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A pulse length analyzer

Paul Scheie

A PULSE LENGTH ANALYZER - SCHEIE

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By

Paul O. Scheie

A Thesis

Submitted in Partial Fulfillment of the

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MASTER OF SCIENCE

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June 3, 1957
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Thesis committee

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CHAPTER I

INTRODUCTION

For experiments involving small scintillators and for experiments involving scintillator telescopes, the circuitry may closely resemble that used with geiger counters. However, if it is desired to make use of the variation in the signal amplitude in a large scintillator, some means must be found to deal with a wide range of amplitudes. The scintillator employed in the experiment to which this work relates is a tank 10 feet in diameter that contains 300 gallons of activated toluene; light of the scintillations is collected by a 16 inch photo-multiplier tube that is located on top of the tank.

A circuit has been developed which will convert the pulses from the photo-multiplier tube into pulses of standard height (ie., amplitude) whose length (ie., duration) is proportional to the logarithm of the height of the original pulse. This circuit is called the "Logarithmic Circuit" in Fig. 1. In such a manner amplitudes extending over a range of somewhat less than 10^4 can be handled. Before the scintillation pulses can be used, however, it is necessary to learn what correspondence exists between the cosmic-ray particles traversing the tank and the scintillation pulses produced by such traversals. In particular, it is essential that the average pulse produced by the traversal of a single, minimum ionizing particle be determined.

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A circuit has been [illegible] pulses from the photo-multiplier [illegible] and height (i.e., amplitude) [illegible] proportional to the [illegible] pulse. This circuit [illegible] Fig. 1. In such a [illegible] of somewhat less [illegible] tillation pulses can be [illegible] learn what correspondence [illegible] ticles traversing the [illegible] duced by such [illegible] that the average [illegible] minimum ionizing [illegible]

In order to obtain this information an experiment was designed to run a geiger counter tray telescope in coincidence with the scintillator. Fig. 1 gives the block diagram of the experiment. If the geiger tube trays are triggered by ionizing particles so that pulses from the traversal arrive at the coincidence circuit within a time of approximately 2 μsec., a "coincidence pulse" of 2 μsec. will be produced. This coincidence pulse opens a gate in the pulse length analyzer. The particle, or particles, triggering the geiger tube trays should also have passed through the scintillator and, therefore, have produced a pulse in the photomultiplier tube. This scintillation pulse is passed through a cathode follower and cable to an amplifier and then to the logarithmic circuit described earlier. The logarithmic pulse is delayed to compensate for delays in the geiger tubes and coincidence circuits, and then it is converted by the pulse length analyzer into an amplitude which is recorded by the 10 channel pulse amplitude analyzer.

By this arrangement it is possible to record the pulses from the scintillator only when the geiger tube telescope is triggered simultaneously. Most of the events in which the telescope is triggered will be the traversals of singly occurring mu-mesons; events with more than one particle will occur with rapidly decreasing probability as the number of particles increases. An analysis of the differential spectrum of scintillator pulses occurring simultaneously with triggering of the telescope will then make it possible

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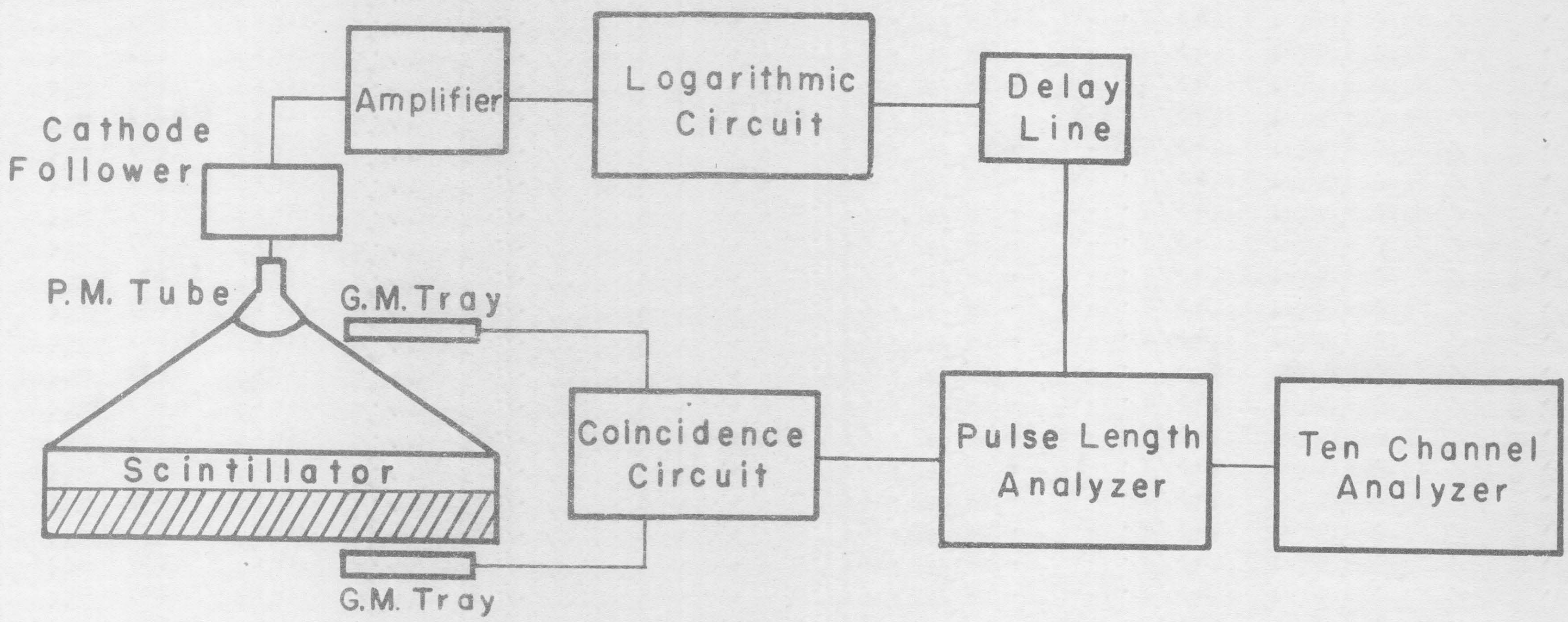

Figure I

Block Diagram For Scintillator Calibration

to identify the average pulse amplitude corresponding to traversal of the scintillator tank by single particles.

The remainder of this paper is concerned with the circuits necessary to make the information from the telescope coincidence and scintillator ready for the 10 channel analyzer. This will require two major steps. The first brings the two pulses together at a gate where the scintillator pulse will be passed only if the gate has been opened by a coincidence pulse from the telescope. The second step is to convert such a gated pulse into a pulse whose height is proportional to the length of the gated pulse. The result of these circuits, then, is to produce a pulse whose amplitude is proportional to the logarithm of the amplitude of the pulse produced in the scintillator, and to produce this pulse only when it occurs in coincidence with the counter telescope.

Fig. 2 shows the arrangement of these circuits. The coincidence pulse triggers a univibrator with variable output pulses in order that the gate can be opened for a predetermined length of time. During this time the delayed pulse from the logarithmic circuit passes through the gate in inverted form. This gated pulse is re-inverted and then made to trigger the linear sweep circuit.

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Fig. [illegible] shows the [illegible] coincidence pulse [illegible] put pulses [illegible] determined [illegible]. [illegible] pulse from the [illegible] in inverted form. [illegible] made to trigger [illegible].

