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A Qualitative Study on an Exploding Wire Fuse

W.D. LaCoss

A QUALITATIVE STUDY ON AN EXPLODING WIRE FUSE LaCOSS

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A QUALITATIVE STUDY ON AN EXPLODING WIRE FUSE

W. D. LaCoss

Submitted in partial fulfillment
of the requirements for the degree
of Master of Science in Electrical Engineering

The University of New Mexico

1961

This thesis, directed and approved by the candidate's committee, has been accepted by the Graduate Committee of the University of New Mexico in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Stuart A. Northrop
Dean

January 11, 1962
Date

Thesis committee

Arnold V. Koehmann
Chairman

W.W. Grannemann

E. L. Jordan

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ABSTRACT

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A QUALITATIVE STUDY ON AN EXPLODING WIRE FUSE

CHAPTER I -- INTRODUCTION

This paper concerns a study of an exploding wire fuse element in a circuit that is pulsed periodically. The associated circuit, in general, is as unique as the fuse: the circuit energy is stored initially in a module of capacitors, and the operation of the fuse is highly dependent not only upon the parameters of the fuse wire, but equally upon the geometry of the capacitor module and the parameters R, L, and C of the circuit.

There are a number of current experimental investigations being conducted that use the rapid discharge of electrical energy from a capacitor energy storage system. Probably the most outstanding of these deals with the control of a thermonuclear reaction. Some of these systems employ fuses with each capacitor; this is to prevent a hazardous explosion in the event one of the storage capacitors should fault by internally shorting which would allow the circuit energy to be dissipated in the shorted capacitor. The general practice, if the individual capacitors are not fused, is to limit the total number of capacitors in the circuit so that a fault would, at most, cause only minor damage in the immediate surroundings. Fuses are not widely used in these circuits for two major reasons:

1. There is very little known about the use of fuses in pulsed circuitry, and further, they are not commercially available.
2. Most experiments involving capacitor discharges require an extremely low-source impedance, and a fuse would introduce resistance and inductance to impede a desirable fast rise time of the initial current pulse.

In designing a fuse to operate in a circuit that is pulsed periodically, there are two distinct evaluations to be made. The first involves an experimental determination of the necessary parameters to provide a fusing action which would rapidly disrupt the fuse link and disconnect the fault element from the rest of the system. The second evaluation involves establishing the normal operating fuse criteria by determining the limiting conditions analogous to the current and voltage ratings that apply to power-line fuses.

This study was undertaken in an effort to develop basic concepts of normal operating fuse criteria that will apply to capacitor energy storage systems. To develop ideas germane to normal fuse operating limits, it was necessary to consider certain principles that preceded this development:

1. A review had to be made of some of the experimental work on the exploding wire phenomenon to indicate a possible application as a fuse.
2. A typical circuit requiring such a fuse had to be described schematically and analyzed to show the dependency between the circuit parameters and the necessary conditions for obtaining a fusing action.

CHAPTER II -- SOME EXPLODING WIRE CHARACTERISTICS

The electrically exploded wire has been studied for many years, both for possible applications, and as a scientific curiousity. The observable phenomena of an electrically exploded wire includes a brilliant flash of light, very high shock fronts, and the generation of extremely high temperatures. Basic instrumentation used to study exploding wires are current and voltage measuring devices. However, high-speed photography, including x-ray photography, and various types of magnetic and electrostatic probes - and to some extent spectroscopy - are also employed.

Although some theories have been proposed to explain the mechanisms that occur when a wire bursts, none are complete. Some parts of the process are still without any explanation. However, the poor understanding of wire explosions has not prevented their use in a variety of applications. The application under consideration in this report is the use of a wire as a protective fuse in a high-energy capacitor storage system.

The basic circuit required for the occurrence of a wire explosion consists of a charged capacitor which is allowed to discharge through a fine wire. Experimental observations have revealed that different mechanisms will occur by simply selecting different wire materials or by changing the circuit parameters (capacitance, inductance, resistance, and initial circuit energy). Yet to choose a set of conditions from the parameters mentioned, to determine a relationship of the wire's change in resistance, change in inductance, and change in temperature as functions of time, presents a formidable problem, capable at most of only a partial solution.

The specific concern of this study is an investigation of the factors involved for normal operating conditions that apply to a wire placed in a circuit to function as a protective fuse. The normal operating mode was chosen as a subject of study because this is the one area that has not been carefully considered in any previous exploding wire fuse applications[1,2]. However, since the fuse is intended to function as an exploding wire, certain physical limitations are implicitely imposed and need to be considered in order to define normal operating conditions.

A qualitative look at the behavior of a wire's resistance as a function of time, with constant current input, is illustrated in Figure 1. Figure 1a is representative of a low resistance material, e.g., Cu, Al, Ag or Au. Figure 1b is representative of a relatively high resistance material, e.g., Fe, W, Ni, or Cu-Ni alloys, et al. On each of the two curves where the slope changes rapidly, it has been shown that phase changes are occurring[3]. The first phase change is from a solid to a liquid, and the second phase is from a liquid to vapor. Since the curves are obtained using a constant current source, the absissa can also be labeled, "$\int i^2 dt$,"[*], which is proportional to time. The iq required to reach the first phase change, i.e., to reach the melting point of the wire, can be represented by the energy relationship

$$dQ = KmC_v dT = r(T)i^2 dt$$

or

$$\int_o^t i^2 dt = Km \int_{T_o}^{T} \frac{C_v dT}{r(T)}$$

Q = Heat in calories
M = Mass in grams
C_V = Specific heat
r = ohms
T = Temperature - °C
i = current

---

[1]Superscript numbers refer to references listed at the end of this report.

[*]The $\int i^2 dt$ is referred to in the literature on this subject as "action." It has the units of ampere-coulombs and will be designated in this report as "iq."

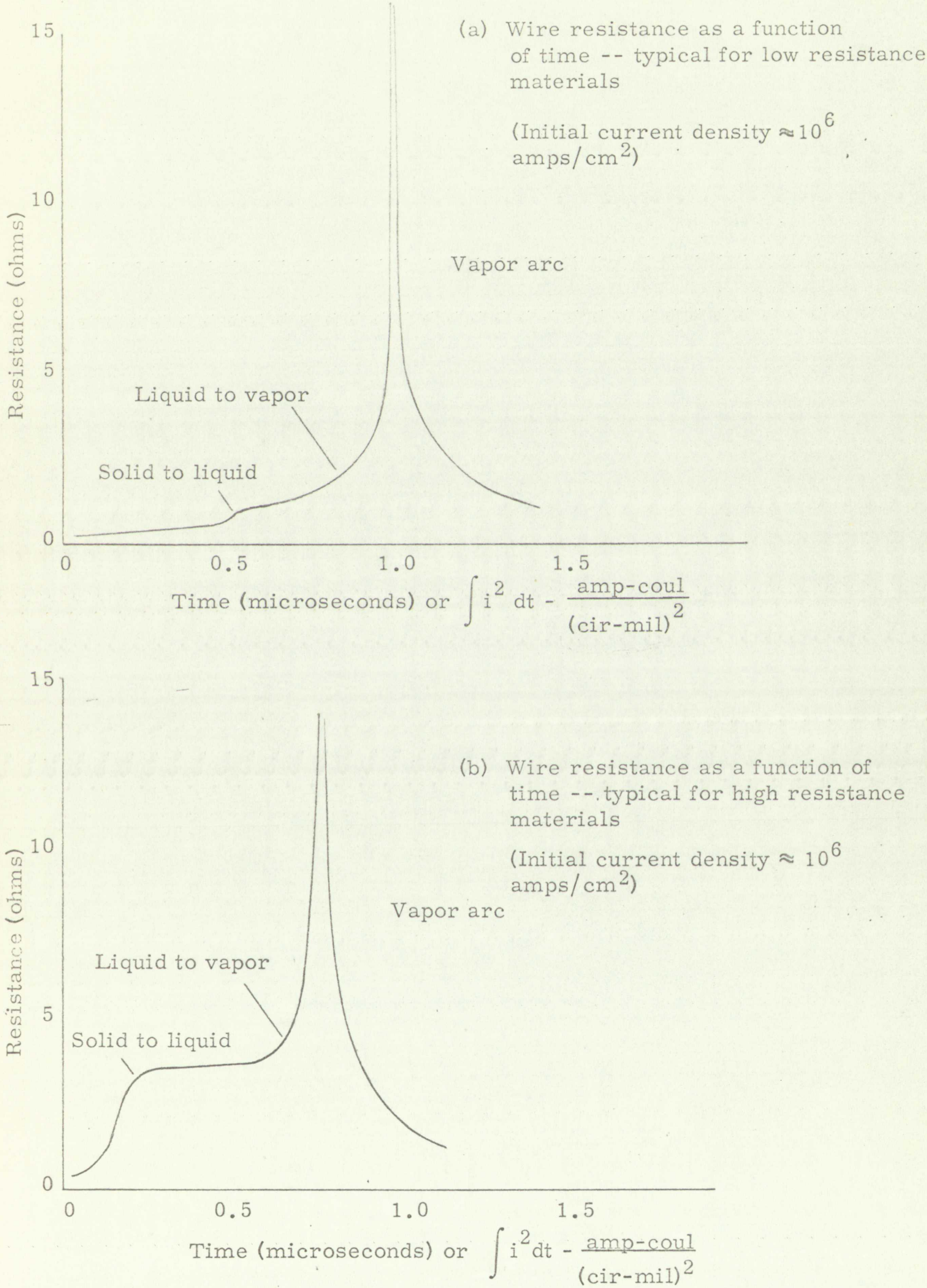

Figure 1. Wire Resistance as a Function of Time, for Constant Current Input

The iq necessary to produce a small temperature rise in the wire can be calculated easily using the above expression when the resistance is a linear function of the temperature and the specific heat for constant volume is nearly constant, or mean specific heat $\overline{C}$ can be used. Near the melting temperature, and into the vapor phase, both r and C_v become very non-linear and the integration impractical. However, experimentally definite values[3] of the iq-to-melt and iq-to-vapor have been established for several materials. These values have been determined using a constant current source, and have been shown to correspond to iq-to-melt and iq-to-vapor values for varying current. These values will apply in any case, provided no appreciable heat loss occurs during the time of the heat input to the wire.

A charged coaxial cable system provides a simple constant current energy source. While the constant current feature is desirable to simplify the analysis, the energy-storage limitation of a coaxial line will only allow very small wires to be exploded.

In order to study the phenomena of large exploding wires, a large high-voltage capacitor energy storage system is required. A typical exploding-wire test circuit is illustrated schematically in Figure 2. The number of capacitors, and the charging voltage, is determined by the required iq-to-vapor for the wire placed in the circuit. Current and voltage measurements will allow observation of the electrical behavior of the circuit during the discharge of the stored energy.

In Figure 3, typical current and voltage wave forms are shown to illustrate the extreme circuit behavior for special conditions.

Figure 3a represents a case where the total stored energy is just sufficient to vaporize the test wire. The total iq is obtained by determining the area under the square of the current waveform. This quantity represents the iq required to vaporize the test wire and has been shown[3] to correspond with the iq-to-vapor values obtained using a constant current source.

