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A Study of an Inverse Feed-Back, Low Frequency, Low-Pass Filter

Ralph A. Nobles

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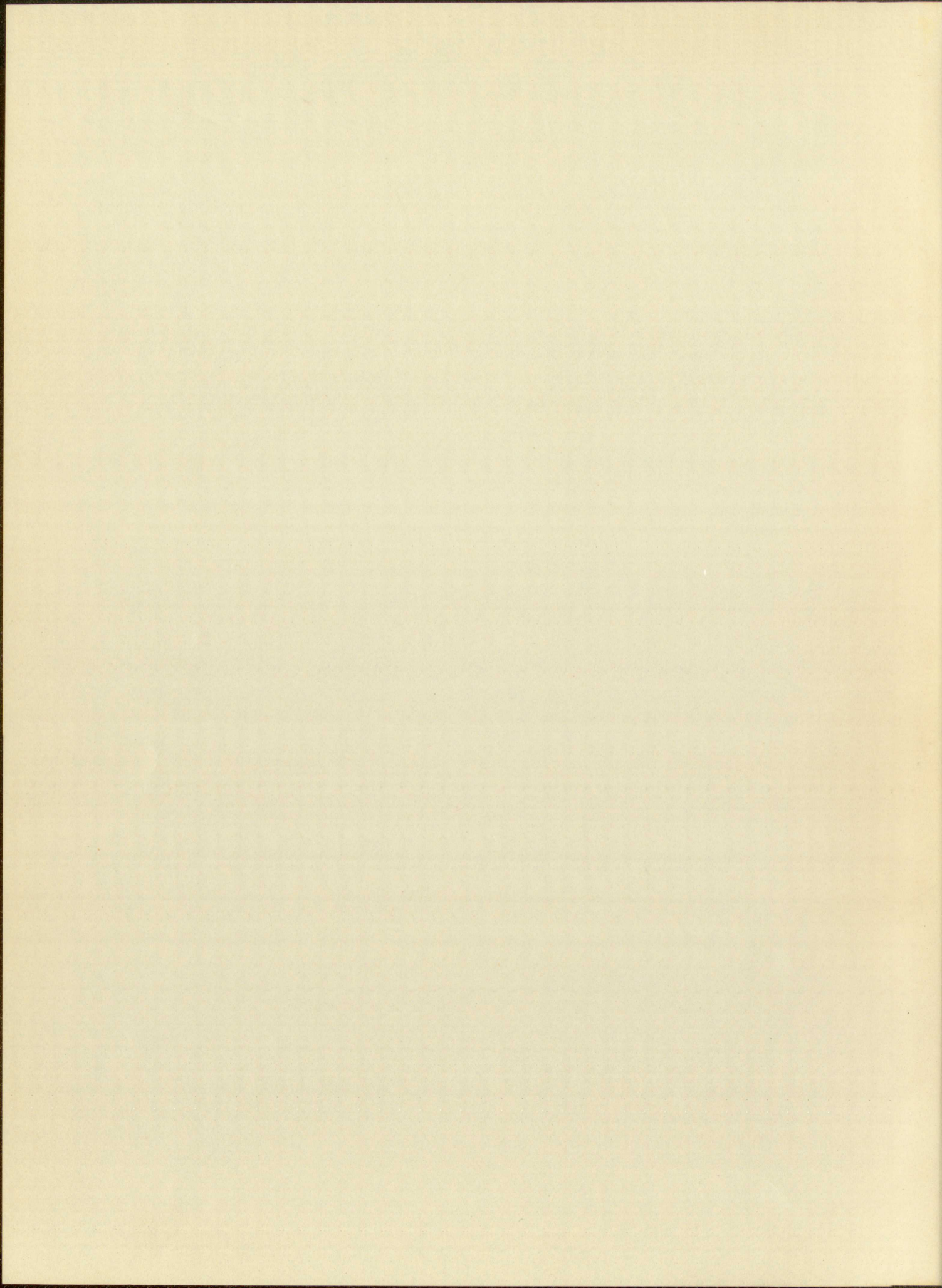
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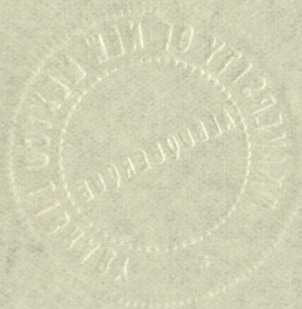
By

Ralph A. Nobles

A thesis submitted in
partial fulfillment of the
requirements for the Degree of
Master of Science in Physics

The University of New Mexico
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A STUDY OF AN INVERSE FEED-BACK,
LOW FREQUENCY, LOW-PASS FILTER.

By

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INTRODUCTION

In connection with other research work in the Physics Department at the University of New Mexico it became of interest to make a detailed study of a resistance-capacitance coupled low-pass filter which incorporated inverse feed-back.

In order to determine the effect of the inverse feed-back on the filters, calculations are made on circuits with and without feed-back. Response curves are calculated for step-wave inputs and for sine-wave inputs, and these calculated curves are then compared with experimental response curves.

CHAPTER I

CALCULATION OF RESPONSE CURVES OF A TWO SECTION R-C SERIES COUPLED FILTER WITHOUT FEED-BACK

The circuit diagram of the two section resistance-capacitance series coupled circuit is given in Fig. 1.



Fig. 1

Two section filter without feed-back

INTRODUCTION

In connection with other research work in the field of the University of New Mexico at Albuquerque, a study of a resistance-capacitance coupled circuit was made. The circuit was of the type in which the response curves are calculated for step-function inputs and the output is compared with the response curves. The circuit is of the type in which the response curves are calculated for step-function inputs and the output is compared with the response curves.

CHAPTER I

CALCULATION OF RESPONSE CURVES OF A TWO-SECTION RC CIRCUIT

The circuit diagram of the two section resistance-capacitance coupled circuit is shown in Fig. 1.

The section time constant is given by

In the following calculation and in all subsequent calculations in this paper, square upper case letters will stand for the Laplace transform¹ of the corresponding lower case letter. Thus E_o will be the Laplace transform of \mathcal{E}_o , I_o the transform of \mathcal{I}_o etc. The symbol Z will be used as the parameter in the Laplace transforms.

Applying Kirchhoff's laws² to the elementary circuits in Fig. 2

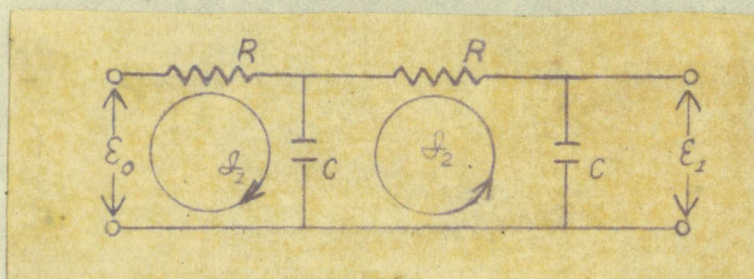


Fig. 2

we obtain the following transform equations:

$$(1.1) \quad E_o = I_1 r + \frac{I_1 + I_2}{cZ}$$

$$(1.2) \quad 0 = I_2 r + \frac{I_1 + I_2}{cZ} + \frac{I_2}{cZ}$$

$$(1.3) \quad E_1 = \frac{I_2}{cZ}$$

Solving these equations for E_1 in terms of E_o gives:

$$(1.4) \quad E_1 = \frac{E_o}{c^2 r^2 Z^2 + 3 r c Z + 1}$$

In the actual circuits, r was 10^7 ohms and c was $1/2$ microfarad, so $rc = 5$, and $r^2 c^2 = 25$. Thus (1.4) becomes:

¹ R. V. Churchill, Modern Operational Mathematics in Engineering, New York: McGraw Hill, 1944, Chapters I, II, III.

² L. Page and N.I. Adams, Principles of Electricity, New York: D. Van Nostrand, 1931, p. 173.

$$(1.5) \quad E_i = \frac{E_o}{25z^2 + 15z + 1} \approx \frac{E_o}{25(z + 0.524)(z + 0.076)}$$

The Laplace transform of a step input, Fig. 2, beginning at $t = 0$ and having an amplitude of \mathcal{E} is:

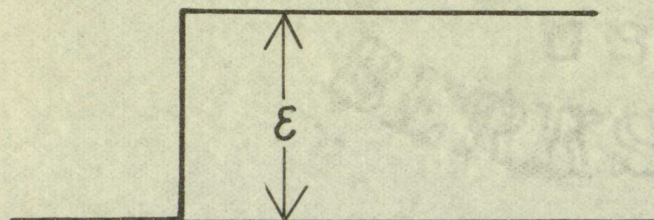


Fig. 3

$$(1.6) \quad E_o = \int_0^{\infty} e^{-zt} f(t) dt = \int_0^{\infty} e^{-zt} \mathcal{E} dt = \left. \frac{-e^{-zt} \mathcal{E}}{z} \right|_0^{\infty} = \frac{\mathcal{E}}{z},$$

where

$$f(t) = \mathcal{E} \quad \text{for } t > 0, \text{ and } f(t) = 0 \quad \text{for } t < 0.$$

Now by the use of the inversion theorem we get:

$$(1.7) \quad \mathcal{E}_i = \frac{1}{2\pi i} \int_c \frac{E_o e^{zt}}{25(z + 0.524)(z + 0.076)} dz$$

where E_o is the Laplace transform of the input wave, which in this case is given by equation 1.6. On substitution of this value for E_o in relation 1.7 we get:

$$(1.8) \quad \mathcal{E}_i = \frac{\mathcal{E}}{25 \cdot 2\pi i} \int_c \frac{e^{zt} dz}{z(z + 0.524)(z + 0.076)}$$

The integral in equation 1.8 has simple poles at $z = 0$, $z = -.524$, and $z = -.076$ (See Fig. 4).



$$(1.5) \quad E = \frac{E_0}{1 + \frac{1}{2} \frac{E_0^2}{E_0^2}} \quad \text{and having an amplitude of } E_0$$

The region outside the cylinder is empty and having an amplitude of E_0 .

Let E_0 be the amplitude of the wave.



$$(1.6) \quad E = \int_0^{\infty} E_0 \exp(-\frac{1}{2} \frac{E_0^2}{E_0^2} z) dz = \frac{E_0}{1 + \frac{1}{2} \frac{E_0^2}{E_0^2}}$$

where

$$(1.7) \quad \frac{1}{2} \frac{E_0^2}{E_0^2} = \frac{1}{2} \frac{E_0^2}{E_0^2} \quad \text{for } z > 0, \text{ and } \frac{1}{2} \frac{E_0^2}{E_0^2} = 0 \quad \text{for } z < 0.$$

Now by the use of the Laplace transform we get

$$(1.7) \quad \mathcal{L}\left\{\frac{1}{2} \frac{E_0^2}{E_0^2}\right\} = \frac{1}{2} \frac{E_0^2}{E_0^2} \quad \text{for } z > 0, \text{ and } \frac{1}{2} \frac{E_0^2}{E_0^2} = 0 \quad \text{for } z < 0.$$

where \mathcal{L} is the Laplace transform of the function $\frac{1}{2} \frac{E_0^2}{E_0^2}$ given by equation (1.6). The Laplace transform of the function $\frac{1}{2} \frac{E_0^2}{E_0^2}$ is given by equation (1.7).

1.7 we get

$$(1.8) \quad \mathcal{L}\left\{\frac{1}{2} \frac{E_0^2}{E_0^2}\right\} = \frac{1}{2} \frac{E_0^2}{E_0^2} \quad \text{for } z > 0, \text{ and } \frac{1}{2} \frac{E_0^2}{E_0^2} = 0 \quad \text{for } z < 0.$$

The integral in equation (1.8) is the Laplace transform of the function $\frac{1}{2} \frac{E_0^2}{E_0^2}$.

$\mathcal{L}\left\{\frac{1}{2} \frac{E_0^2}{E_0^2}\right\} = \frac{1}{2} \frac{E_0^2}{E_0^2}$

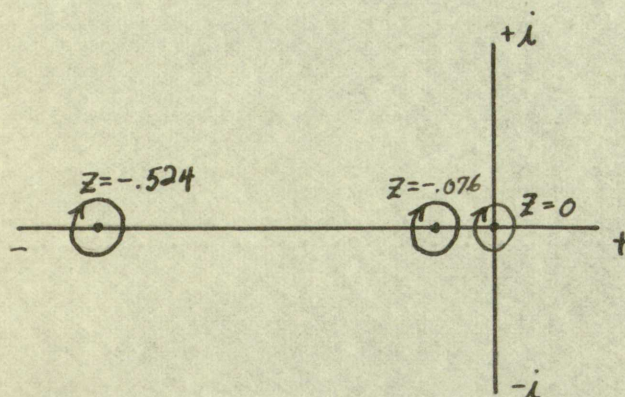


Fig. 4

Poles of the integral in equation 1.8

Evaluation of the expression at these poles gives:

$$(1.9) \quad \mathcal{E}_1 = \frac{\mathcal{E}}{25} \left[\frac{1}{(0.524)(0.076)} + \frac{e^{-.524t}}{(-.524)(-.524+.076)} + \frac{e^{-.076t}}{(-.076)(-.076+.524)} \right]$$

This reduces to:

$$(1.10) \quad \mathcal{E}_1 = \frac{\mathcal{E}}{25} \left[25.1 + 4.28 e^{-.524t} - 29.4 e^{-.076t} \right]$$

Equation 1.10 is then the response of a two section, R-C series coupled filter without feed-back to a step-wave input.

Table 1

Tabulation of \mathcal{E}_1 as a function of t

t (sec)	\mathcal{E}_1
1	0.009 \mathcal{E}
5	0.218 \mathcal{E}
10	0.452 \mathcal{E}
20	0.747 \mathcal{E}
30	0.900 \mathcal{E}
40	0.948 \mathcal{E}
50	0.980 \mathcal{E}
60	0.992 \mathcal{E}
70	1.00 \mathcal{E}

See Fig. 19 for a plot of Table 1.

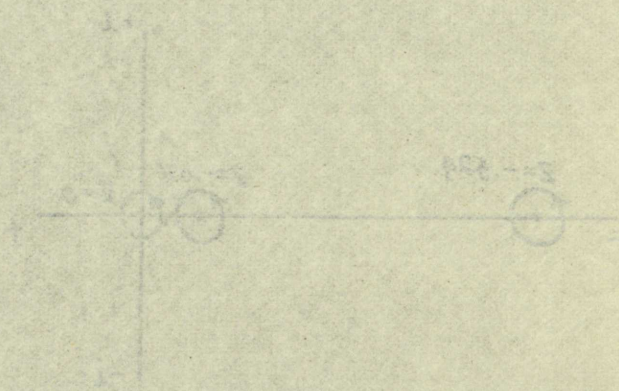


Fig. 1

Table 1. Results of the experiment.

Evaluation of the results of the experiment.

$$(1.9) \quad \epsilon_1 = \frac{1}{2} \left[\frac{1}{(0.25)(0.15)} - \frac{1}{(0.25)(0.15)} - \frac{1}{(0.25)(0.15)} - \frac{1}{(0.25)(0.15)} \right]$$

This reduced set

$$(1.10) \quad \epsilon_1 = \frac{1}{2} \left[\frac{1}{(0.25)(0.15)} + \frac{1}{(0.25)(0.15)} - \frac{1}{(0.25)(0.15)} - \frac{1}{(0.25)(0.15)} \right]$$

Equation 1.10 shows the results of the experiment.

After without feedback to a feedback.

Table 2. Results of the experiment.

1.0	1.5	2.0
0.1	0.2	0.3
0.2	0.3	0.4
0.3	0.4	0.5
0.4	0.5	0.6
0.5	0.6	0.7
0.6	0.7	0.8
0.7	0.8	0.9
0.8	0.9	1.0
0.9	1.0	1.1
1.0	1.1	1.2

See Fig. 19 for a plot of the results.

In the case of a sine wave beginning at $t = 0$ and having an amplitude of \mathcal{E} and a frequency of $\frac{\omega}{2\pi}$ the Laplace transform is given by:

$$\begin{aligned}
 (1.11) \quad E_0 &= \int_0^{\infty} e^{-zt} (\mathcal{E} \sin \omega t) dt = \frac{\mathcal{E}}{2i} \int_0^{\infty} e^{-zt} (e^{+i\omega t} - e^{-i\omega t}) dt \\
 &= \frac{\mathcal{E}}{2i} \left[\frac{e^{(-z+i\omega)t}}{-z+i\omega} \Big|_0^{\infty} + \frac{e^{-(z+i\omega)t}}{z+i\omega} \Big|_0^{\infty} \right] = \frac{\mathcal{E}}{2i} \left[\frac{1}{z-i\omega} - \frac{1}{z+i\omega} \right] \\
 &= \frac{\mathcal{E}\omega}{z^2 + \omega^2}
 \end{aligned}$$

If now in equation 1.7, which was obtained by the use of the inversion theorem, we substitute the Laplace transform of a sine wave (equation 1.11) instead of that of a step-wave we get:

$$\begin{aligned}
 (1.12) \quad \mathcal{E}_1 &= \frac{1}{2\pi i} \int_C \frac{\omega \mathcal{E} e^{zt} dz}{(z^2 + \omega^2)(z + .524)(z + .076)} \\
 &= \frac{\mathcal{E}\omega}{25 \cdot 2\pi i} \int_C \frac{e^{zt} dz}{(z+i\omega)(z-i\omega)(z+.524)(z+.076)}
 \end{aligned}$$

This expression has simple poles at $z = -i\omega$, $z = i\omega$, $z = -.524$ and $z = -.076$.