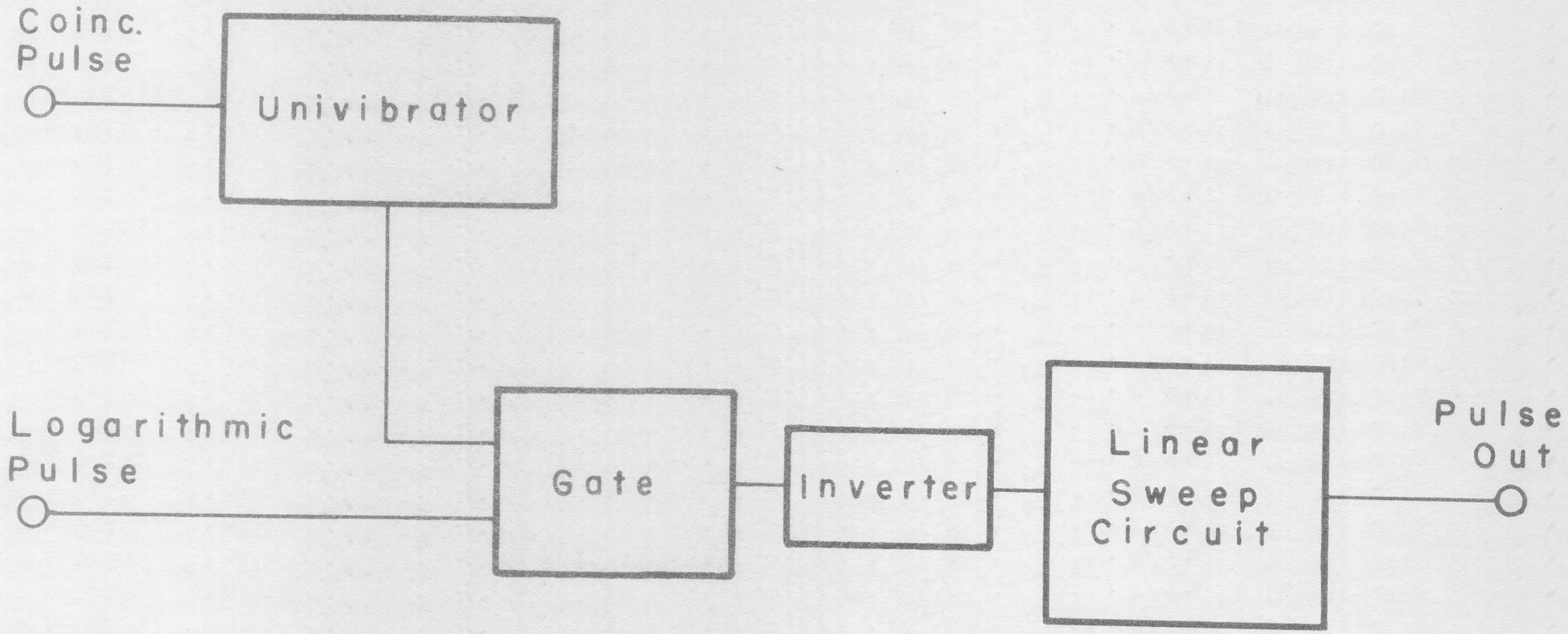

Figure 2

Pulse Length Analyzer

CHAPTER II

THE CIRCUITS

I. THE POWER SUPPLY

The power supply is an integral part of the entire electronic unit. Inasmuch as no part of the circuit requires accurately stabilized potentials, it was possible to employ a simple rectifier-filter supply. Potentials available include -100 v., -300 v., +150 v., +300 v., and +400 v.. The schematic diagram for the complete power supply is contained in Fig. 3.

II. THE UNIVIBRATOR

The first step is to bring together the coincidence pulse from the geiger tube trays and the delayed logarithmic pulse from the scintillator. The circuit to accomplish this is shown in Fig. 4.

The coincidence pulse from the geiger tube trays is used to open the gating circuit to allow the logarithmic pulse from the scintillator to pass through. However, the coincidence pulse is only some 2 microseconds long; whereas the logarithmic pulses extend from 20 to 500 microseconds in length. A univibrator was, therefore, inserted to maintain the gate in an opened condition for times that can be selected. These times range from ten microseconds to 1,000 microseconds, with a separate switch to allow manual control

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The power supply [illegible] of the entire electronic unit. [illegible] accurately stabilized power [illegible] a single rectifier [illegible] diode -100 v., -300v. [illegible] schematic diagram [illegible] in Fig. 3.

The first [illegible] pulse from the [illegible] pulse from the scintillator [illegible] is shown in Fig. [illegible].

The scintillator pulse [illegible] used to open the gating [illegible] pulse from the scintillator [illegible] coincidence pulse [illegible] the longitudinal [illegible] length. A univibrator [illegible] the gate in [illegible] lated. These [illegible] microseconds, [illegible]

