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A Nuclear Magnetic Resonance Absorption Experiment

Richard Runge

Runge — Nuclear Magnetic Resonance Absorption Experiment

A NUCLEAR MAGNETIC RESONANCE ABSORPTION EXPERIMENT

A Thesis

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In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Physics

by

Richard John Runge

June 1949

This thesis, directed and approved by the candidate's committee, has been accepted by the Graduate Committee of the University of New Mexico in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

France P. Scholes
DEAN

May 17 - 1949
DATE

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by

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MASTER OF SCIENCE

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Thesis committee

[illegible]

ACKNOWLEDGEMENT

Much of the theoretical material in the first two chapters was gotten from the work of Hammermesh[5], and indirectly from the work of Schwinger, as well as the papers on the subject by Bloch and his co-workers.[1,2,3] A good deal of the discussion in the first chapter is derivative from the paper of Blombergen, Purcell, and Pound.[4] The notation used from page 12 on is often that of Bloch's.[1]

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1. - F. Bloch, "Nuclear Induction," Phys. Rev., 70:7-8, 1946.

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INTRODUCTION

In nuclear magnetic resonance absorption work, the detection of the absorption of energy by protons situated in a steady strong magnetic field and subject to a weak radiofrequency (RF) magnetic field is attempted. The protons are present in a sample placed inside an RF coil. A radiofrequency current, present in this coil, produces a rapidly oscillating magnetic field which is capable of supplying energy to the system of protons in the steady strong magnetic field. This absorption of energy by the protons alters their relative numerical distribution among the energy levels available to them.

The sample, with the surrounding RF coil, is placed in a bridge circuit and the bridge is balanced. When the large magnetic field is slowly brought to a critical value, dependent on the gyromagnetic ratio of protons and the frequency of the RF field, a condition of resonance is attained. This state of resonance is characterized by a greatly increased absorption of energy from the RF field by the protons. When this occurs, the effective Q of the coil is slightly changed and the bridge is unbalanced.

The detailed discussion of the theory of nuclear magnetic resonance absorption (NMRA) and the description of an attempt at the experimental detection of the effect in

the protons of a paraffin sample form the subject of this thesis. Experiments of this type give the gyromagnetic ratio of the nucleus involved directly. From a knowledge of the nuclear spin, one can then infer the magnetic moment of the nucleus.

The theory of the effect will be considered in the first two chapters following this introduction. The present experimental attempt had to remain unsuccessful, mainly due to the fact that a sufficiently homogeneous magnetic field could not be established with the equipment available. Nevertheless, the experiment is described in detail in Chapters III and IV, in order to give a complete account of the work done and in order to furnish aid for future experiments on NMRA.

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CHAPTER I

PRELIMINARY QUALITATIVE DISCUSSION OF THE THEORY

We shall first consider the steady condition of a system of N non-interacting nuclei present in a strong homogeneous magnetic field of strength H_o oersteds parallel to the z-axis. If each nucleus posesses a spin of $I\hbar$, where I is a fixed nuclear spin quantum number, and in addition a magnetic moment μ, then the result is that a nucleus can have any one of 2I + 1 orientations with respect to the z-axis. Each of these orientations is characterized by a magnetic quantum number, m, in the range $-I \leq m \leq I$. If I is integral so is m; but if I is $\frac{1}{2}$-integral, then m is also. In the case of protons, $I = \frac{1}{2}$, hence $m = \pm\frac{1}{2}$. For those nuclei such that m = I we have a maximum component of spin in the direction of H_o, and hence the lowest possible energy. On the other hand, for those nuclei for which m = -I, we have a maximum component of spin in opposition to H_o and hence the highest available energy. The spacing between the energy levels is constant and equal to $\mu H_o/_I$ ergs.

The distribution of nuclei in the various levels follows Maxwell-Boltzman statistics, the expression for the number of nuclei in the i-th level being:

$$N_i = N \; \exp\left\{\mu H_o \, {}^{i}/IkT\right\} \Big/ \sum_{n=-I}^{+I} \exp\left\{\mu H_o \, {}^{n}/IkT\right\} \quad (1)$$

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where $-I \leqslant i \leqslant I$, k is Boltzmann's constant, and T the spin temperature. From (1) it follows that most nuclei occupy states of lower energy, relative numbers decreasing through successively higher quantum states (smaller i).

If a weak radiofrequency magnetic field of peak value $2H_1$ oersteds and of the form $2H_1 \cos \omega t$ is applied to this system along the x-axis, the strong field H_o being maintained in the + z direction, then the nuclei may absorb energy from this RF field. This would tend to alter their relative distributions among the energy levels available to them. Transitions between levels would follow the same selection rules as those governing similar transitions in atomic spectra, namely $\Delta m = \pm 1$. In case $\Delta m = 1$, we have a case of emission; but if $\Delta m = -1$, we have absorption of energy by the nucleus. In effect, the nucleus absorbs a quantum of energy $h\nu$ equal to $\mu H_o/_I$ from the RF field and is left in a higher energy state (lower m). The condition $h\nu = \mu H_o/_I$ or equivalently $\hbar\omega = \mu H_o/_I$ establishes the condition for resonance between the RF field and the system of nuclei present in the field H_o. Clearly the greatest probability for an absorptive transition will prevail when ω, the frequency of the RF field satisfies,

$$\omega = \mu H_o/_{I\hbar} \quad . \tag{2}$$

where $-I \leq i \leq I$, [illegible]
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$2H_1$ oscillating [illegible]
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Clearly the greatest probability [illegible]
will prevail when [illegible]

$$\omega = \mu H_0/I\hbar \quad (1)$$

If we set $\gamma = \mu/I\hbar$ where γ is the ratio of magnetic moment to spin, or in other words the gyromagnetic ratio of the nucleus, we can then write the resonance condition (2) in the form

$$\omega = \gamma H_o \quad . \tag{3}$$

Equation (3) will occur later on through a different analysis.

If emission were negligible, many nuclei would in time populate the highest energy levels. However, in any actual laboratory situation, the system of nuclei in the field H_o, which we shall refer to as the spin-field system, is not a perfect trap for absorbed energy. The energy absorbed by the spin-field system from the RF field is readily transferred to other degrees of freedom of the system as a whole.

As more energy is absorbed by the spin-field system and higher energy levels become more populated, part of this absorbed energy goes into thermal energy of the nuclear environment. This interaction between the spin-field system and the thermal environment of the nuclei is called the spin-lattice interaction. Associated with this interaction is a characteristic time T_1, called the spin-lattice relaxation time. This time, T_1, is a measure of the rate at which the spin-field system approaches thermal equilibrium with its environment, after a sudden application of the field H_o to

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the sample containing the nuclei in question. Its value, for any given nucleus, will vary depending on the substance used to present the nuclear species as well as the state of the substance, etc. The exact way in which T_1 enters into the theory of NMRA (Nuclear Magnetic Resonance Absorption) will be postponed until Chapter II. T_1 usually ranges from 10^{-2} to 10^2 sec., depending on the substance.

So far in our discussion, we have neglected to consider spin-spin interaction between neighboring nuclei through the medium of their magnetic moments. There may also be a perturbing effect due to electronic magnetic moments such as those present in substances containing paramagnetic ions. These latter, when present, are a serious perturbation; but in many substances they are not present and hence we need only to consider coupling between adjacent nuclei.

For two magnetic dipoles of strength μ, separated by a distance r, the order of magnitude of the interaction energy is μ^2/r^3 ergs. If $H_o\mu \gg \mu^2/r^3$, the spin-field energies will greatly exceed these spin-spin coupling energies and the latter will essentially contribute to a resonance broadening of the former. Bloch[1] calculates the strength of these internuclear fields to be about 1 gauss in order of magnitude, hence their contribution will be small in the presence of a field H_o of several thousand gauss. In absorptive transitions between the principal energy levels of the

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$\Delta t \approx \hbar/\Delta E$

spin-field system, the lifetime against transitions will be of the order of $\hbar r^3/\mu^2$ sec. This suggests a second characteristic time $T_2 = \hbar r^3/\mu^2$ associated with spin-spin coupling. Small values of T_2 imply strong spin-spin coupling and broader lines, while larger values imply weak spin-spin coupling and sharper lines. T_2 will be introduced at the appropriate place in Chapter II.

