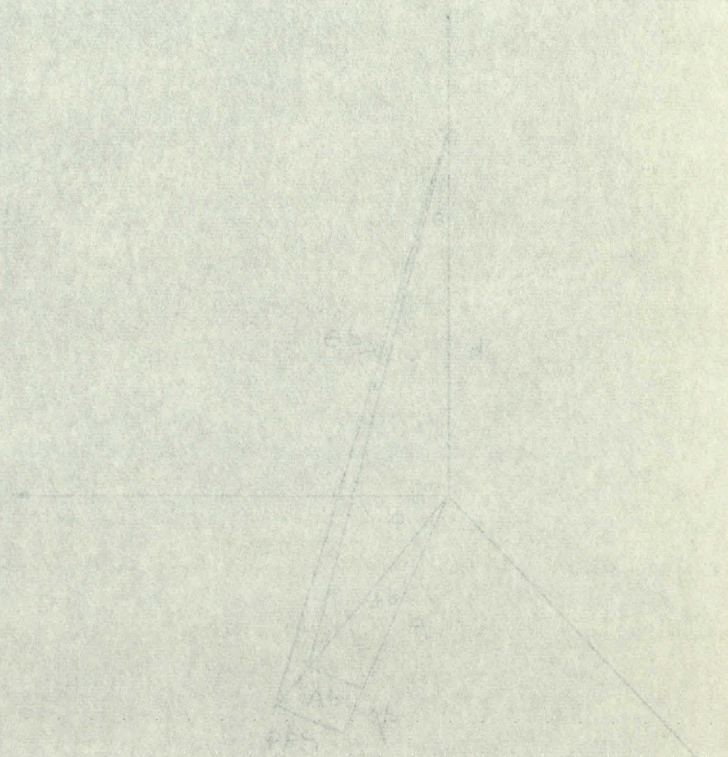


Figure 2. Differential Area Illuminated by a Radar Altimeter

Since discussion will be limited, for the most part, to radar altimeters; and therefore, range information is all that is desired, radar return for pulse systems will be illustrated as an A-type scope presentation where horizontal displacement is along a time axis and consequently directly proportional to aircraft



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altitude. In the following material, an expression for the radar return as displayed on an A-scope will be considered. It is important to emphasize that the expression will represent a mean returned pulse obtained by taking the ensemble average of a large number of individual pulses. To obtain the expression for the mean return pulse, it is necessary to consider first the return from a single, isolated scatterer, and then project the effect to an area-extensive scattering target.

Accordingly, the average power of the signal returned to the radar by the n^{th} scatterer is given by[3]

$$P_{R_n}(t) = \frac{P_T\left(t - \frac{2r_n}{c}\right) G_n \lambda^2 \sigma_n}{(4\pi)^3 \, r_n^{\,4}} \tag{3}$$

where $P_T\left(t - \frac{2r_n}{c}\right)$ = the transmitted pulse function delayed by the propagation time to the n^{th} scatterer

r_n = the range to the n^{th} scatterer from the radar

c = the velocity of light

σ_n = the scattering cross section of the n^{th} scatterer

G_n = the gain of the antenna in the direction of the n^{th} scatterer

λ = the wavelength of the carrier radiation.

When the effect of a large number of scatterers located throughout

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[3] Ibid, p. 230.

the entire illuminated area is considered, the expression for power return becomes:[4]

$$\overline{P_R(t)} = \iint \frac{P_T(t - \frac{2r}{c})\, \lambda^2\, G^2(\theta, \phi)\, \sigma_o(\theta, \phi)\, dA}{(4\pi)^3\, r^4} \tag{4}$$

It is important to mention again that the expression $\overline{P_R(t)}$ represents a mean returned pulse, the mean taken over a large number of returned pulses. The other expressions introduced are the average scattering cross section per unit area, σ_o, and the antenna gain, G, both functions of the coordinates (θ, ϕ) shown in Figure 2, and the differential area, dA, also shown in Figure 2. The differential area, dA, may be expressed as:

$$dA = \rho\, d\rho\, d\phi$$

and since

$$\rho^2 + h^2 = r^2$$

then

$$\rho\, d\rho = r\, dr$$

and

$$dA = r\, dr\, d\phi$$

[4] Ibid, p. 232.

Now $\overline{P_R(t)}$ may be rewritten as[5]

$$\overline{P_R(t)} = \frac{\lambda^2}{(4\pi)^3}\int_h^{\frac{ct}{2}}\int_0^{2\pi} \frac{P_T(t-\frac{2r}{c})\, G^2(\theta,\phi)\, \sigma_0(\theta,\phi)\, d\phi\, dr}{r^3} \tag{5}$$

With respect to range, the limits of integration are from the vertical distance, h, to a point where the return becomes negligible. If t is the time for the signal to travel out to this point and back, the range is given by $\frac{ct}{2}$. If it is now assumed that gain and scattering cross section do not vary with ϕ, the expression becomes:

$$P_R(t) = \frac{\lambda^2}{32\pi^2}\int_h^{\frac{ct}{2}} \frac{P_T(t-\frac{2r}{c})\; G^2(\theta)\; \sigma_0(\theta)\; dr}{r^3} \tag{6}$$

To illustrate how a transmitted pulse is affected by scattering ground, expressions for G^2 and σ_0 will be assumed. A half-wave dipole with $G(\theta) = 1.64\cos^2\theta$ will be taken for the antenna , and the scattering cross section will be approximated by $\cos\theta$. It can be seen from Figure 2 that $\cos\theta = \frac{h}{r}$, therefore

$$G^2(\theta)\sigma_0(\theta) = 2.69\,\frac{h^5}{r^5}$$

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[5] *Ibid*, p.233.

Now the expression for the mean power return becomes:

$$\overline{P_R(t)} = \frac{2.69\,\lambda^2 h^5}{32\pi^2} \int_h^{\frac{ct}{2}} \frac{P_T(t - \frac{2r}{c})\,dr}{r^8} \tag{7}$$

If the transmitted pulse is rectangular with amplitude P_o, and duration, τ, evaluation of the integral is different in two time intervals.[6] The first is the time interval before the trailing edge of the transmitted pulse reaches the ground. The second time interval corresponds to the period after the trailing edge reaches the ground and during which time the illuminated annulus on the ground spreads out. The results are shown below:

$$\overline{P_R'(t)} = 0 \quad , \; t < \frac{2h}{c} \tag{8}$$

$$\overline{P_R(t)} = \frac{2.69\,\lambda^2 h^5 P_o}{32\pi^2} \int_h^{\frac{ct}{2}} \frac{dr}{r^8} \quad , \; \frac{2h}{c} < t < \frac{2h}{c} + \tau \tag{9}$$

$$\overline{P_R(t)} = \frac{\lambda^2 P_o}{83.3\,\pi^2 h^2} \left[1 - \left(\frac{2h}{ct}\right)^7 \right] , \; \frac{2h}{c} < t < \frac{2h}{c} + \tau \tag{10}$$

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[6] Ibid, P. 236

$$\overline{P_R(t)} = \frac{2.69\,\lambda^2 h^5 P_o}{32\pi^2} \int_{\frac{c(t-\tau)}{2}}^{\frac{ct}{2}} \frac{dr}{r^8} \quad , t > \frac{2h}{c} + \tau \qquad (11)$$

$$\overline{P_R(t)} = \frac{1.54\,\lambda^2 h^5 P_o}{\pi^2 c^7} \left[\frac{1}{(t-\tau)^7} - \frac{1}{t^7} \right] \quad , t > \frac{2h}{c} + \tau \qquad (12)$$

The peak of the mean return occurs at $t = \frac{2h}{c} + \tau$ and is given by[7]

$$\overline{P_R(t)} = \frac{\lambda^2 P_o}{83.3\pi^2 h^2} \left[1 - \frac{1}{\left(1 + \frac{c\tau}{2h}\right)^7} \right] \qquad (13)$$

when $t = \frac{2h}{c} + \tau$

To illustrate the behavior of the power return for the situation considered, an example will be taken with h and τ arbitrairily selected as 1000 meters and 1 μ sec. respectively. The plot of the mean power return, Figure 3, is normalized such that the peak return will have a value of unity, other values having proportional magnitudes.

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[7] Ibid, p. 236.

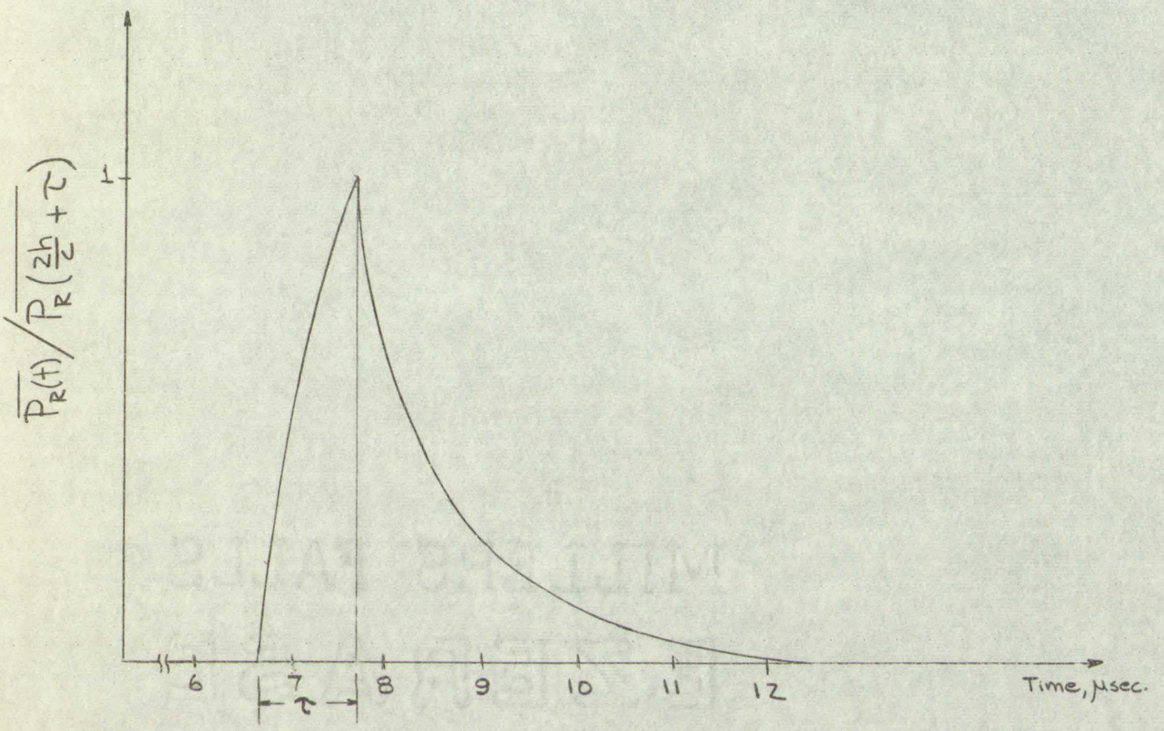

Figure 3. Normalized Mean Power Return

For a given altitude, the location of the peak power return is a function of t only, since it will always occur at $t = \frac{2h}{c} + \tau$. When a weighted delta function is assumed as the transmitted pulse, $\tau = 0$, and the leading edge of the return pulse becomes vertical. In later discussion, the assumption of a transmitted delta function will prove useful, and therefore the mean return pulse associated

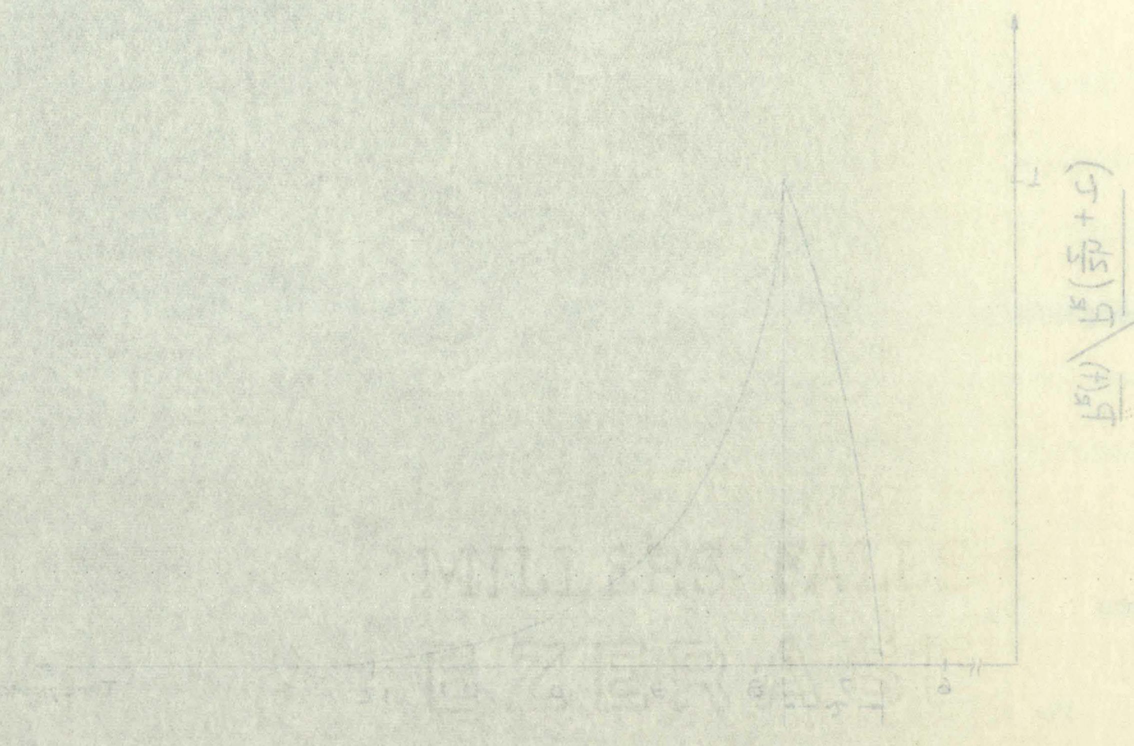

Figure 3. Normalized Mean Power Return

For a given altitude, the location of the peak power return is a function of t only, since it will always occur at $t = \frac{2H}{c} + T$. When a weighted delta function is assumed as the transmitted pulse, $T = 0$, and the leading edge of the return pulse becomes vertical. In later discussion, the assumption of a transmitted delta function will prove useful, and therefore the mean return pulse associated

with a transmitted delta function of weight, P_o, will now be obtained. If the expressions for $G^2(\theta)$ and $\sigma_o(\theta)$ remain unchanged, the expression for $\overline{P_R}(t)$ from Equation 7, is given by

$$\overline{P_R(t)} = \frac{2.69\lambda^2 h^5 P_o}{32\pi^2} \int_h^{r_{max.}} \frac{\delta(t - \frac{2r}{c})\, dr}{r^8} \tag{14}$$

where r_{max} is the range at which return becomes negligible. Solution for $\overline{P_R}(t)$ results in

$$\overline{P_R(t)} = \frac{10.76\,\lambda^2 h^5 P_o}{\pi^2 c^7 t^8} \quad , \; h < t < r_{max} \tag{15}$$

The above return, although ideal in that it results from a transmitted delta function, will be used as an example subsequently when comparable fm conditions are considered, and the returns for the two systems compared.

2.5 The Effect of Scattering Ground on an FM Radar Altimeter.

The assumption of an approximately plane ground surface will also be made while considering the effect of scattering ground on an fm radar altimeter. The coordinate system shown in Figure 2 will remain unchanged and a similar approach to the problem will be made, that is, the effect of isolated scatterers will be determined, and then extended to an area-extensive scattering target.

In Section 2.2, the fm radar return was assumed to be a reflection from a specular type target. If the return from a scattering target is considered, the frequency variation of the return components will be as shown in Figure 4. (target and radar both stationary).