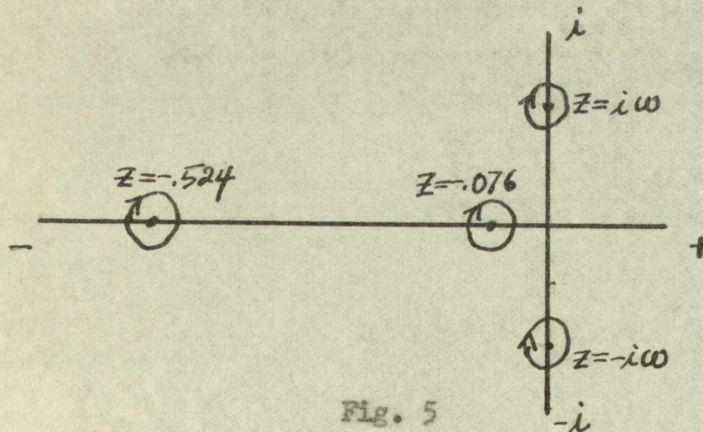


Fig. 5

Poles of the integral in equation 1.12

In the case of a one-way system, the value of β is 1 and a frequency of $\frac{1}{2}$ is obtained. This is the case of a one-way system.

$$(1.11) \quad E_0 = \int_0^\infty E(\omega) d\omega = \frac{1}{2\pi} \int_0^\infty \left[\frac{1}{\omega} + \frac{1}{\omega + i0} \right] d\omega$$

$$= \frac{1}{2\pi} \left[\frac{1}{\omega} + \frac{1}{\omega + i0} \right]_0^\infty = \frac{1}{2\pi} \left[\frac{1}{\omega} + \frac{1}{\omega + i0} \right]_0^\infty = \frac{1}{2\pi} \left[\frac{1}{\omega} + \frac{1}{\omega + i0} \right]_0^\infty$$

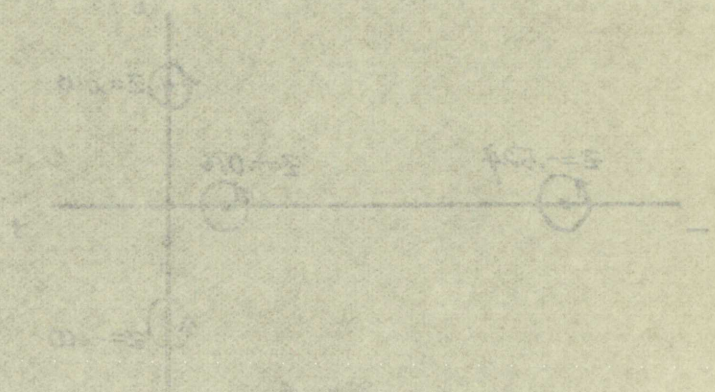
If now, equation (1.11) is used, the value of β is 1 and a frequency of $\frac{1}{2}$ is obtained. This is the case of a one-way system.

$$(1.12) \quad E_0 = \frac{1}{2\pi} \int_0^\infty \left[\frac{1}{\omega} + \frac{1}{\omega + i0} \right] d\omega$$

$$= \frac{1}{2\pi} \left[\frac{1}{\omega} + \frac{1}{\omega + i0} \right]_0^\infty = \frac{1}{2\pi} \left[\frac{1}{\omega} + \frac{1}{\omega + i0} \right]_0^\infty$$

This expression has a value of $\frac{1}{2}$ for $\omega = 0$ and $\omega = \infty$.

$E = -0.06$



Also, the value of E is -0.06 .

Evaluation of the integral at these poles gives:

$$(1.13) \quad \mathcal{E}_1 = \frac{\mathcal{E}\omega}{25} \left[\frac{e^{-i\omega t}}{(-2i\omega)(-i\omega + .524)(-i\omega + .076)} + \frac{e^{i\omega t}}{(2i\omega)(i\omega + .524)(i\omega + .076)} \right] \\ + \frac{\mathcal{E}\omega}{25} \left[\text{transient terms} \right]$$

For $t \gg 1$ sec the transient terms no longer give a contribution and equation 1.13 becomes simply:

$$(1.14) \quad \mathcal{E}_1 = \frac{\mathcal{E}\omega}{25} \left[\frac{e^{-i\omega t}}{(-2i\omega)(-i\omega + .524)(-i\omega + .076)} + \frac{e^{i\omega t}}{(2i\omega)(i\omega + .524)(i\omega + .076)} \right]$$

This expression reduces to:

$$(1.15) \quad \mathcal{E}_1 = \frac{\mathcal{E}}{25} \left[\frac{-.6\omega}{(.0398 - \omega^2)^2 + .36\omega^2} \cos \omega t + \frac{.0398 - \omega^2}{(.0398 - \omega^2)^2 + .36\omega^2} \sin \omega t \right] \\ = \frac{\mathcal{E}}{25} \left[A \cos \omega t + B \sin \omega t \right] = \frac{\mathcal{E}}{25} \left(\sqrt{A^2 + B^2} \right) \left[\sin(\omega t + \delta_\omega) \right]$$

where $A = \frac{-.6\omega}{(.0398 - \omega^2)^2 + .36\omega^2}$, $B = \frac{.0398 - \omega^2}{(.0398 - \omega^2)^2 + .36\omega^2}$

$$\delta_\omega = \tan^{-1} \frac{A}{B}$$

The frequency response in terms of the input amplitude is given by the quantity $\sqrt{A^2 + B^2} / 25$ in equation 1.15.

Evaluation of the integral of eq. (1.12) is

$$(1.13) \quad \mathcal{E}_1 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{j\omega t}}{(-j\omega - \alpha)(-j\omega - \beta)} d\omega$$

$$+ \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{j\omega t}}{(-j\omega - \alpha)(-j\omega - \beta)} d\omega$$

For $t > 0$, we use the contour shown in Fig. 1.13, which encloses the poles at α and β .

Then (1.13) becomes

$$(1.14) \quad \mathcal{E}_1 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{j\omega t}}{(-j\omega - \alpha)(-j\omega - \beta)} d\omega$$

This expression reduces to

$$(1.15) \quad \mathcal{E}_1 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{j\omega t}}{(-j\omega - \alpha)(-j\omega - \beta)} d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{j\omega t}}{(-j\omega - \alpha)(-j\omega - \beta)} d\omega$$

$$\text{where } A = \frac{1}{\alpha - \beta} \left(\frac{1}{-j\omega - \alpha} - \frac{1}{-j\omega - \beta} \right)$$

$$\mathcal{E}_1 = \frac{A}{\alpha - \beta} \left(e^{\alpha t} - e^{\beta t} \right)$$

The frequency response in terms of the input and output is

$$\text{quantity } \sqrt{A^2 + B^2} \text{ is equal to } \dots$$

Table 2

Tabulation of the frequency response of equation 1.15

ω	A	B	$\sqrt{A^2+B^2}/25$
10^{-2}	- 3.75	24.8	1.01
10^{-1}	-13.4	6.6	0.595
10^0	- .47	- .75	3.5×10^{-2}
10^1	-6×10^{-4}	-10^{-2}	$\sim 10^{-4}$
10^2	~ 0	-10^{-4}	~ 0

For a plot of the values of the amplitude of equation 1.15 as a function of ω see Fig. 20.

Table 2
 Tabulation of the frequency response of section 1.1

ω	A	B	$\sqrt{A^2 + B^2}$
10^{-2}	-2.72	2.8	3.91
10^{-1}	-1.4	2.8	3.03
10^0	-	-	-
10^1	-	-	-
10^2	-	-	-

For a plot of the values of the amplitude of section 1.1 as a function of ω see Fig. 20.

CHAPTER II

CALCULATION OF RESPONSE CURVES OF A TWO SECTION R-C SERIES
COUPLED FILTER WITH INVERSE FEED-BACK

The circuit diagram of a two section resistance-capacitance filter with inverse feed-back is given in Fig. 6.

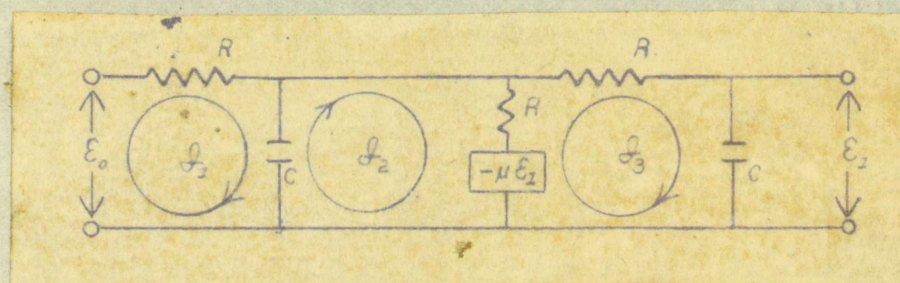


Fig. 6

Two section filter with inverse feed-back

Applying Kirchhoff's laws to the elementary circuits in Fig. 6, we obtain the following transform equations:

$$(2.1) \quad E_o = I_1 R + \frac{I_1 - I_2}{CZ}$$

$$(2.2) \quad \mu E_i = \frac{I_2 - I_1}{CZ} + (I_2 - I_3) R$$

$$(2.3) \quad -\mu E_i = (I_3 - I_2) R + I_3 R + \frac{I_3}{CZ}$$

$$(2.4) \quad E_i = \frac{I_3}{CZ}$$

Solving equations 2.1 through 2.4 for E_i in terms of E_o gives:

$$(2.5) \quad E_i = \frac{E_o}{R^2 C^2 Z^2 + 4RCZ + \mu + 2}$$

CALCULATION OF THE EFFECT OF A CHANGE IN THE

OF THE

The circuit diagram of a two-section network is shown in Fig. 1.

with inverse feedback is shown in Fig. 2.

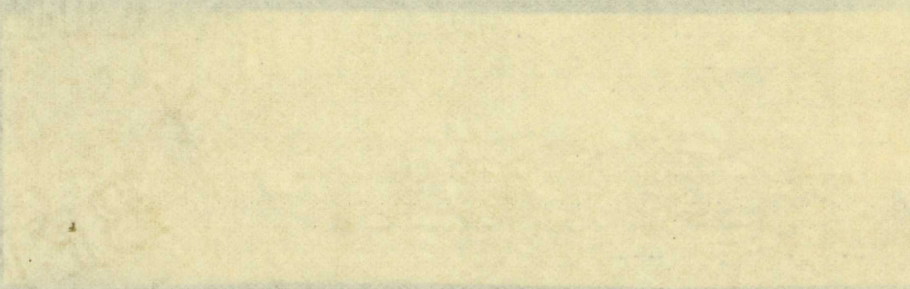


Fig. 1

Two-section filter with inverse feedback

Applying Kirchhoff's laws to the circuit shown in Fig. 1,

we obtain the following equations:

$$(2.1) \quad E_0 = I_1 R + \frac{I_1 - I_2}{C_1}$$

$$(2.2) \quad w E_1 = \frac{I_2 - I_3}{C_2} + (I_2 - I_3) R$$

$$(2.3) \quad -w E_1 = (I_3 - I_4) R + I_3 R + \frac{I_4}{C_3}$$

$$(2.4) \quad E_1 = \frac{I_3}{C_3}$$

Solving equations (2.1) through (2.4) for the current I_1 we obtain

$$(2.5) \quad E_1 = \frac{E_0}{R^2 C_1^2 + R C_2 C_3 + R C_1 C_3 + R C_2 C_1}$$

Now by the use of the inversion theorem and the Laplace transform for a step-wave input (Equation 1.6) we get:

$$(2.6) \mathcal{E}_1 = \frac{1}{2\pi i} \int_c \frac{E_0 e^{zt} dz}{\lambda^2 c^2 z^2 + 4\lambda c z + \mu + 2} = \frac{\mathcal{E}}{2\pi i} \int_c \frac{e^{zt} dz}{z(\lambda^2 c^2 z^2 + 4\lambda c z + \mu + 2)}$$

$$= \frac{\mathcal{E}}{2\pi i} \int_c \frac{e^{zt} dz}{\lambda^2 c^2 z \left(z + \frac{2 + \sqrt{2 - \mu}}{c\lambda} \right) \left(z - \frac{-2 + \sqrt{2 - \mu}}{c\lambda} \right)}$$

Here, as in the previous case, $r = 10^7$ ohms, $C = 1/2$ microfarad, and in addition $\mu = 10$. Thus as before $rc = 5$, and $r^2 C^2 = 25$. Equation 2.6 then becomes:

$$(2.7) \mathcal{E}_1 = \frac{\mathcal{E}}{25 \cdot 2\pi i} \int_c \frac{e^{zt} dz}{\left(z + \frac{2 + i2\sqrt{2}}{5} \right) \left(z - \frac{-2 + i2\sqrt{2}}{5} \right) z}$$

The integral in equation 2.7 has simple poles at $z = 0$, $z = -\frac{2 + i2\sqrt{2}}{5}$, and $z = \frac{-2 + i2\sqrt{2}}{5}$.

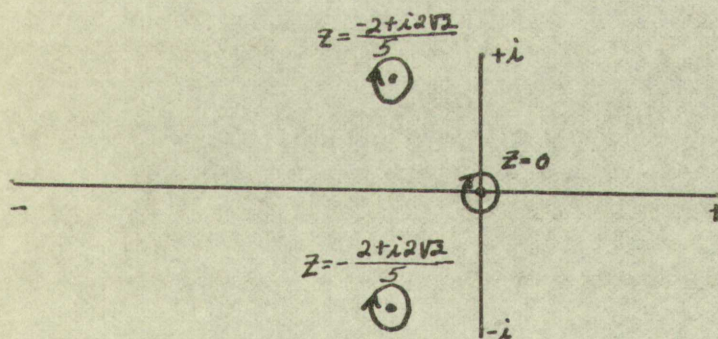


Fig. 7

Poles of the integral in equation 2.7

Evaluation of the integral at these poles gives:

$$(2.8) \mathcal{E}_1 = \mathcal{E} \left[\frac{1}{12} + \frac{e^{-\frac{2+2\sqrt{2}i}{5}t}}{-16+8\sqrt{2}i} + \frac{e^{-\frac{-2+2\sqrt{2}i}{5}t}}{-16-8\sqrt{2}i} \right]$$

Now by the use of the residue theorem and the partial fraction decomposition, we can find the inverse Laplace transform of the function $F(s)$.

$$(2.6) \quad \frac{1}{s^2 + 2s + 2} = \frac{1}{(s+1)^2 + 1} = \frac{1}{(s+1-j)(s+1+j)}$$

$$= \frac{A}{s+1-j} + \frac{B}{s+1+j}$$

Here, as in the previous example, we find the residues A and B by the method of partial fractions. We find that $A = \frac{1}{2j}$ and $B = -\frac{1}{2j}$. Then we have

$$(2.7) \quad \frac{1}{s^2 + 2s + 2} = \frac{1}{2j} \frac{1}{s+1-j} - \frac{1}{2j} \frac{1}{s+1+j}$$

The inverse Laplace transform of the function $F(s)$ is then given by

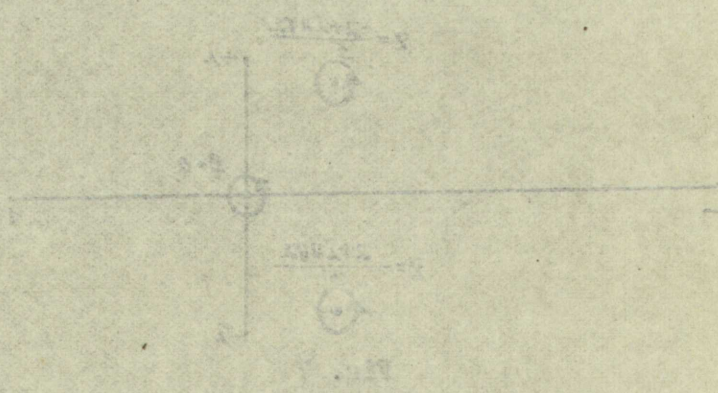
$$f(t) = \frac{1}{2j} e^{-(1-j)t} - \frac{1}{2j} e^{-(1+j)t}$$


Figure 2.1. Poles of the function $F(s)$ in the complex plane.

Evaluation of the integral of the function $F(s)$ over the contour C in the complex plane.

$$(2.8) \quad \frac{1}{s^2 + 2s + 2} = \frac{1}{(s+1-j)(s+1+j)} = \frac{A}{s+1-j} + \frac{B}{s+1+j}$$

By algebraic manipulations equation 2.8 can be put in the form:

$$(2.9) \quad \mathcal{E}_1 = \frac{\mathcal{E}}{12} \left[1 - e^{-4t} (\cos 0.565t + 0.707 \sin 0.565t) \right]$$

Equation 2.9 is then the analytical expression for the response of a two section filter with inverse feed-back to a step-wave input.

Table 3

Tabulation of the values of \mathcal{E}_1 as a function of t .

T (sec)	12 \mathcal{E}_1
0	0 \mathcal{E}
0.5	0.054 \mathcal{E}
1	0.18 \mathcal{E}
1.5	0.345 \mathcal{E}
2	0.522 \mathcal{E}
3	0.828 \mathcal{E}
4	1.018 \mathcal{E}
5	1.103 \mathcal{E}
6	1.104 \mathcal{E}
7	1.073 \mathcal{E}
8	1.036 \mathcal{E}
10	0.993 \mathcal{E}

For a plot of the values in Table 3 see Fig. 19.

If we substitute the Laplace transform of a sine-wave for E_1 in Equation 2.5 and then use the inversion theorem we get:

$$(2.10) \quad \mathcal{E}_1 = \frac{1}{2\pi i} \int_c \frac{E_0 e^{zt} dz}{c^2 \lambda^2 \left(z + \frac{2+2\sqrt{2}i}{c\lambda} \right) \left(z - \frac{-2+2\sqrt{2}i}{c\lambda} \right)}$$

$$= \frac{\mathcal{E} \omega}{2\pi i} \int_c \frac{e^{zt} dz}{25(z^2 + \omega^2) \left(z + \frac{2+2\sqrt{2}i}{5} \right) \left(z - \frac{-2+2\sqrt{2}i}{5} \right)}$$

By algebraic manipulation we obtain

$$(2.9) \quad \left[\frac{3}{2} - 1 \right] e^{-\lambda t} (\cos \omega t + \sin \omega t) = 0$$

Equation 2.9 is then the simplified expression for the response of the

section filter with input $\delta(t)$ and $\omega = 1$.

Table 1

Tabulation of the values of $\delta(t)$ and $\omega(t)$

0.0	0.0000
0.1	0.0000
0.2	0.0000
0.3	0.0000
0.4	0.0000
0.5	0.0000
0.6	0.0000
0.7	0.0000
0.8	0.0000
0.9	0.0000
1.0	0.0000

For a plot of the values in Table 1 see Fig. 1.

If we substitute the values in Table 1 into

Equation 2.9 and then use the Laplace transform we get

$$(2.10) \quad \frac{1}{s} \left(\frac{e^{-\lambda t}}{s^2 + \omega^2} \right) = \frac{1}{s} \left(\frac{e^{-\lambda t}}{s^2 + 1} \right)$$

$$= \frac{1}{s} \left(\frac{e^{-\lambda t}}{s^2 + 1} \right)$$

$$= \frac{\varepsilon \omega}{25 \cdot 2\pi i} \int_c \frac{e^{zt} dz}{(z+i\omega)(z-i\omega)(z+\frac{2+2\sqrt{3}i}{5})(z-\frac{-2+2\sqrt{3}i}{5})}$$

The integral in equation 2.10 has simple poles at $Z = i\omega$, $Z = -i\omega$, $Z = -\frac{2+2\sqrt{3}i}{5}$, and $Z = \frac{-2+2\sqrt{3}i}{5}$.