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CHAPTER II

QUANTITATIVE DISCUSSION OF THE THEORY

The wave-mechanical treatment of non-interacting nuclei with magnetic moment $\vec{\mu}$ in a magnetic field $\vec{H}$ is obtained by considering the Hamiltonian, $\mathcal{H}$, for a nucleus:

$$\mathcal{H} = -\vec{\mu} \cdot \vec{H} \tag{1}$$

We shall assume $\vec{H}$ is independent of the co-ordinates but shall consider $\vec{H}$ a function of the time, t. The spin $\vec{s}$ of the nucleus will be given in terms of $\hbar$ and the spin quantum number I as

$$s = |\vec{s}| = I\hbar \quad (= \sqrt{I(I+1)}\,\hbar \text{ exactly}) \tag{2}$$

Although the magnitude of $\vec{s}$ is fixed, its direction is not. It is convenient to introduce the gyromagnetic ratio, γ, of the nucleus as the ratio of magnetic moment to spin,

$$|\gamma| = \mu/s \qquad \text{or} \qquad \gamma\vec{s} = \vec{\mu} \tag{3}$$

Equations (3) are based on the assumption that spin and magnetic moment are parallel or anti-parallel, and that their magnitudes have a fixed ratio.

From equation (3), we can write equation (1) as

$$\mathcal{H} = -\gamma\vec{s} \cdot \vec{H} \tag{4}$$

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nuclei [illegible]

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Although [illegible]

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$$[illegible]$$

The equation of motion of $\vec{s}$ can be written in the usual way,

$$\hbar/_i \ \dot{\vec{s}} = \left[\mathcal{H}\vec{s} - \vec{s}\,\mathcal{H} \right] \qquad (5)$$

where the right member is the commutator of $\mathcal{H}$ and $\vec{s}$. Equation (5) is an operator equation relating the operator associated with $\dot{\vec{s}}$ to an operator obtained from $\mathcal{H}$ and $\vec{s}$ through their commutator.

Equation (5) can be written as:

$$\hbar/_i \ \dot{\vec{s}} = -\gamma \left[(\vec{s}\cdot\vec{H})\vec{s} - \vec{s}(\vec{s}\cdot\vec{H}) \right] \qquad (6)$$

after using (4). The standard commutation rules for the spin component operators can be used to put the commutator (6) in a useful form, these are:

$$s_x s_y - s_y s_x = i\hbar s_z$$
$$s_y s_z - s_z s_y = i\hbar s_x$$
$$s_z s_x - s_x s_z = i\hbar s_y \qquad (7)$$

Making use of the fact that $\vec{H}$ depends on t only and hence commutes with the operators s_x, s_y, s_z, and then expanding the commutator in (6) we obtain:

$$\begin{aligned}\hbar/_i \ \dot{\vec{s}} = \Big\{ & \vec{i}\,(H_x s_x^2 + H_y s_y s_x + H_z s_z s_x - H_x s_x^2 - H_y s_x s_y - H_z s_x s_z) \\ & + \vec{j}\,(H_x s_x s_y + H_y s_y^2 + H_z s_z s_y - H_x s_y s_x - H_y s_y^2 - H_z s_y s_z) \\ & + \vec{k}\,(H_x s_x s_z + H_y s_y s_z + H_z s_z^2 - H_x s_z s_x - H_y s_z s_y - H_z s_z^2) \Big\}\end{aligned}$$

$$(8)$$

By means of the relations (7), equation (8) can be simplified, giving:

$$\hbar/_i \, \dot{\vec{s}} = -i\hbar\gamma \left\{ \vec{i}\,(H_z s_y - H_y s_z) + \vec{j}(H_x s_z - H_z s_x) + \vec{k}(H_y s_x - H_x s_y) \right\} \quad (9)$$

But the operator equation (9) is simply:

$$\hbar/_i \, \dot{\vec{s}} = -\hbar i\gamma \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ s_x & s_y & s_z \\ H_x & H_y & H_z \end{vmatrix} = -i\hbar\gamma\ (\vec{s} \times \vec{H}) \quad (10)$$

or finally,

$$\dot{\vec{s}} = \gamma\ (\vec{s} \times \vec{H}) \quad (11)$$

An equation can be obtained giving the time dependence of the (quantum mechanical) expectation value of $\vec{s}$. This expectation value of $\vec{s}$ is denoted by $\vec{S}$, it is defined as:

$$\vec{S} = \int \psi^* (\vec{r})\vec{s}\ \psi(\vec{r})\ d\tau \quad (12)$$

where $\psi(\vec{r})$ is a stationary wave function corresponding to the probability that $\vec{s}$ has the orientation $\vec{r}$. The integration is understood to be taken over all orientations $\vec{r}$. Likewise $\dot{\vec{S}}$ can be defined as the expectation value of $\dot{\vec{s}}$, its equation is:

$$\dot{\vec{S}} = \int \psi^* (\vec{r}) \dot{\vec{s}} \, \psi(\vec{r}) \;\; d\tau \tag{13}$$

Multiplying (11) through on the left by ψ^* and on the right by ψ and integrating gives

$$\dot{\vec{S}} = \gamma \, (\vec{S} \times \vec{H}) \tag{14}$$

The steps leading to (14) depend on the fact that ψ is a scalar and that $\vec{H}$, being time dependent only, factors out of the integral. Equation (14) is a vector equation giving the average value of the spins, $\vec{s}$, of the nuclei. It is this average value, taken over a large number of nuclei, which is of interest to us particularly its time variation. The form of (14) is identical to the classical equation of a macroscopic magnetic dipole with gyromagnetic ratio γ in a field $\vec{H}$, spinning about its own axis with an angular momentum $\vec{\Omega}$, namely

$$\dot{\vec{\Omega}} = \gamma (\vec{\Omega} \times \vec{H}) \tag{15}$$

Equation (14) contains the quantum mechanical analogue of Larmor's precession theorem, for setting $H_x = H_y = 0$, and $H_z = H_o$, (uniform field in the +z direction), then the solution of (14) is:

$$\begin{aligned} S_z &= S \cos \theta \\ S_x &= S \sin \theta \sin \omega_o t \\ S_y &= S \sin \theta \cos \omega_o t \end{aligned} \tag{16}$$

Where θ is the constant angle made by $\vec{S}$ with the z-axis (Fig. 1) and where ω_o satisfies

$$\omega_o = \gamma H_o \tag{17}$$

Equations (16) certainly define a precessing vector $\vec{S}$, whose frequency of precession about H_o is given by (17). This is mathematically the same as the resonant condition (3) of Chapter I. The frequency of precession, ω_o, of the quantum mechanical expectation value of $\vec{s}$, namely $\vec{S}$, is called the Larmor precession frequency. The precession of $\vec{S}$ itself is called the quantum mechanical Larmor effect.

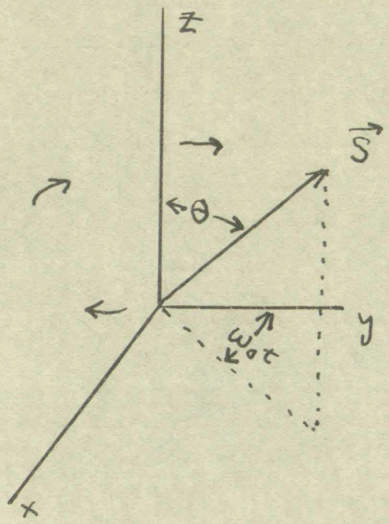

Fig. 1

In the discussion so far, fields due to spin-spin coupling between adjacent nuclei as well as fields due to the spin-lattice interaction have been ignored. These are always present in any real situation and act essentially as strong damping effects. The solutions (16) would have to be modified to include damping terms (decay terms). Actually this damping is so pronounced in most substances that one would expect $\vec{S}$ to rapidly approach a steady alignment in the H_o direction, rather than to precess indefinitely about H_o as equations (16) would indicate.

where θ is the polar angle [illegible]
made by $\vec{E}$ with the z-axis (Fig. [illegible]),
and where ω_p satisfies

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This approach of $\vec{S}$ into alignment with the field, the speed of which is measured by T_1, represents the approach to an equilibrium condition between the spin-field system and the lattice environment, the former releasing its original available energy to the lattice. When no further absorption of energy from the spin-field system is possible, the steady condition reached is thus maintained. In our problem, which is concerned with the possibility of absorption of energy by the spin-field system from an RF field, it can be shown that if conditions for this absorption are satisfactory (i.e., near resonance), then no steady alignment of $\vec{S}$ in one direction will be forthcoming, but rather, that indefinite precession of $\vec{S}$ can be maintained. This continued precession, in spite of damping, can be attributed to a kind of forced oscillation effect due to the presence of the RF field.