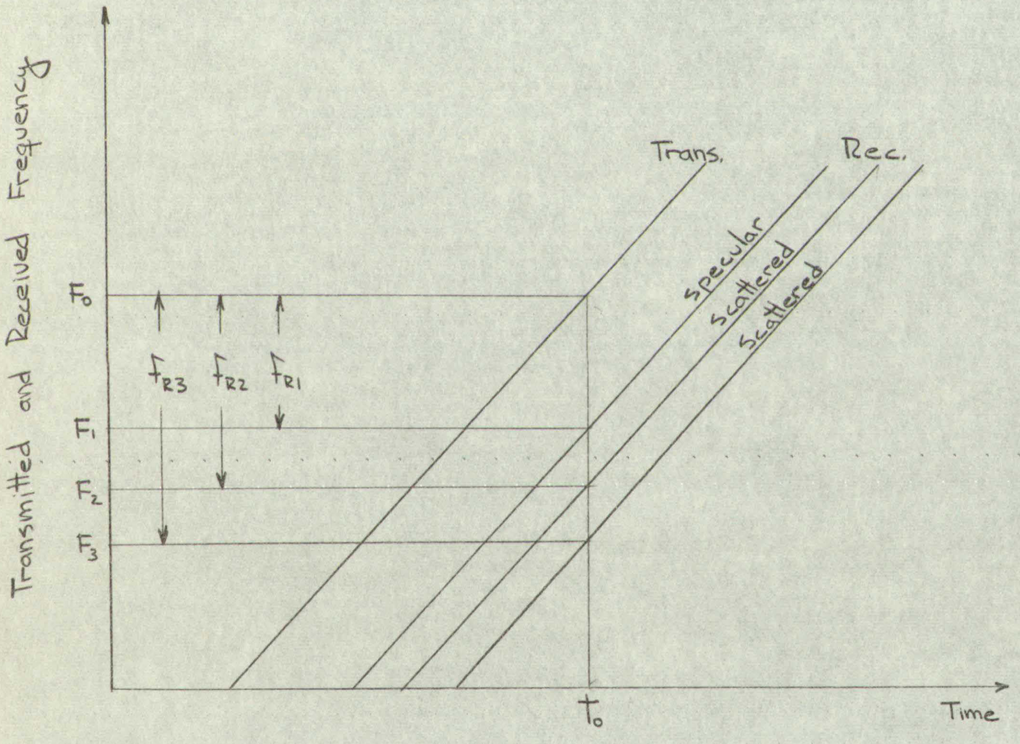

Figure 4. FM Radar Return from a Scattering Target

As Figure 4 illustrates, at any time, t_o, the received signal will be composed of a specular component of frequency, F_1, and a number of scattered components of frequencies, F_2, F_3, ..., F_N (only two scattered components shown for simplicity). The scattered return frequencies will depend on the slant range to each individual scatterer, but will always be at a frequency lower than that of the specular component since the specular return corresponds to the shortest path length (actual aircraft altitude) between radar and ground. If an area-extensive scattering target is now considered, rather than the isolated scatterers discussed above, the received signal frequency at t_o, shown in Figure 4, will be composed of a continuous band of frequencies ranging from the specular return frequency down to the frequency corresponding to negligible return. If, as mentioned in Section 2.2, the transmitted and received signals are mixed and the unwanted high-frequency content is filtered out, the resulting beat note voltage will contain the band of beat frequencies ranging from the difference frequency associated with the specular return, out to the difference frequency corresponding to negligible return. These beat frequencies are in part represented by f_{R1}, f_{R2}, and f_{R3} in Figure 4.

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beat frequencies can be obtained. This spectrum analyzer presentation, corresponds to the pulse radar's A-scope presentation since horizontal displacement is along a frequency axis and once again directly proportional to aircraft altitude. Henceforth, a power spectrum so obtained will be referred to as the return power spectrum, $B(f_R)$, and the beat frequencies will be called return beat frequencies, and usually specified as either specular or scattered beat frequencies.

Subsequently, a spectrum-type presentation will be assumed as the fm altimeter indicator when analogous relationships between pulse and fm radar are considered. It is therefore desirable to obtain an expression for the fm mean return power spectrum, $\overline{B(f_R)}$, representing an average taken over a long period of radar operation, just as the mean return pulse, $\overline{P_R}(t)$, represents a mean taken over a large number of return pulses. However, prior to obtaining $\overline{B(f_R)}$, it is necessary to consider some characteristics of the beat note voltage that result from an fm radar's periodicity.

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The previous discussion on scattered fm radar returns considered only a short time interval of transmission and reception to illustrate the effect of scattering ground. Obviously it is

impossible to vary the transmitted frequency in one direction for an extended period of time, so a periodic modulation is required. One form of modulation that has been employed in fm radar altimeters is triangular modulation, in which the rate of change of carrier frequency remains constant, but the direction of change is periodically reversed.[8] The carrier frequency alternately increases and decreases linearly through a frequency band, ΔF. The time interval between direction changes is given by $1/(2f_m)$ where f_m is the frequency of the triangular modulating wave. This form of modulation along with a scattered return is shown in Figure 5, below. (As before, a stationary radar and target are assumed; and, consequently, no Doppler shift is present. In the following section, relative motion between the radar and the target; and the accompanying Doppler effect will be considered).

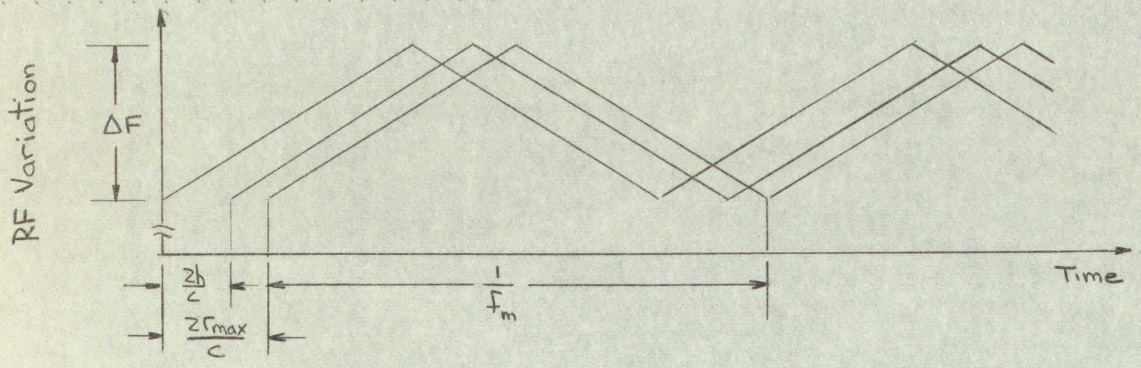

Figure 5. Periodic Modulation for FM Radar

[8]Luck, D. G. C., Frequency Modulated Radar, McGraw-Hill, New York, 1949, p. 15.

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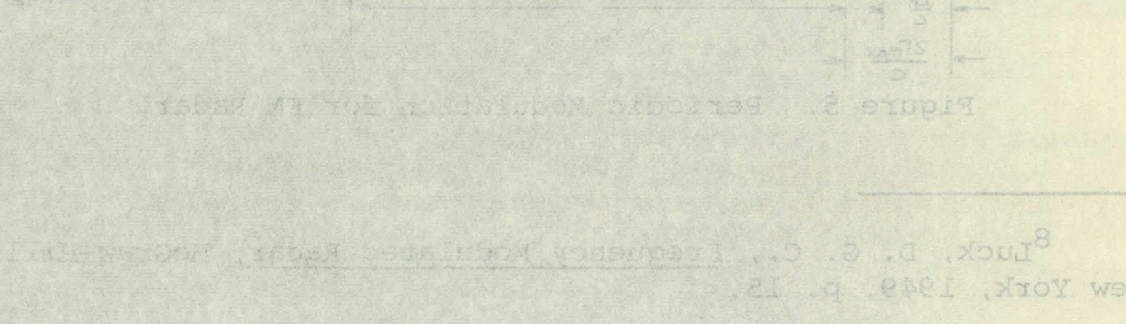

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[8]Luck, D. G. C., Frequency Modulated Radar, McGraw-Hill, New York, 1949, p. 15.

The upsweep portion of a cycle shown in Figure 5 has been discussed in Section 2.5 and illustrated in detail in Figure 4. Inspection of the downsweep portion of a cycle shows that the difference, or beat frequencies retain the same value; the transmitted frequency being lower than the return frequencies by the same amount that it was higher on the upsweep portion. However, the turnaround portion (the time interval beginning at the transmitted frequency turnaround and lasting until the frequency of the return from the most distant scatterer changes direction) required additional consideration. If the values of the beat frequencies are plotted against time, they will vary as shown in Figure 6. For simplicity only three beat frequencies components are shown, the specular beat frequency, $\frac{2h}{c}\dot{F}$, the beat frequency corresponding to the return from the most distant scatterer, $\frac{2r_{max.}}{c}\dot{F}$, and the beat frequency from an intermediate scatterer, $\frac{2r_i}{c}\dot{F}$. Furthermore, the time axis is expanded so as to show only one turnaround.

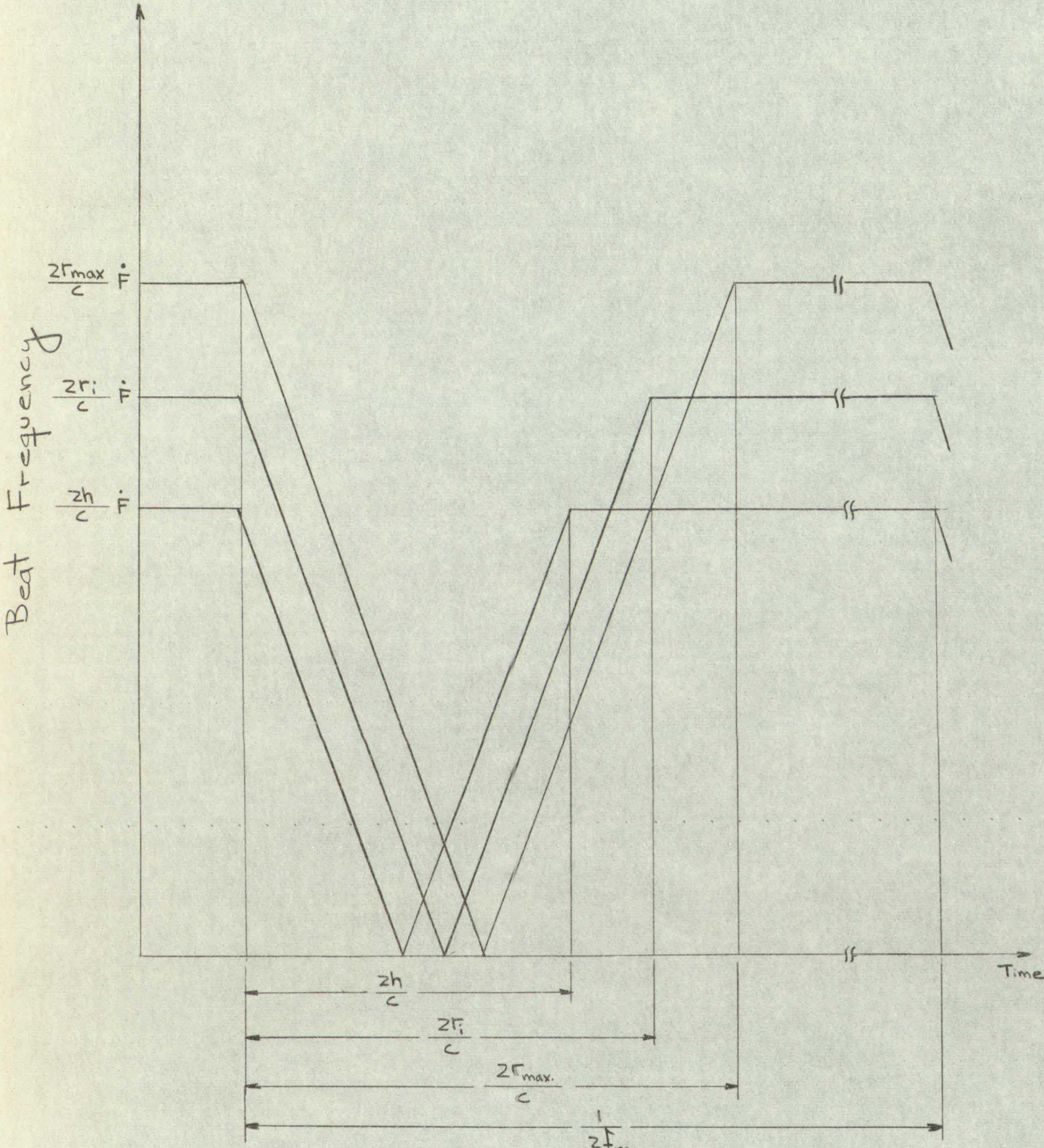

Figure 6. Beat Frequency Variation at Turnaround

Examination of Figure 5 and Figure 6 shows that the turnaround period lasts for a time interval, $2r_{max}/c$, and occurs twice in one modulation period, $1/f_m$. Due to the low modulation frequency commonly used in fm radars, it is usually no problem to satisfy the inequality, $\frac{1}{2f_m} >> \frac{2r_{max}}{c}$ and thereby minimize the duration of the turnaround period. Reference to Figure 6 also shows that beat frequencies lower than $\frac{2h}{c}\dot{F}$, the beat frequency representing actual aircraft altitude, occur for a time interval, $2h-r_{max}/c$ which is less than the specular transit time, $2h/c$.

Taking these facts into consideration, the characteristics of a beat-note voltage resulting from a triangular modulation can be generally described as a signal resulting from the combination of frequency components described in Section 2.5 with an error introduced periodically for an extremely short time interval. Proper selection of the modulating frequency will make the duration of the error-introducing turnaround period negligible with respect to the modulation period.

2.7 Cancellation of Doppler Effect.

One additional topic which should be considered prior to obtaining an expression for the mean return power spectrum, $\overline{B(f_R)}$, is the Doppler effect that will occur when there is relative motion between target and radar. If triangular modulation is

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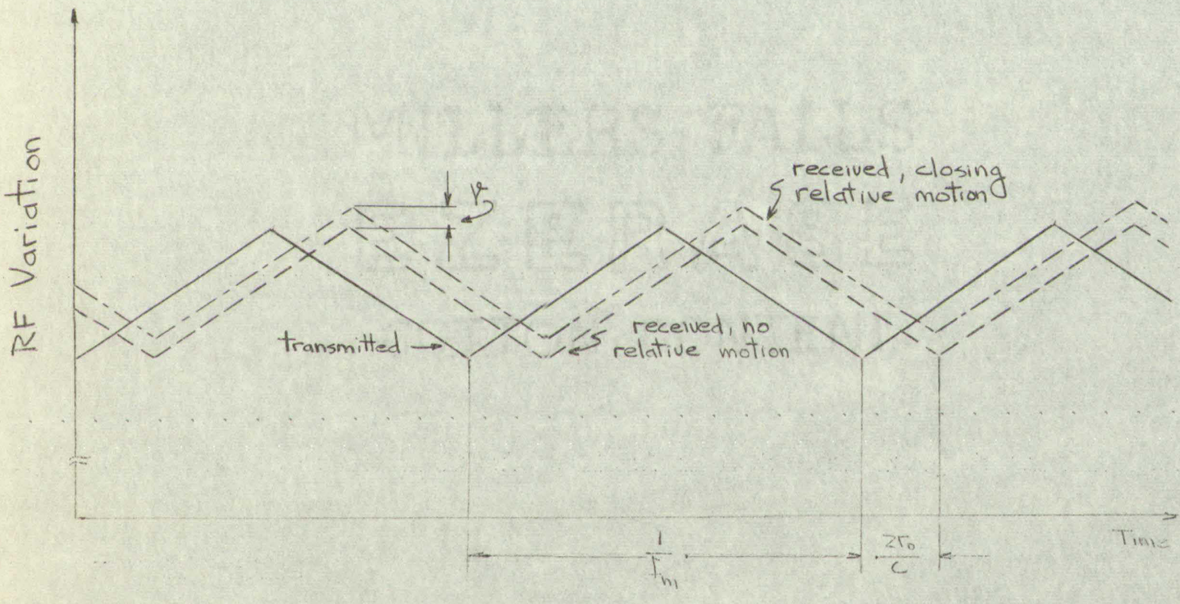
One additional topic which should be considered prior to obtaining an expression for the mean return power spectrum, $\overline{P_R}$, is the Doppler effect that will occur when there is relative motion between target and radar. If triangular modulation is

again assumed, the frequency variation of a return with closing relative motion will be shifted (increased) by an amount, γ , the Doppler frequency. This shift, along with a return resulting from stationary target and radar, is illustrated in Figure 7. For simplicity, only a single scattered return at range, r_o, will be illustrated; the effect being identical for the band of scattered returns that normally occurs. Furthermore, the delay time, $2r_o/c$, is greatly exaggerated for illustrative purposes.