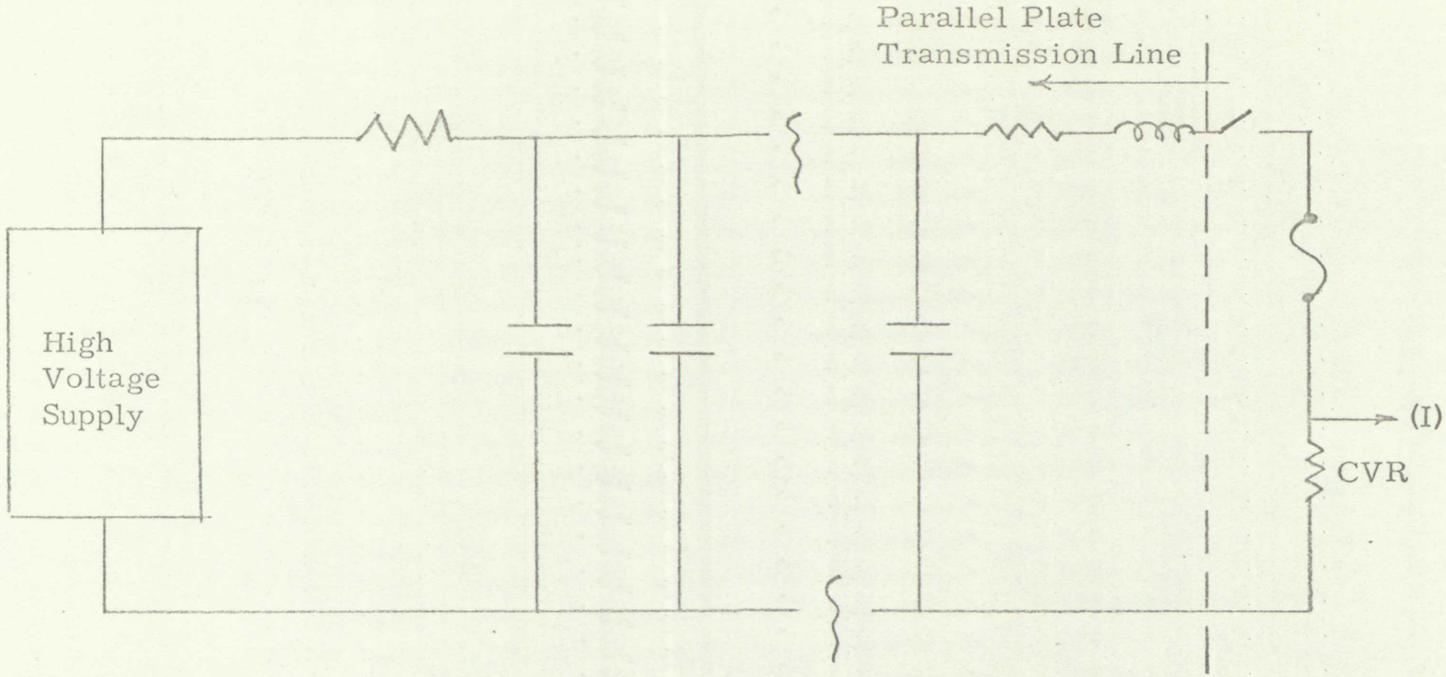

Figure 2. Exploding Wire Test Circuit

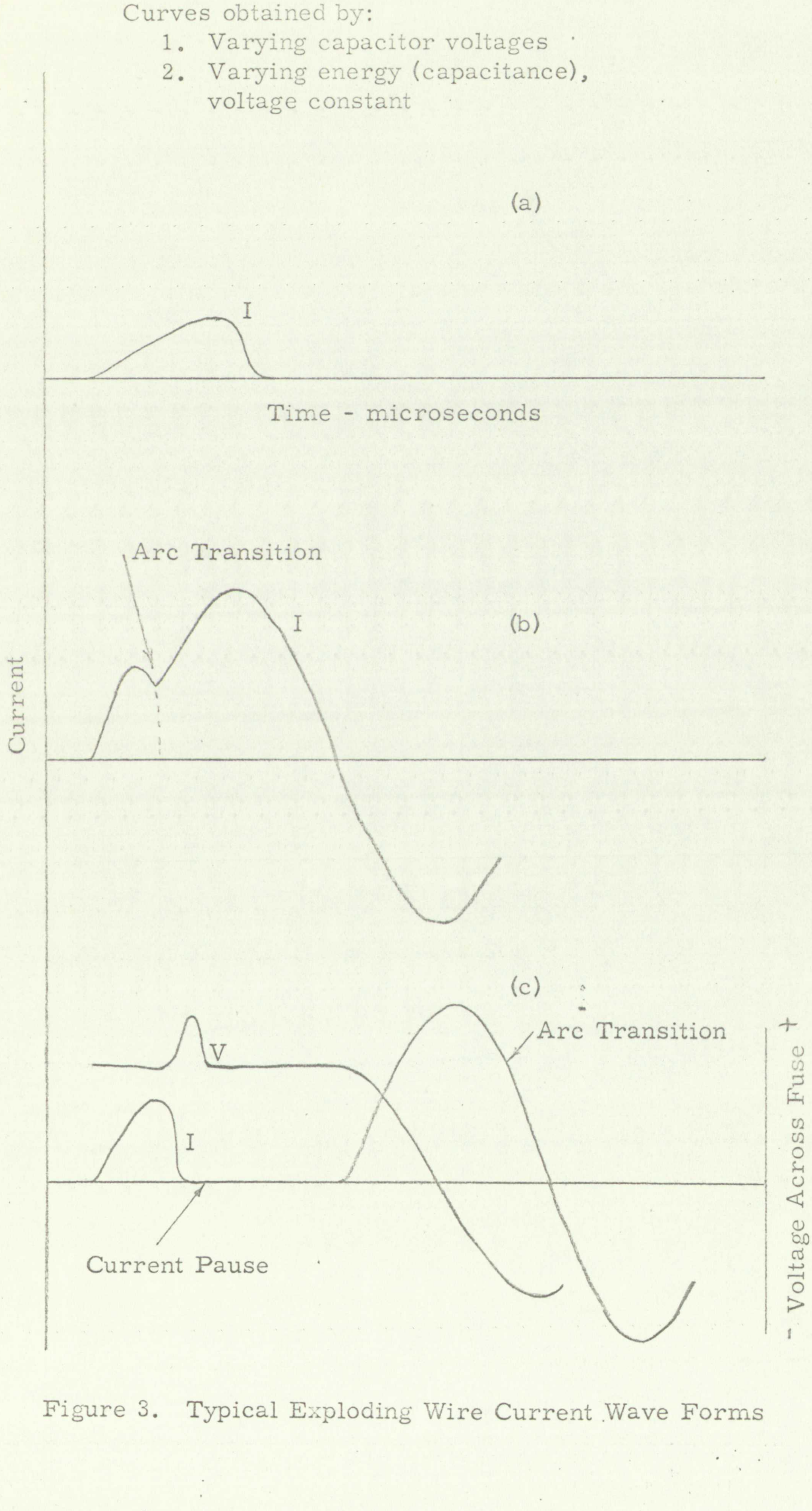

Figure 3. Typical Exploding Wire Current Wave Forms

If the capacitors in the test circuit are charged to a much higher voltage, and all other conditions remain the same as those that produced the waveform in (a), then the circuit current would be similar to that shown in Figure 3b. In this case, the wire is vaporized early during the time of the discharge, and only a small percent of the total stored energy is required. The current flow through the vapor of the exploding wire transfers to a low impedance arc discharge across the gap left by the vaporized wire. It continues to conduct until the remaining energy in the circuit is absorbed by the circuit resistance.

A third case will result if the capacitors are initially charged to a voltage intermediate to the two previous situations. This case is shown in Figure 3c. The wire is vaporized during the first current pulse. The loading effect of the vaporizing wire on the circuit causes the current to go toward zero and pause until an arc across the gap allows the discharge of the remaining energy in the circuit.

With reference to the current waveforms in Figure 3, the iq-to-vapor is the same in each case*. These characteristic waveforms were assumed to be obtained by changing the initial capacitor voltage. A similar set of curves can also be obtained by (1) varying the length and diameter of the test wire; or (2) varying the source resistance and inductance while maintaining a constant capacitor voltage.

---

*When the first current peak is reached, the wire has not yet expanded. If at this time, the current density in the wire is constant, then the iq-to-vapor in each case is approximately the same, provided this current density is less than 10^7 amps/cm^2. Current densities greater than 10^7 amps/cm^2 are difficult to achieve and create anomalous energy effects. It is suggested that the anomalous conditions are due, in part, to predominate skin effect and shock heating.

If the capacitors are charged to a higher voltage, and all other conditions remain the same as those that produced the waveforms in (a), then the circuit current would be similar to that shown in Figure 3b. In this case, the wire is vaporized early during the time of the discharge, and only a small percent of the total stored energy is required. The current flow through the vapor of the exploding wire transfers to a low impedance arc discharge across the gap left by the vaporized wire. It continues to conduct until the remaining energy in the circuit is absorbed by the circuit resistance.

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With reference to the current waveforms in Figure 3, the $\int i^2 dt$ to vapor is the same in each case.* These characteristic waveforms were assumed to be obtained by changing the initial capacitor voltage. A similar set of curves can also be obtained by (1) varying the length and diameter of the test wire, or (2) varying the source resistance and inductance while maintaining a constant capacitor voltage.

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CHAPTER III -- FUSING ACTION AND CAPACITOR BANK COMPATIBILITY

A circuit typical of many capacitor energy storage systems is illustrated schematically in Figure 4. This module, except for the addition of individual fuses to each capacitor, is similar to an exploding wire test circuit. The storage capacitors are connected by parallel plate transmission lines which indicate close coupling. The purpose of the fuses is to electrically disconnect any capacitor that becomes faulty by shorting. In this example, if a capacitor fails by shorting and after the bank is charged, and before the load switch(es) close(s), the load is then just the fuse wire associated with the faulty capacitor. Since a shorted capacitor essentially connects one end of its fuse to ground, the energy in the remaining capacitors will discharge through this fuse.

The requirements, then, for fusing action to occur, are that the short circuit iq be sufficient to cause vaporization of the fuse wire while the current contributed from each individual remaining capacitors must have an iq that is less than an amount required to melt. It is important to know the ratio of the melt-to-vapor* iq. A compilation of these constants, which were determined experimentally, can be found in the literature[2] for various metals.

The melt-to-vapor ratios for the low-resistance materials mentioned previously, were found to be on the order of 1 : 2, while the melt-to-vapor ratios of the higher resistance materials were found to be on the order of 1 : 4. At present, the only significant fact that can be concluded from the melt-to-vapor ratio is that a minimum number of

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*The iq-to-vapor is not considered to correspond to the liquid-vapor phase. It is the iq for near-complete vaporization and is shown in Figure 1 as the vapor-to-arc transition.

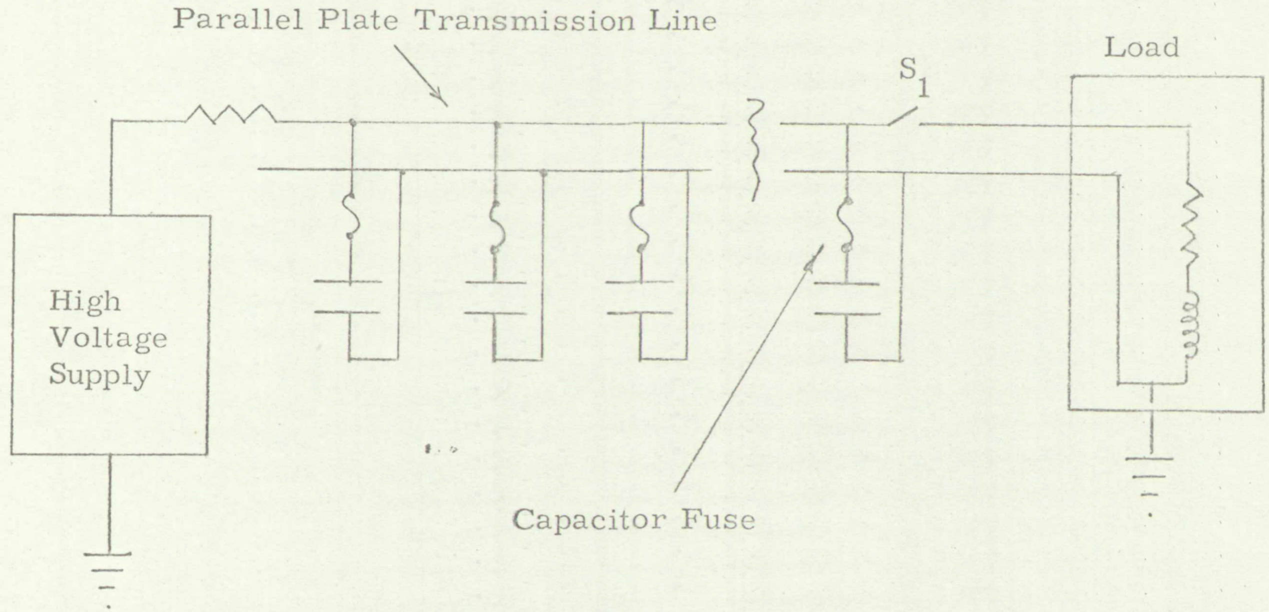

Figure 4. Capacitor Module Circuit Schematic

capacitors are required in the module for fusing action to occur without melting the fuses on the non-fault capacitor. In addition, it is observed that this minimum will differ as the melt-to-vapor ratio differs for various materials.

The important wire parameter that must be specifically determined from the standpoint of fusing action for any selected wire material and for a specific module of capacitors, is the "length" of the fuse wire. With reference to Figure 3, three distinct current waveforms are shown which characterize the extreme possible circuit behavior that can be obtained in an exploding wire circuit. The concept of an exploding wire fuse can be understood by analyzing a hypothetical capacitor module capable of the extreme behavior that has been discussed.

First, a capacitor module must be assumed where the energy initially stored is just sufficient to vaporize the fuse wire when its capacitor suddenly develops an internal short circuit (see Figure 3 a). The majority of this energy is absorbed by the exploding wire if the resistance of the assumed module is negligibly small compared to the effective resistance of the exploding wire fuse. However, some energy will be absorbed by the contact resistance or possible arc within the faulty capacitor. The fractional amount of energy absorbed within the capacitor becomes intolerable when it is sufficient to cause the capacitor case to break open, or the capacitor itself to explode with a devastating force.

To assume that the effective resistance of the exploding fuse is very small compared to the resistance of the internal capacitor fault, is tantamount to assuming that the fuse is infinitely short. This extreme condition determines the maximum number of capacitors that can be close-coupled, without protective fuses, and without an internal capacitor fault resulting in external damage due to the energy from the remaining capacitors in the module. This necessarily implies a low-energy storage system.

If it is next assumed that the infinitely short wires are replaced in such a way that the resistance is equal to the presumed resistance of an internal capacitor short, then the energy of the hypothetical module could be double that just previously assumed before external damage occurs. For any further increase in the length of the wire, it follows that the wire will absorb the greatest share of the available energy. If the energy of the module is further increased, the wire length must also be increased to maintain the circuit behavior shown in Figure 3a, where the energy available is just sufficient to vaporize the wire. (It is implied that this wire will absorb a constant amount of energy per unit length.)

The analysis can be continued by further increasing the module energy and correspondingly increasing wire length until a condition is reached when the supposedly negligible amount of energy absorbed by the capacitor is again sufficient to cause it to break or fly apart.

