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A Transient Thermal Analog of Nuclear Reactors

James R. Hume

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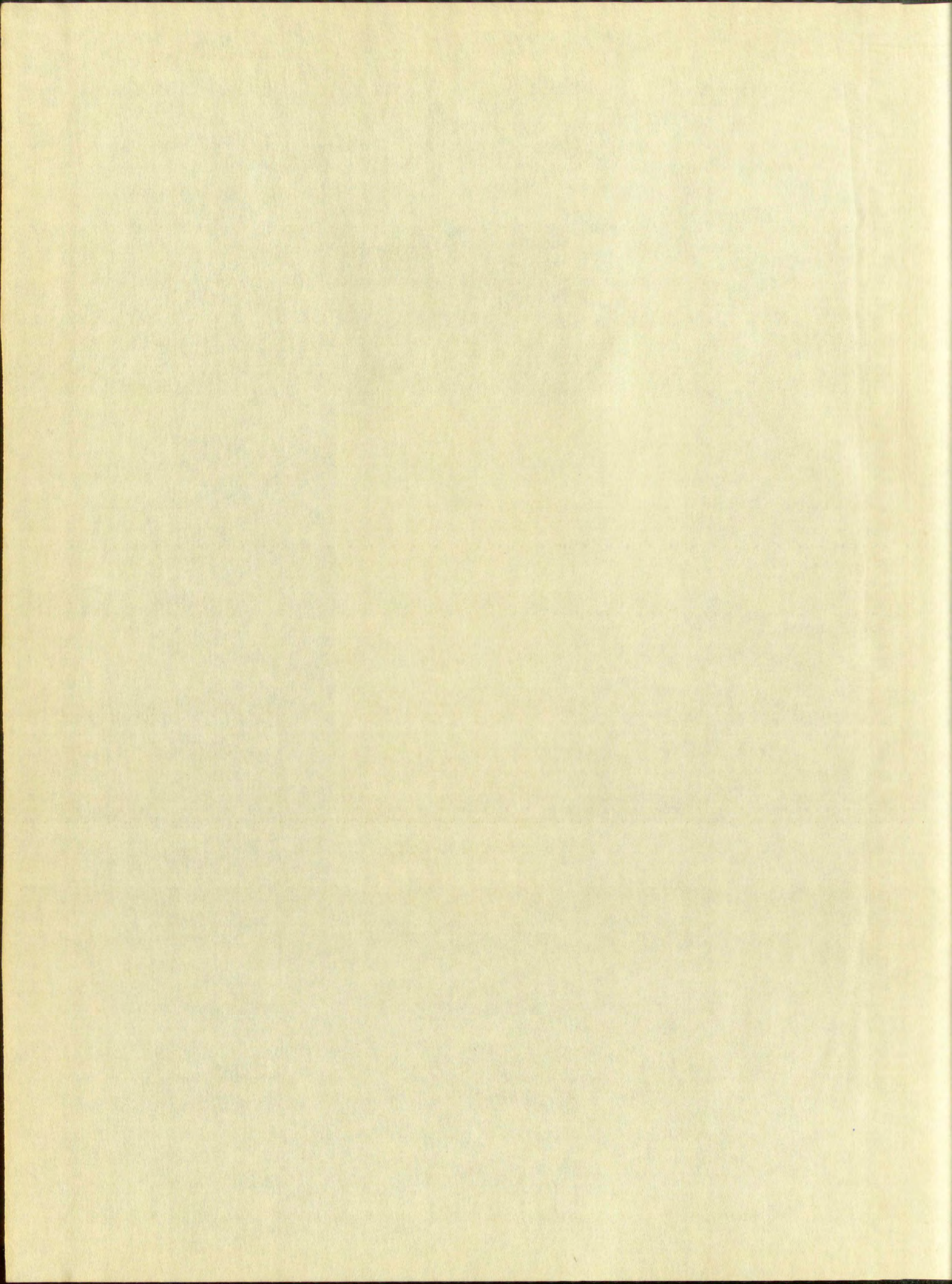
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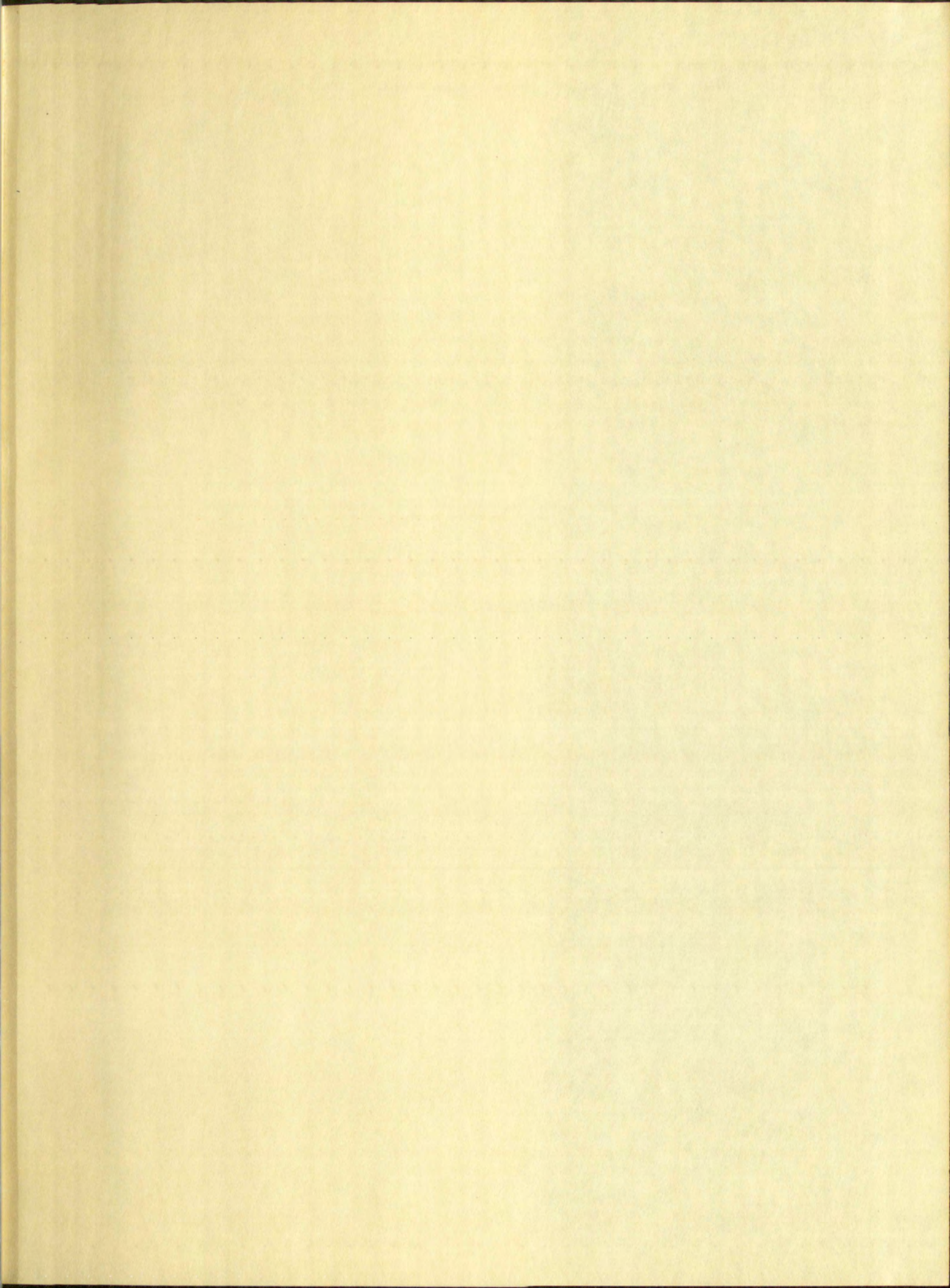
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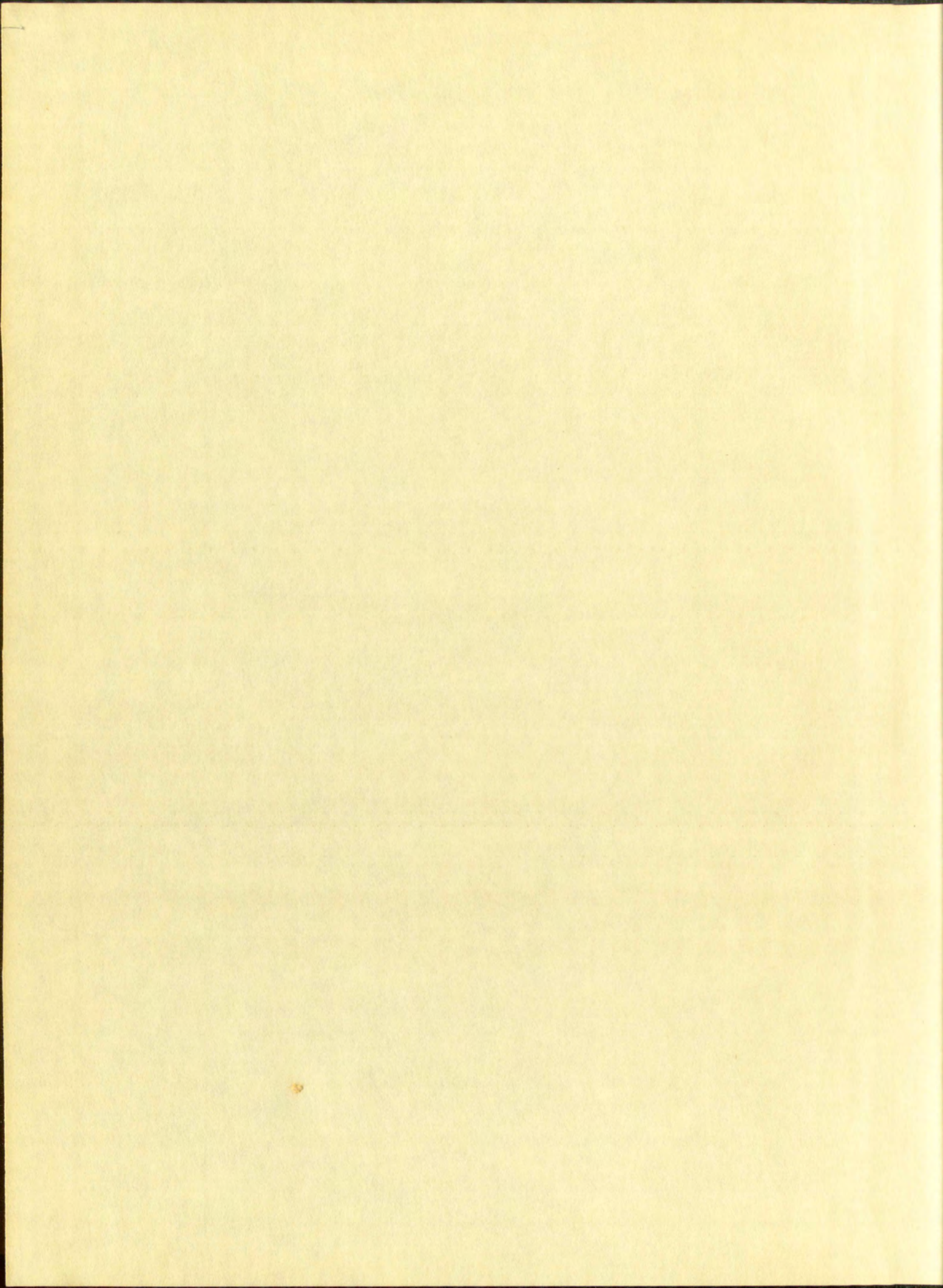
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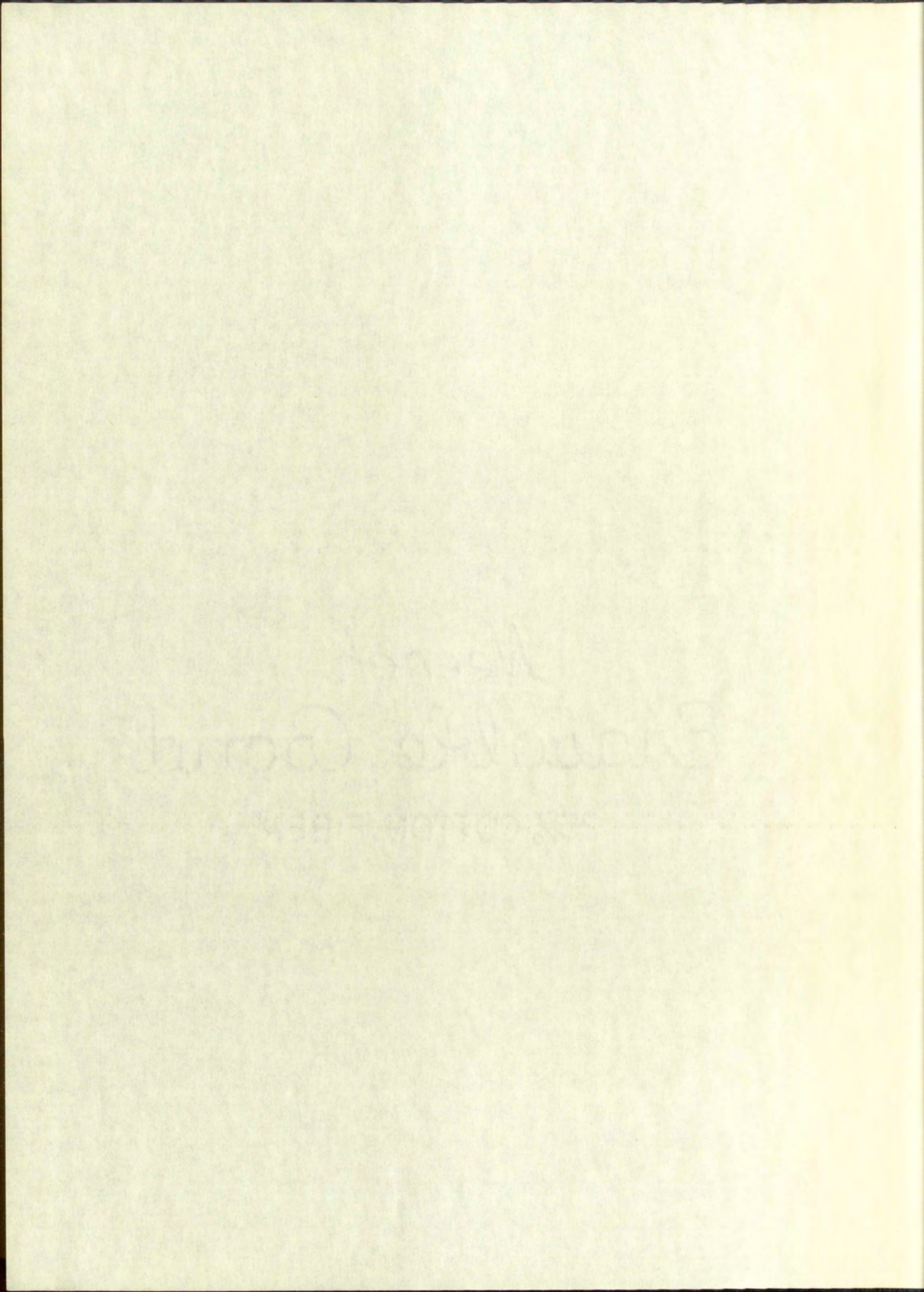
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A TRANSIENT THERMAL ANALOG OF
NUCLEAR REACTORS



By
James R. Hume

A Thesis
Submitted in Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Engineering

The University of New Mexico
1958



A THESIS SUBMITTED TO THE FACULTY OF ENGINEERING
IN CANDIDACY FOR THE DEGREE OF
MASTER OF SCIENCE

BY
JAMES E. HARRIS

A THESIS

SUBMITTED TO THE FACULTY OF ENGINEERING OF THE

UNIVERSITY OF TORONTO IN CANDIDACY FOR THE DEGREE OF

MASTER OF SCIENCE

THE UNIVERSITY OF TORONTO

1951

This thesis, directed and approved by the candidate's committee, has been accepted by the Graduate Committee of the University of New Mexico in partial fulfillment of the requirements for the degree of

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LIST OF NOMENCLATURE

- x = distance in direction of coolant flow
 t = time
 U_g = steady power, BTU/hr
 ΔU_g = step increase in power, BTU/hr
 h = heat transfer coefficient, BTU/f² hr °F
 p = perimeter of fuel element, f
 m = fuel element temperature, °F
 f = bulk coolant temperature, °F
 c = specific heat, BTU/lb °F
 δ = specific weight, lb/f³
 A = cross-sectional area, f²
 w = fluid flow rate, lb/hr
 X = similarity number, non-dimensional distance
 T = similarity number, non-dimensional time
 M = similarity number, non-dimensional metal temperature
 F = similarity number, non-dimensional fluid temperature
 a = similarity number, ratio of heat capacities
 $b = 1 + 1/a$
 $\bar{F} = \bar{F}(X, p)$, where p is complex Laplace variable
 $\bar{M} = \bar{M}(X, p)$, where p is complex Laplace variable
 V = fluid velocity, f/hr
 z = variable of integration

LIST OF SYMBOLS

L = distance in direction of coolant flow

t = time

P = electric power, W

q = heat transfer in power, W

h = heat transfer coefficient, $W/m^2 \cdot ^\circ C$

p = pressure of fluid element, Pa

T = fluid element temperature, $^\circ C$

T_c = bulk coolant temperature, $^\circ C$

T_w = average heat, W/m^2

G = specific weight, W/m^3

A = cross-sectional area, m^2

v = fluid flow rate, m^3/s

X = similarity number, non-dimensional distance

T = similarity number, non-dimensional time

M = similarity number, non-dimensional mass

temperature

F = similarity number, non-dimensional fluid

temperature

a = similarity number, ratio of heat capacities

$\beta = 1/\alpha$

$\beta = \beta(x, y)$, where β is complex Laplace variable

$\beta = \beta(x, y)$, where β is complex Laplace variable

v = fluid velocity, m/s

z = variable of integration

D = inside diameter of circular coolant flow channel,
effective diameter of annular coolant flow
channel, f

k = thermal conductivity, $\text{BTU}/f \text{ hr } ^\circ\text{F}$

G = mass velocity = w/A

μ = dynamic viscosity, $\text{lb hr}/f^2$

K = conversion factor for galvanometer trace
deflection, $^\circ\text{F}/\text{in}$

W = weight of thermocouple

Subscripts m and f refer to metal and fluid properties.

V = weight of cathode

deflection, θ

K = conversion factor for cylindrical wave

μ = dynamic viscosity, lb/ft^2

C = mass velocity, lb/ft^2

E = thermal conductivity, W/m^2

channel, h

effective viscosity of fluid, lb/ft^2

Q = initial velocity of cathode, ft/sec

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Invaluable services were performed by the author's wife, Mary L. Hume, who provided continual encouragement and performed a difficult job of typing to make this paper possible.

JRH

CHAPTER 1

INTRODUCTION

The nuclear reactor is presently receiving considerable attention as a possible source of energy for stationary power plants and propulsion systems. This has stimulated interest in many engineering problems which become very significant in reactor design work. The response of the reactor core heat transfer system to changes in power generation is one of these problems.

In the reactor, steady power level operation is achieved when the chain reaction is just self-sustaining, i.e. the reproduction of neutrons is exactly one. The expression "reactivity" is defined as the percentage by which the actual reproduction of neutrons differs from one. Positive changes in reactivity can result from many factors such as removal of neutron poisons, insertions of fissioning, moderating, or reflecting materials, and as a result of local temperature changes. Changes such as these occur from start-up and shut-down procedures, changing power demands, and accidents. The effect of positive reactivity is to put the reactor on an approximate exponential power rise which continues until the excess reactivity is removed by some means. Under certain conditions the rate of power increase may be quite high, hence the nuclear reactor has the potential for very large and rapid thermodynamic transients.

Knowledge of the transient response of the reactor core heat transfer system is important design information for control system specification, determination of maximum operating

temperatures, analysis of thermal stresses, and complete nuclear analysis.

The typical gas-cooled reactor consists of a large mass of neutron moderating material perforated by channels which extend through the assembly. Centered in the channels are the fuel elements. The coolant gas, which is usually pressurized, flows in the channel between fuel and moderator. It is usual to assume that 90% of the heat produced appears in the fuel and 10% in the moderator. The intensity of heat generation is a function of position in the reactor and may be described by trigonometric or Bessel functions depending on the core geometry.

THE INVESTIGATION

The problem of this investigation is the transient response of the bulk coolant temperature and the fuel temperature of a nuclear reactor as a result of a power excursion. The problem is attacked both theoretically and experimentally.

The theoretical portion includes a similarity analysis of an idealized mathematical model of the fuel and coolant channel. This analysis leads to non-dimensional differential equations for which an analytical solution was obtained. Because this solution is so complex, the numerical solution of another investigator is used for comparison with measurements.

The experimental investigations result in measured temperatures produced by a thermal analog of a reactor fuel and coolant channel. The time variation of the metal temperature was

These are the main results of the present investigation and are presented in the following sections.

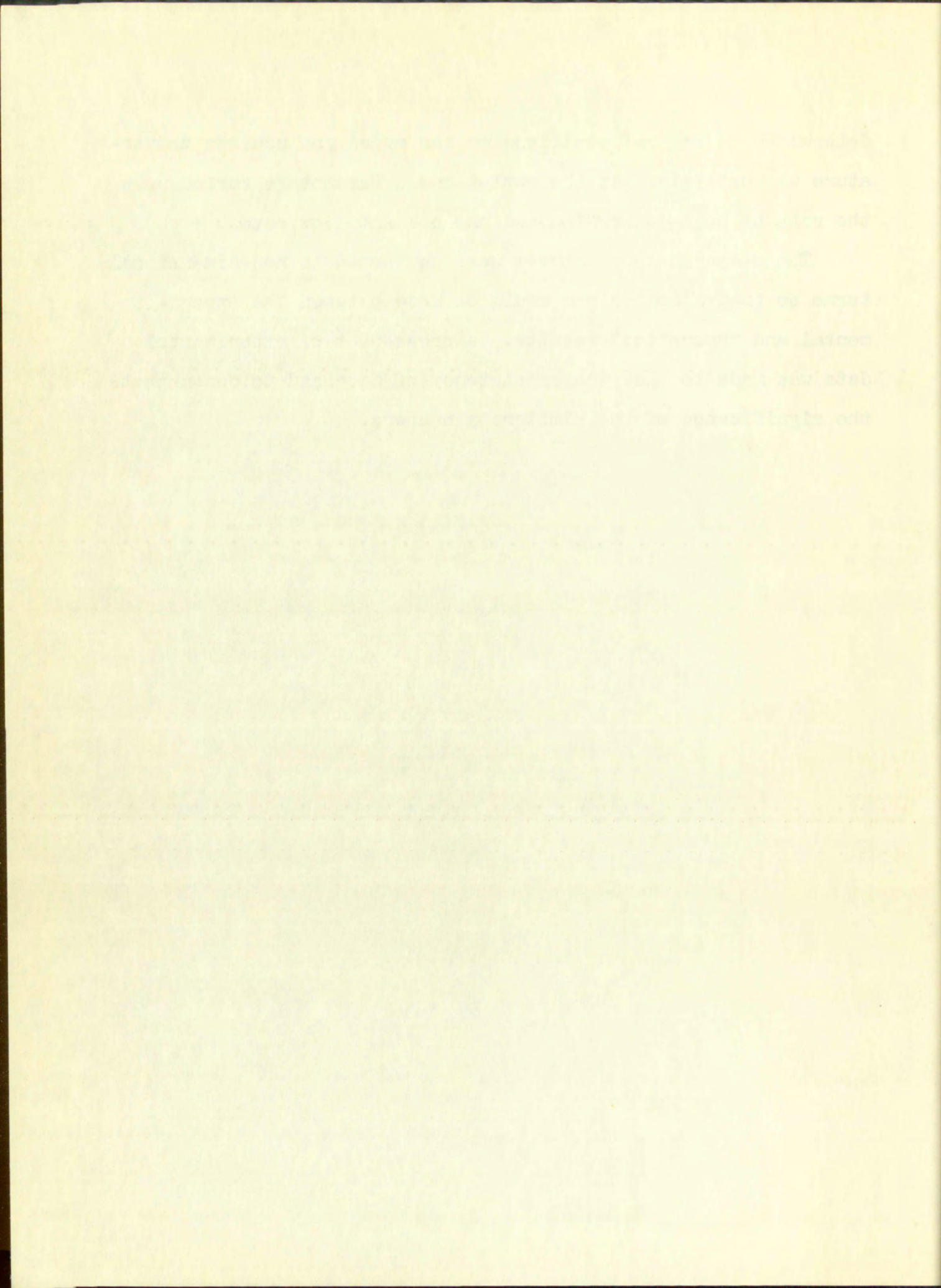
The typical gas-cooled reactor consists of a large mass of gas, usually helium, contained in a vessel which is heated by a nuclear fuel element. The gas is heated by the fuel element and the heat is transferred to the gas by convection. The gas is then used to drive a turbine or to heat a secondary fluid. The gas is cooled by a heat exchanger and the cycle is repeated. The typical gas-cooled reactor is shown in Figure 1. The gas is heated by the fuel element and the heat is transferred to the gas by convection. The gas is then used to drive a turbine or to heat a secondary fluid. The gas is cooled by a heat exchanger and the cycle is repeated.

THE INVESTIGATION

The problem of this investigation is the transient response of the gas-cooled reactor and the fuel temperature of a nuclear reactor as a result of a power excursion. The problem is attacked both theoretically and experimentally. The theoretical portion includes a similarity analysis of an idealized mathematical model of the fuel and coolant channels. This analysis leads to non-dimensional differential equations for which an analytical solution was obtained. Because this solution is so complex, the numerical solution of another three-point problem is used for comparison with the analytical solution. The experimental investigation results in measured temperature curves produced by a thermal analog of a reactor fuel and coolant channels. The time variation of the fuel temperature was

determined at several positions on the model and coolant temperature was determined at the outlet end. Parameters varied were the rate of heat generation and the coolant flow rate.

The measured temperatures were converted to non-dimensional terms so that a comparison could be made between the experimental and theoretical results. A cross-plot of experimental data was made to show the consistency of data and to demonstrate the significance of the similarity numbers.



CHAPTER 2

LITERATURE REVIEW

Analysis of the complete nuclear reactor response to transient changes in reactivity has been presented by several authors in the unclassified literature. Notable among these is Schultz(1)⁺ who lumps spacial effects and makes other simplifying assumptions that permit him to express heat transfer phenomenon in the form of "transfer functions." These are combined with the transfer functions of other reactor processes associated with the transient to give a prediction of the response of the entire nuclear system. This technique omits details of the heat transfer transient to permit solution of the larger problem. A more complex treatment is presented by Howard(2) who considers in some detail the average cell in a heterogeneous reactor. Although the heat transfer process is given considerable attention, spacial variations are averaged to permit more emphasis to be placed on the "neutronics" of the problem.

Exclusive attention is given to the thermodynamic aspects of reactor transients in the paper by Brown(3) who emphasized that the transient analysis must be based on the generalized

⁺Numbers in parentheses refer to the LIST OF REFERENCES in the Appendix.

laws of thermodynamics rather than on a succession of steady state equations. An approximate method for predicting the rise of reactor temperatures following start-up is presented in an article by Bonilla(4). His solution is achieved by numerical solution of finite steps in space and time.

Hellman, Habetler, and Barbrov(5) have set up the thermal transient problem for the whole core of a liquid metal-cooled reactor in the form of a thermal network for numerical solution. Heat generation and transmission are considered as functions of time and two space variables. Temperatures and thermal properties are space and time averaged for each space and time node. This leads to more than 110 finite difference equations and solution was accomplished on a high-speed digital computer. The application of numerical methods to transient problems involving a perturbation in initial coolant temperature or direction of coolant flow has been presented by Dusenberre(6).

Paynter and Takahashi(7) have published a method for computing the response of heat exchangers from an analogy between their equations and those from statistics. The response of a fluid flowing through an insulated pipe subject to a step increase in the inlet temperature has been published by Rizika(8,9) for both compressible and incompressible flow.

By far the most pertinent work is that of Clark, Arpaci, and Treadwell(10,11) which appeared while this investigation was in progress. These authors had derived differential

equations by a similar method and with the same assumptions as those presented in this thesis. Their solutions were also obtained by use of the Laplace transform and are similar. Improvement in the form of the solutions results from the use of non-dimensional equations here and from the use of alternate mathematical methods of inversion. The experimental verification exhibited with the papers by Clark, et al, was contributed by Treadwell(12) and Stuart(13) in their thesis work at Massachusetts Institute of Technology. Their experiments were performed on equipment nearly identical to that used in this investigation and included the measurement of fluid and metal temperatures at two locations for temperature changes on the order of 50°F. Their coolant fluid was water whereas air was used here.

Numerical solutions to the non-dimensional differential equations were calculated by Bankston(14) in connection with this project.

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CHAPTER 3

THEORETICAL INVESTIGATION

A physical system typical of this problem may be described as a solid cylindrical metal fuel rod within a concentric circular channel through the moderating material. The annular space is filled with pressurized gas which flows in the positive x direction. The transient increase in power occurs at time $t = 0$. A cross-section of a differential element is shown in Figure 1.

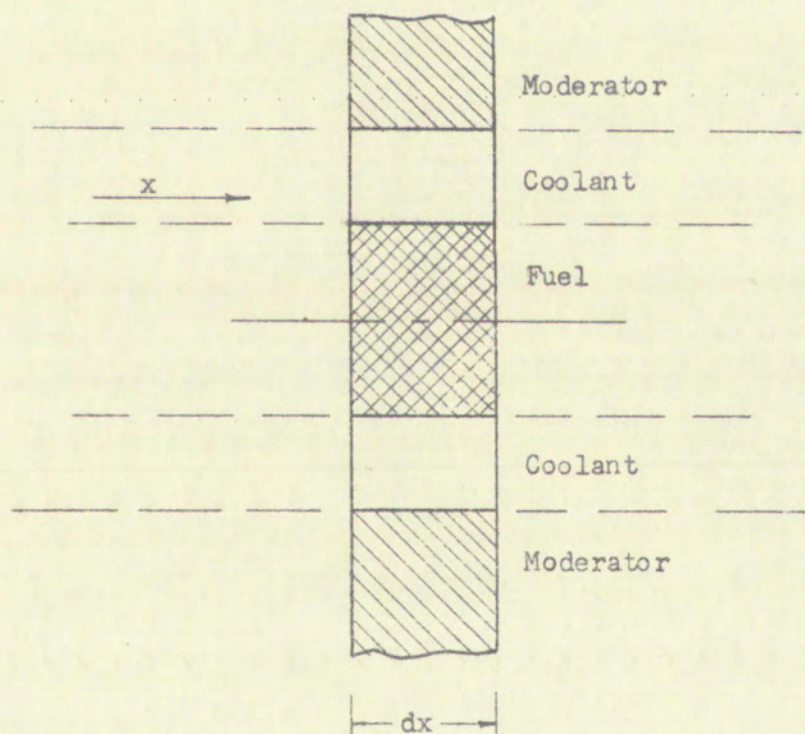
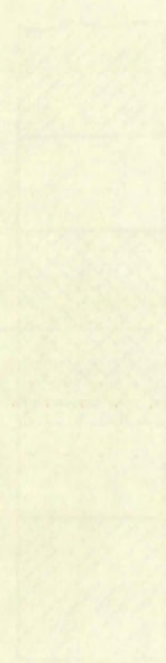


Figure 1 Differential Cross-section of a Fuel Element
and Its Cooling Channel

The first part of the paper is devoted to a review of the literature on the topic. The second part presents the results of the experiments. The third part discusses the results and compares them with the theoretical predictions. The fourth part concludes the paper.



The results of the experiments show that the theoretical predictions are in good agreement with the experimental data. This suggests that the model is valid and can be used to predict the behavior of the system.

The theoretical problem is simplified by the following assumptions:

1. The interface between moderator and coolant is adiabatic.
2. There is a stepwise increase in power with respect to time.
3. There are no radial temperature gradients in either the metal or the fluid, but there is convection heat transfer from the fuel to the gas.
4. There is no axial heat conduction.
5. All metal and fluid properties are constant.
6. The heat transfer coefficient $h \neq f(x,t)$.

DERIVATION OF DIFFERENTIAL EQUATIONS

The differential section in Figure 1 may be regarded as two parts: the fluid part and the metal part. Each of these yields an energy balance equation. For the metal part

$$(U_g + \Delta U_g) dx = h p (m - f) dx + c_m \gamma_m A_m \left(\frac{\partial m}{\partial t} \right) dx \quad 1$$

and for the fluid part

$$h p (m - f) = w c_f df \quad 2$$

where U_g = steady power, BTU/hr

ΔU_g = step increase in power, BTU/hr

h = heat transfer coefficient, BTU/f²hr°F

p = perimeter of fuel element, f

1. The first part of the paper is devoted to a general discussion of the problem.

2. In the second part we shall consider the case of a homogeneous medium.

3. The third part is devoted to the study of the properties of the solutions.

4. In the fourth part we shall discuss the question of the stability of the solutions.

5. The fifth part is devoted to the study of the asymptotic behavior of the solutions.

6. The sixth part is devoted to the study of the properties of the solutions.

7. The seventh part is devoted to the study of the properties of the solutions.

8. The eighth part is devoted to the study of the properties of the solutions.

9. The ninth part is devoted to the study of the properties of the solutions.

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11. The eleventh part is devoted to the study of the properties of the solutions.

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13. The thirteenth part is devoted to the study of the properties of the solutions.

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15. The fifteenth part is devoted to the study of the properties of the solutions.

m = metal temperature, $^{\circ}\text{F}$

f = fluid temperature, $^{\circ}\text{F}$

c = specific heat (at constant pressure for the fluid),
BTU/lb $^{\circ}\text{F}$

δ = specific weight, lb/ ft^3

A = cross-sectional area in radial plane, ft^2

w = fluid flow rate, lb/hr

and the subscripts m and f refer to the metal and fluid parts.

Since $f = f(x, t)$, then

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial t} dt.$$

Also $w = \delta_f A_f V$ where V is the fluid velocity.

Equations 1 and 2 may be rearranged to form

$$\begin{aligned} (m - f) &= \frac{U_g + \Delta U_g}{h_p} - \frac{c_m \delta_m A_m}{h_p} \left(\frac{\partial m}{\partial t} \right) \\ &= \frac{U_g + \Delta U_g}{h_p} - \frac{c_m \delta_m A_m}{h_p} \left(\frac{\partial m}{\partial t} \right) \end{aligned} \quad 3$$

and

$$(m - f) = \frac{w c_f}{h_p} \left(\frac{\partial f}{\partial x} \right) + \frac{c_f \delta_f A_f}{h_p} \left(\frac{\partial f}{\partial t} \right). \quad 4$$

The steady and transient parts of equations 3 and 4 may be separated by the substitution

$$m = m' + m'' \text{ and } f = f' + f''$$

where the single prime denotes the steady state temperature for $t < 0$ when $\frac{\partial m'}{\partial t} = \frac{\partial f'}{\partial t} = 0$, and the double prime denotes the

Let \mathcal{H} be a Hilbert space and let $\mathcal{H}_1, \mathcal{H}_2$ be subspaces of \mathcal{H} . Then $\mathcal{H}_1 \perp \mathcal{H}_2$ if and only if $\mathcal{H}_1 \subset \mathcal{H}_2^\perp$.

Let \mathcal{H} be a Hilbert space and let $\mathcal{H}_1, \mathcal{H}_2$ be subspaces of \mathcal{H} . Then $\mathcal{H}_1 \perp \mathcal{H}_2$ if and only if $\mathcal{H}_1 \subset \mathcal{H}_2^\perp$. Since $\mathcal{H}_1 \perp \mathcal{H}_2$ implies $\mathcal{H}_1 \subset \mathcal{H}_2^\perp$, we have that $\mathcal{H}_1 \perp \mathcal{H}_2$ if and only if $\mathcal{H}_1 \subset \mathcal{H}_2^\perp$.

Let \mathcal{H} be a Hilbert space and let $\mathcal{H}_1, \mathcal{H}_2$ be subspaces of \mathcal{H} . Then $\mathcal{H}_1 \perp \mathcal{H}_2$ if and only if $\mathcal{H}_1 \subset \mathcal{H}_2^\perp$. Also $\mathcal{H}_1 \perp \mathcal{H}_2$ if and only if $\mathcal{H}_1 \subset \mathcal{H}_2^\perp$.

Let \mathcal{H} be a Hilbert space and let $\mathcal{H}_1, \mathcal{H}_2$ be subspaces of \mathcal{H} . Then $\mathcal{H}_1 \perp \mathcal{H}_2$ if and only if $\mathcal{H}_1 \subset \mathcal{H}_2^\perp$. Equations 1 and 2 are equivalent to each other.

Let \mathcal{H} be a Hilbert space and let $\mathcal{H}_1, \mathcal{H}_2$ be subspaces of \mathcal{H} . Then $\mathcal{H}_1 \perp \mathcal{H}_2$ if and only if $\mathcal{H}_1 \subset \mathcal{H}_2^\perp$. For $\mathcal{H}_1 \perp \mathcal{H}_2$ implies $\mathcal{H}_1 \subset \mathcal{H}_2^\perp$ and $\mathcal{H}_1 \subset \mathcal{H}_2^\perp$ implies $\mathcal{H}_1 \perp \mathcal{H}_2$.