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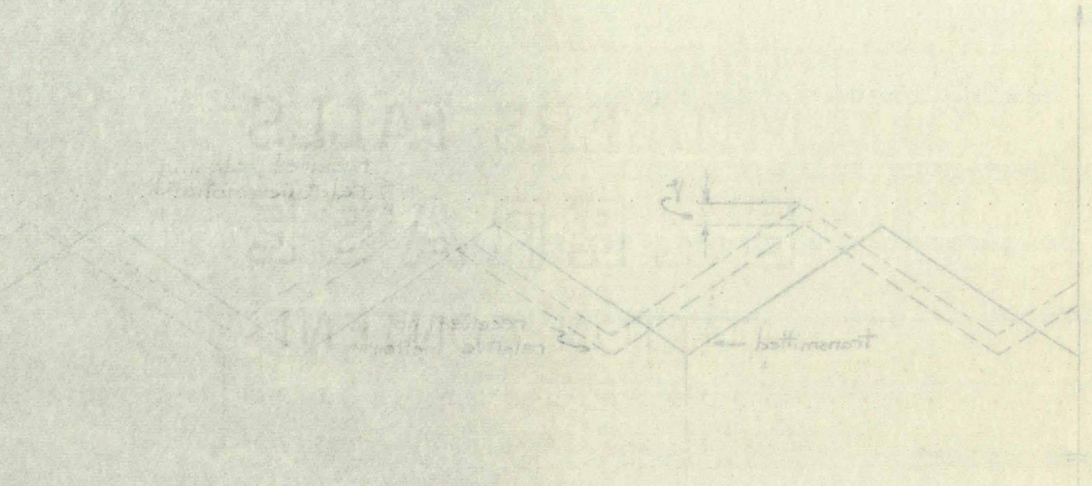

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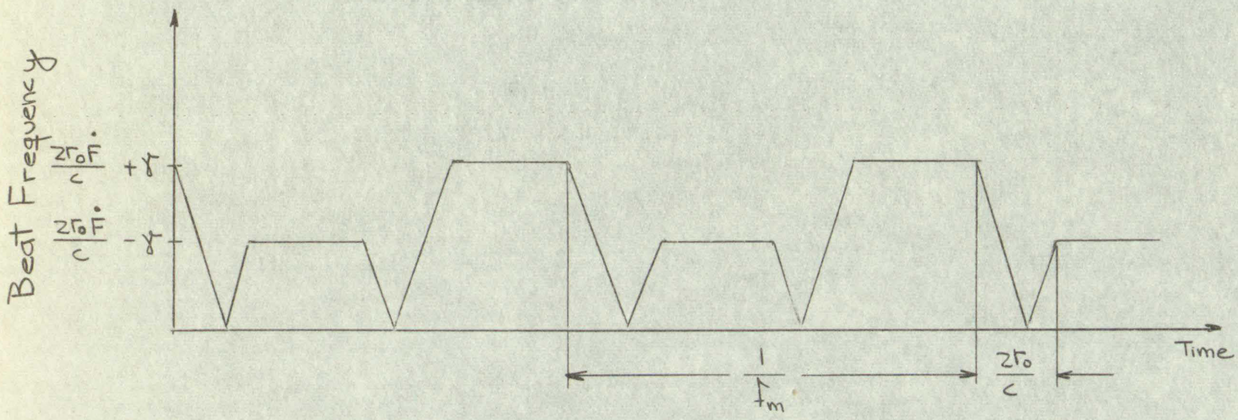

Figure 8. Beat Frequency Variation When Relative Motion Between Radar and Target Exists

It can be seen in Figure 8, that on alternate half cycles the beat frequency takes on values differing by plus and minus γ from the frequency that would result if no relative motion was present. The beat note voltage is then a sinusoidal one whose frequency periodically changes from

$$\frac{2r_o}{c}\dot{F} - \gamma \text{ to } \frac{2r_o}{c}\dot{F} + \gamma$$

(It should be remembered that the modulation period, $1/f_m$, is usually much greater than the turnaround time, $2r_o/c$, although the illustration does not show it as such).

When an area-extensive target is considered, rather than a single, isolated scatterer, the characteristics of the beat note voltage will generally be as described in Section 2.6 with the exception that on alternate half cycles of modulation, each frequency component in the band will shift in frequency by an amount equal to twice the Doppler frequency for that particular component. When an expression for the frequency spectrum of a beat note voltage of this nature is obtained in the following section, it will, as in the pulse radar, represent a mean return spectrum obtained over many cycles of modulation. It follows then, that an averaging of the scattered return components over a long period of radar operation, results in the cancellation of the Doppler effect when a triangular or similar symmetrical form of modulation is employed.

2.8 Derivation of an Expression for the FM Mean Return Power Spectrum.

In Section 2.6 the application of the fm beat voltage to a spectrum analyzer range indicator was suggested as being comparable to a pulse radar A-scope, since in both cases, horizontal displacement would be proportional to altitude. It is now desirable to obtain an expression for the fm mean return power spectrum, and consider the variables that effect it. The

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expression for the total power returned to an fm radar altimeter within a band of return beat frequencies is obtained in the same manner as the expression for the mean return pulse, $\overline{P_R}(t)$, Equation 4, and is given by[9]

$$dW_R = \overline{B(f_R)}\, df_R = \int \frac{\lambda^2 W_t\, G^2(\theta,\phi)\, \sigma_o(\theta,\phi)\, dA}{(4\pi)^3 r^4} \tag{16}$$

where W_R = the received power

$\overline{B(f_R)}$ = the mean return power spectrum

f_R = the return beat frequency

λ = the transmitted wave length, and is a function of time

W_t = the constant transmitter power

(θ,ϕ) = the coordinates shown in Figure 2

r, dA, $G(\theta,\phi)$, and $\sigma_o(\theta,\phi)$ are the same as before

Again it merits mentioning that the above expression represents a mean obtained by averaging over many modulation periods.

Determination of $\overline{B(f_R)}$ requires that dA be expressed in terms of a coordinate along the contour on the ground giving constant Doppler shift, and a coordinate normal to this contour.[10]

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As illustrated in Section 2.7, proper selection of the modulating wave-form will result in cancellation of the Doppler shift in which case a simple geometric integration of the illuminated area can be made. Then, as in Section 2.4, $dA = \rho\, d\rho\, d\phi$ and the expression for the mean return power spectrum becomes[11]

$$\overline{B(f_R)}\, df_R = \int \frac{\lambda^2 W_t\, G^2(\theta,\phi)\, \sigma_o(\theta,\phi)\, \rho\, d\rho\, d\phi}{(4\pi)^3\, r^4} \tag{17}$$

Since $r^2 = \rho^2 + h^2$ (Figure 2), $\rho\, d\rho$ is equal to rdr. From Equation 2, the return beat frequency, f_R, is

$$f_R = \frac{2r}{c}\dot{F}$$

therefore

$$r = \frac{c f_R}{2\dot{F}} \tag{18}$$

and

$$dr = \frac{c}{2\dot{F}}\, df_R \tag{19}$$

Using these relationships, $\overline{B(f_R)}df_R$ can be written as

$$B(f_R)\, df_R = \int_0^{2\pi} \frac{\lambda^2 W_t\, G^2(\theta,\phi)\, \sigma_o(\theta,\phi)}{(4\pi)^3} \left(\frac{2\dot{F}}{c}\right)^2 \frac{df_R}{f_R^{\,3}}\, d\phi \tag{20}$$

---

[11] Ibid, p. 17.

The limits are such because the integration is to be taken with respect to $d\phi$. To simplify the integration, two approximations will be made. A symmetrical antenna pattern and a uniform scattering ground will again be assumed so that G and σ_o can be considered as constant with respect to ϕ. The wavelength, λ, is a function of time only, so it can be taken out in front of the integral. The return beat frequency, f_R, is not a function of ϕ when a uniform scattering ground is assumed, so all that remains behind the integral is $d\phi$. The mean return power spectrum can therefore be written.

$$\overline{B(f_R)} = \frac{\lambda^2 W_t G^2(\theta)\sigma_o(\theta)\dot{F}^2}{8\pi^2 c^2 f_R^3} , \quad \frac{2h\dot{F}}{c} < f_R < \frac{2r_{max}\dot{F}}{c} \qquad (21)$$

The limits are such, since as mentioned before, f_R varies from the specular return beat frequency, $2h\dot{F}/c$, out to a frequency corresponding to negligible return, $\frac{2r_{max}\dot{F}}{c}$. For further simplification of $\overline{B(f_R)}$, the expression for $G^2(\theta)\sigma_o(\theta)$ used in the pulse case will now be applied to the above, that is

$$G^2(\theta)\sigma_o(\theta) = \frac{2.69\,h^5}{r^5}$$

and since

$$r = \frac{c f_R}{2\dot{F}}$$

$$G^2(\theta)\sigma_o(\theta) = \frac{86\,\dot{F}^5 h^5}{c^5 f_R^5}$$

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$$\overline{B(f_R)} = \frac{\lambda^2 W_t\, G^2(\theta)\,\sigma_o(\theta)\, F^2}{8\pi^2 c^2\, t_s^3} \qquad \frac{2hF}{c} < f_R < \frac{2r_{max}F}{c} \qquad (21)$$

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$$G^2(\theta)\,\sigma_o(\theta) = \frac{2.59\,h^3}{r^3}$$

and since

$$r = \frac{c f_R}{2F}$$

$$G^2(\theta)\,\sigma_o(\theta) = \frac{8(2.59)F^3 h^3}{c^3 f_R^3}$$

Inserting this expression in Equation 21 yields

$$\overline{B(f_R)} = \frac{10.76\,\lambda^2\,W_t\,\dot{F}^7 h^5}{\pi^2 c^7 f_R^{\,8}} \quad , \frac{2h\dot{F}}{c} < f_R < \frac{2r_{max.}\dot{F}}{c} \tag{23}$$

Comparison of the above expression with the return for a transmitted delta function in the pulse case (Equation 15) shows that both returns vary as an inverse eighth function of the independent variable. If both radar returns are expressed as functions of range, Equations 15 and 23 become, respectively

$$\overline{P_R(r)} = \frac{10.76\,\lambda^2 h^5 P_o c}{\pi^2 r^8} \quad , h < r < r_{max} \tag{24}$$

$$\overline{B(r)} = \frac{10.76\,\lambda^2 h^5 W_t c}{\pi^2 \dot{F} r^8} \quad , h < r < r_{max.} \tag{25}$$

If the above functions are normalized so that their peak values are unity, the expressions for the returns are identical, and in both cases the true altitude is given by the leading edge of the return.

The fact that the expression for the mean returns of the two radar systems are identical when properly manipulated suggests a basis of comparing pulse and fm radar. That is, the range

presentations, as functions of time for the pulse radar and frequency for the fm radar, can be thought of as analagous expressions for the two systems. If additional relationships of this nature are obtained, they will permit a comparison of pulse and fm radar systems. The mechanics of such a comparison will be discussed in the following chapter.

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CHAPTER III

ANALOGOUS RELATIONS BETWEEN PULSE AND FM RADAR ALTIMETERS

The relationships used in the comparison of pulse and fm radar altimeters will be approached in the following manner. Similarly shaped returns will be assumed as presentations on the pulse radar A-scope and the fm radar spectrum analyzer. From this initial assumption several methods of comparison can be employed. For example, the corresponding radar system characteristics necessary for a presentation to occur can be compared, or the effect of varying particular radar system parameters can be considered. Another method of comparison is indicated by the nature of the range presentations' independent variables, time and frequency. For, if Fourier transformation technique is applied to analogous functions, the resulting expressions will represent new system properties and can also be considered analogous.

The additional relationships required will be obtained either by using examples, or in terms of general functions. Then, when comparisons of specific systems are made, the appropriate functions can be substituted for the examples or general expressions, and as system parameters are varied, and various

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The additional relationships required will be obtained either by using examples, or in terms of general functions. Then, when comparisons of specific systems are made, the appropriate functions can be substituted for the examples or general expressions, and as system parameters are varied, and various

type targets are considered, the corresponding effects on each radar system can be determined.

For ease in handling, the subscripts used to obtain specific expressions in previous material will be dropped. For example, a pulse return will be denoted, p(t), and a return power spectrum, B(f). For further simplification, functions of time will be denoted by lower case letters, functions of frequency by upper case letters.

3.1 Gating the Return Pulse Analogous to Filtering the FM Return Power Spectrum.

If the range presentations for the two radar systems are generally represented by the sketch in Figure 9, below, then, as discussed previously, the actual aircraft altitude is given by the value of the time delay, t_o, for the pulse radar, and the specular frequency, f_o, for the fm radar.

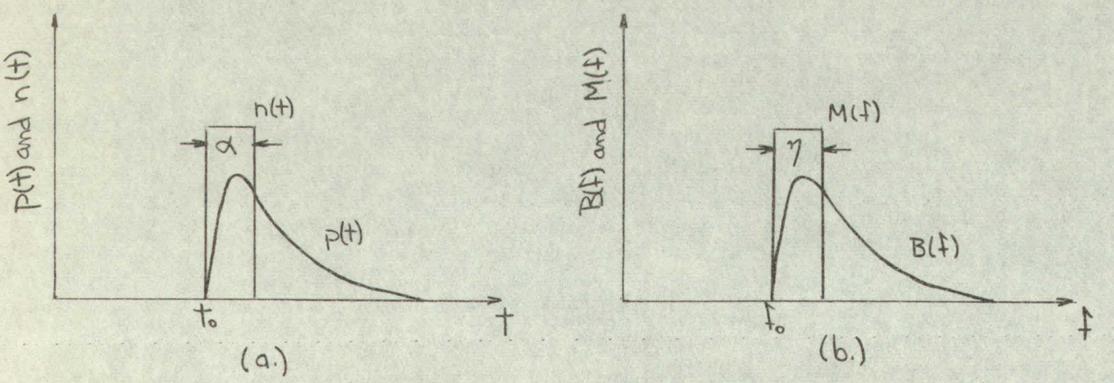

Figure 9. General Radar Range Returns for Pulse Radar (a.) and FM Radar (b)

If the position of the return pulses' leading edge is determined by a movable gate of fixed duration, α, the fm analogy will be the use scanning filter with fixed bandwidth, η, to determine the leading edge of the return power spectrum. The gate and filter are represented by the functions, n(t) and M(f) respectively, in Figure 9. As a result of the gating and filtering, the modified radar returns are given by the expressions

$$\begin{aligned} q(t) &= n(t)\ p(t) \\ &= p(t) \quad , t_o < t < t_o + \alpha \end{aligned} \tag{26}$$

and

$$\begin{aligned} C(f) &= M(f)\ B(f) \\ &= B(f) \quad , f_o < f < f_o + \eta \end{aligned} \tag{27}$$

where q(t) is the pulse through the gate, and C(f) is the spectrum in the filter. The pulse radar A-scope and the fm radar spectrum analyzer will display the range presentations, q(t) and C(f) as shown in Figure 10 below.

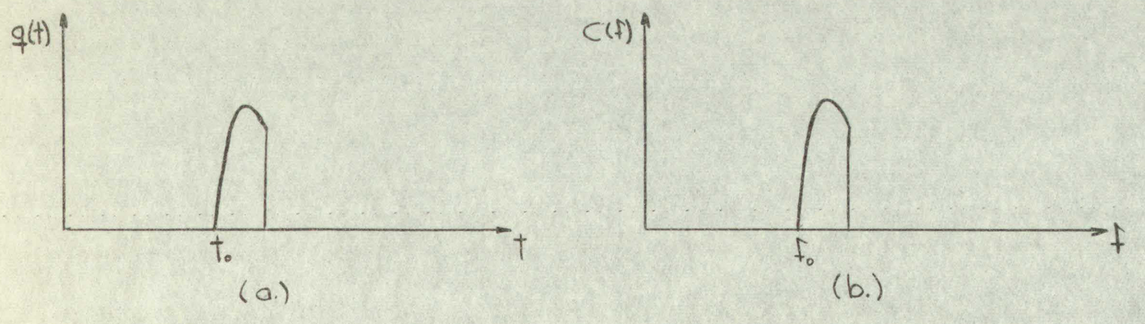

Figure 10. Gated Return Pulse (a.) and Filtered Return Spectrum (b.)