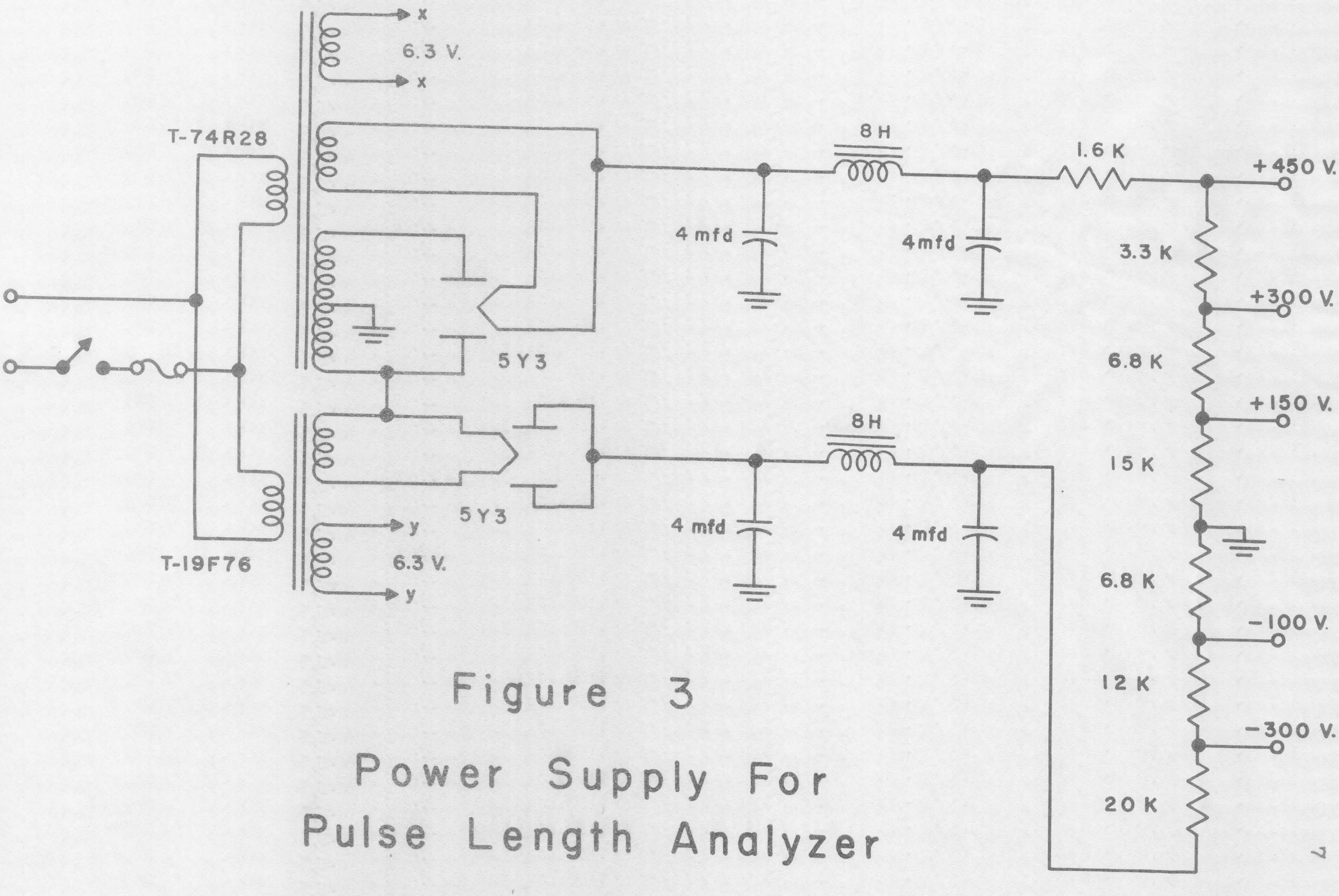

Figure 3

Power Supply For
Pulse Length Analyzer

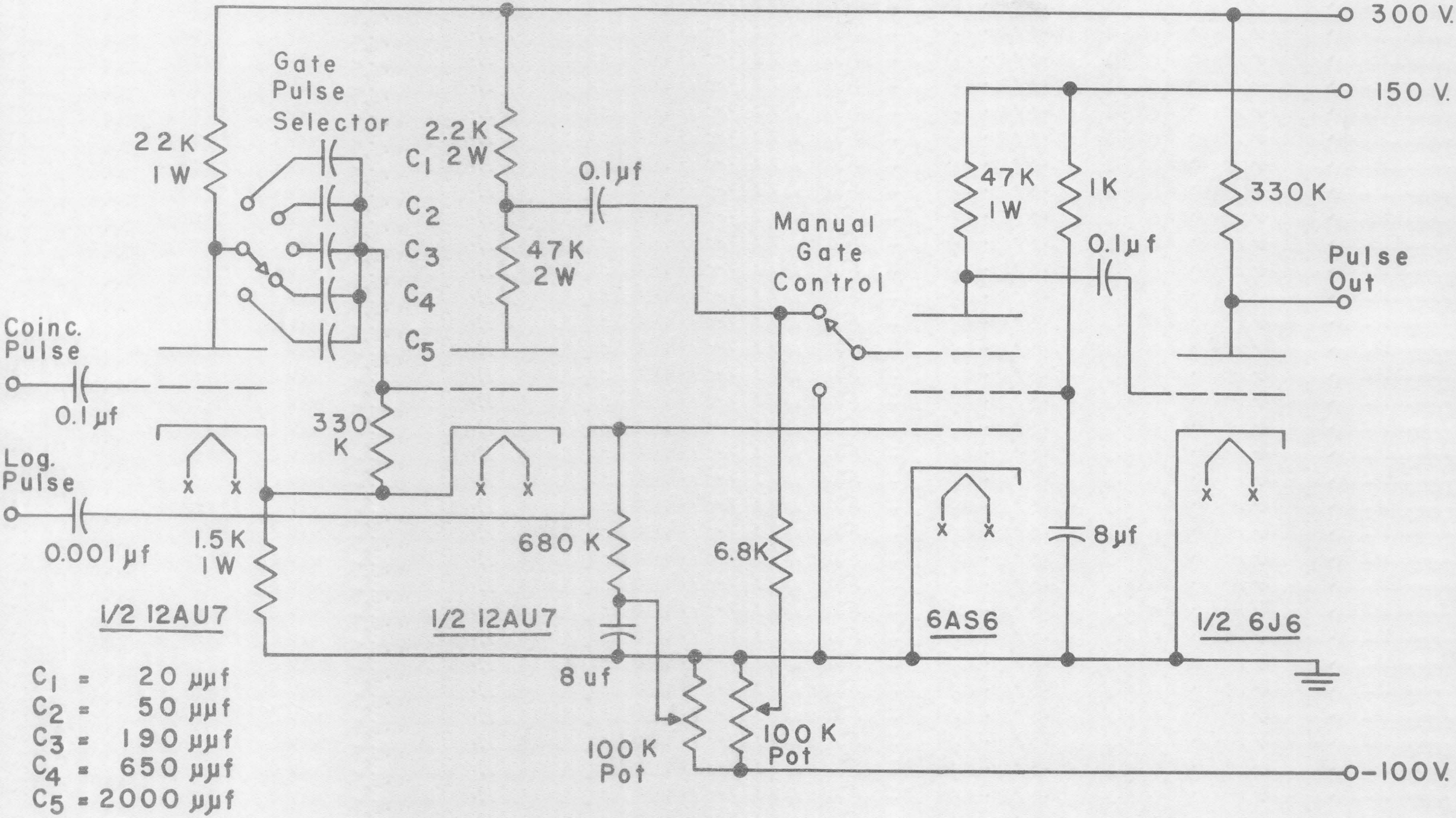

Figure 4
Univibrator, Gate, And Inverter

of the gate (See Fig. 4). One could, of course, make the length long enough to include, in one setting, all possible pulses from the logarithmic circuit, but this would increase the dead-time in the case of fast counting. This would also lead to the possibility of more than one scintillation pulse being counted for a single coincidence of the telescope. Thus, the variable-length coincidence pulse makes it possible to choose an optimum length for various ranges and rates of scintillator pulses.

The univibrator employed was cathode coupled. It was found that this model gave the sharpest pulses over the entire range desired, and it also eliminated any direct capacitance-coupled path between the input and the output.

It was noted that the output length was considerably affected by the input length. This was, however, discounted as being of little consequence since, in this particular circuit arrangement, the input pulse from the coincidence circuit would always be the same length. The capacitances, as given in Table I, were determined with input pulses from the coincidence circuit by which the univibrator would normally be activated.

III. THE GATE AND INVERTER

The gate tube used was a 6AS6. The pulse from the coincidence-univibrator is placed on the third grid. This grid can insure cut off, but even if it is positive, the nature of the pulse on the plate is still controlled by the

of the gate (see Fig. 4). One could, of course, make the length long enough to include, in each setting, all possible pulses from the logarithmic circuit, but this would increase the dead-time in the case of fast counting. This would also lead to the possibility of more than one scintillation pulse being counted for a single coincidence of the telescope. Thus, the variable-length coincidence pulse makes it possible to choose an optimum length for various ranges and rates of scintillator pulses.

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TABLE I

UNIVIBRATOR PULSES WITH CORRESPONDING CAPACITANCES

Capacitance (μμf)	Pulse Out (μsec.)
C_1 = 20	10
C_2 = 50	30
C_3 = 190	100
C_4 = 650	300
C_5 = 2,000	1,000

first grid. The pulse from the logarithmic circuit is placed on the first grid. If grid three is positive as the result of a coincidence pulse, there will be produced at the plate of the gating tube a pulse whose length is the same as that of the logarithmic pulse but whose polarity is reversed. An inverter follows to provide a positive pulse to trigger the sweep circuit. Fig. 5 gives the wave shapes from the input through the inverter.

IV. THE LINEAR SWEEP CIRCUIT

There are two simple methods for measuring a pulse length. One consists of turning on a high frequency oscillator for the duration of the pulse and measuring the number of oscillations occurring in this period; if the frequency of the oscillator is known, the length can be readily calculated. In the other method the length of the pulse is converted into an amplitude which can then be fed into an ordinary scaler or pulse-amplitude analyzer. The second method was the one employed in this work and the actual circuit used is shown in Fig. 6. Wave shapes for the circuit are given in Fig. 7.