Let \mathcal{H} be a Hilbert space and let $\mathcal{H}_1, \mathcal{H}_2$ be subspaces of \mathcal{H} . Then $\mathcal{H}_1 \perp \mathcal{H}_2$ if and only if $\mathcal{H}_1 \subset \mathcal{H}_2^\perp$. End

The above theorem is a special case of the following theorem. Let \mathcal{H} be a Hilbert space and let $\mathcal{H}_1, \mathcal{H}_2$ be subspaces of \mathcal{H} . Then $\mathcal{H}_1 \perp \mathcal{H}_2$ if and only if $\mathcal{H}_1 \subset \mathcal{H}_2^\perp$.

where $\mathcal{H}_1 \perp \mathcal{H}_2$ means that $\mathcal{H}_1 \subset \mathcal{H}_2^\perp$ and $\mathcal{H}_2 \subset \mathcal{H}_1^\perp$. The above theorem is a special case of the following theorem. Let \mathcal{H} be a Hilbert space and let $\mathcal{H}_1, \mathcal{H}_2$ be subspaces of \mathcal{H} . Then $\mathcal{H}_1 \perp \mathcal{H}_2$ if and only if $\mathcal{H}_1 \subset \mathcal{H}_2^\perp$.

transient part. Then

$$m' - f' + m'' - f'' = \frac{U_g}{hp} + \frac{\Delta U_g}{hp} - \frac{c_m \partial m^A m}{hp} \left(\frac{\partial m''}{\partial t} \right) \quad 5$$

and

$$m' - f' + m'' - f'' = \frac{w c_f}{hp} \left(\frac{\partial f'}{\partial x} + \frac{\partial f''}{\partial x} \right) + \frac{c_f \partial f^A f}{hp} \left(\frac{\partial f''}{\partial t} \right) \quad 6$$

But when $t < 0$, $m' - f' = \frac{U_g}{hp}$ 7

$$m' - f' = w c_f \left(\frac{\partial f'}{\partial x} \right). \quad 8$$

By combining equations 5, 6, 7, and 8, the differential equations for $t \geq 0$ become

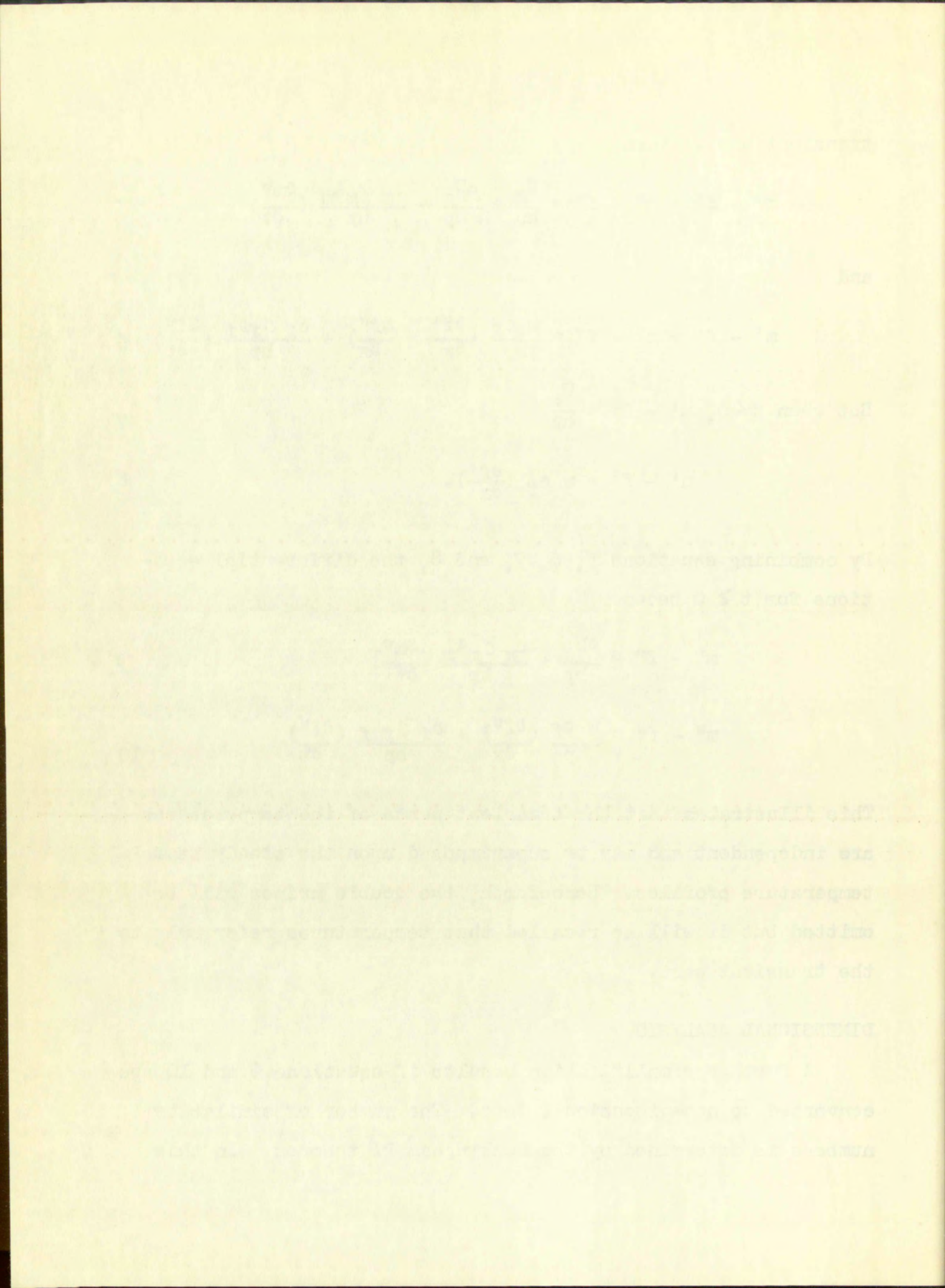
$$m'' - f'' = \frac{\Delta U_g}{hp} - \frac{c_m \partial m^A m}{hp} \left(\frac{\partial m''}{\partial t} \right) \quad 9$$

$$m'' - f'' = \frac{w c_f}{hp} \left(\frac{\partial f''}{\partial x} \right) + \frac{c_f \partial f^A f}{hp} \left(\frac{\partial f''}{\partial t} \right) \quad 10$$

This illustrates that the transient parts of the temperatures are independent and may be superimposed upon the steady state temperature profiles. Henceforth, the double primes will be omitted but it will be recalled that temperatures refer only to the transient part.

DIMENSIONAL ANALYSIS

A further simplification results if equations 9 and 10 are converted to non-dimensional form. The number of similarity numbers is determined by the Buckingham Pi theorem. In this



problem the number of variables $n = 9$: (m) , (f) , (x) , (t) , $(w c_f)$, $(c_f \delta_f A_f)$, $(c_m \delta_m A_m)$, (ΔU_g) , and (hp) . The number of primary independent units $p = 4$: (mass), (length), (time), and (temperature). Then the number of similarity numbers to be expected is $n - p = 5$. Since m and f of the original nine variables are dependent, then there should be 2 dependent and 3 independent similarity numbers.

An inspection of the differential equations 9 and 10 indicates that two of the independent similarity numbers are

$$X = \frac{xhp}{wc_f} \quad \text{and} \quad T = \frac{thp}{c_f \delta_f A_f}$$

where X and T are non-dimensional lengths and time. The third independent similarity number appears later in the derivation. The two dependent similarity numbers are expressed initially as $m = K_m M$ and $f = K_f F$ where M and F are non-dimensional temperatures. Then

$$\left(\frac{\partial m}{\partial t}\right) = K_m \left(\frac{\partial M}{\partial T}\right) \left(\frac{\partial T}{\partial t}\right) = K_m \left(\frac{hp}{c_f \delta_f A_f}\right) \left(\frac{\partial M}{\partial T}\right)$$

$$\left(\frac{\partial f}{\partial x}\right) = K_f \left(\frac{\partial F}{\partial X}\right) \left(\frac{\partial X}{\partial x}\right) = K_f \left(\frac{hp}{wc_f}\right) \left(\frac{\partial F}{\partial X}\right)$$

$$\left(\frac{\partial f}{\partial t}\right) = K_f \left(\frac{hp}{c_f \delta_f A_f}\right) \left(\frac{\partial F}{\partial T}\right)$$

Substituting the above into 9 and 10 results in

$$K_m M - K_f F = \frac{\Delta U_g}{hp} - K_m \left(\frac{c_m \delta_m A_m}{c_f \delta_f A_f}\right) \left(\frac{\partial M}{\partial T}\right)$$

$$K_m M - K_f F = K_f \left(\frac{\partial F}{\partial T} \right) + K_f \left(\frac{\partial F}{\partial X} \right)$$

It is now apparent that the third similarity number ought to be

$$a = \frac{c_m \delta_m A_m}{c_f \delta_f A_f} \text{ and for the simplest form } K_m = K_f = \frac{\Delta U_g}{hp}.$$

Now the differential equations to be solved are:

$$M - F = 1 - a \frac{\partial M}{\partial T} \quad 11$$

$$M - F = \frac{\partial F}{\partial X} + \frac{\partial F}{\partial T} \quad 12$$

where the similarity numbers are defined as $M \equiv \frac{mhp}{\Delta U_g}$, $F \equiv \frac{fhp}{\Delta U_g}$,

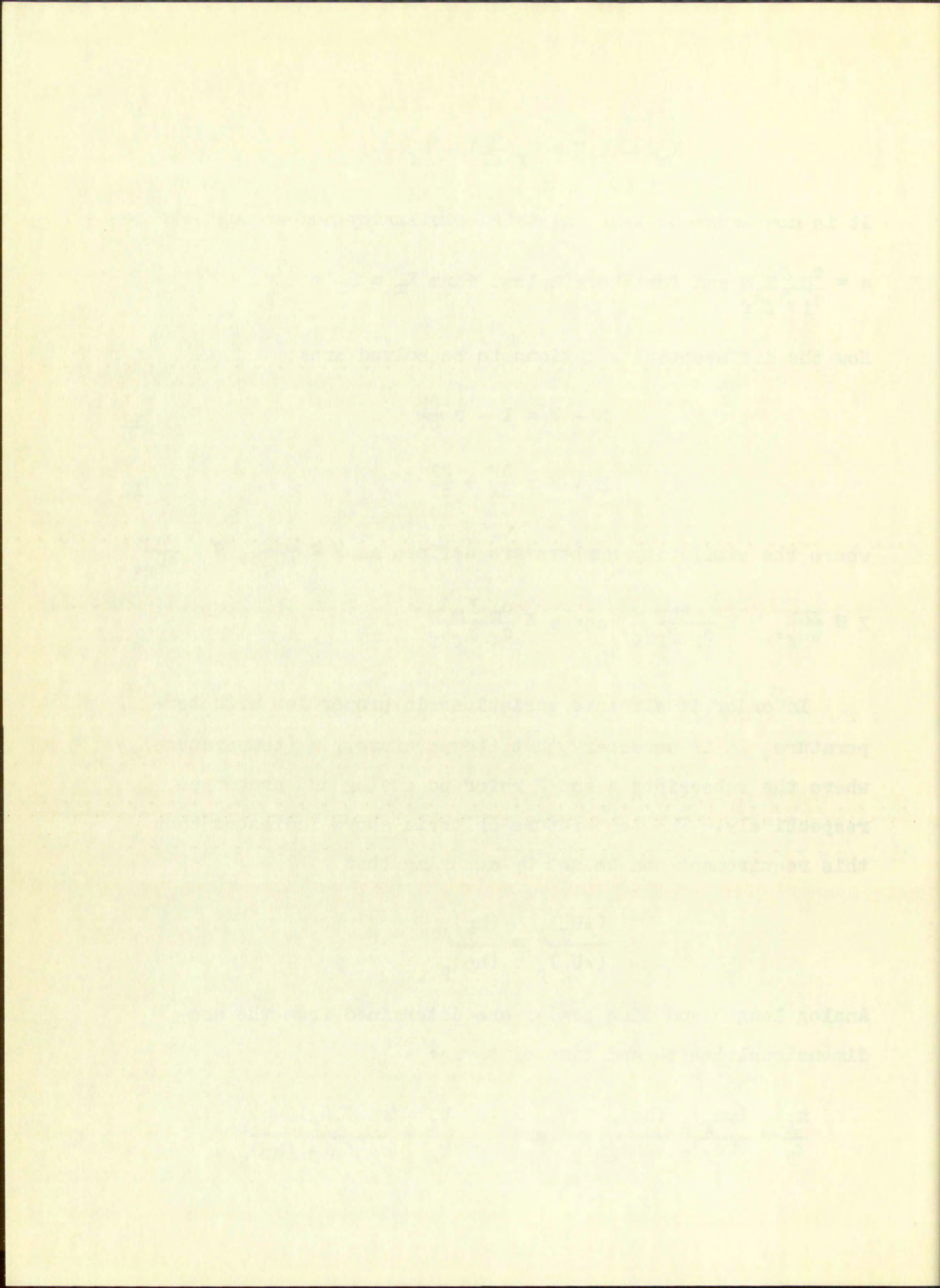
$$X \equiv \frac{xhp}{wc_f}, \quad T \equiv \frac{thp}{c_f \delta_f A_f}, \text{ and } a \equiv \frac{c_m \delta_m A_m}{c_f \delta_f A_f}.$$

In order to simulate variations in properties with temperature, it is necessary that $(\text{temperature})_A = (\text{temperature})_P$ where the subscripts A and P refer to analog and prototype respectively. The temperature criteria above indicates that this requirement can be met by assuring that

$$\frac{(\Delta U_g)_A}{(\Delta U_g)_P} = \frac{(hp)_A}{(hp)_P}.$$

Analog length and time scales are determined from the non-dimensional length and time criteria:

$$\frac{x_A}{x_P} = \frac{(wc_f)_A}{(wc_f)_P} \frac{(hp)_P}{(hp)_A} \quad \text{and} \quad \frac{t_A}{t_P} = \frac{(c_f \delta_f A_f)_P}{(c_f \delta_f A_f)_A} \frac{(hp)_P}{(hp)_A}.$$



ANALYTICAL SOLUTION

Equations 11 and 12 may be solved explicitly for M and F to obtain

$$\frac{\partial^2 F}{\partial T^2} + \frac{\partial^2 F}{\partial XT} + (1 + 1/a)\frac{\partial F}{\partial T} + (1/a)\frac{\partial F}{\partial X} - (1/a) = 0 \quad 13$$

and

$$\frac{\partial^2 M}{\partial T^2} + \frac{\partial^2 M}{\partial XT} + (1 + 1/a)\frac{\partial M}{\partial T} + (1/a)\frac{\partial M}{\partial X} - (1/a) = 0 \quad 14$$

Equations 13 and 14 may be operated on by the Laplace transform method in which the needed transformations are found in the references(15,16,17). The following transforms will be used:

$$L[1 - e^{-T/a}] = \frac{a}{p(p + 1/a)}$$

$$L[g(X,T)] = \int_0^\infty e^{-pt}[g(X,T)dT] = \bar{g}(X,p)$$

$$L\left[\frac{\partial^2 g}{\partial T^2}\right] = p^2\bar{g} - p[g(X,+0)] - \frac{\partial g}{\partial T}(X,+0)$$

$$L\left[\frac{\partial^2 g}{\partial X \partial T}\right] = \frac{\partial}{\partial X}[p\bar{g} - g(X,+0)]$$

$$L\left[\frac{\partial g}{\partial T}\right] = p\bar{g} - g(X,+0)$$

$$L\left[\frac{\partial g}{\partial X}\right] = \frac{\partial \bar{g}}{\partial X}$$

$$L[1/a] = 1/ap$$

The initial conditions imposed on this problem are:

a. $F(X,+0) = 0$

c. $M(X,+0) = 0$

b. $\frac{\partial F}{\partial T}(X,+0) = 0$

d. $\frac{\partial M}{\partial T}(X,+0) = 1/a$

Hence the transformed equation 13 is

$$p^2 \bar{F} + \frac{\partial}{\partial X}(p \bar{F}) + (1 + 1/a)p \bar{F} + (1/a) \frac{\partial \bar{F}}{\partial X} - 1/ap = 0$$

or

$$\frac{\partial \bar{F}}{\partial X} + \frac{p^2 + p + p/a}{p + 1/a} \bar{F} = \frac{1/a}{p(p + 1/a)} \quad 15$$

and the transformed equation 14 is

$$p^2 \bar{M} - 1/a + \frac{\partial}{\partial X}(p \bar{M}) + (1 + 1/a)p \bar{M} + (1/a) \frac{\partial \bar{M}}{\partial X} - 1/ap = 0$$

or

$$\frac{\partial \bar{M}}{\partial X} + \frac{p^2 + p + p/a}{p + 1/a} \bar{M} = \frac{1/a(p + 1)}{p(p + 1/a)}. \quad 16$$

Equations 15 and 16 may be readily solved as ordinary differential equations. The boundary condition, $F(T, 0) = 0$, $\bar{F}(p, 0) = 0$, provides for the transformed solution of equation 15:

$$\bar{F}(X, p) = \frac{1/a}{p^2(p + 1 + 1/a)} [1 - \exp - \frac{p(p + 1 + 1/a)X}{(p + 1/a)}]. \quad 17$$

A suitable boundary condition for equation 16 may be obtained from equation 11 where $F(0, T) = 0$ so that

$$\frac{\partial M}{\partial T}(0, T) + (1/a) M(0, T) = 1/a.$$

Operating on this by the Laplace transform yields

$$p \bar{M}(0, p) - M(0, +0) + (1/a) \bar{M}(0, p) = (1/ap) \text{ or } \bar{M}(0, p) = \frac{1/a}{p(p + 1/a)}$$

since $M(X, +0) = 0$. Then the transformed solution for equation 16 is

22

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$$\bar{M}(X,p) = \frac{1/a(p+1)}{p^2(p+1+1/a)} - \left[\frac{1/a^2}{p^2(p+1+1/a)(p+1/a)} \right] \exp - \frac{p(p+1+1/a)}{p+1/a} X. \quad 18$$

The inversion of equations 17 and 18 is simplified by expanding the first fractions in each into partial fractions and by performing the division in the exponential. It is convenient to let $b \equiv 1 + 1/a$. Then

$$\bar{F}(X,p) = -\frac{1}{ab^2p} + \frac{1}{abp^2} + \frac{1}{ab^2(p+b)} - \frac{1}{ap^2(p+b)} \left[\exp - pX - X + \frac{X/a}{p+1/a} \right] \quad 19$$

and

$$\bar{M}(X,p) = \frac{1}{a^2b^2p} + \frac{1}{abp^2} - \frac{1}{a^2b^2(p+b)} - \frac{1/a^2}{p^2(p+b)(p+1/a)} \left[\exp - pX - X + \frac{X/a}{p+1/a} \right] \quad 20$$

Difficulty arises in obtaining the inverse solution because of the term $\exp \frac{X/a}{p+1/a}$. However, this may be expanded into the power series $1 + \frac{(X/a)(p+1/a)^{-1}}{1!} + \frac{(X/a)^2(p+1/a)^{-2}}{2!} + \dots + \frac{(X/a)^n(p+1/a)^{-n}}{n!}$ $n = 1, 2, 3, \dots$

An alternate approach to this difficulty is to arrange terms involving p such that the inverse is one of the Bessel functions(10,11). However, difficulty arises later when performing

$$f(p, q) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(i p x) \exp(-i q x) dx = \delta(p - q)$$

The inversion of equation 17 and 18 is simplified by expanding the first two terms in each into partial fractions and by performing the division in the exponential. It is convenient to let $b = 1/\lambda$. Then

$$f(x, p) = -\frac{1}{a^2 p^2} + \frac{1}{a^2 p} + \frac{1}{a^2 (p + b)}$$

$$f(x, p) = -\frac{1}{a^2 p^2} + \frac{1}{a^2 p} + \frac{1}{a^2 (p + b)}$$

and

$$f(x, p) = -\frac{1}{a^2 p^2} + \frac{1}{a^2 p} + \frac{1}{a^2 (p + b)}$$

$$f(x, p) = -\frac{1}{a^2 p^2} + \frac{1}{a^2 p} + \frac{1}{a^2 (p + b)}$$

Equation 19 is obtained by expanding the first two terms of the term $\exp \frac{1}{2} \frac{x^2}{a^2}$. However, this may be expanded into the

$$1 + \frac{1}{2} \frac{x^2}{a^2} + \frac{1}{24} \frac{x^4}{a^4} + \frac{1}{720} \frac{x^6}{a^6} + \dots$$

$$1 + \frac{1}{2} \frac{x^2}{a^2} + \frac{1}{24} \frac{x^4}{a^4} + \frac{1}{720} \frac{x^6}{a^6} + \dots$$

An alternate approach to this difficulty is to express terms involving p such that the division in the second term of equation 17, however, difficulty arises later when performing

the integration in the convolution method. Some assistance might be provided here by referring to the work by Wheelon and Robacker(18).

The terms in equation 19 and 20 must now be properly grouped.

$$\begin{aligned}\bar{F}(X,p) = & -\frac{1}{ab^2p} + \frac{1}{abp^2} + \frac{1}{ab^2(p+b)} \\ & - \frac{e^{-X}}{a} \left[\left(\frac{e^{-pX}}{p^2} \right) \left(\frac{1}{p+b} \right) \right] \\ & - \frac{e^{-X}}{a} \left[\left(\frac{e^{-pX}}{p^2} \right) \left(\frac{1}{p+b} \right) \sum_1^{\infty} \frac{(X/a)^n (p+1/a)^{-n}}{n!} \right]\end{aligned}\quad 21$$

$$\begin{aligned}\bar{M}(X,p) = & \frac{1}{a^2b^2p} + \frac{1}{abp^2} - \frac{1}{a^2b^2(p+b)} \\ & - \frac{e^{-X}}{a} \left[\left(\frac{e^{-pX}}{p^2} \right) \left(\frac{1}{p+b} \right) \left(\frac{1}{p+1/a} \right) \right] \\ & - \frac{e^{-X}}{a} \left[\left(\frac{e^{-pX}}{p^2} \right) \left(\frac{1}{p+b} \right) \left(\frac{1}{p+1/a} \right) \sum_1^{\infty} \frac{(X/a)^n (p+1/a)^{-n}}{n!} \right]\end{aligned}\quad 22$$

The inverse functions required are as follows:

		<u>Restrictions</u>
$L^{-1}[(p+1/a)^{-n}] = \frac{T^{n-1}e^{-T/a}}{(n-1)!}$		$T > 0$
$L^{-1}[1/p]$	$= 1$	$T > 0$
$L^{-1}[1/p^2]$	$= T$	$T > 0$
$L^{-1}\left[\frac{1}{p+b}\right]$	$= e^{-bT}$	$T > 0$
$L^{-1}\left[\frac{1}{p+1/a}\right]$	$= e^{-T/a}$	$T > 0$
$L^{-1}\left[\frac{e^{-pX}}{p^2}\right]$	$= 0$ $= T - X$	$0 < T < X$ $T > X$

The integration in the denominator is done by the method of partial fractions. The result is

The terms in equation 19 and 20 are now to be grouped.

grouped.

$$F(x, p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-ipx}}{p^2 + a^2} dp = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-ipx}}{(p+ia)(p-ia)} dp$$

$$= \frac{1}{2\pi} \left(\int_{-\infty}^{\infty} \frac{e^{-ipx}}{(p+ia)(p-ia)} dp \right)$$

$$= \frac{1}{2\pi} \left(\int_{-\infty}^{\infty} \frac{e^{-ipx}}{(p+ia)(p-ia)} dp \right)$$

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$$= \frac{1}{2\pi} \left(\int_{-\infty}^{\infty} \frac{e^{-ipx}}{(p+ia)(p-ia)} dp \right)$$

$$= \frac{1}{2\pi} \left(\int_{-\infty}^{\infty} \frac{e^{-ipx}}{(p+ia)(p-ia)} dp \right)$$

The inverse Laplace transform is as follows:

Partial Fractions

$$F(s) = \frac{1}{(s+ia)(s-ia)} = \frac{A}{s+ia} + \frac{B}{s-ia}$$

$$F(s) = \frac{1}{(s+ia)(s-ia)} = \frac{A}{s+ia} + \frac{B}{s-ia}$$

$$F(s) = \frac{1}{(s+ia)(s-ia)} = \frac{A}{s+ia} + \frac{B}{s-ia}$$

$$F(s) = \frac{1}{(s+ia)(s-ia)} = \frac{A}{s+ia} + \frac{B}{s-ia}$$

$$F(s) = \frac{1}{(s+ia)(s-ia)} = \frac{A}{s+ia} + \frac{B}{s-ia}$$

$$F(s) = \frac{1}{(s+ia)(s-ia)} = \frac{A}{s+ia} + \frac{B}{s-ia}$$

The restrictions on the term $(T - X)$ implies that the inverse solutions must be handled separately for each of two time intervals: $0 \leq T \leq X$ and $T > X$. Referring to the definitions, $T > X$ is equivalent to $t > x/V$ where V is the fluid velocity. This has the physical significance that until $t = x/V$ no fresh fluid has reached point x and the temperatures are independent of x . After $t = x/V$, fluid which was not in the channel at $t = 0$ influences temperatures at x and the temperatures are dependent on x .

The solutions of 19 and 20 for the first time interval are

$$F(X, T) = T/ab + 1/ab^2[e^{-bT} - 1] \quad 23$$

and

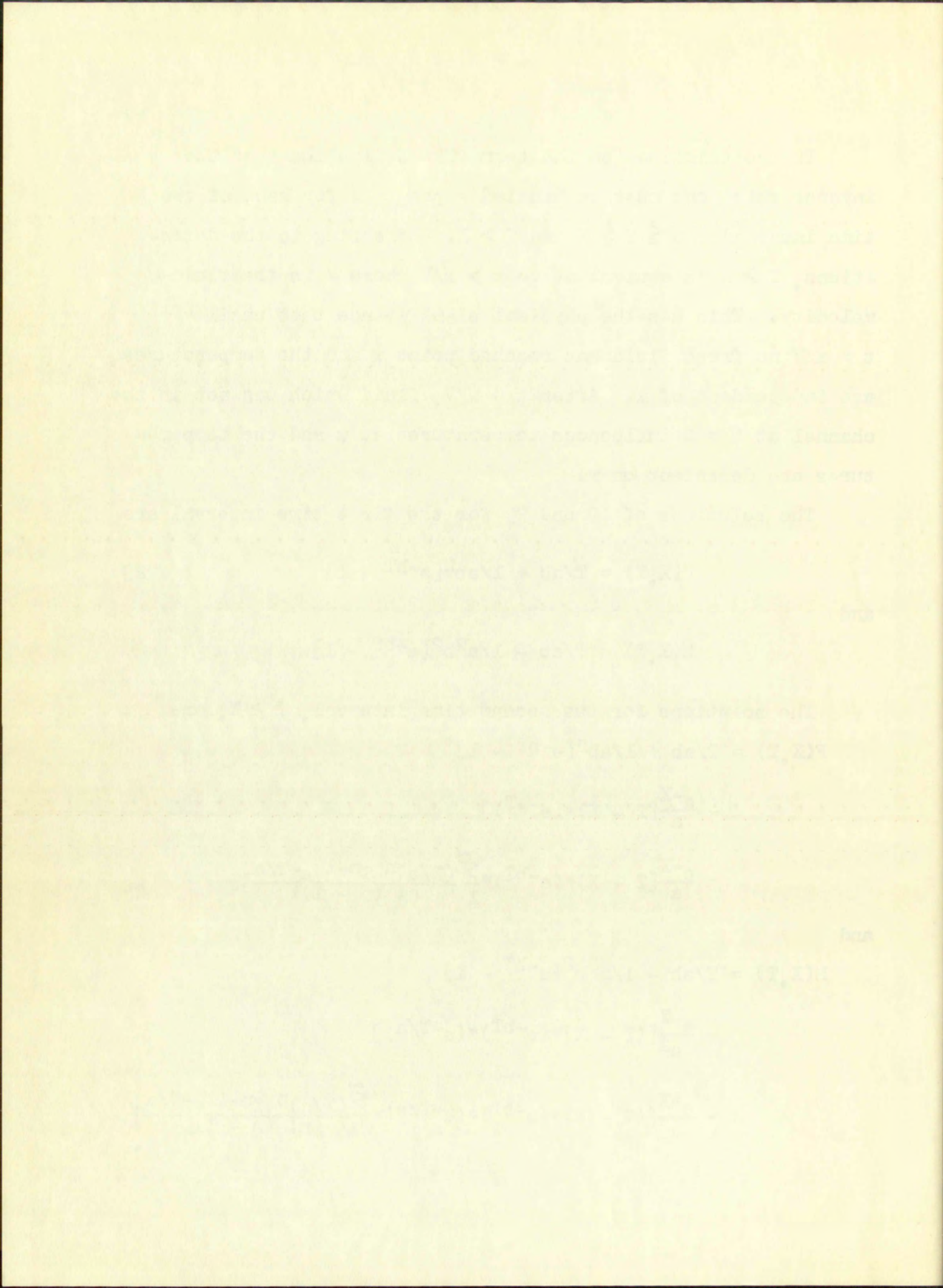
$$M(X, T) = T/ab - 1/a^2b^2[e^{-bT} - 1]. \quad 24$$

The solutions for the second time interval, $T > X$, are

$$\begin{aligned} F(X, T) = & T/ab + 1/ab^2[e^{-bT} - 1] \\ & - \frac{e^{-X}}{a}[T - X](e^{-bT}) \\ & - \frac{e^{-X}}{a}[T - X](e^{-bT}) * \sum_{n=1}^{\infty} \frac{(X/a)^n T^{n-1} e^{-T/a}}{n! (n-1)!} \end{aligned} \quad 25$$

and

$$\begin{aligned} M(X, T) = & T/ab - 1/a^2b^2[e^{-bT} - 1] \\ & - \frac{e^{-X}}{a^2}[(T - X)(e^{-bT})(e^{-T/a})] \\ & - \frac{e^{-X}}{a^2}[(T - X)(e^{-bT})(e^{-T/a}) * \sum_{n=1}^{\infty} \frac{(X/a)^n T^{n-1} e^{-T/a}}{n! (n-1)!}] \end{aligned}$$



where the asterisks indicate that the terms so connected are to be treated by the method of convolution. This is a technique for the inverse transformation for the product of functions and is discussed by Churchill(16), pages 36-40 and by Pipes(17), pages 525-526. The following equations illustrate the general convolution technique:

$$\bar{g}_1(X,p) \bar{g}_2(X,p) = g_1(X,T) * g_2(X,T)$$

$$g_1(X,T) * g_2(X,T) = \int_0^T g_1(X,T-z) g_2(X,z) dz$$

$$= \int_0^X g_1(X,T-z) g_2(X,z) dz$$

$$+ \int_X^T g_1(X,T-z) g_2(X,z) dz$$

In this problem, the inverse transformation of $\frac{e^{-pX}}{p^2}$ is involved in all the convolution operations. When $T < X$ the inverse is zero and therefore the integral from 0 to X is zero. Equations 25 and 26 are restricted in application to $T > X$ and the convolution integral has the limits X to T.

The convolutions required for equations 25 and 26 are now performed.

In equation 25

$$(T-X) * (e^{-bT}) = \int_X^T (T-z-X) e^{-bz} dz$$

$$= (T-X) \int_X^T e^{-bz} dz - \int_X^T ze^{-bz} dz$$

where the asterisk indicates that the terms are convoluted and
 as indicated by the method of convolution. This is a convolution
 and the inverse transformation for the product of functions and
 is discussed by (2.10-11), pages 3-4 and 17-18 (17).
 pages 23-24. The following equations illustrate the general
 convolution theorem.

$$\begin{aligned} \bar{f}_1(x) \bar{f}_2(x) &= \bar{f}_1(x) * \bar{f}_2(x) \\ \bar{f}_1(x) \bar{f}_2(x) &= \int_0^T \bar{f}_1(x, \tau) \bar{f}_2(x, \tau) d\tau \\ &= \int_0^T \bar{f}_1(x, \tau) \bar{f}_2(x, \tau) d\tau \\ &= \int_0^T \bar{f}_1(x, \tau) \bar{f}_2(x, \tau) d\tau \end{aligned}$$

In this problem, the inverse transformation of $\frac{e^{-px}}{p}$ is involved
 in all the convolution operations. When $T < X$ the inverse is
 zero and therefore the integral from 0 to X is zero. Equations 25
 and 26 are verified in application to $T > X$ and the convolu-
 tion integral has the limits X to T .
 The convolution is repeated for equations 25 and 26 and now
 performed.

In equation 25

$$\begin{aligned} (T - X) e^{-px} &= \int_X^T (T - \tau) e^{-p\tau} d\tau \\ &= (T - X) \int_X^T e^{-p\tau} d\tau \end{aligned}$$

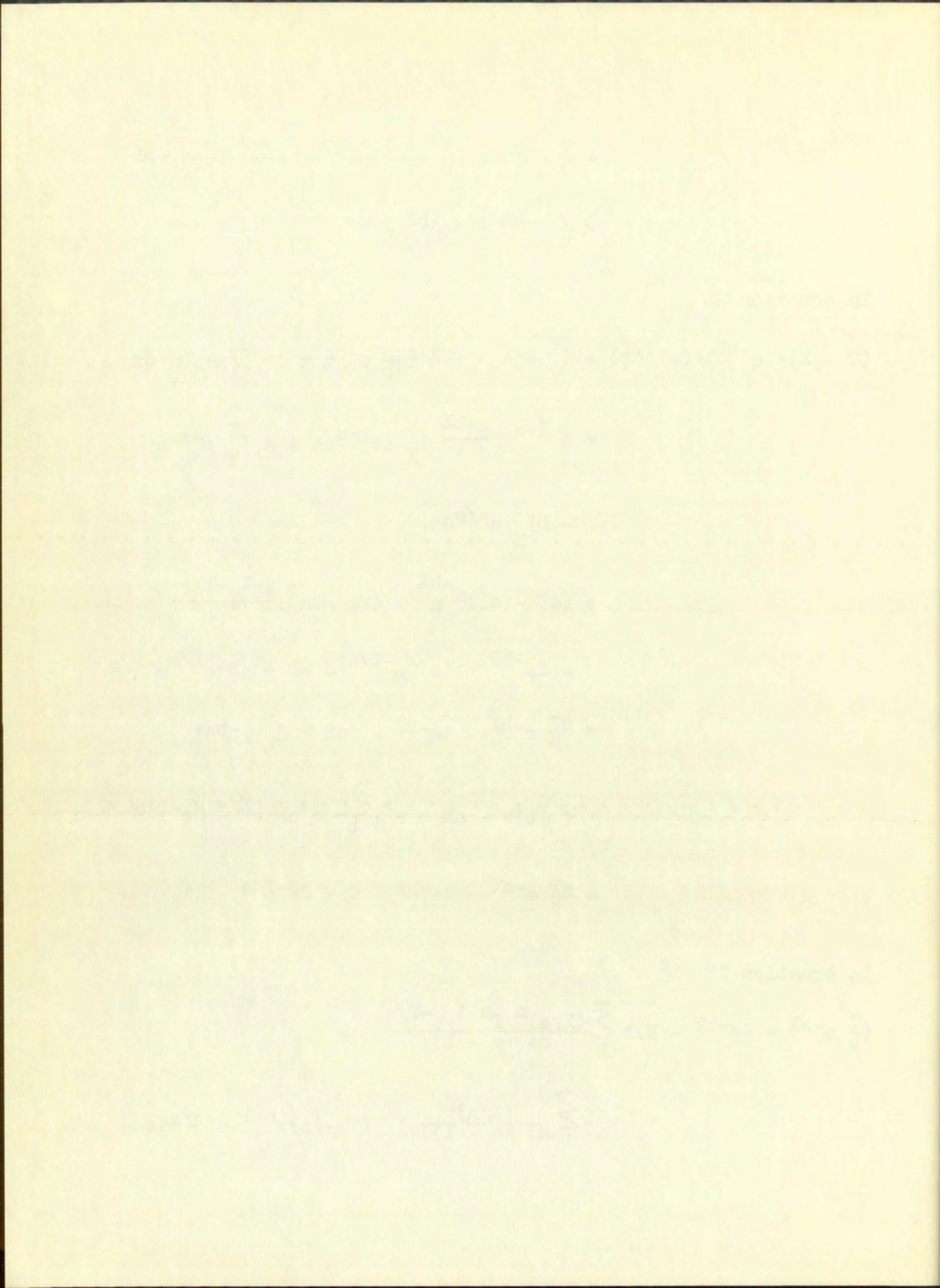
$$\begin{aligned}
&= \frac{T}{b} e^{-bX} + \left[\frac{X}{b} + \frac{1}{b^2} \right] e^{-bT} - \left[\frac{2X}{b} + \frac{1}{b^2} \right] e^{-bX} \\
&= \frac{T}{b} e^{-bX} + Ae^{-bT} - B
\end{aligned}$$

In equation 26

$$\begin{aligned}
(T - X)(e^{-bT})(e^{-T/a}) &= \int_X^T e^{-(T-z)/a} \left[\frac{z}{b} e^{-bX} + Ae^{-bz} - B \right] dz \\
&= e^{-T/a} \left[\frac{e^{-bX}}{b} \int_X^T ze^{z/a} dz + A \int_X^T e^{-z} dz \right. \\
&\quad \left. - B \int_X^T e^{z/a} dz \right] \\
&= (aT - a^2) \frac{e^{-bX}}{b} - (Xa - a^2) \frac{e^{-X} e^{-T/a}}{b} \\
&\quad - Ae^{-bT} + Ae^{-X} e^{-T/a} - aB + aB e^{X/a} e^{-T/a} \\
&= \frac{aT}{b} e^{-bX} - Ae^{-bT} - (aB + \frac{a^2}{b} e^{-bX}) \\
&\quad + \left[(Ab - aX + a^2) \frac{e^{-X}}{b} + aB e^{X/a} \right] e^{-T/a} \\
&= \frac{aT}{b} e^{-bX} - Ae^{-bT} - C + De^{-T/a}
\end{aligned}$$

In equation 25

$$\begin{aligned}
\left(\frac{T}{b} e^{-bX} + Ae^{-bT} - B \right) * \sum_{n=1}^{\infty} \frac{(X/a)^n T^{n-1} e^{-T/a}}{n! (n-1)!} &= \\
&= \frac{e^{-bX}}{b} \sum_{n=1}^{\infty} \frac{(X/a)^n}{n! (n-1)!} \int_X^T (T-z) z^{n-1} e^{-z/a} dz
\end{aligned}$$



$$\begin{aligned}
& + A \sum_{n=1}^{\infty} \frac{(X/a)^n}{n! (n-1)!} \int_X^T e^{-b(T-z)} z^{n-1} e^{-z/a} dz \\
& - B \sum_{n=1}^{\infty} \frac{(X/a)^n}{n! (n-1)!} \int_X^T z^{n-1} e^{-z/a} dz
\end{aligned}$$

Without evaluating integrals, this is of the form

$$= \frac{e^{-bX}}{b} \Sigma_1 + A \Sigma_2 - B \Sigma_3$$

where Σ_1 , Σ_2 and Σ_3 involve combinations of series.