An important aspect of the previous analysis was omitted which would preclude an experimental duplication to the extent hypothesized. At first, a small module was assumed indicating some minimum energy requirements. As the wire length was increased, so was the system energy increased to maintain the specified current pulse. When the energy in the system is on the order of twenty thousand joules, it still represents a relatively small capacitor bank. An example of such a bank would be seven 15 μf capacitors charged to 20k volts to store 21k joules. However, a 20k joule wire explosion would not be very practical in a protective fuse application. An electrical explosion of this magnitude is violent and can be quite destructive to the immediate surroundings.

The foregoing analysis was based on the assumption that in the event of a capacitor fault, practically all of the energy in the module would be absorbed by the exploding wire fuse. In contrast to this assumption, circuit behavior illustrated by Figure 3b indicates that practically all of the module energy would be absorbed by the fault resistance of the capacitor. This is due to the formation of a low-impedance arc

in place of the bursting wire which allows the circuit current to ring out.

For fuse action like that described, it is evident that adequate protection cannot be obtained when the energy in the system is sufficient to break open a capacitor. Given specific circuit parameters - voltage, capacity, and fuse-wire length - which are capable of producing a fuse action like that shown in Figure 3a , the fusing action shown in Figure 3b is obtained if a much shorter fuse wire is used, and the circuit voltage and capacity remain the same. In the latter case, the energy in the circuit is in excess of that required to fully vaporize the wire. In addition, the gap left by the exploding wire does not have sufficient spacing to hold off the remaining circuit voltage.

Fusing action can be obtained as illustrated in Figure 3c if the fuse electrode spacing is great enough to prevent an immediate vapor-to-arc conduction transition after the wire bursts. In this case, the energy in the circuit is much greater than that required to vaporize the fuse wire. However, since the fuse resistance is tending to become infinite, the current is tending toward zero, which thereby brings about a momentary current pause.

At present, it can be said that the energy dissipated during the first current pulse, in this case, can be made no greater, having a high energy system, than the energy dissipated during the current pulse shown in Figure 3a, which has a low energy system. The controlling parameters are the initial capacitor voltage and the length of the fuse which are the same for both types of fuse action. The capacitor energy is greater in one case with the module having a greater number of capacitors. Unfortunately, the current pause indicated by Figure 3c is a phenomenon that exists momentarily. The current pause is followed by a discharge due to a subsequent arc formation between the fuse electrodes. However, it has been demonstrated[1,2] experimentally that within limitations, the occurrance of a restrike can be prevented.

It is necessary to prevent a current restrike in an exploding wire fuse application since the restrike would allow the discharge of all of the energy in the capacitor module. This condition can be achieved providing the wire length and voltage dependence can be experimentally determined.

The foregoing arguments indicate qualitatively that the length of a wire in a fuse application is to be considered dependent upon the conditions that prevail only during a fault. No attempt was made to ascertain theoretically, or from the literature, the dependency of other wire parameters, such as diameter and material, on the circuit behavior during a fault. This neglect was intentional and is believed to be justified since it can be shown that these parameters must be determined by considering the system in absence of a fault where the module is discharging into its proper load.

A semi-quantitative analysis on the circuit shown in Figure 4 can be performed to indicate the influence of normal circuit operation on determining the parameters of the capacitor fuse. The circuit is basically composed of capacitance, inductance, and resistance.

For purposes of the analysis, it will be assumed that the inductance and resistance of the load are linear and very much greater than the R and L of the fuses (in the suggested parallel combination). This assumption implies that the impedance due to the fuses will have a negligible effect in determining the resonant frequency of the circuit.

An equation for the circuit can be written:

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0,$$

where

R = Load resistance,

L = Load inductance, and

C = Total capacitance of module.

[illegible] energy [illegible]. [illegible] can be achieved providing [illegible] voltage dependence can be experimentally determined.

The preceding [illegible] has indicated only that [illegible] its application [illegible] dependent upon the conditions [illegible]. No attempt was made to ascertain theoretically or from the literature, the dependency of other wire parameters, such as diameter and material, on the current existing during a fault. This neglect was intentional and is believed to be justified since it can be shown that these parameters must be determined by considering the system in presence of a fault where the module is discharging into the proper load.

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An equation for the circuit can be written:

[illegible]

where

[illegible]

The solutions of the differential equation for the critical and over-damped cases could be obtained; however, they are of no interest. The only case of interest is when $R < 2\sqrt{\frac{L}{C}}$, where $\int i^2 dt$ is maximum. Then, for initial conditions, when $t = 0$, $I_o = 0$, and $V_o = \frac{q_o}{C}$,

by transformation,

$$RI(S) + SLI(S) + \frac{1}{C}\left(\frac{I(S)}{S} - \frac{q_o}{S}\right) = 0,$$

$$I(S) = \frac{V_o}{L}\left[\frac{1}{\left(S + \frac{R}{2L}\right)^2 + \left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)}\right],$$

$$I(S) = \frac{V_o}{L}\left[\frac{1}{(S + \alpha)^2 + \beta^2}\right], \text{ and}$$

$$i(t) = \frac{V_o}{L}\left[\frac{1}{\beta}\,\epsilon^{-\alpha t}\sin\beta t\right].$$

Where

$$\beta = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}, \text{ and}$$

$$\alpha = \frac{R}{2L},$$

the iq of the circuit is then

$$\int i^2 dt = \int_o^\infty \left[\frac{V_o}{L\beta}\,\epsilon^{-\alpha t}\sin\beta t\right]^2 dt, \text{ and}$$

$$iq = \frac{C\,V_o^2}{2R}.$$

The resulting expression shows that the circuit iq is a function of both the initial capacitor bank energy, $E_o = \frac{1}{2} C V_o^2$, and the load resistance. With all capacitors connected in a close coupled arrangement, the total circuit current can be taken as the sum of the currents from each capacitor, i.e.,

$$i_t = i_1 + i_2 + --- + i_n.$$

Where

$$\int i_t^2 \, dt = \int (i_1 + i_2 + --- + i_n)^2 \, dt$$

for

$$i_1 = i_2 = --- = i_n, \text{ then}$$

$$\int i_1^2 \, dt = \frac{1}{n^2} \int i_t^2 \, dt.$$

The last expression indicates the relationship of the total circuit iq to the iq on a per-capacitor basis. When the capacitors are discharged, the current through a fuse will cause it to heat. The fuse temperature rise can be calculated by this expression:

$$\int_o^\infty i_1^2 \, dt = \frac{K\overline{C}m}{r_o} \int_{T_o}^{T_2} \frac{d\tau}{1 + \alpha(\tau - T_o)},$$

where

$$r = r_o + r_o \alpha (\tau - T_o) \text{ and,}$$

$$\int_o^\infty i_1^2 dt = \frac{1}{n^2} \int_o^\infty i_t^2 dt, \text{ with}$$

K = Proportionality constant-joule per calorie,

r_o = fuse resistance at T_o,

$\overline{C}$ = mean heat capacity,

M = mass, and

α = temperature coefficient of resistance.

Since the load resistance remains constant, the circuit iq will not be influenced by a relatively small change in resistance of the fuses. Therefore, a means of determining the individual iq requires only that the initial energy $\left(\frac{1}{2} CV^2\right)$ be known and the load resistance, "R". (The latter can be treated as a parameter for any specific capacitor bank and allowed to vary from some minimum to a maximum resistance.)

On the right side of the expression for the fuse temperatures rise, $\frac{m}{r}$, are the parameters of the fuse wire. Since $m = \rho L a$ and $r = \sigma L/a$, where

ρ = density of fuse material ,

L = length,

a = area, and

σ = resistivity,

then

$$\frac{m}{r} = \frac{\rho a^2}{\sigma} .$$

The result indicates that the iq is independent of the length of the fuse wire but depends upon the area to the second power.

On integrating the expression,

$$\int_0^{\infty} i^2 dt = \frac{K\overline{C}\rho a^2}{\sigma} \int_{T_o}^{T} \frac{d\tau}{1 + \alpha(\tau - T_o)} ,$$

$$iq = \frac{K\overline{C}\rho a^2}{\sigma} \left[\frac{\log 1 + \alpha(\tau - T_o)}{\alpha}\right]_{\tau = T_o}^{\tau = T} ,$$

$$iq = \frac{K\overline{C}\rho a^2}{\sigma\alpha}\left[\log 1 + \alpha(T - T_o)\right] ,$$

then it follows that

$$(T - T_o) = \frac{EXP\left(\frac{\alpha\sigma iq}{K\overline{C}\rho a^2}\right) - 1}{\alpha}.$$

(In calculating the temperature rise it is assumed that there is no appreciable heat loss during the current discharge.)

For normal operation, the temperature rise that the fuse will undergo must not exceed the melting point of the fuse material. Therefore, minimizing the temperature rise for a maximum iq would require an optimum selection of wire parameters shown in the exponent of the temperature equation. Metals considered to be good electrical conductors have densities and heat capacities that do not differ greatly; however, their temperature coefficient of resistance and resistivity do.

It can be determined, then, that relatively low-resistance materials will provide the optimum parameters for minimum temperature rise with maximum iq. This conclusion could be obtained more readily by reexamining Figure 1. The iq-to-melt is shown to be larger for the low-resistance materials which have comparable diameters. Of these, copper is preferred both because of its resistivity properties and its economy.

The iq-to-melt as a function of wire area is illustrated for copper in Figure 5. This curve was calculated using the constant, iq-to-melt = $0.030 \frac{\text{amp-coul}}{(\text{cir-mil})^2}$, as reported in the literature[2]. This constant was experimentally determined.

The relationship between capacitor iq, (which is a function of the capacitor energy and equivalent load resistance) and the fuse wire diameter has now been established. It should be pointed out that the derived equation for "temperature increase" cannot be used with accuracy in plotting the curve in Figure 5. However, for copper, this equation is quite accurate for temperatures as high as 600°C since α and $\overline{C}$ do not vary significantly.

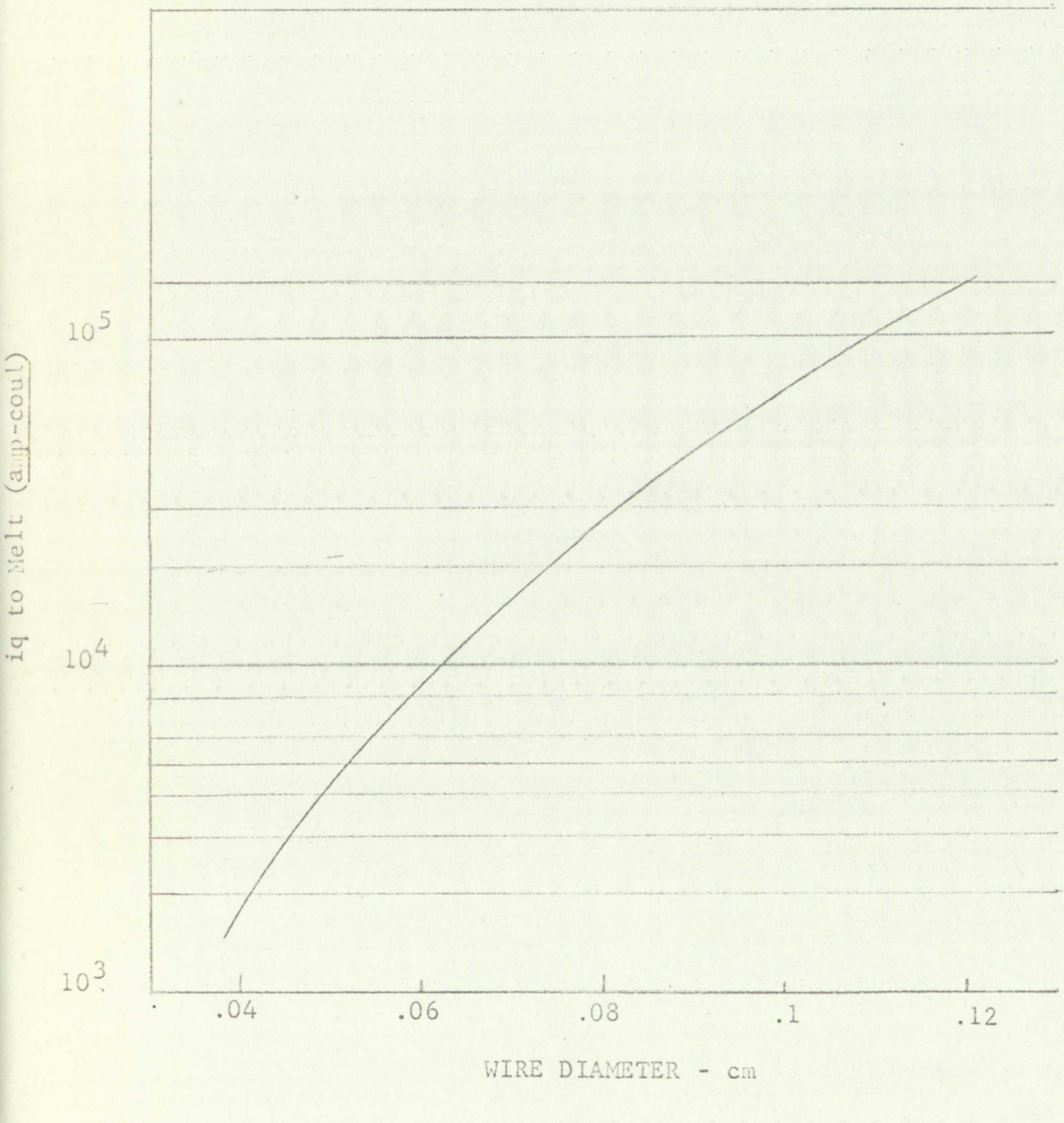


Figure 5. Copper Wire iq-to-Melt as a Function of Wire Diameter

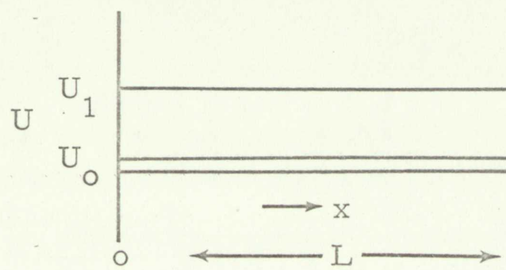
CHAPTER IV -- METHODS OF HEAT DISSIPATION FOR A FUSE DURING NORMAL OPERATION

The conditions have now been established which set an upper bound on the energy transfer in a capacitor energy storage system using protective fuses. Establishment of an upper bound is a necessary condition, but it is not sufficient when the occurrance of charging and discharging the capacitors is to be repeated periodically. The energy absorbed by the fuse in the form of heat must be dissipated prior to subsequent pulses. Since resistance increases with temperature, if the fuse is not allowed sufficient time to cool, the next current discharge may produce a temperature rise in excess of the upper bound.