The average nuclear magnetic moment per unit volume of a sample of nuclei is denoted by $\vec{M}$. $\vec{M}$ and $\vec{S}$ are simply related, as are $\vec{\mu}$ and $\vec{s}$, namely

$$\vec{M} = h \gamma \vec{S} \tag{18}$$

where γ is the gyromagnetic ratio of the nuclear species in question and n is the concentration of the species (no. per c.c.). If a given sample contains different kinds of nuclei, each with a distinct Larmor frequency in the field, and if resonant conditions hold for one particular nuclear species,

This suppression of lattice vibrations [illegible] with the result, the speed of which [illegible] governed by T_1, [illegible] an equilibrium condition between the spin-field system and the lattice environment, the former [illegible] available energy to the lattice. [illegible] of energy [illegible] condition [illegible]. In [illegible] is concerned with the [illegible] or absorption of energy by the spin-field system [illegible] if conditions [illegible] near resonance, [illegible] tion will be [illegible] cession of S can be maintained. [illegible] in spite of damping, [illegible] oscillation [illegible].

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$$\omega = \gamma \, H$$

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then off-resonance contributions to $\vec{M}$ can be neglected. Hence $\vec{M}$, as defined by (18) can be considered as referring to a single nuclear type.

Combining equations (14) and (18) yields

$$\dot{\vec{M}} = \gamma (\vec{M} \times \vec{H}) \quad . \tag{19}$$

Equation (19) gives the time dependence of the average moment per unit volume. In the solution of equation (19) for any real system in which spin-spin and spin-lattice fields cannot be ignored, it is convenient to revise (19) in such a way that the form of $\vec{H}$ is left so as to include only externally applied fields. These perturbing fields are not explicitly introduced into $\vec{H}$.

One adds terms of the type

$$-(M_z - M_o)/T_1 \tag{20}$$

and

$$-M_x/T_2 \quad ; \quad -M_y/T_2 \tag{21}$$

to the right side of the M_z, M_x, and M_y component equations obtained from (19) respectively (see equations (24) below). The field $\vec{H}$ to be inserted into (19) is simply the applied external field. In (20), M_o is the static equilibrium value attained by $\vec{M}$ in a steady field H_o. It is related to H_o by

the formula

$$M_o = X_o H_o \qquad , \qquad (22)$$

where X_o is the static nuclear paramagnetic susceptibility given by the Curie law:

$$X_o = nI(I + 1)\,\gamma^2 \hbar^2 / 3kT \qquad (23)$$

for the case $\mu H_o << kT$. T_1 and T_2 in (20) are simply the two relaxation times previously discussed.

Writing out the equations in component form after introducing the terms (20) and (21) gives

$$\dot{M}_z = \gamma(M_x H_y - M_y H_x) - (M_z - M_o)/T_1 \qquad (24a)$$

$$\dot{M}_x = \gamma(M_y H_z - M_z H_y) - M_x/T_2 \qquad (24b)$$

$$\dot{M}_y = -\gamma(M_x H_z - M_z H_x) - M_y/T_2 \qquad (24c)$$

in which H_z, H_x, and H_y are the externally applied fields.

The justification of such a procedure in treating the problem is not rigorous, however good results are obtained. In addition, equations (24) show a direct relation between the two relaxation times T_1 and T_2 and the decay constants for M_z, M_x, and M_y.

Before proceeding to the case where H_x and H_y are introduced as weak RF fields, let us first consider the solution of equations (24) when $H_z = H_o$, while $H_x = H_y = 0$.

the formula

$$\tau_c = \kappa \eta_0 [illegible]$$

where τ_c is the [illegible] number [illegible] [illegible] given by the [illegible] law

$$\tau_0 = [illegible] (1 + [illegible]) [illegible] \qquad [illegible]$$

For the case [illegible] [illegible] [illegible] [illegible] that two relaxation times [illegible] [illegible].

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$$\dot{N}_1 = [illegible]$$

$$\dot{N}_2 = [illegible] \qquad [illegible]$$

$$\dot{N}_3 = [illegible] \qquad [illegible]$$

in which N_1, N_2, and N_3 [illegible] [illegible]

The [illegible] [illegible] [illegible] problem is not [illegible], [illegible] [illegible] In addition, equations [illegible] [illegible] [illegible] the two relaxation times [illegible] [illegible] [illegible] for N_1, N_2, and N_3.

Before [illegible] [illegible] [illegible] [illegible] introduced [illegible] [illegible] [illegible] solution of [illegible] [illegible] [illegible] $= 0$.

The equations (24) become:

$$\dot{M}_z = -(M_z - M_o)\ /T_1$$

$$\dot{M}_x = \gamma M_y H_o - M_x/T_2$$

$$\dot{M}_y = -\gamma M_x H_o - M_y/T_2 \qquad (25)$$

The equation in M_z has the solution:

$$M_z = M_o + (M_z(o) - M_o)\, e^{-t/T_1} \qquad (26)$$

while the equation in M_x becomes:

$$\ddot{M}_x + (\omega_o^2 + \frac{1}{T_2^2})\, M_x + {}^2/T_2\, \dot{M}_x = 0 \qquad (27)$$

In equation (27), $\omega_o = \gamma H_o$ as before, the term in $\dot{M}_x$ plainly shows damping of M_x due to spin-spin coupling. The solution of (27) is:

$$M_x = e^{-t/T_2}\ M_x(o) \cos \omega_o t \qquad (28a)$$

with a similar solution for M_y namely,

$$M_y = -e^{-t/T_2}\, M_y(o) \sin \omega_o t \qquad (28b)$$

Hence, if initially $M_z(0) \neq M_o$ and not both $M_x(0)$ and $M_y(0)$

equal zero, then the x and y components of $\vec{M}$ will rapidly damp out, oscillating with a frequency ω_o, while M_z will approach M_o exponentially. In other words, $\vec{M}$ will spiral about the field, finally coming into alignment with it.

The solution of equations (24) for the case of a weak applied RF field in addition to the steady field H_o is, of course, the problem concerned in the NMRA experiment. If the uniform z-field, H_z, is slowly varied with the time, the results will be sensibly the same as though H_z were constant; so that the equations will hold, as H_z is varied slowly about the resonant value $H_o = \omega_o/\gamma$. Let $H_z = H_o$, while $H_x = 2H_1 \cos \omega t$. This oscillating field can be regarded as the superposition of two fields rotating in opposite directions, i.e.,

(a) $\quad H_x = H_1 \cos \omega t \quad , \quad H_y = -H_1 \sin \omega t$

and

(b) $\quad H_x = H_1 \cos \omega t \quad , \quad H_y = H_1 \sin \omega t$

If $\gamma > 0$, the field (a) will rotate in the same direction as $\vec{M}$. However, if $\gamma < 0$, then the field (b) will rotate in the same direction as $\vec{M}$.

In the first case $\vec{M}$ and the field (a) will be in phase while $\vec{M}$ and (b) will be out of phase. If ω is close to the resonant frequency ω_o, the effect of (b) will be

negligible and the effective field will be given by (a). Our concern is with the case $\gamma > 0$ and hence the field (a). Inserting $H_z = H_o$ and (a) into (24) gives:

$$\dot{M}_z = -\gamma H_1 (M_x \sin\omega t + M_y \cos\omega t) - (M_z - M_o)/T_1 \qquad (29a)$$

$$\dot{M}_x = \gamma(M_y H_o + M_z H_1 \sin\omega t) - M_x/T_2 \qquad (29b)$$

$$\dot{M}_y = -\gamma(M_x H_o - M_z H_1 \cos\omega t) - M_y/T_2 \qquad (29c)$$

Introducing two quantities u and v through the equations:

$$M_x = u\cos\omega t - v\sin\omega t$$

$$M_y = -u\sin\omega t - v\cos\omega t \qquad (30)$$

as well as the constants:

$$\alpha = 1/\gamma H_1 T_1 \quad ,$$

$$\beta = 1/\gamma H_1 T_2 \quad ,$$

$$\delta = \frac{\gamma H_o - \omega}{\gamma H_1} = \frac{\omega_o - \omega}{\gamma H_1} \qquad (31)$$

and letting $\tau = \gamma H_1 t$ be the new independent variable, equations (29) become:

$$\frac{dM_z}{d\tau} + \alpha M_z - v = \alpha M_o \tag{32a}$$

$$\frac{dv}{d\tau} + \beta v - \delta u + M_z = 0 \tag{32b}$$

$$\frac{du}{d\tau} + \beta u + \delta v = 0 \tag{32c}$$

As long as energy is supplied to the spin-field system by the RF field as rapidly as it disappears, a steady condition of dynamic equilibrium may be assumed. This amounts to assuming that M_z, u and v do not vary with time, so that $\dot{M}_z$, $\dot{u}$ and $\dot{v}$ all vanish. Equations (32) become:

$$\beta u + \delta v = 0$$

$$\beta v - \delta u + M_z = 0$$

$$M_z - M_o = v/\alpha \tag{33}$$

which have solutions,

$$u = M_z \, \delta/(\delta^2 + \beta^2)$$

$$v = -M_z \, \beta/(\delta^2 + \beta^2) \tag{34}$$

$$M_z = M_o (\delta^2 + \beta^2)/(\delta^2 + \beta^2 + \beta/\alpha)$$

When the value of M_z is inserted the equations for u and v and the values for α, β and δ given by (31) are used,

equations (34) become:

$$M_z = M_o \frac{(\omega - \omega_o)^2 + T_2^{-2}}{(\omega - \omega_o)^2 + T_2^{-2} + \frac{T_1}{T_2}\left(\frac{\omega_o H_1}{H_o}\right)^2} \qquad (35a)$$

$$u = \frac{M_o(\omega_o - \omega)\,\omega_o\, H_1/H_o}{(\omega - \omega_o)^2 + T_2^{-2} + \frac{T_1}{T_2}\left(\frac{\omega_o H_1}{H_o}\right)^2} \qquad (35b)$$

$$v = \frac{-M_o\omega_o\, H_1/H_o T_2}{(\omega - \omega_o)^2 + T_2^{-2} + \frac{T_1}{T_2}\left(\frac{\omega_o H_1}{H_o}\right)^2} \qquad (35c)$$