In equation 26

$$\begin{aligned}
& \left(\frac{aT}{b} e^{-bX} - A e^{-bT} - C + D e^{-T/a} \right) * \sum_{n=1}^{\infty} \frac{(X/a)^n}{n! (n-1)!} \frac{T^{n-1} e^{-T/a}}{1} = \\
& = \frac{a}{b} e^{-bX} \Sigma_1 - A \Sigma_2 - C \Sigma_3 \\
& + D \sum_{n=1}^{\infty} \frac{(X/a)^n}{n! (n-1)!} \int_X^T e^{-(T-z)/a} z^{n-1} e^{-z/a} dz \\
& = \frac{a}{b} e^{-bX} \Sigma_1 - A \Sigma_2 - C \Sigma_3 + D \Sigma_4
\end{aligned}$$

The following solutions for the second time interval are obtained by substituting the results of the convolutions in equations 25 and 26

$$\begin{aligned}
F(X, T) &= T/ab + 1/ab^2 (e^{-bT} - 1) \\
&- \frac{e^{-X}}{a} \left[(T + \Sigma_1) \frac{e^{-bX}}{b} + A(e^{-bT} + \Sigma_2) - B(1 + \Sigma_3) \right]
\end{aligned}$$

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1. The first part of the paper is devoted to a discussion of the general theory of the problem.

2. The second part is devoted to a discussion of the special case of the problem.

3. The third part is devoted to a discussion of the numerical solution of the problem.

In equation (1)

$$\frac{d^2 x}{dt^2} + \frac{dx}{dt} + x = 0 \quad (1)$$

where x is the displacement of the mass from its equilibrium position, t is the time, and m is the mass of the body.

The initial conditions are $x(0) = 0$ and $\dot{x}(0) = 0$.

The solution of equation (1) is given by the following expression:

The following expression for the displacement $x(t)$ is obtained by substituting the value of ω into equation (1) and (2).

Equations (1) and (2) are

$$F(x, t) = \frac{1}{\omega} \sin \omega t + \frac{1}{\omega^2} \cos \omega t$$

where ω is the angular frequency of the oscillations.

$$F(X,T) = T/ab + 1/ab^2(e^{-bT} - 1) - \frac{e^{-X}}{ab} \left[(T + \Sigma_1)e^{-bX} + (X + 1/b)(e^{-bT} + \Sigma_2) - (2X + 1/b)(1 + \Sigma_3) \right] \quad 27$$

and

$$\begin{aligned} M(X,T) &= T/ab - 1/a^2b^2(e^{-bT} - 1) \\ &\quad - \frac{e^{-X}}{a^2} \left[(T + \Sigma_1)\frac{a}{b} e^{-bX} - A(e^{-bT} + \Sigma_2) - C(1 + \Sigma_3) + D(e^{-T/a} + \Sigma_4) \right] \\ &= T/ab - 1/a^2b^2(e^{-bT} - 1) - \frac{e^{-X}}{ab} \left[(T + \Sigma_1)e^{-bX} - (X/a + 1/ab)(e^{-bT} + \Sigma_2) - e^{-bX} [(2X + 1/b) + a](1 + \Sigma_3) + [e^{-X}(X/a + 1/ab - X + a) + e^{-bX}(2X + 1/b)](e^{-T/a} + \Sigma_4) \right]. \end{aligned} \quad 28$$

All of the above integrals are of the form

$$\int z^n e^{az} dz = e^{az} \left[\frac{z^n}{a} - \frac{n}{a^2} z^{n-1} + \frac{n(n-1)}{a^3} z^{n-2} - \dots + (-1)^{n-1} \frac{n!}{a^n} z + (-1)^n \frac{n!}{a^{n+1}} \right],$$

so that Σ_1 , Σ_2 , Σ_3 and Σ_4 involve combinations of series which would be difficult to evaluate without a digital computer. For that reason a numerical solution of these equations was not attempted.

$$T(X, Y) = T(X, Y) + \lambda \frac{\partial}{\partial \lambda} T(X, Y) = \lambda \frac{\partial}{\partial \lambda} T(X, Y) + T(X, Y)$$

$$T(X, Y) = T(X, Y) + \lambda \frac{\partial}{\partial \lambda} T(X, Y) = \lambda \frac{\partial}{\partial \lambda} T(X, Y) + T(X, Y)$$

and

$$T(X, Y) = T(X, Y) + \lambda \frac{\partial}{\partial \lambda} T(X, Y) = \lambda \frac{\partial}{\partial \lambda} T(X, Y) + T(X, Y)$$

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$$T(X, Y) = T(X, Y) + \lambda \frac{\partial}{\partial \lambda} T(X, Y) = \lambda \frac{\partial}{\partial \lambda} T(X, Y) + T(X, Y)$$

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ALL of the above integrals are of the form

$$\int_0^1 x^a (1-x)^b dx = \frac{a! b!}{(a+b+1)!}$$

$$\int_0^1 x^a (1-x)^b dx = \frac{a! b!}{(a+b+1)!}$$

so that I_1 , I_2 , I_3 and I_4 involve combinations of series which

would be difficult to evaluate without a digital computer. For

that reason a numerical solution of these equations was not

attempted.

It has not been possible to show that this solution converges to a steady state solution for $T \rightarrow \infty$ because of the appearance of T in the infinite series terms. However, a partial check of the final solution is provided by substituting $X = 0$ which gives back the boundary conditions $F(0, T) = 0$ and $M(0, T) = 1 - e^{-T/a}$.

The evaluation of the infinite series terms would amount to a tremendous undertaking without a high-speed digital computer. The series terms derived here can probably be manipulated to match the series terms given by Clark et al, (10, 11) which were computed for a range of parameters on the M.I.T.-I.B.M. digital computer.

In gas-cooled reactors, the value of "a" may be quite large for the low-power research models but becomes smaller and approaches one for power reactors. Equations 27 and 28 indicate that the effect of decreasing "a" is to increase the initial rate of temperature response.

NUMERICAL SOLUTION

The analytical solutions of equations 11 and 12 are so complex that a numerical solution is of more value when a digital computer is not available. Solution of the equations by a finite difference method was accomplished by Bankston(14) as a special problem for the Department of Mechanical Engineering, University of New Mexico. This investigation was in progress at that time and parameters were chosen to be

compatible with the experimental equipment to be described later.

In Bankston's calculation the coolant flow selected was 10 cfm of standard air at a pressure of 75 psig. The power transient was 30.9 watts/inch. Bulk fluid properties were evaluated at 300°F and film properties for calculating the heat transfer coefficient were evaluated at 400°F. The heat transfer coefficient was evaluated from

$$\frac{hD}{k} = 0.23 \left(\frac{DG}{\mu} \right)^{0.8} \left(\frac{c_p \mu}{k} \right)^{0.4} \quad 29$$

where new symbols are

D = inside tube diameter

k = thermal conductivity

G = mass velocity = w/A_f

μ = viscosity

From the above specifications, the following values for the similarity numbers were computed:

$M = 3.88 \times 10^{-3} m$ where m is the metal temperature in °F;

$F = 3.88 \times 10^{-3} f$ where f is the fluid temperature in °F;

$X = 0.437 x$ where x is distance on the tube in feet;

$T = 19.7 t$ where t is time in seconds; and

$$a = 265 = \frac{\delta m c_m A_m}{\delta f c_f A_f}$$

The relatively large value of "a" causes the temperature change during the first time interval to be only 2.88°F at $x = 6$ feet where the steady state temperature ($t \rightarrow \infty$) is 932°F. It is apparent that solution may begin with the

comparable with the experimental equipment to be described later.

In addition, a calibration of the constant flow system was

made of standard air at a pressure of 15 mm. The power

consumption was 30.5 watt-hours. Both these properties were

evaluated at 300°K and these properties for calculating the heat

transfer coefficient were evaluated at 300°K. The heat trans-

fer coefficient was evaluated from

$$\frac{h}{k} = 0.22 \left(\frac{G}{k} \right)^{0.6} \left(\frac{c_p \mu}{k} \right)^{0.4}$$

where the symbols are

h = heat transfer

k = thermal conductivity

G = mass velocity = w/A

c_p = velocity

From the above specifications, the following values for the

dimensionless numbers were computed:

$Pr = 3.16 \times 10^{-3}$ where n is the metal temperature in °C

$Pe = 5.16 \times 10^{-3}$ where t is the fluid temperature in °C

$X = 0.477$ where x is distance on the tube in feet

$T = 19.7$ where t is time in seconds; and

$$a = 265 = \frac{h}{k} \frac{A}{w}$$

The relatively large value of "a" causes the temperature

change during the first time interval to be only 0.38°K as

$x = 0$ feet where the steady state temperature ($t = \infty$) is

332°K. It is apparent that solution may begin with the

equations for the second time interval and assumed initial metal and fluid temperatures of zero.

The convergence criteria for determining the size of Δx and Δt was not found but on the basis of judgement, convenience and an estimate of the error, the choice was $\Delta x = 1/2$ foot and $\Delta t = 5$ seconds.

The calculations used a finite difference rectangular grid in the X, T plane with nodes at intervals of ΔX and ΔT . The nodal temperatures represented the average temperature of a section of length ΔX during the time interval ΔT . The finite difference equations were derived from the non-dimensional differential equations 11 and 12, and involved both M and F at four adjacent nodes. The solution starts with the initial and boundary conditions which provide M and F at three of the nodes. The two finite difference equations provide M and F at the fourth node. The procedure was continued to $X = 2.622$ and $T = 2500$.

The method employed is straightforward and relatively simple but the actual work is long and tedious. Since each calculation depends on all previous calculations, computation errors are a serious threat. The major disadvantage of this type of numerical calculation is that the entire process may have to be repeated in order to demonstrate the effect of a new ratio of heat capacities, "a." Without convergence criteria the accuracy of the results is slightly uncertain.

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The results of this numerical calculation are compared with experimentally determined temperatures in a later section.

The results of this numerical calculation are shown in Table 1.

Experimentally determined temperature is 1.57 K.

CHAPTER 4

EXPERIMENTAL INVESTIGATION

The theoretical investigation in the preceding chapter resulted in analytical and numerical solutions for the metal and bulk temperatures as functions of length and time. The necessary assumptions and complexity of the analytical solution and the time-consuming quality of the numerical solution suggest experimental methods. A thermal analog of the reactor coolant channel has the advantage that actual thermodynamic changes in properties are included. This may lead to fewer restrictions than required by the theoretical methods and give greater confidence in the results. A disadvantage is that construction of the model, data reduction, and analysis of results are also quite time consuming.

APPARATUS

The analog investigated here was a thin-walled metal tube which was heated by its resistance to the flow of electrical current. The coolant was air at 75 psig. Temperatures were detected by thermocouples placed at various places along the tube and at the coolant exit. Thermocouple outputs were recorded on an oscillograph. The power supply was a d.c. arc welder and an approximate step function change in heat generation was produced by completing the circuit with a knife switch.

An overall picture of the heated tube and associated equipment is shown in Figure 2. Details of the lower end of the heated tube are shown in Figure 3. This picture shows two

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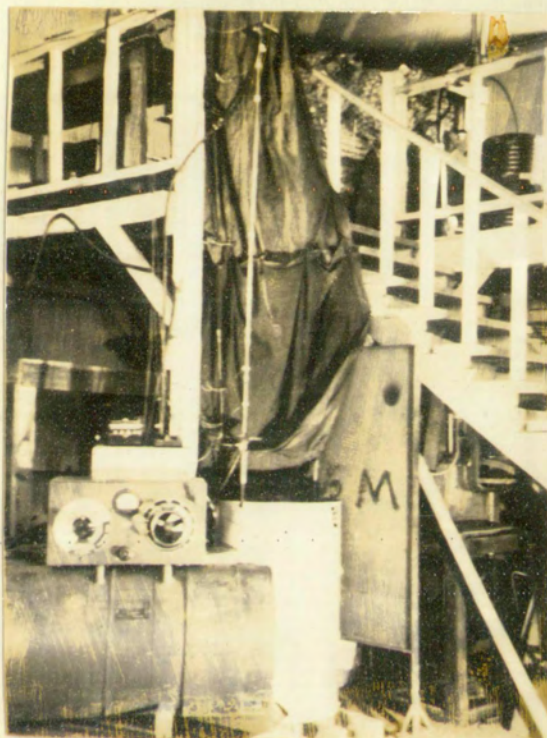
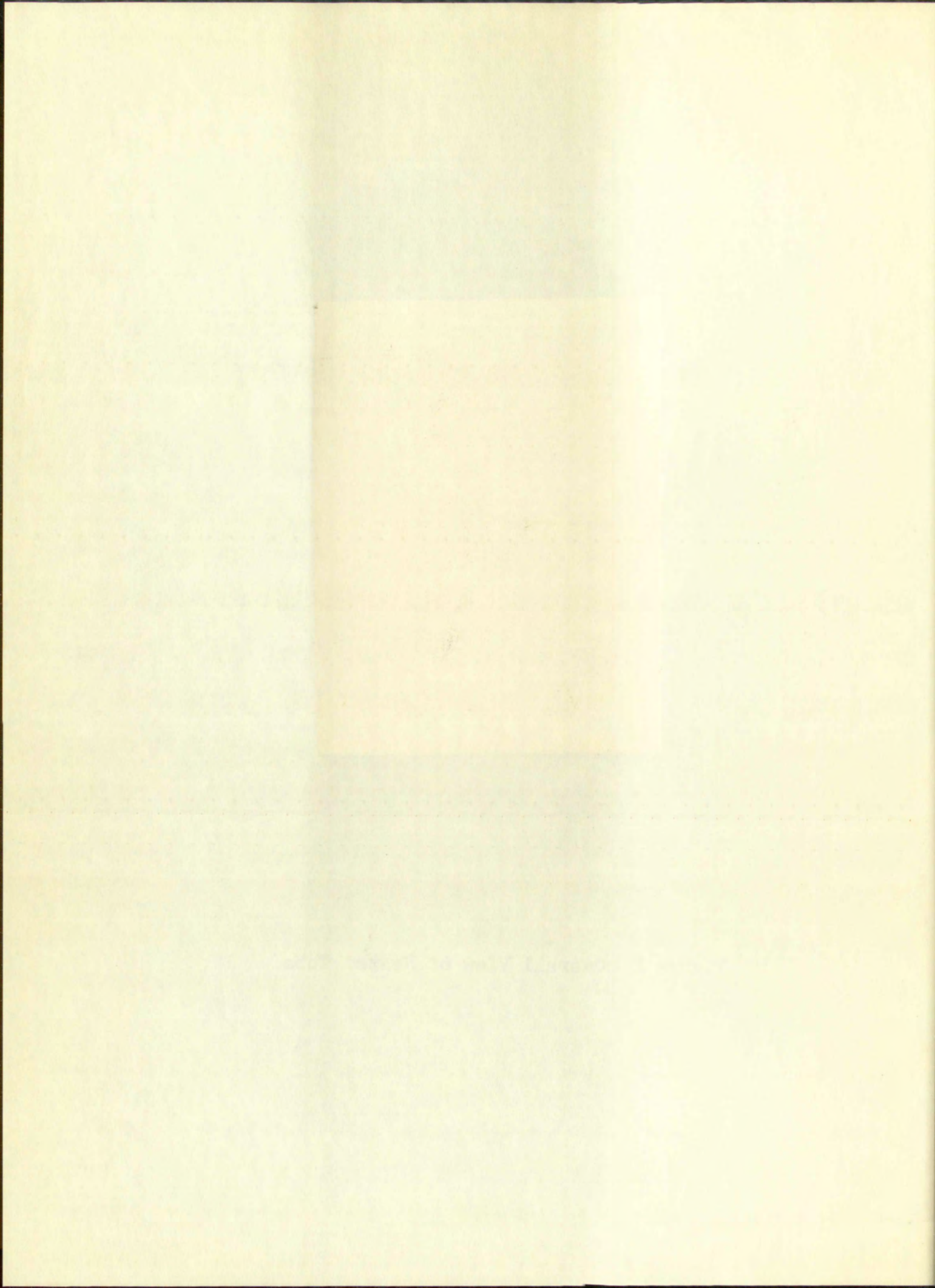


Figure 2 Overall View of Heated Tube



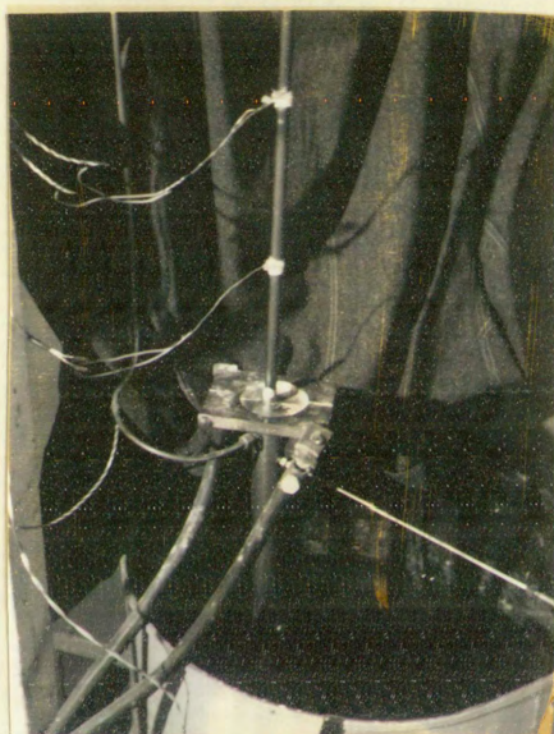


Figure 3 Detail of Electrode and Thermocouples

Figure 3. Detail of electrode and transducer.

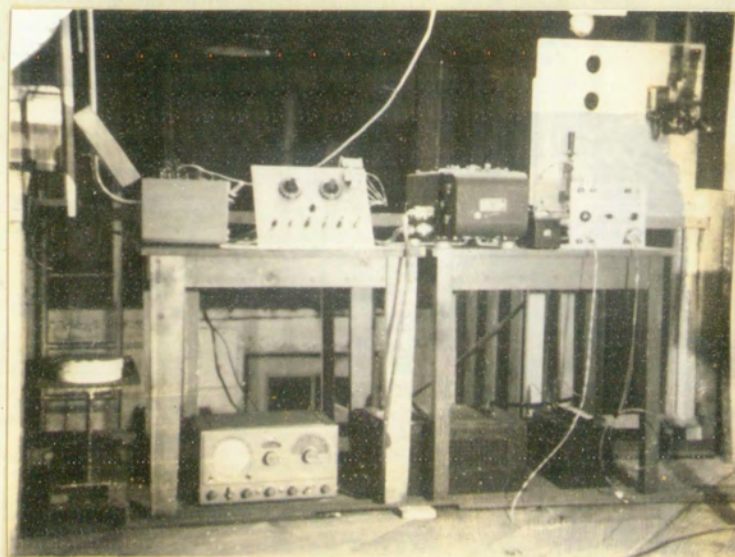
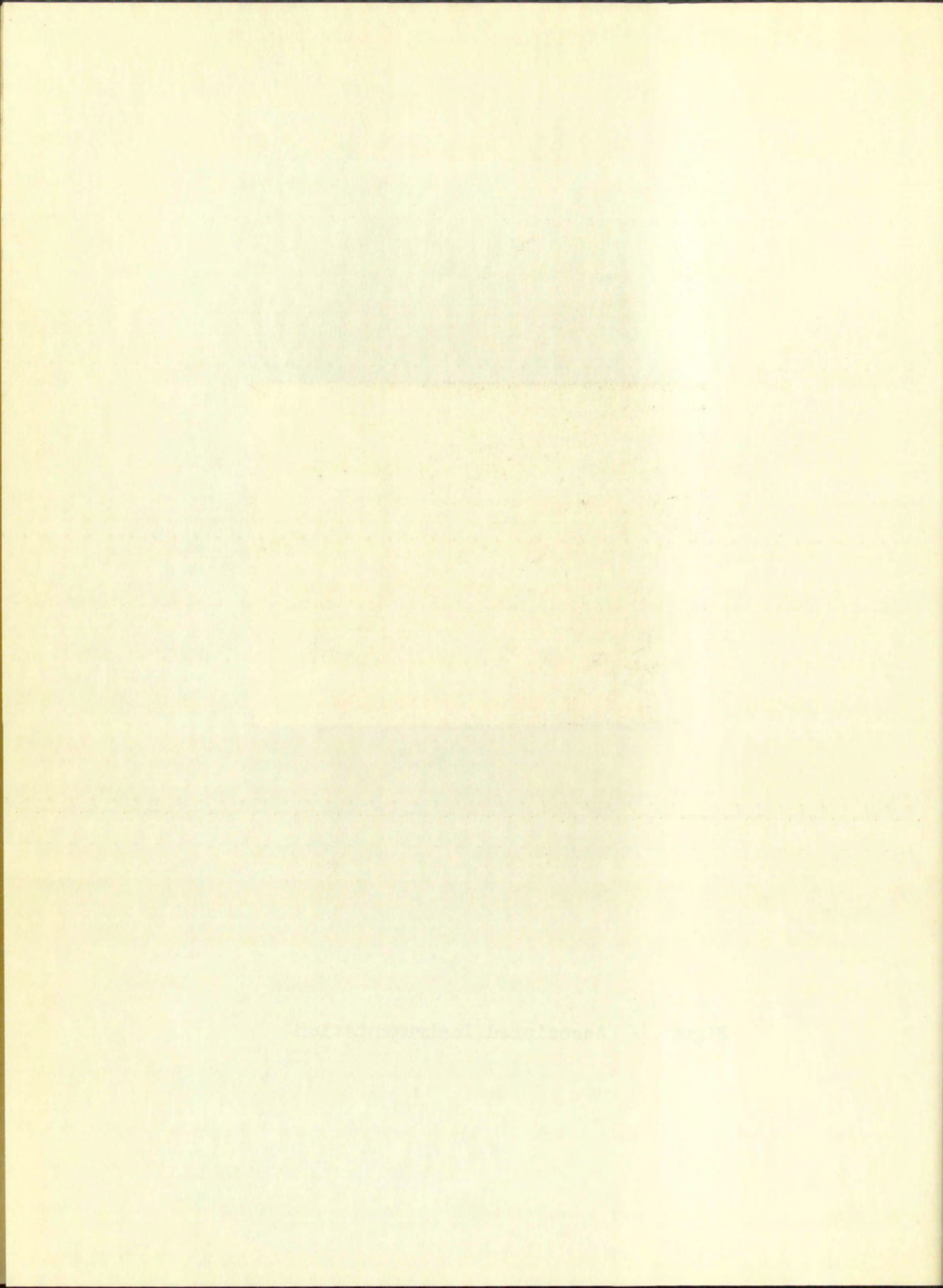


Figure 4 Associated Instrumentation



thermocouples installed on the tube, one of the electrical contacts, and the coolant temperature probe. Additional instrumentation is shown in an overall view in Figure 4.

The experimental data obtained were for three levels of heat generation and for three coolant flow rates. Other variables were maintained in agreement with the specification for the numerical solution. Four runs were made for the heat generation level and flow rate corresponding to the numerical solution specification. Two runs each were made of the other four experimental conditions.

Heat Exchanger -- The test section was an Inconel metal tube 6 feet long, 1/2 inch o.d. and 0.035 inch wall thickness. This simulated the reactor fuel element. Inconel was chosen because it has a relatively high electrical resistivity (100μ - ohm-cm) which does not change appreciably with changes in temperature. It also has high strength properties at high temperatures.

A three foot length of tube at the inlet end was left unheated for the development of a stable turbulent flow pattern. This corresponds to $L/D \cong 85$ which is more than sufficient to minimize starting effects on the heat transfer coefficient.

Air System -- The coolant for the analog was air supplied by the compressed air system in the Mechanical Engineering shop. Two pressure regulators reduced and regulated the inlet air pressure to the heated tube. One of the regulators also incorporated an oil and water separator which removed most of the

moisture from the coolant, however, there was evidence of a slight amount of moisture in the test section.

The flow rate was determined by measuring the pressure drop across a sharp-edged orifice. The design of the orifice and its discharge coefficients were taken from the publication by Grace and Lapple(19). A 36-inch water manometer with a least division of 0.1 inch was used to obtain accurate differential pressure readings. Variations in flow rate during a run were observed to be on the order of 3%.

Inlet pressure to the test section was measured with a conventional pressure gauge. Pressure variations were on the order of 5%. Inlet air temperature was monitored by a mercury thermometer inserted in a well in the inlet piping.

The downstream end of the test section terminated in a 12 inch length of 1 inch stainless steel pipe. Access to this chamber was provided by two tapped holes; one located $3/4$ inch and the other about $8-3/4$ inch downstream from the end of the heated tube. The original intent of this chamber was to obtain a better measurement of the coolant bulk temperature by insuring mixing in an unheated region. Lack of time prevented utilizing the chamber except for the mounting of the coolant temperature probe in the upstream access hole.

Air leaving the mixing chamber passed through about 50 feet of coiled $1/2$ inch copper tubing which was immersed in a barrel of water. Thus, hot air leaving the analog was cooled to room temperature before reaching the flow-rate controlling valve at

distance from the coolant, however, there was evidence of a slight amount of leakage in the test section.

The flow rate was determined by measuring the pressure drop across a sharp-edged orifice. The design of the orifice and the discharge coefficient were taken from the published data by Grace and Lapley (1955). A 3/8-inch water manometer with a least division of 0.1 inch was used to obtain accurate differential pressure readings. Variations in flow rate during a run were observed to be on the order of 1%.

Static pressure in the test section was measured with a conventional pressure tap. Pressure variations were on the order of 0.1 inch. Inlet air temperature was monitored by a thermistor inserted in a wall in the inlet piping.

The downstream end of the test section terminated in a 12 inch length of 1 inch stainless steel pipe. Access to this chamber was provided by two tapped holes; one located 3/4 inch and the other about 5-1/4 inch downstream from the end of the heated tube. The orifice inlet of this chamber was so placed as to give a better measurement of the coolant bulk temperature by insuring mixing in an unheated region. Lack of time prevented utilizing the chamber except for the monitoring of the coolant temperature probe in the upstream access hole.

After leaving the mixing chamber passed through about 10 feet of coiled 1/2 inch copper tubing which was immersed in a bath of water. Thus, after leaving the mixing chamber cooled to room temperature before passing the flow-rate measuring valve at

the end of the system.

Electrical Power System -- The electrical power circuit diagram is shown in Figure 14 in the Appendix.

Power for heating the analog was provided by a DC welder power supply. Since this was an AC-DC motor-generator set, isolation from ground was provided. Current measurement was accomplished with a new 50 mv.-200 ampere shunt and a 0-100 millivoltmeter with a least division of one millivolt. Voltage was measured with a 0-30 voltmeter with a least division of 0.5 volts. Both meters were calibrated against a standard cell prior to the experiments.

Circuit closing was done with a knife-switch. The circuit was opened with a circuit breaker which also provided protection against accidental shorts. Difficulty was experienced in obtaining a consistent power level from the generator. This was overcome by providing an attendant to continually adjust the welder controls during the experimental runs. Some power fluctuation resulted but the effect is significant only during the first few seconds of each run.

An effort was made to nullify the effect of the heat capacity associated with the electrical contacts on the tube. The contactor developed was intended to be self-heating so that there would be a negligible temperature gradient between the tube and the contact and hence no heat loss.

the end of the system.

Electrical power lines -- The electrical power circuit

Diagram is shown in Figure 1 in the Appendix.

Power for heating the anode was provided by a DC heater

power supply. Since this was an AC-DC heater-generator set,

isolation from ground was provided. Heater resistance was

accomplished with a new 50 ohm, 200 watt resistor and a 0-100

millivoltmeter with a least division of one millivolt. Voltage

was measured with a 0-30 voltmeter with a least division of 0.5

volts. Both meters were calibrated against a standard cell

prior to the experiments.

Circuit closing was done with a knife-switch. The circuit

was opened with a circuit breaker which also provided protec-

tion against accidental shorts. Difficulty was experienced in

obtaining a constant power level from the generator. This was

overcome by providing an attendant to continually adjust the

valve controls during the experimental runs. Some power

fluctuation resulted but the effect is significant only during

the first few seconds of each run.

An effort was made to nullify the effect of the heat

capacity associated with the electrical contacts on the tube.

The contactor developed was intended to be self-heating so that

there would be a negligible temperature gradient between the

tube and the contact and hence no heat loss.

A view of this contactor is shown in Figure 3. A 3 inch disc of Incoloy with a concentric hole of $1/4$ inch radius was machined to a thickness of $1/32$ inch for all radii greater than $1/2$ inch. The thickness at the edge of the hole which contacted the tube was $1/8$ inch. Between the edge of the hole and the $1/2$ inch radius, the thickness decreased uniformly to $1/32$ inch. The disc was then split across a diameter and the two halves soldered to copper blocks. In use, the two halves of the disc were assembled around the Inconel tube and clamped in place by studs through the copper blocks. Electrical leads were attached to the copper so that current would flow through the disc to the tube and the disc would be heated by the I^2R loss within it.

The effectiveness of this contact was not fully investigated.

Thermocouples -- Chromel-alumel thermocouples were chosen for measuring tube temperatures at the lower end of the tube because of their suitability for high temperature applications. Because of the superior thermal e.m.f., iron-constantan thermocouples were used for the three measurements on the upper portion of the tube where lower temperatures were expected. All thermocouples were made from 26 gage wire.

The attachment of the thermocouples to the tube was the subject of some investigation. Attempts to provide a metallic bond failed because of difficulties in welding the dissimilar materials and because of stray electrical effects. The following technique was finally used: First, a thin layer of seriesen

A view of this contact is shown in Figure 3. The hole of the tube was machined to a thickness of $1/32$ inch for all radii greater than $1/2$ inch. The thickness at the edge of the hole which contacted the tube was $1/8$ inch. Between the edge of the hole and the $1/2$ inch radius, the thickness decreased uniformly to $1/32$ inch. The disc was then split across a diameter and the two halves soldered to copper blocks. In use, the two halves of the disc were assembled around the inner tube and clamped in place by bands through the copper blocks. Electrical leads were attached to the upper so that current would flow through the disc to the tube and the disc would be heated by the I²R loss within it.

The effectiveness of this contact was not fully investigated. ~~Thermocouples~~ -- Chromel-iron thermocouples were chosen for measuring tube temperatures at the lower end of the tube because of their stability for high temperature applications. Because of the superior thermal e.m.f., iron-constantan thermocouples were used for the three measurements on the upper portion of the tube where lower temperatures were expected. All thermocouples were made from 30 gage wire. The attachment of the thermocouples to the tube was the subject of some investigation. Attempts to provide a sealable bond failed because of difficulties in welding the dissimilar materials and because of stray electrical effects. The following technique was finally used: First, a thin layer of acetone

ceramic cement was applied in a half-inch-wide strip three-quarters of the way around the tube. Each wire of the thermocouple pair was placed on the tube so that it was electrically insulated by the ceramic layer. The properly cleaned ends were twisted tightly and welded together. The joint was clipped so as to leave about $1/8$ inch of the welded junction. This junction was then bent up to lay against the tube. A strip of $1/2$ inch glass tape was tightly wrapped twice around the tube over the thermocouple joint and tied at the back. A piece of 26 gauge wire was wrapped once around the tube over the junction and fastened by twisting the ends together at the back. The wire held the thermocouple tightly against the tube.

Impedance matching considerations required that the thermocouple circuit have a minimum electrical resistance. Since the thermocouple materials have relatively high resistance, only a minimum length was used. Rough calculations indicated that the end of a six-inch length would not be affected by heat conduction from the hot junction. Copper leads were used beyond the end of the six inch length and the lead-to-thermocouple splice became the reference junction. The reference junction temperature was assumed to be at room temperature.

The adequacy of the thermocouple installation was demonstrated by steady state tests using a thermometer well clamped on to the tube over the thermocouple. Temperature readings were compared at equilibrium and during the cooling period

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following removal of power. Excellent agreement of the temperatures indicated by the thermocouple and the thermometer was observed. This data is shown in Table 1 in the Appendix.

Fluid temperature was detected by means of an iron-constantan thermocouple probe constructed by Pratt and Whitney Aircraft Co. The probe consists of a 1/8 inch diameter stainless steel tube which encases the thermocouple wire. The probe was installed in the upper access hole of the mixing chamber. It was inserted so that the thermocouple junction was centered in the stream of air leaving the heated tube.

Oscillograph -- The output e.m.f. of the various thermocouples was detected by the galvanometers in a Hathway Type S-12A Oscillograph. The galvanometer traces were recorded on photographic paper moving at a speed of 1/2 inch/sec. A time marker system within the oscillograph provided marks on the paper at intervals of 1/10 second so that the time dependence of all temperatures could be accurately determined. The time marker was compared to a 60 cycle/sec signal and found to be extremely accurate.

CALIBRATION

Independent calibration of the individual galvanometers was not possible since they have a relatively small internal impedance (about 7 ohms). When connected to a thermocouple the power developed within them is greatly influenced by the impedance of the external circuit. This made it necessary to

calibrate the oscillograph galvanometer while they were connected with their thermocouples. To accomplish this, a switching circuit was devised so that a known e.m.f could be applied in series with the galvanometer-thermocouple circuit. A portion of the switching circuit is shown schematically in Figure 15 in the Appendix. This part is typical of the six independent circuits required for the six thermocouples.

The source of known e.m.f. was obtained by constructing a voltage divider circuit. The schematic diagram of this calibrating circuit is shown in Figure 16 in the Appendix. A 12 volt storage battery was used as the prime source of electrical power. Ten voltage steps were provided by using five taps in a series resistance circuit and a high-low range switch.

The actual output voltage of this circuit would depend upon the output impedance. Since each galvanometer-thermocouple circuit has a different impedance, it was necessary to put the circuit to be calibrated into a leg of a Wheatstone bridge. The bridge is balanced by adjusting resistors which shunt the galvanometer circuit. The voltage divider then feeds the bridge which, when balanced, always constitutes the same impedance. Since the other three legs of the bridge are fixed, the voltage across the galvanometer is also fixed.

The voltage corresponding to each step was determined by comparison on a Leeds and Northrup No. 8662 Portable Precision Potentiometer.

calibrate the new E-type galvanometer with the new standard cell. To accomplish this, a balancing circuit was devised so that a known e.m.f. could be applied in series with the galvanometer-thermometer circuit. A portion of the balancing circuit is shown schematically in Figure 1 in the Appendix. This part is typical of the six independent circuits required for the six thermometers.

The source of known e.m.f. was obtained by connecting a voltage divider circuit. The cathode circuit of this cell, balancing circuit is shown in Figure 1 in the Appendix. A 10 volt standard battery was used as the prime source of electrical power. Ten voltage steps were provided by using five pairs of series resistance elements and a high-low range switch.

The actual output voltage of this circuit would depend upon the output impedance. Since each galvanometer-thermometer circuit has a different impedance, it was necessary to put the circuit to be calibrated into a leg of a Wheatstone bridge. The bridge is balanced by adjusting resistors which show the galvanometer circuit. The voltage divider then feeds the bridge which, when balanced, always connected the same impedance. Since the other three legs of the bridge are fixed, the voltage across the galvanometer is also fixed.

The voltage corresponding to each step was determined by comparison on a Leeds and Northrup No. 9605 portable Weston potentiometer.

The resistance of the fluid temperature probe was about the same magnitude as the resistance of the galvanometer. It was evident that only half of the power resulting from the thermal e.m.f. would be developed in the galvanometer. This was overcome by the use of a low-level current amplifier designed by Summers(20) which matched impedances and provided some amplification. A schematic of this amplifier is presented in Figure 17 in the Appendix.

After carefully determining the output voltages for each step of the calibrating circuit, the galvanometer-thermocouple circuits were calibrated in turn with the corresponding galvanometer deflection being recorded on the oscillograph record. A typical calibration plot of deflection versus applied e.m.f. is shown in Figure 18 in the Appendix. It will be noted that the response is essentially linear over the range of interest. The assumption that the e.m.f. output of the thermocouples used is directly proportional to temperature is within the accuracy required here. Then the conversion from galvanometer trace deflection to temperature is simply $\text{temperature} = K(\text{deflection})$ where K must be determined for each circuit. The K's determined are tabulated in Table 2 in the Appendix.

EXTERNAL HEAT LOSSES

The external surface of the heated tube was not insulated and heat transferred from this surface was found to be appreciable. This loss was found experimentally by measuring the equilibrium temperature at each thermocouple location when a

measured amount of power was delivered to the tube and there was no coolant flow. Under these conditions, essentially all of the input power is dissipated at the outer tube surface. Conduction loss at the ends is quite small since the cross-section is small. Data from this experiment is presented in Table 3 in the Appendix. A plot of surface temperature versus heat loss is shown in Figure 19 in the Appendix. Curves are presented for several distances down the tube from the inlet end. Heat loss is somewhat greater at the upper end of the tube because of the greater natural convection velocity.

During this experiment, the thermal e.m.f. was measured on the precision potentiometer and was also recorded on the oscillograph. This provided an additional check on galvanometer calibration and is indicated in Figure 18 in the Appendix.

EXPERIMENTAL TESTS

An outline of the procedure followed in performing an experimental run follows: The coolant flow conditions were carefully set up. The oscillograph was started and the knife-switch closed to start heat generation in the tube. Galvanometer traces were observed on the viewing screen of the oscillograph and were photographically recorded. When no further deflection could be detected, the oscillograph was shut off. The portable precision potentiometer was connected to the calibrating input terminals of the thermocouples switching circuit (Figure 15) and the thermal e.m.f. was measured

The amount of power was reduced to the value and time
was no longer fixed. Under these conditions, essentially all
of the input power is dissipated at the lower tube surface.
Conduction loss at the ends is quite small since the cross-
section is small. Data from this experiment is presented in
Table 2 in the Appendix. A plot of surface temperature versus
heat loss is shown in Figure 12 in the Appendix. Curves are
presented for several distances from the tube from the inlet
end. Heat loss is somewhat greater at the upper end of the
tube because of the greater natural convection velocity.
During this experiment, the thermal conductivity was measured
on the precision potentiometer and was also recorded on the
oscillograph. This provided an additional check on natural
convection velocity and is indicated in Figure 13 in the Appendix.

