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Static Output Feedback: A Survey

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Abstract

This paper reviews the static output feedback problem in the control of linear, time-invariant (LTI) systems. It includes analytical and computational methods and presents in a unified fashion, the knowledge gained in the decades of research into this most important problem.

1 Introduction

The output feedback problem is probably the most important open question in control engineering. Simply stated, the problem is as follows: Given a linear, time-invariant system, find a static output feedback so that the closed-loop system has some desirable characteristics, or determine that such a feedback does not exist. This paper attempts to survey the state of knowledge concerning the output feedback problem. The paper has two main parts: the first involves the study of the time-invariant plant described by

$$\dot{x}(t) = Ax(t) + Bu(t); \quad y(t) = Cx(t) \tag{1.1}$$

under the influence of static output feedback of the form

$$u(t) = Ky(t) + v(t). \tag{1.2}$$

The closed-loop system is

$$\dot{x} = (A + BKC)x(t) + Bv(t) \equiv A_c x(t) + Bv(t). \tag{1.3}$$

We take the state $x(t) \in \mathbb{R}^n$, the control input $u(t) \in \mathbb{R}^m$, and the output $y(t) \in \mathbb{R}^p$. The case where a dynamical output compensator of order $n_f \leq n$ is used may be brought back to the static output feedback case as discussed in [1]. The second part of this paper involves the solution of various coupled matrix design equations of the sort obtained in pole-placement and LQ design using output feedback, game theory, and elsewhere. Such coupled systems of equations are currently "solved" using iterative numerical techniques.

We recall here a few mathematical definitions which will be used in this paper. We say that a rational function $H(s)$ is Bounded-Input-Bounded-Output-Stable (BIBO) stable or that it belongs to H^∞ if it is proper, with all its poles in the left-half-plane (LHP). We let $\mathcal{S}$ denote the set of matrices whose entries are in H^∞. A Unit in $\mathcal{S}$ is a member of $\mathcal{S}$ whose inverse is also in $\mathcal{S}$. A matrix is said to be epic if it has full row rank and monic if it has full column rank. In what follows, A' or A^T denote the transpose of any matrix A, and the controller is either $u = -Ky + v$ or $u = Ky + v$, as introduced in any given section.

The paper is organized as follows: Section 2 contains a discussion of stabilizability using static output feedback. The section also includes design procedures such as the Covariance assignment and the decision methods. The Pole placement problem is presented in Section 3 and the eigenstructure assignment is discussed in section 4. Section 5 is devoted to the Linear Quadratic Regulator problem with output feedback. Our conclusions are presented in section 6

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2 Stabilizability By Static Output Feedback

In this section, we discuss the problem of stabilizing an open-loop unstable system with static output feedback.

2.1 Necessary Conditions

We first identify the cases where static output feedback can not stabilize an open-loop unstable system. In order to state these conditions, we recall the following theorems.

Theorem 2.1 [2] **The Parity-Interlacing-Property (PIP)** A linear system $H(s)$ is stabilizable with a stable compensator $C(s)$ or strongly stabilizable with $C(s)$ if and only if the number of real poles of $H(s)$, counted according to their McMillan degree, between any pair of real blocking zeros in the right-half-plane is even. We then say that the plant $H(s)$ satisfies the PIP. ■

Note that in the SISO case, the PIP fails to hold for many real systems. On the other hand, as observed in [2], the PIP holds generically in the MIMO case.

Theorem 2.2 [3] A linear system $H(s)$ is stabilizable with a stable compensator $C(s)$ which has no real unstable zeros if and only if 1) $H(s)$ satisfies the PIP, and 2) The number of real blocking zeros of $H(s)$ between any two real poles of $H(s)$ is even. We then say that $H(s)$ satisfies the even PIP. ■

Using Theorem 2.2, *a necessary condition for static output stabilizability is that the plant $H(s)$ satisfies the even PIP.*

2.2 Sufficient Conditions

We start out by discussing the simple case of SISO systems, of relative degree $n^* \leq 1$, and which are minimum phase. A simple root-locus argument then shows that such systems are stabilizable with a large enough static output feedback.

2.3 Design Approaches and Limitations

In the case of SISO systems, graphical approaches (root-locus, Nyquist) are used to answer both the existence and the design questions of stabilizing static output controllers. In addition, there exist some necessary and sufficient algebraic tests [4], [5] for the existence of stabilizing output feedbacks. These tests however, require some preliminary derivations (finding roots, eigenvalues) which are just as complicated as the graphical methods. In addition, they are not easily extendable to the MIMO case, although some specialized cases may be resolved using the Multivariable Nyquist criterion [6]. The work in [7], also presents a complete characterization of strictly-proper

SISO systems related to each other with static output feedback. In fact, it states that such systems must share the same zeros and the same breakaway points.