M_z is given directly by (35a). At resonance, i.e., when $\omega = \omega_o$, M_z has the constant value:

$$M_z = \frac{M_o}{1 + T_1T_2(\gamma H_1)^2} \qquad (36)$$

On the other hand, inspection of (35b) shows that u = 0, while from (35c) it follows that v_{res} is a maximum given by:

$$v_{res} = \frac{-M_oT_2\,\gamma H_1}{1 + T_1T_2\,\gamma^2 H_1^{\,2}} \qquad (37)$$

Hence at resonance the equations for M_x and M_y are found from

(30 and (37) to be:

$$M_x = \frac{M_o T_2 \gamma H_1}{1 + T_1 T_2 \gamma^2 H_1^{\,2}} \sin \omega_o t \tag{38a}$$

$$M_y = \frac{M_o T_2 \gamma H_1}{1 + T_1 T_2 \gamma^2 H_1^{\,2}} \cos \omega_o t \tag{38b}$$

which give a steady precession of $\vec{M}$ making a fixed angle with H_o.

As resonance is approached, by allowing H_z to slowly approach H_o, $\vec{M}$ will begin to precess about H_z as indicated by equations (30) and (35). The amount of energy absorbed from the RF field, near resonance, will be directly indicated by the extent $\vec{M}$ deviates from alignment in the direction of H_z. From (30) it follows that the component of $\vec{M}$ normal to H_z is given by $\sqrt{u^2 + v^2}$. At resonance u = 0, hence this normal component is only v. Therefore the magnitude of v is an indication of the amount of absorption of energy by the spin-field system from the RF field.

Then in order to achieve maximum absorption at resonance it is desirable to make v_{res} as large as possible. Inspection of (37) shows that for a fixed γ, T_1 and T_2, v_{res} depends critically on H_1. Calculating the derivative of v_{res} with respect to H_1 and setting the result equal to zero gives the result that v_{res} is a maximum, when H_1 is

equal to H_1^*, where:

$$H_1^* = {}^1/_\gamma \sqrt{T_1T_2} \tag{39}$$

For fields H_1 far from H_1^*, absorption will be poor. This of course is important in any bridge type detector, since the change in effective $1/Q$ depends upon the size of the absorption.

The value of v_{res} for $H_1 = H_1^*$, which is denoted by v^*, and hence the maximum value of the $\vec{M}$ component normal to H_z is given from (37) and (39) as:

$$v^* = \frac{-M_o}{2}\sqrt{{}^{T_2}/_{T_1}} \tag{40}$$

The ratio of v_{res} for $H_1 = kH^*$ to v^* can be obtained from (37) and (40), denoting this ratio by $R(k)$ one obtains

$$R(k) = {}^{2k}/_{(1 + k^2)} \tag{41}$$

Equation (41) gives essentially a universal resonance curve for the absorption. If we set $R(k) = \frac{1}{2}$ and solve (41), k values equal to 0.26795 and 3.7205 are obtained. To state the matter in approximate terms, the size of v_{res} is decreased by one-half provided H_1 is about four times greater than H_1^* or one-fourth as great as H_1^*. This shows insensitivity to variations in H_1 as far as the magnitude of the absorption is concerned.

equal to r_1^2, where

[illegible]

For field H_1 [illegible] of course [illegible] the change in [illegible] absorption.

The value of [illegible] v^2, and hence the [illegible] H_s is given from (37) and (38) as:

[illegible]

The [illegible] (37) and (40), [illegible]

[illegible] (41)

Equation (41) [illegible] curve for the absorption [illegible] (1)). [illegible] value [illegible] the [illegible] by one-half [illegible] or one-fourth [illegible] variations in H_s [illegible] is concerned.

Denoting by $\Delta\omega$ the quantity $\omega_o - \omega$ and putting $H_1 = H_1^*$, equation (35c) becomes:

$$v(\Delta\omega) = \frac{2v^*}{2 + T_2^2 \ \Delta\omega^2} \tag{42}$$

The ratio of $v(\Delta\omega)$ to v^*, denoted by $K(\Delta\omega)$ is then given by:

$$K(\Delta\omega) = \frac{2}{2 + T_2^2 \ \Delta\omega^2} \tag{43}$$

K is a measure of the decrease of v from resonance, even though H_1 is optimum. Setting $K(\Delta\omega) = \frac{1}{2}$ to get the "half-width" for v at optimum H_1 and solving for $\Delta\omega$ gives a frequency spread:

$$\Delta\omega_{\frac{1}{2}} = \pm \sqrt{2}/T_2 \tag{44}$$

Since in many cases $T_2 = 10^{-4}$ sec., it follows that in general $\Delta\omega_{\frac{1}{2}}$ is of the order of 1.4×10^4 or $\Delta\nu_{\frac{1}{2}} = 2330$ cycles per sec. Equation (44) gives the deviation of ω from ω_o necessary to diminish v by one-half when H_1 is optimum (i.e., when $H_1 = H_1^*$).

On the other hand, keeping ω as the fixed quantity ($\omega = \omega_o$), then the variation of H_z from H_o, namely $\Delta H_{\frac{1}{2}}$, which would diminish v by one-half is given by:

[illegible] $H_1 = H_1^*$, equation [illegible] becomes:

$$[illegible]$$

The ratio of [illegible] by:

$$[illegible]$$

K is a measure of the [illegible] though H_1 is [illegible] "half-width" for [illegible] a frequency [illegible]:

$$\Delta\omega_{\frac{1}{2}} = [illegible]$$

[illegible] general [illegible] per sec. Equation [illegible] measurement [illegible] when $H_1 = H_1^*$.

On the other hand, [illegible] $(\omega = \omega_0)$, [illegible] would [illegible]

$$\Delta H_{\frac{1}{2}} = \Delta\omega_{\frac{1}{2}}/\gamma \tag{45}$$

hence:

$$\Delta H_{\frac{1}{2}} = \pm \sqrt{2}/\gamma T_2 \tag{46}$$

Taking $T_2 = 10^{-4}$ and taking the value $\gamma = 2.672 \times 10^4$ for protons gives a value $\Delta H_{\frac{1}{2}} = 0.53$ gauss. This figure is an indication of the sensitivity of the method to inhomogenieties present in the field H_z. The small value of $\Delta H_{\frac{1}{2}}$ would indicate a sizeable amount of absorption from only a small part of a proton sample in a field which was too inhomogeneous.

Inserting for H_1 in (38a,b) the optimum value given by (39) yields for M_x and M_y at resonance:

$$M_x = \frac{M_o}{2}\sqrt{T_2/T_1}\,\sin\omega_o t$$

$$M_y = \frac{M_o}{2}\sqrt{T_2/T_1}\,\cos\omega_o t \tag{47}$$

Comparing equations (47) with the equations (a) for the rotating field:

$$H_x = H_1 \cos\omega_o t$$

$$H_y = -H_1 \sin\omega_o t \tag{48}$$

shows that $\vec{M}_{xy}$, the component of $\vec{M}$ perpendicular to H_z rotates about H_z in the same direction as the field (48), i.e., clockwise, and lags this field by 90°.