Fig. 13

EXPERIMENTAL TYPE

In addition to the procedure followed in performing the
experimental run follows the coolant flow conditions were
carefully set up. The oscillograph was started and the knife-
switch closed to start heat generation in the tube. Galvano-
meter traces were observed on the viewing screen of the
oscillograph and were photographically recorded. When no
further calibration could be obtained, the oscillograph was
cut off. The potentiometer potentiometer was connected
to the calibration input terminals of the potentiometer
input circuit (Figure 14) and the thermal conductivity was measured.

directly for each thermocouple. Input power was recorded.

A total of six experimental tests were made. Two runs were made for each test.

The inlet air pressure was 75 psig in all of the tests. This results in a similarity number "a" of 265 and corresponds to the "a" used in computing the numerical solution. The tests involved significant variations in the input power and air flow rate only. The following is a summary of the experimental conditions for each test:

Test No. 1, $w = 0.865$ lb/min, input power = 30.9 watts/in.

Test No. 2, $w = 0.708$ lb/min, input power = 30.9 watts/in.

Test No. 3, $w = 0.500$ lb/min, input power = 30.9 watts/in.

Test No. 4, $w = 0.865$ lb/min, input power = 13.9 watts/in.

Test No. 5, $w = 0.865$ lb/min, input power = 41.6 watts/in.

Test No. 6, $w = 0.865$ lb/min, input power = 30.9 watts/in.

DATA REDUCTION

After developing the oscillograph record, selected times after the beginning of the power transient were located. A set of proportional dividers were set up to give the proper conversion from deflection to temperature as indicated by Table 2. The temperature from each thermocouple circuit was determined for the selected times and recorded. The compilation of these temperatures is presented in Tables 4 and 5 in the Appendix.

The averaged data from Table 4 was plotted on graph paper as temperature versus distance on the tube. Smooth curves were drawn through points for the same time after the beginning

directly for each thermocouple. The first was recorded
A total of six experimental runs were made, each run
was made for each test.
The inlet air pressure was 75 psia at the inlet.
This results in a density of 0.075 lb/ft³ and a viscosity
of 1.2 x 10⁻⁴ lb/ft-sec. The inlet air temperature was 700°R.
Involved adjustment variations in the inlet gas velocity
flow rate only. The following is a summary of the experimental
conditions for each test:

- Test No. 1, $v = 0.85$ ft/sec, inlet power = 30.2 watts
- Test No. 2, $v = 0.70$ ft/sec, inlet power = 30.2 watts
- Test No. 3, $v = 0.50$ ft/sec, inlet power = 30.2 watts
- Test No. 4, $v = 0.35$ ft/sec, inlet power = 30.2 watts
- Test No. 5, $v = 0.25$ ft/sec, inlet power = 30.2 watts
- Test No. 6, $v = 0.15$ ft/sec, inlet power = 30.2 watts

DATA REDUCTION

After developing the calibration curves, the data were
after the beginning of the power transient were ignored. A set
of proportional dividers were set up to give the proper
also from section to temperature as indicated by Table 1.
The temperature from each thermocouple circuit was determined
for the selected times and then averaged. The composition of the
temperature is presented in Table 1 and 2 in the Appendix.
The averaged data from Table 1 was plotted on graph paper
as temperature versus distance on the x-axis. Smooth curves
were drawn through points for each time after the beginning

of the transient. The purpose of "smoothing" the data was to minimize scattering and eliminate erratic readings. A typical example of this procedure is shown in Figure 20 in the Appendix where the smoothed curves for Test 2 are presented.

The conversion to dimensionless temperature, time and distance was required next. The values for thermal properties used in the numerical solution were assumed to be adequate. The heat transfer coefficient was previously evaluated from

$$\frac{hD}{k} = 0.23 \left(\frac{DG}{\mu} \right)^{0.8} \left(\frac{c_p \mu}{k} \right)^{0.4} \quad 29$$

where $G = w/A_f$. A flow rate correction for h can be made on the basis of equation 29 which yields the relation

$$\frac{h_1}{h_2} = \left(\frac{w_1}{w_2} \right)^{0.8} \quad 30$$

The proper ΔU_g to use in computing the dimensionless temperatures is determined by

$$\Delta U_g = (\text{input power}) - (\text{external power loss}) \quad 31$$

where the external power loss was determined from the "36 inch" curve of Figure 19 using the "smoothed" temperature at the location of interest. This technique is somewhat arbitrary but it may be argued that considerably more data and analytical evidence would be required to justify a different procedure.

The non-dimensional data may now be calculated using the above modifications. The resulting similarity numbers, along

at the terminals. The purpose of "averaging" the data was to
minimize the effect of random errors in the readings. A typical
example of this procedure is shown in Figure 20 in the appendix
where the recorded curves for Test 2 are presented.

The conversion to dimensionless temperature, time and dis-
tance was performed next. The values for thermal properties
used in the theoretical solution were taken to be constants.
The heat transfer coefficient was previously evaluated from

$$h = \frac{q}{A(T_s - T_f)} \quad (20)$$

where $q = W/\Delta t$. A flow rate correction for h can be made on
the basis of equation 20 which yields the relation

$$\frac{h}{h_0} = \left(\frac{W}{W_0} \right)^{0.8} \quad (21)$$

The proper h_0 to use in computing the dimensionless
temperatures is determined by

$$h_0 = (input\ power) - (exit\ power) \quad (22)$$

where the external power loss was determined from the "50 inch"
curve of Figure 19 using the "measured" temperature at the
location of interest. This technique is somewhat arbitrary but
it may be argued that considerably more data and analytical
evidence would be required to justify a different procedure.
The one-dimensional data may now be calculated using the
above modifications. The resulting statistical numbers, along

with the "smoothed" temperature and external heat loss data are tabulated in Table 6 in the Appendix.

The measured fluid temperatures may be converted to non-dimensional form by a similar technique. In this case, fluid temperature was measured only at $x = 72$ inches. The external heat loss for correcting ΔU_g was obtained by using a temperature obtained by extrapolating the smoothed metal temperature to $x = 72$ inches and using the curve for "36 inches" in Figure 19 .

ERRORS

Estimates of the response time of the thermocouples was facilitated by information found in a text by Giedt(21). By extrapolation of data presented in his Table 14.1, the response of the coolant thermocouple was estimated to be $t = 0.3$ seconds where t is the time required for a thermocouple to reach 63.2% of a step change in temperature. The response of the tube thermocouples can be computed from

$$t = Wc/hA$$

32

where t = time to reach 63.2% of a step change, hr.

W = weight of the thermocouple, lbs.

$$= \gamma L \pi d^2 / 4$$

c = specific heat, BTU/lb^oF

h = coefficient of heat transfer, BTU/h f² ^oF

A = surface area of thermocouple, f²

$$= \pi dL.$$

with the "known" temperature and assumed that the temperature
 tabulated in Table 6 in the Appendix.
 The measured fluid temperatures may be converted to true
 dimensional form by a surface correction. In Table 6, fluid
 temperature was assumed only at a 1/2 inch from the surface.
 Heat loss for convection h_c was obtained by using a temperature
 obtained by extrapolating the measured mean temperature
 to a 1/2 inch and using the curve for 1/2 inch in Figure 10.

RESULTS

Estimation of the response time of the thermocouple was
 facilitated by information found in a paper by Kestel.¹
 Extrapolation of data presented in his Table 1, the response
 of the coolant thermocouple was estimated to be 0.1
 seconds where t is the time required for a thermocouple to
 reach 63.2% of a step change in temperature. The response of
 the tube thermocouple can be accepted from

$$t = 0.0014 W^2 / K$$

where t = time to reach 63.2% of a step change, sec.
 W = weight of the thermocouple, lbm.
 K = $3.14 \times 10^{-6} / (h_c A)$
 h_c = coefficient of heat transfer, $Btu/hr \cdot ft^2 \cdot ^\circ F$
 A = surface area of thermocouple, ft^2
 $t = 0.0014 W^2 / K$

This reduces to $t = \delta dc/4h$. The following values were assumed:

$$\delta = 540 \text{ lb/f}^3$$

$$d = 0.016 \text{ in.}$$

$$c = 0.11 \text{ BTU/lb } ^\circ\text{F}$$

$$h = 250 \text{ BTU/h f}^2 \text{ } ^\circ\text{F}$$

Substitution of these values results in $t = 7.9 \times 10^{-5} \text{ hr} = 0.28$ seconds.

Trace deflections were measured to an accuracy of about $\pm 1/64$ inches. The data reduction error = $1/64K$, ranges from ± 4 to ± 18 $^\circ\text{F}$.

This column is a 5 digit. The following values were

obtained:

$$b = 0.00119$$

$$d = 0.00119$$

$$e = 0.0119$$

$$h = 0.00119$$

Inspection of these values reveals that $b = 0.00119$ and $d = 0.00119$

are equal.

These relations were assumed to be accurate of about

1 inch. The data reduction error is 1 inch, ranges from

1 to 2 inches.

CHAPTER 5

RESULTS

A summary of the results of Bankston's numerical solution is presented in Figure 5 by a graph of the similarity number M versus X for several values of T . This graph provides the necessary information for making a comparison between theory and analog results.

Tests 4, 5, and 6 investigated the effect of varying the magnitude of ΔU_g . The results of this series of tests is compiled in non-dimensional form in Table 6. The correlation of this data is shown in Figures 6, 7, and 8 where non-dimensional metal temperature, M , is plotted versus non-dimensional time, T . The three figures are for three particular values of non-dimensional distance, X .

According to the dimensional analysis, the effect of varying the flow rate should be accounted for by the similarity number X . This was investigated by Tests 1, 2, and 3. The non-dimensional data is also compiled in Table 6. Figures 9, 10 and 11 are plots of M versus T for tests 1, 2, and 3 respectively. Each graph contains the curves for three values of X . Superimposed on the experimental curves are the theoretical curves for the same values of X . These theoretical curves are drawn from information presented by Figure 6 as mentioned above.

The averaged fluid temperatures from Tests 1 and 6 were converted to non-dimensional temperature, F , and compiled in Table 7 in the Appendix. Figure 12 is a plot of F versus T

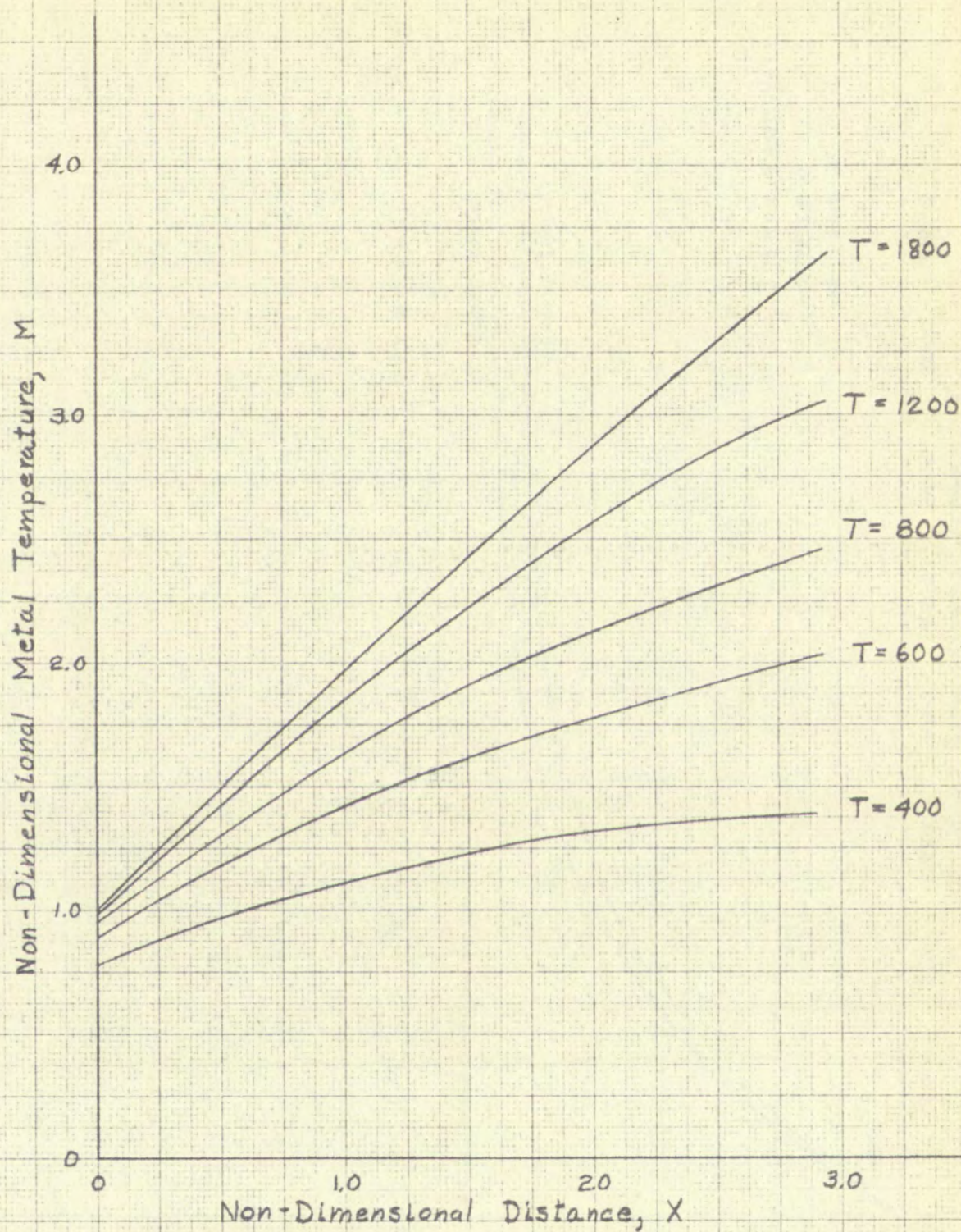


FIGURE 5 SUMMARY OF METAL TEMPERATURE
FROM NUMERICAL SOLUTION

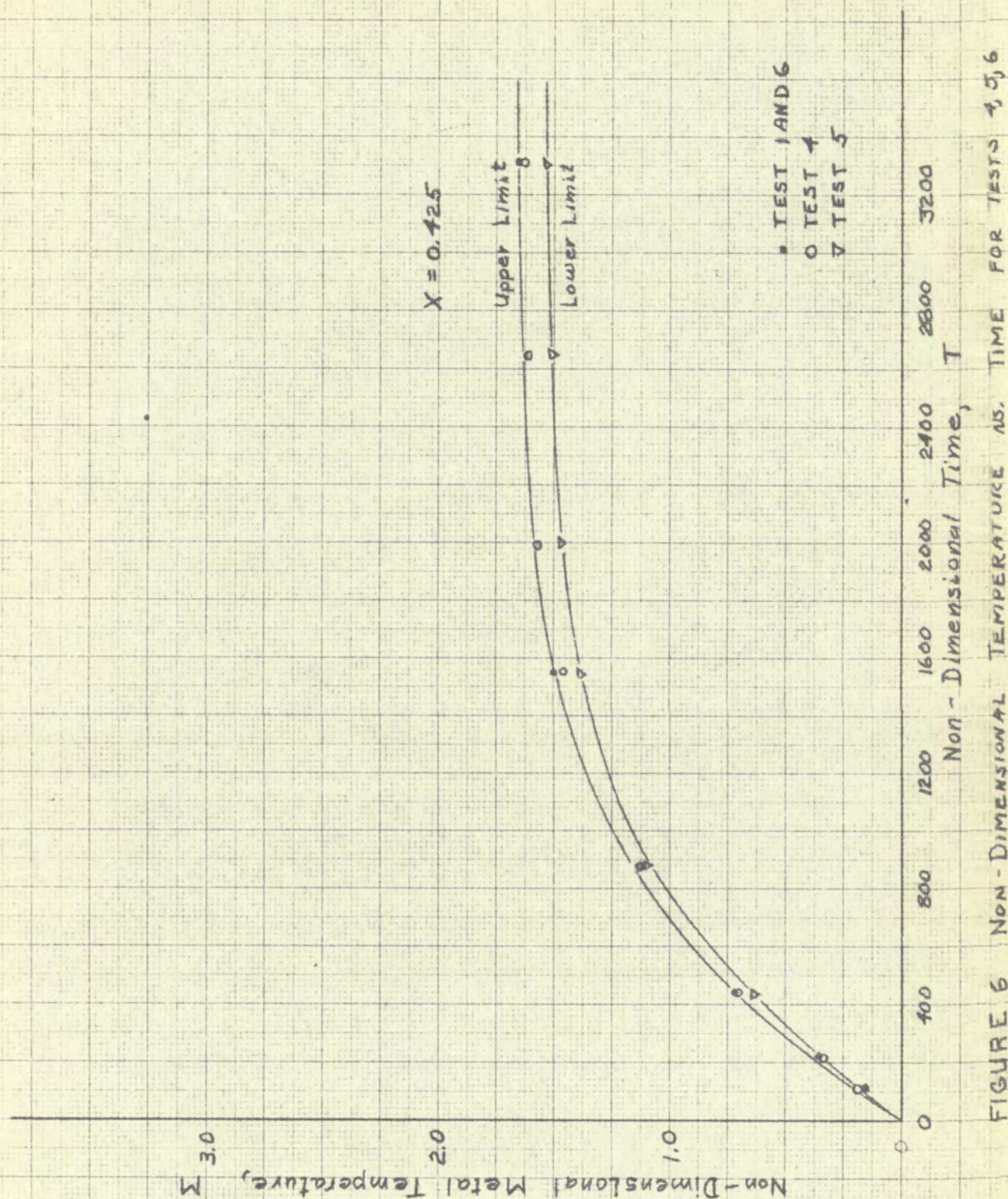


FIGURE 6 NON-DIMENSIONAL TEMPERATURE VS. TIME FOR TESTS 4, 5, 6

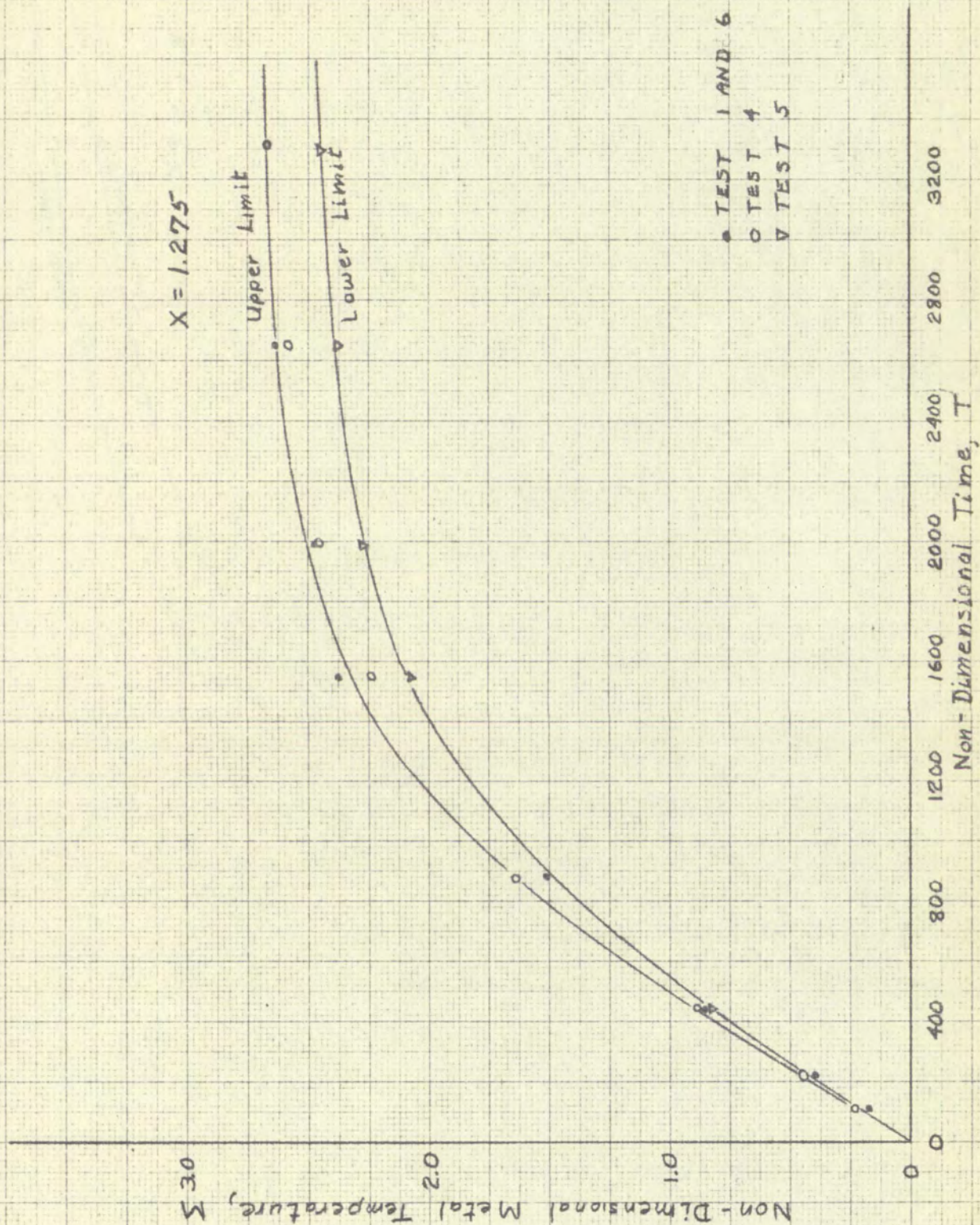
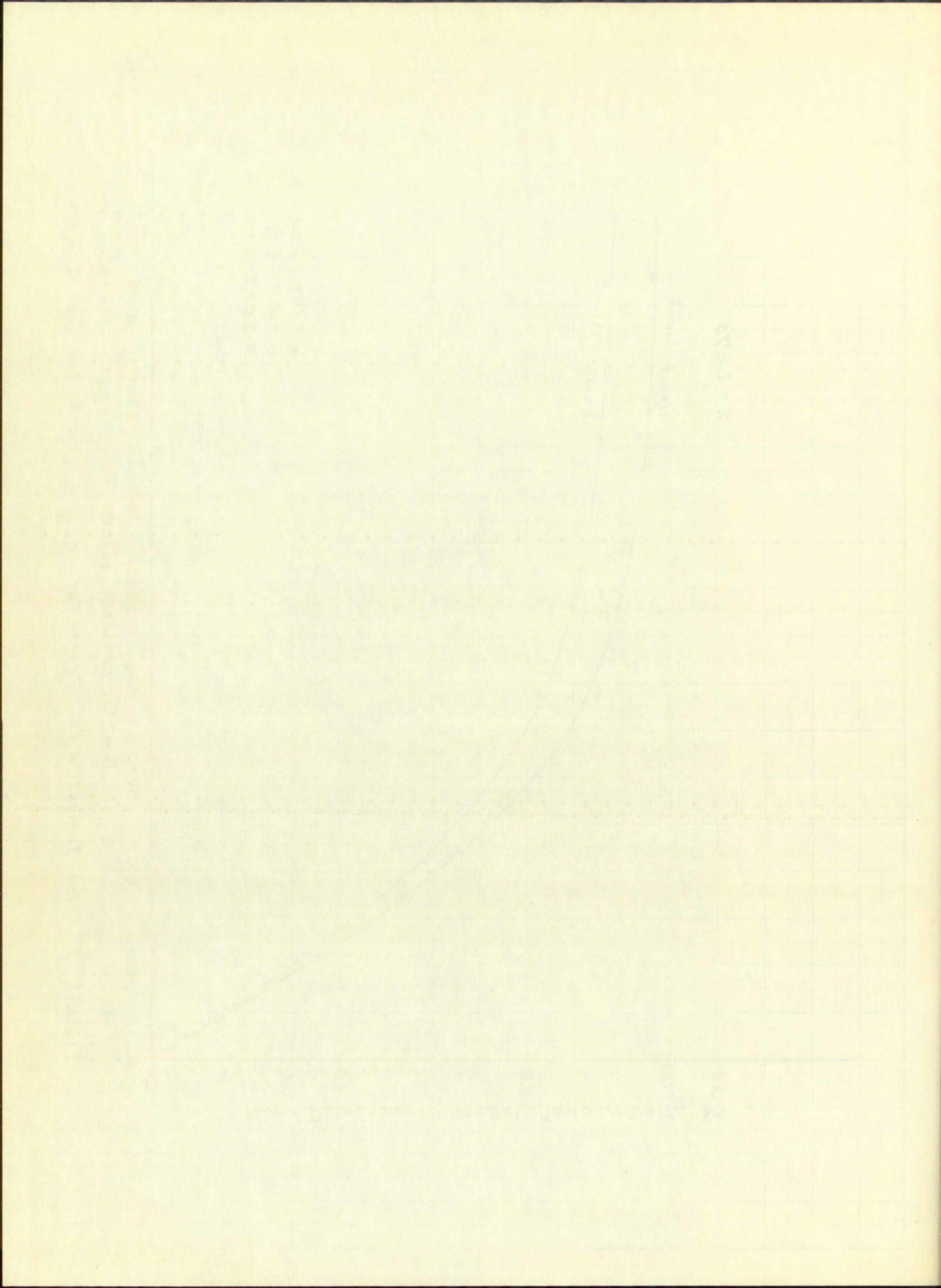


FIGURE 7 Non-Dimensional Temperature vs Time for Tests 4, 5, 6



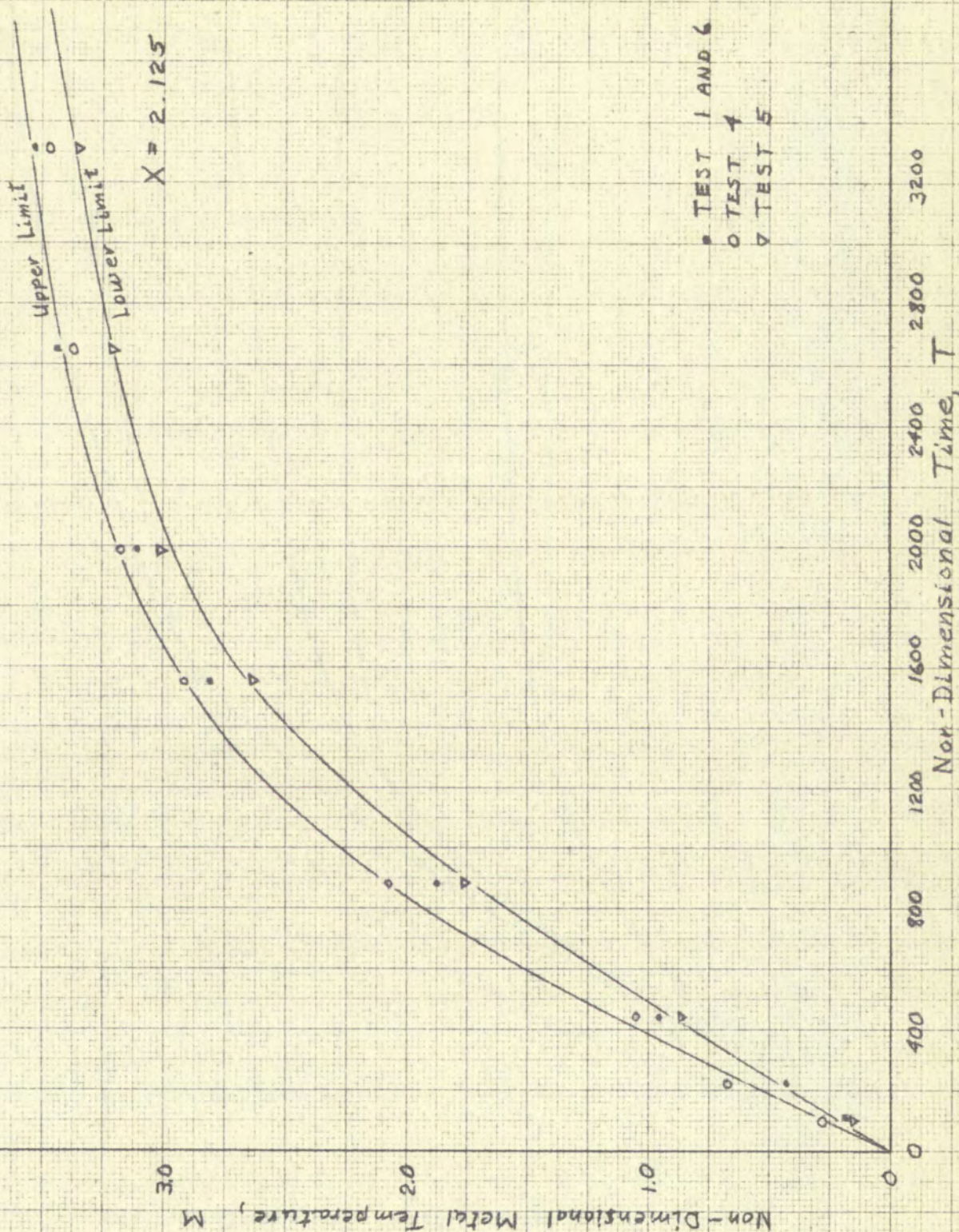
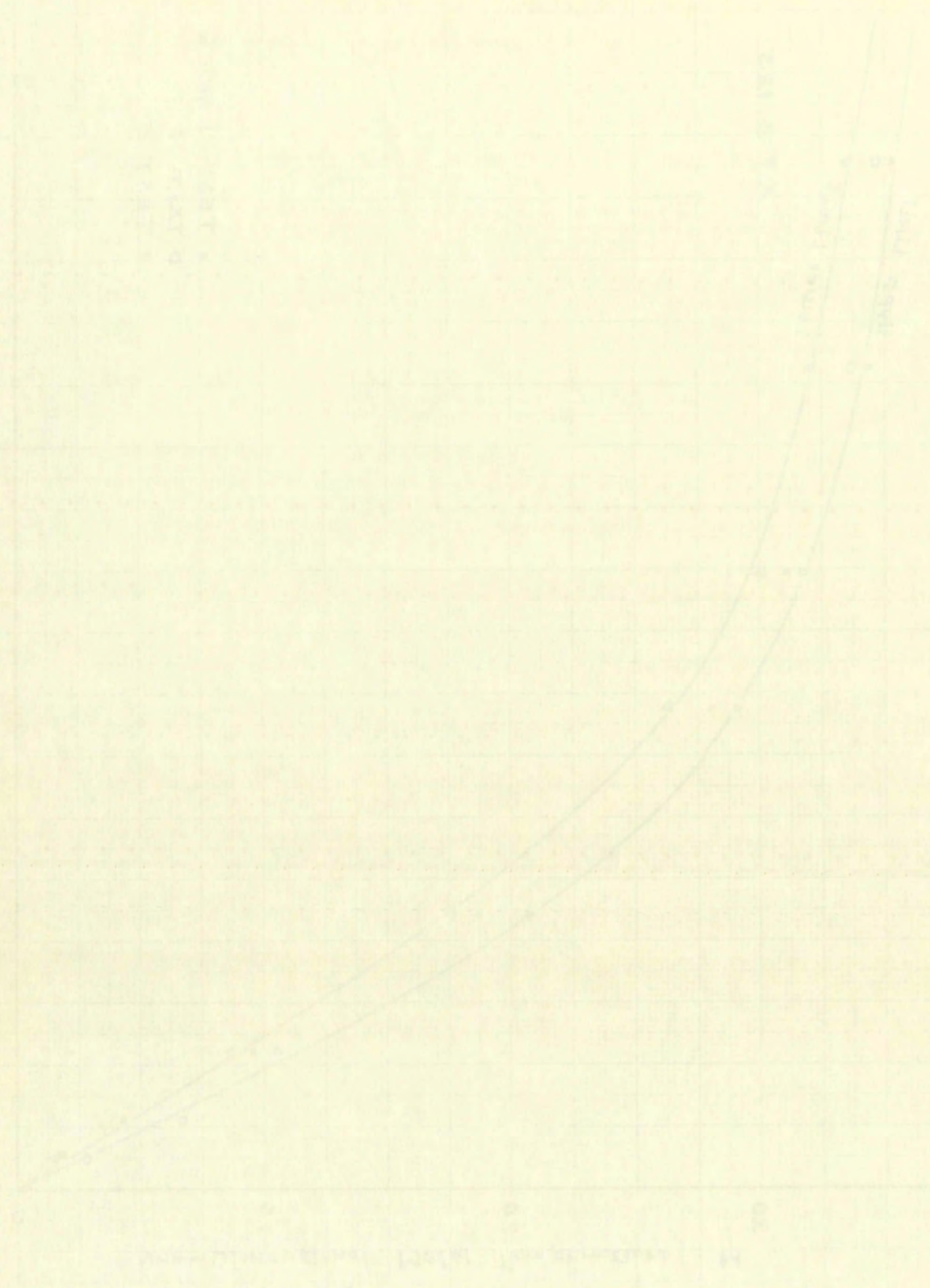


FIGURE 8 NON-DIMENSIONAL TEMPERATURE VS. TIME FOR TESTS 4, 5, 6



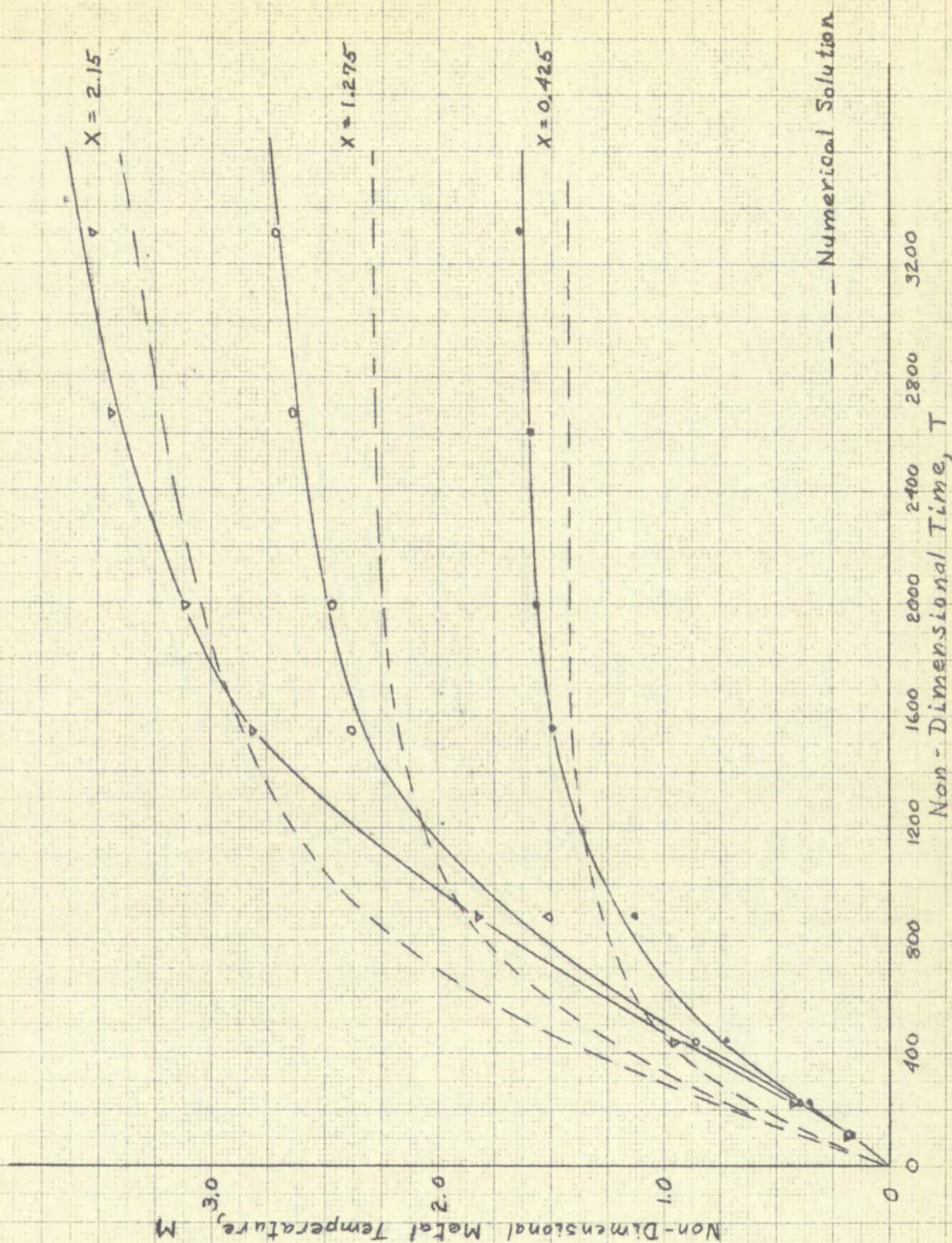
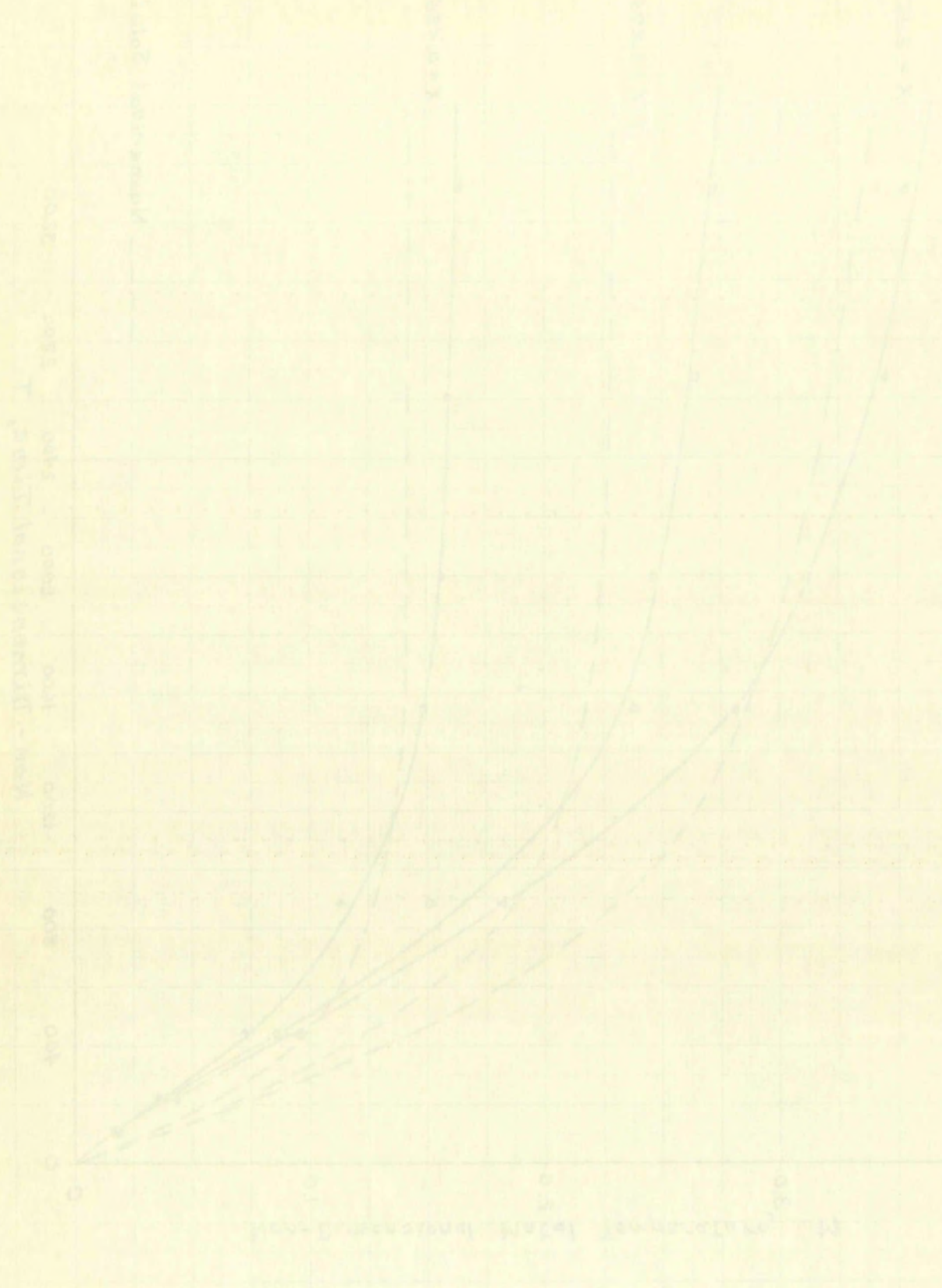


FIGURE 9 SUMMARY OF METAL TEMPERATURES FOR TEST 1

Fig. 1. Dependence of the rate of polymerization on the concentration of the initiator.