If the position of the return pulse's leading edge is deter- mined by a movable gate or fixed [illegible] the analogy will be the use scanning filter with fixed bandwidth, to deter- mine the leading edge of the return power spectrum. The gate and filter are represented by the functions r(t) and M(f) respectively in Figure 9. As a result of the gating and filtering, the modi- fied radar returns are given by the expressions

$$q(t) = r(t)\, p(t)$$

$$= p(t), \quad t_o < t < t_o + a \qquad (26)$$

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$$C(f) = M(f)\, B(f)$$

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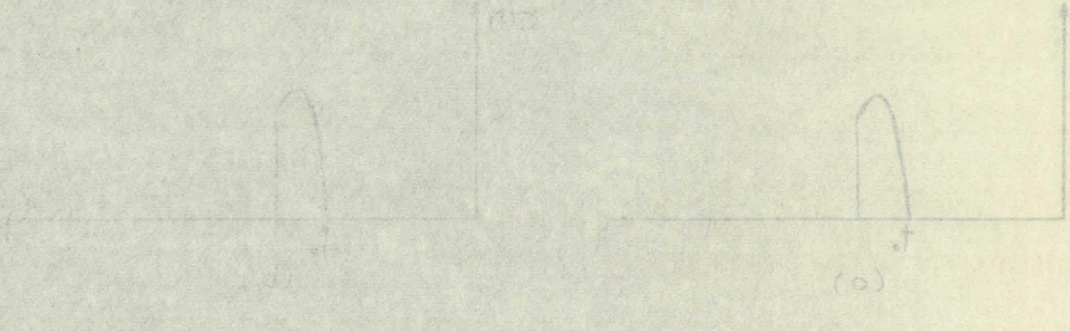

Figure 10. Gated Return Pulse (a) and Filtered Return Spectrum (b)

The relationship between the duration of the gate, α, and the bandwidth of the filter, η, can best be approached by considering the range increment, ϵ, corresponding to the magnitudes of α and η. That is $\epsilon = \frac{c\alpha}{2}$ for the pulse radar and $\epsilon = \frac{c\eta}{2\dot{F}}$ for the fm radar. Then, for the indicator presentations to be proportionally altered,

$$\frac{c\alpha}{2} = \frac{c\eta}{2\dot{F}}$$

or

$$\alpha = \frac{\eta}{\dot{F}} \tag{28}$$

The value of $\dot{F}$ is fixed for the type fm systems to be considered, so for a given gate length, α, the corresponding filter bandwidth, η, can be determined, and vice versa.

The gate or scanning filter can be locked-on the leading edge of each system's return by using servo-type followers in the following manner. If the output of the gate circuit or scanning filter is fed back to the positioning controls of the gate or filter through a servo that shifts the position of the gate, in time, and the filter, in frequency, to the left if an input to the servo exists, and to the right if no input exists, then the gate or filter will automatically follow the leading edge of the radar return. Because of fading and noise, this gate or filter will follow, more or less closely, the leading edge, with consequent variations in the error of the indication.

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$$\frac{c\eta}{2F} = \frac{c\alpha}{2}$$

or

$$\alpha = \frac{\eta}{F} \qquad (28)$$

The value of F is fixed for the type fm systems to be considered, so for a given gate length, α, the corresponding filter bandwidth, η, can be determined, and vice versa.

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3.2 Fourier Transformation of the Mean Return Pulse and the FM Mean Return Spectrum.

Previous discussion has been limited to the return functions, p(t) and B(t), and alterations of these functions. Additional relationships may be obtained between the two systems if Fourier transformation is employed. That is, the direct Fourier transformation of p(t) results in the return pulse power spectrum, $|P(f)|$, while an inverse Fourier transformation of $|B(f)^{\frac{1}{2}}|$ yields a time function that will be called the fm beat note voltage, b(t), and considered the input to the spectrum analyzer range indicator. Actually, since phase information is not available, there are an infinite number of time functions that have a power spectrum, B(f); however, the lack of phase information has no effect on the properties of b(t) to be considered in this paper. The functions $|P(f)|$ and b(t) will be considered analogous since they are obtained by transformation of related expressions.

To illustrate the relationship between $|P(f)|$ and b(t), unit amplitude delta function range presentations corresponding to a range, r_o, will be assumed for both radar systems. The delta function will be delayed in time by $t_o = {}^{2r_o}/c$ on the pulse radar A-scope, and will represent a single beat frequency

$f_o = 2r_o\dot{F}/c$, on the fm spectrum analyzer. Although the delta function presentation represents a hypothetical situation, it will be useful, not only to demonstrate the relationship between transformed analogous functions, but also to obtain additional relationships between pulse and fm radar systems. Figure 11 shows the assumed range presentations.

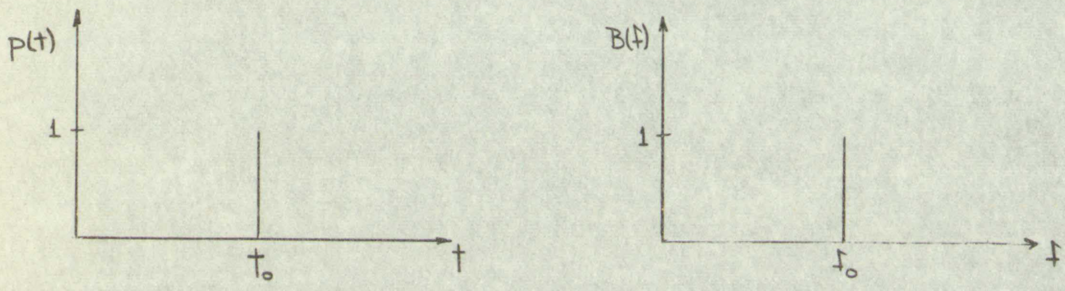

Figure 11. Delta Function Range Presentations

The return pulse power spectrum and the fm beat note voltage are now obtained by Fourier transformation and sketched in Figure 12.

$$|P(f)| = \left| \int_{-\infty}^{\infty} \delta(t - t_o)\, e^{-j2\pi ft}\, dt \right|$$

$$|P(f)| = \left| e^{-j2\pi ft_o} \right| \tag{29}$$

$$|P(f)| = 1$$

and

$$b(t) = \int_{-\infty}^{\infty} \delta(f+f_o)e^{j2\pi ft}\,df + \int_{-\infty}^{\infty} \delta(f-f_o)e^{j2\pi ft}\,df$$

$$b(t) = 2\cos(2\pi f_o t) \qquad (30)$$

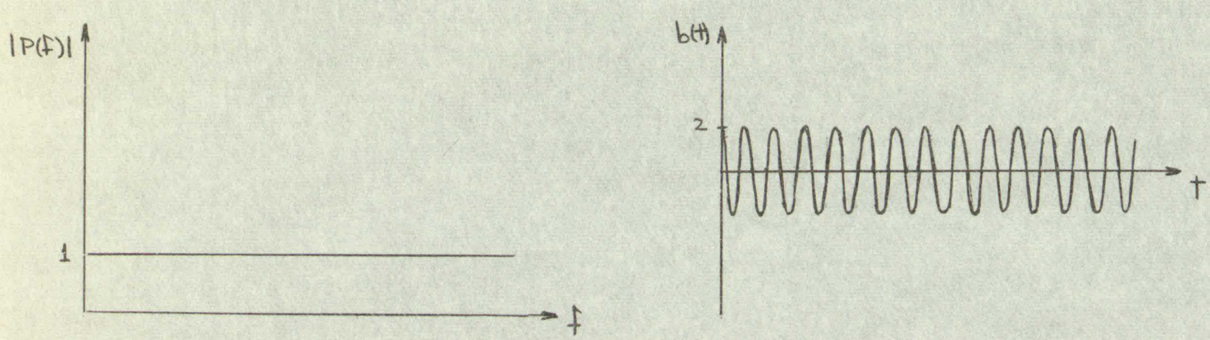

Figure 12. Pulse Power Spectrum, $|P(f)|$, and FM Beat Note Voltage, $b(t)$

An analogy between $\left|P(f)\right|$ and $b(t)$, as it stands, has little value, but an alteration of the two functions, similar to the operation performed on $p(t)$ and $B(f)$ in Section 3.1, is of interest and is discussed in the following section.

3.3 Pulse Radar Receiver Bandwidth Analogous to FM Beat Note Voltage Sampling Time.

If the pulse radar's receiver is assumed to be a linear system with transfer function, $D(f)$, then the response to the return pulse, $p(t)$, is

$$H(f) = \left|P(f)\right| D(f) \qquad (31)$$

Since $|P(f)|$ in a pulse system corresponds to b(t) in an fm system, the pulse receiver transfer function, D(f), will correspond to a time function multiplier, k(t), in the fm case. The general effect of k(t) can be thought of as an alteration of the fm beat note voltage given by

$$g(t) = b(t)\, k(t) \tag{32}$$

If the transfer function, D(f), ideally represents a square receiver bandpass, then it is given by

$$D(f) = u(f - f_1) - u\left[f - (f_1 + \beta)\right] \tag{33}$$

where f_1 = the lower frequency cutoff

β = the receiver bandwidth

The analogous fm beat note voltage multiplier, k(t), is then represented by

$$k(t) = u(t) - u(t - \tau) \tag{34}$$

where the time, τ, is comparable to the pulse receiver bandwidth, β.

If the functions D(f) and k(t) are applied to the return pulse power spectrum $|P(f)|$, and the fm beat note voltage, b(t), obtained for the delta function returns in Section 3.2, the

Since $|P(f)|$ in a pulse system corresponds to $\Lambda(t)$ in an fm system, the pulse receiver transfer function, $D(f)$ will correspond to a time function multiplier, $K(t)$, in the fm case. The general effect of $K(t)$ can be thought of as an alteration of the fm beat note voltage given by

$$g(t) = k(t)\,\Lambda(t) \qquad (32)$$

If the transfer function, $D(f)$, ideally represents a square receiver bandpass, then it is given by

$$D(f) = u(f - f_1) - u\left[f - (f_1 + \beta)\right] \qquad (33)$$

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The analogous fm beat note voltage multiplier, $k(t)$, is then represented by

$$k(t) = u(t) - u(t - \tau) \qquad (34)$$

where the time, τ, is comparable to the pulse receiver bandwidth, β.

If the functions $D(f)$ and $k(t)$ are applied to the ideal pulse power spectrum $|P(f)|$, and the fm beat note voltage, $\Lambda(t)$, obtained for the delta function returns in section 3.2, the

resulting functions, denoted H(f) and g(t), will be given by

$$H(f) = |P(f)| = 1, \quad 0 < f < B$$
$$= 0, \quad \text{elsewhere} \tag{35}$$

and

$$g(t) = b(t) = 2\cos(2\pi f_0 t), \quad 0 < t < \tau$$
$$= 0, \quad \text{elsewhere} \tag{36}$$

The pulse receiver lower cutoff frequency, f_1, was taken as zero, thereby representing application of the receiver bandwidth at the video amplifier, a permissable operation as long as the system is assumed linear. This then allows H(f) to represent the power spectrum of the pulse indicator presentation. The functions, H(f) and g(t), are shown in Figure 13, below

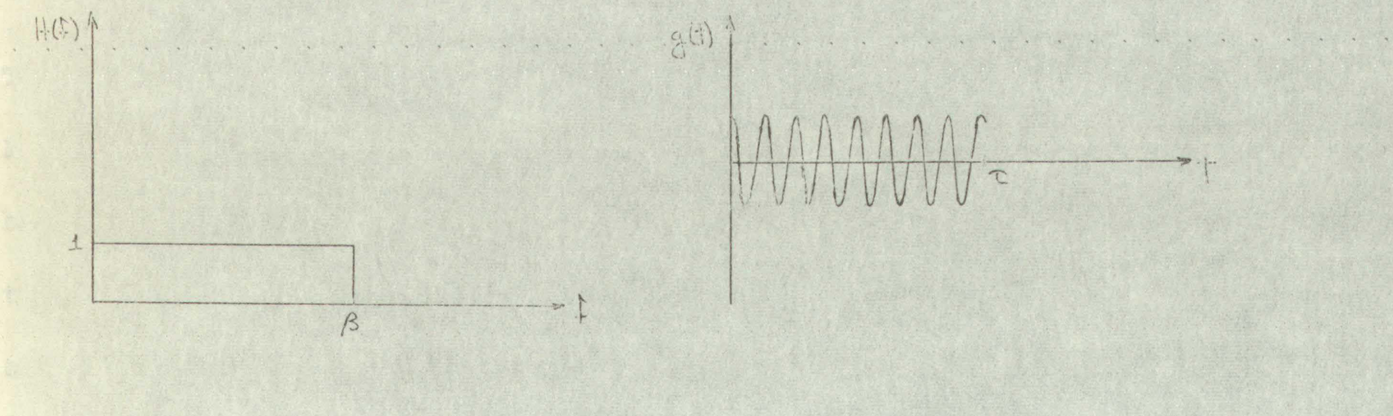

Figure 13. Pulse Presentation Power Spectrum, H(f), and Altered FM Beat Note Voltage, g(t)

The function, g(t), can be thought of as observing the beat note voltage for a time, τ. That is, the fm mean return power spectrum is obtained by determining the frequency content of the fm beat note voltage, b(t), over time, τ, rather than the infinite time interval employed in Equation 30, Section 3.2.

To determine how the original range information is affected by the above operations, h(t) and G(f), the pulse and fm radar indicator presentations, respectively, must be obtained. Using Fourier transformation, they are found to be

$$h(t) = 2\beta \frac{\sin 2\pi\beta(t-t_o)}{2\pi\beta(t-t_o} \tag{37}$$

and

$$G(f) = \frac{\tau}{\pi} \frac{\sin \pi(f_o - f)\tau}{\pi(f_o - f)\tau} \tag{38}$$

The proper delay time t_o, for h(t) was obtained by using $P(f) = e^{-j2\pi f t_o}$ rather than $|P(f)| = 1$. It could also have been obtained by using $|P(f)| = 1$, and keeping in mind that the results would require shifting to the right in time by an amount, t_o. Figure 14 shows a comparison of the original range information functions, p(t) and B(f), and the indicator presentations, h(t) and G(f), resulting from alteration by D(f) and k(t). The indicator presentations are normalized to facilitate comparison. As Figure 14 illustrates, the pulse indicator

presentation has successive zero values at $1/2\beta$ intervals from t_o. Similarly, the fm indicator presentation has successive zero values at $1/\tau$ intervals on either side of f_o. And, as Equations 37 and 38 indicate, both functions are inversely proportional to their respective independent variables.

A point that warrants investigation is the possibility of a direct relationship between the pulse receiver bandwidth, , and the fm beat note "observation" time, τ . One approach is the effect of varying the magnitude of the bandwidth and time interval. For, if τ is shortened, the corresponding pulse effect is a decrease in β , and in both systems a loss of information results. Hypothetically then, a zero τ will correspond to a receiver passing no frequencies, and in both cases all information is lost; on the other hand, an infinite τ and β results in no loss of information. Although examination from this point of view yields comparable results, a general fixed relationship between β and τ is difficult due to the nature of the variables. That is, β is a fixed system parameter determined in the design phase of a radar system, while τ represents a time interval over which the beat note voltage is observed when obtaining a mean return power spectrum; it may also be a design parameter in some automatic systems. However, some additional comments about τ , that will be useful subsequently, can be made.

presentation has successive zero values at $1/\Delta f$ intervals from t_o. Similarly, the fm indicator presentation has successive zero values at $1/\tau$ intervals on either side of f_o. And, as Equations 37 and 38 indicate, both functions are inversely proportional to their respective independent variables.

A point that warrants investigation is the possibility of a direct relationship between the pulse-position indicator and the fm beat note "observation" time, τ. One approach is the effect of varying the magnitude of the bandwidth and time interval. For, if τ is shortened, the [illegible] effect is a decrease in Δf, and in both systems a loss of information results. Hypothetically then, a zero Δf will correspond to a receiver passing no frequencies, and in both cases all information is lost; on the other hand, an infinite τ and Δf results in no loss of information. A rigorous examination from this point of view yields comparable results, but a general relationship between Δf and τ is difficult due to the nature of the variables. That is, Δf is a fixed system parameter determined in the design phase of a radar system, while τ represents a time interval over which the beat note voltage is observed when obtaining a mean return power spectrum. It may also be a design parameter in some automatic systems. However, some additional comments about τ, that will be useful subsequently, can be made.