2.3.1 Parameterization Methods

In this section, we list some parameterization results that are potentially useful in solving the static output feedback problem.

Theorem 2.3 [2] A compensator $C(s) = N_c(s)D_c(s)^{-1}$ where $N_c(s)$ and $D_c(s)$ are in $\mathcal{S}$ internally stabilizes the plant $H(s) = N_p(s)D_p(s)^{-1}$ if and only if $N_c(s)N_p(s) + D_c(s)D_p(s)$ is a Unit of $\mathcal{S}$. Moreover, the set of all stabilizing compensators of $H(s)$ is given by

$$\mathcal{C} = \left\{C(s) = [N_c(s) + D_p(s)Q(s)][D_c(s) - N_p(s)Q(s)]^{-1}\right\}$$

$\forall Q(s) \in \mathcal{S}$. ■

It can then be argued that a necessary and sufficient condition for the static output stabilizability problem is that there exists a $Q(s) \in \mathcal{S}$ such that $K = [N_c(s) + D_p(s)Q(s)][D_c(s) - N_p(s)Q(s)]^{-1}$ is a constant matrix. Unfortunately, this and other so-called necessary and sufficient conditions are non-testable. We illustrate this point using the next approach: In [8] another necessary and sufficient condition was stated as follows.

Theorem 2.4 Given the system (1.1), and let $E_i = C^\dagger C$, where superscript "$\dagger$" denotes the Moore-Penrose inverse. Then, the system is stabilizable with static output feedback $K = R^{-1}(L + B'P)E_i$ if and only if, there exist matrices $Q > 0$, $R > 0$ and L of compatible dimensions such that the algebraic equation

$$A'P + PA - E_i(PB + L')R^{-1}(B'P + L)E_i + Q = 0 \qquad (2.4)$$

has a unique solution $P > 0$. ■

The problem resides in the fact that one can not easily choose the matrices $Q > 0$, $R > 0$ and L, nor can we easily solve for P.

2.3.2 Covariance Assignment and Stabilizability

The basic idea behind the covariance assignment methods [9], [10], is that given a stochastic system $\dot{x} = Ax + Bu + \Gamma w$; $y = Cx$ and a static output feedback $u = Ky$, the steady-state covariance matrix $X = \lim_{t\to\infty} E\{x(t)x(t)^T\}$ can be assigned a given matrix value by looking for solutions for K in the Lyapunov equation

$$(A + BKC)X + X(A + BKC)^T + \Gamma W \Gamma^T = 0 \qquad (2.5)$$

where $W > 0$ is the covariance matrix of the zero-mean, white-noise process $w(t)$, i.e. $E\{w(t)w(\tau)^T\} = W\delta(t - \tau)$. The key point is that for a given X, equation (2.5) is **linear** in the unknown output feedback matrix K. From Lyapunov stability theory, we also know that if $P > 0$, then any K which satisfies the matrix inequality

$$(A + BKC)^T P + P(A + BKC) < 0 \qquad (2.6)$$

results in a closed-loop system which is asymptotically stable. For a fixed P, inequality (2.6) is a **Linear Matrix Inequality** (LMI) in the matrix K [11]. The LMI in (2.6) is convex in K so that convex programming techniques can be used to numerically find a K whenever $P > 0$ is given. From (2.6), one can easily show that a necessary condition for static output stabilizability is that the two matrix inequalities,

$$B^\perp(AP + PA^T)(B^\perp)^T < 0 \qquad (2.7)$$
$$(C^T)^\perp(A^T P^{-1} + P^{-1}A)((C^T)^\perp)^T < 0 \qquad (2.8)$$

be satisfied by some $P > 0$, where $B^\perp$ and $(C^T)^\perp$ are full-rank matrices, orthogonal to B and C^T respectively. In [12], it is shown that the converse is also true, that is if there exists a $P > 0$ which satisfies inequalities (2.7) and (2.8), then there exists a stabilizing static output feedback K, given by

$$\begin{aligned} K &= -R^{-1}B^T PQ^{-1}C^T(CQ^{-1}C^T)^{-1} + S^{\frac{1}{2}}L(CQ^{-1}C^T)^{\frac{-1}{2}} \\ S &= R^{-1}\left[I - B^T PQ^{-1}[Q - C^T(CQ^{-1}C^T)^{-1}C]^{-1}QPBR^{-1}\right] \end{aligned}$$

where $S > 0$, and L is any matrix which satisfies $\| L \| < 1$, P is any positive-definite matrix which satisfies (2.7, 2.8), and

$$A^T P + PA + Q - PBR^{-1}B^T P = 0 \qquad (2.9)$$

for some positive-definite Q and R. Unfortunately, finding a positive-definite P, solution of (2.9) and which satisfies inequalities (2.7) and (2.8) is an open problem.