It is instructive to look at the condition of resonance from the point of view of its effect on the RF voltage across the coil used to produce the RF field. Assuming that the coil has zero resistance, if the voltage across the coil is $E_o \sin \omega_o t$, then the current will be $E_o/L\omega_o \cos \omega_o t$, where L is the inductance. The actual field present (equivalent to the rotating field (48) at resonance) is in the x-direction, which we take to be the coil axis. This field is proportional to the current. The x-component of $\vec{M}$, at resonance and with optimum H_1, will induce a small voltage across the coil. This voltage, V(t) will be proportional to $-\dot{M}_x$ and hence vary as $-\cos \omega_o t$, and thus will lead the applied voltage $E_o \sin \omega_o t$ by 90°. The presence of coil resistance introduces a slight phase shift.

If the coil has N turns and cross sectional area A, the total flux Φ due to M_x is:

$$= 4\pi N A M_x \tag{49}$$

hence the induced voltage (in volts) will be:

$$V(t) = -10^{-8}\,\dot{\Phi} = \frac{-2\pi NA}{10^8}\,\omega_o M_o \sqrt{T_2/T_1}\,\cos \omega_o t \tag{50}$$

in view of (47).

The magnitude of the peak value of this induced voltage will be:

$$V_{peak} = \frac{2\pi NA\,\omega_o^2}{10^8}\sqrt{T_2/T_1}\;\chi_o \tag{51}$$

in view of (22).

Using (23) we get:

$$V_{peak} = \frac{2\pi NA\,\omega_o^2}{3kT\;10^8}\sqrt{T_2/T_1}\;nI(I+1)\,\gamma\hbar^2 \tag{52}$$

CHAPTER III

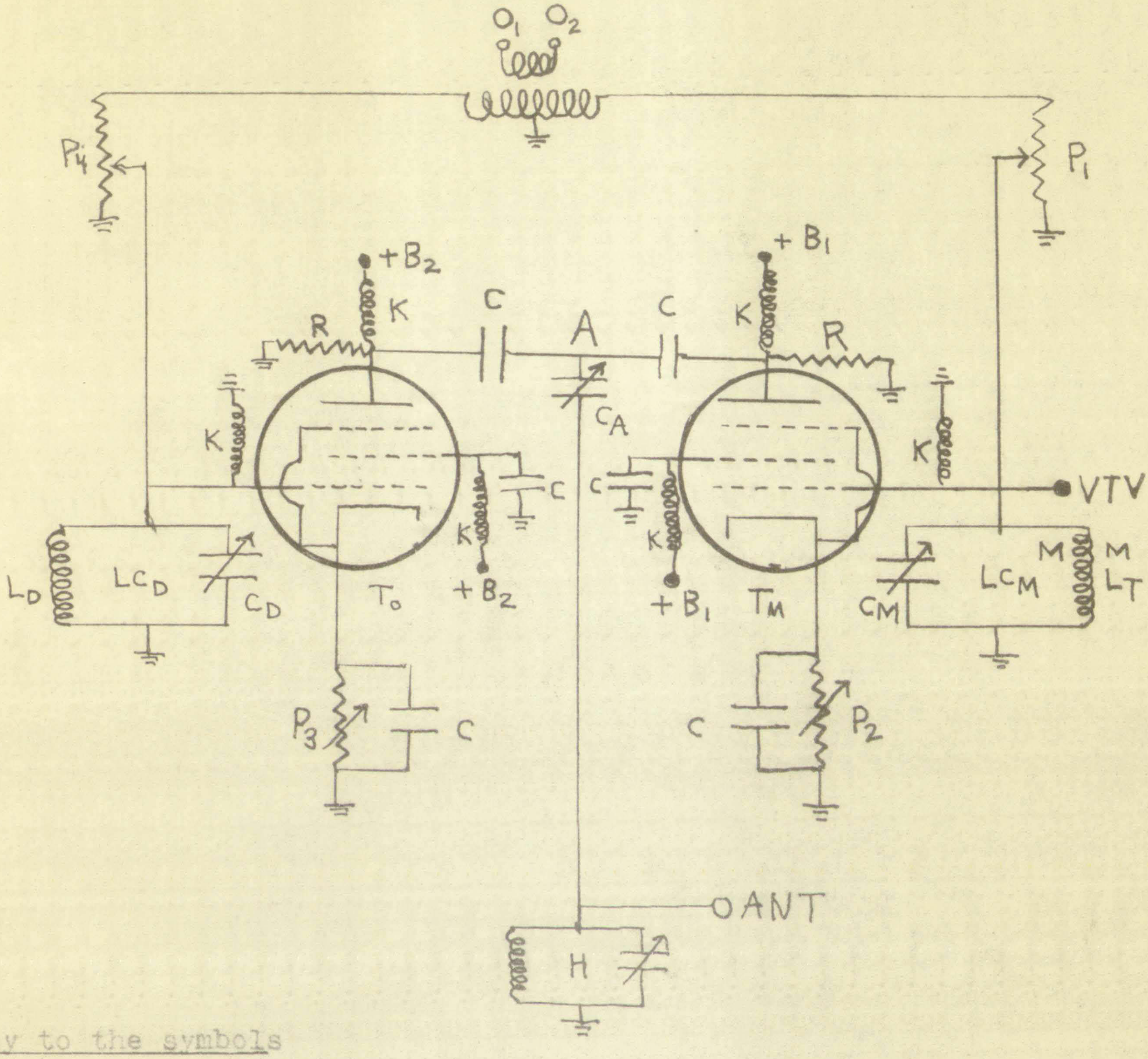
DESCRIPTION OF THE APPARATUS AND OF THE METHOD

The experimental arrangement for attempting to detect the NMRA signal at resonance consisted of a well shielded RF bridge circuit (Fig. 1) with two tuned (LC) circuits in the arms of the bridge. One tuned circuit (LC_D) was adapted from a transmitter tuning unit TU-9-B and was completely enclosed in the aluminum chassis provided with the unit. This "dummy" element was intended to provide balance against another tuned circuit (LC_M) which contained the toroidally wound RF coil with paraffin sample situated in the field of the electromagnet.

The two arms of the bridge were supplied with RF current from the ends of a secondary coil whose center tap was grounded and which was driven by a crystal oscillator at 8 megacycles/sec. Thus the signals in the alternate arms of the bridge were out of phase by 180°. Adjustment of the amplitudes of the signal voltages applied to the two tuned circuits in alternate arms of the bridge was possible through potentiometers P_1 and P_4.

The two tuned circuits were tuned to resonance with the RF current and the voltages appearing on the input sides of these elements were applied to the grids of two 6AU6 pentodes (T_M and T_o). The output, off the plate, of these two tubes

Fig. 1 - The Bridge Circuit



Key to the symbols

L_T - Toroidal RF coil
L_D - Dummy RF coil
C_M - Variable condensor in magnetic arm
C_D - Variable dummy condensor
K - RF chokes, 20-30 henries
C - 470 mmf.
LC_M - Tuned circuit with paraffin sample in magnetic field
LC_D - Dummy tuned circuit
A - Junction point for signals in bridge arms
P_i - Variable potentiometers
ANT- Receiver antenna binding post
VTV- Lead to Vacuum tube voltmeter (see Fig. 2)

O_1, O_2-Oscillator tank
R - 1 megohm
T_M, T_O -6AU6 pentodes
M-Magnet pole faces

was fed through small condensors to a common point A, (see Fig. 1). The object, in balancing the bridge, was to make the signals appearing from alternate sides at A equal in amplitude by adjusting the tuning and the potentiometers P_1.. P_4. Since these signals were out of phase by 180°, one could expect zero signal at A when the signal amlitudes from the two sides were equal. From A, a variable condensor is led to the antenna post of a receiver. The point of contact is bypassed to ground through a tuned circuit (H) at resonance with the RF current. The purpose of the tuned circuit was to shunt harmonics and noise to ground, these harmonics originating in the pentodes. The two pentodes themselves served the purpose of decoupling the two sides of the bridge more effectively than direct contact at A would have allowed. In addition, these tubes provide some amplification. Since the inter-electrode capacitance of the tubes is very small, very little direct coupling between the input points of the two tuned circuits occurs. On the other hand, no loss of signal strength occurs with this arrangement.

These tubes introduce, however, distortion (& harmonics) due to the fact that the tubes are near cut-off during part of the cycle. These harmonics can be shunted to ground through the tuned circuit (H) while the fundamental signal finds the tuned circuit of higher impedance.

In order to balance the bridge initially, the receiver

is set at maximum RF and AF gain while direct contact with the antenna post is postponed (actually the lead from A is placed in very close proximity with the antenna post). The BFO* of the receiver is switched on and the audio output is fed into earphones or an AF voltmeter. Approach to balance is indicated by decreased intensity of the audio output. The lead from A is brought into closer proximity with the antenna post until actual contact is made. Further adjustment can then be made until the audio output is a few millivolts, as indicated by the AF* voltmeter. The BFO can then be switched off, provided the balance is reasonably steady.