From a qualitative point of view one can analyze the general operation of the circuit as follows. The incoming positive pulse arrives at the switching vacuum tube (5963) whose first section is normally conducting. The positive pulse turns on the first section which, in turn, cuts off

first grid, the pulse that is [illegible] and
placed on the first grid [illegible]
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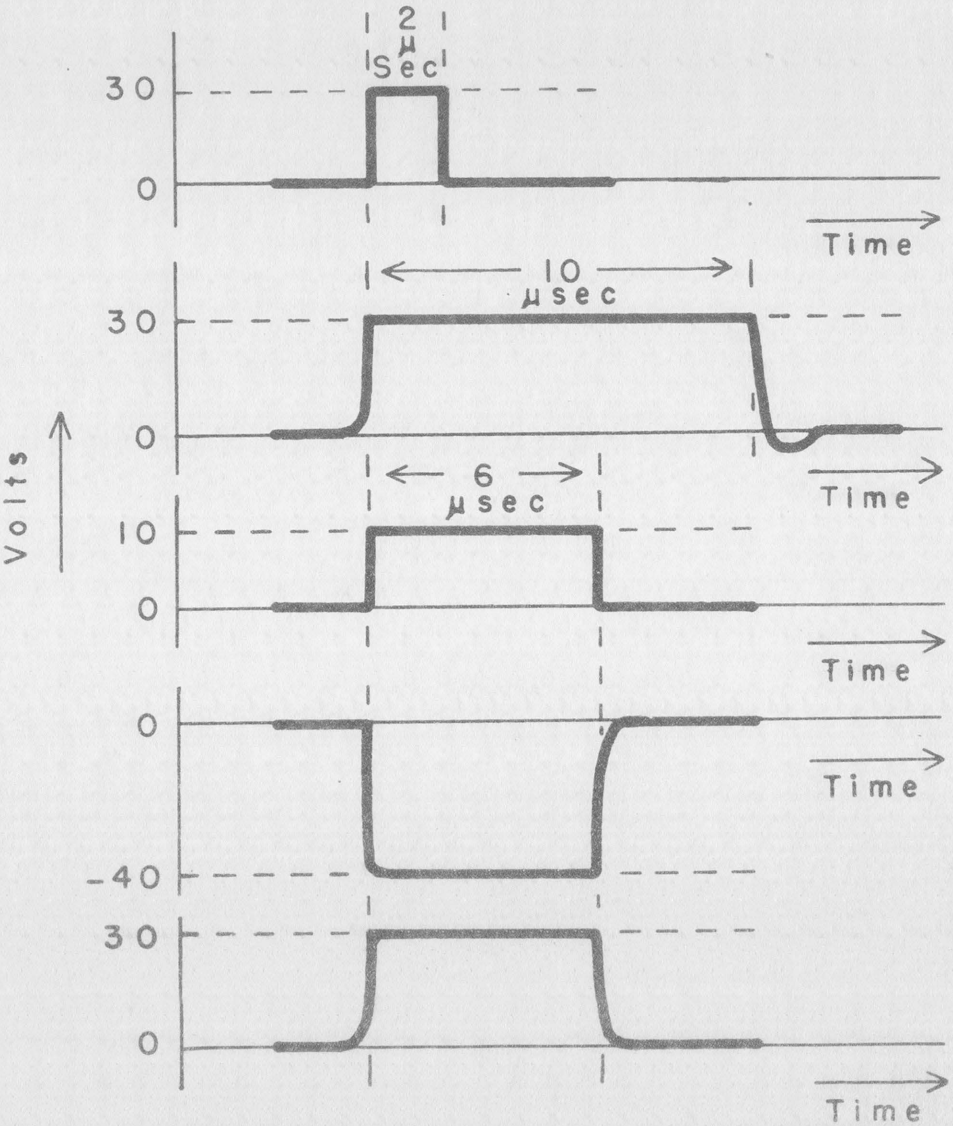

Figure 5

Wave Shapes
Input Through inverter

(a) Coincidence Pulse
(b) Univibrator Pulse
(c) Logarithmic Pulse
(d) Gated Pulse
(e) Inverted Pulse