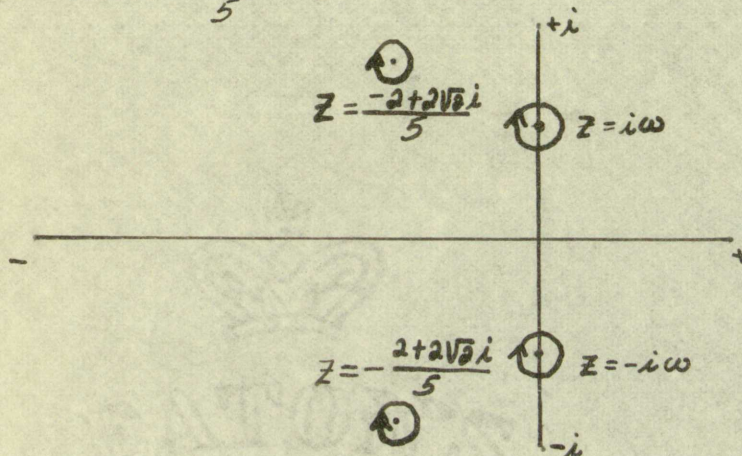


Fig. 8

Poles of the integral in equation 2.10

Evaluation of the integral at these poles gives:

$$(2.11) \mathcal{E}_1 = \frac{\varepsilon \omega}{25} \left[\frac{e^{-i\omega t}}{(-i\omega)(-i\omega + \frac{2+2\sqrt{3}i}{5})(-i\omega + \frac{2-2\sqrt{3}i}{5})} + \frac{e^{i\omega t}}{(i\omega)(i\omega + \frac{2+2\sqrt{3}i}{5})(i\omega + \frac{2-2\sqrt{3}i}{5})} + \frac{e^{\frac{-2-2\sqrt{3}i}{5}t}}{(\frac{-2-2\sqrt{3}i}{5} + i\omega)(\frac{-2-2\sqrt{3}i}{5} - i\omega)(\frac{-2-2\sqrt{3}i}{5} + \frac{2-2\sqrt{3}i}{5})} + \frac{e^{\frac{-2+2\sqrt{3}i}{5}t}}{(\frac{-2+2\sqrt{3}i}{5} + i\omega)(\frac{-2+2\sqrt{3}i}{5} - i\omega)(\frac{-2+2\sqrt{3}i}{5} + \frac{2+2\sqrt{3}i}{5})} \right]$$

For $t \gg 1$ equation 2.11 reduces to:

$$\frac{e^{5x}}{25} \left[\frac{\omega^3}{i\pi 25} - \frac{1}{2} \right] = \frac{e^{5x}}{25} \left[\frac{\omega^3}{i\pi 25} - \frac{1}{2} \right]$$

The integral in equation 2.11 is also equal to $\frac{1}{2} \int_{-\infty}^{\infty} f(\omega) d\omega$, and $\frac{1}{2} \int_{-\infty}^{\infty} f(\omega) d\omega = \frac{1}{2} \int_{-\infty}^{\infty} f(\omega) d\omega$.

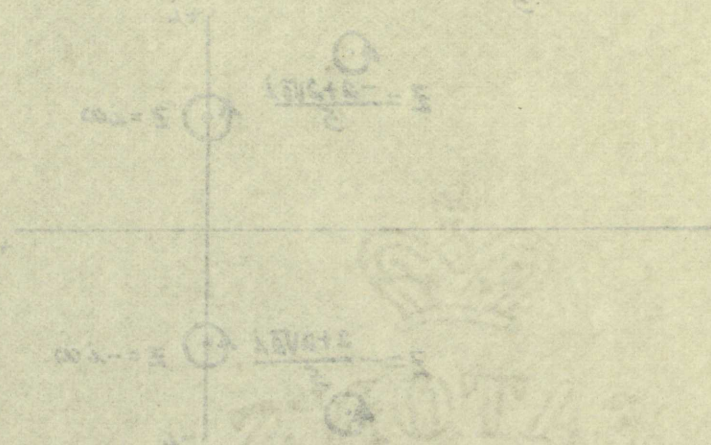


Figure 2.11: Poles and zeros of the integrand in equation 2.11.

Evaluation of the integral in equation 2.11.

$$(2.11) \int_{-\infty}^{\infty} f(\omega) d\omega = \frac{1}{2} \int_{-\infty}^{\infty} f(\omega) d\omega = \frac{1}{2} \int_{-\infty}^{\infty} f(\omega) d\omega$$

$$\frac{e^{5x}}{25} \left[\frac{\omega^3}{i\pi 25} - \frac{1}{2} \right] = \frac{e^{5x}}{25} \left[\frac{\omega^3}{i\pi 25} - \frac{1}{2} \right]$$

$$\left[\frac{e^{5x}}{25} \left(\frac{\omega^3}{i\pi 25} - \frac{1}{2} \right) \right] = \frac{e^{5x}}{25} \left(\frac{\omega^3}{i\pi 25} - \frac{1}{2} \right)$$

For $t > 0$, the integral in equation 2.11 is equal to

$$(2.12) \quad \mathcal{E}_1 = \frac{\mathcal{E}\omega}{25} \left[\frac{e^{-i\omega t}}{(-2i\omega)(-i\omega + \frac{2+2\sqrt{2}i}{5})(-i\omega + \frac{2-2\sqrt{2}i}{5})} + \frac{e^{i\omega t}}{(2i\omega)(i\omega + \frac{2+2\sqrt{2}i}{5})(i\omega + \frac{2-2\sqrt{2}i}{5})} \right]$$

Equation 2.12 can further be reduced to:

$$(2.13) \quad \mathcal{E}_1 = \frac{\mathcal{E}}{25} \left[-A \cos \omega t - B \sin \omega t \right] = \frac{\mathcal{E}}{25} \sqrt{A^2 + B^2} \left[-\sin(\omega t + \delta_\omega) \right]$$

$$= \frac{\mathcal{E}}{25} \sqrt{A^2 + B^2} \left[\sin(\omega t + \delta_\omega + \pi) \right]$$

where $A = \frac{\frac{16\omega}{5}}{(2\omega^2 - \frac{24}{25})^2 + \frac{64}{25}\omega^2}$, $B = \frac{2(2\omega^2 - \frac{24}{25})}{(2\omega^2 - \frac{24}{25})^2 + \frac{64}{25}\omega^2}$,

and $\delta_\omega = \tan^{-1} \frac{A}{B}$

The frequency response of equation 2.13 is of course given by the factor $\sqrt{A^2 + B^2}/25$.

Table 4

Frequency response of equation 2.13 as a function of ω

ω (rad/sec)	$\sqrt{A^2 + B^2}/25$
0.01	8.4×10^{-2}
0.1	8.3×10^{-2}
1.0	4.2×10^{-2}
2	1.04×10^{-2}
10	4×10^{-4}
100	0

$$(2.12) \quad \frac{E}{\omega} = \frac{E}{\omega} \left[\frac{1}{(1 - \omega^2)^2 + 4\zeta^2 \omega^2} \right]$$

$$+ \frac{E}{\omega} \left[\frac{1}{(1 - \omega^2)^2 + 4\zeta^2 \omega^2} \right]$$

Equation 2.12 can be written as:

$$(2.13) \quad \frac{E}{\omega} = \frac{E}{\omega} \left[\frac{1}{(1 - \omega^2)^2 + 4\zeta^2 \omega^2} \right]$$

$$\left[\frac{1}{(1 - \omega^2)^2 + 4\zeta^2 \omega^2} \right] = \frac{1}{\sqrt{A^2 + B^2}}$$

$$\text{where } A = \frac{1}{\omega} \left[\frac{1}{(1 - \omega^2)^2 + 4\zeta^2 \omega^2} \right] \text{ and } B = \frac{1}{\omega} \left[\frac{1}{(1 - \omega^2)^2 + 4\zeta^2 \omega^2} \right]$$

$$\text{and } \frac{A}{B} = \frac{1}{\omega}$$

The frequency response of equation (2.12) is of the form:

$$\frac{1}{\sqrt{A^2 + B^2}}$$

Frequency response of equation (2.12) is plotted as follows:

ω (rad/sec)	$\frac{1}{\sqrt{A^2 + B^2}}$
0	1.0
0.5	0.8
1.0	0.7
1.5	0.6
2.0	0.5
2.5	0.4
3.0	0.3
3.5	0.2
4.0	0.1

A plot of the frequency response of equation 2.13 is given in Fig. 20.

The phase shift in equation 2.13 follows in Table 5.

Table 5

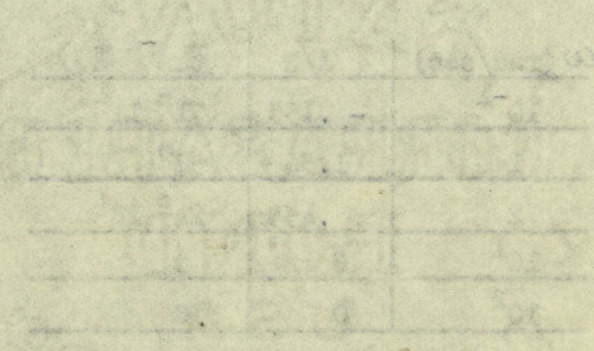
Phase shift of equation 2.13 as a function of ω

ω (rad/sec)	A/B	$\tan^{-1} \frac{A}{B}$
10^{-1}	-.155	$171^{\circ} 11'$
1	1.54	57°
2	.455	$24^{\circ} 28'$
10^1	0	0
10^2	0	0

See Fig. 21 for a plot of the phase shift of equation 2.13.

A plot of the frequency spectrum of the signal is shown in Fig. 1. The figure shows the frequency spectrum of the signal, which is a function of the frequency f and the time t .

Figure 1. Frequency spectrum of the signal.



See Fig. 2 for a plot of the signal spectrum, which is a function of the frequency f and the time t .

CHAPTER III

CALCULATION OF RESPONSE CURVES OF FOUR SECTION FILTER
WITHOUT FEED-BACK

The circuit diagram of a four section resistance-capacitance series coupled circuit without feed-back is given in Fig. 9.

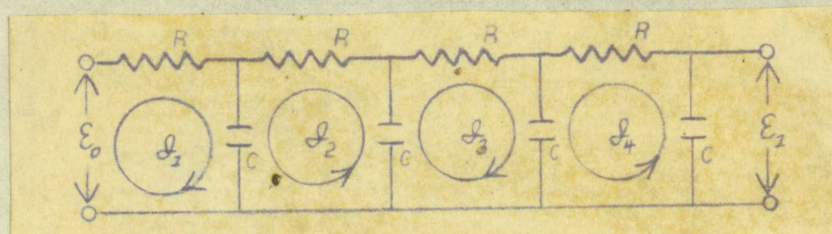


Fig. 9

Four section R-C series coupled filter

Analysis of the circuit in Fig. 9 leads to the following five transform equations:

$$(3.1) \quad E_0 = I_1 r + \frac{I_1 + I_2}{Cz}$$

$$(3.2) \quad 0 = I_2 r + \frac{I_2 + I_1}{Cz} + \frac{I_2 + I_3}{Cz}$$

$$(3.3) \quad 0 = I_3 r + \frac{I_3 + I_2}{Cz} + \frac{I_3 + I_4}{Cz}$$

$$(3.4) \quad 0 = I_4 r + \frac{I_4 + I_3}{Cz} + \frac{I_4}{Cz}$$

$$(3.5) \quad E_1 = \frac{I_4}{Cz}$$

Solving these equations for E_1 in terms of E_0 gives:

$$(3.6) \quad E_1 = \frac{-E_0}{a^4 - a^3 - 3a^2 + 2a + 1}$$

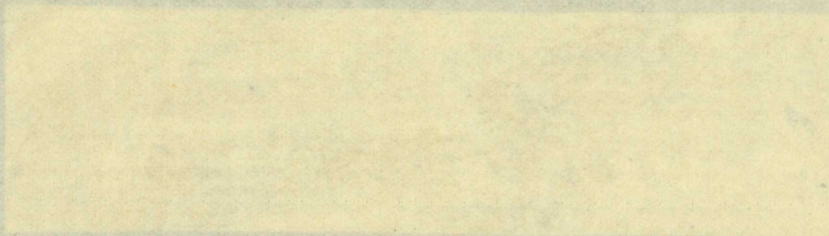
PROBLEM 1

Calculate the voltage across the 10 ohm resistor in the circuit shown.

Let V_x be the voltage across the 10 ohm resistor.

The circuit diagram is shown below.

Section coupled circuit analysis is used to solve the problem.



Let V_x be the voltage across the 10 ohm resistor.

For section 1-2, KVL gives

Analysis of the circuit in Fig. 1 leads to the following equations:

Equations:

$$(3.1) \quad E_s = I_1 R_1 + \frac{I_1 I_2}{C_1}$$

$$(3.2) \quad 0 = I_2 R_2 + \frac{I_2 I_1}{C_2} + \frac{I_2 I_3}{C_3}$$

$$(3.3) \quad 0 = I_3 R_3 + \frac{I_3 I_1}{C_4} + \frac{I_3 I_2}{C_5}$$

$$(3.4) \quad 0 = I_4 R_4 + \frac{I_4 I_1}{C_6} + \frac{I_4}{C_7}$$

$$(3.5) \quad E_1 = \frac{I_1}{C_8}$$

Solving these equations for I_1 in terms of E_s gives

$$(3.6) \quad E_1 = \frac{E_s}{\omega^2 - \omega_0^2 - j\omega\alpha}$$

where $a = rcz + 2$

and since here also $rc = 5$, (see Chapter I, page 2) we have:

$$(3.7) \quad a = 5z + 2$$

Substitution of this value for a in equation 3.6 gives:

$$(3.8) \quad E_1 = \frac{-E_0}{625z^4 + 875z^3 + 375z^2 + 50z + 1}$$

The denominator in equation 3.8 can be factored approximately by the use of Horner's method.³ On factoring, equation 3.8 becomes:

$$(3.9) \quad E_1 \approx \frac{-E_0}{625(z + 0.024)(z + 0.225)(z + 0.72)(z + 0.43)}$$

Now by using the Laplace transform of a step-wave input from equation 1.6 and equation 3.9 in the inversion theorem we get:

$$(3.10) \quad \mathcal{E}_1 = \frac{-E}{625 \cdot 2\pi i} \int_c \frac{e^{zt} dz}{z(z + 0.024)(z + 0.225)(z + 0.72)(z + 0.43)}$$

This expression has simple poles at $z = 0$, $z = -0.024$, $z = -0.225$, $z = -0.72$, and $z = -0.43$.

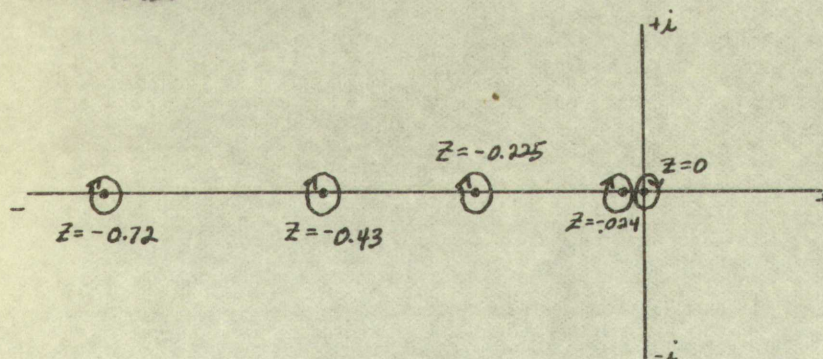


Fig. 10

Poles of the ~~integral~~ ^{integrand} in equation 3.10

³ E. J. Wilczynski and H. E. Slaughter, College Algebra, New York: Allyn and Bacon, 1916, p. 164.

where $\alpha = 1.55 + i$

and since here also $\alpha = 1.55 + i$ (see Appendix), we have

$$(3.7) \quad \alpha = 1.55 + i$$

Substitution of this value for α in (3.6) gives

$$(3.8) \quad E_1 = \frac{-E_0}{0.25 + 1.875i + 0.125i^2 + 0.125i^3}$$

The denominator in equation (3.8) is the characteristic equation of the system

of Horner's method, or, in other words, the characteristic equation of the system

$$(3.9) \quad E_1 = \frac{-E_0}{0.25(1 + 7.5i + 0.5i^2 + 0.5i^3)}$$

Now by using the known characteristic equation of a system, we can find the roots of the system

and equation (3.9) is the characteristic equation of the system

$$(3.10) \quad E_1 = \frac{-E_0}{0.25(1 + 7.5i + 0.5i^2 + 0.5i^3)}$$

This expression has three poles at $s = 0$, $s = -1.55 + i$, and $s = -1.55 - i$.

$s = -0.75$, and $s = -0.75$.

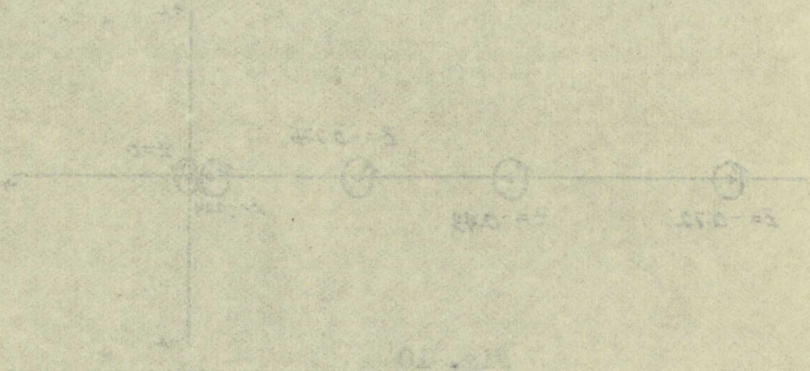


Fig. 10. Poles of the transfer function (3.10).