To establish an upper bound for the repetition rate, complete knowledge of the mode of heat dissipation from the fuse wire is required. Considering the three principal means of heat transfer - conduction radiation and convection - it is forbiddingly difficult to account for all three simultaneously. Consequently, each method of heat transfer will be examined individually, hopefully to find the outstanding contributor in special instances.

Heat Transfer by Conduction

Heat flow by conduction will be considered by assuming the following model:



The length of the fuse wire is L. The temperature at the ends of the wire, o^- and L^+, are assumed zero. The wire is insulated so that no heat loss can occur except by conduction through the ends. The initial condition, established by current flow through this conductor, is assumed to be a function of the distance along the wire. In this case, $f(x) = U$, is a constant. That is to say, the temperature rise is instantaneous with no losses occuring during the heat input.

In practice, this is a realizable condition since the electrical discharge occurs in a matter of tenths of milliseconds. The complete solution of the one-dimensional heat-flow equation will yield the temperature-time distribution as a function of distance along the length of the wire. It follows then, that

$$U_t = k^2 U_{xx},$$

where

U_t = Partial derivative of temperature with respect to time,

U_{xx} = second partial derivative of temperature with respect to distance, and

k^2 = diffusivity.

The boundary conditions and initial conditions are:

$$U(o, t) = U(L, t) = 0 \text{ for } t \geq o \text{ and}$$
$$U(x, o) = f(x) = U_1, \ o<x<L.$$

Then

$$U(x, t) = U \text{ (transient)} + U \text{ (steady state)},$$

$$\text{but } U \text{ (steady state)} = 0.$$

A solution, by separation of variables, is obtained by assuming the product solution

$$U(x, t) = X(x)\ T(t),$$

$$U_t = XT', \text{ and}$$

$$U_{xx} = X''T.$$

Then

$$XT' = k^2\ X''T \text{ (dividing by XT)}$$

$$\frac{T'}{T} = k^2\ \frac{X''}{X}.$$

The variables X and T are separated in that the left side is a function of T alone and the right side is a function of X only. Each side must independently be equal to the same constant. Neither zero or a positive constant satisfies the boundary condition; therefore, a negative constant is used:

$$\frac{T'}{k^2T} = -\beta^2$$

$$T' + k^2\beta^2T = 0$$

$$T(t) = \epsilon^{-k^2\beta^2 t}$$

$$\frac{X''}{X} = -\beta^2$$

$$X'' + \beta^2 X = 0$$

$$X(x) = c \sin \beta X$$

$$X(0) = X(L) = 0 = \sin \beta X$$

$$\beta L = n\pi$$

$$\beta = \frac{n\pi}{L}$$

Then for every integral value of n, another value of β and a different solution is obtained:

$$X(x) = C_n \sin \frac{n\pi x}{L}.$$

Hence the solution that satisfies the boundary condition is

$$U(x, t) = \sum_{n=1}^{\infty} C_n \sin \frac{n\pi}{L} x \, \epsilon^{\frac{-n^2 \pi^2 k^2}{L^2} t}$$

at

$$t = o, \quad U(x, o) = f(x).$$

$$f(x) = \sum_{1}^{\infty} C_n \sin \frac{n\pi}{L} x.$$

The c_n's are seen to be the sine Fourier coefficients of f(x).

For $f(x) = U_1$ a constant,

$$C_n = \frac{2}{L} \int_o^L U_1 \sin \frac{n\pi}{L} x \, dx$$

$$= \frac{2U_1}{L} \left(\frac{L}{n\pi}\right) \left(- \cos \frac{n\pi x}{L}\right)\Bigg|_o^L$$

$$= \frac{2}{L} U_1 \left(\frac{L}{n\pi}\right) (1 - \cos n\pi).$$

On substituting the Fourier coefficients,

$$U(x, t) = \sum_{n=1}^{\infty} \frac{2U_1}{n\pi} \left[1 - \cos n\pi\right] \sin \frac{n\pi}{L} x \quad \epsilon^{-k^2 \left[\frac{n\pi}{L}\right]^2 t},$$

and the final expression is

$$U(x, t) = \frac{4U_1}{\pi} \sum_{1}^{\infty} \frac{1}{2n-1} \sin \frac{(2n-1)\pi}{L} x \, \epsilon^{-k^2 \left[\frac{(2n-1)\pi}{L}\right]^2 t}.$$

Using the resulting expression, the temperature can be obtained as a function of time along the wire where cooling takes place the slowest, namely at $\frac{L}{2}$; then letting

$$\frac{k^2 \pi^2}{L^2} = \frac{1}{\lambda},$$

$$U\left(\frac{L}{2}, t\right) = \frac{4U_1}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin \frac{(2n-1)\pi}{2L} L \, \epsilon^{\frac{-(2N-1)^2}{\lambda} t}$$

$$= \frac{4U_1}{\pi} \sum_{n=1}^{\infty} \frac{-1^{(n+1)}}{2n-1} \epsilon^{\frac{-(2n-1)^2}{\lambda}}, \text{ and}$$

$$U\left(\frac{L}{2}, t\right) = \frac{4U_1}{\pi} \left[\epsilon^{-\frac{t}{\lambda}} - \frac{1}{3} \epsilon^{-\frac{9t}{\lambda}} + \frac{1}{5} \epsilon^{-\frac{25t}{\lambda}} - - - - \right].$$

In order to check the solution, let t = 0 and

$$U\left(\frac{L}{2}, 0\right) = \frac{4U_1}{\pi} \left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + - - - - \right].$$

The infinite sum [4] of the series:

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} - - - - - = \frac{\pi}{4},$$

then

$$U\left(\frac{L}{2}, 0\right) = \frac{4U_1}{\pi} \left(\frac{\pi}{4}\right) = U_1,$$

which correctly is the initial condition at t = 0

A graphical representation of the temperature at $\frac{L}{2}$ is more descriptive than the series expression, but one other characteristic of this expression must be examined first. A particular value of time must be assumed so that the temperature at $\frac{L}{2}$ has nearly reached the steady

state, $U \approx 0$. A finite time will be used which is equivalent to a five time constant decay for the first exponential term in the series:

$$U\left(\frac{L}{2}, 5\lambda\right) = \frac{4U_1}{\pi}\left[\epsilon^{-5} - \frac{1}{3}\ \epsilon^{-45} + - - - -\right].$$

Considering only the first term of the series (since $\epsilon^{-5} >> \epsilon^{-45}$), it can be said that the temperature will be 0.67 percent of the initial condition.

Since $t = 5\lambda$,

then $L^2 = \frac{k^2\pi^2 t}{5}$, represents the time, as a function of length "L", for the center of the wire to cool from the initial condition to 0.67 percent of this initial condition. Using constants for copper,

$$k^2 = \frac{K}{\overline{C}\rho},$$

with

K = thermal conductivity - 0.92 cal/cm-sec °C,
$\overline{C}$ = specific heat - 0.093 cal/gm °C,
ρ = density - 8.9 gm/cm^3,

it follows then that $t = (.454)\ L^2$.

Figure 6 represents a graph of this expression. It illustrates, for any length of wire over the range shown, and given the initial condition of an instantaneous uniform temperature rise, that if the only heat loss is due to conduction, then the time involved for the temperature to return to approximately ambient (5 time constants), is related to the square of the length of wire.

Exploding-wire fuses for high-voltage capacitors may vary from 20 cm to greater than 35 cm in length. The minimum length is determined by the voltage stand-off capability during a capacitor fault. The

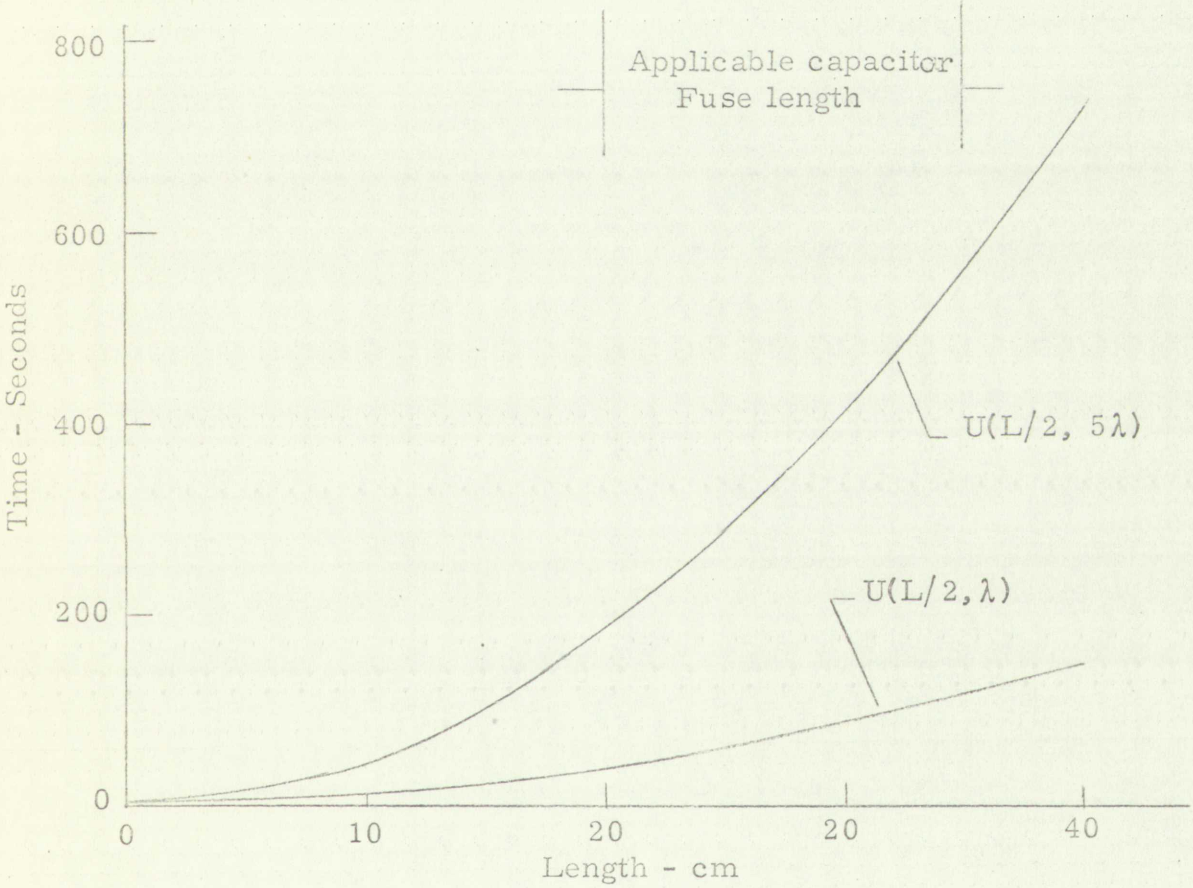


Figure 6. Time-vs-Length on Cooling of Fuse to Ambient

maximum length is only limited by practical considerations. As an example, a fuse 30 cm long would require 408 seconds, or approximately 6.8 minutes, to cool down to essentially ambient before it could be pulsed a second time.

For later comparison, the temperature-time history at the center of a 30 cm fuse wire is shown in Figure 7. The initial temperature was taken to be 400°C, and ambient 0°C. Before exploring any further aspects of heat loss by conduction, heat loss due to radiation will be considered.