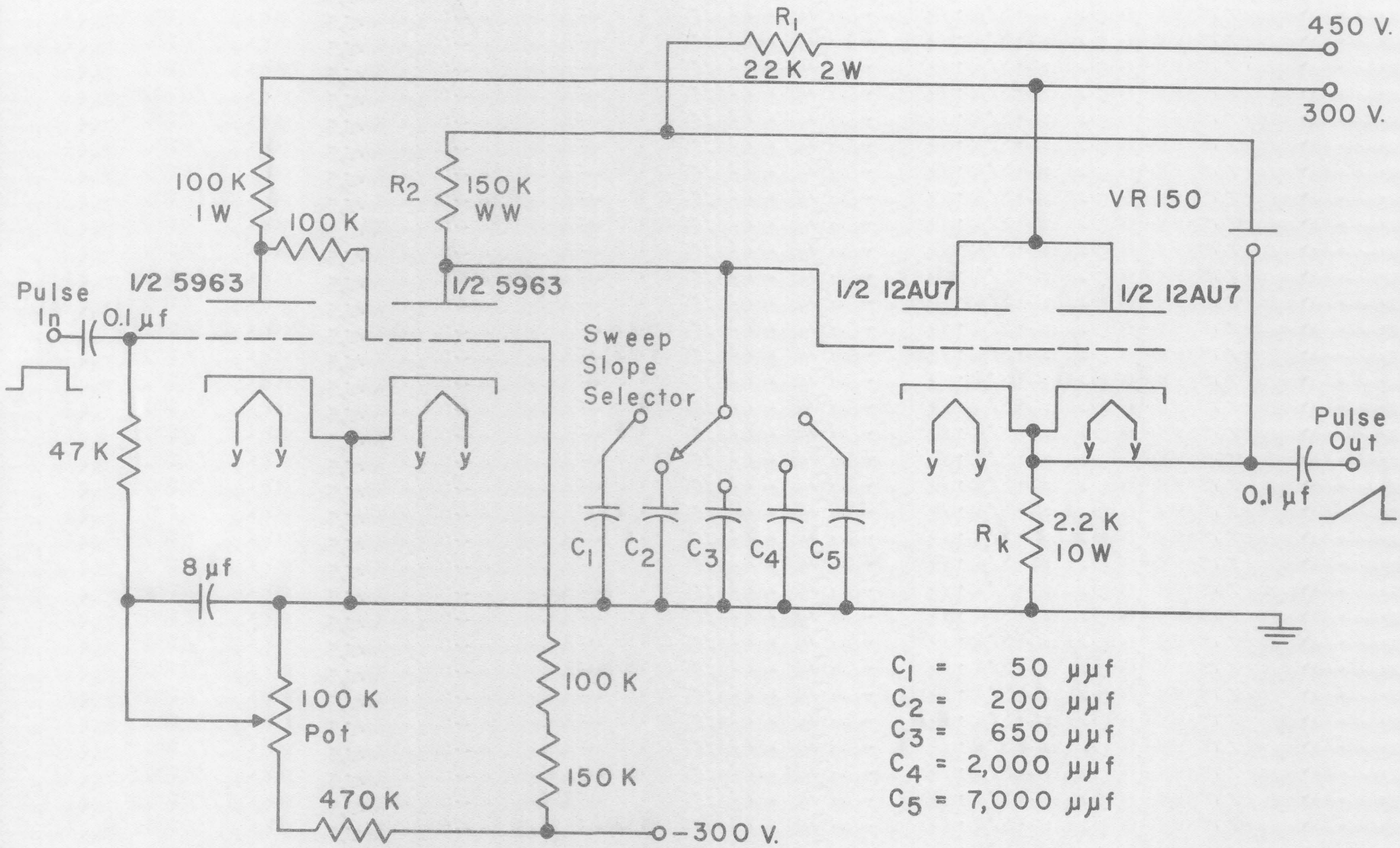

Figure 6
Linear Sweep Circuit

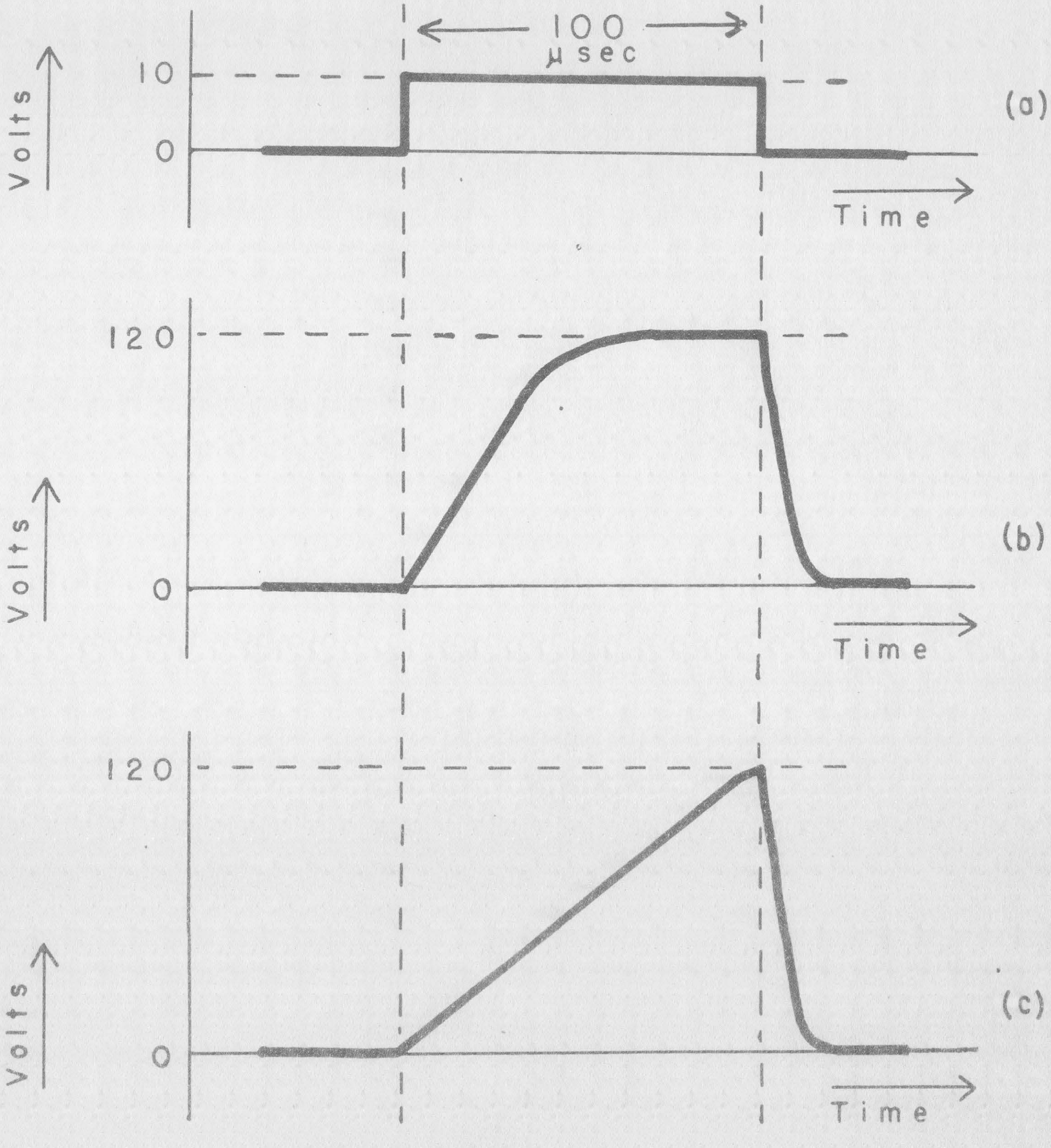

Figure 7

Wave Shapes For Linear Sweep Circuit

(a) Input Pulse

(b) Sweep Pulse — No Feedback

(c) Sweep Pulse

the second. Now, with the second stage conducting, the plate potential is relatively low, in the neighborhood of five volts for this circuit. When it is cut off, the plate potential cannot rise immediately to the supply potential because current must still flow through the plate resistor to charge the capacitor, C. This produces a finite rise-time for the plate potential; and, if the remainder of the circuit were omitted, the wave shape would appear as indicated in Fig. 7b. If the cathode follower and the VR tube are added as a form of feedback, the potential towards which the capacitor charges is maintained at approximately the same value. To the degree that this potential remains constant, so does the charging current to the capacitor remain constant. As a result, the potential of the capacitor should increase at a constant rate. The circuit employs several values of the charging capacitors that can be chosen to give a charging rate that is optimum for the range of pulse lengths being investigated.

The circuit will now be analyzed in a more quantitative fashion. A simplified schematic diagram is given in Fig. 8. The equations which describe the operation of the circuit are the following:

$$E_o = (i_b + i_1)R_K + V_o + (i_1 + i_2)R_1 \tag{1}$$

$$E_o = i_2 R_2 + e_c + (i_1 + i_2)R_1 \tag{2}$$

$$E_b = e_b + (i_b + i_1)R_K \tag{3}$$

$$e_g = e_c - e_K \tag{4}$$

$$di_b = g_m\, de_g + \frac{1}{r_p}\, de_b \tag{5}$$

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Fig. 9. The equations which [illegible]

circuits are the following:

$$E_o = [illegible]$$

$$E_1 = [illegible]$$

$$E_2 = [illegible]$$

$$e_g = [illegible]$$

$$di_p = [illegible]$$

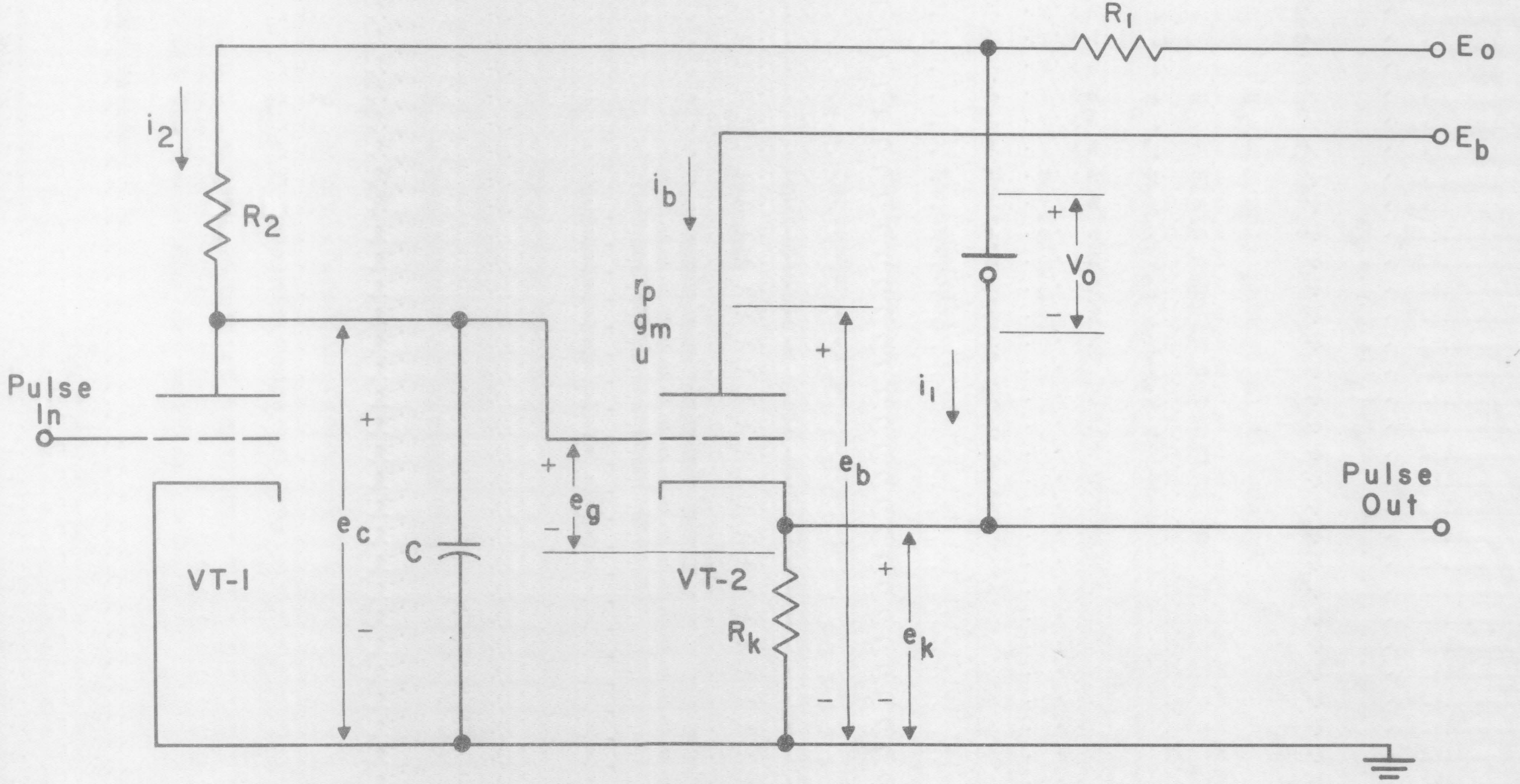

Figure 8

Simplified Sweep Circuit

$$de_c = i_2 \frac{dt}{C} \tag{6}$$

$$e_k = (i_1 + i_b) R_k \tag{7}$$

where the symbols indicate the following:

E_o - Supply potential to charge capacitor

E_b - Plate supply potential for VT-2

V_o - Potential across VR tube

e_b - Instantaneous plate potential of VT-2 (relative to cathode)

e_g - Instantaneous grid potential of VT-2 (relative to cathode)

e_k - Instantaneous cathode potential of VT-2

e_c - Instantaneous potential across capacitor (also instantaneous grid potential of VT-2 relative to ground)

i_1 - Instantaneous current through resistance, R_1

i_2 - Instantaneous current through resistance, R_2

i_b - Instantaneous plate current of VT-2

R_1 - Dropping resistor

R_2 - Charging resistor

R_k - Resistance of cathode resistor of VT-2

r_p - Dynamic plate resistance of VT-2

μ - Amplification factor of VT-2

g_m - Transconductance of VT-2

C - Capacitance of capacitor to be charged

If all the equations are written in terms of small changes, they become

$$(di_b + di_1) R_k + (di_1 + di_2) R_1 = 0 \tag{1}$$

$$di_2 R_2 + de_c + (di_1 + di_2) R_1 = 0 \qquad (2)$$

$$de_b + (di_b + di_1) R_K = 0 \qquad (3)$$

$$de_g + de_K - de_c = 0 \qquad (4)$$

$$di_b - g_m \, de_g - \frac{1}{r_p} de_b = 0 \qquad (5)$$

$$de_c - i_2 \frac{dt}{C} = 0 \qquad (6)$$

$$de_K - (di_1 + di_b) R_K = 0 \qquad (7)$$

These equations are solved for i_2 by first obtaining di_b/dt and di_1/dt in terms of i_2 and di_2/dt, and then solving the resulting differential equation for i_2.

Subtracting equation (2) from (1) and substituting e_c from equation (6) gives

$$\frac{di_2}{dt} = \frac{1}{R_2}\left[\frac{di_b}{dt} + \frac{di_1}{dt}\right] R_K - \frac{i_2}{C} \qquad (8)$$

di_b/dt is obtained by substituting for de_g and de_b from equations (4) and (3) into (5) so that

$$di_b - g_m(de_c - de_K) + \frac{R_K}{r_p}(di_b + di_1) = 0$$

Substituting for de_k and de_c from (7) and (6) gives

$$\frac{di_b}{dt} = \frac{1}{[r_p + (\mu+1)R_K]}\left[\frac{\mu i_2}{C} - (\mu+1) R_K \frac{di_1}{dt}\right] \qquad (9)$$

From equation (2) one can solve for di_1 to obtain

$$di_1 = -\frac{1}{R_1}\left[di_2 R_1 + de_c + di_2 R_2\right]$$

Substituting for de_c from (6) gives

$$di_1 = -\frac{1}{R_1}\left[di_2 (R_1 + R_2) + \frac{i_2 \, dt}{C}\right]$$

Solving for di_1/dt

$$\frac{di_1}{dt} = -\left[\frac{di_2}{dt} + \frac{di_2}{dt}\frac{R_2}{R_1} + \frac{i_2}{R_1 C}\right] \tag{10}$$

Substituting equation (9) and then (10) into (8) yields

$$\frac{di_2}{dt}\left[R_1 R_2 r_p + R_1 R_2 R_k(\mu+1) + R_1 R_k r_p + R_2 R_k r_p\right] + \frac{i_2}{C}\left[R_1 r_p + R_1 R_k + R_k r_p\right] = 0 \tag{11}$$

Now by setting

$$\alpha = \left[R_1 R_2 r_p + R_1 R_2 R_k(\mu+1) + R_1 R_k r_p + R_2 R_k r_p\right]$$

and

$$\beta = \left[R_1 r_p + R_1 R_k + R_k r_p\right]\frac{1}{C}$$

then

$$\alpha \frac{di_2}{dt} + \beta i_2 = 0$$

$$i_2 = i_{20}\, e^{-\frac{\beta}{\alpha}t} \tag{12}$$

$$\frac{di_2}{dt} = -\frac{\beta}{\alpha}\, i_{20}\, e^{-\frac{\beta}{\alpha}t} \tag{13}$$

At this point it is now possible to find e_k, which is the desired variable since this is the output potential. Substituting first equation (9) and then (10) into (7) gives

$$\frac{de_K}{dt} = \frac{-R_K r_p (R_2 + R_1)}{R_1\left[r_p + (\mu+1)R_K\right]}\frac{di_2}{dt} + \frac{R_K}{R_1 C}\frac{\mu R_1 - r_p}{r_p + (\mu+1)R_K}\, i_2$$

Whereupon, substituting for i_2 and di_2/dt from (12) and (13) leaves

$$\frac{de_K}{dt} = \left[\frac{C r_p \beta (R_1 + R_2)}{\alpha} + \mu R_1 - r_p\right]\frac{R_K\, i_{20}\, e^{-\frac{\beta}{\alpha}t}}{C R_1\left[r_p + (\mu+1)R_K\right]}$$