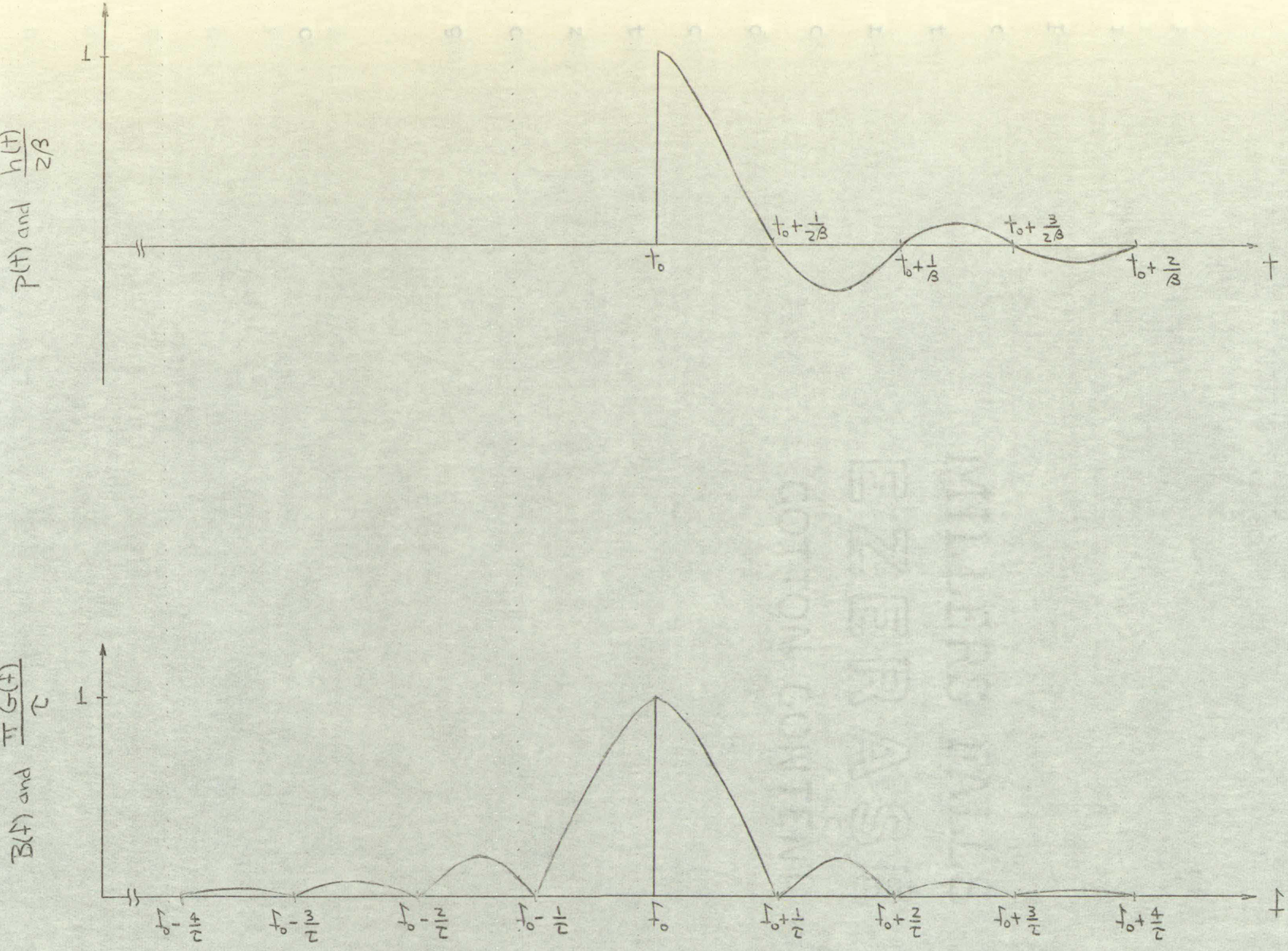

Figure 14. Delta Function Range Information, p(t) and B(f), and Indicator Presentation, h(t) and G(f), after Alteration by D(f) and k(t)

If the time τ is represented in terms of K modulation cycles, then

$$\tau = \frac{K}{f_m}$$

As previously mentioned in Section 2.7, a long period of fm radar operation is necessary when determining the mean return power spectrum, not only to obtain a true mean, but also for cancellation of the Doppler effect. The number, K, then represents the number of cycles observed in order to satisfy the above requirements. The magnitude of K is somewhat vague, depending on the particular system considered and the degree of accuracy desired, among other things. However, consideration of the indicator presentation, G(f), yields some information as to the magnitude of K. If τ is replaced by K/f_m, G(f) will have successive zero values at f_m/K intervals from f_o. For minimum distortion of the original radar return, $\delta(f - f_o)$, f_o should be much greater than f_m/K. That is

$$f_o >> \frac{f_m}{K}$$

or

$$K >> \frac{f_m}{f_o}$$

But f_o is the specular beat frequency given by $f_o = 2h\dot{F}/c$, so

$$K >> \frac{c f_m}{2h\dot{F}}$$

For the triangular modulation considered in Section 2.6, $\dot{F} = 2\Delta F f_m$, resulting in

$$K >> \frac{1}{2\left(\frac{2h}{c}\right)\Delta F} \tag{39}$$

Again selection of K depends on degree of accuracy desired; however, the above expression indicates the dependency of K on the signal transit time, and the transmitted frequency deviation. Furthermore, an indication of the accuracy obtained can be determined by plotting the G(f) in Figure 14 for various values of K, and observing the resulting distortion. Although the above relationships were obtained by considering altered delta functions as range presentations, they will be useful when more realistic radar return is considered.

One additional consideration is the application of the gating and filtering operation discussed in Section 3.1 to the range presentations of Figure 14. This would then eliminate the undesired portion of the presentations that theoretically exists out to infinite time and frequency. Such a procedure suggests a more practical approach to the subject of pulse-fm radar analogies, and will be discussed in the following section.

3.4 Systematic Application of Analogous Radar System Functions.

The analogies between pulse and fm radar that have been obtained were considered when the opportunity presented itself, rather than in a chronological sequence of system operation. The practical application of such an analogy would be to consider the pulse-fm relationships, step-by-step, from the time a return signal is received at the antenna, to the time it becomes in indicator presentation. As an example, the analogous functions and operations that have been considered will be generally illustrated in conjunction with simplified block diagrams of the two radar systems. Figure 15 shows the situation to be considered.

As the block diagrams illustrate, each system receives a finite section of return signal to be displayed on the radar indicators. After mixing and filtering, the fm signal becomes the beat note voltage that has been discussed previously. The effect of a finite length signal is represented at this point by the k(t) mentioned in Section 3.3. (Obviously, operation by k(t) could have been considered at the antenna; however, by considering it at this point, the results are consistent with Section 3.3). After operation by k(t), the altered beat note voltage is represented by g(t). The analogous operation in pulse radar is the effect of the receiver band width, D(f),

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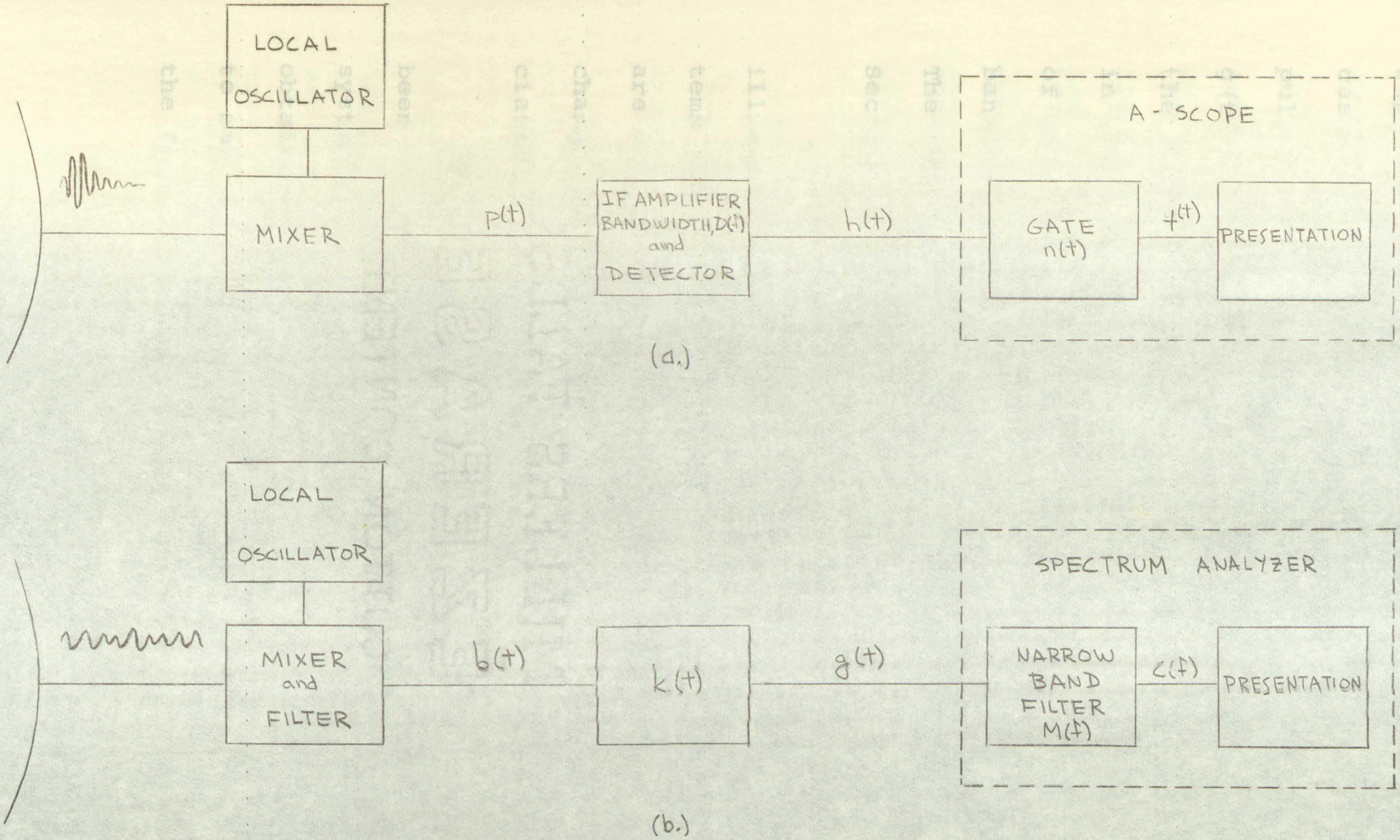

Figure 15. Chronological Sequence of Analogous Operations For Pulse Radar (a.) and FM Radar (b.)

represented here by the IF amplifier bandwidth (which in most cases will be the limiting bandwidth factor). The resulting pulse, h(t), then corresponds to the altered beat note voltage, g(t). The functions, h(t) and B(f), are next operated on by the radar indicators. For the pulse radar, the use of a gate in the indicator circuit, n(t), results in the presentation of g(t). The analogous fm operation is the use of a narrow band pass filter, M(f), resulting in the presentation of C(f). The operation by n(t) and M(f) was discussed in more detail in Section 3.1.

The example considered here, although extremely simplified, illustrates the use of analogies when entire pulse and fm systems are considered. When more extensive system operations are considered, the same procedure is applicable; the major change being the additon of more system functions and the associated transformations.

The discussion of pulse - fm analogies, to this point, has been limited to either ideal or general terrain returns and system functions, although the mean returns for each system obtained in Chapter II indicated that radar terrain return was, to say the least, of a complex nature. It follows then that the Fourier transformation of more realistic terrain returns

will, in general, be an involved process. Therefore, the use of radar return approximations would be advantageous in comparing the two systems, if the approximations were sufficiently accurate, and if an analogous relationship existed between the two systems' return approximations. The following material will consider possible approximations for pulse and fm radar terrain return. However, rather than attempt to discuss the two approximations simultaneously, the fm case will first be considered; then an analogous approximation for a pulse radar altimeter will be discussed.

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CHAPTER IV

APPROXIMATION OF THE FM MEAN RETURN SPECTRUM

As suggested in the latter part of Section 3.4, an approximation of the fm mean return spectrum would be advantageous due to the complex properties of radar return from scattering targets. However, prior to obtaining such an approximation, additional discussion of the properties of scattered terrain return, as applied to an fm radar altimeter, is required. In the following section the general characteristics of scattered terrain return will be discussed, and in Section 4.2 the statistical properties of scattered terrain return will be considered.

4.1 General Characteristics of the Beat Note Voltage and Return Power Spectrum Resulting from a Scattering Target.

In Section 2.8 an expression for the mean return of an fm radar altimeter operating above a scattering ground was obtained. Circular symmetry of the ground backscatter was assumed, and the scattering properties of the illuminated terrain were then approximated by assuming the average scattering cross section per unit area to be $\sigma_o(\theta) = C\cos\theta$. Furthermore, the expression $\overline{B(f_R)}$ was seen to have a finite upper limit corresponding to the beat frequency component associated with the scatterer

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at distance, r_{max}, where return becomes negligible. At that time the magnitude of r_{max} was not discussed. However, in the following material a definite value of r_{max}, relative to the true altitude, h, will be required. The selection of r_{max} is somewhat arbitrary in that it is dependent on the radar antenna beamwidth, the scattering properties of the ground target, and the degree of accuracy desired. These factors taken into consideration, a value of r_{max} equal to one and a half times the true altitude, h, will be selected for the example previously considered. The expression for $\overline{B(f_R)}$ in Section 2.8, Equation 23, becomes (the subscript R will again be dropped)

$$\overline{B(f)} = \frac{10.76\,\lambda^2 W_t \dot{F}^7 h^5}{\pi^2 c^7 f^8} \quad , \frac{2h\dot{F}}{c} < f < \frac{3h\dot{F}}{c} \tag{40}$$

If $\overline{B(f)}$ is normalized it will appear as shown in Figure 16 below.

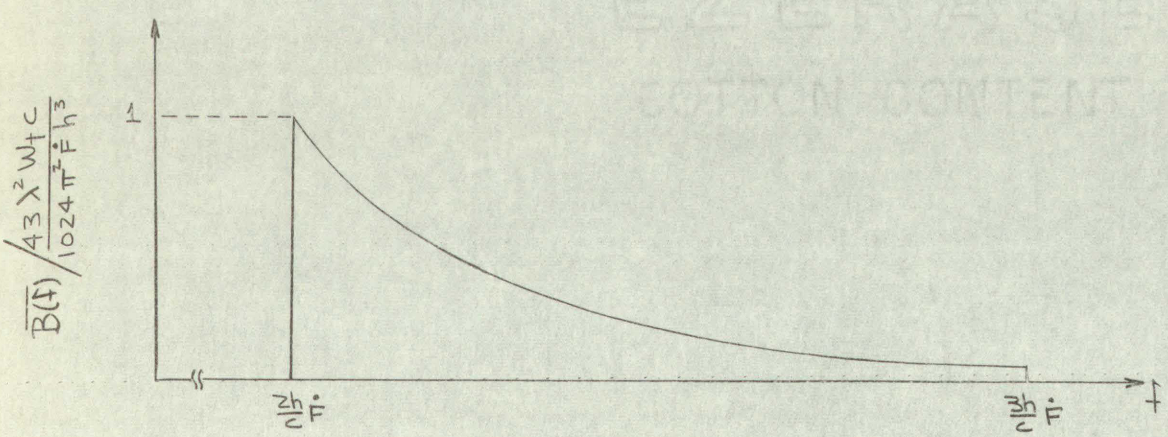

Figure 16. Normalized Mean Return Power Spectrum

To substantiate the selection of $r_{max} = 1.5h$ in the example, reference to experimental terrain return for pulse radars indicates that return generally becomes negligible at smaller angles of incidence than the 48.3 degree angle of incidence corresponding to $r = 1.5h$.[12] Therefore, the theoretical expression given above is conservative when compared to actual experimental data. In addition, it might be interesting to note that for the theoretical example considered, the magnitude of the beat frequency component occurring at $f = \frac{3h\dot{F}}{c}$ is 3.88% of the maximum beat frequency component occurring at $f = \frac{2h\dot{F}}{c}$. Furthermore, the amount of power returned to the radar in the frequency band, $\frac{2h}{c}\dot{F} < f < \frac{3h}{c}\dot{F}$ is 94% of the power returned when r_{max} is infinite rather than 1.5h.

The beat note voltage associated with the power spectrum shown in Figure 15 can be generally described as an oscillatory time function that is the sum of the sinusoids having frequencies in the band, $\frac{2h}{c}\dot{F} < f < \frac{3h}{c}\dot{F}$, the magnitude of each sinusoid being proportional to the square root of the normalized power value at that sinusoid's frequency. The statistical properties of this beat note voltage are examined in the following section.