At this point we have approximate balance (perfect balance is out of the question, due to slight leakages, etc.) and the experiment can be begun. If the bridge and the receiver are well shielded (all chassis are connected to a single ground), the receiver should pick up very little signal from any source other than the lead coming in from A. If a significant fraction of the total signal picked up comes from sources other than the lead from A, the possibility of sensitive balance of the bridge is small. In any RF bridge (even at about 8 megacycles) this is always a problem since unshielded elements and leads radiate and pick up radiation.

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* - BFO is an abbreviation for "Beat Frequency Oscillator".
* - AF stands for "Audio Frequency".

is set at maximum [illegible]

the antenna post [illegible]

placed in very close [illegible]

ends of the receiver [illegible]

fed into [illegible]

is indicated by [illegible]

The lead from [illegible]

antenna post [illegible]

ment can then be made [illegible]

volts, as indicated by [illegible]

be switched off [illegible]

At this point [illegible]

balance is out of the [illegible]

and the experiment can be [illegible]

receiver are [illegible]

single [illegible]

signal from [illegible]

If a [illegible]

comes from [illegible]

of sensitive [illegible]

bridge [illegible]

since [illegible]

ation.

[illegible]

The bridge can never be balanced as far as noise is concerned, hence the receiver must have some arrangement whereby incoming noise is suppressed. The receiver used was provided with a crystal filter for this purpose. This is an important factor. A bridge of this type might be balanced as far as input signal of a single frequency was concerned, but this is of no use in detecting slight changes in the degree of balance if these latter were below the noise level in magnitude.

The "magnetic" tuned circuit, LC_M, was connected from its ungrounded side to a vacuum tube voltmeter arrangement VTV (see Fig. 2) which provided means of measuring the peak value of the RF voltage across the coil. From the coil inductance, the RF current could then be measured and from the coil dimensions, the field $2H_1$ acting on the protons in the sample could be calculated. With no incoming signal from the oscillator, the bias voltage on the tube T_V (see Fig. 2) is set by means of R_1 until zero current is indicated on milliameter I. When a steady RF voltage is applied from the oscillator, the tube passes current and I no longer reads zero. By means of the potentiometer R_2, increment of bias of a few millivolts can be added until I indicates no current again. The millivoltmeter, ΔV, which measures this bias increment directly, gives the peak value of this RF voltage directly.

20

The bridge [illegible] for [illegible] is concerned, [illegible] whereby [illegible] are provided with a crystal [illegible] on important [illegible] to far as input [illegible] and this is of the use in [illegible] of balance if [illegible] magnitude.

The [illegible] from its ungrounded side to a [illegible] VTV (see Fig. 21) [illegible] value of the [illegible] that the attenuation, the [illegible] coil [illegible] sample would be [illegible] the oscillator [illegible] is set by means of [illegible] on millimeter [illegible] oscillator, [illegible] zero. By means of [illegible] of a [illegible] again. The [illegible] crement [illegible] directly.

Once the bridge is sufficiently balanced to detect the expected signal, the current in the field coil of the electromagnet is turned on and set at an appropriate value to give a field strength H not too close to the resonant value $H_o = \omega_o/\gamma$. $\omega_o/2\pi$ is the steady frequency of the oscillator, which in our experiment was 7.975 megacycles. It is convenient to modulate the strong z-field H_z with a weak 60 cycle modulating field so that as resonance is approached, the total field H_z' sweeps through the resonant point at a 60 cycle rate. This is achieved by winding about a hundred turns of wire about the yoke of the electromagnet and passing a small 60 cycle current through it (see Fig. 4). We thus obtain a total z-field H_z' of the form $H_z' = H_z + \overline{H} \cos 377t$, where $\overline{H}$ is a few gauss in magnitude. The field H_z is brought up towards the resonant value H_o very slowly, the modulating field being on all this time. The 60 cycle variation of this field is so slow in comparison with the RF field that it can be regarded as sensibly constant from the point of view of the theory developed in the first two chapters.

When H_z is about $\frac{1}{2}$ gauss away from H_o, the resonance point is passed through 120 times per sec. by the system and increased absorption of energy from the RF field by the spin-field system should occur just that many times per second. This suddenly increased absorption of energy by the protons

Once the values of [illegible] for

the external [illegible]

electromagnet [illegible]

to give a fixed [illegible]

value H_0 = [illegible]

oscillatory [illegible]

It is [illegible]

[illegible] 50 cycle [illegible] field [illegible]

quenched, [illegible]

point [illegible]

a [illegible]

and [illegible]

At time [illegible]

N [illegible]

H_0 is [illegible]

the [illegible]

variation [illegible]

RF field [illegible]

the point [illegible]

observed.

[illegible]

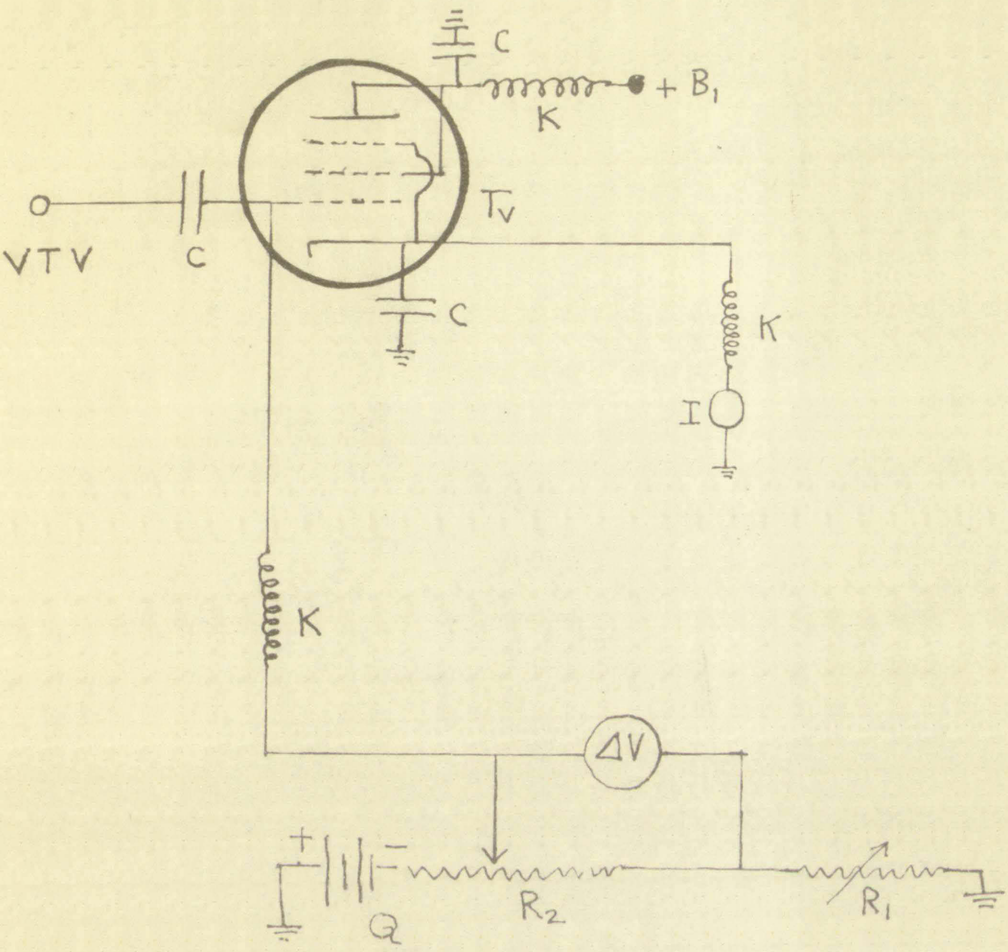
point [illegible]

increase [illegible]

field [illegible]

This [illegible]