Then, substituting for α and β gives

$$\frac{de_K}{dt} = \frac{R_1 R_K}{C}\frac{\left[r_p^2 + r_p R_1 + r_p R_2 \mu + r_p R_K \mu + R_2 R_K \mu(\mu+1)\right] i_{20}\, e^{-\frac{\beta}{\alpha}t}}{\left[r_p R_1 R_2 + r_p R_1 R_K + r_p R_K R_2 + R_1 R_2 R_K(\mu+1)\right]\left[r_p + (\mu+1)R_K\right]}$$

By setting

$$\gamma = \frac{i_{20} R_1 R_K}{C}\frac{r_p^2 + r_p R_1 + r_p R_2 \mu + r_p R_K \mu + R_2 R_K \mu(\mu+1)}{\left[r_p R_1 R_2 + r_p R_1 R_K + r_p R_2 R_K + R_1 R_2 R_K(\mu+1)\right]\left[r_p + (\mu+1)R_K\right]} \qquad (14)$$

the equation becomes

$$\frac{de_K}{dt} = \gamma\, e^{-\frac{\beta}{\alpha}t}$$

which, when integrated gives

$$e_K = e_{K0} + \frac{\alpha}{\beta}\gamma\left[1 - e^{-\frac{\beta}{\alpha}t}\right]$$

and if $\frac{\beta}{\alpha}t \ll 1$, the exponential function can be expanded and only the first order term retained so that

$$e_K = e_{K0} + \frac{\alpha}{\beta}\gamma\frac{\beta}{\alpha}t$$

or

$$e_K = e_{K0} + \gamma t \qquad (15)$$

From this it is evident that γ is the slope of the sweep curve.

If a particular set of values from the circuit are substituted in equation (14) it is found that

$$\gamma = \frac{71.8}{81.8} \frac{i_{20}}{C}$$

$$\gamma = 0.88 \frac{i_{20}}{C}$$

where $r_p = 3.5k$ and $\mu = 20$.

i_{20} is given by

$$i_{20} = \frac{e_{K0} + V_0 - e_{c0}}{R_2}$$

and for this particular circuit this is 166/150,000 volts/ohm. Therefore

$$\gamma = \frac{71.8}{81.8} \times \frac{166}{150,000} \times \frac{1}{C}$$

$$\gamma = \frac{0.976}{C} \times 10^{-3} \quad \frac{\text{Volts}}{\text{second}}$$

Figure 9 shows the relation between the measured input pulse lengths and the corresponding output pulse amplitudes for the ranges that could easily be checked. Table II gives the expected slopes as calculated from the above expression for the corresponding capacitances as well as the actual slopes measured at the cathode of the cathode follower by an oscilloscope. In actual practice a trimmer capacitor was added where necessary to provide fine adjustment.

The value for γ is critically sensitive to only a few of the parameters of the circuit, since in practice $R_2 > R_1 > r_p$, R_k. If the terms which do not appreciably affect γ be neglected, then one is left with

A_{20} is given by

$$A_{20} = \frac{[illegible]}{[illegible]}$$

and for this particular electrode [illegible]

Therefore

$$\gamma = \frac{[illegible]}{[illegible]}$$

$$\gamma = \frac{[illegible]}{C}$$

Figure 8 shows the relation between the [illegible] put pulse lengths and [illegible] tudes for the [illegible] gives the expected [illegible] pression for [illegible] actual slopes [illegible] by an oscilloscope. [illegible] was added [illegible]

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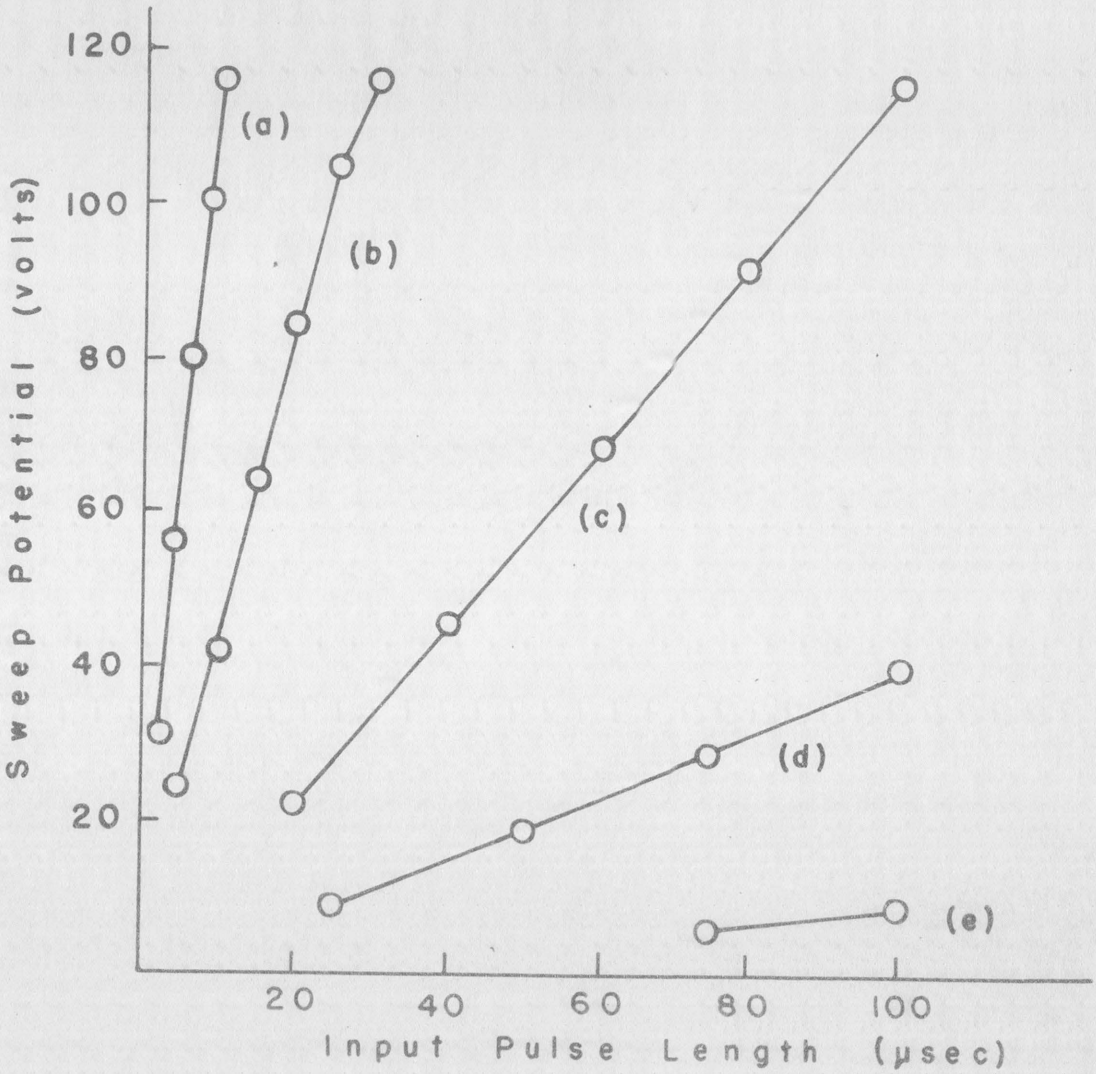

(a)	C_1 = 50 μμf	Slope = 11.6 v/μsec
(b)	C_2 = 200 μμf	Slope = 3.86 v/μsec
(c)	C_3 = 650 μμf	Slope = 1.16 v/μsec
(d)	C_4 = 2000 μμf	Slope = 0.400 v/μsec
(e)	C = 7000 μμf	Slope = 0.114 v/μsec

Figure 9

Input Pulse Length vs Sweep Pulse Amplitude

TABLE II

ACTUAL AND EXPECTED SLOPES
FROM THE LINEAR SWEEP CIRCUIT WITH
CORRESPONDING CAPACITANCES

Maximum Rise Time (μsec)	C (μμf)	Expected Slope Volts/μsec.	Actual Slope Volts/μsec.
1000	7000	0.139	0.114
300	2000	0.488	0.400
100	650	1.50	1.16
30	200	4.88	3.86
10	50	13.9	11.6

$$\gamma = \frac{i_{20} R_1 R_k}{C} \frac{r_p R_2 \mu + R_2 R_k \mu(\mu+1)}{[R_1 R_2 r_p + R_1 R_2 R_k (\mu+1)][r_p + (\mu+1) R_k]}$$

which, upon further simplification, reduces to

$$\gamma = \frac{i_{20}}{C} \frac{R_k \mu}{r_p + (\mu+1) R_k} = 0.88 \frac{i_{20}}{C} \tag{17}$$

It can be noted here that the value of the slope is insensitive to small variations in R_1. In fact, the remaining expression for γ is that for the initial slope of a capacitor charging to the potential $[V_0 + e_k - e_c]$ through the resistance R_2 multiplied by the constant, 0.88.