1 - linear dependence; 2 - quadratic dependence; 3 - cubic dependence; 4 - higher order dependence.

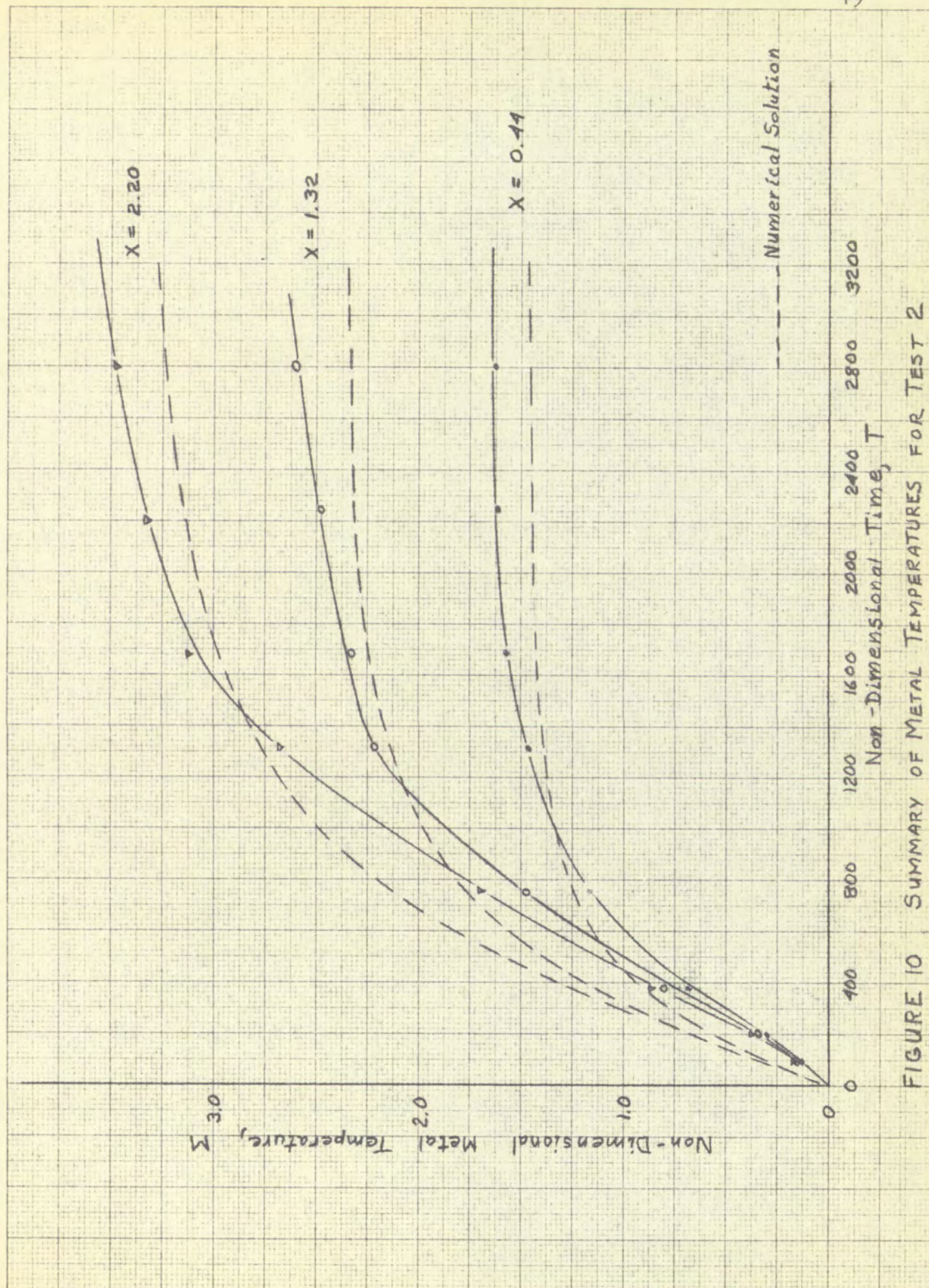
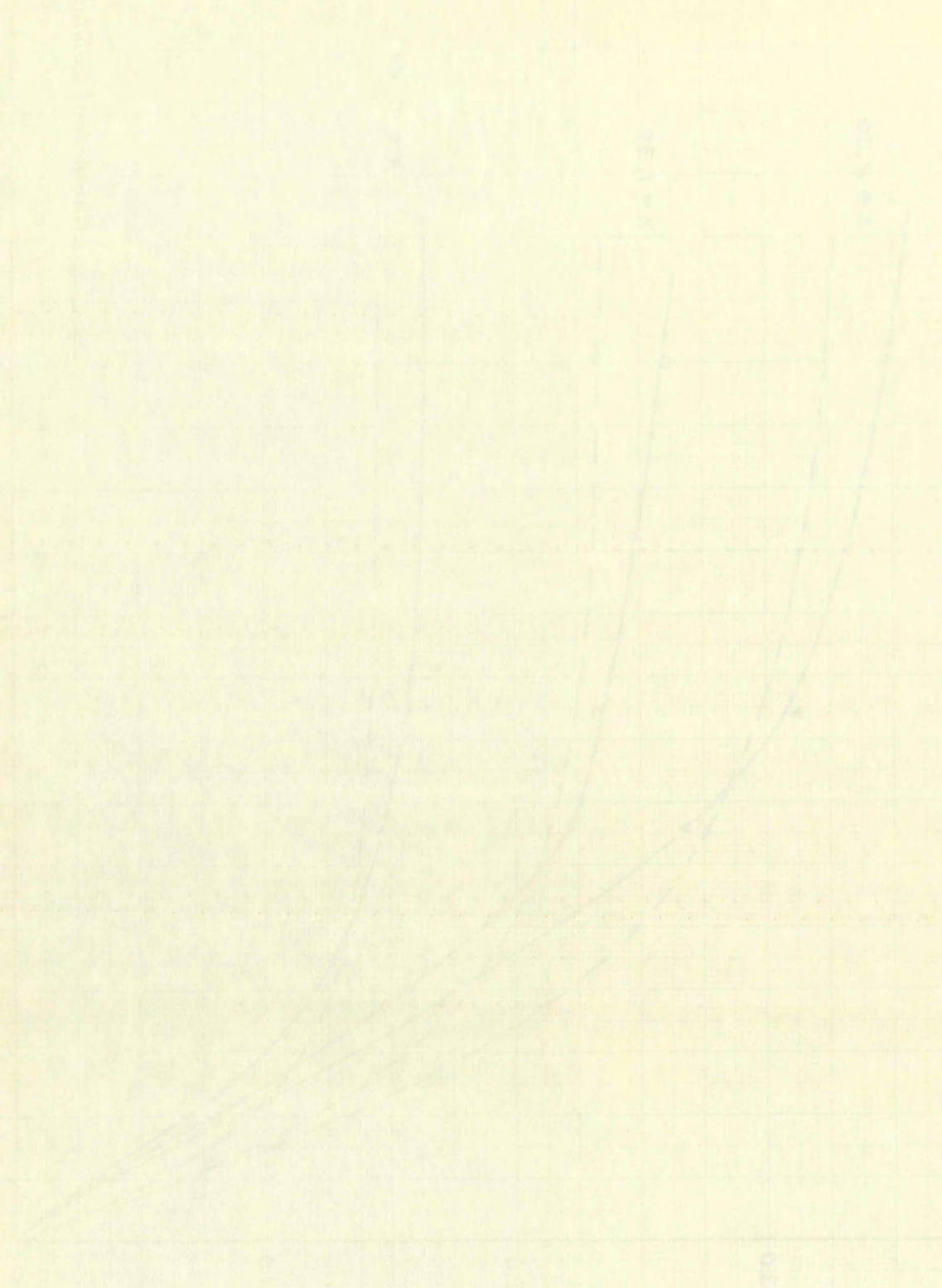


FIGURE 10 SUMMARY OF METAL TEMPERATURES FOR TEST 2



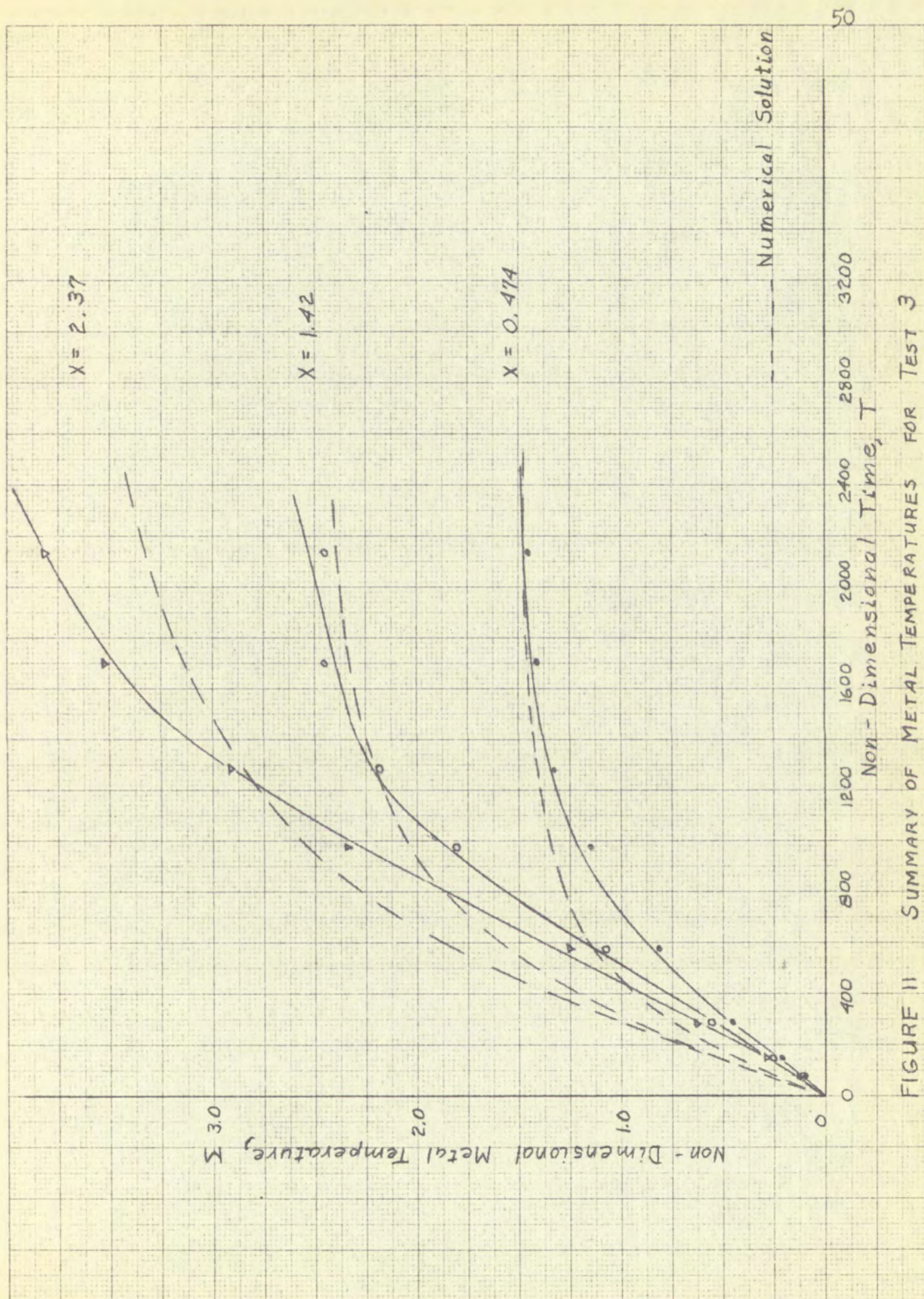


FIGURE 11 SUMMARY OF METAL TEMPERATURES FOR TEST 3

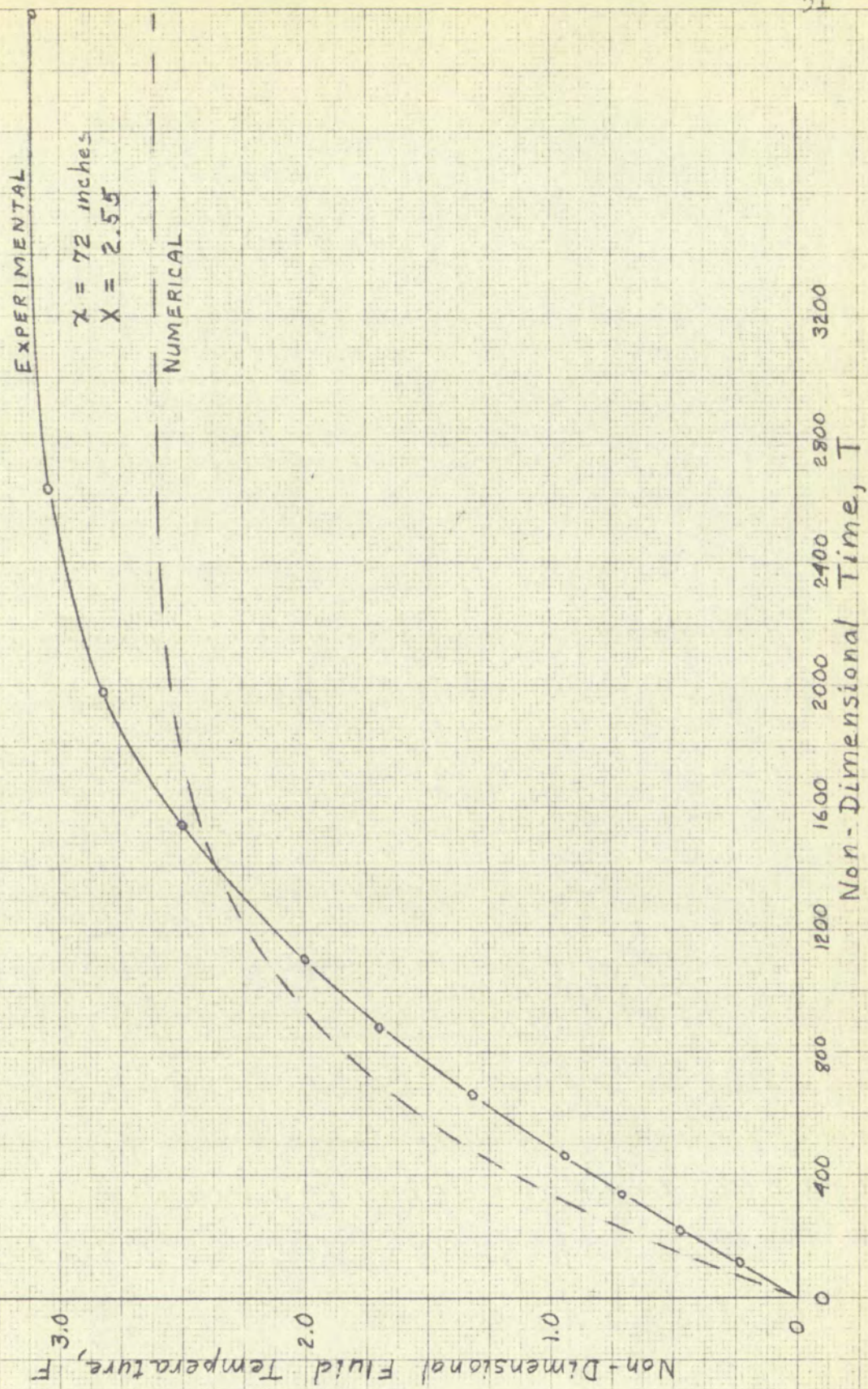
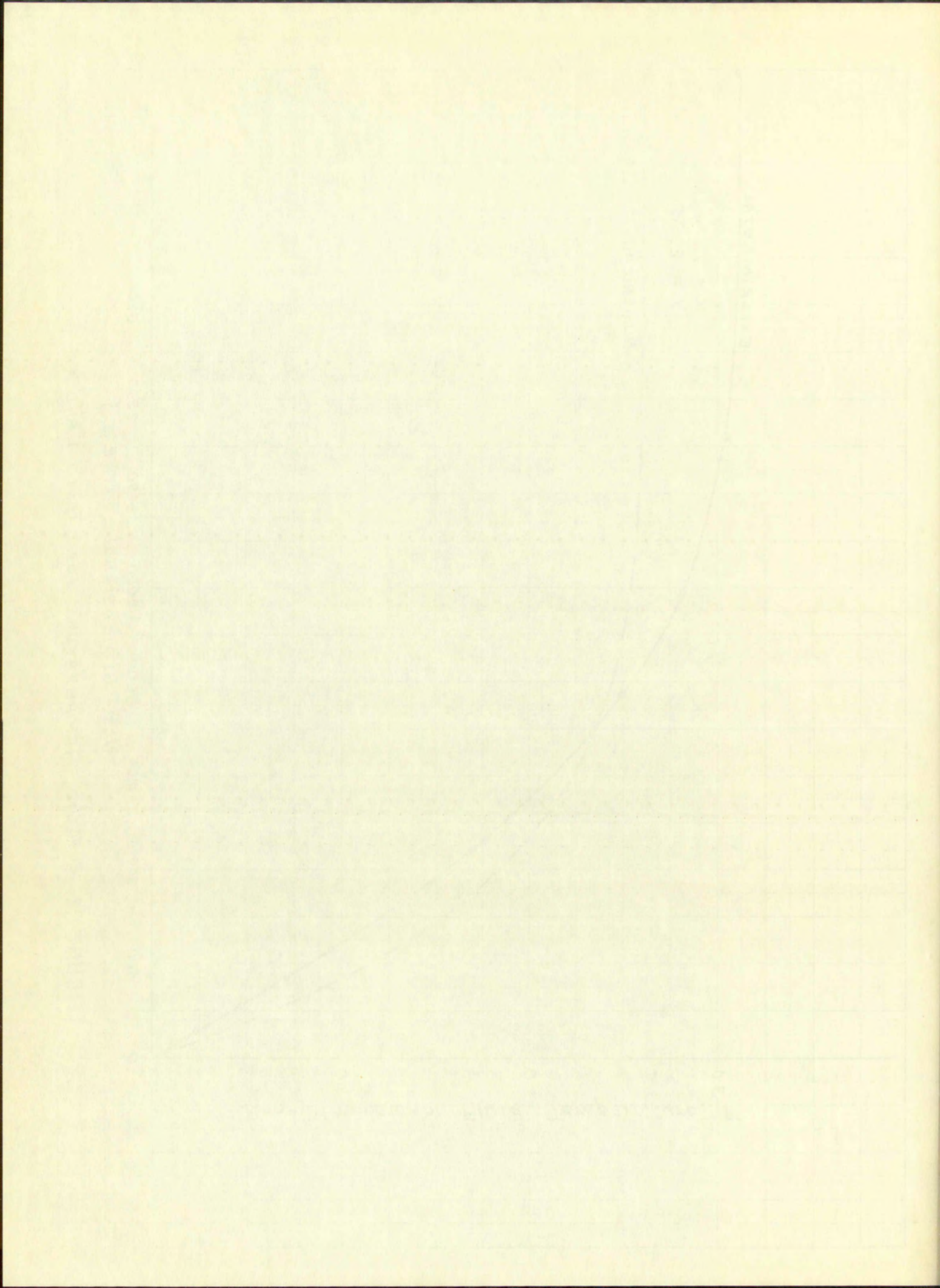


FIGURE 12 FLUID TEMPERATURE FOR TESTS 1 AND 6



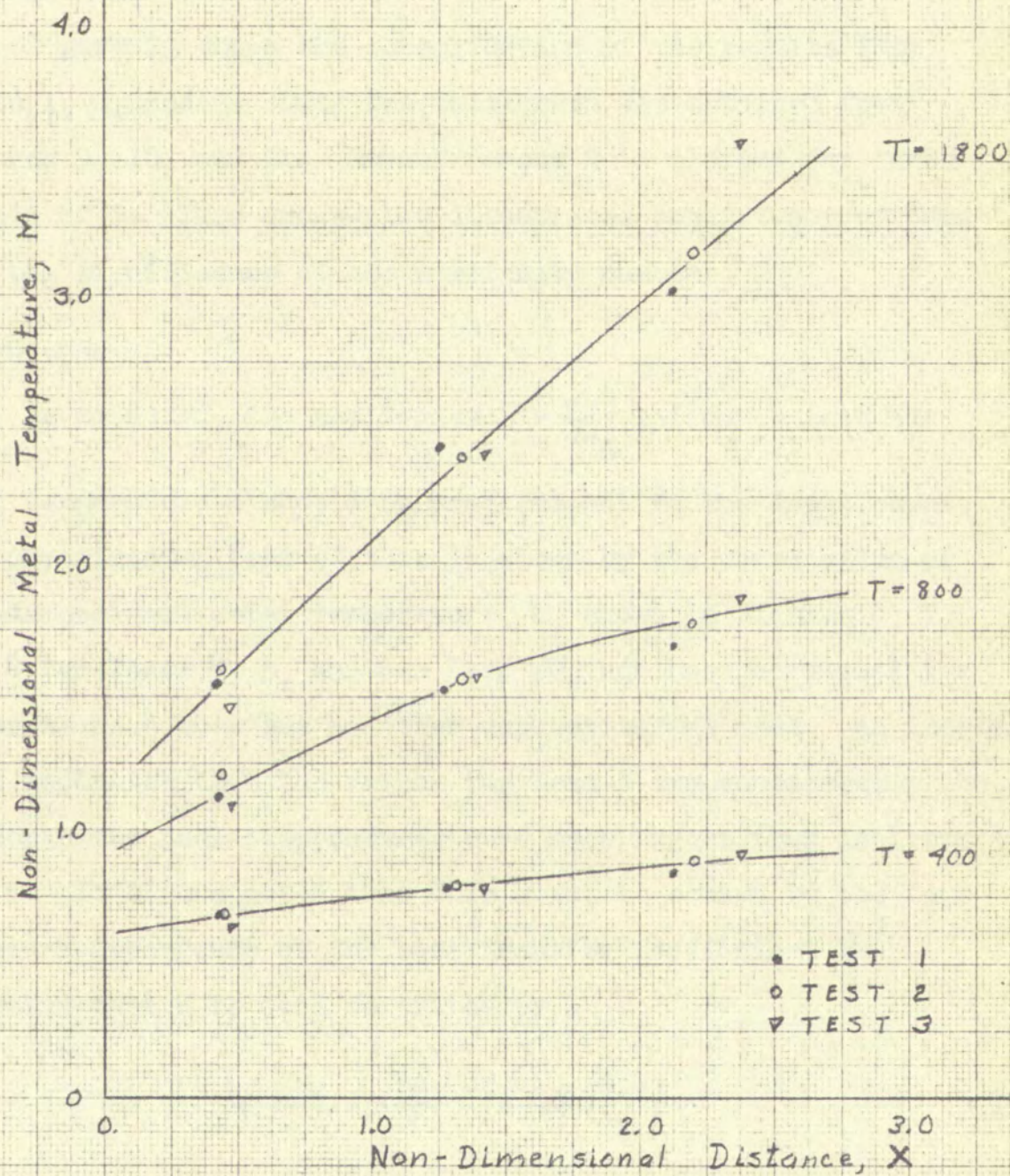


FIGURE 13 CROSS-PLOT OF RESULTS
FROM TESTS 1, 2, 3

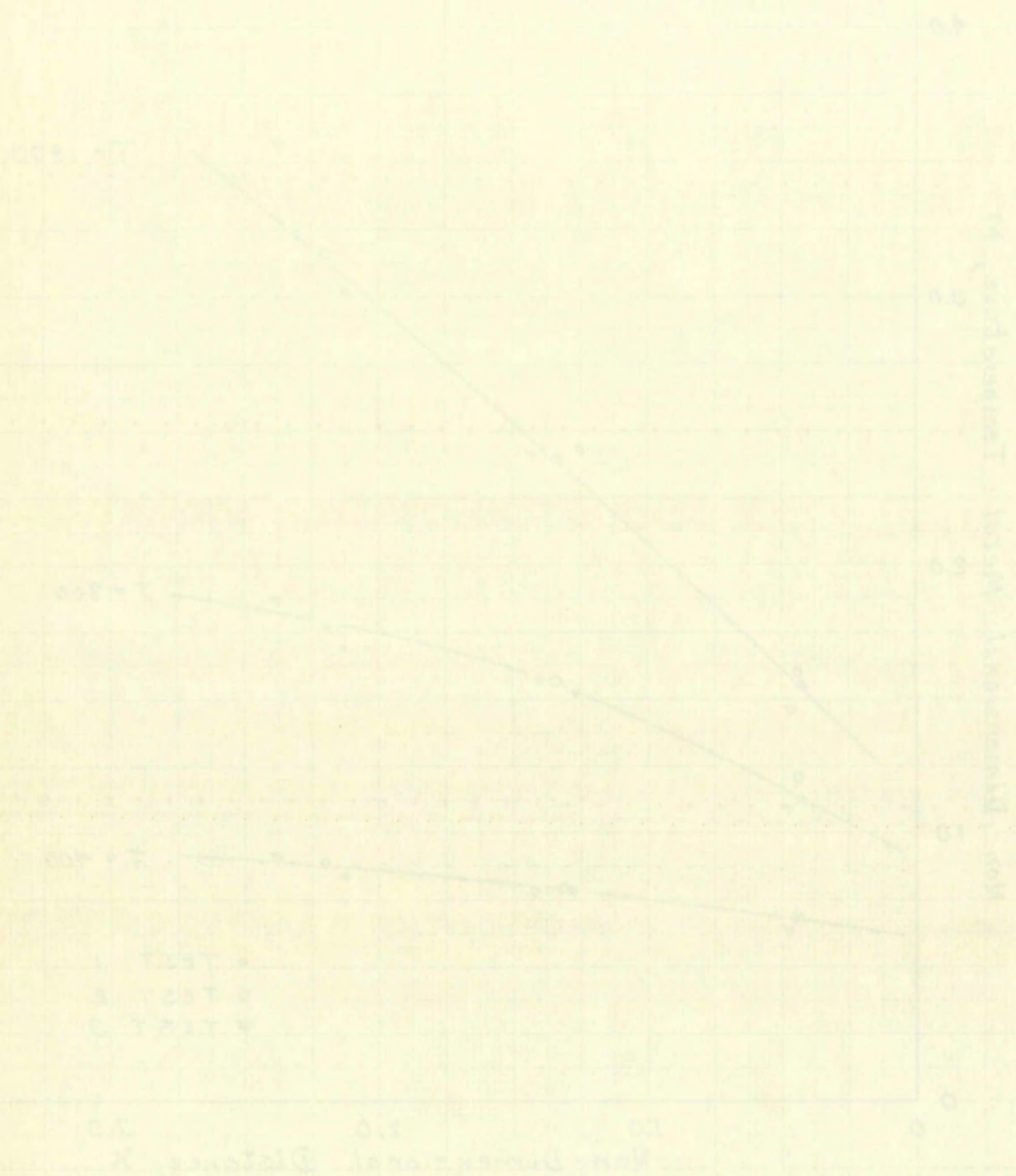


FIGURE 12. Change of Non-dimensional Distance X from Tests 1, 2, 3.

for $X = 2.55$ which corresponds to $x = 72$ inches for those tests. The theoretical curve for the same X is plotted with the experimental results.

Figure 13 shows the consolidation of the results from Tests 1, 2, and 3. Data for this graph was obtained from Figures 9, 10, and 11. Here M versus X is plotted for three values of T . This cross-plot illustrates consistency of data and the significance of the similarity numbers.

DISCUSSION

As expected, the similarity $M = \frac{mhp}{\Delta U_g}$ indicates that the real temperature m should be proportional to the input power. The experimental test of this is shown by the correlation of non-dimensional metal temperature, M , shown in Figures 6, 7, and 8 for Tests 4, 5, and 6. Here ΔU_g has been corrected for external heat loss but no other correction was made. An inspection indicates that the values for Test 5 are consistently low although the real temperatures were much higher than in Tests 4 and 6. It is suspected that this might be caused by the temperature dependence of the heat transfer coefficient h . Assuming that h is well described by

$$\frac{hD}{k} = 0.23 \left(\frac{DG}{\mu} \right)^{0.8} \left(\frac{c_p \mu}{k} \right)^{0.4}, \quad 29$$

the temperature dependence may be attributed to k and μ by

$$h \propto \frac{(k)^{0.6}}{(\mu)^{0.4}},$$

The experimental curves for the heat transfer coefficient h are plotted with the experimental results.

Figure 13 shows the correlation of the results from Tests 1, 2, and 3. Data for this graph was obtained from Figures 9, 10, and 11. Here h versus X is plotted for three values of T . This comparison illustrates consistency of data and the reliability of the similarity numbers.

As expected, the similarity number $N = \frac{hL}{k} \frac{\rho c_p}{\mu}$ indicates that the heat transfer coefficient h should be proportional to the input power. The experimental test of this is shown by the correlation of non-dimensional heat transfer coefficient N , shown in Figures 9, 10, and 11. Data U_0 has been corrected for external heat loss but no other correction was made. It is noted that the values for Test 5 are consistently low although the heat transfer coefficient was much higher than in Tests 1 and 2. It is suggested that this might be caused by the dependence of the heat transfer coefficient h on the input power P as well as by the input power P .

$$\frac{h}{k} = 0.25 \left(\frac{P}{L^2} \right)^{0.8} \left(\frac{\rho c_p}{\mu} \right)^{0.1}$$

The relationship between h and P may be attributed to h and P as

$$h = \frac{0.25}{L^2} P^{0.8} \left(\frac{\rho c_p}{\mu} \right)^{0.1}$$

and

$$\frac{h_4}{h_5} = \left(\frac{\mu_5}{\mu_4}\right)^{0.4} \left(\frac{k_4}{k_5}\right)^{0.6} \quad . \quad 32$$

The variables are evaluated as follows:

$$\mu_4 = 0.021 \text{ at } 200^\circ\text{F}$$

$$\mu_5 = 0.029 \text{ at } 600^\circ\text{F}$$

$$k_4 = 0.0184 \text{ at } 200^\circ\text{F}$$

$$k_5 = 0.0260 \text{ at } 600^\circ\text{F}$$

Substitutions of these values in equation 32 gives

$$h_4 = h_5(1.38)^{0.4}(0.707)^{0.6}$$

or

$$h_4 = 0.925 h_5.$$

The dimensionless M is directly proportional to h. Therefore, a deviation of approximately 7-1/2% between the curves for Test 4 and Test 5 is explained by the temperature dependence of h. This is about the magnitude of the deviation in Figures 6, 7, and 8. Taking the above into account, the agreement between tests is sufficient to justify the method by which ΔU_g was corrected for external heat loss.

The principal results of the investigation are illustrated by Figures 9, 10, 11, and 12. The departure of the theoretical curves from the experimental curves is characterized by an

$$\frac{1}{\sigma^2} = \frac{1}{\sigma^2} + \frac{1}{\sigma^2}$$

The variables are defined as follows:

$$M_1 = 0.025 \text{ at } 200^\circ\text{C}$$

$$M_2 = 0.025 \text{ at } 200^\circ\text{C}$$

$$M_3 = 0.015 \text{ at } 200^\circ\text{C}$$

$$M_4 = 0.025 \text{ at } 200^\circ\text{C}$$

Substitutions of these values in equation (1) give

$$M_1 = 0.025 \text{ at } 200^\circ\text{C}$$

or

$$M_1 = 0.025 \text{ at } 200^\circ\text{C}$$

The dimensionless M is directly proportional to the rate of

a reaction of a substance with a substance which is

Test 1 and Test 2 are explained by the same mechanism

of M . This is about the same as the rate of reaction of

6, 7, and 8. Taking the above into account, the reaction

from tests is sufficient to explain the results of

was corrected for external heat loss.

The principal results of a series of experiments

by Figures 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 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1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 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1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603, 1604, 1605, 1606, 1607, 1608, 1609, 1610, 1611, 1612, 1613, 1614, 1615, 1616, 1617, 1618, 1619, 1620, 1621, 1622, 1623, 1624, 1625, 1626, 1627, 1628, 1629, 1630, 1631, 1632, 1633, 1634, 1635, 1636, 1637, 1638, 1639, 1640, 1641, 1642, 1643, 1644, 1645, 1646, 1647, 1648, 1649, 1650, 1651, 1652, 1653, 1654, 1655, 1656, 1657, 1658, 1659, 1660, 1661, 1662, 1663, 1664, 1665, 1666, 1667, 1668, 1669, 1670, 1671, 1672, 1673, 1674, 1675, 1676, 1677, 1678, 1679, 1680, 1681, 1682, 1683, 1684, 1685, 1686, 1687, 1688, 1689, 1690, 1691, 1692, 1693, 1694, 1695, 1696, 1697, 1698, 1699, 1700, 1701, 1702, 1703, 1704, 1705, 1706, 1707, 1708, 1709, 1710, 1711, 1712, 1713, 1714, 1715, 1716, 1717, 1718, 1719, 1720, 1721, 1722, 1723, 1724, 1725, 1726, 1727, 1728, 1729, 1730, 1731, 1732, 1733, 1734, 1735, 1736, 1737, 1738, 1739, 1740, 1741, 1742, 1743, 1744, 1745, 1746, 1747, 1748, 1749, 1750, 1751, 1752, 1753, 1754, 1755, 1756, 1757, 1758, 1759, 1760, 1761, 1762, 1763, 1764, 1765, 1766, 1767, 1768, 1769, 1770, 1771, 1772, 1773, 1774, 1775, 1776, 1777, 1778, 1779, 1780, 1781, 1782, 1783, 1784, 1785, 1786, 1787, 1788, 1789, 1790, 1791, 1792, 1793, 1794, 1795, 1796, 1797, 1798, 1799, 1800, 1801, 1802, 1803, 1804, 1805, 1806, 1807, 1808, 1809, 1810, 1811, 1812, 1813, 1814, 1815, 1816, 1817, 1818, 1819, 1820, 1821, 1822, 1823, 1824, 1825, 1826, 1827, 1828, 1829, 1830, 1831, 1832, 1833, 1834, 1835, 1836, 1837, 1838, 1839, 1840, 1841, 1842, 1843, 1844, 1845, 1846, 1847, 1848, 1849, 1850, 1851, 1852, 1853, 1854, 1855, 1856, 1857, 1858, 1859, 1860, 1861, 1862, 1863, 1864, 1865, 1866, 1867, 1868, 1869, 1870, 1871, 1872, 1873, 1874, 1875, 1876, 1877, 1878, 1879, 1880, 1881, 1882, 1883, 1884, 1885, 1886, 1887, 1888, 1889, 1890, 1891, 1892, 1893, 1894, 1895, 1896, 1897, 1898, 1899, 1900, 1901, 1902, 1903, 1904, 1905, 1906, 1907, 1908, 1909, 1910, 1911, 1912, 1913, 1914, 1915, 1916, 1917, 1918, 1919, 1920, 1921, 1922, 1923, 1924, 1925, 1926, 1927, 1928, 1929, 1930, 1931, 1932, 1933, 1934, 1935, 1936, 1937, 1938, 1939, 1940, 1941, 1942, 1943, 1944, 1945, 1946, 1947, 1948, 1949, 1950, 1951, 1952, 1953, 1954, 1955, 1956, 1957, 1958, 1959, 1960, 1961, 1962, 1963, 1964, 1965, 1966, 1967, 1968, 1969, 1970, 1971, 1972, 1973, 1974, 1975, 1976, 1977, 1978, 1979, 1980, 1981, 1982, 1983, 1984, 1985, 1986, 1987, 1988, 1989, 1990, 1991, 1992, 1993, 1994, 1995, 1996, 1997, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 208

initial overstatement and a final understatement of temperatures. The cross-over occurs at a time corresponding to about half of the time required to reach equilibrium which is achieved in the same time for both. Considering that the numerical computation assumed fluid properties at a median temperature of 300°F, the comparison is good.