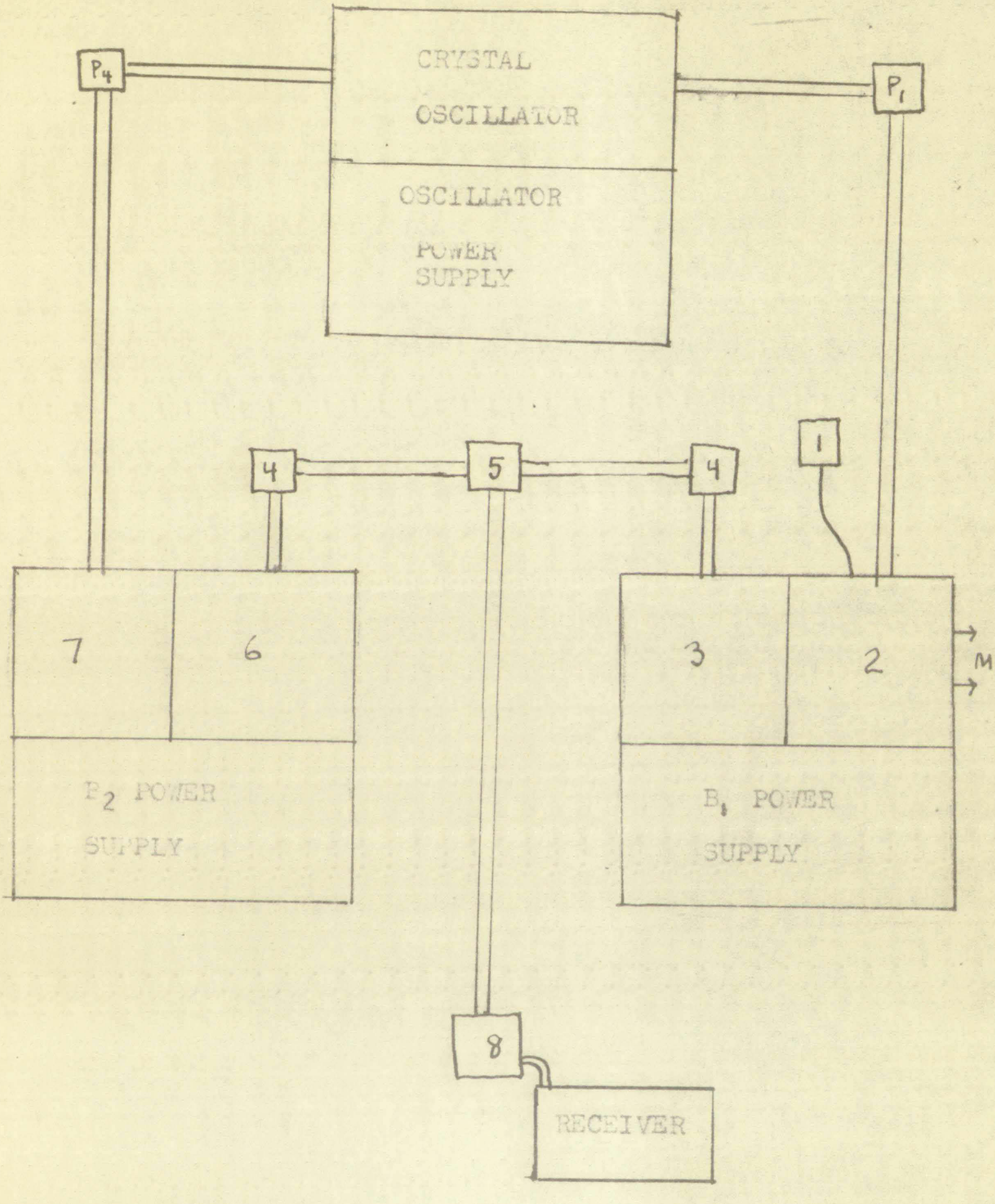
Fig. 2- Vacuum Tube Voltmeter Curcuit



Key to the symbols

T_v - 6AU6 Pentode
ΔV - Millivoltmeter for measuring bias increment
R_1 - Biasing rheostat
R_2 - Bias increment rheostat
Q - 67 volt battery
I - Milliameter
C - 470 mmf.
K - RF choke coils, 40-50 henries
VTV - Input lead from bridge (see Fig. 1)

Fig. 3 - Block diagram of the Bridge Circuit showing the shielding precautions.



Key to the symbols on page 33.

Key to the Symbols of Fig. 3

1. - Controls for biasing T_V in the VT voltmeter circuit. Milliameter and Millivoltmeter (I and ΔV, see Fig. 2).

2. - T_V of the VT voltmeter, the variable condensor C_M and P_2.

M - Leads from C_M to the toroidal coil L_M (in Magnet gap).

3. - The tube, T_M, and associated circuit.

4. - Elbow joints.

5. - The junction point A (see Fig. 1) and the variable condensor C_A.

6. - The tube, T_O, and associated circuit. The potentiometer P_3.

7. - The dummy tuned circuit, LC_D.

8. - The tuned circuit H (see Fig. 1)

in the spin-field system should change the effective Q of the coil by a very small amount and hence detune the circuit LC_M by a slight amount. This would impose a pulse-shaped amplitude modulation on the RF signal appearing at the point A from the magnetic side of the bridge. The bridge should be thrown off balance at this rate, assuming the signal coming in from the dummy side is constant. Thus a signal should appear at the antenna post, taking the form of 120 cycle per sec. pulse modulated RF signal. The receiver should amplify and detect this signal and put out the 120 cycle pulse in the audio output. Note that the pulses are not evenly spaced unless H_z is exactly at resonance and that the BFO is off. If the voltage source for the 60 cycle field is connected to the horizontal plates of a CRO* while the audio output of the receiver is fed into the vertical amplifier, we should expect a sharp peak or pulse on the trace of the CRO when H_z approached close to H_o. If this pulse appears in the middle of the scope screen, $H_z = H_o$. Hence if the field H_z and the oscillator frequency $\omega_o/2\pi$ were accurately measured under these conditions, we could calculate γ from the relation $\gamma = \omega_o/H_o$.

Since the bridge has a tendency to become unbalanced due to changes in temperature, mechanical vibrations, etc., it is necessary in the course of the experiment to check the

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* - CRO stands for "Cathode Ray Oscilliscope".

In the high-field [illegible]

the coil [illegible]

by [illegible]

amplitude [illegible]

4 from [illegible]

be thrown off [illegible]

coming in [illegible] the [illegible]

should appear [illegible]

cycle per sec. [illegible]

should equally [illegible]

pulse to the [illegible]

evenly [illegible]

CRO is off. If [illegible] is

connected to the [illegible]

output of [illegible]

we should [illegible]

CRO when [illegible]

in the middle of [illegible]

field [illegible]

measured [illegible]

the relation [illegible]

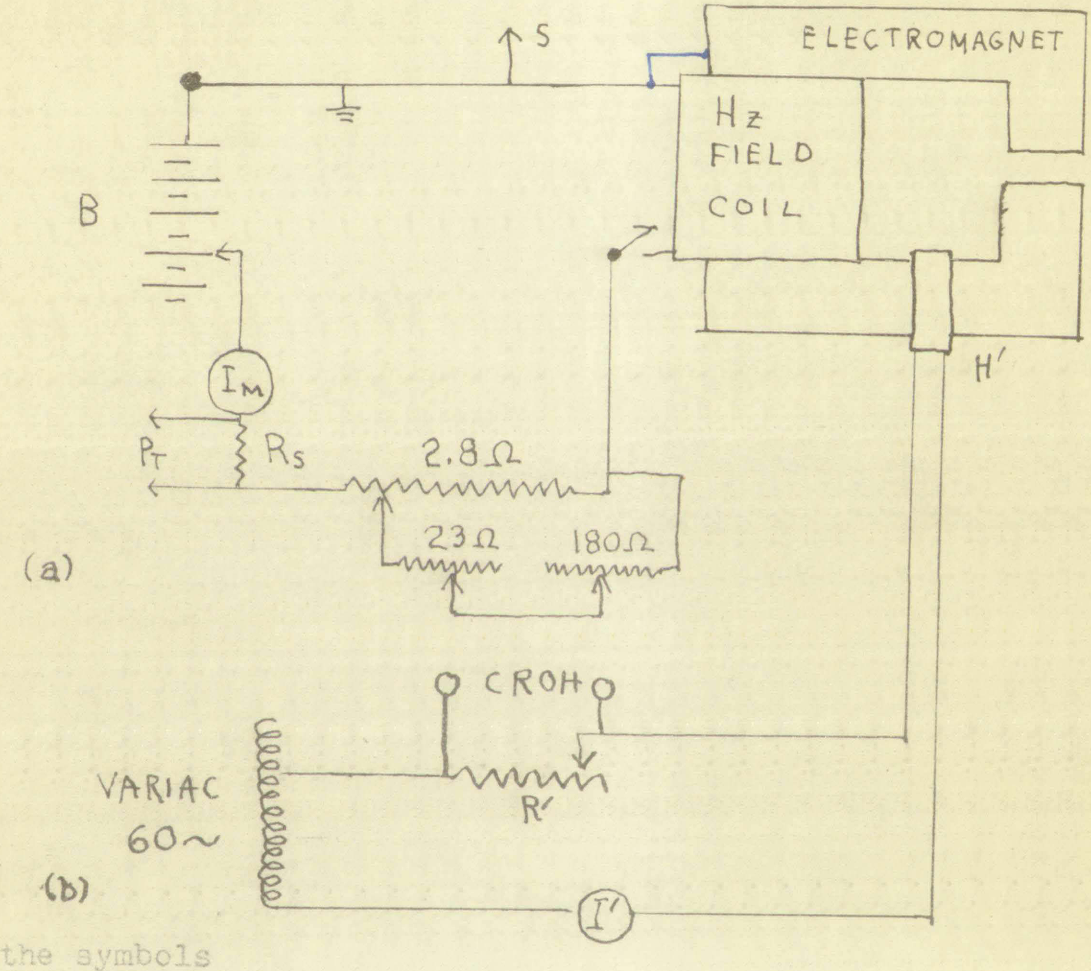
[illegible]

due to changes in [illegible]

it is necessary in the [illegible]

[illegible]

Fig. 4 - The electromagnet circuit and the circuit of the 60 cycle modulating field.