Evaluating the integral in equation 3.10 by the theory of residues we get:

$$(3.11) \quad \mathcal{E}_1 = \frac{-\mathcal{E}}{625} \left[\frac{1}{(.024)(.225)(.43)(.72)} + \frac{e^{-.024t}}{(-.024)(-.024+.225)(-.024+.43)(-.024+.72)} \right. \\ \left. + \frac{e^{-.225t}}{(-.225)(-.225+.024)(-.225+.43)(-.225+.72)} + \frac{e^{-.72t}}{(-.72)(-.72+.024)(-.72+.225)(-.72+.43)} \right. \\ \left. + \frac{e^{-.43t}}{(-.43)(-.43-.024)(-.43+.225)(-.43+.72)} \right]$$

This reduces to:

$$(3.12) \quad \mathcal{E}_1 \approx \frac{-\mathcal{E}}{625} \left[597 - 716 e^{-.024t} + 213 e^{-.225t} + 13.25 e^{-.72t} - 101 e^{-.43t} \right]$$

Table 6

Tabulation of the values of equation 3.12 as a function of t

t (sec)	\mathcal{E}_1
10	0.067 \mathcal{E}
20	0.248 \mathcal{E}
30	0.392 \mathcal{E}
50	0.609 \mathcal{E}
100	0.851 \mathcal{E}
150	0.924 \mathcal{E}
200	0.946 \mathcal{E}

The fact that the asymptotic value of \mathcal{E}_1 is 0.957 \mathcal{E} instead of 1.000 \mathcal{E} is due to inaccuracy in factoring the denominator of equation 3.8.

For a plot of equation 3.12 see Fig. 22. Equation 3.12 is the analytical expression for the response of a four section filter, without feed-back, to a step-wave input.

Evaluating the integral in equation 2.11 by the method of residues:

Let:

$$(2.11) \quad \oint_{\Gamma} \frac{z^{-3}}{z^2 + 1} dz = \oint_{\Gamma} \frac{z^{-3}}{(z-i)(z+i)} dz$$

$$= \oint_{\Gamma} \frac{z^{-3}}{(z-i)(z+i)} dz = \oint_{\Gamma} \frac{z^{-3}}{(z-i)(z+i)} dz$$

$$= \oint_{\Gamma} \frac{z^{-3}}{(z-i)(z+i)} dz = \oint_{\Gamma} \frac{z^{-3}}{(z-i)(z+i)} dz$$

This reduced to:

$$(2.12) \quad \oint_{\Gamma} \frac{z^{-3}}{z^2 + 1} dz = \oint_{\Gamma} \frac{z^{-3}}{(z-i)(z+i)} dz$$

Table 1

Values of the integrals in equation 2.12 for various values of ϵ

ϵ (sec)	$\oint_{\Gamma} \frac{z^{-3}}{z^2 + 1} dz$
0.1	0.0000
0.2	0.0000
0.3	0.0000
0.4	0.0000
0.5	0.0000
0.6	0.0000
0.7	0.0000
0.8	0.0000
0.9	0.0000
1.0	0.0000

The fact that the integrals in equation 2.12 are zero for all values of ϵ is due to the fact that the integrand is an odd function of z .

1.000 £ is the amount of money in the bank at the end of the year.

For a plot of ϵ versus $\oint_{\Gamma} \frac{z^{-3}}{z^2 + 1} dz$, see Figure 2.13.

analytical expression for the function $f(z) = \frac{1}{z^2 + 1}$ is given by:

food-back, to a food-back.

The analytical expression for the response of the filter to a sine-wave input is given by using the Laplace transform from equation 1.11 together with relation 3.8 in the inversion theorem, giving:

$$(3.13) \quad \mathcal{E}_1 = \frac{\mathcal{E}\omega}{2\pi i \cdot 625} \int_C \frac{e^{zt} dz}{(z+i\omega)(z-i\omega)(z+.024)(z+.225)(z+.72)(z+.43)}$$

This expression has simple poles at $Z = -i\omega$, $Z = i\omega$, $Z = -.024$, $Z = -.225$, $Z = -.72$, and $Z = -.43$.

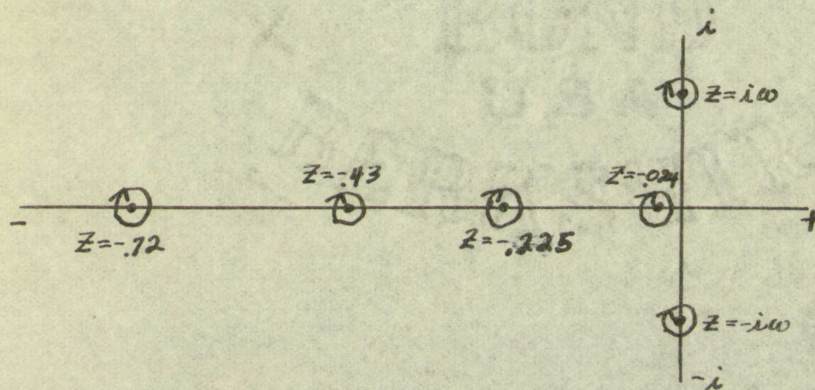


Fig. 11

Poles of the integral in equation 3.13

Evaluation of the integral by residue theorem gives:

$$(3.14) \quad \mathcal{E}_1 = \frac{\mathcal{E}\omega}{625} \left[\frac{e^{-i\omega t}}{(-2i\omega)(-i\omega+.024)(-i\omega+.225)(-i\omega+.43)(-i\omega+.72)} \right. \\ + \frac{e^{i\omega t}}{(2i\omega)(i\omega+.024)(i\omega+.225)(i\omega+.43)(i\omega+.72)} \\ + \frac{e^{-.024t}}{(-.024+i\omega)(-.024-i\omega)(-.024+.225)(-.024+.43)(-.024+.72)} \\ + \frac{e^{-.225t}}{(-.225+i\omega)(-.225-i\omega)(-.225+.024)(-.225+.43)(-.225+.72)} \\ \left. + \frac{e^{-.43t}}{(-.43+i\omega)(-.43-i\omega)(-.43+.024)(-.43+.225)(-.43+.72)} \right]$$

The simplified expression for the wave function is
 wave-wave function is given by the following expression
 together with relation 2.1 for the function $\psi(x)$

$$(2.13) \quad \psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{\psi}(k) e^{ikx} dk$$

This expression is the Fourier transform of the function $\tilde{\psi}(k)$
 $\tilde{\psi}(k) = -\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi(x) e^{-ikx} dx$

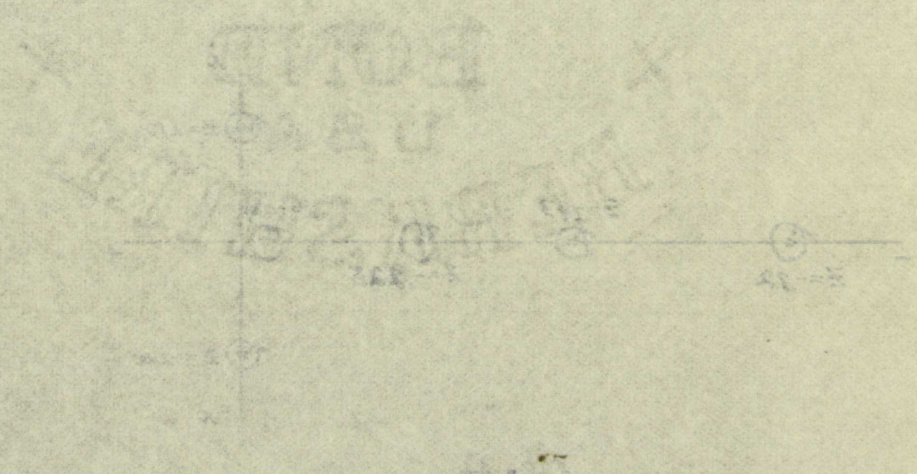


Figure 2.1: Plot of the function $\psi(x)$

Evaluation of the integral by means of the residue theorem

$$(2.14) \quad \psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{\psi}(k) e^{ikx} dk$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{k^2 + 1} e^{ikx} dk$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{\infty} \frac{1}{k^2 + 1} e^{ikx} dk \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{\infty} \frac{1}{k^2 + 1} e^{ikx} dk \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{\infty} \frac{1}{k^2 + 1} e^{ikx} dk \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{\infty} \frac{1}{k^2 + 1} e^{ikx} dk \right]$$

$$+ \frac{e^{-.72t}}{(-.72+iu)(-.72-iu)(-.72+.024)(-.72+.225)(-.72+.43)}$$

For $t \gg 1$, that is after the transients have died out, equation 3.14 reduces to:

$$(3.15) \quad \mathcal{E}_1 = \frac{\mathcal{E}}{625} \left[A \cos \omega t + B \sin \omega t \right]$$

$$= \frac{\mathcal{E}}{625} \sqrt{A^2 + B^2} \left[\sin(\omega t + \delta_\omega) \right]$$

where

$$A = \frac{(1.4\omega^3 - 0.083\omega)}{\omega^8 + .76\omega^6 + .131\omega^4 + .0049\omega^2 + .0000029}$$

$$B = \frac{(\omega^4 - .6\omega^2 + .0017)}{\omega^8 + .76\omega^6 + .131\omega^4 + .0049\omega^2 + .0000029}$$

and

$$\delta_\omega = \tan^{-1} \frac{A}{B}$$

The frequency response of equation 3.15 is of course given by the factor $\sqrt{A^2 + B^2}$

Table 7

Tabulation of the frequency dependent amplitude of equation 3.15

ω (rad/sec)	A	B	$\sqrt{A^2 + B^2}$
10^{-2}	-2.45×10^2	4.83×10^2	5.4×10^2
10^{-1}	-1.06×10^2	$-.66 \times 10^2$	1.24×10^2
1	0.695	.21	.725
10^1	1.32×10^{-6}	10^{-4}	$\sim 10^{-4}$
10^2	0	10^{-8}	0
10^{-3}	-2.86×10^1	5.87×10^2	5.87×10^2

See Fig. 23 for a plot of the frequency response of equation 3.15.

$$E = \frac{(-2\alpha + i\omega) \cdot 22 + 0.22 \cdot 22 + 22 \cdot 22 + 22 \cdot 22}{\dots}$$

For $\omega \gg 1$, find in the limit the frequency response of the system.

reduce to:

$$(3.12) \quad E = \frac{2}{22} \left[A \cos \omega t + B \sin \omega t \right]$$

$$E = \frac{2}{22} \sqrt{A^2 + B^2} \left[\cos(\omega t + \phi) \right]$$

where

$$A = \frac{(1 + 0.22\omega^2)}{\omega^2 + 1.76\omega + 1.31\omega^2 + 0.0099\omega^4 + 0.000047\omega^6}$$

$$B = \frac{(\cos^2 - 0.22\omega^2 + 0.22)}{\omega^2 + 1.76\omega + 1.31\omega^2 + 0.0099\omega^4 + 0.000047\omega^6}$$

and

$$\phi = \tan^{-1} \frac{B}{A}$$

The frequency response of equation (3.12) is given by the ratio

$$\text{for } \sqrt{A^2 + B^2}$$

Plotting of the frequency response of the system.

ω (rad/sec)	E	ϕ (deg)
10^{-2}	0.0000	0.00
10^{-1}	0.0000	0.00
1	0.0000	0.00
10^1	0.0000	0.00
10^2	0.0000	0.00
10^3	0.0000	0.00
10^4	0.0000	0.00

See Fig. 23 for a plot of the frequency response of the system.

CHAPTER IV

CALCULATION OF RESPONSE CURVES OF A FOUR SECTION FILTER
WITH FEED-BACK

The circuit diagram of the four section resistance-capacitance series coupled filter with inverse feed-back is given in Fig. 12.

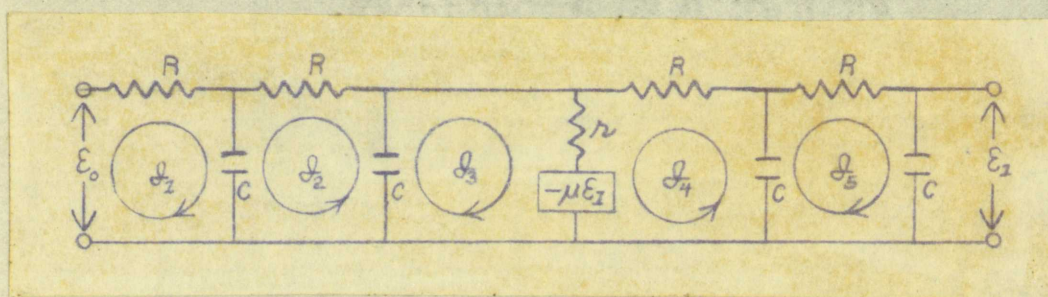


Fig. 12

Four section inverse feed-back filter

Analysis of the circuit in Fig. 12 leads to the following set of transform equations:

$$(4.1) \quad E_0 = I_1 R + \frac{I_1 + I_2}{cZ}$$

$$(4.2) \quad 0 = I_2 R + \frac{I_1 + I_2}{cZ} + \frac{I_2 + I_3}{cZ}$$

$$(4.3) \quad \mu E_1 = (I_3 + I_4) r + \frac{I_3 + I_4}{cZ}$$

$$(4.4) \quad \mu E_1 = (I_4 + I_5) r + I_4 R + \frac{I_4 + I_5}{cZ}$$

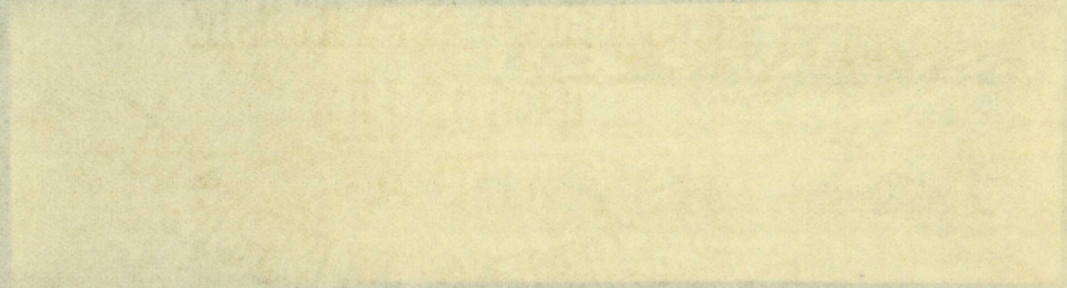
$$(4.5) \quad 0 = I_5 R + \frac{I_4 + I_5}{cZ} + \frac{I_5}{cZ}$$

$$(4.6) \quad E_1 = \frac{I_5}{cZ}$$

ANALYSIS OF THE CIRCULAR CROWN

By J. H. ...

The crown is divided into two parts, the upper part being of length l_1 and the lower part of length l_2 . The crown is supported at the ends by two vertical supports, the distance between which is $2a$. The crown is subjected to a uniformly distributed load of intensity w per unit length.



For section I-I in the crown, the following equations are obtained:

Analysis of the crown is made in two parts, the upper part of length l_1 and the lower part of length l_2 .

Four equations:

$$(1.1) \quad E_1 = I_1 R + \frac{I_1 l_1}{c_1}$$

$$(1.2) \quad 0 = I_2 R + \frac{I_2 l_2}{c_2} + \frac{I_1 l_1}{c_1}$$

$$(1.3) \quad w E_1 = (I_1 + I_2) w + \frac{I_1 l_1}{c_1}$$

$$(1.4) \quad w E_2 = (I_1 + I_2) w + \frac{I_2 l_2}{c_2}$$

$$(1.5) \quad 0 = I_2 R + \frac{I_2 l_2}{c_2} + \frac{I_1 l_1}{c_1}$$

$$(1.6) \quad E_1 = \frac{I_1}{c_1}$$

The values of the constants used in the experimental set-up on this filter were as follows: $R = 10^7$ ohms, $c = 1/2$ microfarad, $\mu = 10$ and $r = 2 \times 10^7$ ohms. The resistance r was always chosen equal to $n \times 10^7$ ohms, where n is one-half of the number of filter sections. This was done in order to match the impedance of the filter sections to the feed-back section.

On solving equations 4.1 through 4.6 for E_1 in terms of E_0 one gets:

$$(4.7) \quad E_1 = \frac{2E_0}{2(RcZ)^4 + 15(RcZ)^3 + 35(RcZ)^2 - \mu(RcZ) - 2\mu + 4}$$

Since $Rc = 5$ and $\mu = 10$, equation 4.7 becomes:

$$(4.8) \quad E_1 = \frac{2E_0}{1250Z^4 + 1875Z^3 + 875Z^2 + 185Z + 24}$$

Factoring the denominator of 4.8 by Horner's Method gives:

$$(4.9) \quad E_1 = \frac{E_0}{625(Z + .429)(Z + .850)(Z + .1105 + i.2012)(Z + .1105 - i.2012)}$$

The analytic expression for the response of the filter to a step-wave input is obtained by using the transform of a step-wave from equation 1.6 and equation 4.9 in the inversion theorem, giving:

$$(4.10) \quad \mathcal{E}_1 = \frac{\mathcal{E}}{625 \cdot 2\pi i} \int_c \frac{e^{z\tau} dz}{z(z + .429)(z + .850)(z + .1105 + i.2012)(z + .1105 - i.2012)}$$

This expression has simple poles at $Z = 0$, $Z = -.850$, $Z = -.429$, $Z = -.1105 - i.2012$, and $Z = -.1105 + i.2012$.

The value of the constant α is determined by the condition that this linear wave is periodic, $\alpha = 2\pi$. The constant α is determined by the condition that the wave is periodic, $\alpha = 2\pi$. This was done in order to obtain a value for α which is the two-body constant.

In solving equation (4.7) for E_1 , we obtain:

$$(4.7) \quad E_1 = \frac{2E_0}{2(RCZ)^2 + 12(RCZ)^2 + 32(RCZ)^2 + 12(RCZ)^2 + 12(RCZ)^2 + 12(RCZ)^2}$$

Since $\alpha = 2$ and $\mu = 10$, equation (4.7) becomes:

$$(4.8) \quad E_1 = \frac{2E_0}{1.250E^2 + 18.75E^2 + 8.75E^2 + 1.25E^2 + 1.25E^2 + 1.25E^2}$$

Factoring the denominator of (4.8) we obtain:

$$(4.9) \quad E_1 = \frac{E_0}{0.25(2 + 1.25)(2 + 1.25)(2 + 1.25)(2 + 1.25)(2 + 1.25)(2 + 1.25)}$$

The analytic expression for the response of the linear system is obtained by using the technique of a partial fraction expansion and equation (4.9) in the Laplace domain.

$$(4.10) \quad E_1 = \frac{E_0}{0.25(2 + 1.25)(2 + 1.25)(2 + 1.25)(2 + 1.25)(2 + 1.25)(2 + 1.25)}$$

This expression has single terms of $\frac{1}{s + 1.25}$ and $\frac{1}{s + 1.25}$. $s = -1.25 - 1.25j$ and $s = -1.25 + 1.25j$.