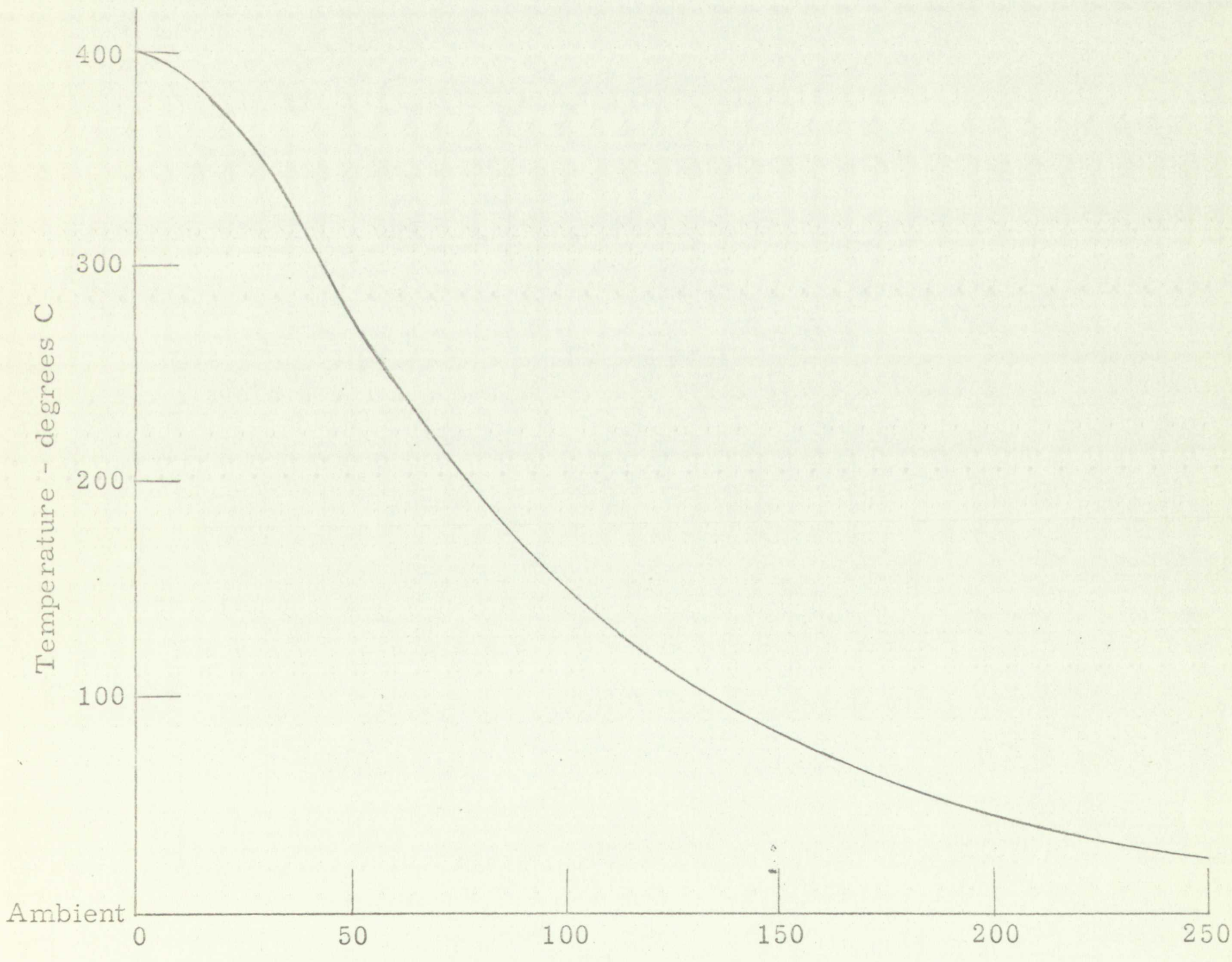


Figure 7. Heat Transfer by Conduction; Temperature as a Function of Time; Using Copper Wire, 30 cm Length; U(L/2, t)

Heat Transfer by Radiation

Radiation was defined by Maxwell as the transfer of heat from a hot body to a cooler body without appreciable heating of the intervening space. It is now known that thermal radiation is a means of transferring energy by electromagnetic waves; in nature, it is similar to light.

The Stefan-Boltzmann law pertaining to total radiation from a black body states that the total rate of heat radiation is proportional to the absolute temperature to the fourth power; stated as an equation, $E = \sigma T^4$, where the black body is defined as the maximum emitter for all wave lengths and σ is a constant value called Stefan's constant, which is

$$5.735 \times 10^{-5} \ \mathrm{erg/cm^2/sec/degree^4}.$$

The equation, $E = \sigma T^4$, refers specifically to emission and not to the net loss.

If the black body is surrounded by a black surface at temperature T_a, the gain from the surroundings is σT_a^4, and the net loss of energy per sq cm per sec. is

$$E_{net} = \sigma (T^4 - T_a^4).$$

The emissivity of a black-body is taken as unity. Any other surface will then be a perfect imitator of a black body, and emit P percent as much energy in all wave lengths as would a black body at the same temperature. Other surfaces are usually called grey bodies. Their emissivity, denoted as "ϵ," is always less than unity. The expression finally arrived at is then

$$E = \epsilon \sigma (T^4 - T_a^4),$$

where

E = heat radiation in ergs/sec/sq cm,

ϵ = total emissivity,

σ = Stefan's constant erg/sq cm/sec/degree^4, and

T = temperature - absolute.

In order to derive a temperature-time relationship of heat loss due to radiation for the pulsed fuse, the following assumptions will be applied: (1) the losses are due only to radiation, (2) the initial temperatures "T" along the wire is uniform, and (3) the temperature of the surfoundings is T_a. The rate of heat loss per unit surface area could be expressed:

$$\frac{\partial Q}{\partial t} = -\epsilon\sigma\left(T^4 - T_a{}^4\right).$$

The heat capacity of the fuse is

$$C_v = \frac{1}{m}\frac{\partial Q}{\partial t},$$

and for constant volume, total differentials will replace the partial derivities. By substitution, dQ is eliminated. On separating variables, the equation simplifies to the following:

$$\frac{dT}{T^4 - T_a{}^4} = K_1 dt,$$

where

$$K_1 = \frac{-\epsilon\sigma}{mC} \quad \text{(for unit surface area).}$$

To simplify K_1, it will be put in terms of the wire parameters. For a cylindrical wire, the ratio of the area to mass $\frac{A}{m}$ is,

Surface area $= \pi DL$; and

Mass $= \dfrac{\pi D^2 L \rho}{4}$,

where ρ = density,

L = length of wire, and

D = diameter.

It follows that

$$\frac{A}{m} = \frac{4\rho}{D},$$

and

$$K_1 = -\frac{4\epsilon\sigma}{\rho D\overline{C}}.$$

If each side of the equation is integrated, the following equations are derived:

$$\int \frac{dT}{T^4 - T_a^4} = \int K_1 dt, \text{ and}$$

$$K_1 \int dt = K_1 t + c = \int \frac{dT}{T^4 - T_a^4}.$$

To obtain a solution of the expression on the right, a partial fraction expansion is used:

$$\frac{1}{T^4 - T_a^4} = \frac{1}{\left(T^2 + T_a^2\right)(T + T_a)(T - T_a)}$$

$$= \frac{AT + B}{T^2 + T_a^2} + \frac{C}{T + T_a} + \frac{D}{T - T_a}.$$

Therefore,

$$1 = (AT + B)\left(T^2 - T_a^{\ 2}\right) + C\left(T^2 + T_a^{\ 2}\right)(T - T_a) + D\left(T^2 + T_a^{\ 2}\right)(T + T_a),$$

and equating coefficients of like powers gives the following expressions:

$$A = 0,$$

$$B = -\frac{1}{2T_a^{\ 2}},$$

$$C = -\frac{1}{4T_a^{\ 3}}, \text{ and}$$

$$D = \frac{1}{4T_a^{\ 3}}.$$

Then,

$$\int \frac{dT}{T^4 - T_a^{\ 4}} = -\frac{1}{2T_a^{\ 2}} \int \frac{dT}{T^2 + T_a^{\ 2}} - \frac{1}{4T_a^{\ 3}} \int \frac{dT}{T + T_a} + \frac{1}{4T_a^{\ 3}} \int \frac{dT}{T - T_a}.$$

Then,

$\displaystyle\int \frac{dT}{T^2 + T_a^{\ 2}}$, by substituting $T = T_a \tan\theta$, is

$$\int \frac{d\theta}{T_a} = \frac{\theta}{T_a},$$

and

$$\theta = \frac{1}{T_a} \tan^{-1} \frac{T}{T_a}.$$

Then,

$$-\frac{1}{2T_a^{\,2}}\int\frac{dT}{T^2+T_a^{\,2}} = -\frac{1}{2T_a^{\,3}}\tan^{-1}\frac{T}{T_a}\ .$$

Solution of the remaining two integral expressions,

then, is:

$$-\frac{1}{4T_a^{\,3}}\int\frac{dT}{T+T_a} = -\frac{1}{4T_a^{\,3}}\log T+T_a\text{, and}$$

$$\frac{1}{4T_a^{\,3}}\int\frac{dT}{T-T_a} = \frac{1}{4T_a^{\,3}}\log T-T_a.$$

It follows then, that,

$$\int\frac{dT}{T^4-T_a^{\,4}} = \frac{1}{4T_a^{\,3}}\left[\log\frac{T-T_o}{T+T_a} - 2\tan^{-1}\frac{T}{Ta}\right].$$

The result where c is a constant of integration is:

$$K_1t+c = \frac{1}{4T_a^{\,3}}\left[\log\frac{T-T_a}{T+T_a} - 2\tan^{-1}\frac{T}{T_a}\right].$$

Solving for the constant c, the initial condition at $t = 0$ is $T = T_1$.

Then

$$c = \frac{1}{4T_a^{\,3}}\left[\log\frac{T_1-T_a}{T+T_a} - 2\tan^{-1}\frac{T_1}{T_a}\right].$$

The final expression with time as a function of temperature is as follows:

$$t = \frac{1}{4T_a^3K_1}\left[\log\frac{T - T_a}{T + T_a} - 2\tan^{-1}\frac{T}{T_a} - \log\frac{T_1 - T_a}{T_1 + T_a} + 2\tan^{-1}\frac{T_1}{T_a}\right].$$

Since $K_1 = \dfrac{-4\epsilon\sigma}{\rho D\overline{C}}$,

then

$$t = \frac{\rho D\overline{C}}{16\epsilon\sigma T_a^3}\left[\log\frac{T + T_a}{T - T_a} + 2\tan^{-1}\frac{T}{T_a} - \log\frac{T_1 + T_a}{T_1 - T_a} - 2\tan^{-1}\frac{T_1}{T_a}\right].$$

The resulting expression, which represents heat loss due to radiation, is seen to be independent of the length of the fuse wire, but dependent upon the diameter. This was an anticipated result due to the original imposed restrictions.

In order to determine the significance of the radiation expression, a graphical representation helps considerably. As an example, initial conditions will be assumed to be approximately those that would apply in a practical application.[2] It will be assumed that the initial temperature, T_1, is 700 degree Kelvin and T_a = 300 degree Kelvin. The wire diameter will be treated as a parameter in order to determine the influence of wire diameter on heat dissipation. The values for the remaining constants required for a numerical solution of the radiation equation are:

Initial Temperature	T_1	$= 700°K$
Ambient Temperature	T_a	$= 300°K$
Density of Copper	ρ	$= 8.9$ gm/cm^3
Mean Specific Heat	$\overline{C}$	$= 0.093$ cal/gm - °C
Stephan's Constant	σ	$= 5.7 \times 10^{-5} \times 2.39 \times 10^{-8}$ cal/cm^2 sec deg^4
Emissivity	ϵ	$= 0.568$ (for oxidized C_u, 200 - 600°C)*
Wire Diameter	D	= 0.1 cm, 0.05 cm.