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[12]Edison, A. R., Moore, R. K., Warner, B. D., "Radar Return at Near-Vertical Incidence, Summary Report," Technical Report EE-24, University of New Mexico Engineering Experiment Station, September, 1959.
1959

4.2 Statistical Properties of the Beat Note Voltage Resulting from a Scattering Target.

Previous discussion has dealt exclusively with the use of a spectrum analyzer as an fm radar range indicator, since it readily allowed comparison between pulse and fm radar return presentations. Another method of determining range employed by fm radar altimeters incorporates the use of a frequency measuring device called an "averaging cycle-rate counter."[13] The counter produces a current directly proportional to the frequency of the input signal, which in this application would be the beat note voltage, b(t). An ammeter calibrated in terms of distance then serves as the aircraft altitude indicator. For example, if the ideal beat note voltage illustrated in Figure 12 is the counter input, the ammeter indicator will read $r_o = cf_o/2\dot{F}$, where f_o is the frequency of the beat note voltage and $c/2\dot{F}$ is a constant for the fm radar considered. When a more realistic beat note voltage consisting of a band of frequency components, as discussed above, is considered, the counting procedure is not as straightforward as the above discussion. This situation will now be considered.

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[13] Luck, _op. cit._, p. 91.

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If the beat note voltage having the power spectrum shown in Figure 16 is applied to the input of an fm radar counter, it is not readily apparent what the counter output will be. However, previous research in pulse radar terrain return indicates a method of approaching the problem. In this study, a return pulse was treated as a sample taken from a continuous r-f wave. The r-f voltage was then to exhibit characteristics of Gaussian shot noise, and was treated as such.[14] This method of approach is equally applicable to an fm radar's beat note voltage, the only departure being that the frequency band is shifted by mixer action when the beat note voltage is obtained. Furthermore, the beat note voltage is a continuous wave, so no assumption regarding sampling is necessary.

Application of a particular property of Gaussian shot noise will allow prediction of the fm counter reading when a beat note voltage having a power spectrum such as the one illustrated in Figure 16 is the fm counter input. That is, the expected frequency, $\overline{f}$, of the counter is given by[15]

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[14] Edison, A.R., "Radar Return Statistics at Near-Vertical Incidence," Technical Report EE-35, University of New Mexico Engineering Experiment Station, October, 1960.

[15] Rice, S. O., "Mathematical Analysis of Random Noise," Section 3.3, Bell System Technical Journal, Vols. 23,24, also Dover publication S262, edited by Nelson Wax.

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$$\bar{f} = \left[\frac{\int_{f_1}^{\frac{3}{2}f_1} f^2 \, \overline{B(f)} \, df}{\int_{f_1}^{\frac{3}{2}f_1} \overline{B(f)} \, df} \right]^{\frac{1}{2}} \tag{41}$$

where $\overline{B(f)}$ is the fm mean return power spectrum

f_1 is the lower beat frequency limit, $\frac{2h\dot{F}}{c}$

$3/2\ f_1$ is the upper beat frequency limit, $\frac{3h\dot{F}}{c}$

Solution of $\bar{f}$ for the spectrum of Figure 15 results in

$$\bar{f} = 1.1363\ f_1$$

The counter indicator would therefore be expected to indicate an altitude

$$\bar{h} = \frac{c\bar{f}}{2\dot{F}} = \frac{c(1.1363\ f_1)}{2\dot{F}}$$

or

$$\bar{h} = 1.1363\ h$$

That is, the expected indicated altitude would be 13.63% higher than the actual aircraft altitude, the error resulting from the use of a counter, and the scattering nature of the ground target.

To substantiate the application of Rice's expression for expected frequency to fm radar beat note voltages, an experiment designed to count the number of zero crossings of a complex current is discussed in Section 4.4. The experiment is also useful in determining the validity of the return spectrum

approximation that is discussed in the following section.

4.3 Discrete Spectrum Approximation of the FM Return Power Spectrum.

As previously indicated, an approximation of the FM return power spectrum may be advantageous if the approximation is valid, and an analogous approximation for a pulse radar altimeter exists. Such an approximation will now be considered. The accuracy of the approximation will be determined by assuming a counter indicator, applying Rice's expression for expected frequency to the approximation, and comparing the results with those obtained for the continuous spectrum.

The fm mean return power spectrum illustrated in Figure 15 is a continuous function in the frequency range of interest, and therefore has an infinite number of frequency components within this range. The approximation of this continuous spectrum to be considered at this time is a discrete spectrum composed of a finite number of equally spaced frequency components located in the same frequency range. The magnitude of each frequency component has the same value as the magnitude of that frequency component in the continuous spectrum. The interval between discrete components is determined, as will be shown later, by the degree of accuracy desired. The approximation by

a discrete spectrum can be generally represented by the illustration in Figure 17.

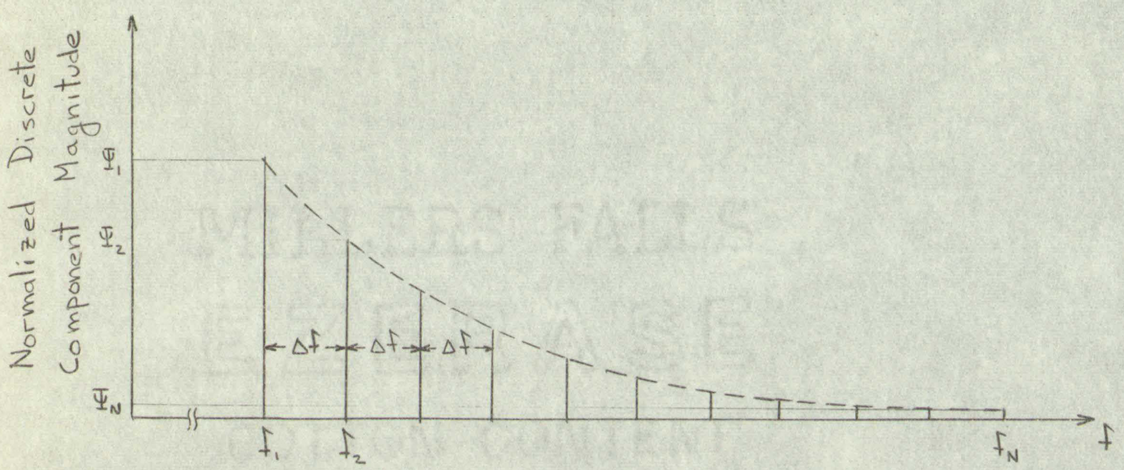

Figure 17. Approximation by a Discrete Spectrum

For the example that has been considered previously, the upper and lower frequency limits, f_1 and f_N, respectively, are

$$f_1 = \frac{2h}{c} \dot{F}$$

$$f_N = \frac{3h}{c} \dot{F} = 3/2\ f_1$$

where N is the total number of discrete components. The selection of equally spaced discrete components results in

$$\Delta f = \frac{f_N - f_1}{N-1} = \frac{f_1}{2(N-1)} \tag{42}$$

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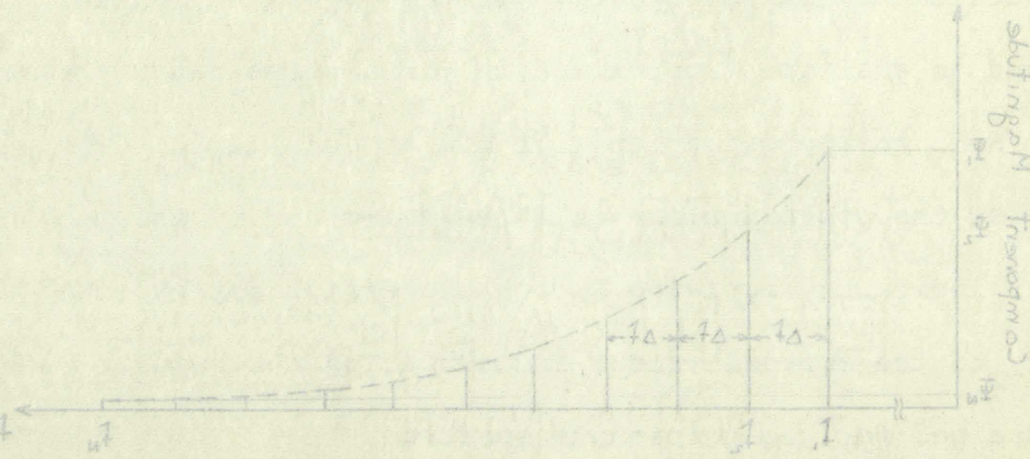

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$$\Delta f = \frac{f_N - f_1}{N-1} = \frac{f_1}{2(N-1)} \tag{42}$$

The value of each frequency component can then be described in terms of f_1 and N as follows

$$f_1 = f_1 + (0)\,\Delta f = f_1$$

$$f_2 = f_1 + \Delta f = f_1 \left[1 + \frac{1}{2(N-1)}\right]$$

$$f_3 = f_1 + 2\Delta f = f_1 \left[1 + \frac{1}{N-1}\right]$$

. . . .

$$f_n = f_1 + (n-1)\,\Delta f = f_1 \left[1 + \frac{n-1}{2(N-1)}\right] \tag{43}$$

. . . .

$$f_N = f_1 + (N-1)\,\Delta f = f_1 \left[1 + \frac{1}{2}\right] = \frac{3}{2} f_1$$

The magnitudes, Ψ_1, Ψ_2, Ψ_3, ...Ψ_n of the frequency components for the approximation of the normalized continuous spectrum are given by

$$\Psi_1 = \left[\frac{f_1}{f_1}\right]^8 = 1$$

$$\Psi_2 = \left[\frac{f_1}{f_2}\right]^8 = \frac{1}{\left[1 + \frac{1}{2(n-1)}\right]^8}$$

. . . .

$$\Psi_n = \left[\frac{f_1}{f_n}\right]^8 = \frac{1}{\left[1 + \frac{n-1}{2(N-1)}\right]^8} \tag{44}$$

. . . .

$$\Psi_N = \frac{f_1}{f_N} = \frac{1}{(1+\frac{1}{2})^8} = \left(\frac{2}{3}\right)^8$$

A comparison between the continuous spectrum and the approximation can be made by considering a summation process that is analogous to the use of Rice's expression on the continuous spectrum. That is, the expected frequency, $\bar{f}_\Psi$, for the discrete function is given by (the Ψ subscript is used to differentiate from the expected frequency of the continuous case which henceforth will be denoted $\bar{f}_b$)

$$\bar{f}_\Psi = \left[\frac{\sum_{n=1}^{N} f_n^2 \Psi_n}{\sum_{n=1}^{N} \Psi_n} \right]^{\frac{1}{2}}$$

For the case discussed, the expected frequency is given by

$$\bar{f}_\Psi = \left[\frac{\sum_{n=1}^{N} f_1^2 \left[1 + \frac{n-1}{2(N-1)}\right]^2 \left[1 + \frac{n-1}{2(N-1)}\right]^{-8}}{\sum_{n=1}^{N} \left[1 + \frac{n-1}{2(N-1)}\right]^{-8}} \right], N > 1$$

which after slight manipulation becomes

$$\bar{f}_\Psi = \frac{f_1}{2(N-1)} \left[\frac{\sum_{n=1}^{N} \frac{1}{[n + (2N-3)]^6}}{\sum_{n=1}^{N} \frac{1}{[n + (2N-3)]^8}} \right]^{\frac{1}{2}}, N > 1 \qquad (45)$$

The above represents a general expression for the expected frequency of a discrete spectrum. For a fixed number of discrete components, N, the expected frequency of the resulting approximation can be obtained by performing the indicated operation. The table below shows the expected frequency calculated for several values of N.

Total Number of Discrete Components, N	Expected Frequency, $\bar{f}$
2	1.0232 f_1
3	1.0577 f_1
5	1.0904 f_1
6	1.0986 f_1
11	1.1164 f_1
21	1.1261 f_1

The accuracy of this type approximation can now be determined by comparing the expected frequencies obtained above to the expected frequency, $\bar{f}_b$, obtained for the continuous spectrum in Section 4.2. A comparison can best be made by determining the percent deviation of $\bar{f}_{\Psi}$ from $\bar{f}_b$ for different values of N, as shown in Figure 18, on the following page.

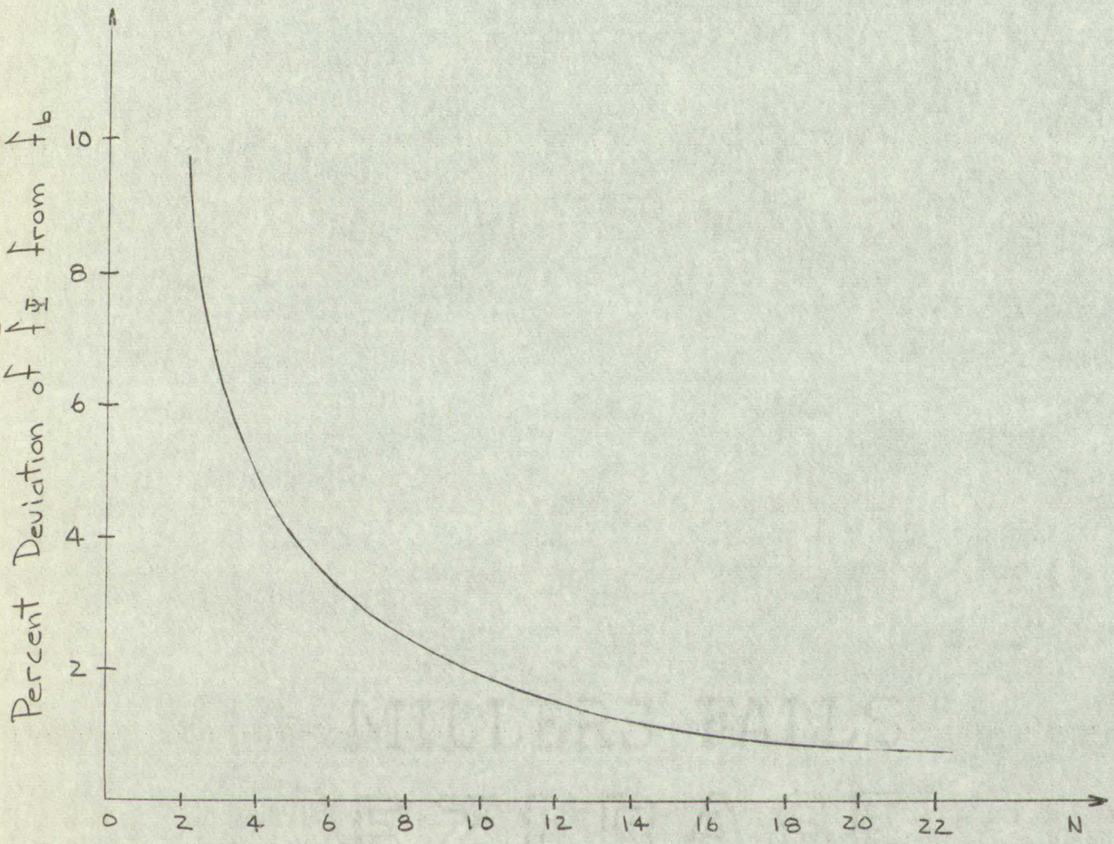

Figure 18. Percent Deviation of $\overline{f}_{\Psi}$ from $\overline{f}_b$ as N Varies

Figure 18 illustrates, as would be suspected, that as the number of discrete components, N, increases, the expected frequency of the approximation, $\overline{f}_{\Psi}$, approaches the expected frequency of the continuous spectrum, $\overline{f}_b$, given by Rice's expression. The percent deviation can also be considered a percent

error introduced by the approximation; then for a given allowable error, the number of discrete components required for an approximation can be determined from the curve in Figure 18. It should be remembered, however, that Figure 18 is applicable only to the approximation of this particular power spectrum. For approximation of other fm return power spectrum, a different curve, corresponding to Figure 18, would have to be obtained.