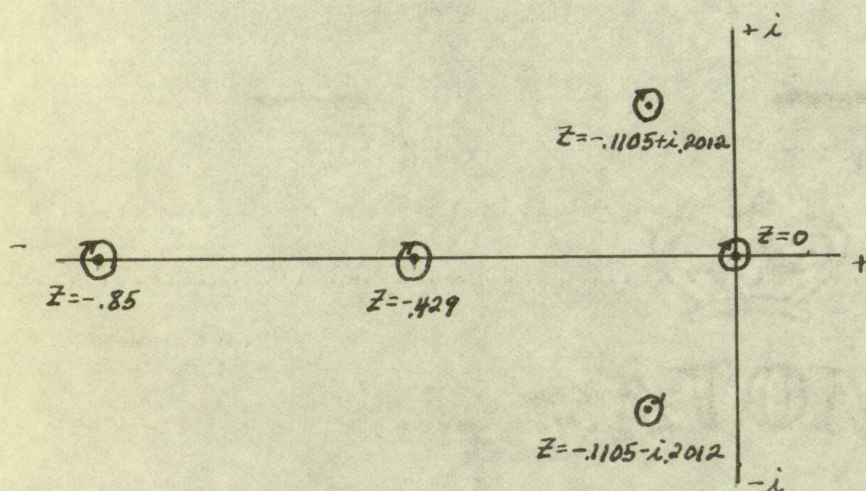


Fig. 13

Poles of the integral in equation 4.10

Evaluation of the integral at these poles gives:

$$\begin{aligned}
 (4.11) \quad \mathcal{E}_1 = & \frac{\mathcal{E}}{625} \left[\frac{1}{(.429)(.850)(.1105 + j.2012)(.1105 - j.2012)} \right. \\
 & + \frac{e^{-.429t}}{(-.429)(-.429 + .850)(-.429 + .1105 + j.2012)(-.429 + .1105 - j.2012)} \\
 & + \frac{e^{-.850t}}{(-.850)(-.850 + .429)(-.850 + .1105 + j.2012)(-.850 + .1105 - j.2012)} \\
 & + \frac{e^{(-.1105 - j.2012)t}}{(-.1105 - j.2012)(-.1105 - j.2012 + .429)(-.1105 - j.2012 + .850)(-.1105 - j.2012 + .1105 + j.2012)} \\
 & \left. + \frac{e^{(-.1105 + j.2012)t}}{(-.1105 + j.2012)(-.1105 + j.2012 + .429)(-.1105 + j.2012 + .850)(-.1105 + j.2012 + .1105 - j.2012)} \right]
 \end{aligned}$$

2. 100. 1000

1000

1000

1000

1000

1000

Evaluation of the integral of the function

$$(4.11) \quad \int_{-\infty}^{\infty} f(x) dx = \frac{1}{2\pi i} \int_{\gamma} f(z) dz$$

$$+ \frac{1}{2\pi i} \int_{\gamma} f(z) dz = \frac{1}{2\pi i} \int_{\gamma} f(z) dz$$

$$+ \frac{1}{2\pi i} \int_{\gamma} f(z) dz = \frac{1}{2\pi i} \int_{\gamma} f(z) dz$$

$$+ \frac{1}{2\pi i} \int_{\gamma} f(z) dz = \frac{1}{2\pi i} \int_{\gamma} f(z) dz$$

$$+ \frac{1}{2\pi i} \int_{\gamma} f(z) dz = \frac{1}{2\pi i} \int_{\gamma} f(z) dz$$

This reduces to:

$$(4.12) \quad \mathcal{E}_1 = \frac{\mathcal{E}}{625} \left[52.1 - 39.05 e^{-.429t} + 4.76 e^{-.85t} \right] \\ + \frac{\mathcal{E}}{625} e^{-.1105t} \left[-17.92 \cos .2012t - 73.15 \sin .2012t \right]$$

Equation 4.12 is the analytic expression for the reaction of a four section filter with feed-back to a step-wave input.

Table 8

Tabulation of the values of equation 4.12

t (sec)	625 \mathcal{E}_1
0	-0.22 \mathcal{E}
1	-0.14 \mathcal{E}
2	-0.68 \mathcal{E}
3	2.44 \mathcal{E}
5	13.48 \mathcal{E}
10	64.2 \mathcal{E}
15	107.0 \mathcal{E}
20	117.9 \mathcal{E}
30	104.2 \mathcal{E}
50	104.6 \mathcal{E}

See Fig. 22 for a plot of the values of equation 4.12.

The response of this filter to a sine-wave input is obtained by using the transform of a sine-wave from equation 1.11 and relation 4.9 in the inversion theorem, giving:

$$(4.13) \quad \mathcal{E}_1 = \frac{\mathcal{E}\omega}{625 \cdot 2\pi i} \int_C \frac{e^{z\tau} dz}{(z+i\omega)(z-i\omega)(z+.429)(z+.85)(z+.1105+.2012i)(z+.1105-.2012i)}$$

This volume for

$$(A.12) \quad \epsilon_1 = \frac{6}{625} \left[52.1 - 3202 \epsilon - 4.76 \epsilon^2 \right]$$

$$+ \frac{6}{625} \left[-12.92 \text{ Geo } 2000 - 73.15 \text{ Geo } 4000 \right]$$

Equation A.12 is the ratio expression for the reaction

tion after with loss back of a step-down ratio.

Table A

Table A.1. Values of ϵ_1 and ϵ_2

0.00	0.00
0.01	0.01
0.02	0.02
0.03	0.03
0.04	0.04
0.05	0.05
0.06	0.06
0.07	0.07
0.08	0.08
0.09	0.09
0.10	0.10
0.11	0.11
0.12	0.12
0.13	0.13
0.14	0.14
0.15	0.15
0.16	0.16
0.17	0.17
0.18	0.18
0.19	0.19
0.20	0.20

See Fig. 32 for a plot of the volume ratio ϵ_1/ϵ_2 .

The volume of this liquid is measured by the method of

using the relation between the volume of the liquid and the

in the inverse process, that is

$$(A.13) \quad \epsilon_1 = \frac{6 \cos \theta}{625 \pi r^2} \int_0^{\theta} \sin^2 \theta' d\theta'$$

This expression has simple poles at $Z = -j\omega$, $Z = j\omega$, $Z = -.429$, $Z = -.85$, $Z = -.1105 - j.2012$, and $Z = -.1105 + j.2012$.

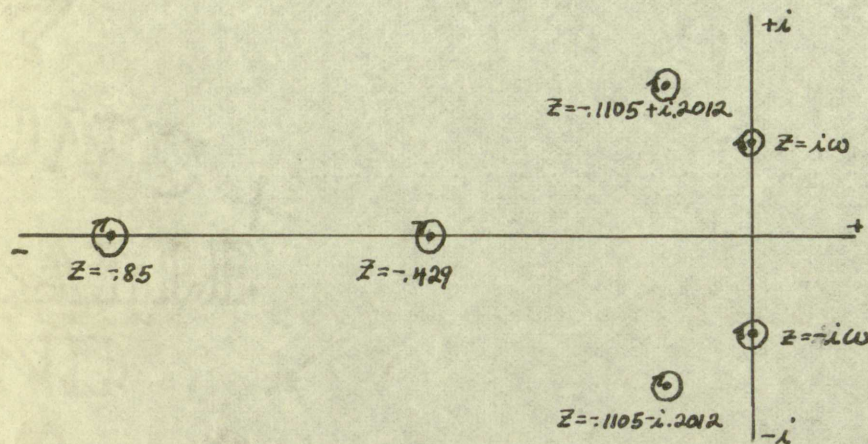


Fig. 14

Poles of the integral in equation 4.13

Evaluation of the integral at these poles gives:

$$\begin{aligned}
 (4.14) \quad \mathcal{E}_1 = & \frac{\mathcal{E}}{625} \left[\frac{(1.5\omega^3 - .1475\omega)2\cos\omega t + (\omega^4 + 0.0191 - 0.699\omega^2)2\sin\omega t}{(1.5\omega^3 - .1475\omega)^2 + (\omega^4 - 0.699\omega^2 + 0.0194)^2} \right. \\
 & + 33.55 \frac{\omega e^{-.429t}}{\omega^2 + .184} - 8.13 \frac{\omega e^{-.85t}}{\omega^2 + .7225} \\
 & \left. + 2\omega e^{-.1105t} \left\{ \frac{(.00584 - .085\omega^2)2\cos.2012t + (.00158 + \omega^2 0.0786)2\sin.2012t}{(.00584 - .085\omega^2)^2 + (.00158 + \omega^2 0.0786)^2} \right\} \right]
 \end{aligned}$$

For $t \gg 1$ equation 4.14 reduces to:

$$\begin{aligned}
 (4.15) \quad \mathcal{E}_1 = & \frac{\mathcal{E}}{625} \left[A \cos\omega t + B \sin\omega t \right] \\
 = & \frac{\mathcal{E}}{625} \sqrt{A^2 + B^2} \left[\sin(\omega t + \delta_\omega) \right]
 \end{aligned}$$

This expression has a pole at $z = -1$, $z = i$, $z = -i$.

$$z = -1, z = i, z = -i, \text{ and } z = \infty.$$

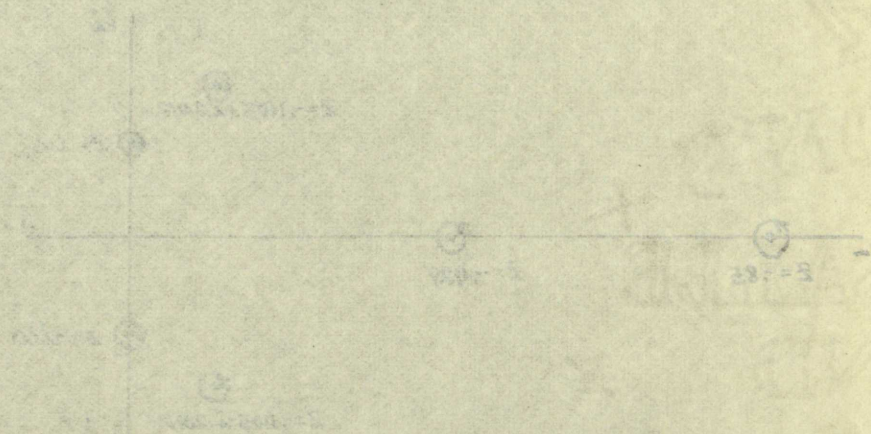


Fig. 1

Table of the residues of the function $f(z)$.

Evaluation of the integral of the function $f(z)$.

$$I = \frac{1}{2\pi i} \oint_C f(z) dz = \frac{1}{2\pi i} \left[\text{Residue at } z = -1 + \text{Residue at } z = i + \text{Residue at } z = -i \right]$$

$$= \frac{1}{2\pi i} \left[\frac{1}{z+1} + \frac{1}{z-i} + \frac{1}{z+i} \right]$$

$$= \frac{1}{2\pi i} \left[\frac{1}{-1+1} + \frac{1}{i-i} + \frac{1}{-i+i} \right]$$

For $\epsilon \gg 1$ expression (A.12) becomes:

$$(A.12) \quad \left[\frac{1}{2\pi i} \left(\frac{1}{z+1} + \frac{1}{z-i} + \frac{1}{z+i} \right) \right]$$

$$= \frac{1}{2\pi i} \left[\frac{1}{z+1} + \frac{1}{z-i} + \frac{1}{z+i} \right]$$

where

$$A = \frac{3\omega^3 - .295\omega}{(1.5\omega^3 - .1475\omega)^2 + (\omega^4 - .699\omega^2 + .0191)^2}$$

$$B = \frac{2\omega^4 + 0.0382 - 1.398\omega^2}{(1.5\omega^3 - .1475\omega)^2 + (\omega^4 - .699\omega^2 + .0191)^2}$$

$$\delta_\omega = \tan^{-1} \frac{A}{B}$$

Table 9

Tabulation of the frequency dependent factor in equation 4.15

ω	$\sqrt{A^2 + B^2}$
10^{-2}	105.3
10^{-1}	100.0
1.0	1.44
2.0	0.124
10^1	2.02×10^{-4}
10^2	2×10^{-8}

See Fig. 23 for a plot of the values of equation 4.15.

where

$$A = \frac{3\omega^2 - 1.995\omega}{(1.5\omega^2 - 1.995\omega)^2 + (\omega^2 - 1.995\omega + 0.01)^2}$$

$$B = \frac{3\omega^2 + 0.005\omega - 1.995\omega}{(1.5\omega^2 - 1.995\omega)^2 + (\omega^2 - 1.995\omega + 0.01)^2}$$

$$\delta\omega = \frac{B}{A}$$

The values of the frequency components are reported in Table 1.

ω	$\delta\omega$
1.0	0.000
1.5	0.000
2.0	0.000
2.5	0.000
3.0	0.000
3.5	0.000
4.0	0.000

See Fig. 2 for a plot of the frequency components.

CHAPTER V

CONSIDERATION OF THE PROBLEM OF THE RESPONSE OF A 2N
SECTION RESISTANCE-CAPACITANCE SERIES COUPLED FILTER
WITH INVERSE FEED-BACK

The circuit diagram of a 2N section resistance-capacitance series coupled filter is given in Fig. 15.

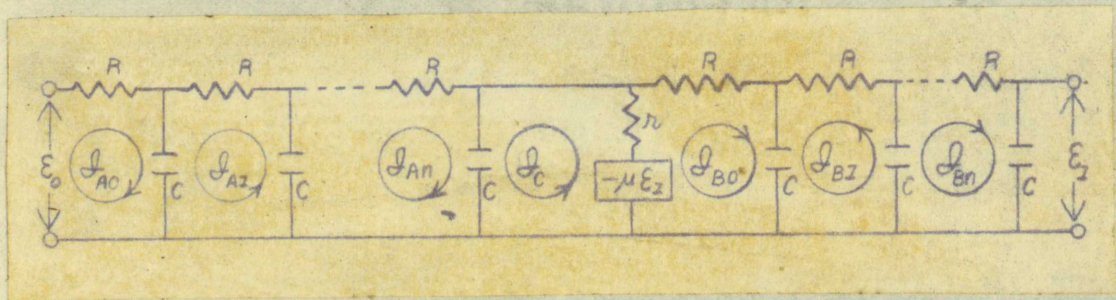


Fig. 15

2N section filter with inverse feed-back

Analysis of the circuit in Fig. 15 leads to the following set of transform equations:

$$(5.1) \quad E_0 = I_{A0} \left(R + \frac{1}{cZ} \right) + \frac{I_{A1}}{cZ}$$

$$(5.2) \quad 0 = I_{A1} \left(R + \frac{2}{cZ} \right) + I_{A0} \frac{1}{cZ} + I_{A2} \frac{1}{cZ}$$

$$(5.3) \quad 0 = I_{A(n-1)} \left(\frac{1}{cZ} \right) + I_{An} \left(R + \frac{2}{cZ} \right) + \frac{I_c}{cZ}$$

$$(5.4) \quad \mu E_1 = I_c \left(r + \frac{1}{cZ} \right) + I_{B0} r + \frac{I_{An}}{cZ}$$

$$(5.5) \quad \mu E_1 = I_c R + I_{B0} \left(2R + \frac{1}{cZ} \right) + \frac{I_{B1}}{cZ}$$

COMPARISON OF TWO METHODS OF ANALYSIS

SECTION RESISTANCE-CAPACITANCE METHOD

By the method of

The circuit diagram of the element is shown in Fig. 7.1. The element is given in Fig. 7.1.

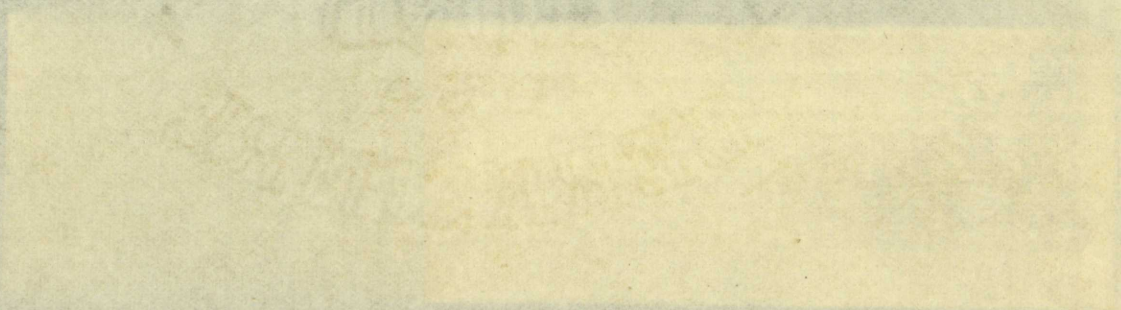


Fig. 7.1

2N section element with linear parameters

Analysis of the element in Fig. 7.1 leads to the following set of equations:

Form equations:

$$(2.1) \quad E_o = I_{a0} \left(R + \frac{1}{C_2} \right) + \frac{I_{a1}}{C_2}$$

$$(2.2) \quad 0 = I_{a1} \left(R + \frac{1}{C_2} \right) + I_{a0} \frac{1}{C_2} + I_{a2} \frac{1}{C_2}$$

$$(2.3) \quad 0 = I_{a(n-1)} \left(\frac{1}{C_2} \right) + I_{a(n)} \left(R + \frac{1}{C_2} \right) + \frac{I_{a(n+1)}}{C_2}$$

$$(2.4) \quad W E_1 = I_{a0} \left(R + \frac{1}{C_2} \right) + I_{a1} \frac{1}{C_2} + \frac{I_{a2}}{C_2}$$

$$(2.5) \quad W E_1 = I_{a0} R + I_{a0} \left(R + \frac{1}{C_2} \right) + \frac{I_{a1}}{C_2}$$

$$(5.6) \quad 0 = I_{B1} \left(R + \frac{2}{c\bar{z}} \right) + I_{B0} \frac{1}{c\bar{z}} + \frac{I_{B2}}{c\bar{z}}$$

$$(5.7) \quad 0 = \frac{I_{B(n-1)}}{c\bar{z}} + I_{Bn} \left(R + \frac{2}{c\bar{z}} \right)$$

$$(5.8) \quad E_1 = \frac{I_{Bn}}{c\bar{z}}$$

Now one can set⁴ $I_{An} = Ae^{n\theta} + Be^{-n\theta}$ and $I_{Bn} = Ce^{n\theta} + De^{-n\theta}$ where A, B, C, and D are arbitrary independent constants so that $I_{A0} = A + B$, $I_{AK} = Ae^{K\theta} + Be^{-K\theta}$, $I_{B0} = C + D$, $I_{BK} = Ce^{K\theta} + De^{-K\theta}$. Substituting then the values of the I's above in equation 5.2 we get:

$$(5.9) \quad 0 = (Ae^{\theta} + Be^{-\theta}) \left(R + \frac{2}{c\bar{z}} \right) + (A+B) \frac{1}{c\bar{z}} + (Ae^{2\theta} + Be^{-2\theta}) \frac{1}{c\bar{z}}$$

Equation 5.6 can be rewritten as:

$$(5.10) \quad 0 = \left[e^{\theta} \left(R + \frac{2}{c\bar{z}} \right) + \frac{1}{c\bar{z}} + e^{2\theta} \frac{1}{c\bar{z}} \right] A + \left[e^{-\theta} \left(R + \frac{2}{c\bar{z}} \right) + \frac{1}{c\bar{z}} + e^{-2\theta} \frac{1}{c\bar{z}} \right] B$$

Since A and B are arbitrary independent constants,

$$(5.11) \quad e^{\theta} \left(R + \frac{2}{c\bar{z}} \right) + \frac{1}{c\bar{z}} + \frac{e^{2\theta}}{c\bar{z}} = 0$$

This can be written as:

$$(5.12) \quad \theta = \text{arc cosh} \left(-\frac{cR\bar{z} + 2}{2} \right)$$

A similar argument using equation 5.6 instead of equation 5.2 will likewise lead to equation 5.12, indicating that only one θ is needed in the solution.