*Condon, E. U., and Odishaw, H., Handbook of Physics, McGraw-Hill Book Company, Inc., New York, 1958, (6-16)

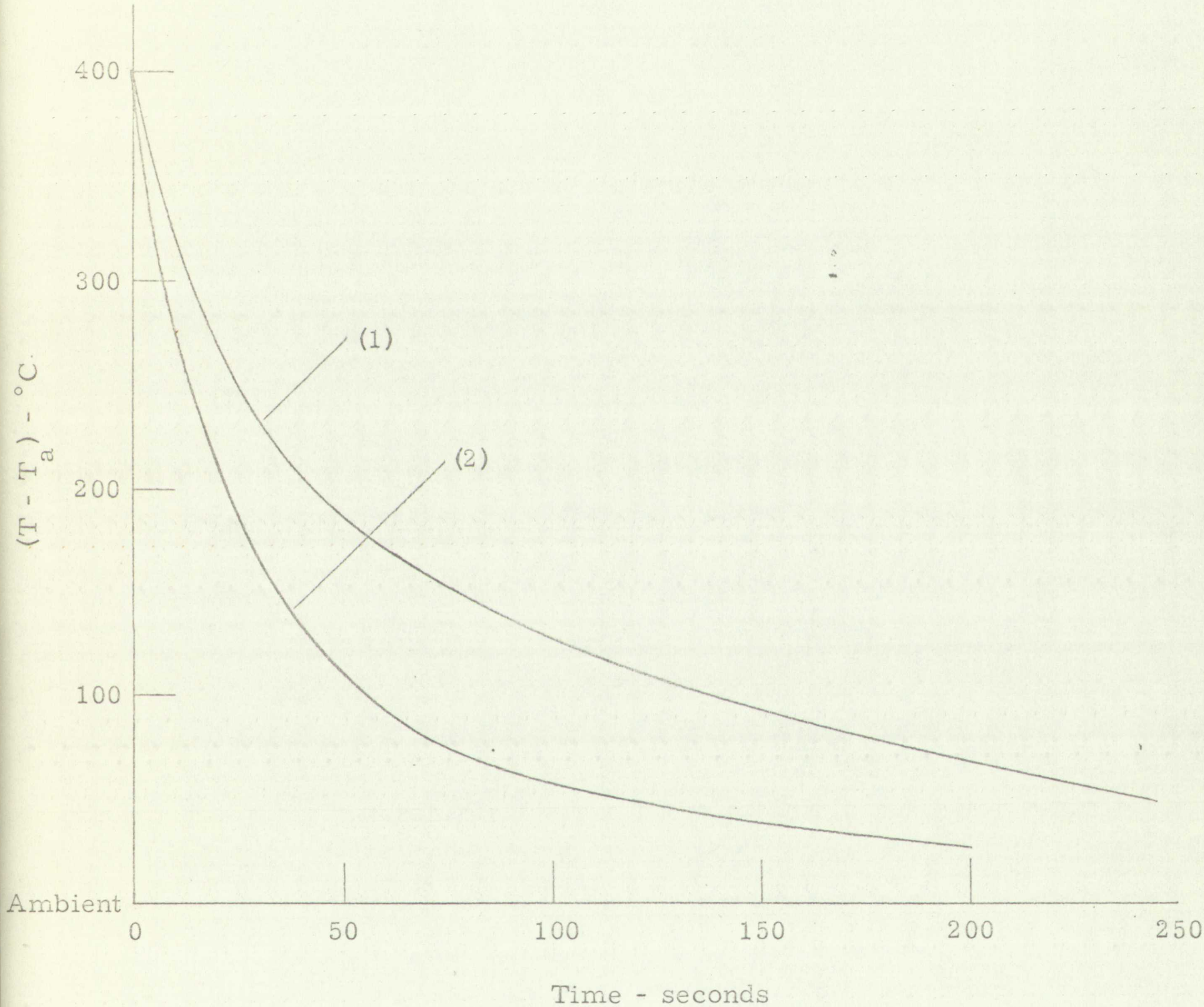


Figure 8. Heat Transfer by Radiation; Temperature as a Function of Time; Copper Wire Diameter: (1) 0.1 cm; (2) 0.05 cm

A graph of the radiation equation with the above values, is shown as Figure 8. Before any further comparison of the methods of heat dissipation can be made, the heat transfer by convection will have to be considered.

Heat Transfer by Convection

Heat transfer by natural convection is exceedingly complex and, as indicated in the literature, it is not completely understood. However, a great deal of effort, both theoretical and experimental, has been expended since the late 1800's to develop satisfactory formulas representing convection.

Newton's Law of Cooling describes convection as the rate of heat loss which is proportional to the temperature difference between the body and its surroundings. In the usual symbols, Newton's Law of Cooling is written:

$$\frac{dQ}{dt} = -k(T - T_a), \text{ where } k$$

involves at least these two factors: (1) the nature of the surface, and (2) the surface area.

It is well known that Newton's Law of Cooling is only satisfactory for small temperature differences between the body and surroundings; it also applies fairly accurately for large temperature differences for forced air cooling. A search of the literature on heat transfer revealed that in the case of small cylinders, the heat transfer coefficient very closely approximates a one-fourth power law:

$$\frac{dQ}{dt} = -h(\Delta T)^{1/4} A(\Delta T),$$

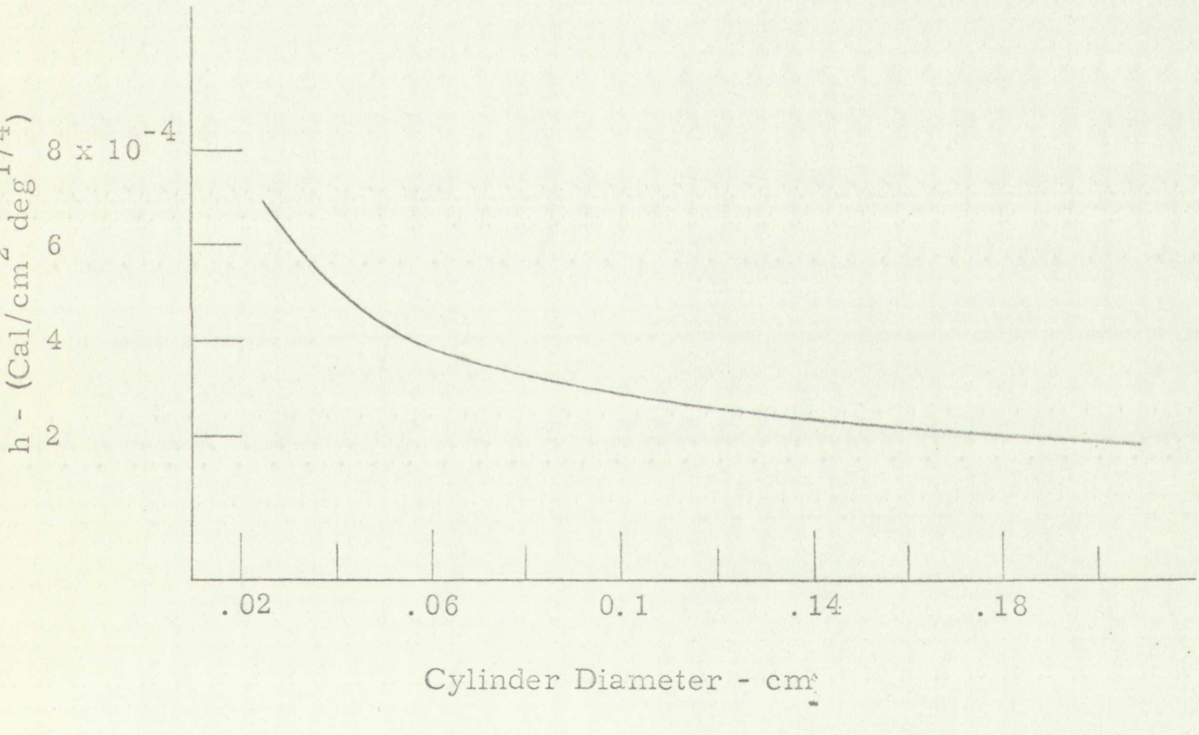


Figure 9. Convection Heat Transfer Coefficient h, as a Function of Wire Diameter; Ambient Temperature Range 0-200°C Derived from Data by Moore[5].

where

h = heat transfer coefficient for a horizontal cylinder in air, and

A = surface area.

Based on the foregoing, the value of h corresponding to the temperature difference between the cylinder and ambient can be determined. This heat transfer coefficient is shown plotted in Figure 9 as a function of cylinder diameter. Data on Figure 9 is derived from the correlation curves in Moore.[5] This curve is based upon experimental data and is reasonably accurate without any modification if the ambient is between zero and 200°C. Now a solution of heat loss by convection for the fuse wire can be pursued.

Assuming that there are no losses due to radiation or conduction, the rate of heat transfer is:

$$\frac{dQ}{dt} = -hA(\Delta T)^{5/4}.$$

By definition,

$$\frac{dQ}{dT} = C_v m.$$

Then, it follows that:

$$\frac{dT}{(\Delta T)^{5/4}} = -\frac{hA}{m\overline{C}}\ dt, \text{ and}$$

$$\int \frac{dT}{(T - T_a)^{5/4}} = -4(T - T_a)^{-1/4} = -\frac{hA}{m\overline{C}}\ t + c.$$

Again,

$$\frac{A}{m} = \frac{4}{D\rho}, \text{ and initial conditions are:}$$

at

$$t = 0,\ T = T_1.$$

Then,

$$c = -4(T_1 - T_a)^{-1/4}.$$

Making the substitution,

$$(T - T_a)^{-1/4} = \frac{h}{D\rho\overline{C}}\ t + 4(T_1 - T_a)^{-1/4}, \text{ and}$$

solving for t, the following equation is derived:

$$t = \left[(T - T_a)^{-1/4} - (T_1 - T_a)^{-1/4}\right] \Big/ \frac{h}{D\rho\overline{C}}.$$

The equation above will determine the relation of time as a function of temperature due to heat loss by convection. Figure 10 shows a plot of this equation for two wire diameters using the following values:

Wire Diameter	$D = 0.1$ cm, $h = 2.85 \times 10^{-4}$
	$D = 0.05$ cm, $h = 4.25 \times 10^{-4}$
Specific Heat	$\overline{C} = 0.093$ cal/gm-°C
Density of Copper	$\rho = 8.9$ gm/cm^3
Temperature	$T_1 = T_a + 400$ deg C

The three principal means of heat transfer have been considered independently over a range of wire values. In the case of heat loss by conduction, it was found that although the rate of heat transfer is independent of the wire diameter for a given initial temperature, it is highly dependent upon the length. For both radiation and convection, the rate of heat loss is directly dependent on the wire diameter, but independent of length.

In order to study the combined effects of all three methods of heat transfer, all parameters of the wire must be specified. It would seem

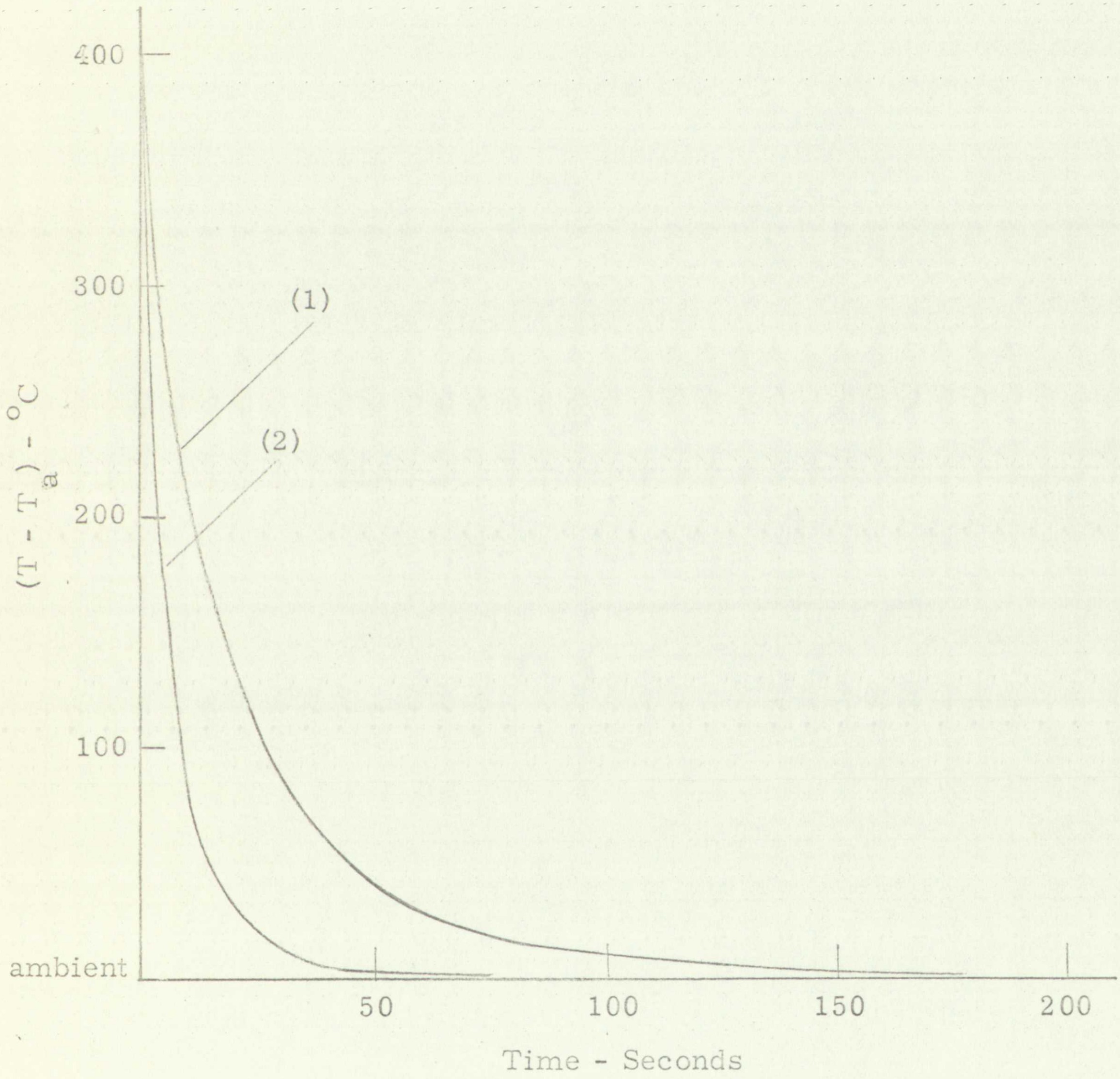


Figure 10. Heat Transfer by Convection; Temperature as a Function of Time for Copper Wire of Diameter (1) 0.1 cm and (2) 0.05 cm

that this would require many combinations of wire dimensions. However, on a practical basis, it can be said that if the fuse wire was assumed to be comparatively long, this would imply that the associated capacitor is either charged to a very high voltage or is a very large capacitor, or both. In either event, a wire with a relatively large diameter would be required for large discharge currents. On the other hand, shorter wires would imply the use of relatively low voltage, or small capacitors, thus calling for the use of small diameter wires because of the small discharge currents.