In addition to obtaining an expected frequency, the discrete spectrum will yield an approximation of the beat note voltage if Fourier transformation is applied. That is, if the discrete spectrum is denoted, $\overline{B'(f)}$, it is described by

$$\overline{B'(f)} = \sum_{n=1}^{N} \Psi_n \, \delta(f \pm f_n) \tag{46}$$

Then, the associated beat note voltage approximation, $b'(t)$, is given by

$$b'(t) = \int_{-\infty}^{\infty} \sum_{n=1}^{N} \left|\Psi_n^{\frac{1}{2}}\right| \delta(f \pm f_n) e^{j2\pi f t} \, df$$

$$b'(t) = 2 \sum_{n=1}^{N} \left|\Psi_n^{\frac{1}{2}}\right| \cos(2\pi f_n)t \tag{47}$$

The beat note voltage approximation, $b'(t)$, can be manipulated in the same manner as the actual beat note voltage, $b(t)$, and will be analogous to the Fourier transformation of a return pulse approximation when pulse radar altimeters are considered in Chapter V.

In the following section, an experiment designed to examine the concept of an expected frequency will be discussed. The results of the experiment will also indicate the validity of the expected frequency obtained from the discrete approximation.

4.4 Determination of the Expected Frequency by Experiment.

Previous discussion concerning an fm radar's beat note voltage has utilized S. O. Rice's expression for the expected number of zero crossings of Gaussian shot noise current. This application was partially justified by extending a study performed on pulse radar terrain return to the fm radar situation. To further substantiate the use of Rice's expression, and also investigate the validity of the approximation in Section 4.3, the following experiment was performed.

Six audio oscillators were used to represent the frequency components of the discrete spectrum discussed in the last section. The oscillator frequencies were set at 10 cps intervals from 100 to 150 cps. The oscillator output voltages were set proportional to the square root of the corresponding discrete

The beat note voltage approximation, [illegible] can be manipulated in the same manner as the actual beat note voltage, [illegible]. This will be analogous to the Fourier transformation of [illegible] pulse approximation when pulse radar altimeters are considered in Chapter V.

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Six audio oscillators were used to represent the frequency components of the discrete spectrum discussed in the last section. The oscillator frequencies were set at 10 cps intervals from 100 to 150 cps. The oscillator input voltages were proportional to the square root of the corresponding discrete

component power magnitudes. Each oscillator was connected in a series with a 100k ohm isolating resistor forming one branch of a parallel circuit connected to a 3.6k ohm load resistor. (See Figure 19). This arrangement represented an approximation in which Δf = 10cps, N = 6, f_{I} = 100. The voltage waveform in the load resistor corresponds to the beat note voltage resulting from the spectrum approximation. The expected frequency of the approximated beat note voltage, using the summation process discussed previously, is given by

$$\bar{f}_{\Psi} = \frac{1}{2(N-1)} \left[\frac{\sum\limits_{n=1}^{N} \frac{1}{[n+(2N-3)]^6}}{\sum\limits_{n=1}^{N} \frac{1}{[n+(2N-3)]^8}} \right]^{\frac{1}{2}} = 10 \left[\frac{\sum\limits_{n=1}^{N} \frac{1}{(n+9)^6}}{\sum\limits_{n=1}^{N} \frac{1}{(n+9)^8}} \right]^{\frac{1}{2}} \tag{48}$$

which is evaluated to

$$\bar{f} = 109.86 \text{ cps}$$

Therefore, the expected frequency, $\bar{f}_x$, obtained by counting the zero crossings of the load resistor voltage waveform should yield comparable results. (The x subscript on $\bar{f}_x$ indicates that the frequency is obtained by experiment.)

The experimental expected frequency, $\bar{f}_x$, was obtained in the following manner: The load resistor voltage was amplified and clipped, resulting in a train of rectangular pulses of various lengths, which in turn was used to trigger a negative

component power magnitudes. Each oscillator was connected in a series with a 100K ohm isolating resistor forming one branch of a parallel circuit connected to a 2.5K ohm load resistor. (See Figure 19). This arrangement represented an approximation in which $\Delta f = 10$cps, $N = 6$, $f_1 = 100$. The voltage waveform in the load resistor corresponds to the beat note voltage resulting from the spectrum approximation. The expected frequency of the approximated beat note voltage, using the summation process discussed previously, is given by

$$\bar{f}_\Psi = \frac{1}{2(N-1)}\left[\frac{\sum_{i=1}^{N}\frac{1}{[i+(2N-5)]^{4}}}{\sum_{i=0}^{N}\frac{1}{[i+(2N-5)]^{6}}}\right] = \frac{1}{G}\left[\frac{\sum_{i=1}^{N}\frac{1}{(N+3)^{4}}}{\sum_{i=1}^{N}\frac{1}{(N+4)}}\right] \qquad (48)$$

which is evaluated to

$$\bar{f} = 100.36 \text{ cps}$$

Therefore, the expected frequency, $\bar{f}$, obtained by counting the zero crossings of the load resistor voltage waveform should yield comparable results. (The x subscript on $\bar{f}_x$ indicates that the frequency is obtained by experiment.)

The experimental expected frequency, $\bar{f}_x$, was obtained in the following manner: The load resistor voltage was amplified and clipped, resulting in a train of rectangular pulses of various lengths, which in turn was used to trigger a one-shot

pulse generator. The nagative pulses, each representing one cycle, were then fed into an electronic counter, which counted the number of pulses over a known time interval and therefore, gave the frequency, $\overline{f}_x$. The experimental setup and the waveforms occurring at different points in the circuit are illustrated in Figure 19.

The number of cycles counted over an 81.8 second time interval was 8960, which corresponds to a frequency, $\overline{f}_x = 109.54$ cps. When compared to the expected frequency predicted by the summation process, $\overline{f}_\Psi = 109.86$ cps, it can be seen that the difference is less than 0.3% of $\overline{f}_\Psi$. The above experiment was also performed for approximations of three other continuous spectra, representing fm return power spectra obtained using different values of $\sigma_o(\theta)$, and in each case the difference between $\overline{f}_x$ and $\overline{f}_\Psi$ was less than 1% of $\overline{f}_\Psi$.

The results of this experiment are useful for several reasons: (1) They indicate that the expression for expected number of zero crossings of a Gaussian shot noise current is applicable to the beat note voltages existing in an fm radar altimeter working against a scattering ground target. (2) The results show that the summation process discussed in Section 3.6 is analogous to Rice's expression for a continuous spectrum. (3) The experiment shows that as long as a frequency measuring

pulse generator. The negative pulses, each representing one cycle, were then fed into an electronic counter which counted the number of pulses over a known time interval and therefore gave the frequency f_x. The experimental setup and the wave-forms occurring at different points in the circuit are illustrated in Figure 19.

The number of cycles counted over an 81.6 second time interval was 8960, which corresponds to a frequency $\bar{f}_x$ = 109.8 cps. When compared to the expected frequency predicted by the summation process, $\bar{f}_T$ = 109.58 cps, it can be seen that the difference is less than 0.3% of $\bar{f}_T$. The above experiment was also performed for approximations of three other non-uniform spectra, representing fm return power spectra obtained using different values of $\sigma_i(\theta)$, and in each case the difference between $\bar{f}_x$ and $\bar{f}_T$ was less than 1% of $\bar{f}_T$.

The results of this experiment are useful for several reasons: (1) They indicate that the expression for expected number of zero crossings of a Gaussian shot noise current is applicable to the beat note voltages existing in an fm radar altimeter working against a scattering ground target. (2) The results show that the summation process proposed in Section 1.4 is analogous to Rice's expression for a continuous spectrum. (3) The experiment shows that as long as a frequency measuring

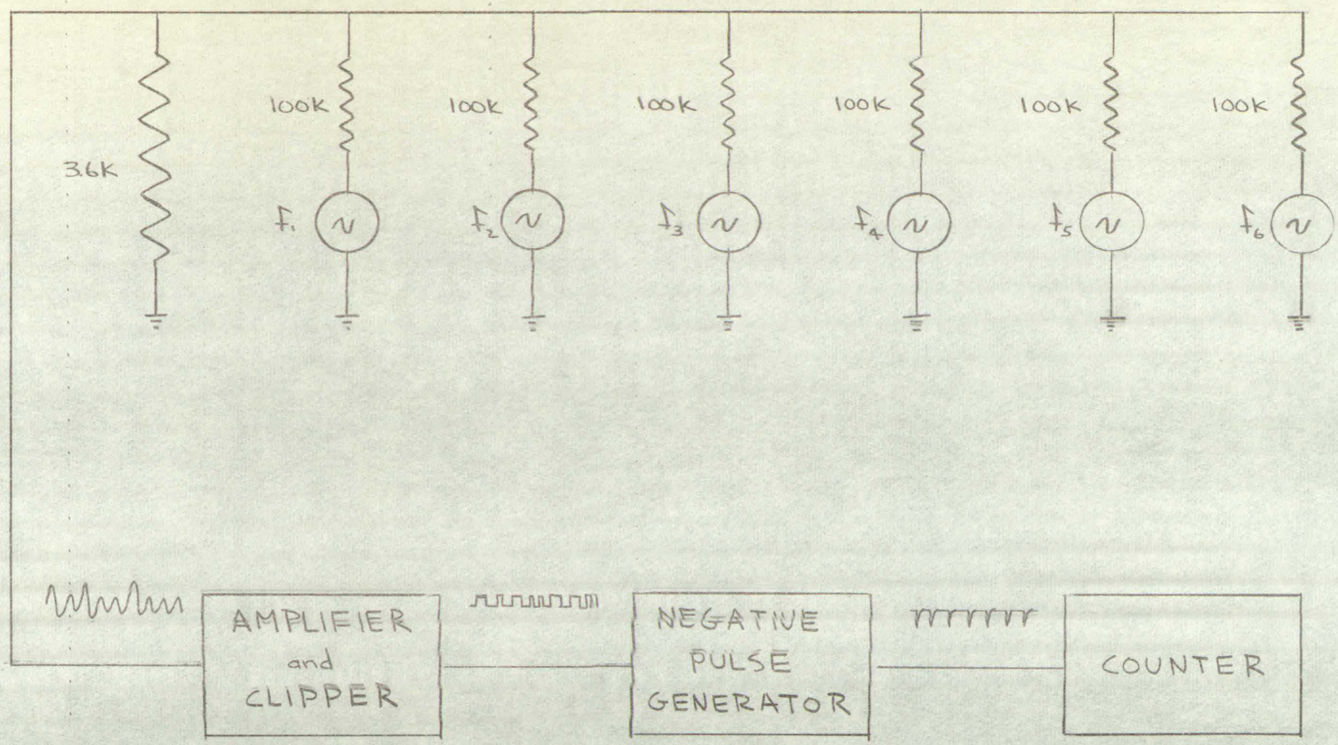

Figure 19. Experimental Layout for Determining $\overline{f}_x$

device, such as a cycle-rate counter, is the source of radar range information, the approximation of a continuous spectrum by a discrete spectrum with a relatively small number of frequency components is valid. Furthermore, the approximation can be as accurate as desired, since a curve similar to Figure 18 can be used to determine how many discrete components are necessary to keep within a given allowable error.

It has been convenient to neglect pulse radar altimeters while the approximation of an fm return power spectrum was considered. It is now desirable to obtain a pulse radar analogy corresponding to the spectrum approximation in the same manner that previous analogies were obtained.

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CHAPTER V

PULSE RADAR APPROXIMATION ANALOGOUS TO THE FM SPECTRUM APPROXIMATION

In Chapter III, the basis of an analogy between pulse and fm radar altimeters was established by considering the similarity of range presentations for the two systems under identical operating conditions. Furthermore, it was stated that similar alteration of related functions would result in additional analogies. Consequently, the approximation of an fm radar's return power spectrum will correspond to the approximation of a pulse radar's return pulse discussed in the following section.

5.1 Delta Function Approximation of the Mean Return Pulse.

If a return pulse, $p(t)$, is approximated by a series of weighted delta functions equally spaced throughout the pulse duration, the approximation is analogous to the approximation of a continuous return power spectrum by a discrete spectrum. Figure 20 illustrates the delta function approximation of a general return pulse, $p(t)$, of duration, $t_N - t_1$.

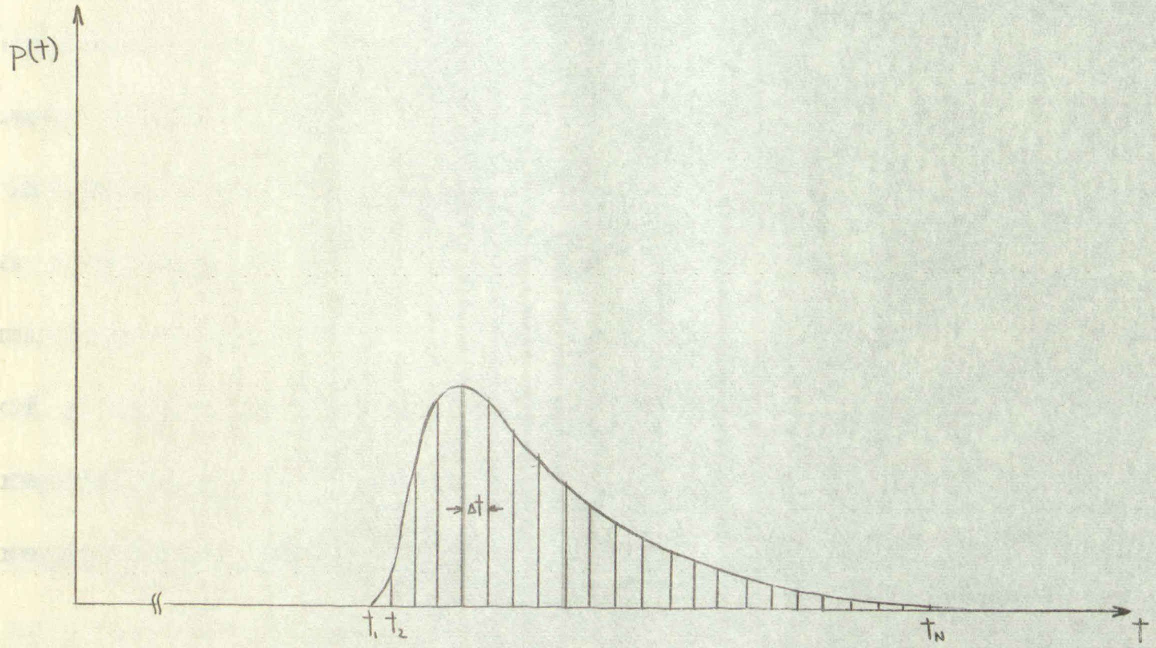

Figure 20. Delta Function Approximation

As in the fm case, the time interval between delta functions is

$$\Delta t = \frac{t_N - t_1}{N-1}$$

and the time, t_n, is

$$t_n = t_1 + (n-1)\,\Delta t$$

The delta function approximation, p'(t), can now be written as

$$p'(t) = \sum_{n=1}^{N} p(t_n)\,\delta(t - t_n) \qquad (49)$$

It was emphasized in Section 2.4 that the theoretical expression for the mean return pulse, $\overline{P_R}(t)$, represents the mean taken over a large number of return pulses. Experimental values of the mean return pulse have been obtained by taking a photographic record of actual radar terrain return, and determining the mean for various delay times from the leading edge of a return pulse.[16] The results of such an experiment can be represented by the above summation process, where each $p(t_n)$ represents the mean taken for a delay time, t_n. That is

$$p(t_n) = \frac{1}{K} \sum_{k=1}^{K} p_k(t_n) \tag{50}$$

where p_k represents the amplitude of the $k^{\underline{th}}$ pulse at delay time, t_n. Inserting this expression into the expression for $p'(t)$ results in

$$\overline{p'(t)} = \sum_{n=1}^{N} \left[\frac{1}{K} \sum_{k=1}^{K} p_k(t_n) \right] \delta(t - t_n) \tag{51}$$

This expression for $\overline{p'}(t)$, representing the experimental determination of a mean return pulse, can be considered analogous to the discrete spectrum approximation of an fm radar's mean return

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[16] Edison, Moore, and Warner, <u>op. cit.</u>

power spectrum, since in both cases the analogous range presentations were the functions approximated. Furthermore, the expected frequency of the beat note voltage, associated with the fm discrete spectrum, involved a transformation from the frequency domain to the time domain, so the analogous operation for the pulse system will be a transformation of the delta function pulse approximation to the frequency domain. This transformation is discussed in the following section.