The consistency of data is partially shown by the agreement among tests in Figures 6, 7, and 8. A comprehensive demonstration of consistency is presented by the cross-plot in Figure 13. In non-dimensional terms, Figure 13 indicates that the similarity theory, experimental data and calculations are probably valid since the results are consistent and are comparable to the numerical solution.

initial overstatement and a final understatement of temperature.

Thus, the mean over course of a time corresponding to about half of the time resulted in some adjustment which is achieved in the same time for both. Considering that the numerical comparison agreed fairly precisely at a mean temperature of

300° F, the comparison is good. The consistency of data is partially shown by the agreement among tests in Figures 6, 7, and 8. A comprehensive demonstration of consistency is presented by the enclosed in Figure 12. In non-dimensional terms, Figure 12 indicates that the similarity theory, experimental data and calculations are probably valid since the results are consistent and are comparable to

the numerical solution.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions can be drawn from this investigation:

1. Transient performance of a heat exchanger with an internal source can be correlated by use of the similarity numbers $M = \frac{mhp}{\Delta U_g}$, $F = \frac{fhp}{\Delta U_g}$, $X = \frac{fhp}{\Delta U_g}$,

$$T = \frac{thp}{c_f \delta_f A_f}, \text{ and } a = \frac{c_m \delta_m A_m}{c_f \delta_f A_f}.$$

2. The use of an average temperature for fluid properties gives a numerical solution which overstates initial temperatures and understates final temperatures for the conditions of this investigation.
3. The assumption of constant fluid properties may lead to significant error when temperature changes are 400°F or greater.
4. The analytical solution must be further simplified before it becomes useful as a design tool.

An analytical solution of this problem for design application has still not been achieved and it is recommended that further efforts be directed toward the simplification of the results obtained in this investigation. The approach to this problem might be the division of the problem into the special cases of small "a" and large "a," small "X" and large "X," etc.

CHAPTER 2

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions can be drawn from this investigation:

1. The

1. Theoretical performance of a heat exchanger with an

internal heater can be correlated by use of the

$$\text{Nusselt number, } N_u = \frac{hD}{k_f}, \quad \text{Prandtl number, } Pr = \frac{c_p \mu}{k_f}, \quad \text{Reynolds number, } Re = \frac{\rho u D}{\mu}$$

$$T = \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right), \quad \text{and } \phi = \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right)$$

2. The use of an average temperature for fluid properties

also gives a numerical solution which overstates

initial temperatures and understates final temperatures

thus for the conditions of this investigation.

3. The assumption of constant fluid properties may

lead to significant error when temperature changes

are 400% or greater.

4. The analytical solution must be further simplified

before it becomes useful as a design tool.

An analytical solution of this problem for design appli-

cation has still not been achieved and it is recommended that

further efforts be directed toward the simplification of the

results obtained in this investigation. The approach to this

problem might be the division of the problem into the special

cases of small "a" and large "a", small "X" and large "X", etc.

It should be borne in mind, however, that the analytical solution may not represent the physical problem very closely unless temperature dependent effects are accounted for. This suggests that a more sophisticated numerical method might be more useful, but a digital computer should be used to reduce the labor.

Additional work on the thermal analog discussed here might be rewarding. The similarity numbers derived appears to be quite significant, and it is believed that the analog approach should give good results which would be valuable in design studies. Further analog results would also be of assistance in deriving, as well as verifying, new theoretical approaches.

Some refinements are recommended in the present apparatus. The starting point should be the development of a better method for installing thermocouples on the tube. A low-heat-capacity insulation around the heated tube would reduce external heat losses and improve the precision of the results. The possibility of measuring bulk coolant temperature at several points might also be investigated. A more stable power source and a larger air supply tank might contribute to slight improvements in data.

APPENDIX

- (1) Board, U. S. Federal of Public Health
 (2) Board, U. S. Federal of Public Health

Behavior of a ...

of the ...

- (3) Board, U. S. Federal of Public Health

and Process ...

Science Conference, Philadelphia

No. ...

- (4) Board, U. S. Federal of Public Health

approximate ...

of the ...

- (5) Board, U. S. Federal of Public Health

of the ...

approximate ...

No. ...

1955-1960, August 1960

- (6) Board, U. S. Federal of Public Health

to Pipes and ...

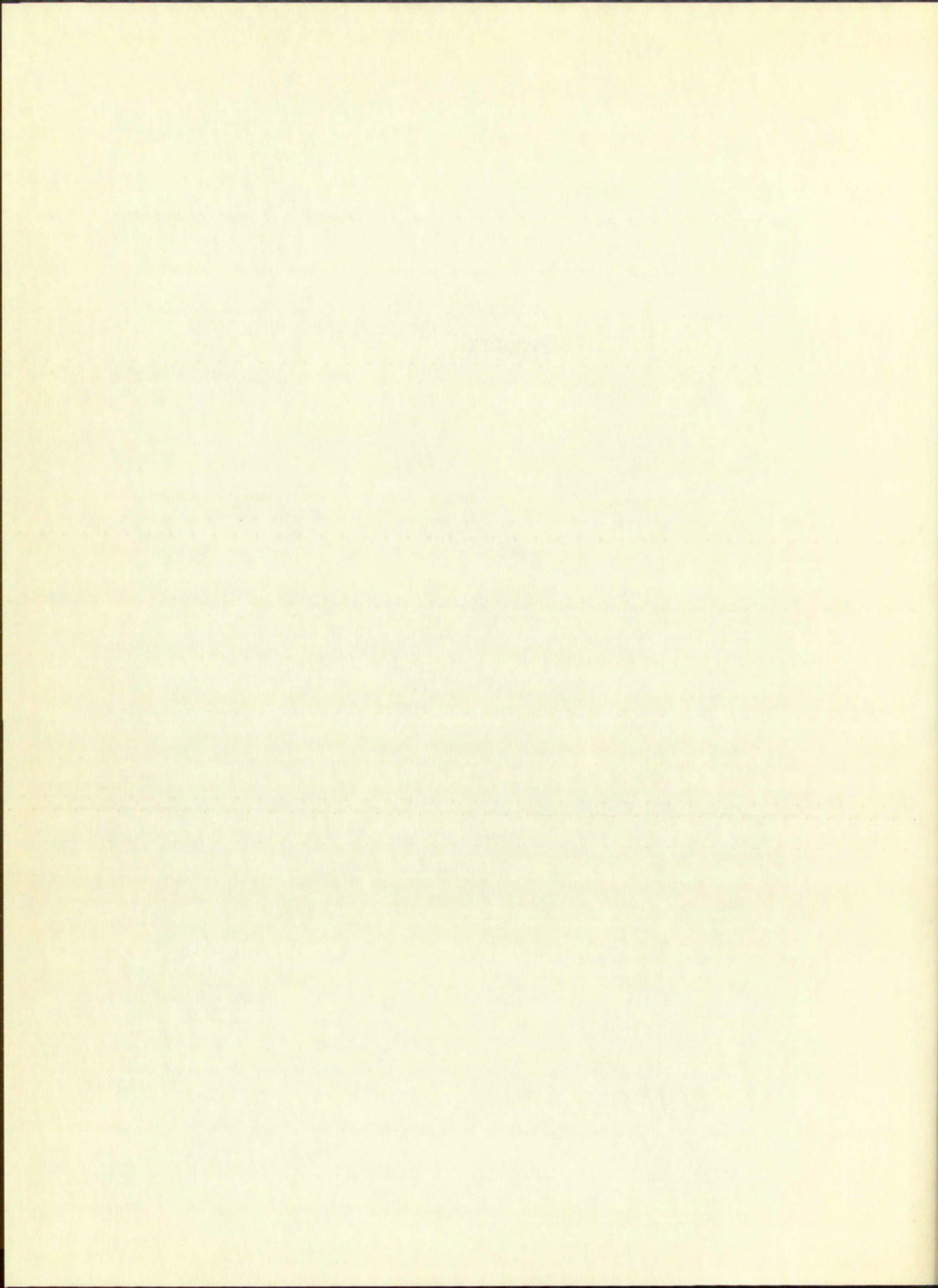
approximate ...

- (7) Board, U. S. Federal of Public Health

approximate ...

approximate ...

1951, 1952, 1953, 1954



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COMPARISON OF THERMOMETER IN SPECIAL WELL

AND TUBE THERMOCOUPLES

Test A - Comparison with iron-constantan thermocouple at $x = 36$

inches.

Date: 14 June 1958

Thermometer in well °F	Chromel-Alumel TC in well °F	Iron-Constantan TC on tube °F
equilibrium		
598	592	592
605	600	607
tube cooling		
	535	520
readings taken	500	482
in succession		
	230	225
	220	218
	213	210
	207	207

THE UNIVERSITY OF CHICAGO

PHYSICS DEPARTMENT

EXPERIMENTAL DATA		THEORY	
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20
21	22	23	24
25	26	27	28
29	30	31	32
33	34	35	36
37	38	39	40
41	42	43	44
45	46	47	48
49	50	51	52
53	54	55	56
57	58	59	60
61	62	63	64
65	66	67	68
69	70	71	72
73	74	75	76
77	78	79	80
81	82	83	84
85	86	87	88
89	90	91	92
93	94	95	96
97	98	99	100

TABLE 1 (Continued)

COMPARISON OF THERMOMETER IN SPECIAL WELL

AND TUBE THERMOCOUPLES

Test B - Comparison with chromel-alumel thermocouple at x = 60 inches.

Date: 17 June 1958

Thermometer in well °F	Chromel-Alumel TC in well °F	Chromel-Alumel TC on tube °F
equilibrium		
640	642	662
640	642	662
tube cooling		
	337	322
readings		
taken	303	294
in		
succession	288	283
	278	275
	270	270
	262	259

THE HISTORY OF THE
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English in 1630 to the present time

By
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Author of "The History of the City of New York"
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TABLE 2
 FACTOR K FOR CONVERTING OSCILLOGRAPH
 TRACE DEFLECTION TO TEMPERATURE

Circuit	Factor K = °F/inch	Probable Error, °F (Assuming $\pm 1/64$ " reading accuracy)
Galv. No. 2 - T.C. at 1/2" or 6"	657	± 10
Galv. No. 3 - T.C. at 12"	577	± 9
Galv. No. 4 - T.C. at 36"	593	± 9
Galv. No. 5 - T.C. at 60"	1130	± 18
Galv. No. 6 - T.C. at 66" or 71-1/2"	1040	± 16
Galv. No. 10 with amplifier - Fluid Temp. Probe	235	± 4

TABLE 1. 1-1. OPERATIONAL DATA

TABLE 1. 1-1. OPERATIONAL DATA

Operating time (hr)	Operating time (hr)	Operating time (hr)
1.0	1.0	1.0
2.0	2.0	2.0
3.0	3.0	3.0
4.0	4.0	4.0
5.0	5.0	5.0
6.0	6.0	6.0
7.0	7.0	7.0
8.0	8.0	8.0
9.0	9.0	9.0
10.0	10.0	10.0
11.0	11.0	11.0
12.0	12.0	12.0
13.0	13.0	13.0
14.0	14.0	14.0
15.0	15.0	15.0
16.0	16.0	16.0
17.0	17.0	17.0
18.0	18.0	18.0
19.0	19.0	19.0
20.0	20.0	20.0
21.0	21.0	21.0
22.0	22.0	22.0
23.0	23.0	23.0
24.0	24.0	24.0
25.0	25.0	25.0
26.0	26.0	26.0
27.0	27.0	27.0
28.0	28.0	28.0
29.0	29.0	29.0
30.0	30.0	30.0
31.0	31.0	31.0
32.0	32.0	32.0
33.0	33.0	33.0
34.0	34.0	34.0
35.0	35.0	35.0
36.0	36.0	36.0
37.0	37.0	37.0
38.0	38.0	38.0
39.0	39.0	39.0
40.0	40.0	40.0
41.0	41.0	41.0
42.0	42.0	42.0
43.0	43.0	43.0
44.0	44.0	44.0
45.0	45.0	45.0
46.0	46.0	46.0
47.0	47.0	47.0
48.0	48.0	48.0
49.0	49.0	49.0
50.0	50.0	50.0
51.0	51.0	51.0
52.0	52.0	52.0
53.0	53.0	53.0
54.0	54.0	54.0
55.0	55.0	55.0
56.0	56.0	56.0
57.0	57.0	57.0
58.0	58.0	58.0
59.0	59.0	59.0
60.0	60.0	60.0
61.0	61.0	61.0
62.0	62.0	62.0
63.0	63.0	63.0
64.0	64.0	64.0
65.0	65.0	65.0
66.0	66.0	66.0
67.0	67.0	67.0
68.0	68.0	68.0
69.0	69.0	69.0
70.0	70.0	70.0
71.0	71.0	71.0
72.0	72.0	72.0
73.0	73.0	73.0
74.0	74.0	74.0
75.0	75.0	75.0
76.0	76.0	76.0
77.0	77.0	77.0
78.0	78.0	78.0
79.0	79.0	79.0
80.0	80.0	80.0
81.0	81.0	81.0
82.0	82.0	82.0
83.0	83.0	83.0
84.0	84.0	84.0
85.0	85.0	85.0
86.0	86.0	86.0
87.0	87.0	87.0
88.0	88.0	88.0
89.0	89.0	89.0
90.0	90.0	90.0
91.0	91.0	91.0
92.0	92.0	92.0
93.0	93.0	93.0
94.0	94.0	94.0
95.0	95.0	95.0
96.0	96.0	96.0
97.0	97.0	97.0
98.0	98.0	98.0
99.0	99.0	99.0
100.0	100.0	100.0

TABLE 3

EXTERNAL HEAT LOSS MEASUREMENT

Test No. 1Date: 17 June 1958

Input Power, watts/inch	Metal Temp. at $x = 1/2"$ °F	Metal Temp. at $x = 12"$ °F	Metal Temp. at $x = 36"$ °F	Metal Temp. at $x = 60"$ °F	Metal Temp. at $x = 72-1/2"$ °F
0.69	56	88	92	90	43
1.47	118	175	178	178	91
3.10	189	287	303	305	157
4.89	272	404	427	440	228
7.59	357	523	550	575	317
10.0	430	625	657	708	398

Test No. 2Date: 21 June 1958

Input Power, watts/inch	Metal Temp. at $x = 6"$ (°F)	Metal Temp. at $x = 12"$ (°F)	Metal Temp. at $x = 36"$ (°F)	Metal Temp. at $x = 60"$ (°F)	Metal Temp. at $x = 66"$ (°F)
1.34	157	153	160	156	150
2.28	233	230	240	154	236
4.08	365	355	369	390	390
7.03	528	506	530	608	590

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TABLE 4.

RAW DATA - METAL TEMPERATURES

Flow Rate: 0.865 #/minPower Input: 30.9 watts/inInlet Air Temp.: 82°F; Reference Junction (Room Temp.): 85°FDate: 21 June 1958Test No. 1

Time sec	Temp. (°F) at x = 6"			Temp. (°F) at x = 12"			Temp. (°F) at x = 36"			Temp. (°F) at x = 60"			Temp. (°F) at x = 66"		
	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg
5	55	51	53	30	38	34	32	45	39	42	52	47	40	50	45
10	95	102	99	80	75	78	100	105	103	100	108	104	95	105	100
15	126	137	132	121	122	122	140	157	149	140	148	144	145	150	148
20	181	176	179	167	164	166	198	210	204	200	205	203	205	220	213
30	230	223	227	227	223	225	283	284	284	282	282	282	310	295	303
40	268	255	262	275	267	271	345	340	343	358	355	357	390	385	388
50	287	272	280	305	293	299	390	392	391	420	410	415	455	440	448
70	312	297	305	340	322	331	456	444	450	510	490	500	540	520	530
90	325	308	317	355	335	345	482	468	475	575	545	560	600	575	588
120	317	321	319	350	348	349	492	488	490	595	580	588	615	600	608
150	327	315	321	360	342	351	498	490	494	608	580	594	620	595	608
measured	306	335	321	334	329	332	472	473	473	598	585	592	610	588	599

TABLE 4 (Continued)

RAW DATA - METAL TEMPERATURES

Date: 21 June 1958Test No. 2Flow Rate: 0.708 #/minPower Input: 30.9 watts/inInlet Air Temp.: 85°F; Reference Junction (Room Temp.): 86°F

Time sec	Metal Temp. (°F) at x = 6"			Metal Temp. (°F) at x = 12"			Metal Temp. (°F) at x = 36"			Metal Temp. (°F) at x = 60"			Metal Temp. (°F) at x = 66"		
	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg
5	-	40	-	38	28	32	44	40	42	50	45	48	55	40	48
10	-	82	-	85	78	81	88	97	93	100	90	95	105	95	100
15	-	127	-	135	130	133	148	160	154	165	155	160	170	160	165
20	-	181	-	177	165	171	227	210	219	220	205	213	230	215	223
30	-	243	-	242	237	240	313	298	306	310	295	303	335	305	320
40	-	275	-	288	282	285	387	375	381	395	370	383	415	390	403
50	-	296	-	326	306	317	438	422	430	475	445	460	480	450	465
70	-	323	-	362	346	354	502	480	491	578	540	559	575	540	558
90	-	345	-	378	358	368	532	520	526	630	605	618	620	590	605
120	-	332	-	378	380	279	532	542	540	670	695	683	670	630	650
150	-	320	-	382	372	279	552	548	550	675	725	700	665	640	653
measured	347	351	349	349	389	269	531	543	537	667	685	676	667	685	676

TABLE 4 (Continued)

RAW DATA - METAL TEMPERATURES

Flow Rate: 0.50 #/min

Power Input: 30.9 watts/in

Inlet Air Temp.: 87°F; Reference Junction (Room Temp.): 86°F

Date: 21 June 1958

Test No. 3

Time Sec	Metal Temp. (°F) x = 6"			Metal Temp. (°F) x = 12"			Metal Temp. (°F) x = 36"			Metal Temp. (°F) x = 60"			Metal Temp. (°F) x = 66"		
	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg
5	35	40	38	37	33	35	48	45	47	40	50	45	45	45	45
10	68	82	75	74	80	77	100	102	101	85	100	93	85	105	95
15	102	120	111	122	124	123	162	168	165	145	160	153	155	150	153
20	125	153	139	160	169	165	208	218	213	190	212	201	200	215	208
30	183	198	191	241	234	238	318	304	311	295	300	298	305	305	305
40	223	235	229	302	285	294	405	382	394	390	380	385	395	385	390
50	246	263	255	341	323	332	470	445	458	485	465	475	485	470	478
70	274	302	288	400	383	392	553	537	545	620	615	618	595	600	598
90	294	321	308	427	412	420	605	592	599	710	700	705	685	675	680
120	300	335	318	445	438	442	643	627	635	775	735	755	735	740	738
150	308	304	306	448	438	443	663	640	652	825	770	797	775	745	760
measured	445	450	448	477	478	478	647	655	651	821	830	826	802	795	799

number	772	773	774	775	776	777	778	779	780	781	782	783	784	785	786	787	788	789	790	791	792	793	794	795	796	797	798	799	800
140	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268
150	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297
160	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326
170	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355
180	356	357	358	359	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384
190	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401	402	403	404	405	406	407	408	409	410	411	412	413
200	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441	442
210	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	460	461	462	463	464	465	466	467	468	469	470	471
220	472	473	474	475	476	477	478	479	480	481	482	483	484	485	486	487	488	489	490	491	492	493	494	495	496	497	498	499	500
230	501	502	503	504	505	506	507	508	509	510	511	512	513	514	515	516	517	518	519	520	521	522	523	524	525	526	527	528	529
240	530	531	532	533	534	535	536	537	538	539	540	541	542	543	544	545	546	547	548	549	550	551	552	553	554	555	556	557	558
250	559	560	561	562	563	564	565	566	567	568	569	570	571	572	573	574	575	576	577	578	579	580	581	582	583	584	585	586	587
260	588	589	590	591	592	593	594	595	596	597	598	599	600	601	602	603	604	605	606	607	608	609	610	611	612	613	614	615	616
270	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	632	633	634	635	636	637	638	639	640	641	642	643	644	645
280	646	647	648	649	650	651	652	653	654	655	656	657	658	659	660	661	662	663	664	665	666	667	668	669	670	671	672	673	674
290	675	676	677	678	679	680	681	682	683	684	685	686	687	688	689	690	691	692	693	694	695	696	697	698	699	700	701	702	703
300	704	705	706	707	708	709	710	711	712	713	714	715	716	717	718	719	720	721	722	723	724	725	726	727	728	729	730	731	732
310	733	734	735	736	737	738	739	740	741	742	743	744	745	746	747	748	749	750	751	752	753	754	755	756	757	758	759	760	761
320	762	763	764	765	766	767	768	769	770	771	772	773	774	775	776	777	778	779	780	781	782	783	784	785	786	787	788	789	790
330	791	792	793	794	795	796	797	798	799	800	801	802	803	804	805	806	807	808	809	810	811	812	813	814	815	816	817	818	819
340	820	821	822	823	824	825	826	827	828	829	830	831	832	833	834	835	836	837	838	839	840	841	842	843	844	845	846	847	848
350	849	850	851	852	853	854	855	856	857	858	859	860	861	862	863	864	865	866	867	868	869	870	871	872	873	874	875	876	877
360	878	879	880	881	882	883	884	885	886	887	888	889	890	891	892	893	894	895	896	897	898	899	900	901	902	903	904	905	906
370	907	908	909	910	911	912	913	914	915	916	917	918	919	920	921	922	923	924	925	926	927	928	929	930	931	932	933	934	935
380	936	937	938	939	940	941	942	943	944	945	946	947	948	949	950	951	952	953	954	955	956	957	958	959	960	961	962	963	964
390	965	966	967	968	969	970	971	972	973	974	975	976	977	978	979	980	981	982	983	984	985	986	987	988	989	990	991	992	993
400	994	995	996	997	998	999	1000	1001	1002	1003	1004	1005	1006	1007	1008	1009	1010	1011	1012	1013	1014	1015	1016	1017	1018	1019	1020	1021	1022

1000 1001 1002 1003 1004 1005 1006 1007 1008 1009 1010 1011 1012 1013 1014 1015 1016 1017 1018 1019 1020 1021 1022 1023 1024 1025 1026 1027 1028 1029 1030 1031 1032 1033 1034 1035 1036 1037 1038 1039 1040 1041 1042 1043 1044 1045 1046 1047 1048 1049 1050 1051 1052 1053 1054 1055 1056 1057 1058 1059 1060 1061 1062 1063 1064 1065 1066 1067 1068 1069 1070 1071 1072 1073 1074 1075 1076 1077 1078 1079 1080 1081 1082 1083 1084 1085 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1096 1097 1098 1099 1100 1101 1102 1103 1104 1105 1106 1107 1108 1109 1110 1111 1112 1113 1114 1115 1116 1117 1118 1119 1120 1121 1122 1123 1124 1125 1126 1127 1128 1129 1130 1131 1132 1133 1134 1135 1136 1137 1138 1139 1140 1141 1142 1143 1144 1145 1146 1147 1148 1149 1150 1151 1152 1153 1154 1155 1156 1157 1158 1159 1160 1161 1162 1163 1164 1165 1166 1167 1168 1169 1170 1171 1172 1173 1174 1175 1176 1177 1178 1179 1180 1181 1182 1183 1184 1185 1186 1187 1188 1189 1190 1191 1192 1193 1194 1195 1196 1197 1198 1199 1200 1201 1202 1203 1204 1205 1206 1207 1208 1209 1210 1211 1212 1213 1214 1215 1216 1217 1218 1219 1220 1221 1222 1223 1224 1225 1226 1227 1228 1229 1230 1231 1232 1233 1234 1235 1236 1237 1238 1239 1240 1241 1242 1243 1244 1245 1246 1247 1248 1249 1250 1251 1252 1253 1254 1255 1256 1257 1258 1259 1260 1261 1262 1263 1264 1265 1266 1267 1268 1269 1270 1271 1272 1273 1274 1275 1276 1277 1278 1279 1280 1281 1282 1283 1284 1285 1286 1287 1288 1289 1290 1291 1292 1293 1294 1295 1296 1297 1298 1299 1300 1301 1302 1303 1304 1305 1306 1307 1308 1309 1310 1311 1312 1313 1314 1315 1316 1317 1318 1319 1320 1321 1322 1323 1324 1325 1326 1327 1328 1329 1330 1331 1332 1333 1334 1335 1336 1337 1338 1339 1340 1341 1342 1343 1344 1345 1346 1347 1348 1349 1350 1351 1352 1353 1354 1355 1356 1357 1358 1359 1360 1361 1362 1363 1364 1365 1366 1367 1368 1369 1370 1371 1372 1373 1374 1375 1376 1377 1378 1379 1380 1381 1382 1383 1384 1385 1386 1387 1388 1389 1390 1391 1392 1393 1394 1395 1396 1397 1398 1399 1400 1401 1402 1403 1404 1405 1406 1407 1408 1409 1410 1411 1412 1413 1414 1415 1416 1417 1418 1419 1420 1421 1422 1423 1424 1425 1426 1427 1428 1429 1430 1431 1432 1433 1434 1435 1436 1437 1438 1439 1440 1441 1442 1443 1444 1445 1446 1447 1448 1449 1450 1451 1452 1453 1454 1455 1456 1457 1458 1459 1460 1461 1462 1463 1464 1465 1466 1467 1468 1469 1470 1471 1472 1473 1474 1475 1476 1477 1478 1479 1480 1481 1482 1483 1484 1485 1486 1487 1488 1489 1490 1491 1492 1493 1494 1495 1496 1497 1498 1499 1500 1501 1502 1503 1504 1505 1506 1507 1508 1509 1510 1511 1512 1513 1514 1515 1516 1517 1518 1519 1520 1521 1522 1523 1524 1525 1526 1527 1528 1529 1530 1531 1532 1533 1534 1535 1536 1537 1538 1539 1540 1541 1542 1543 1544 1545 1546 1547 1548 1549 1550 1551 1552 1553 1554 1555 1556 1557 1558 1559 1560 1561 1562 1563 1564 1565 1566 1567 1568 1569 1570 1571 1572 1573 1574 1575 1576 1577 1578 1579 1580 1581 1582 1583 1584 1585 1586 1587 1588 1589 1590 1591 1592 1593 1594 1595 1596 1597 1598 1599 1600 1601 1602 1603 1604 1605 1606 1607 1608 1609 1610 1611 1612 1613 1614 1615 1616 1617 1618 1619 1620 1621 1622 1623 1624 1625 1626 1627 1628 1629 1630 1631 1632 1633 1634 1635 1636 1637 1638 1639 1640 1641 1642 1643 1644 1645 1646 1647 1648 1649 1650 1651 1652 1653 1654 1655 1656 1657 1658 1659 1660 1661 1662 1663 1664 1665 1666 1667 1668 1669 1670 1671 1672 1673 1674 1675 1676 1677 1678 1679 1680 1681 1682 1683 1684 1685 1686 1687 1688 1689 1690 1691 1692 1693 1694 1695 1696 1697 1698 1699 1700 1701 1702 1703 1704 1705 1706 1707 1708 1709 1710 1711 1712 1713 1714 1715 1716 1717 1718 1719 1720 1721 1722 1723 1724 1725 1726 1727 1728 1729 1730 1731 1732 1733 1734 1735 1736 1737 1738 1739 1740 1741 1742 1743 1744 1745 1746 1747 1748 1749 1750 1751 1752 1753 1754 1755 1756 1757 1758 1759 1760 1761 1762 1763 1764 1765 1766 1767 1768 1769 1770 1771 1772 1773 1774 1775 1776 1777 1778 1779 1780 1781 1782 1783 1784 1785 1786 1787 1788 1789 1790 1791 1792 1793 1794 1795 1796 1797 1798 1799 1800 1801 1802 1803 1804 1805 1806 1807 1808 1809 1810 1811 1812 1813 1814 1815 1816 1817 1818 1819 1820 1821 1822 1823 1824 1825 1826 1827 1828 1829 1830 1831 1832 1833 1834 1835 1836 1837 1838 1839 1840 1841 1842 1843 1844 1845 1846 1847 1848 1849 1850 1851 1852 1853 1854 1855 1856 1857 1858 1859 1860 1861 1862 1863 1864 1865 1866 1867 1868 1869 1870 1871 1872 1873 1874 1875 1876 1877 1878 1879 1880 1881 1882 1883 1884 1885 1886 1887 1888 1889 1890 1891 1892 1893 1894 1895 1896 1897 1898 1899 1900 1901 1902 1903 1904 1905 1906 1907 1908 1909 1910 1911 1912 1913 1914 1915 1916 1917 1918 1919 1920 1921 1922 1923 1924 1925 1926 1927 1928 1929 1930 1931 1932 1933 1934 1935 1936 1937 1938 1939 1940 1941 1942 1943 1944 1945 1946 1947 1948 1949 1950 1951 1952 1953 1954 1955 1956 1957 1958 1959 1960 1961 1962 1963 1964 1965 1966 1967 1968 1969 1970 1971 1972 1973 1974 1975 1976 1977 1978 1979 1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023 2024 2025 2026 2027 2028 2029 2030 2031 2032 2033 2034 2035 2036 2037 2038 2039 2040 2041 2042 2043 2044 2045 2046 2047 2048 2049 2050 2051 2052 2053 2054 2055 2056 2057 2058 2059 2060 2061 2062 2063 2064 2065 2066 2067 2068 2069 2070 2071 2072 2073 2074 207

TABLE 4 (Continued)

RAW DATA - METAL TEMPERATURES

Flow Rate: 0.865 #/min Power Input: 13.9 watts/in Inlet Air Temp.: 85°F ; Reference Junction (Room Temp.): 88°F Date: 23 June 1958Test No. 4

Time sec	Metal Temp. ($^{\circ}\text{F}$) $x = 6''$			Metal Temp. ($^{\circ}\text{F}$) $x = 12''$			Metal Temp. ($^{\circ}\text{F}$) $x = 36''$			Metal Temp. ($^{\circ}\text{F}$) $x = 60''$			Metal Temp. ($^{\circ}\text{F}$) $x = 66''$		
	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg
5	20	15	18	24	20	22	20	23	22	32	30	31	25	30	28
10	42	25	31	42	35	39	45	46	46	52	45	49	53	55	54
15	56	35	47	61	56	59	71	73	72	73	60	67	80	82	81
20	70	47	59	82	77	80	96	96	96	93	92	93	110	110	110
30	90	56	73	112	102	107	137	133	135	142	123	133	158	147	153
40	106	67	87	130	123	127	165	165	165	168	159	164	197	200	199
50	111	68	90	142	133	138	188	190	189	185	182	184	232	220	226
70	116	75	96	155	138	147	213	206	210	220	208	214	268	258	253
90	122	80	101	156	145	151	218	216	217	235	223	229	278	280	279
120	122	80	101	157	146	153	230	228	229	248	242	245	302	282	292
150	125	-	-	165	-	-	234	-	-	260	-	-	312	-	-
measured	140	135	138	155	150	153	225	226	226	284	276	280	298	294	296

TABLE 4 (Continued)

RAW DATA - METAL TEMPERATURES

Date: 23 June 1958Test No. 5Flow Rate: 0.865 #/minPower Input: 41.6 watts/inInlet Air Temp.: 91°F; Reference Junction (Room temp.): 90°F

Time sec	Metal Temp. (°F) x = 6"			Metal Temp. (°F) x = 12"			Metal Temp. (°F) x = 36"			Metal Temp. (°F) x = 60"			Metal Temp. (°F) x = 66"		
	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg
5	37	41	39	40	46	43	57	61	59	32	42	37	50	52	51
10	93	87	90	98	93	96	132	131	132	110	112	111	128	125	127
15	136	126	131	150	148	149	200	202	201	180	175	178	205	193	199
20	173	158	166	203	194	199	267	266	267	243	238	241	268	265	267
30	220	207	214	274	268	271	375	370	373	364	362	363	398	382	390
40	250	237	244	322	322	322	446	444	445	470	463	467	495	478	487
50	270	257	264	356	354	355	502	495	499	455	552	554	568	550	559
70	293	266	280	392	386	389	566	551	559	655	655	655	662	638	650
90	305	261	283	412	402	407	598	581	590	710	703	707	712	690	701
120	308	272	290	418	406	412	615	597	606	740	732	735	732	712	722
150	308	273	290	422	-	-	615	-	-	757	-	-	740	-	-
measured	402	402	402	440	447	444	618	610	614	795	780	788	785	781	783

TABLE 4 (Continued)

RAW DATA - METAL TEMPERATURES

Date: 23 June 1958Test No. 6Flow Rate: 0.865 #/minPower Input: 39.9 watts/inInlet Air Temp.: 97°F; Reference Junction (Room Temp.): 92°F

Time Sec	Metal Temp. (°F) x = 6"			Metal Temp. (°F) x = 12"			Metal Temp. (°F) x = 36"			Metal Temp. (°F) x = 60"			Metal Temp. (°F) x = 66"		
	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg
5	42	33	38	31	30	31	47	42	45	25	32	29	45	42	44
10	80	73	77	67	70	69	97	97	97	75	90	82	95	93	94
15	120	121	121	111	110	111	157	150	154	128	148	138	147	156	152
20	152	146	149	148	145	147	210	202	206	197	193	195	212	202	207
30	195	188	192	200	200	200	282	276	279	266	275	270	300	294	297
40	218	222	220	230	239	235	335	334	335	338	350	344	365	362	364
50	237	237	237	257	264	261	382	376	379	402	396	399	432	412	422
70	251	256	254	286	293	290	429	426	428	473	485	479	512	493	503
90	252	266	259	298	307	303	445	460	448	508	538	523	542	550	546
120	256	268	262	300	313	307	453	466	460	518	563	541	542	568	555
150	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
measured	310	317	315	345	350	348	477	482	480	610	613	612	620	621	621

TABLE 5

RAW DATA - FLUID EXIT TEMPERATURE

Refer to Table 4 for Test Conditions

Time sec	Test No. 1			Test No. 2			Test No. 3		
	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg
5	55	50	53	39	48	44	50	46	48
10	85	102	94	104	103	104	105	103	104
15	145	148	147	160	155	158	152	168	160
20	187	189	188	206	201	204	200	200	200
30	266	266	266	288	280	284	294	280	287
40	327	322	325	347	345	346	372	355	364
50	373	363	368	402	392	397	440	418	429
70	438	418	428	468	462	465	539	507	523
90	475	453	465	507	505	506	591	564	578
120	490	480	485	542	537	540	642	623	633
150	500	490	495	555	552	554	668	655	662
measured	503	510	507	568	569	569	690	685	688

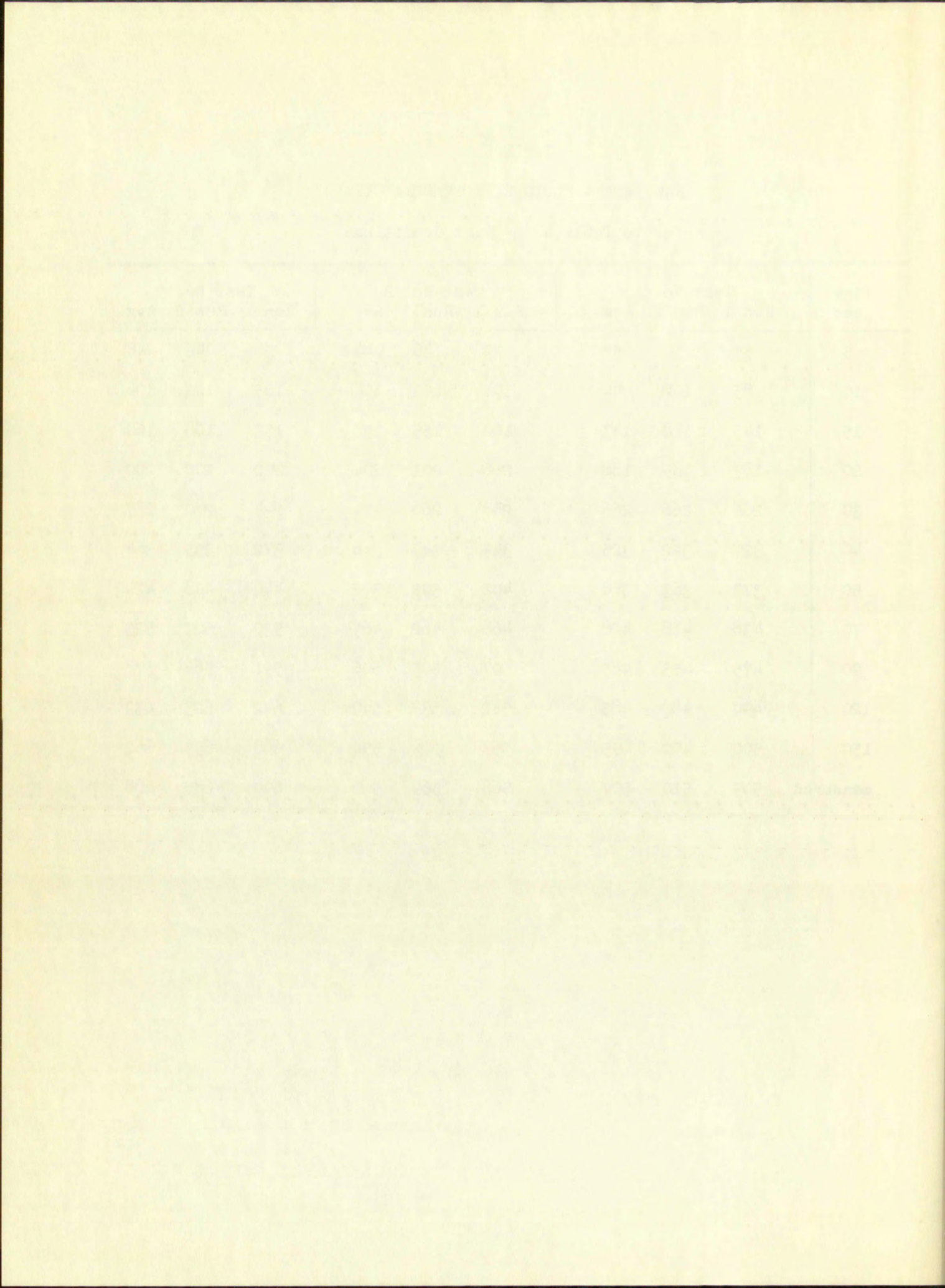


TABLE 5 (Continued)