The deviation of the actual slopes from the calculated ones was a matter of some concern. Several possibilities for partial explanation can be found. In the first place, considerable difficulty was encountered in measuring the initial current and the total rise time accurately. On the basis of several different values obtained from several different instruments it appears that the uncertainty in all measured potentials is approximately 10 per cent. In addition to this, the component resistances and capacitances had tolerances of only 10 per cent. In one case the substitution of a supposedly equal-valued capacitor for C resulted in a change in sweep time of almost 20 per cent.

By letting $V = e_{k0} + V_0 - e_{c0}$

and $dV = 0.1 V$

the root-mean-square deviation of can be calculated from

$$\gamma = \frac{4 \ldots}{c} \ldots \text{[illegible]}$$

which, upon further [illegible] reduces to

$$\gamma = \frac{4 \ldots}{c} \ldots \text{[illegible]}$$

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tive to small variations in R [illegible] ex-
pression for γ [illegible]
changing to the potential [illegible]
ance R_s multiplied by [illegible].

The deviation [illegible]
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for partial explanation [illegible],
considerable difficulty [illegible]
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basis of several [illegible]
Fenwal instruments [illegible] that the uncertainty in the
measured potentials is approximately [illegible]. In addi-
tion to this, the component [illegible]
tolerances of only 10 per cent. [illegible]
of a supposedly [illegible]
change in sweep time of [illegible].

By letting $V = e_m$ [illegible]

and

$$dV = d/V \ldots \text{[illegible]}$$

the root-mean-square [illegible]

$$\overline{e^2} = \sum_{i=1}^{w} \left[\frac{\partial f(x_i)}{\partial x_i}\right]^2 \overline{\Delta x_i^2} \quad [1]$$

Using this on equation (16) gives for $\overline{d\gamma^2}$

$$\overline{d\gamma^2} = \left[\frac{\mu}{r_p/R_k + \mu + 1}\right]^2 \left[\frac{V}{R_2 C}\right]^2 \left[\frac{\overline{dV^2}}{V^2} + \frac{\overline{dR_2^2}}{R_2^2} + \frac{\overline{dC^2}}{C^2} + \frac{\overline{dr_p^2}}{R_k^2} + \frac{r_p^2}{R_k^4}\overline{dR_k^2}\right]$$

and for the particular value C = 0.002 mfd, $d\gamma$ is

$$d\gamma = 0.136 \times 10^6 \ \frac{\text{volts}}{\text{second}}$$

The measured value is 0.400 ± 0.04 volts/microsecond and that calculated is 0.488 volts/microsecond. Thus, the uncertainty in the calculated value because of propagation of errors is larger than the difference between the calculated and the observed values.

One might expect a systematic deviation to appear because of stray capacitances in leads, in the 6J6, and in the VR tube; but the figures obtained do not indicate such a deviation. If it does exist it is overshadowed by the deviation calculated above.

There also remains the possibility of stray inductances producing a lag in the rise time. With the current through the cathode changing at a rate of 10^6 volts/second, an inductance of one microhenry would produce a back emf of 1 volt. However, although the cathode resistor was wire-wound and would seem to be a good source of the emf, there

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[1] W. Edwards Deming, Statistical Adjustment of Data (New York: John Wiley & Sons, Inc., 1944), p. 40.

was no appreciable change noted when a carbon two-watt resistor was put in its place. Of course, the current of 75 ma. going through this resistor heated the carbon quite rapidly and could have changed its resistance.

Another point of interest was the fact that the total increase in the rising potential was limited to 120 volts. Whenever the potential neared this point a slight rounding off occurred and then the potential leveled off.

The answer is found to lie with the existence of grid current in the cathode follower which, up to now, has been assumed to be negligible.

In order to check for grid current a resistor of 2.2 K was inserted in the grid circuit. When the capacitor, C, was charged the potential across this resistor, as measured on an oscilloscope, was 2 volts which corresponds to a grid current of about 1 ma.

This grid current produces a potential drop of 150 volts across R_2. The charging current, i_2, is also of the order of 1 ma. as shown in equation (16). Therefore, when both currents flow through R_2 a potential drop of 300 volts is produced across R_2 which sets a maximum of 150 volts for e_c since the supply potential is 450 volts. The maximum is reduced somewhat by the current in the VR tube which maintains a potential drop across R_1.

Grid current also has another effect on the output pulse. Because the maximum e_k is 120 volts while $e_k + V_o$

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Another point of difference [illegible] was that the increase in the [illegible] potential was limited to 150 volts. Whenever the potential reached this value a sudden [illegible] off occurred and then the potential leveled off.

The answer is found [illegible] with the [illegible] of grid current in the cathode follower which, at first, had been assumed to be negligible.

In order to check for grid current, a resistor of [illegible] K was inserted in the grid circuit. When the [illegible] was charged the potential across this resistor, as measured on an oscilloscope, was [illegible] volts which represents a grid current of about 1 [illegible].

This grid current produced a potential of [illegible] volts across R_g. The charging current, i_c, is also of the order of 1 ma. and in the same direction. Therefore, when both currents flow through R_k a potential of [illegible] volts is produced across R_k which sets a maximum of 150 volts for e_o since the supply is limited to 300 volts. This maximum is reduced somewhat by the current in the [illegible] which maintains a potential drop across R_k.

Grid current also has another effect on the [illegible] pulse. Because the [illegible]

attempts to reach 450 volts due to the current through R_2 the VR tube draws so little current it will no longer regulate. Thus, i_2 no longer remains constant and the slight rounding off results on the top of the pulse, leaving the rest quite linear.

The rate with which the output pulse returns to zero is a function of the first tube of the sweep circuit. The 5963 was picked as giving the sharpest drop from among a group of similar tubes that included a 12AT7, a 12AU7, and a 12AX7.

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CHAPTER III

CONCLUSION

A circuit has been developed to perform the required function of producing a pulse whose height is accurately proportional to the length of an input pulse. A required gating function is also included.

An elementary analysis of the circuit predicts the observed behavior of the circuit within expected limits of error.

Some difficulty may be encountered in counting very long pulses. If the counter input time constant is small, the long pulses will be differentiated. This could be remedied by first differentiating and then re-amplifying the abrupt drop at the end of the sweep pulse; but, since this drop is not linear, one stands to lose some of the linearity just obtained in the sweep circuit. This is especially true for short pulses where the drop at the end is most non-linear.

The author is indebted to Professor John R. Green for suggesting this analyzer and for much advice while it was being constructed.

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