Because of the foregoing, the combined cooling effects will now be considered, using as an example a wire 0.1 centimeters in diameter and 30 centimeters long. The cooling curves for conduction, radiation, and convection are replotted for the wire dimensions stated on Figure 11 using an initial $(T_1 - T_a)$ of 400° C.

It is almost immediately obvious from this set of curves that cooling by conduction is insignificant and that heat transfer is due mainly to convection. However, it also appears that an accurate cooling curve for this particular wire should take into account both convection and radiation. This was done by using previous calculations to set up the following expression:

$$\frac{dT}{dt} = - \frac{4h}{D\rho \overline{C}} (T - T_a)^{5/4} - \frac{4\epsilon\sigma}{\rho D\overline{C}} \left(T^4 - T_a^{\,4}\right).$$

A graphical solution of this equation was obtained with the aid of an analog computer. It is shown redrawn in Figure 11. From this curve, it can be seen that radiation losses are noticeable, but not too significant.

The main purpose of the rigorous attempt to determine the attributal mechanism of cooling, is to allow the prediction of a permissible repetition of the initial conditions without the danger of overheating the fuse. In addition to this, it is fairly clear that from the standpoint of normal fuse operation--i.e., charging the module and discharging it into the

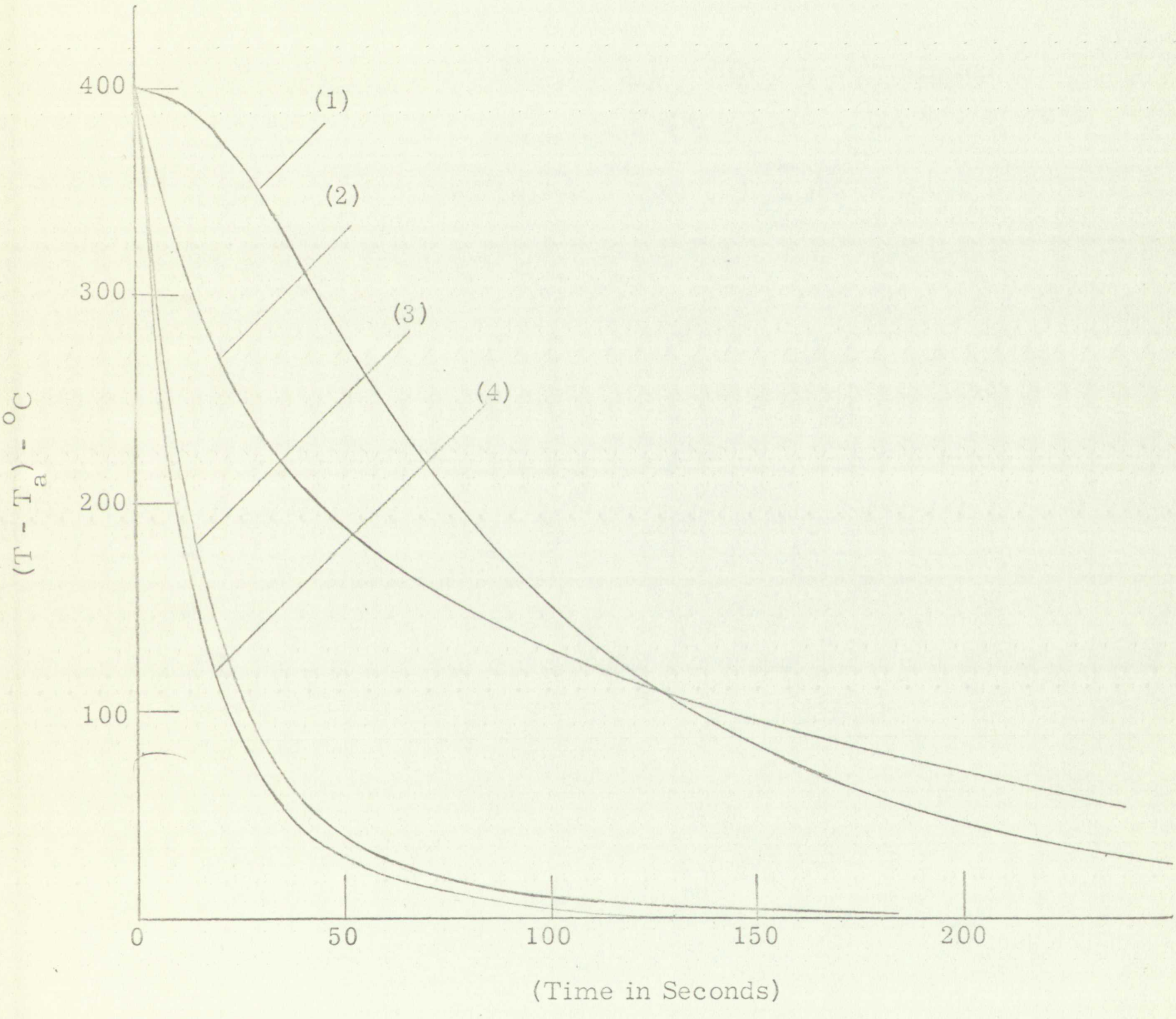


Figure 11. Comparative Graph of Heat Transfer by: (1) Conduction; (2) Radiation; (3) Convection; (4) Composite of Radiation and Convection. Wire Dimensions: Diameter, 0.1 cm; Length 30 cm

load--the only important fuse dimension is the diameter. Although Figure 11 illustrates only one particular wire size it is possible to predict, by examining the cooling curves of 0.05 cm-dia. wire shown in Figures 6, 8, and 10, that similar conclusions would follow.

CHAPTER V -- METHODS TO DETERMINE REPETITION RATE BOUNDARIES

The repetition rate can be obtained directly from the composite curve shown in Figure 12 as follows: On the condition that the fuse is to cool essentially to ambient after each pulse, then when the current discharge is such that it raises the temperature of the fuse to 400°C above ambient, the repetition rate would be one pulse per 130 seconds. Since the rate of cooling is only a function of the state of the wire, the identical conditions exist whether the temperature is reached due to cooling, or due to the heat input. This holds true provided that the heat input occurs in a fraction of a second.

It can then be seen that if the current discharge is such that it raises the temperature of the fuse to 300°C above ambient, the repetition rate would be one pulse per 126 seconds. Similarly, repetition rates for initial temperatures of 200 and 100 degree would be one pulse per 120 seconds, and one pulse per 108 seconds, respectively, as indicated on Figure 12.

A careful examination of the cooling curve suggests that for any initial condition other than $T_1 = 400$, the steep portion of the curve would be used if the heat-input pulse was repeated prior to the time required for the fuse wire to cool to ambient temperature. This is illustrated by the following example which uses the derived temperature equation to establish the initial condition and the cooling curve to establish the final condition prior to each recurrence of heat input.

The temperature equation is:

$$(T - T_o) = \frac{1}{\alpha} \quad \mathrm{EXP}\, \frac{\alpha\, \sigma_0\, iq}{K\bar{C}\rho a^2} - \frac{1}{\alpha},$$

where

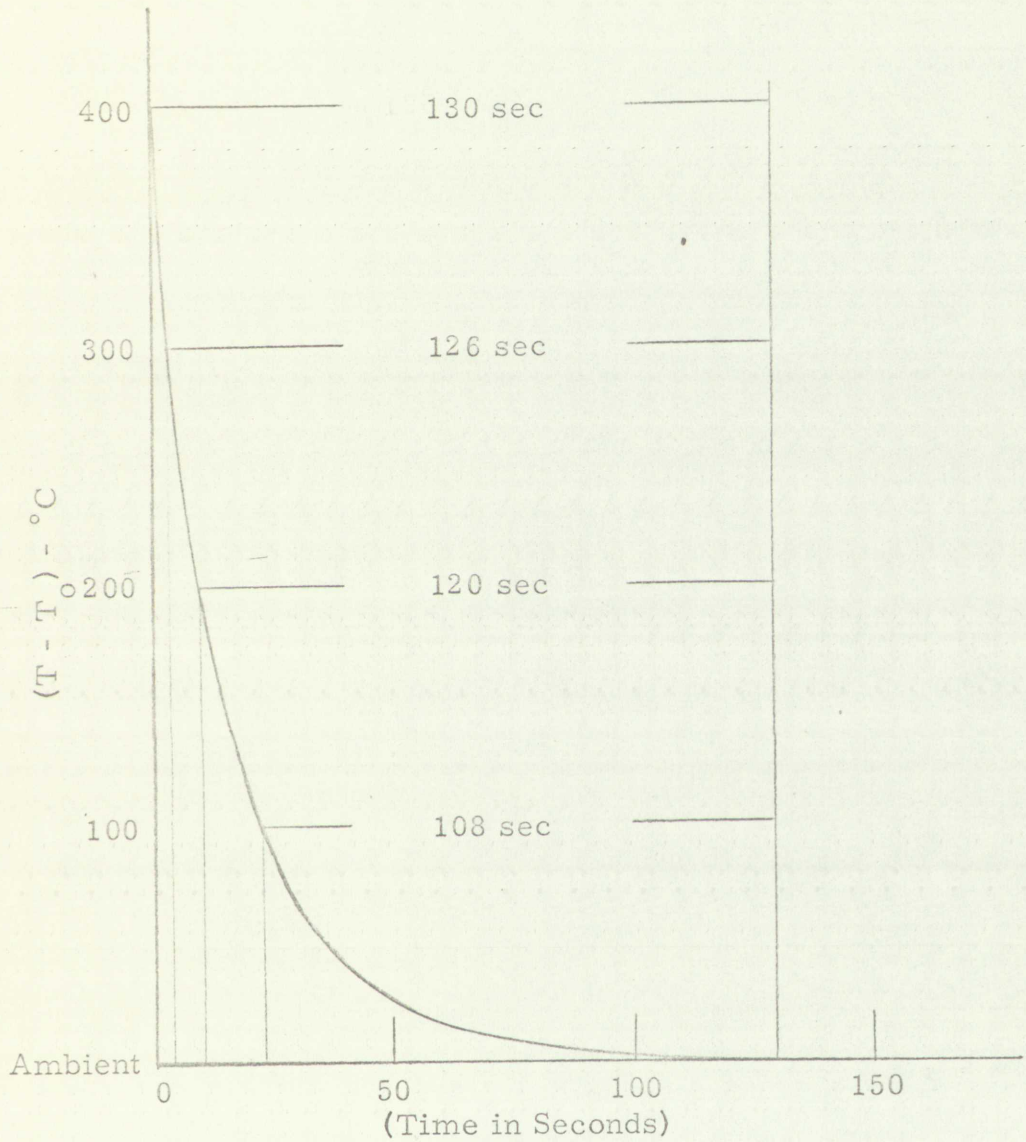


Figure 12. Composite Cooling Curve for Combined Radiation and Convection Cooling (Redrawn from Figure 11)

$T - T_o$ = temperature rise of the wire

α = 0.004 ohm/ohm-°C

σ_o = 1.74×10^{-6} ohm-cm; for T_o (ambient assumed 27°C)

$\overline{C}$ = 0.093 cal/gm-°C

ρ = 8.9 gm/cm^3

a^2 = 6.16×10^{-5} cm^4 (area of 0.1 cm-diameter wire squared)

K = 4.18 Joule/calorie

This example further assumes an iq* equal to 2×10^4 ampere-coulombs and a repetition rate of one pulse per 50 seconds. A solution is shown by the following tabulation:

Pulse No.		time (cooling)	T_o	$\sigma_o \times 10^{-6}$ $\sigma_o(1 + \alpha T_o)$	T
1.	Input	0	0	1.74	234
	Final	50			20
2.	Input	50	20	1.88	280
	Final	100			23
3.	Input	100	23	1.90	287
	Final	150			23
4.		Values repeat			

A plot of this data (see Figure 13) shows the temperature history of the fuse for repeated pulses. To compare the repetition rate, a related plot of the time required for the fuse wire to cool to ambient temperature is also shown (see Figure 13b). It now remains to develop the concepts in establishing the maximum allowable repetition rate for all possible input conditions within the limitations of the fuse.

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*The iq is considered constant, depending only on the initial capacitor energy and equivalent resistance of the load. The relatively small resistance of the fuse wire will have no affect on the iq.