RAW DATA - FLUID EXIT TEMPERATURE

Refer to Table 4 for Test Conditions

Time sec	Test No. 4			Test No. 5			Test No. 6		
	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg
5	25	25	25	72	73	73	58	56	57
10	47	48	48	143	144	144	120	117	119
15	68	69	69	206	206	206	172	172	172
20	80	89	84	265	264	265	218	222	220
30	122	126	124	362	360	361	287	290	289
40	148	156	152	432	437	435	350	353	352
50	168	175	172	488	488	488	396	402	399
70	194	196	195	552	564	558	455	466	461
90	210	212	211	592	600	596	488	521	505
120	224	226	225	618	632	625	-	-	-
150	228	-	-	632	642	635	-	-	-
measured	240	241	241	660	640	650	518	518	507

Date		Description		Amount	
1911	Jan 1	Balance		100.00	
1911	Jan 15	Received from A. B. C.		50.00	
1911	Feb 1	Received from D. E. F.		25.00	
1911	Mar 1	Received from G. H. I.		75.00	
1911	Apr 1	Received from J. K. L.		100.00	
1911	May 1	Received from M. N. O.		150.00	
1911	Jun 1	Received from P. Q. R.		200.00	
1911	Jul 1	Received from S. T. U.		250.00	
1911	Aug 1	Received from V. W. X.		300.00	
1911	Sep 1	Received from Y. Z. A.		350.00	
1911	Oct 1	Received from B. C. D.		400.00	
1911	Nov 1	Received from E. F. G.		450.00	
1911	Dec 1	Received from H. I. J.		500.00	
1911	Total			2000.00	

TABLE 6
RESULTS - METAL TEMPERATURES
Test No. 1 and 6 consolidated

	X = 0.425 x = 12"			X = 1.275 x = 36"			X = 2.125 x = 60"		
Dimensionless Time, T	Smoothed Temp. (°F)	External Heat Loss (watts/in)	Dimensionless Temp., M	Smoothed Temp. (°F)	External Heat Loss (watts/in)	Dimensionless Temp., M	Smoothed Temp. (°F)	External Heat Loss (watts/in)	Dimensionless Temp., M
110	36	0.2	0.156	40	0.3	0.174	43	0.3	0.187
220	81	0.6	0.356	90	0.7	0.396	98	0.7	0.432
440	161	1.4	0.725	189	1.7	0.861	210	1.9	0.963
880	244	2.4	1.14	313	3.2	1.51	368	4.0	1.82
1545	312	3.2	1.50	442	5.3	2.38	511	6.7	2.81
1990	326	3.4	1.57	466	5.8	2.47	548	7.5	3.11
2650	334	3.5	1.62	490	6.2	2.64	585	8.3	3.44
3310	338	3.6	1.65	495	6.3	2.67	594	8.5	3.53

TABLE 6 (Continued)

RESULTS - METAL TEMPERATURES

Test No. 2

Non-Dimensional Time, T	X = 0.440 x = 12"			X = 1.32 x = 36"			X = 2.20 x = 60"		
	Smoother Temp., Of	External Heat Loss, (watts/in)	Non-Dimensional Temp., M	Smoother Temp., Of	External Heat Loss, (watts/in)	Non-Dimensional Temp., M	Smoother Temp., Of	External Heat Loss, (watts/in)	Non-Dimensional Temp., M
94	35	0.2	0.13	41	0.3	0.15	47	0.3	0.18
187	82	0.6	0.31	91	0.7	0.34	99	0.7	0.37
374	180	1.6	0.70	204	1.9	0.80	220	2.0	0.87
748	287	2.9	1.17	353	3.9	1.48	394	4.5	1.70
1310	353	3.8	1.47	467	5.8	2.22	550	7.5	2.68
1680	370	4.1	1.57	500	6.4	2.33	660	8.8	3.12
2240	380	4.2	1.62	523	6.9	2.48	628	9.4	3.33
2800	382	4.2	1.63	538	7.3	2.60	645	9.8	3.48

Station No.	Date	1950		1951		1952		1953		1954	
		Time	Temp.	Time	Temp.	Time	Temp.	Time	Temp.	Time	Temp.
100	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
101	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
102	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
103	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
104	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
105	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
106	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
107	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
108	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
109	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
110	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
111	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
112	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
113	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
114	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
115	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
116	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
117	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
118	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
119	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0
120	10/10	0800	61.0	1000	61.0	1200	61.0	1400	61.0	1600	61.0

TABLE 6 (Continued)
RESULTS - METAL TEMPERATURES

Test No. 3

	X = 0.474 x = 12"			X = 1.42 x = 36"			X = 2.37 x = 60"		
Non-Dimensional Time, T	Smoothed Temp., θ_f	External Heat Loss, (watts/in)	Non-Dimensional Temp., M	Smoothed Temp., θ_f	External Heat Loss, (watts/in)	Non-Dimensional Temp., M	Smoothed Temp., θ_f	External Heat Loss, (watts/in)	Non-Dimensional Temp., M
71	37	0.2	0.10	40	0.3	0.11	44	0.3	0.13
142	77	0.5	0.22	88	0.6	0.25	98	0.7	0.28
284	157	1.3	0.46	188	1.6	0.56	210	1.9	0.63
568	269	2.6	0.82	342	3.7	1.08	387	4.3	1.26
993	360	3.9	1.15	508	6.6	1.80	600	8.7	2.34
1280	403	4.6	1.33	576	8.1	2.18	678	10.7	2.91
1700	423	4.9	1.41	617	9.1	2.45	730	13.0	3.52
2130	432	5.1	1.45	645	9.8	2.44	767	13.5	3.82

TABLE 1

Year	1960-61		1961-62		1962-63		1963-64		Total
	Area (ha)	Yield (kg/ha)	Area (ha)	Yield (kg/ha)	Area (ha)	Yield (kg/ha)	Area (ha)	Yield (kg/ha)	
1960	100	1.2	100	1.5	100	1.8	100	2.1	4.6
1961	120	1.4	120	1.7	120	2.0	120	2.3	5.4
1962	140	1.6	140	1.9	140	2.2	140	2.5	6.2
1963	160	1.8	160	2.1	160	2.4	160	2.7	7.0
1964	180	2.0	180	2.3	180	2.6	180	2.9	7.8
1965	200	2.2	200	2.5	200	2.8	200	3.1	8.6
1966	220	2.4	220	2.7	220	3.0	220	3.3	9.4
1967	240	2.6	240	2.9	240	3.2	240	3.5	10.2
1968	260	2.8	260	3.1	260	3.4	260	3.7	11.0
1969	280	3.0	280	3.3	280	3.6	280	3.9	11.8
1970	300	3.2	300	3.5	300	3.8	300	4.1	12.6
1971	320	3.4	320	3.7	320	4.0	320	4.3	13.4
1972	340	3.6	340	3.9	340	4.2	340	4.5	14.2
1973	360	3.8	360	4.1	360	4.4	360	4.7	15.0
1974	380	4.0	380	4.3	380	4.6	380	4.9	15.8
1975	400	4.2	400	4.5	400	4.8	400	5.1	16.6
1976	420	4.4	420	4.7	420	5.0	420	5.3	17.4
1977	440	4.6	440	4.9	440	5.2	440	5.5	18.2
1978	460	4.8	460	5.1	460	5.4	460	5.7	19.0
1979	480	5.0	480	5.3	480	5.6	480	5.9	19.8
1980	500	5.2	500	5.5	500	5.8	500	6.1	20.6
1981	520	5.4	520	5.7	520	6.0	520	6.3	21.4
1982	540	5.6	540	5.9	540	6.2	540	6.5	22.2
1983	560	5.8	560	6.1	560	6.4	560	6.7	23.0
1984	580	6.0	580	6.3	580	6.6	580	6.9	23.8
1985	600	6.2	600	6.5	600	6.8	600	7.1	24.6
1986	620	6.4	620	6.7	620	7.0	620	7.3	25.4
1987	640	6.6	640	6.9	640	7.2	640	7.5	26.2
1988	660	6.8	660	7.1	660	7.4	660	7.7	27.0
1989	680	7.0	680	7.3	680	7.6	680	7.9	27.8
1990	700	7.2	700	7.5	700	7.8	700	8.1	28.6
1991	720	7.4	720	7.7	720	8.0	720	8.3	29.4
1992	740	7.6	740	7.9	740	8.2	740	8.5	30.2
1993	760	7.8	760	8.1	760	8.4	760	8.7	31.0
1994	780	8.0	780	8.3	780	8.6	780	8.9	31.8
1995	800	8.2	800	8.5	800	8.8	800	9.1	32.6
1996	820	8.4	820	8.7	820	9.0	820	9.3	33.4
1997	840	8.6	840	8.9	840	9.2	840	9.5	34.2
1998	860	8.8	860	9.1	860	9.4	860	9.7	35.0
1999	880	9.0	880	9.3	880	9.6	880	9.9	35.8
2000	900	9.2	900	9.5	900	9.8	900	10.1	36.6
2001	920	9.4	920	9.7	920	10.0	920	10.3	37.4
2002	940	9.6	940	9.9	940	10.2	940	10.5	38.2
2003	960	9.8	960	10.1	960	10.4	960	10.7	39.0
2004	980	10.0	980	10.3	980	10.6	980	10.9	39.8
2005	1000	10.2	1000	10.5	1000	10.8	1000	11.1	40.6
2006	1020	10.4	1020	10.7	1020	11.0	1020	11.3	41.4
2007	1040	10.6	1040	10.9	1040	11.2	1040	11.5	42.2
2008	1060	10.8	1060	11.1	1060	11.4	1060	11.7	43.0
2009	1080	11.0	1080	11.3	1080	11.6	1080	11.9	43.8
2010	1100	11.2	1100	11.5	1100	11.8	1100	12.1	44.6
2011	1120	11.4	1120	11.7	1120	12.0	1120	12.3	45.4
2012	1140	11.6	1140	11.9	1140	12.2	1140	12.5	46.2
2013	1160	11.8	1160	12.1	1160	12.4	1160	12.7	47.0
2014	1180	12.0	1180	12.3	1180	12.6	1180	12.9	47.8
2015	1200	12.2	1200	12.5	1200	12.8	1200	13.1	48.6
2016	1220	12.4	1220	12.7	1220	13.0	1220	13.3	49.4
2017	1240	12.6	1240	12.9	1240	13.2	1240	13.5	50.2
2018	1260	12.8	1260	13.1	1260	13.4	1260	13.7	51.0
2019	1280	13.0	1280	13.3	1280	13.6	1280	13.9	51.8
2020	1300	13.2	1300	13.5	1300	13.8	1300	14.1	52.6
2021	1320	13.4	1320	13.7	1320	14.0	1320	14.3	53.4
2022	1340	13.6	1340	13.9	1340	14.2	1340	14.5	54.2
2023	1360	13.8	1360	14.1	1360	14.4	1360	14.7	55.0
2024	1380	14.0	1380	14.3	1380	14.6	1380	14.9	55.8
2025	1400	14.2	1400	14.5	1400	14.8	1400	15.1	56.6
2026	1420	14.4	1420	14.7	1420	15.0	1420	15.3	57.4
2027	1440	14.6	1440	14.9	1440	15.2	1440	15.5	58.2
2028	1460	14.8	1460	15.1	1460	15.4	1460	15.7	59.0
2029	1480	15.0	1480	15.3	1480	15.6	1480	15.9	59.8
2030	1500	15.2	1500	15.5	1500	15.8	1500	16.1	60.6
2031	1520	15.4	1520	15.7	1520	16.0	1520	16.3	61.4
2032	1540	15.6	1540	15.9	1540	16.2	1540	16.5	62.2
2033	1560	15.8	1560	16.1	1560	16.4	1560	16.7	63.0
2034	1580	16.0	1580	16.3	1580	16.6	1580	16.9	63.8
2035	1600	16.2	1600	16.5	1600	16.8	1600	17.1	64.6
2036	1620	16.4	1620	16.7	1620	17.0	1620	17.3	65.4
2037	1640	16.6	1640	16.9	1640	17.2	1640	17.5	66.2
2038	1660	16.8	1660	17.1	1660	17.4	1660	17.7	67.0
2039	1680	17.0	1680	17.3	1680	17.6	1680	17.9	67.8
2040	1700	17.2	1700	17.5	1700	17.8	1700	18.1	68.6
2041	1720	17.4	1720	17.7	1720	18.0	1720	18.3	69.4
2042	1740	17.6	1740	17.9	1740	18.2	1740	18.5	70.2
2043	1760	17.8	1760	18.1	1760	18.4	1760	18.7	71.0
2044	1780	18.0	1780	18.3	1780	18.6	1780	18.9	71.8
2045	1800	18.2	1800	18.5	1800	18.8	1800	19.1	72.6
2046	1820	18.4	1820	18.7	1820	19.0	1820	19.3	73.4
2047	1840	18.6	1840	18.9	1840	19.2	1840	19.5	74.2
2048	1860	18.8	1860	19.1	1860	19.4	1860	19.7	75.0
2049	1880	19.0	1880	19.3	1880	19.6	1880	19.9	75.8
2050	1900	19.2	1900	19.5	1900	19.8	1900	20.1	76.6
2051	1920	19.4	1920	19.7	1920	20.0	1920	20.3	77.4
2052	1940	19.6	1940	19.9	1940	20.2	1940	20.5	78.2
2053	1960	19.8	1960	20.1	1960	20.4	1960	20.7	79.0
2054	1980	20.0	1980	20.3	1980	20.6	1980	20.9	79.8
2055	2000	20.2	2000	20.5	2000	20.8	2000	21.1	80.6
2056	2020	20.4	2020	20.7	2020	21.0	2020	21.3	81.4
2057	2040	20.6	2040	20.9	2040	21.2	2040	21.5	82.2
2058	2060	20.8	2060	21.1	2060	21.4	2060	21.7	83.0
2059	2080	21.0	2080	21.3	2080	21.6	2080	21.9	83.8
2060	2100	21.2	2100	21.5	2100	21.8	2100	22.1	84.6
2061	2120	21.4	2120	21.7	2120	22.0	2120	22.3	85.4
2062	2140	21.6	2140	21.9	2140	22.2	2140	22.5	86.2
2063	2160	21.8	2160	22.1	2160	22.4	2160	22.7	87.0
2064	2180	22.0	2180	22.3	2180	22.6	2180	22.9	87.8
2065	2200	22.2	2200	22.5	2200	22.8	2200	23.1	88.6
2066	2220	22.4	2220	22.7	2220	23.0	2220	23.3	89.4
2067	2240	22.6	2240	22.9	2240	23.2	2240	23.5	90.2
2068	2260	22.8	2260	23.1	2260	23.4	2260	23.7	91.0
2069	2280	23.0	2280	23.3	2280	23.6	2280	23.9	91.8
2070	2300	23.2	2300	23.5	2300	23.8	2300	24.1	92.6
2071	2320	23.4	2320	23.7	2320	24.0	2320	24.3	93.4
2072	2340	23.6	2340	23.9	2340	24.2	2340	24.5	94.2
2073	2360	23.8	2360	24.1	2360	24.4	2360	24.7	95.0
2074	2380	24.0	2380	24.3	2380	24.6	2380	24.9	95.8
2075	2400	24.2	2400	24.5	2400	24.8	2400	25.1	96.6
2076	2420	24.4	2420	24.7	2420	25.0	2420	25.3	97.4
2077	2440	24.6	2440	24.9	2440	25.2	2440	25.5	98.2
2078	2460	24.8	2460	25.1	2460	25.4	2460	25.7	99.0
2079	2480	25.0	2480	25.3	2480	25.6	2480	25.9	99.8
2080	2500	25.2	2500	25.5	2500	25.8	2500	26.1	100.6
2081	2520	25.4	2520	25.7	2520	26.0	2520	26.3	101.4
2082	2540	25.6	2540	25.9	2540	26.2	2540	26.5	102.2
2083	2560	25.8	2560	26.1	2560	26.4	2560	26.7	103.0
2084	2580	26.0	2580	26.3	2580	26.6	2580	26.9	103.8
2085	2600	26.2	2600	26.5	2600	26.8	2600	27.1	104.6
2086	2620	26.4	2620	26.7	2620	27.0	2620	27.3	105.4
2087	2640	26.6	2640	26.9	2640	27.2	2640	27.5	106.2
2088	2660	26.8	2660	27.1	2660	27.4	2660	27.7	107.0
2089</									

TABLE 6 (Continued)
RESULTS - METAL TEMPERATURES

Test No. 4

	X = 0.425 x = 12"			X = 1.275 x = 36"			X = 2.125 x = 60"		
Dimensionless Time, T	Smoother Temp. (°F)	External Heat Loss (watts/in)	Dimensionless Temp., M	Smoother Temp. (°F)	External Heat Loss (watts/in)	Dimensionless Temp., M	Smoother Temp. (°F)	External Heat Loss (watts/in)	Dimensionless Temp., M
110	20	0.1	0.19	25	0.1	0.24	30	0.2	0.29
220	36	0.2	0.35	46	0.3	0.45	51	0.4	0.68
440	70	0.5	0.70	89	0.6	0.89	105	0.7	1.06
880	112	0.8	1.13	156	1.3	1.64	189	1.7	2.08
1545	140	1.2	1.46	204	1.8	2.24	252	2.4	2.91
1990	150	1.2	1.57	219	2.0	2.45	270	2.6	3.18
2650	153	1.3	1.61	229	2.1	2.58	282	2.8	3.38
3310	154	1.3	1.62	235	2.2	2.67	287	2.9	3.47

REPORT

ON THE

STATE OF

NAME	AGE	SEX	RELATION	EDUCATION	RELIGION	INDUSTRY	RESIDENCE	DATE
JOHN	25	M	S	HIGH SCHOOL	METHODIST	TEACHER	NEW YORK	1900
MARY	22	F	D	HIGH SCHOOL	CATHOLIC	SEWING	NEW YORK	1900
JOHN	20	M	S	HIGH SCHOOL	METHODIST	TEACHER	NEW YORK	1900
MARY	18	F	D	HIGH SCHOOL	CATHOLIC	SEWING	NEW YORK	1900
JOHN	15	M	S	HIGH SCHOOL	METHODIST	TEACHER	NEW YORK	1900
MARY	12	F	D	HIGH SCHOOL	CATHOLIC	SEWING	NEW YORK	1900
JOHN	10	M	S	HIGH SCHOOL	METHODIST	TEACHER	NEW YORK	1900
MARY	8	F	D	HIGH SCHOOL	CATHOLIC	SEWING	NEW YORK	1900
JOHN	5	M	S	HIGH SCHOOL	METHODIST	TEACHER	NEW YORK	1900
MARY	3	F	D	HIGH SCHOOL	CATHOLIC	SEWING	NEW YORK	1900

1900	1901	1902	1903	1904	1905	1906	1907	1908	1909	1910	1911	1912	1913	1914	1915	1916	1917	1918	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939	1940	1941	1942	1943	1944	1945	1946	1947	1948	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033	2034	2035	2036	2037	2038	2039	2040	2041	2042	2043	2044	2045	2046	2047	2048	2049	2050	2051	2052	2053	2054	2055	2056	2057	2058	2059	2060	2061	2062	2063	2064	2065	2066	2067	2068	2069	2070	2071	2072	2073	2074	2075	2076	2077	2078	2079	2080	2081	2082	2083	2084	2085	2086	2087	2088	2089	2090	2091	2092	2093	2094	2095	2096	2097	2098	2099	2100	2101	2102	2103	2104	2105	2106	2107	2108	2109	2110	2111	2112	2113	2114	2115	2116	2117	2118	2119	2120	2121	2122	2123	2124	2125	2126	2127	2128	2129	2130	2131	2132	2133	2134	2135	2136	2137	2138	2139	2140	2141	2142	2143	2144	2145	2146	2147	2148	2149	2150	2151	2152	2153	2154	2155	2156	2157	2158	2159	2160	2161	2162	2163	2164	2165	2166	2167	2168	2169	2170	2171	2172	2173	2174	2175	2176	2177	2178	2179	2180	2181	2182	2183	2184	2185	2186	2187	2188	2189	2190	2191	2192	2193	2194	2195	2196	2197	2198	2199	2200	2201	2202	2203	2204	2205	2206	2207	2208	2209	2210	2211	2212	2213	2214	2215	2216	2217	2218	2219	2220	2221	2222	2223	2224	2225	2226	2227	2228	2229	2230	2231	2232	2233	2234	2235	2236	2237	2238	2239	2240	2241	2242	2243	2244	2245	2246	2247	2248	2249	2250	2251	2252	2253	2254	2255	2256	2257	2258	2259	2260	2261	2262	2263	2264	2265	2266	2267	2268	2269	2270	2271	2272	2273	2274	2275	2276	2277	2278	2279	2280	2281	2282	2283	2284	2285	2286	2287	2288	2289	2290	2291	2292	2293	2294	2295	2296	2297	2298	2299	2300	2301	2302	2303	2304	2305	2306	2307	2308	2309	2310	2311	2312	2313	2314	2315	2316	2317	2318	2319	2320	2321	2322	2323	2324	2325	2326	2327	2328	2329	2330	2331	2332	2333	2334	2335	2336	2337	2338	2339	2340	2341	2342	2343	2344	2345	2346	2347	2348	2349	2350	2351	2352	2353	2354	2355	2356	2357	2358	2359	2360	2361	2362	2363	2364	2365	2366	2367	2368	2369	2370	2371	2372	2373	2374	2375	2376	2377	2378	2379	2380	2381	2382	2383	2384	2385	2386	2387	2388	2389	2390	2391	2392	2393	2394	2395	2396	2397	2398	2399	2400	2401	2402	2403	2404	2405	2406	2407	2408	2409	2410	2411	2412	2413	2414	2415	2416	2417	2418	2419	2420	2421	2422	2423	2424	2425	2426	2427	2428	2429	2430	2431	2432	2433	2434	2435	2436	2437	2438	2439	2440	2441	2442	2443	2444	2445	2446	2447	2448	2449	2450	2451	2452	2453	2454	2455	2456	2457	2458	2459	2460	2461	2462	2463	2464	2465	2466	2467	2468	2469	2470	2471	2472	2473	2474	2475	2476	2477	2478	2479	2480	2481	2482	2483	2484	2485	2486	2487	2488	2489	2490	2491	2492	2493	2494	2495	2496	2497	2498	2499	2500	2501	2502	2503	2504	2505	2506	2507	2508	2509	2510	2511	2512	2513	2514	2515	2516	2517	2518	2519	2520	2521	2522	2523	2524	2525	2526	2527	2528	2529	2530	2531	2532	2533	2534	2535	2536	2537	2538	2539	2540	2541	2542	2543	2544	2545	2546	2547	2548	2549	2550	2551	2552	2553	2554	2555	2556	2557	2558	2559	2560	2561	2562	2563	2564	2565	2566	2567	2568	2569	2570	2571	2572	2573	2574	2575	2576	2577	2578	2579	2580	2581	2582	2583	2584	2585	2586	2587	2588	2589	2590	2591	2592	2593	2594	2595	2596	2597	2598	2599	2600	2601	2602	2603	2604	2605	2606	2607	2608	2609	2610	2611	2612	2613	2614	2615	2616	2617	2618	2619	2620	2621	2622	2623	2624	2625	2626	2627	2628	2629	2630	2631	2632	2633	2634	2635	2636	2637	2638	2639	2640	2641	2642	2643	2644	2645	2646	2647	2648	2649	2650	2651	2652	2653	2654	2655	2656	2657	2658	2659	2660	2661	2662	2663	2664	2665	2666	2667	2668	2669	2670	2671	2672	2673	2674	2675	2676	2677	2678	2679	2680	2681	2682	2683	2684	2685	2686	2687	2688	2689	2690	2691	2692	2693	2694	2695	2696	2697	2698	2699	2700	2701	2702	2703	2704	2705	2706	2707	2708	2709	2710	2711	2712	2713	2714	2715	2716	2717	2718	2719	2720	2721	2722	2723	2724	2725	2726	2727	2728	2729	2730	2731	2732	2733	2734	2735	2736	2737	2738	2739	2740	2741	2742	2743	2744	2745	2746	2747	2748	2749	2750	2751	2752	2753	2754	2755	2756	2757	2758	2759	2760	2761	2762	2763	2764	2765	2766	2767	2768	2769	2770	2771	2772	2773	2774	2775	2776	2777	2778	2779	2780	2781	2782	2783	2784	2785	2786	2787	2788	2789	2790	2791	2792	2793	2794	2795	2796	2797	2798	2799	2800	2801	2802	2803	2804	2805	2806	2807	2808	2809	2810	2811	2812	2813	2814	2815	2816	2817	2818	2819	2820	2821	2822	2823	2824	2825	2826	2827	2828	2829	2830	2831	2832	2833	2834	2835	2836	2837	2838	2839	2840	2841	2842	2843	2844	2845	2846	2847	2848	2849	2850	2851	2852	2853	2854	2855	2856	2857	2858	2859	2860	2861	2862	2863	2864	2865	2866	2867	2868	2869	2870	2871	2872	2873	2874	2875	2876	2877	2878	2879	2880	2881	2882	2883	2884	2885	2886	2887	2888	2889	2890	2891	2892	2893	2894	2895	2896	2897	2898	2899	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TABLE 6 (Continued)
RESULTS - METAL TEMPERATURES

Test No. 5

	X = 0.425 x = 12"			X = 1.275 x = 36"			X = 2.125 x = 60"		
Dimensionless Time, T	Smoothed Temp., $^{\circ}\text{F}$	External Heat Loss (watts/in)	Dimensionless Temp., M	Smoothed Temp., ($^{\circ}\text{F}$)	External Heat Loss (watts/in)	Dimensionless Temp., M	Smoothed Temp. ($^{\circ}\text{F}$)	External Heat Loss, (watts/in)	Dimensionless Temp., M
110	42	0.3	0.14	45	0.3	0.15	48	0.3	0.15
220	97	0.7	0.32	119	0.9	0.39	130	1.1	0.43
440	188	1.6	0.63	229	2.2	0.77	255	2.5	0.87
880	318	3.3	1.10	420	4.9	1.52	472	5.9	1.76
1545	388	4.4	1.39	535	7.2	2.07	635	9.5	2.63
1990	407	4.7	1.47	575	8.1	2.28	696	10.8	3.00
2650	415	4.8	1.50	593	8.5	2.38	715	12.0	3.21
3310	421	4.9	1.53	606	8.8	2.45	732	12.5	3.35

Table 1

Summary of data for the first part of the study

Time (min)	Temperature (°C)	Pressure (atm)	Flow rate (L/min)	Concentration (g/L)	Yield (%)	Purity (%)	Recovery (%)
110	25	0.1	1.0	0.5	95	98	95
220	30	0.2	1.5	0.8	92	95	92
330	35	0.3	2.0	1.2	90	93	90
440	40	0.4	2.5	1.5	88	91	88
550	45	0.5	3.0	1.8	85	88	85
660	50	0.6	3.5	2.2	82	85	82
770	55	0.7	4.0	2.5	80	83	80
880	60	0.8	4.5	2.8	78	81	78
990	65	0.9	5.0	3.2	75	78	75
1100	70	1.0	5.5	3.5	72	75	72

TABLE 7

RESULTS - FLUID TEMPERATURES

Average data from Test 1 and 6

Dimensionless Time, T	Averaged Temp. ($^{\circ}$ F)	External Heat Loss (watts/in)	Dimensionless Temp., F
110	55	0.3	0.238
220	107	0.8	0.473
330	159	1.3	0.715
440	204	2.0	0.94
660	278	3.0	1.32
880	339	4.3	1.69
1100	384	5.4	2.00
1540	445	7.0	2.50
1980	485	8.0	2.82
2640	508	8.8	3.05
measured	513	9.0	3.11

TABLE 1
 RESULTS OF FIELD INVESTIGATION
 Average Data from Tests 1 and 2

Classification Type 1 Temp. °F	Average Temp. °F	Internal Heat Loss (watts/m ²)	Classification Type 2 Temp. °F
111	33	0.5	0.500
122	37	0.6	0.600
133	41	0.7	0.700
144	45	0.8	0.800
155	49	0.9	0.900
166	53	1.0	1.000
177	57	1.1	1.100
188	61	1.2	1.200
199	65	1.3	1.300
200	69	1.4	1.400
211	73	1.5	1.500
222	77	1.6	1.600
233	81	1.7	1.700
244	85	1.8	1.800
255	89	1.9	1.900
266	93	2.0	2.000
277	97	2.1	2.100
288	101	2.2	2.200
299	105	2.3	2.300
300	109	2.4	2.400
311	113	2.5	2.500
322	117	2.6	2.600
333	121	2.7	2.700
344	125	2.8	2.800
355	129	2.9	2.900
366	133	3.0	3.000
377	137	3.1	3.100
388	141	3.2	3.200
399	145	3.3	3.300
400	149	3.4	3.400
411	153	3.5	3.500
422	157	3.6	3.600
433	161	3.7	3.700
444	165	3.8	3.800
455	169	3.9	3.900
466	173	4.0	4.000
477	177	4.1	4.100
488	181	4.2	4.200
499	185	4.3	4.300
500	189	4.4	4.400
511	193	4.5	4.500
522	197	4.6	4.600
533	201	4.7	4.700
544	205	4.8	4.800
555	209	4.9	4.900
566	213	5.0	5.000
577	217	5.1	5.100
588	221	5.2	5.200
599	225	5.3	5.300
600	229	5.4	5.400
611	233	5.5	5.500
622	237	5.6	5.600
633	241	5.7	5.700
644	245	5.8	5.800
655	249	5.9	5.900
666	253	6.0	6.000
677	257	6.1	6.100
688	261	6.2	6.200
699	265	6.3	6.300
700	269	6.4	6.400
711	273	6.5	6.500
722	277	6.6	6.600
733	281	6.7	6.700
744	285	6.8	6.800
755	289	6.9	6.900
766	293	7.0	7.000
777	297	7.1	7.100
788	301	7.2	7.200
799	305	7.3	7.300
800	309	7.4	7.400
811	313	7.5	7.500
822	317	7.6	7.600
833	321	7.7	7.700
844	325	7.8	7.800
855	329	7.9	7.900
866	333	8.0	8.000
877	337	8.1	8.100
888	341	8.2	8.200
899	345	8.3	8.300
900	349	8.4	8.400
911	353	8.5	8.500
922	357	8.6	8.600
933	361	8.7	8.700
944	365	8.8	8.800
955	369	8.9	8.900
966	373	9.0	9.000
977	377	9.1	9.100
988	381	9.2	9.200
999	385	9.3	9.300
1000	389	9.4	9.400