Key to the symbols

B - Storage batteries
I_M - D.C. Ammeter (10 amp. range)
I′ - A.C. Milliameter (200 ma. range)
R_S - Standard precision resistance (1.00000± ohm)
H′ - Field coil of 60 cycle modulating field
P_T - Leads to potentiometer
R′ - Rheostat for varying 60 cycle field strength
CROH - Leads to horizontal plates of CRO
S - Lead to ground magnet steel

balance frequently. Because of the fact that when the system is far from resonance, signals coming in from both sides of the bridge are of the unmodulated continuous wave type, the audio output would not register any unbalance. Hence it is necessary from time to time to switch in the BFO, in which case unbalance shows up as a beat note trace on the CRO screen. It is advisable to put a sensitive AF voltmeter across the audio output of the receiver for the purpose of making a more sensitive determination.

The circuit of the electromagnet consists of the coil itself (made of #12 copper wire) in series with variable resistors and an ammeter for approximate measurement of current strength. The EMF is supplied by storage batteries, capable of delivering a few amperes at steady values. In addition, a small 1 ohm standard precision resistance is inserted into the circuit. Voltages across this resistance, as measured by a potentiometer, give an accurate value of the current when needed.

The magnet itself was capable of delivering fields of about 4500 gauss at 6.5 amps. and was made in the laboratory out of parallel 4"x8"x$\frac{1}{2}$" sheets of soft steel.

The block-diagram of the bridge (Fig. 3) gives an indication of the spatial separation and arrangement of the various units. The long leads in the bridge arms connecting the separated units were made of thick bus bar surrounded by

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1" brass tubing and held rigid by polystyrene plugs in the ends. Complete shielding and considerable separation of the units was found to be necessary in order that balance could be successfully achieved. Earlier work in constructing the bridge, in which the chassis containing the units were in close contact and with a less elaborate shielding arrangement, showed the necessity of making the bridge as described above.

Calibration curves for both the electromagnet large field and the modulating field were obtained using standard methods (i.e., measuring total change of flux through a known coil and by rms voltage induced in a known coil).

The electromagnet was placed on an adjustable support with the air gap between pole faces horizontally situated. The shielded chassis (No. 2 in Fig. 3) containing the variable condensor C_M was placed very close to the air gap and short leads were run out through a $1\frac{1}{2}$ inch hole in this chassis into the air gap of the magnet. The toroidal RF coil (L_M) was placed on a support in the center of the air gap and its terminals soldered to these two leads. This put the coil in parallel with the condenser to form the tuned circuit LC_M. The fact that this coil is toroidal in shape and that the analysis in Chapter II was carried out for an RF field along the x-axis is of no consequence, as a moment's reflection will show. Although the yoke of the magnet is of square cross-section, the pole faces themselves are circular and

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about an inch deep. Copper screen attached on either side of the hole in this chassis was wrapped closely around the air gap, shielding the whole set up. The yoke of the magnet itself was then at ground potential.

CHAPTER IV

DESCRIPTION OF THE EXPERIMENT AND CONCLUSION

The essential aspects of the experimental method have been indicated in the previous chapter. Here we shall describe the experiment performed and discuss, in the way of conclusions, the reasons for its failure.

Before turning on the large field of the electromagnet, the apparatus was properly connected and the audio output of the receiver was fed into an earphone headset. The BFO of the receiver was switched in and the receiver antenna post placed close to the input lead from the point A. At first the beat note was clearly and loudly heard, with the receiver at low AF gain, diminishing in intensity as balance was approached.

Balance was attained by varying the gain of the two bridge tubes using the cathode biasing potentiometers P_2 and P_4 and by adjusting the condensers in the two tuned circuits. After some time, a stable condition of balance was achieved and direct metallic contact was made between the input lead from A and the antenna post. The effect of this was to greatly increase the intensity of the beat note because of the fact that this direct connection of the input stage of the receiver loaded the oscillator and bridge circuit and because of the increased size of the signal reaching the receiver.

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A new balance could be gotten by trial and error adjustment of the bridge. At close balance, the intensity of the beat note was practically inaudible against noise. The audio output of the receiver was measured with a sensitive AF voltmeter until only a few millivolts were registered. The sensitivity of the receiver was such that at this point balance was achieved to about 10 or 20 microvolts. Undesirable noise could be cut down by switching in the crystal filter of the receiver. When steady balance had been maintained with little adjustment for about a half hour, the BFO was switched off and the AF output of the receiver was fed into the V-amplifier of the CRO.

In the meantime, both the main field and modulating field of the magnet had been turned on. It had to be established that variations in the strengths of these fields had no effect on the balance of the bridge.

The RF voltage across the toroidal coil was then measured, using the VT voltmeter arrangement.

At the beginning of a run, the field strength H_z was put far below the resonant value H_o. For a frequency of 7.975 megacycles and for $\gamma = 2.66 \times 10^4$, this turns out to be $H_o = 1880$ gauss. Then a steady approach towards resonance was made, varying the 2.8 ohm rheostat (Fig. 4, Ch. III) a turn at a time. For each turn in this rheostat, the 180 ohm resistor was very slowly varied from zero to 180 ohms. The

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run was stopped when H_z was safely above 1880 gauss. The CRO, swept horizontally by a 60 cycle voltage drop in the modulating field circuit, was closely observed during these runs. Since the resonance width is so sharp (Equation 44, Ch. II), it would be very easy to pass through resonance so quickly that nothing would be observed on the CRO screen. The signal, indicating passage through resonance should have appeared on the CRO screen as a pulse gliding rapidly across this screen.

Many such runs as these were attempted, our apparatus giving no positive results. During each run, the balance of the bridge was frequently checked by switching in the BFO of the receiver. The swing of the modulating field was varied from one-tenth to about 2 gauss. The signal never clearly and unmistakably appeared in any of the runs. The reason for the failure of the experiment is discussed below.

Attempts were made, by successive refinement of the balance, to reduce the residual degree of unbalance of the bridge to such an extent that the incoming signal exceeded this by a detectable amount. In the experiment itself, the maximum voltage across the RF coil was about 20 millivolts. Upon consideration of the toroidal coil dimensions and well known formulas for the inductance of such coils (Terman, Radio Engineering, 3rd ed.), we found the inductance of this coil to be 4.85 microhenries. Using this and the 20 millivolt

figure, we obtain for the peak value of the RF field $H_p = 2H_1 = 97 \times 10^{-6}$ gauss. Using the values $T_2 = 10^{-4}$ and $T_1 = 10^{-2}$ given by Bloch for paraffin and the value $\gamma = 2.66 \times 10^4$ for protons, equation (39) of Chapter II gives for H_1^* the value $H_1^* = 3.78 \times 10^{-2}$ gauss. The ratio of $\frac{1}{2}97 \times 10^{-6}$ to this optimum value is k = .0157. In view of equation (41) of Chapter II, we obtain R(k) = .0314, ignoring the k^2 in the denominator. This means that we lost at least 90% of our signal in being so far away from optimum H_1 strength.

The chief reason for the failure of this experimental attempt must be found in inhomogeneities in the field, H_o, due to differences throughout the steel of the pole faces as well as curvature at the edges of the pole faces. This inhomogeneity was never less than 2 gauss. At resonance we calculated a value for H_o = 1880 gauss. This amounts to a deviation of only 0.1%, yet even this much inhomogeneity could easily diminish the signal strength by several orders of magnitude.

The best we could expect is that only a small portion of the paraffin sample would be at resonance at any instant. But if only a small region of the sample is at resonance, the toroidal coil acts as an exceedingly inefficient pick-up coil. This loss in signal is very difficult to estimate quantitatively, not knowing the exact distribution of inhomogeneities in the field H_o.

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If we use equation (52) of Chapter II to compute the optimum expected signal using the coil data and the value of γ for protons, the result is about 30 microvolts. The inhomogeneities mentioned above were so large that our signal could easily have been lost in the noise of our circuit.