⁴ H. S. Carslaw and J. C. Jaeger, Operational Methods in Applied Mathematics, London: Oxford Press, 1941, p. 42-43.

$$(2.6) \quad 0 = I_0 \left(R + \frac{2}{3} \right) + I_{\infty} \left(R + \frac{2}{3} \right) + \frac{1}{c^2} + \frac{I_{\infty}}{c^2}$$

$$(2.7) \quad 0 = \frac{I_{\infty}}{c^2} + I_{\infty} \left(R + \frac{2}{3} \right)$$

$$(2.8) \quad F_1 = \frac{I_{\infty}}{c^2}$$

Now one can set $I_{\infty} = I_0 e^{H_0} + B e^{-H_0}$ and $I_{\infty} = C e^{H_0} + D e^{-H_0}$.
 A, B, C, and D are arbitrary independent constants.
 $I_{\infty} = e^{H_0} (A e^{H_0} + B e^{-H_0}) = e^{2H_0} A + e^{-H_0} B$
 then the value of the I_{∞} is given by equation (2.8) and

$$(2.9) \quad 0 = (A e^{2H_0} + B e^{-H_0}) \left(R + \frac{2}{3} \right) + (A e^{2H_0} + B e^{-H_0}) \frac{1}{c^2} + \frac{1}{c^2} + \frac{B e^{-H_0}}{c^2}$$

Equation (2.9) can be rewritten as:

$$(2.10) \quad 0 = \left[e^{2H_0} \left(R + \frac{2}{3} \right) + \frac{1}{c^2} + \frac{B e^{-H_0}}{c^2} \right] A + \left[e^{2H_0} \left(R + \frac{2}{3} \right) + \frac{1}{c^2} + \frac{B e^{-H_0}}{c^2} + \frac{B e^{-H_0}}{c^2} \right] B$$

Since A and B are arbitrary independent constants,

$$(2.11) \quad e^{2H_0} \left(R + \frac{2}{3} \right) + \frac{1}{c^2} + \frac{B e^{-H_0}}{c^2} = 0$$

This can be written as:

$$(2.12) \quad \theta = \cos \cos \left(-\frac{2R+2}{2} \right)$$

A similar argument using equation (2.10) instead of equation (2.9) will also lead to equation (2.12). Therefore, both equations (2.11) and (2.12) lead to the solution.

* H. S. Gansler and J. C. Wether, Gravitational Waves in General Relativity, London, Oxford, New York, 1973, p. 100.

Rewriting equations 5.1 through 5.8 in terms of the exponentials gives:

$$(5.13) \quad E_0 = A\left(R + \frac{1}{cZ} + \frac{e^\theta}{cZ}\right) + B\left(R + \frac{1}{cZ} + \frac{e^{-\theta}}{cZ}\right)$$

$$(5.14) \quad 0 = A\left(\frac{e^{(n-1)\theta}}{cZ} + e^{n\theta}R + \frac{2e^{n\theta}}{cZ}\right) + B\left(\frac{e^{-(n-1)\theta}}{cZ} + e^{-n\theta}R + \frac{2e^{-n\theta}}{cZ}\right)$$

$$(5.15) \quad 0 = A\frac{e^{n\theta}}{cZ} + B\frac{e^{-n\theta}}{cZ} + C\lambda + D\lambda + I_c\left(\lambda + \frac{1}{cZ}\right) - \mu E_i$$

$$(5.16) \quad 0 = C\left(2R + \frac{1}{cZ} + \frac{e^\theta}{cZ}\right) + D\left(2R + \frac{1}{cZ} + \frac{e^{-\theta}}{cZ}\right) + I_c R - \mu E_i$$

$$(5.17) \quad 0 = C\left(\frac{e^{(n-1)\theta}}{cZ} + R e^{n\theta} + \frac{2e^{n\theta}}{cZ}\right) + D\left(\frac{e^{-(n-1)\theta}}{cZ} + e^{-n\theta}R + \frac{2e^{-n\theta}}{cZ}\right)$$

$$(5.18) \quad 0 = C\frac{e^{n\theta}}{cZ} + D\frac{e^{-n\theta}}{cZ} - E_i$$

This is a set of six linear algebraic equations with constant coefficients, in six unknowns, A, B, C, D, I, and E_i . Hence one can solve for E_i in terms of E_0 . However, the form of this solution is so complicated as to preclude the comparison with experimental results. For that reason it was decided not to complete these calculations in this paper.

rewriting equations 2.1 through 2.6 in terms of the exponential

given:

$$(2.15) \quad E = A(R + \frac{1}{C_2} + \frac{C_2}{C_1}) + B(R + \frac{1}{C_2} + \frac{C_2}{C_1})$$

$$(2.16) \quad 0 = A(\frac{C_1^{(n-1)0}}{C_2} + e^{-m_0} R + \frac{2C_2^{m_0}}{C_1}) + B(\frac{C_1^{(n-1)0}}{C_2} + e^{-m_0} R + \frac{2C_2^{m_0}}{C_1})$$

$$(2.17) \quad 0 = A(\frac{C_1^{m_0}}{C_2} + B \frac{C_1^{m_0}}{C_2} + C_2 + D_2 + I_2(n + \frac{1}{C_2}) - \mu E$$

$$(2.18) \quad 0 = e(2R + \frac{1}{C_2} + \frac{C_2}{C_1}) + D(2R + \frac{1}{C_2} + \frac{C_2}{C_1}) + I_2 R - \mu E$$

$$(2.19) \quad 0 = e(\frac{C_1^{(n-1)0}}{C_2} + R e^{-m_0} + \frac{2C_2^{m_0}}{C_1}) + D(\frac{C_1^{(n-1)0}}{C_2} + e^{-m_0} R + \frac{2C_2^{m_0}}{C_1})$$

$$(2.20) \quad 0 = e \frac{C_1^{m_0}}{C_2} + D \frac{C_1^{m_0}}{C_2} - E$$

This is a set of six linear algebraic equations with constant coefficients, in six unknowns, A, B, C, D, I, and E. Hence one can solve for E, in terms of E. However, the form of this solution is so complicated as to preclude the comparison with experimental results. For that reason it was decided not to complete these calculations in this paper.

CHAPTER VI

EXPERIMENTAL INVESTIGATION OF TWO AND FOUR-SECTION FILTERS

WITH AND WITHOUT INVERSE FEED-BACK

Two and four-section filters having circuit constants equal to those in the circuits in Chapters I through V were investigated for step-wave inputs. A diagram of the inverse feed-back amplifier⁵ and the filter sections is given in Fig. 18. For investigation of filters without inverse feed-back switch SW was opened; for inverse feed-back it was closed.

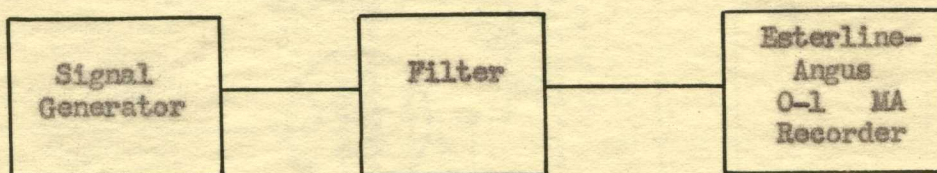


Fig. 16

Block diagram of the experimental set-up

The signal generator for the step-wave input signal was merely a battery and a DPDT switch.

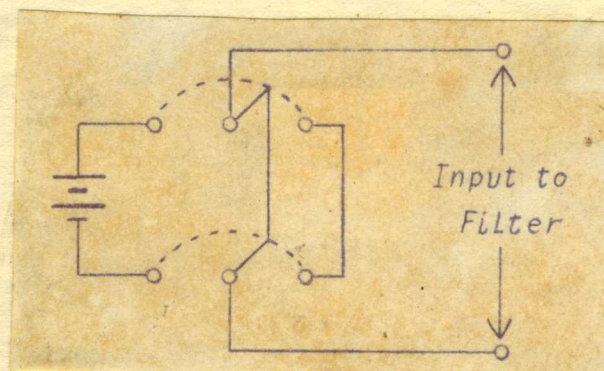


Fig. 17

Step-wave signal generator

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Radiation Laboratory Series, New York: McGraw Hill, 1948, Vol. 18, p. 486, with modifications.

EXPERIMENTAL INVESTIGATION OF THE EFFECT OF THE
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 The two low-pass filters in the circuit are of the
 type in the circuit in the first case, and in the second
 case-wave input. A circuit diagram of the low-pass filter
 the filter section is shown in Fig. 1. The filter is
 without lattice feed-back and is connected to the input
 was closed.

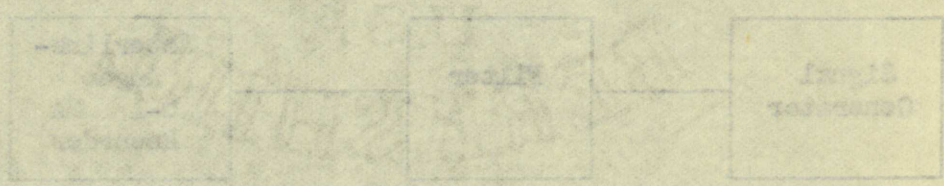


Fig. 1
 Block diagram of the experimental setup
 The signal generator for the case-wave input is connected to the
 and a DPT switch.

A plot of the experimental data obtained for a step-wave input signal into a two section filter without feed-back is given in Fig. 19, and that for a two section filter with feed-back is also plotted in Fig. 19. Experimental response data of four section filters with and without inverse feed-back are given in Fig. 22.

A plot of the experimental data obtained for a two-
signal line a few minutes after the start of the
and that for a two-signal line with a single
Fig. 12. Experimental curves for a two-signal line with
without inverse feedback and with a single



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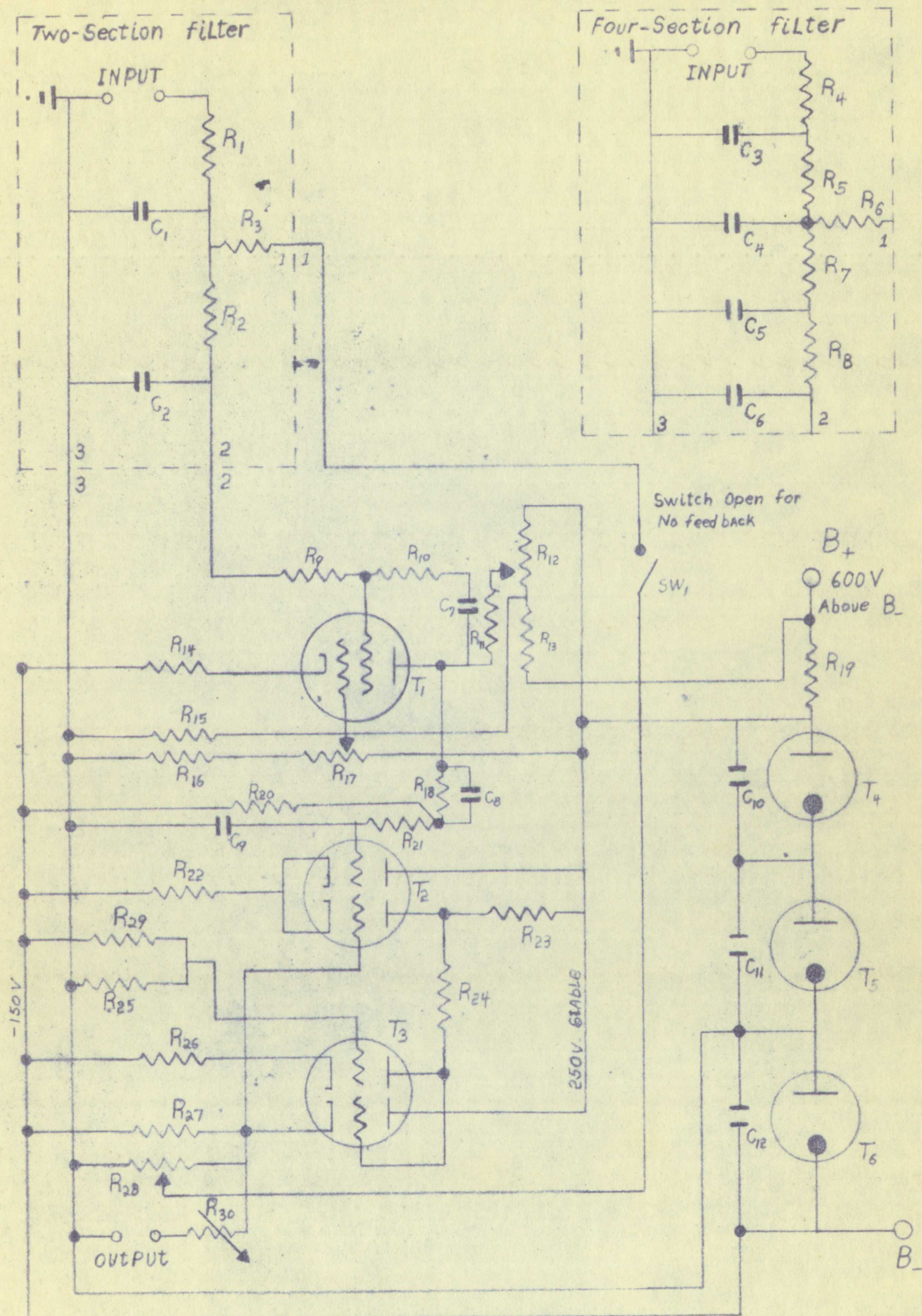


Fig. 18 Inverse feed-back Amplifier with filter section

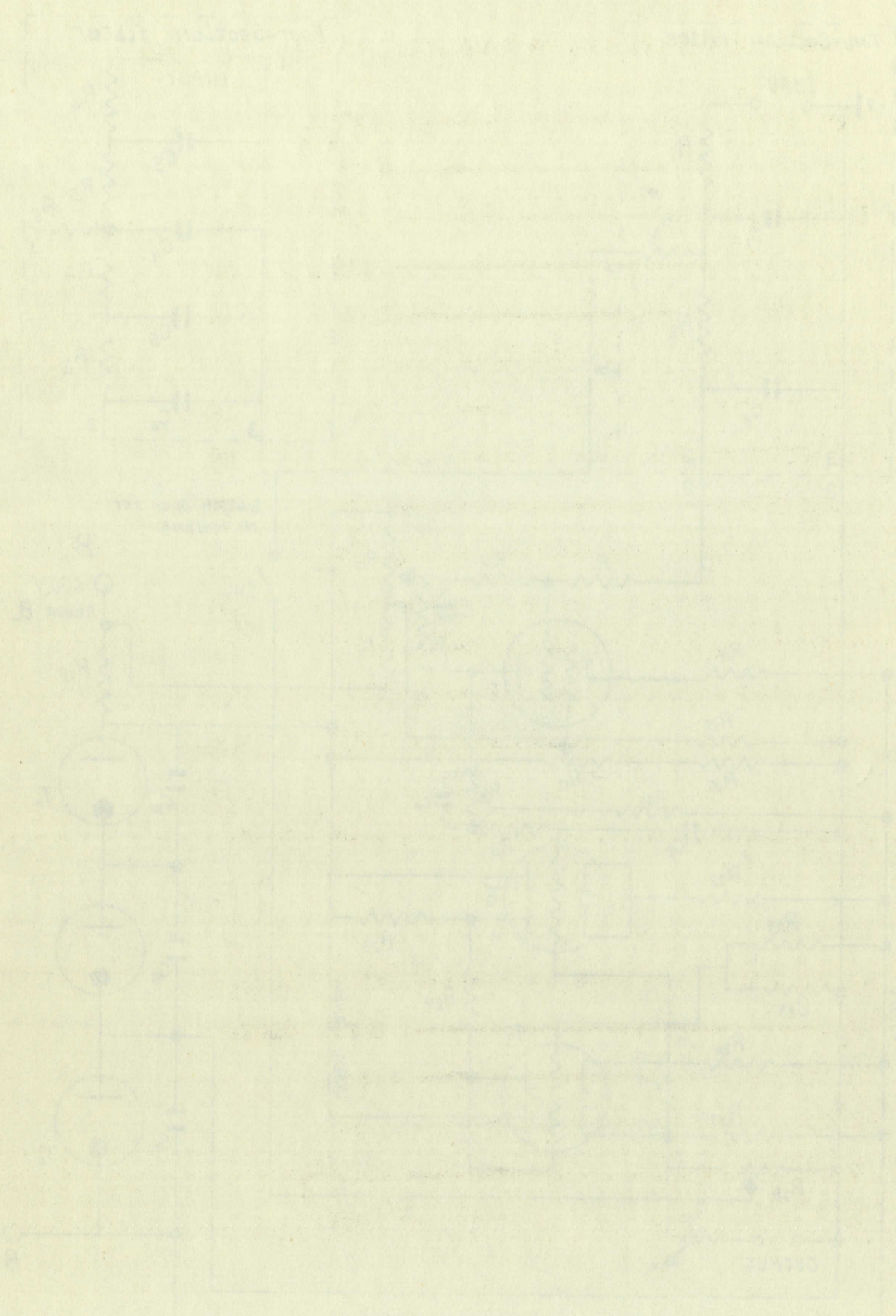


Fig. 17. Low-pass filter amplifier with filter section.