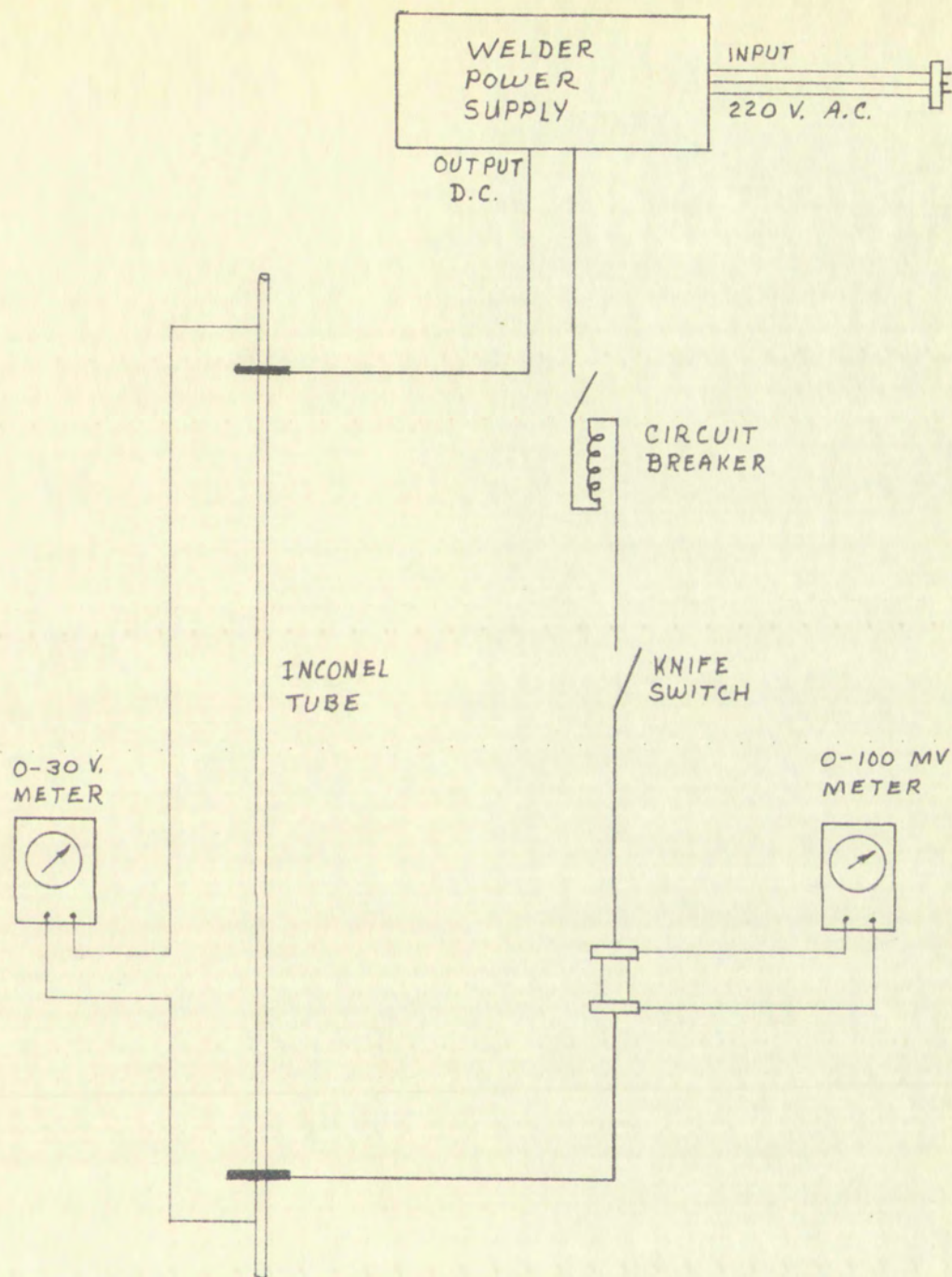


FIGURE 14 ELECTRICAL POWER CIRCUIT

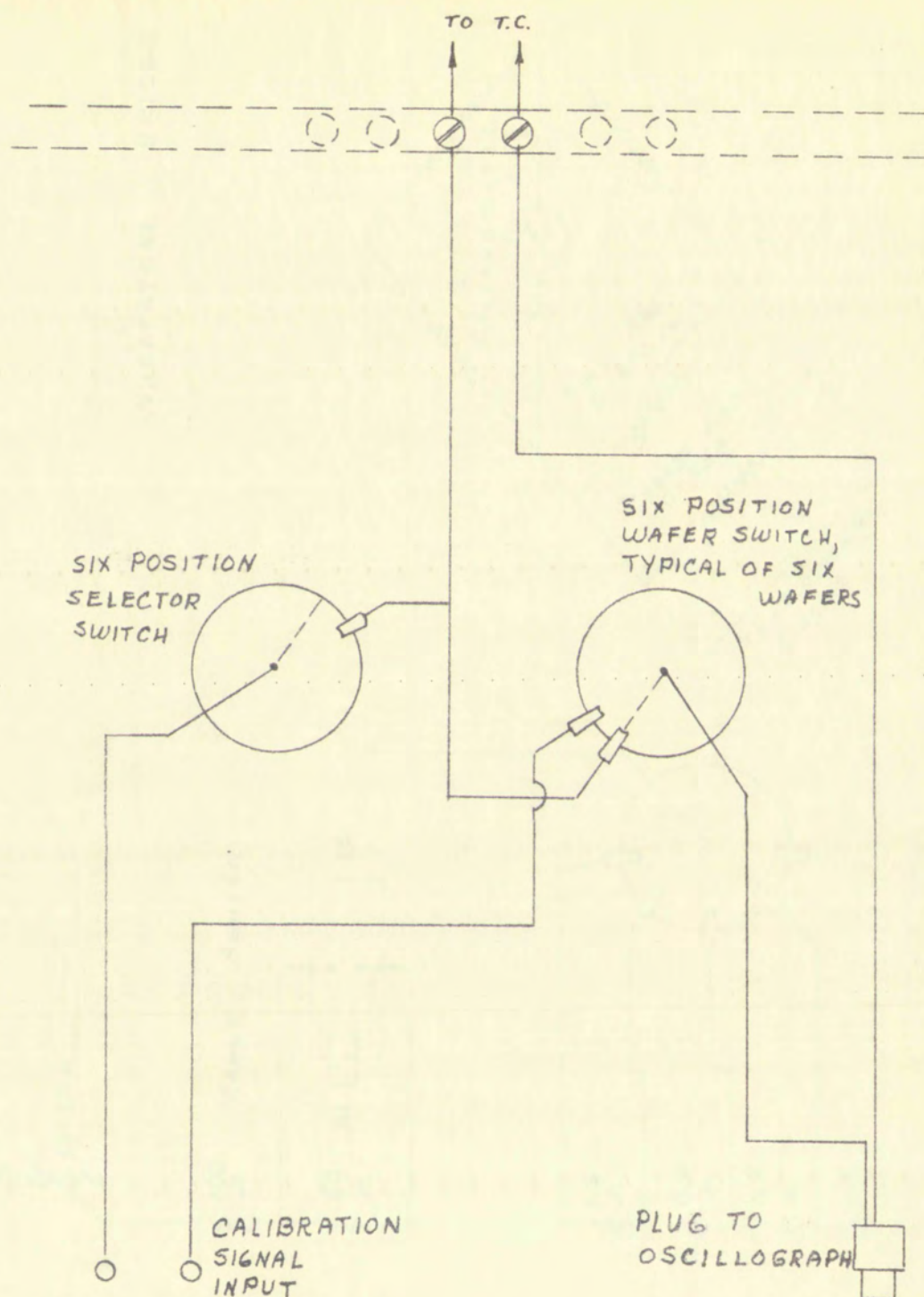


FIGURE 15 TYPICAL PORTION OF T.C. SWITCHING CIRCUIT



FIGURE 1
 Comparison of the
 results of the
 two methods

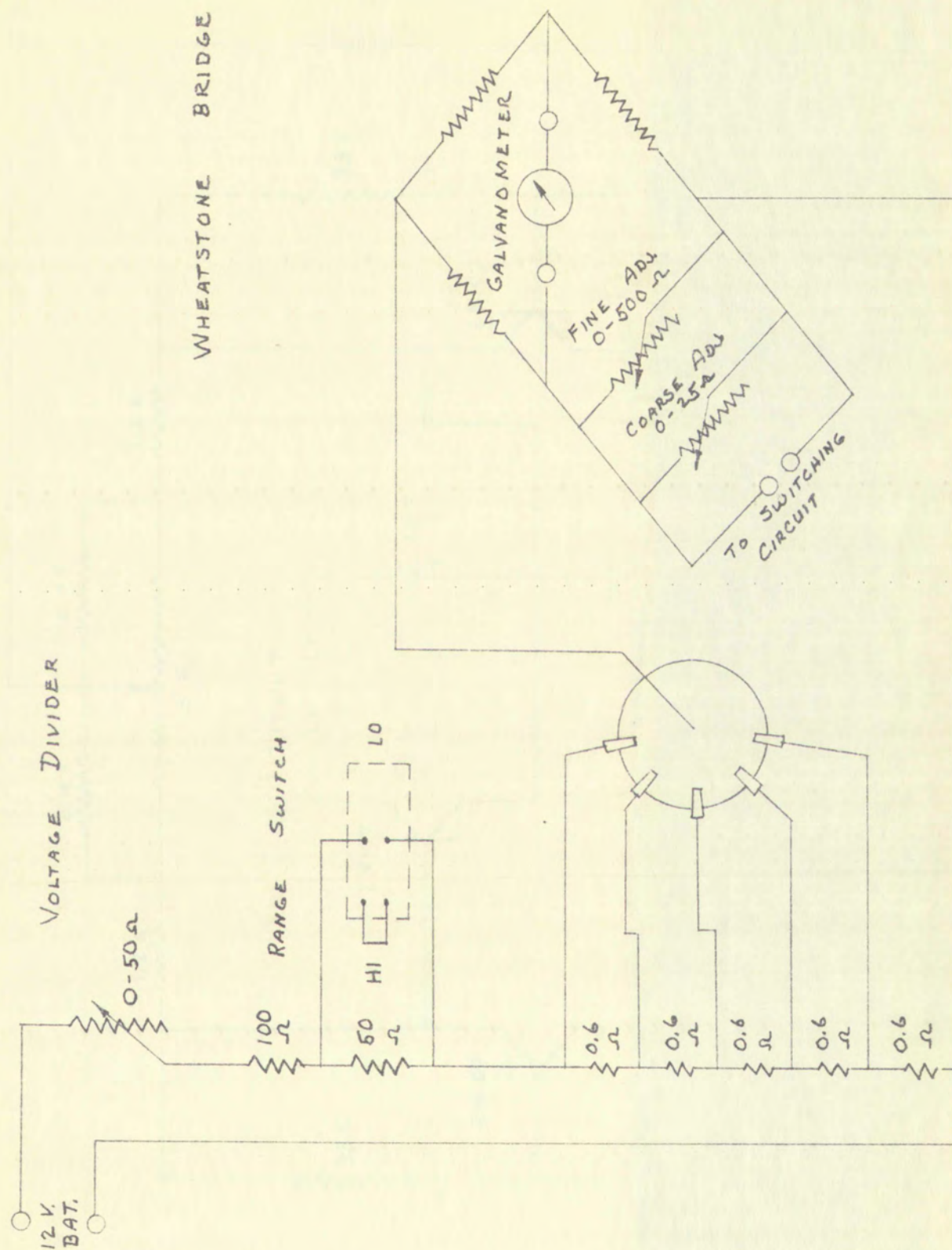
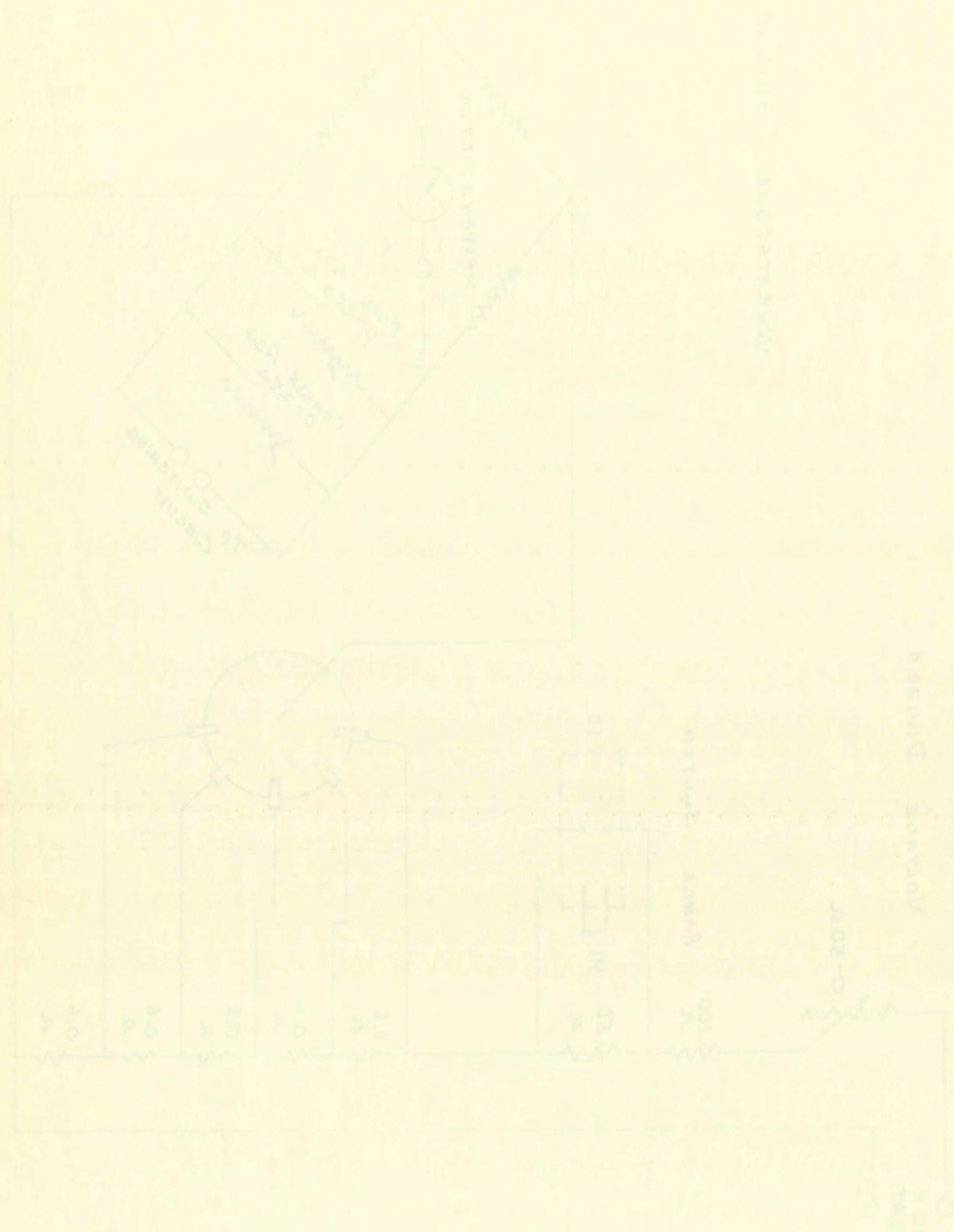


FIGURE 16 CALIBRATION CIRCUIT SCHEMATIC

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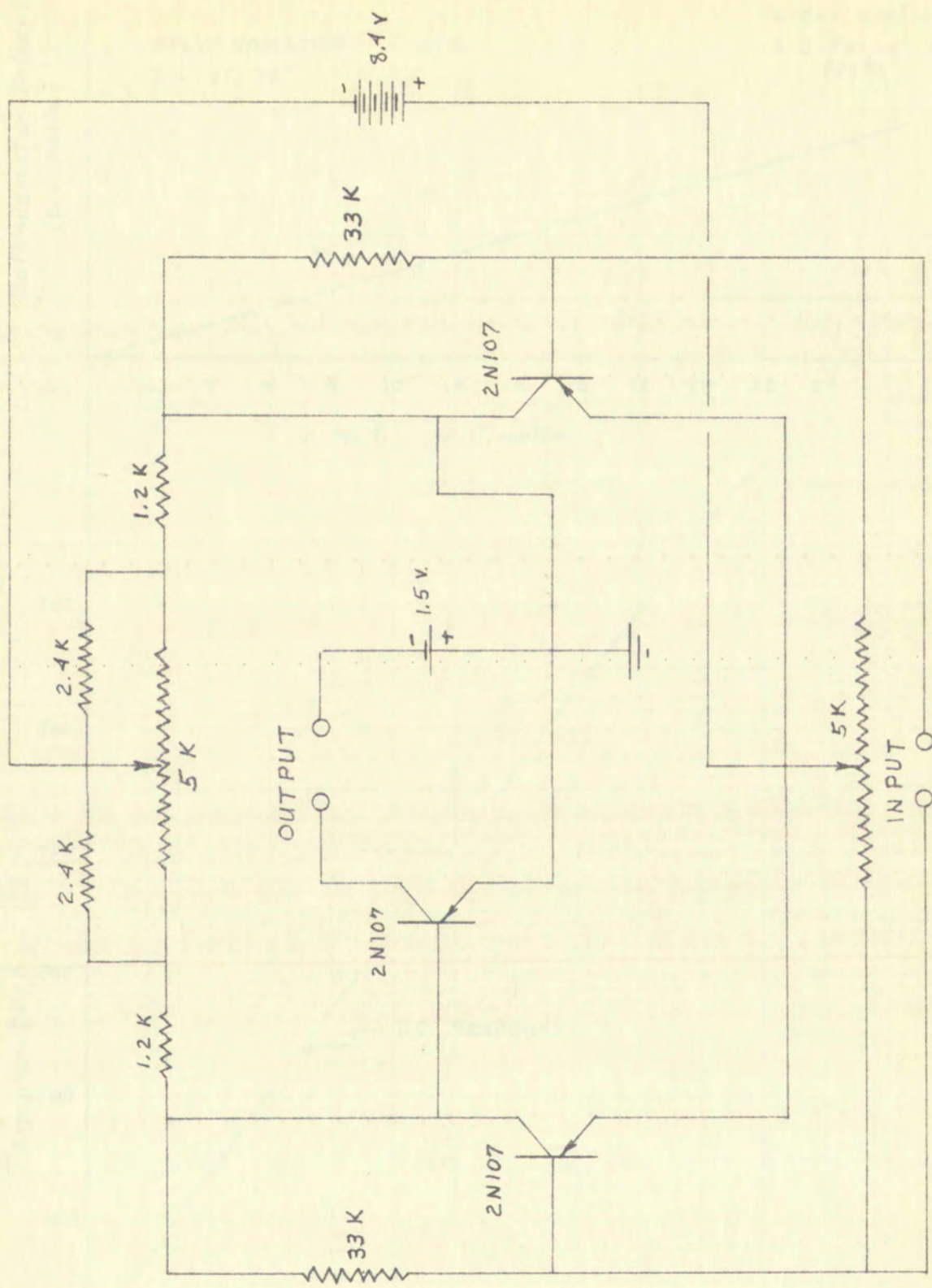
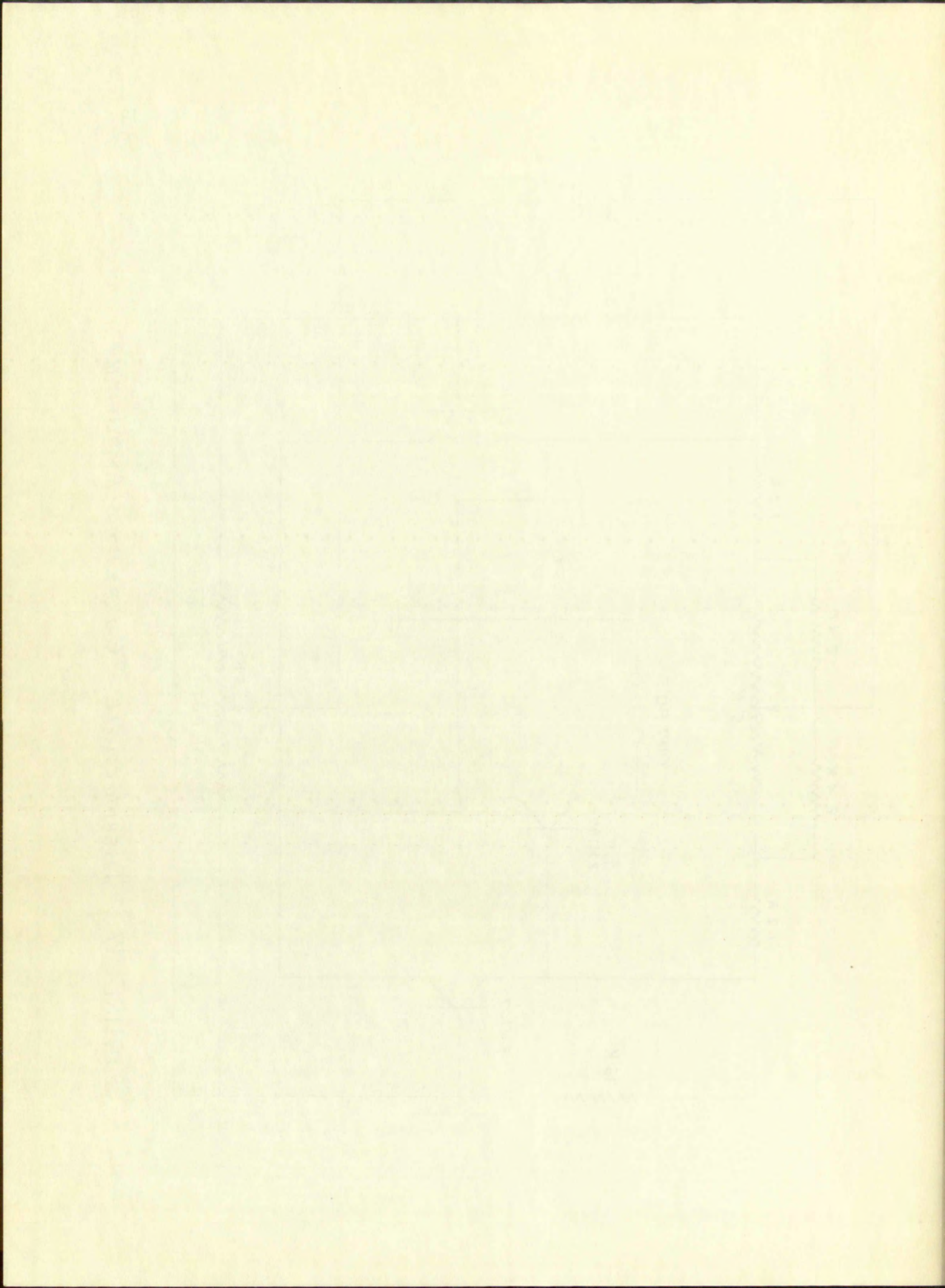


FIGURE 17 THERMOCOUPLE CURRENT AMPLIFIER



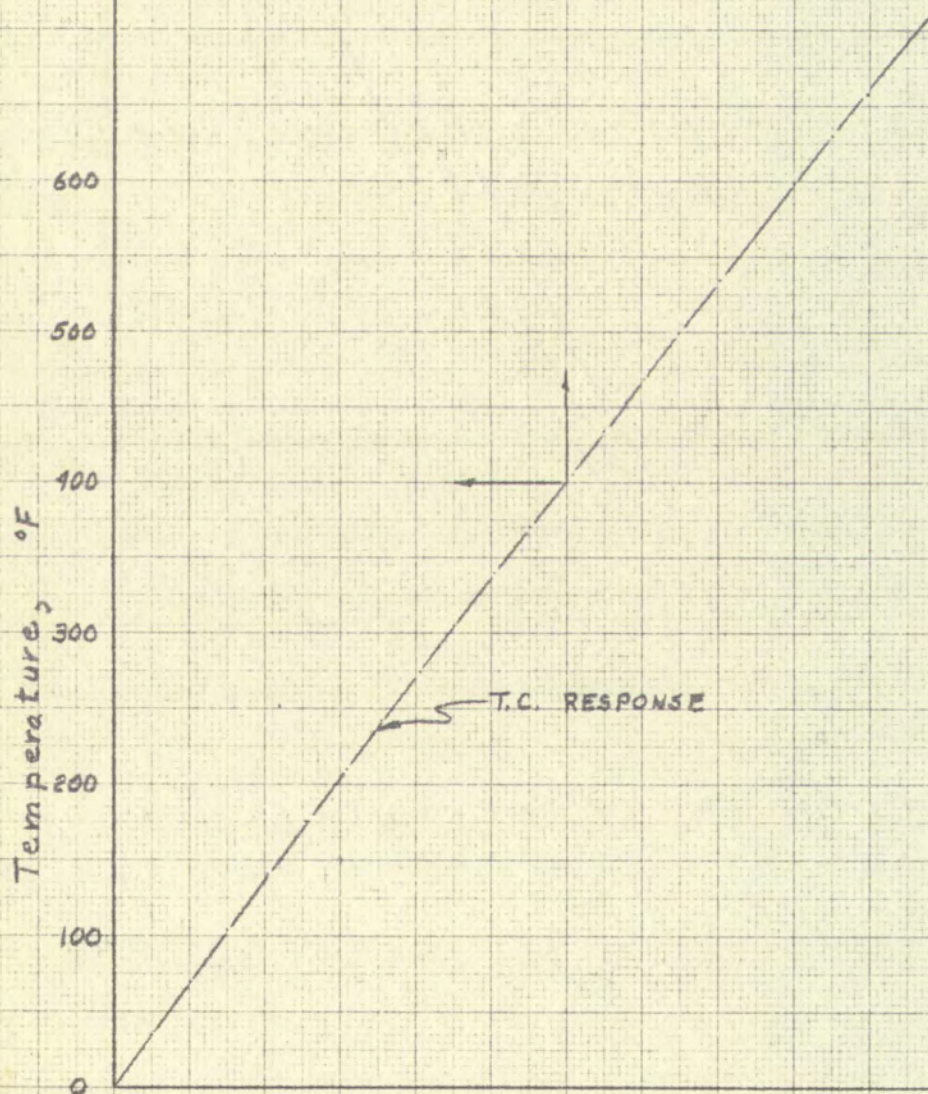
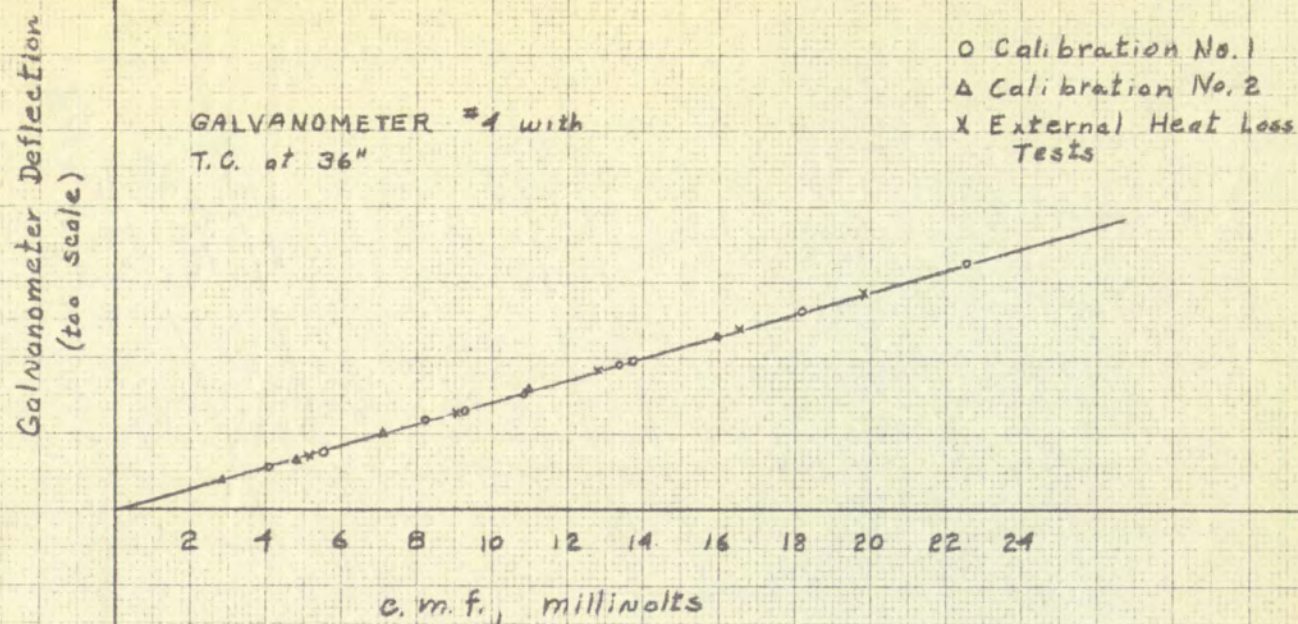


FIGURE 18 TYPICAL GALVANOMETER CALIBRATION
AND THERMOCOUPLE e.m.f.

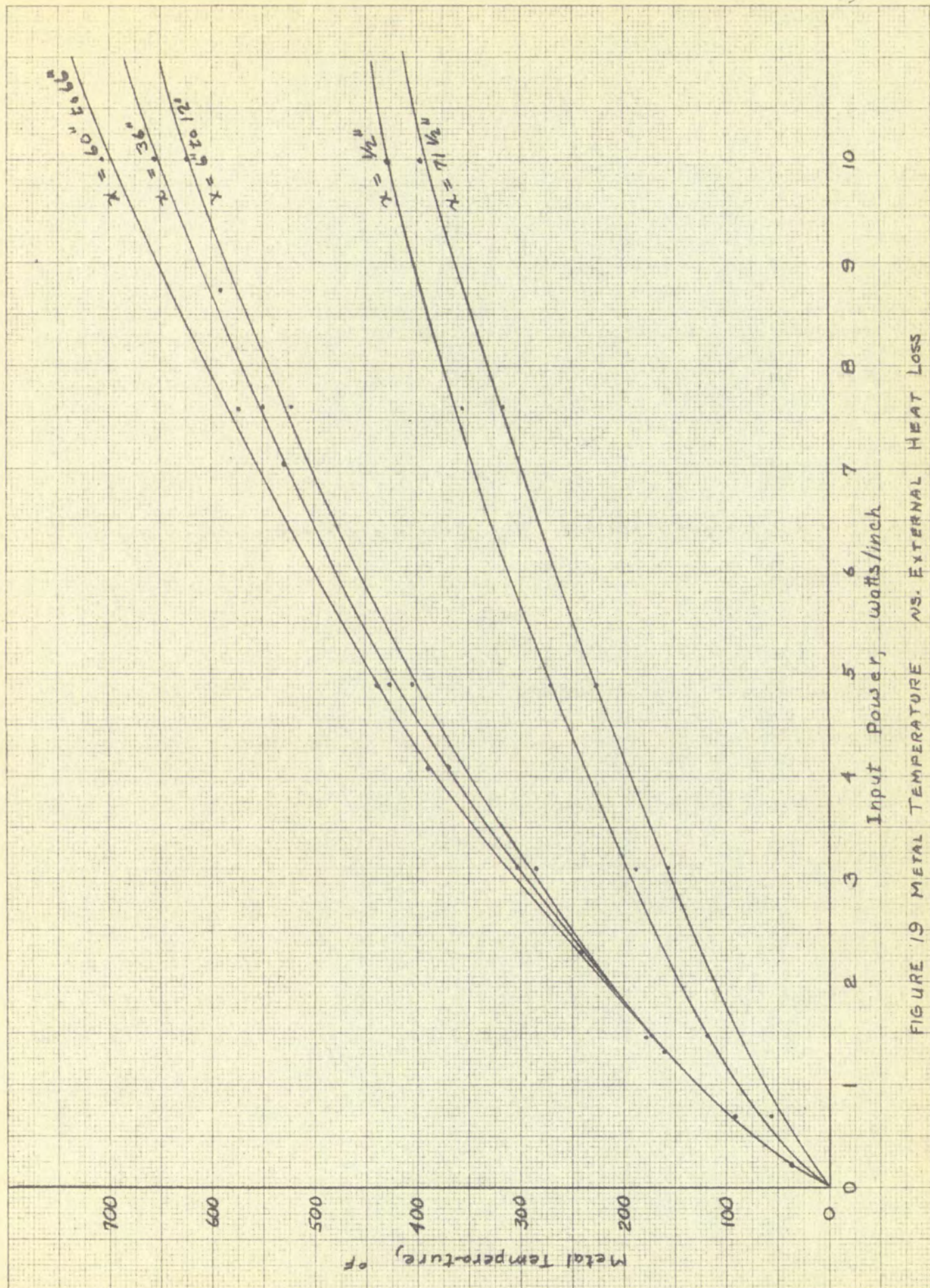


FIGURE 19 METAL TEMPERATURE VS. EXTERNAL HEAT LOSS



800

700

600

500

400

TEMPERATURE °F

300

200

100

0

X=0
X=0

12" 0.44

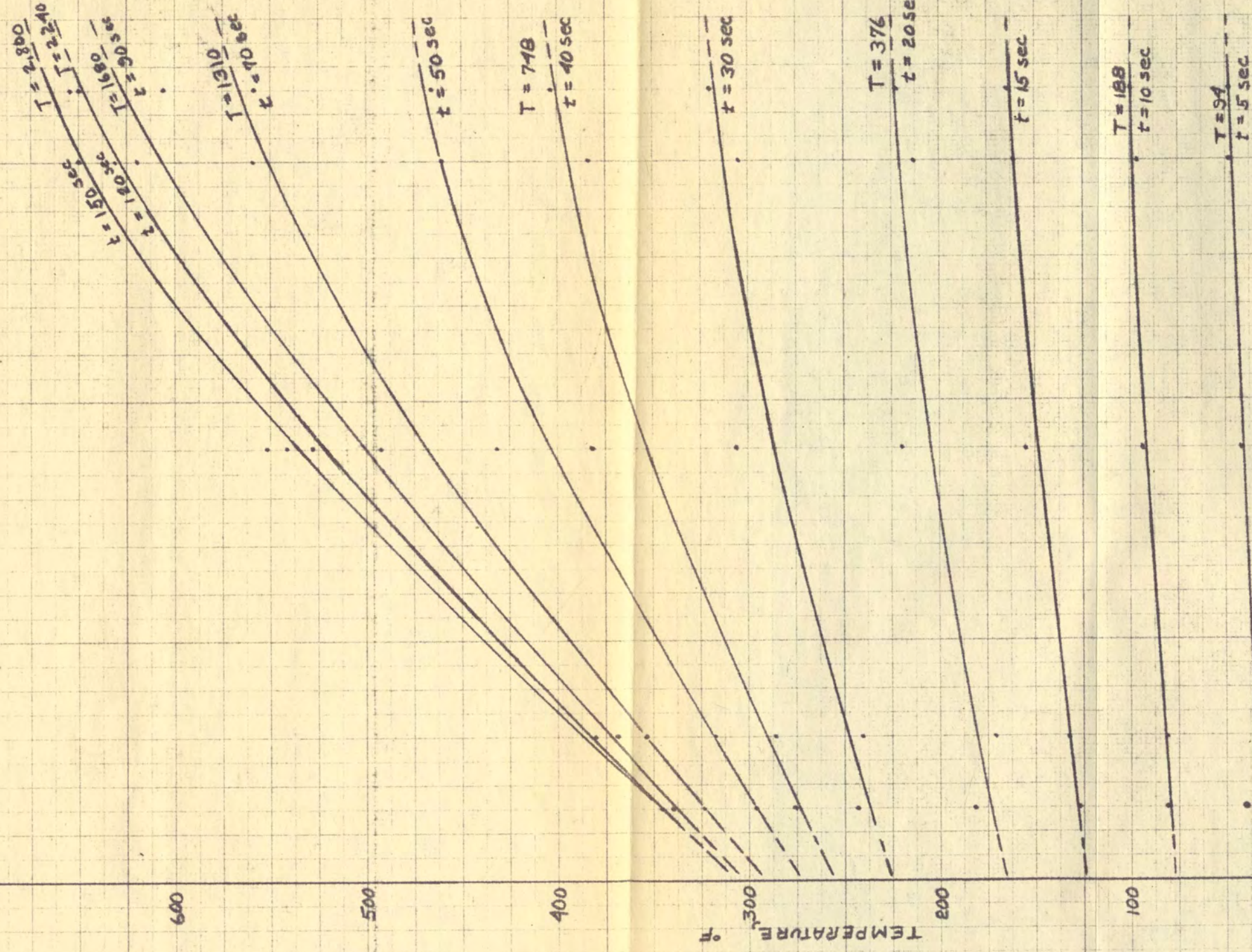
24"

36" 1.32

48"

60" 2.20

72"

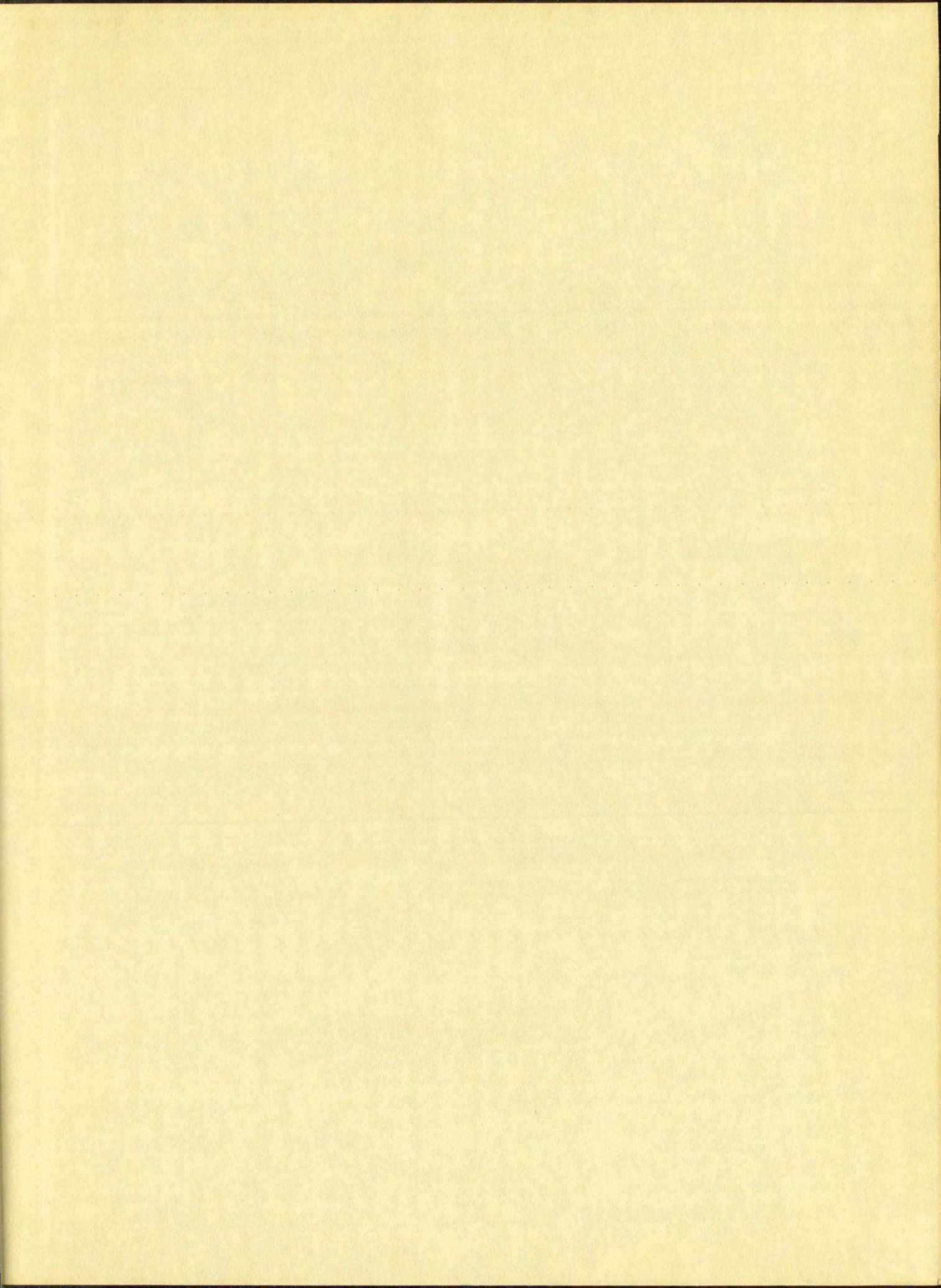


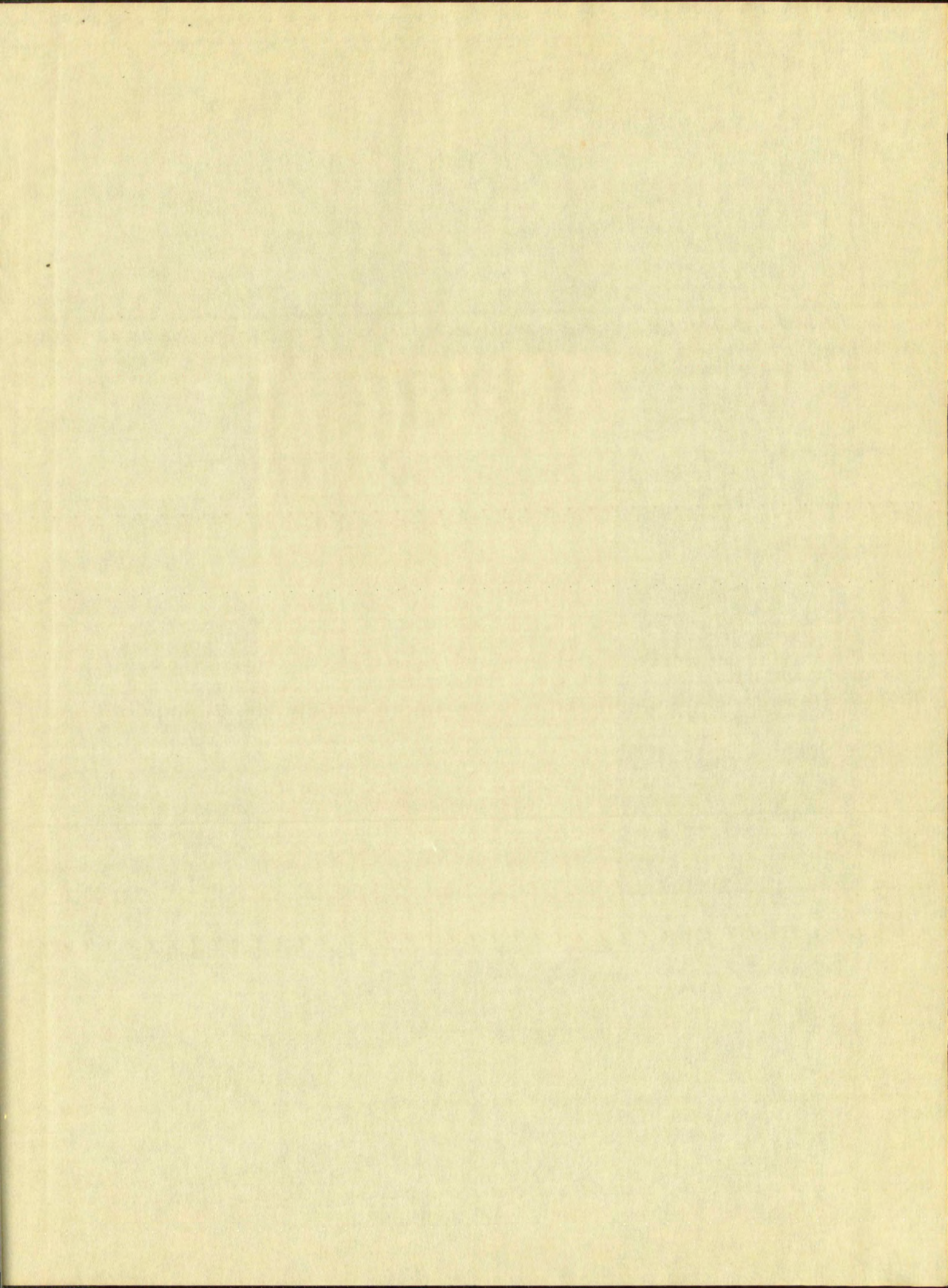
DISTANCE DOWNSTREAM

FIGURE 20 DATA SMOOTHING FOR TEST 2

Wesman
Enacible Bond

25% COTTON FIBER





IMPORTANT!

Special care should be taken to prevent loss or damage of this volume. If lost or damaged, it must be paid for at the current rate of typing.

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