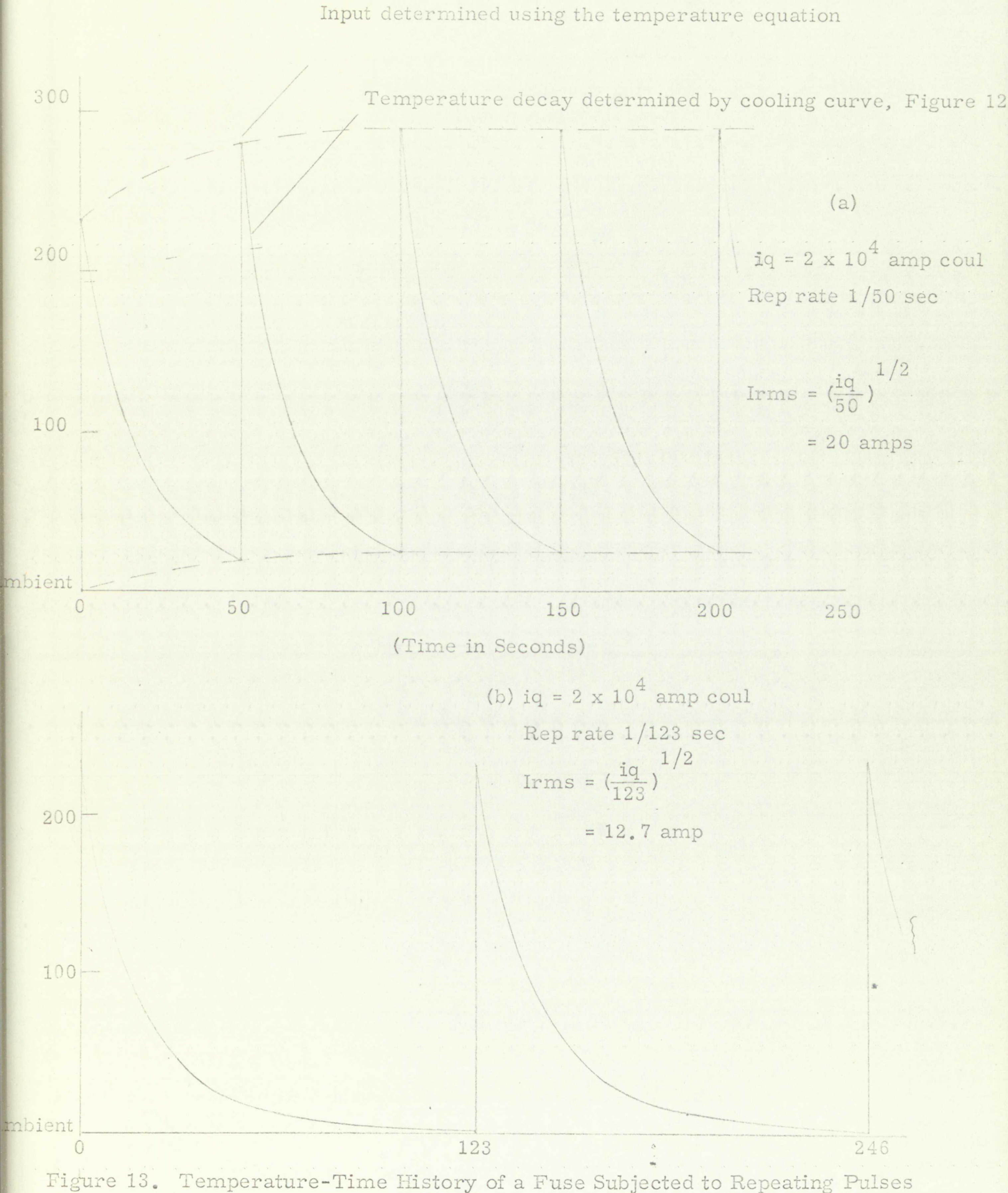

Figure 13. Temperature-Time History of a Fuse Subjected to Repeating Pulses

In the absence of deterioration, a wire would carry indefinitely a steady current only slightly below a current that causes the wire to melt. However, since copper begins to oxidize[10] at steady temperatures in excess of 250°C, permanent wire deformation results from currents that will raise the wire temperature above this value. Since this is true, the first fact that must be established is the maximum temperature the fuse wire can withstand with no appreciable degradation. In previous examples, a maximum temperature increase of 400°C was arbitrarily assumed for illustration purposes. However, the author previously conducted experimental fuse tests[6] and found that in pulsed fuse applications, deformation due to oxidation only became noticable for repeated peak temperatures in excess of 400°C for copper. With reference to Figure 12, it can be seen that the wire temperature is above 250°C for approximately six seconds, when the initial temperature is 400°C. A fuse subjected to repeated temperature cycling such as this, would in time fail prematurely. However, if the rate of deformation is low, the useful life of the fuse is long. In any specific application, the maximum pulse condition must be established experimentally according to the desired fuse life.

In order to derive boundary values for pulse repetition rates as a function of the initial capacitor energy and load resistance, it will be assumed that the peak fuse temperature should not exceed 400°C. (This temperature limitation in most cases may be conservative, but the following procedure would be similar for any other temperature limit.)

Solving the temperature equation on page 41 for iq, produces this equation:

$$iq = \frac{K\overline{C}\rho a^2}{\alpha \sigma_o} \log (\alpha T - \alpha T_o + 1)$$

Using the same values for the constants on the right side of this equation as they appear on page 46, except for $T - T_o$,

where

$T = 400$ (the maximum temperature limit)

and

T_o = a parameter,

the data in the following table was obtained by allowing T_o to take on values corresponding to the cooling time (Figure 12) between repeated pulses.

Then

τ-sec	T_o	$\sigma \times 10^{-6}$	iq
130	0	1.74	29,000
80	10	1.81	27,400
60	16	1.85	26,400
50	26	1.92	25,160
40	41	2.02	23,100
30	64	2.18	21,900
20	116	2.55	15,700
10	200	3.13	9,810

This data is shown plotted in Figure 14 as a line of constant (maximum) temperature for iq as a function of the time between repeated pulses. The area of operation where the fuse degradation will be no greater than the assumed maximum (where iq = 29,000 and the repetition rate = 130 sec) can be determined by comparing the repeated current pulse to an equivalent steady state current.

The capacitor current pulse derived earlier is

$$i = \frac{V}{L\beta} (e^{-\alpha t} \sin\beta t).$$

It follows then that the effective current (I_{rms}) is

$$\left(\frac{1}{\tau} \int_0^\tau i^2 \, dt \right)^{1/2}.$$

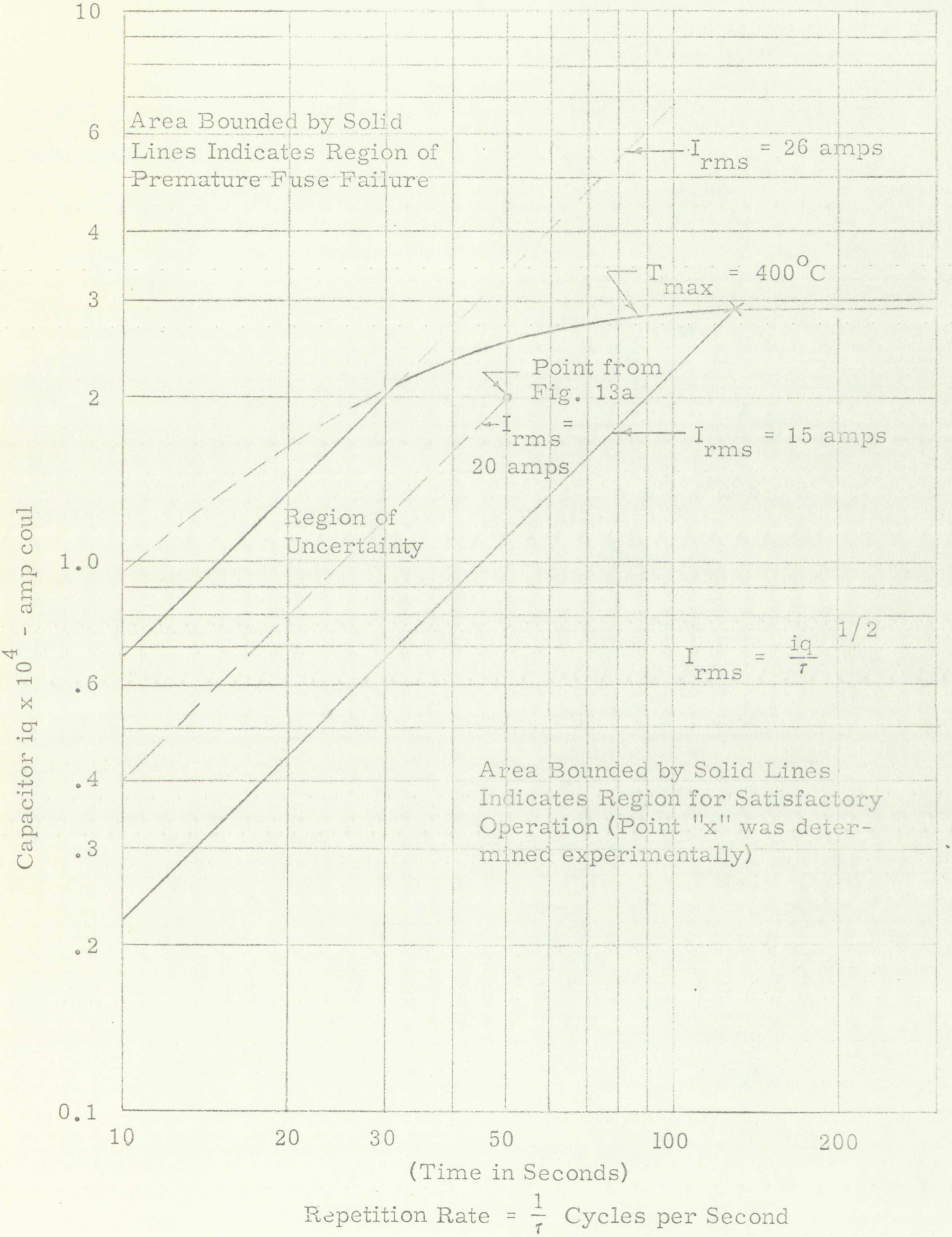

Figure 14. Capacitor iq - vs - Repetition Rate as a Function of
(1) Peak Temperature Obtained per Pulse
(2) Lines of Equivalent Current, Expressed in I_{rms} for a 0.1 cm, Diameter Copper Wire

In this expression, $\frac{1}{\tau}$ is the repetition rate. Since the time for the actual current pulse to occur is infinitely small compared to τ,

$$I_{rms} \text{ is then } \left(\frac{1}{\tau}\int_0^{\infty} i^2\, dt\right)^{1/2} \text{ or } \left(iq/\tau\right)^{1/2}.$$

The equivalent steady state (DC) current for iq - 29,000 and repetition rate of 130 seconds is approximately 15 amperes. The line for 15 amperes is shown plotted on Figure 14 and it represents all combinations of pulse repetitions that would result in fuse degradation no greater than that due to an iq of 29,000 repeated at 130 second intervals.

A region of operation that will positively result in premature failure can be determined by considering all pulse operating conditions that correspond to a steady temperature of 250°C due to constant DC current. According to the derived cooling curves, heat dissipation by connection is predominant for a wire temperature of 250°C. Equating the power input to power dissipated due to heat transfer by convection, the following expression can be solved for current.

That is,

$$I^2 r = K\,A\,h\,(T - T_o)^{5/4},$$

where

$T = 250^{\circ}C$,

$T_o = 27^{\circ}C$ (ambient),

$K = 4.18$ Joule per calorie,

A = wire surface area

r = wire resistance, and

$h = 2.85 \times 10^{-4}$ (heat transfer coefficient for a 0.1 cm dia. wire).

The ratio of surface area to resistance, $\frac{A}{r}$ is $\frac{\pi^2 D^3}{4\sigma}$, where $\sigma = \sigma_o (1 + \alpha\Delta T)$.

Then the expression for current is,

$$I = \left[\frac{K h \pi^2 D^3 (\Delta T)^{5/4}}{\sigma_o (1 + \alpha \Delta T)} \right]^{\frac{1}{2}}$$

Solving this equation for current using the values shown above, the result is, I = 26 amperes. This constant current line is also shown plotted on Figure 14. It represents a boundary for all combinations of iq and repetition rates above which premature fuse failure is certain to occur. The extent of fuse deformation in the region between the extreme boundary lines is uncertain. Satisfactory operation in this area must be determined experimentally. However, the experimental testing can be minimized with the use of additional boundary lines.

For example, if the conditions for temperature cycling the wire as indicated in Figure 13a were reproduced experimentally with no appreciabe degradation occurring, then a boundary line could be constructed by relating the equivalent DC current to this test point. The operating point of Figure 13a is shown plotted on Figure 14, together with the line of equivalent constant current. Additional test points in the uncertain region for both satisfactory and unsatisfactory operation can be established, and these will completely define the fuse limitation for periodic operation.

CHAPTER VI -- SUMMARY

The general requirements for an exploding wire to function as a protective fuse in a high energy capacitor module depend on two distinct determinations. The first requirement is derived from the circuit and wire characteristics when a fault occurs. The fusing action is characterized by a current pause which predominately depends upon the wire length and voltage across the exploding fuse. This voltage-length dependency in any particular capacitor module constitutes, in a sense, a fuse voltage rating. At present, this fuse voltage rating for a specific capacitor module can only be determined experimentally by taking into account the existing fault circuit constants and the exploding wire behavior.

The normal operation of a fused capacitor module establishes the normal fuse current requirement. There is no well-defined way to determine a fuse current rating in pulsed circuitry. Instead, the integral of the current squared, designated iq, affords a direct correlation between energy transfer and energy absorbed by the fuse during a current discharge. The correlation is established by this energy equation:

$$\int i^2 r dt = mc \int dt,$$

which is a function of the fuse, and

$$CV^2/2R = \int i^2 dt,$$

which is a parameter of the energy storage system.

Energy absorbed by the fuse during a discharge is dissipated due to heat transfer by convection, and to some extent by radiation. Permanent deformation will result if the fuse is overheated. The temperature limit where fuse deformation becomes excessive must be determined experimentally. Then the maximum allowable pulse repetition rate can be determined analytically as a function of the energy input per pulse.

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