CIRCUIT CONSTANTS FOR FEED-BACK AMPLIFIER (Fig. 8)

R1,2,3,4,5,7,8,-----	10M	$\frac{1}{2}$ W
R6-----	20M	$\frac{1}{2}$ W
R 10,9,27,25,29-----	100K	1W
R 14-----	200K	1W
R 11,22-----	150K	1W
R 12-----	50K	1W
R 13-----	33K	1W
R 19-----	4K	5W
R 20,14-----	47K	1W
R 18,24,26,16-----	1M	1W
R 21-----	4.7M	$\frac{1}{2}$ W
R 23-----	570K	1W
R 30-----	10K	Pot.
R 28-----	1M	Pot.
R 17-----	2K	Pot.
C 1,2,3,4,5,6,-----	$\frac{1}{2}$ MF.	400V.
C7,8-----	100MMF	Mica
C9-----	0.03MF	400V.
C 10,11-----	8 MF	800V.
C 12-----	8 MF.	450V.
T1-----	6V6	
T2,3-----	6SL7	
T4,6-----	VR 150	
T5-----	VR105	

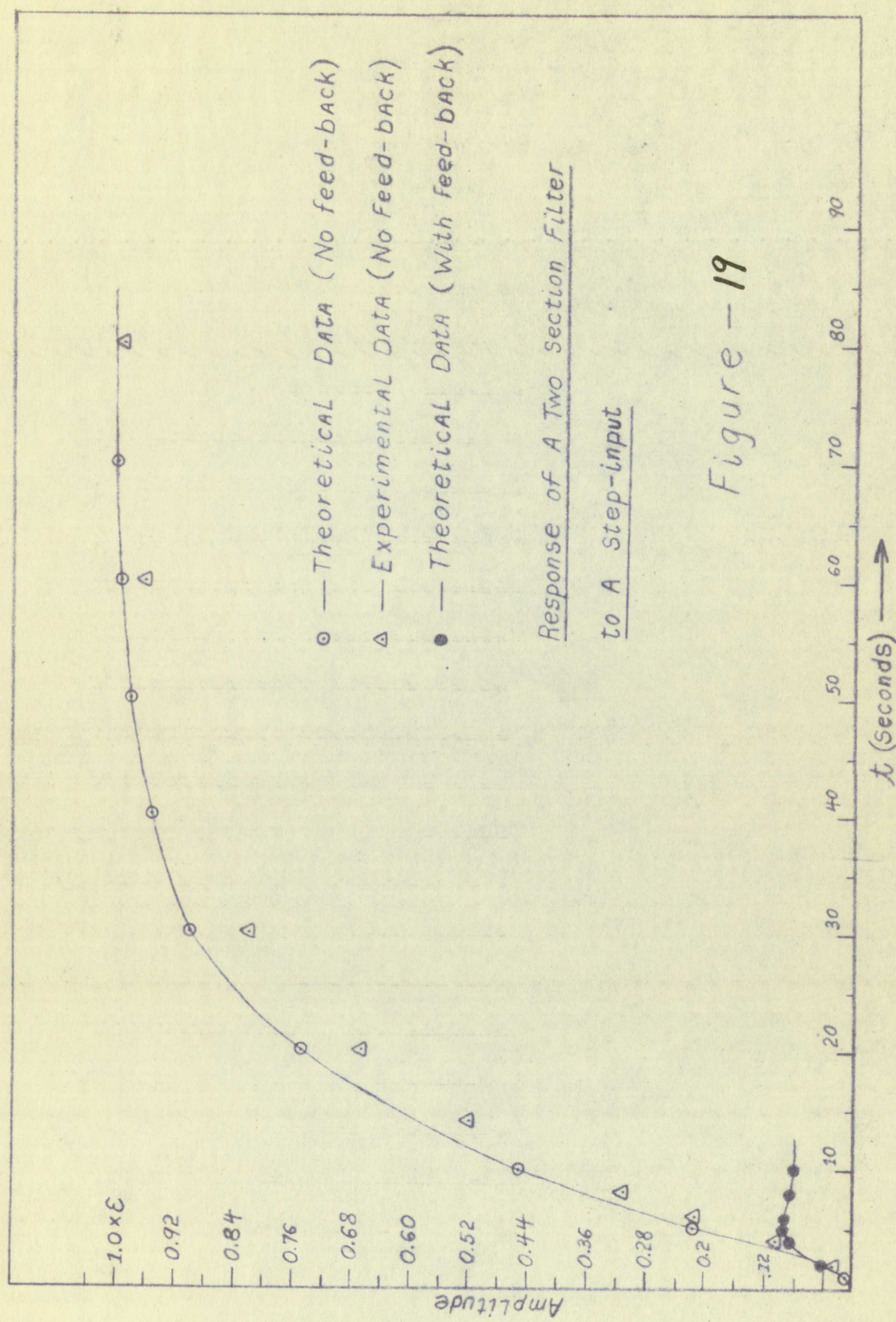
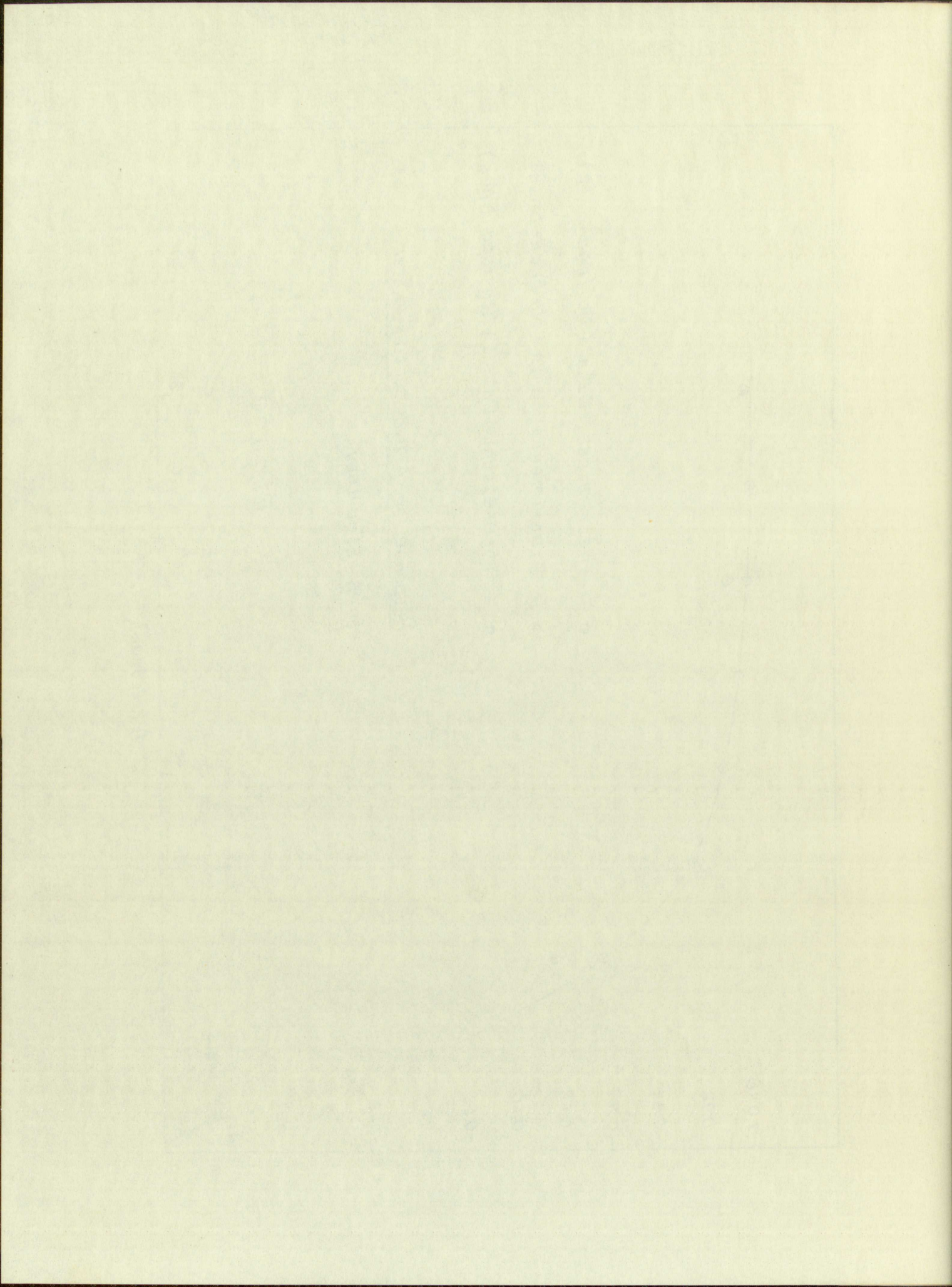
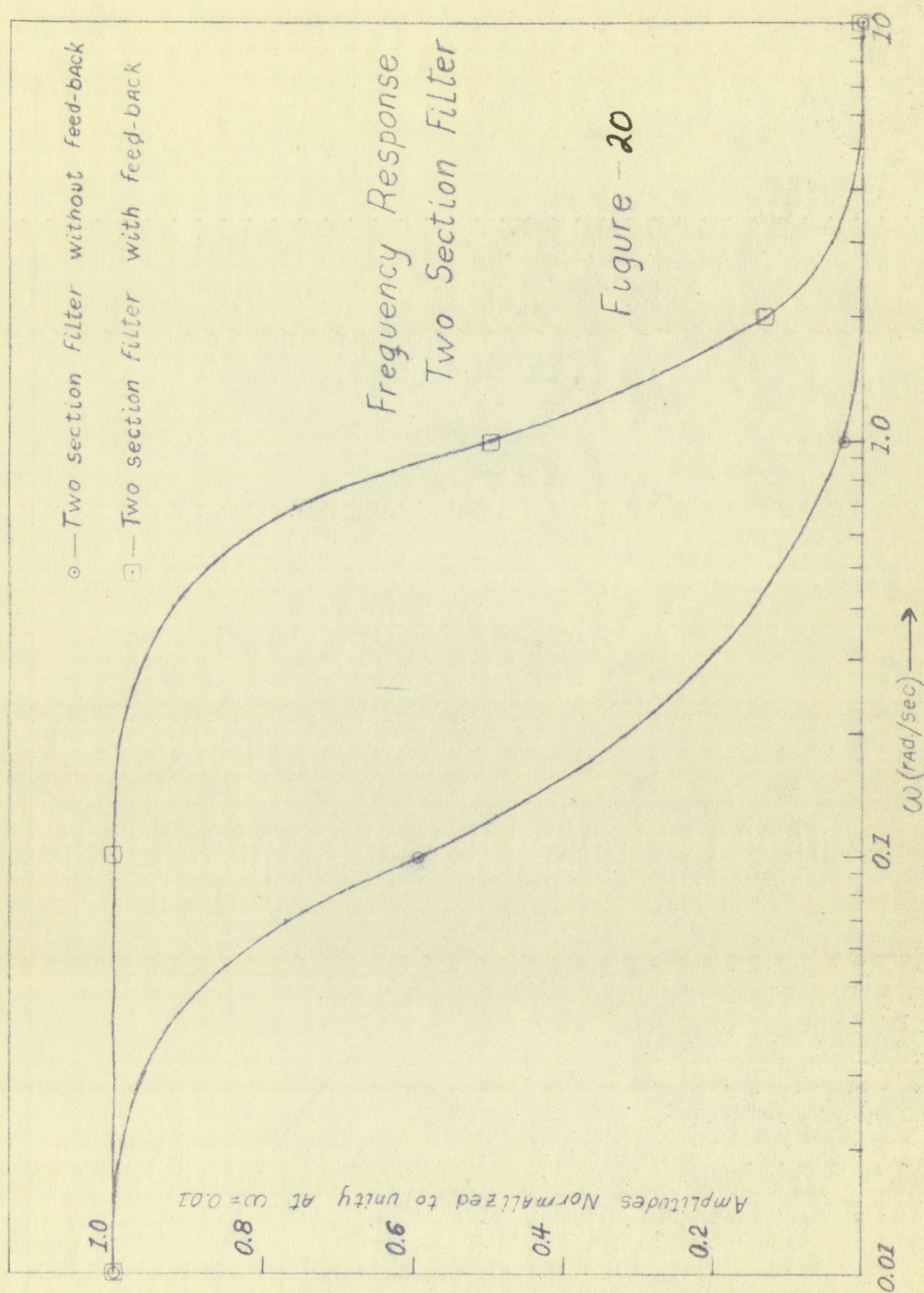
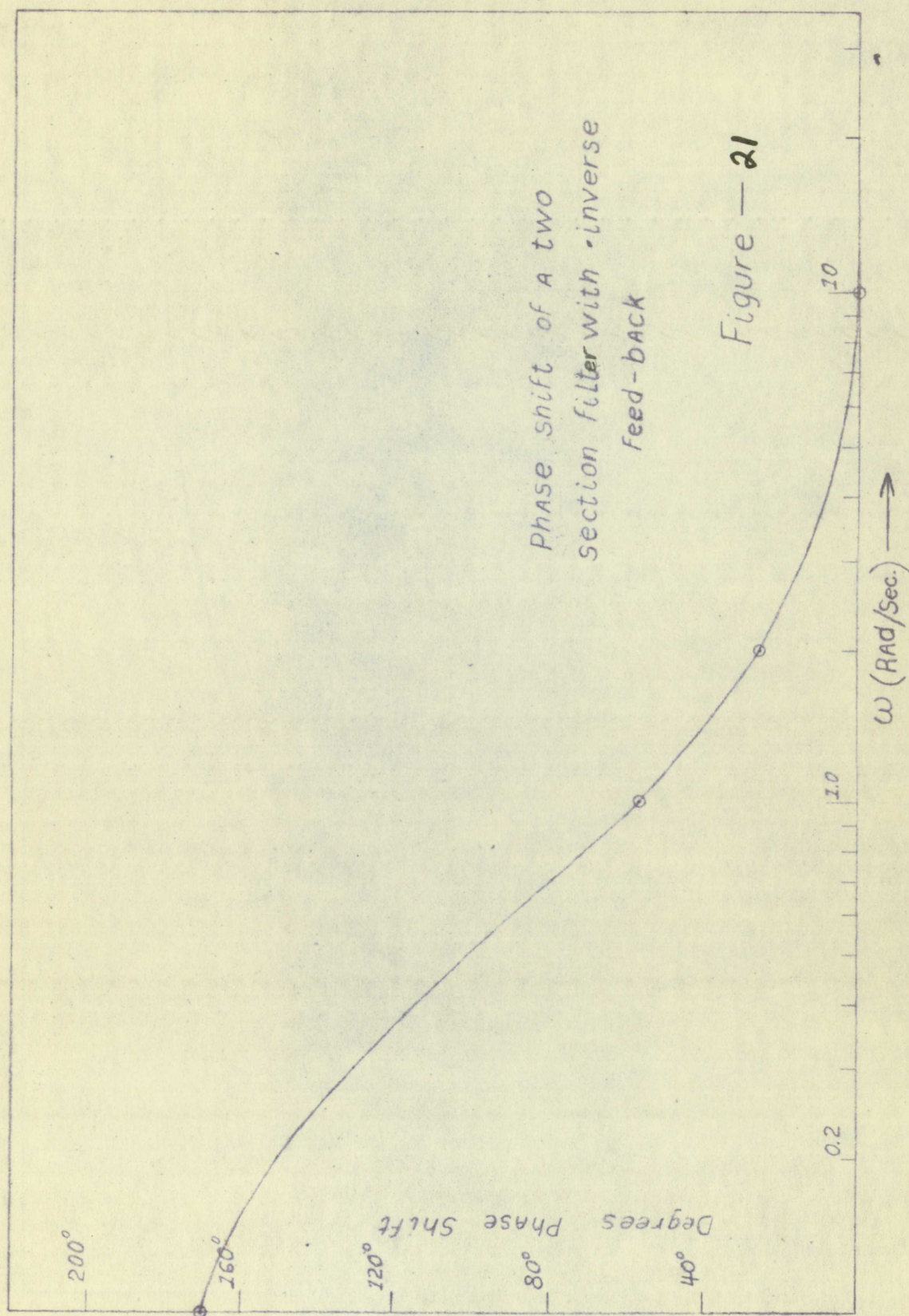
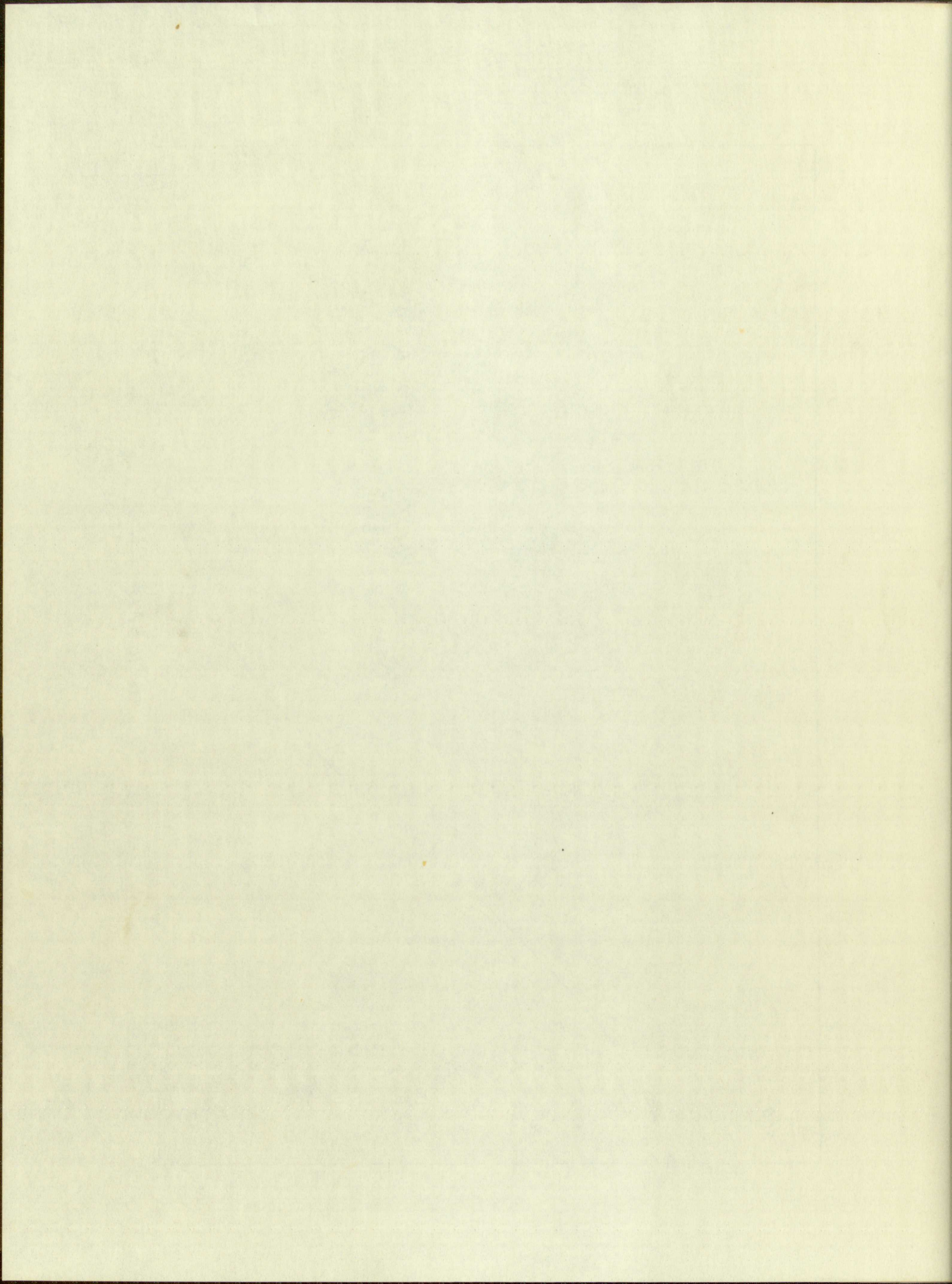