5.2 Effect of Receiver Transfer Function on the Mean Return Approximation.

The expression for the mean return pulse does not take into consideration the distortion of the return imposed by the receiving system. Therefore, if $\overline{p'(t)}$ is to be compared with experimental results, the effect of the receiver transfer function on $\overline{p'(t)}$ will have to be considered before the comparison can be made. If the receiver is a linear system with transfer function, A(f), the response to $\overline{p'(t)}$ will be

$$\overline{q'(t)} = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \sum_{n=1}^{N} p(t_n)\, \delta(t-t_n)\, e^{-j2\pi f t}\, dt \right] A(f)\, e^{j2\pi f t}\, df$$

$$\overline{q'(t)} = \int_{-\infty}^{\infty} \left[\sum_{n=1}^{N} p(t_n)\, e^{-j2\pi f t_n} \right] A(f)\, e^{j2\pi f t}\, df$$

$$\overline{q'(t)} = \sum_{n=1}^{N} p(t_n) \int_{-\infty}^{\infty} A(f)\, e^{j2\pi f(t-t_n)}\, df$$

$$\overline{q'(t)} = \sum_{n=1}^{N} p(t_n)\; a(t-t_n) \quad , \; t > t_n \tag{52}$$

The expression for $\overline{q'}(t)$ represents the function with which comparison to an experimental mean return is made. As previously mentioned, the transformation of p'(t) given by

$$\mathfrak{F}\left[\overline{p'(t)}\right] = \sum_{n=1}^{N} p(t_n)\, e^{-j2\pi f t_n} \tag{53}$$

is analogous to the beat note voltage approximation discussed in Section 4.3, and subsequently used in the experimental determination of an fm radar counter's expected frequency.

An additional point that should be noted is that the delta function approximation of not only the mean return pulse, but also the individual return pulses, can be thought of as impulse sampling, if the time interval between delta functions is properly selected. That is, one form of the sampling theorem

states that if a signal that is a magnitude-time function is sampled instantaneously at regular intervals and at a rate slightly higher than twice the highest significant signal frequency, then the samples contain all of the information of the original signal.[17] The maximum frequency component of the radar return can be determined by considering the extent of the radio-frequency spectrum for a particular condition of operation, and then an appropriate selection of Δt will allow reproduction of the original return pulses and the associated mean return pulse if sampling techniques are employed.

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[17] Black, H. S., Modulation Theory, D. Van Nostrund Co., New York, 1953.

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CHAPTER VI

APPLICATION OF AN FM RADAR ALTIMETER'S COUNTER-INDICATOR TO RADAR TERRAIN RETURN STUDIES

Previous discussion comparing pulse and fm radar return has assumed each system's range indicator as a "presentation-type" device, that is a pulse radar A-scope and an fm radar spectrum analyzer. In this chapter an approach to the study of radar-terrain return using an fm radar counter-indicator is suggested.

A major problem in the study of radar terrain return is the determination of an appropriate expression for the average scattering cross section per unit area, σ_o , of a particular ground target. In Chapter II, an expression for σ_o was assumed to permit comparison of pulse and fm radar altimeters without becoming involved in complex ground target properties, but at that time no indication was given as to the methods employed or difficulties encountered in determining an expression for σ_o . Studies performed have used theoretical expressions for the mean return pulse and those obtained from a photographic record of actual pulse radar terrain return to investigate the properties of σ_o .[18]

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The analogous fm radar study of σ_o, using an averaging cycle-rate counter for range information, is of a less complex nature, and, as would be expected, yields less information. However, the information obtained can be useful as a supplement to pulse radar investigation of σ_o, and is essential if an fm radar altimeter using a counter indicator is to be the source of a particular system's range information. The following discussion considers the use of fm radar to investigate σ_o.

The expected frequency, and consequently range indication, of an fm radar counter indicator, given by Equation 41, is shown below

$$\bar{f}_R = \left[\frac{\int_0^\infty f_R^2 \, \overline{B(f_R)} \, df_R}{\int_0^\infty \overline{B(f_R)} \, df_R} \right]^{\frac{1}{2}}$$

and since $\overline{B(f_R)}$ for a uniform scattering ground is, from Equation 21, given by

$$\overline{B(f_R)} = \frac{\lambda^2 W_t \, G^2(\theta) \, \sigma_o(\theta) \, \dot{F}^2}{8\pi^2 c^2 f_R^3} \quad , \quad \frac{2h\dot{F}}{c} < f_R < \frac{2r_{max}\dot{F}}{c}$$

$\overline{f}_R$ can be written as

$$\overline{f}_R = \left[\frac{\dfrac{\lambda^2 W_t \dot{F}^2}{8\pi^2 c^2} \displaystyle\int_{\frac{2h\dot{F}}{c}}^{\frac{2r_{max}\dot{F}}{c}} \frac{f_R^{\,2}\, G^2(\theta)\, \sigma_o(\theta)}{f_R^{\,3}}\, df_R}{\dfrac{\lambda^2 W_t \dot{F}^2}{8\pi^2 c^2} \displaystyle\int_{\frac{2h\dot{F}}{c}}^{\frac{2r_{max}\dot{F}}{c}} G^2(\theta)\, \sigma_o(\theta)\, df_R} \right]^{\frac{1}{2}} \qquad (54)$$

The expression $G^2(\theta)$ and $\sigma(\theta)$ can be written as function of f_R using the relationship

$$\theta = \cos^{-1}\left(\frac{h}{r}\right) = \cos^{-1}\left(\frac{2h\dot{F}}{c f_R}\right)$$

The expression for $\overline{f}_R$ then becomes

$$\overline{f}_R = \left[\frac{\displaystyle\int_{\frac{2h\dot{F}}{c}}^{\frac{2r_{max}\dot{F}}{c}} \frac{G^2(f_R)\, \sigma_o(f_R)\, df_R}{f_R}}{\displaystyle\int_{\frac{2h\dot{F}}{c}}^{\frac{2r_{max}\dot{F}}{c}} G^2(f_R)\, \sigma_o(f_R)\, df_R} \right]^{\frac{1}{2}} \qquad (55)$$

where the antenna gain, $G^2(f_R)$, is a known radar system parameter, and $\sigma_o(f_R)$ is the theoretical expression whose validity

will be examined by comparison with experimental results. If the expression for $\overline{f}_R$ is not readily integrable, approximation of $\overline{f}_R$ using discrete rather than continuous functions can be employed as in Section 4.3. The percent error resulting from such an approximation can be determined by considering the general shape of the curve of Figure 18, and determining the number of discrete components, N, required before the change in values of $\overline{f}_R$ becomes insignificant for increased N.

The solution of the expression for $\overline{f}_R$, whether exact or approximate, yields a theoretical expected frequency, and consequently expected altitude indication of an fm radar altimeter in level flight over the terrain described by $\sigma_o(f_R)$. Comparison of the theoretical $\overline{f}_R$ and the beat frequency actually measured by an fm radar's averaging cycle rate-counter is then an indication of the validity of the theoretical σ_o describing the ground target. Obviously, considerably more information about the ground's scattering properties is obtained when the range indication is a "scope-type" presentation. For example, the return signal variation with angle of incidence, the leading edge of the return, and the time duration (pulse radar) or frequency extent (fm radar) of the return signal are all factors which are shown on a pulse radar A-scope or an fm radar spectrum

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The solution of the expression for $\bar{f}_R$, whether exact or approximate, yields a theoretical expected frequency, and consequently expected altitude indication of an fm radar altimeter in level flight over the terrain described by $\sigma_r(\theta_r)$. Comparison of the theoretical $\bar{f}_R$ and the beat frequency actually measured by an fm radar's averaging cycle rate-counter is then an indication of the validity of the theoretical σ_r describing the ground target. Obviously, considerably more information about the ground's scattering properties is obtained when the range indication is a "scope-type" presentation. For example the return signal variation with angle of incidence, the leading edge of the return, and the time duration (pulse radar) or frequency extent (fm radar) of the return signal are all factors which are shown on a pulse radar A-scope or an fm radar spectrum

analyzer, but cannot be determined from a counter reading. However, if an fm altimeter counter is to be used as the source of altitude information for a particular system, the error that will exist due to scattering must be determined, and the above method is the least expensive and most practical way of experimentally investigating the error present. In addition, the simplicity and economy of conducting such an experiment might warrant its use, in a supplementary manner, in the pulse radar investigation of σ_o.

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CHAPTER VII

CONCLUSIONS

The expressions for the mean return signals of pulse and fm radar altimeters, obtained under similar operating conditions, have suggested a method of comparing the two systems from the standpoint of range presentations. That is, the relationship between the A-scope presentation of a return pulse and the spectrum analyzer presentation of an fm radar's return power spectrum was considered as a basis from which the two systems could be compared. Additional relationships between pulse and fm radar altimeters were obtained by considering alteration and Fourier transformation of the range presentations. As a result of these operations, gating the return pulse was considered analogous to filtering the fm return spectrum, the return pulse power spectrum was associated with the fm radar beat note voltage, and the pulse receiver bandwidth and the beat note voltage sampling time were considered as analogous functions. Figure 21 illustrates, in terms of general functions, the relationships discussed. Extension of these relationships to complete systems then indicated a more practical approach to a comparison of pulse and fm radar altimeters.

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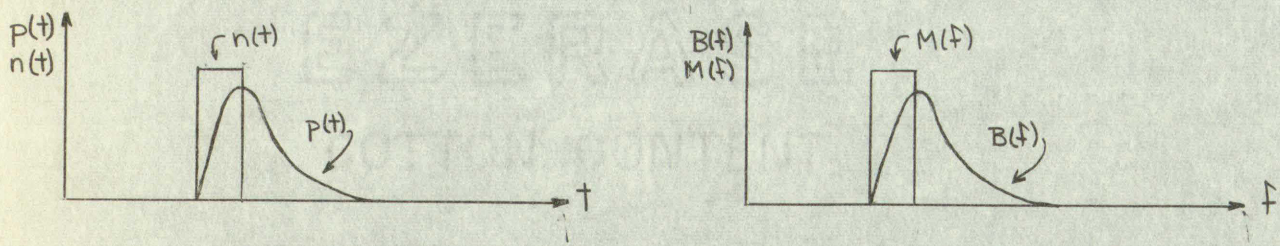
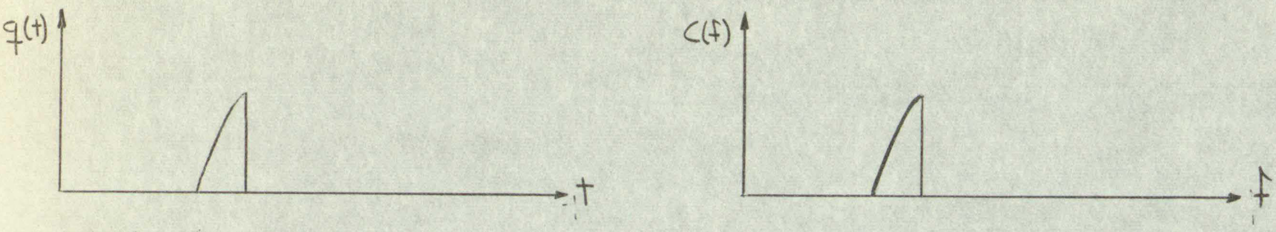
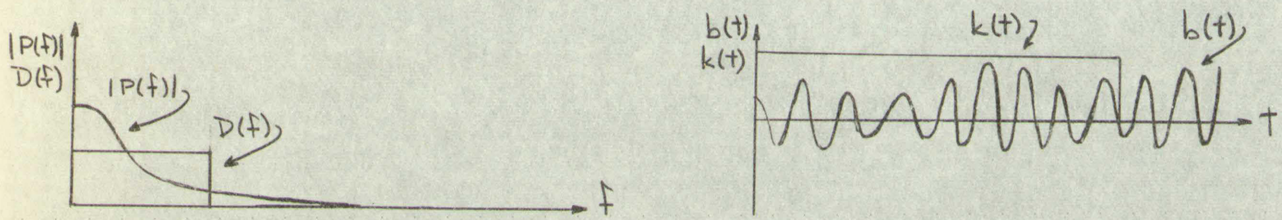
PULSE SYSTEM FM SYSTEM



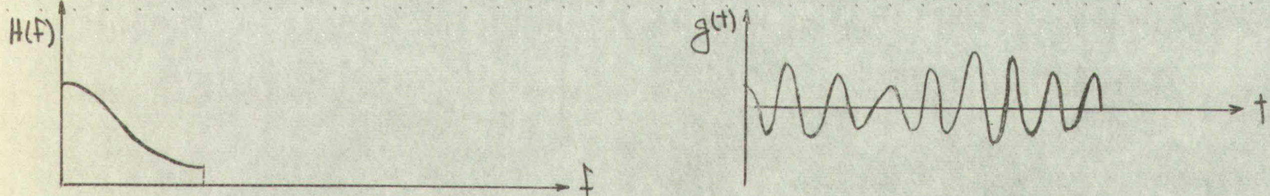
Return Pulse, p(t), Return Spectrum, B(f), and
Position of Gate, n(t), and Scanning Filter, M(f)



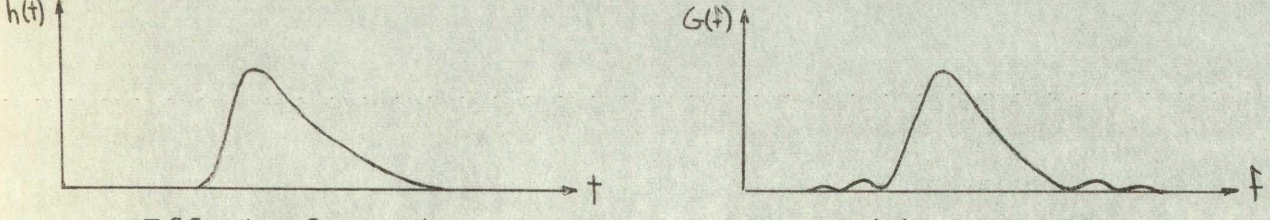
Pulse Through Gate, q(t), and Spectrum in Scanning Filter, C(f)



Pulse Return Power Spectrum, |P(f)|, and Receiver Band Pass Function, D(f); Beat Note Voltage, b(t), and Sampling Time, k(t)



Return Pulse Power Spectrum Through
Receiver, H(f), and Sampled Beat Note Voltage, g(t)



Effect of Receiver on Return Pulse, h(t), and Effect
of Finite Sampling Time on Return Spectrum, G(f)

Figure 21. Pulse-FM Radar Relationships

While considering the above pulse-fm radar relationships, it was convenient to consider either ideal or general terrain returns and system functions, although previous discussion had indicated the complex nature of radar terrain return. Therefore, the use of approximations appeared to be advantageous in a comparison of the two systems, provided the approximations were related and could be practically employed.

For fm radar return, a discrete spectrum approximation of the continuous return spectrum was suggested. The accuracy of the approximation was theoretically investigated by considering the altitude indication expected from an altimeter using a cycle-rate counter as a data-converting element. Experimental results verified the predicted altitude indication, and consequently, substantiated application of the approximation to fm radar return studies using a counter-type altimeter. In addition, it was noted that Fourier transformation of the discrete spectrum resulted in an expression for the voltage waveform examined in the previously mentioned experiment. This expression represented the approximation of the beat note voltage associated with the discrete spectrum.

The pulse radar approximation corresponding to the approximation of an fm radar's return spectrum was the representation of the return pulse by a series of weighted delta functions.

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The pulse radar approximation corresponding to the approximation of an fm radar's return spectrum was the representation of the return pulse by a series of weighted delta functions.

It was shown that if the weight of a delta function was obtained by averaging the magnitude of a large number of individual return pulses at a particular delay time from the leading edge of the pulse, then the ensemble of delta functions represented the experimental determination of the mean return pulse. In addition, it was noted that if the time interval between delta functions was properly selected, the approximation represented an adequate sampling of pulse terrain return. Fourier transformation of the ensemble of delta functions, analogous to the fm beat note voltage approximation, represented the power spectrum of the pulse approximation, and was seen to have application when the effect of the receiver transfer function was considered.

Investigation of fm radar terrain return with counter-type altimeters was then considered. The method suggested was similar to previous pulse radar studies in that the general approach was a comparison of theoretical and experimental radar return. More specifically, a comparison of the theoretical altitude indication of a cycle-rate counter, obtained in terms of the scattering cross section per unit area, and a counter altimeter's actual altitude indication would indicate the validity of the expression for scattering cross section. An

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investigation performed in this manner, in addition to being the most practical approach to analysis of counter-type fm radar altimeters, was also suggested as a possible supplement to pulse radar studies. Justification of this supplementary application was offered from the standpoint of simplicity and economy.

It should again be noted that this comparison of pulse and fm radar altimeters and terrain return has devoted considerably more attention to fm systems, since the major portion of previous radar return studies has dealt with pulse radar systems.

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