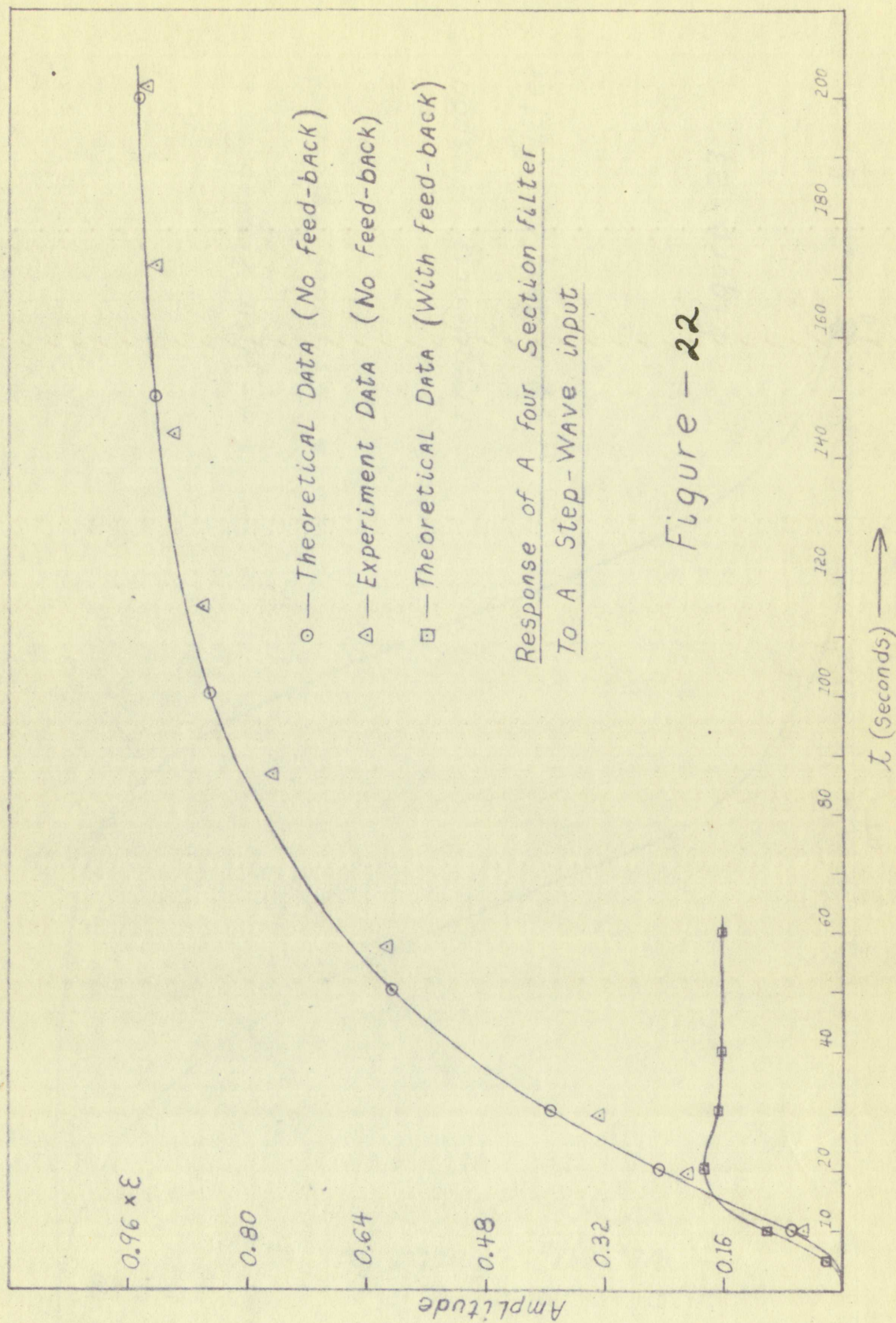
Figure-19

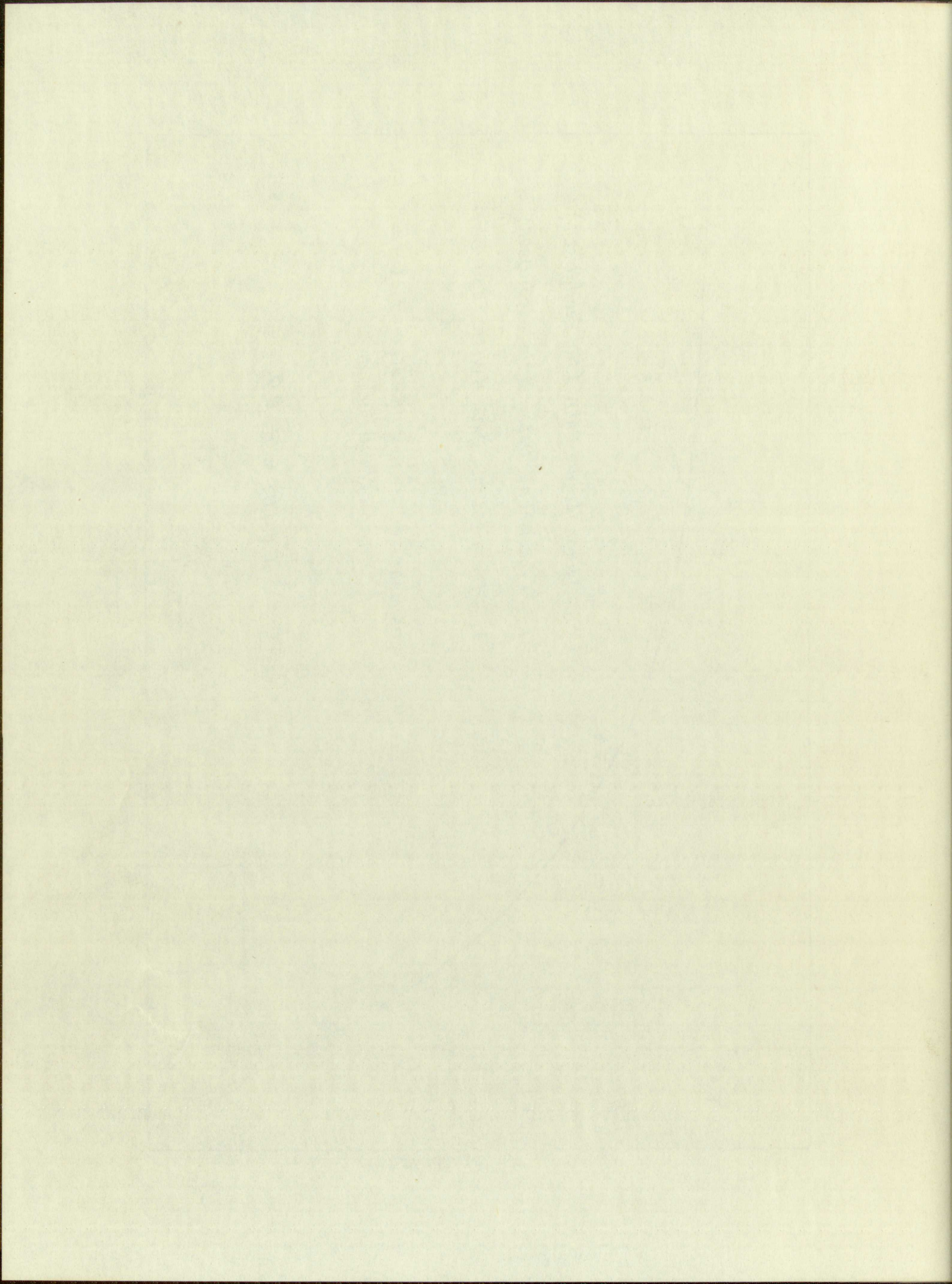


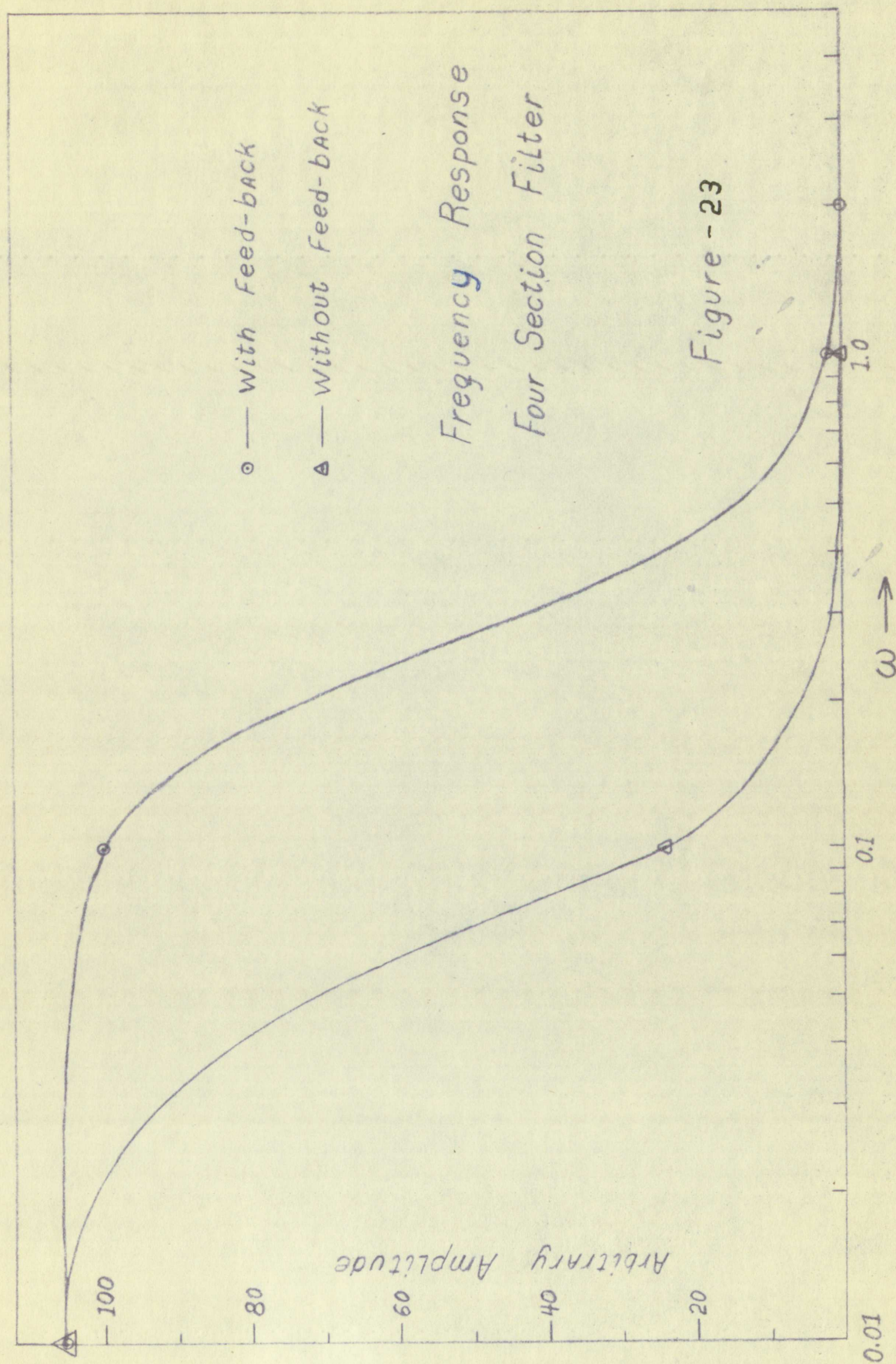












CONCLUSIONS

The experimental data agree very well with the theoretical curves. The slight shift of the experimental points to the right of the theoretical step-wave response curves is probably due to stray capacitances that were not included in the theoretical calculations. The inclusion of inverse feed-back speeds up the reaction time of the filters to a step-wave input by about a factor of ten, and causes the voltage to overshoot its final equilibrium value. The cut-off frequency of the filters with inverse feed-back is about a factor of 5 to 10 higher than the same filter without feed-back.

The most valuable characteristic of filters with negative feed-back, however, is to be seen in the greatly improved sharpness of the cut-off in their frequency response curves.

ACKNOWLEDGMENTS

I am indebted to Professor Victor H. Regener, as it was he who first suggested a thorough investigation of these circuits. Much of the circuitry used in the experimental part of the investigation was designed and built by Professor Regener. I am also indebted to Professor Roy Thomas for many valuable discussions and a continued interest in the work. I am also grateful to him for a clear presentation of the theory of Laplace Transformations.

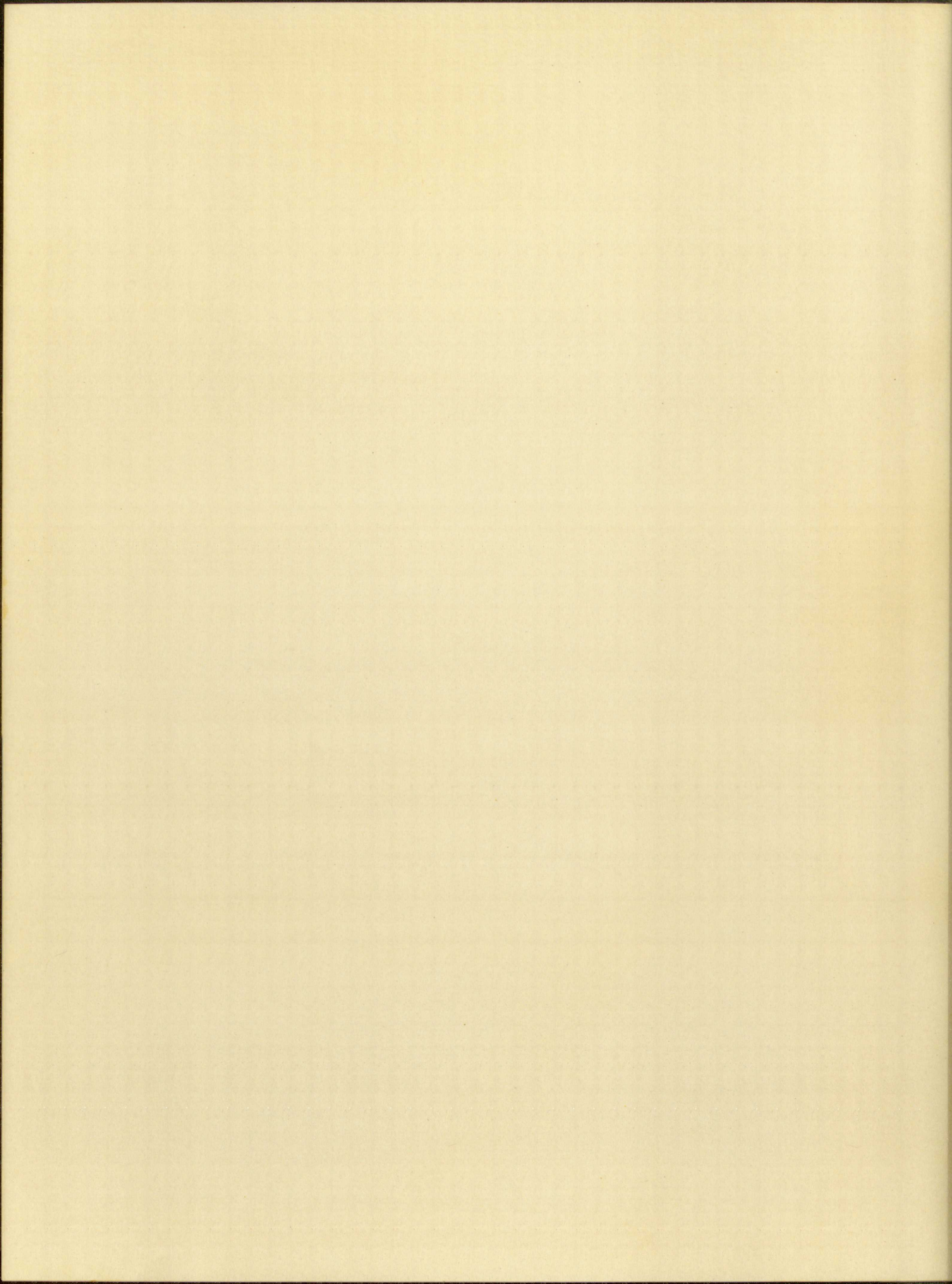
1890
March 20th

Dear Mr. [illegible]

[The body of the letter is mostly illegible due to fading. It appears to be a formal correspondence.]

Yours faithfully,
[Signature]

D.T.





IMPORTANT!

Special care should be taken to prevent loss or damage of this volume. If lost or damaged, it must be paid for at the current rate of typing.

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