2.3.3 Decision Methods

In [13], the usage of decision methods to the study of the output feedback problem was introduced. These methods can also be modified in order to find stabilizing compensators. The basic idea behind this approach can be decomposed into the following steps: First obtain a set of inequalities to be satisfied by the elements k_{ij} of the unknown gain matrix K. These inequalities may be obtained from the usual stability tests. Second, successively eliminate k_{ij} by introducing more inequalities and equalities, until we finally end up with a set of inequalities and equalities to be satisfied by one entry of K, e.g. k_{rs}. Third, check the truth of these one variable sentences and find a range of possible values (if possible) for k_{rs}. Then, one can unfold back using the range just found, in order to find possible ranges of values for all entries k_{ij} of K. There are two main criticisms of the decision methods: the first being that these and other algorithmic approaches do not provide any insight into the solution, and the second being that they are time-consuming and complicated even for simple problems.

3 Pole Placement With Output Feedback

Here, it is desired to select the gain K to place the poles in the closed-loop system (1.3) at desired symmetric locations (i.e. closed under complex conjugation).

3.1 Sufficient Conditions

In [14] it was shown that if (1.1) is minimal, then almost any K will yield a cyclic $\bar{A} = (A + BKC)$, i.e. one such that $sI - A - BKC$ has only one non-unity invariant polynomial. Moreover, for almost any choice of a vector q, we can make $\{\bar{A}, Bq\}$ controllable. Then, we can apply the scalar design formulae to obtain a gain matrix k such that $\det(sI - \bar{A} + Bqk)$ is the desired closed-loop polynomial. In [15, 16], this approach was exploited to show that if (A, B, C) is minimal with B and C of full rank, then $\max(m, p)$ poles are assignable. Davison and Wang [17] and Kimura [18] showed that indeed, under these conditions, $\min(n, m + p - 1)$ poles are assignable generically (i.e. for almost all B and C). This translates into the sufficient condition for generic pole assignability that

$$m + p \geq n + 1. \qquad (3.10)$$

An alternate proof of this was offered in [6, 19]. Another sufficient condition for generic pole assignability was given in [20]

as $m+p+\beta > n+1$; $m > \beta$; $p \geq \alpha$, with α and β the controllability and observability indices respectively. If (1.1) is minimal with B of full rank and A_d is the desired closed-loop plant matrix, then another sufficient condition for pole assignability was given in [21] as $(A-A_d)(I-C^{\dagger}C)=0$. This may be interpreted as a condition that any differences between the actual and the desired plant matrices occur in the perpendicular of $\mathcal{N}(C)$ (with $\mathcal{N}(.)$ representing the Null space).

3.2 Necessary Conditions

In [22] a necessary and sufficient condition for generic pole assignability with a **complex** gain matrix K was established as

$$mp \geq n, \tag{3.11}$$

however, *simple* counter-examples show that this is only necessary for the case of **real** K. In [23], the necessary condition was strengthened to (3.11) plus full rank of the so-called *Plücker matrix*. Reference [24] defined (1.1) as locally completely assignable (for a given K) if, for every desired set of small changes $\delta\sigma_i$ in the poles σ_i of $(A+BKC)$, there exists a δK such that $[A+B(K+\delta K)C]$ has poles at $(\sigma_i+\delta\sigma_i)$. A necessary and sufficient (but non-testable) condition for this to occur was given in terms of the independence of the closed-loop Markov parameter matrices.

3.3 Design Approaches and Limitations

In [15, 16, 17], an explicit "Ackermann-type" formula was given for K in terms of various matrices constructed from (A,B,C) and the desired poles. In [20, 18] a different approach which relates closely to the eigenstructure assignment techniques in the next section was used. References [6] and [23] used the Grassman space (i.e. exterior algebra). In [25] an algorithm was given to assign the eigenvalues arbitrarily close to desired values for the case $m+p>n$. The Hessenberg form was used to solve two single-input problems. First, $p-1$ poles were placed, then $n-p+1$ poles were placed without disturbing the first poles assigned (c.f. [26]). A discussion on the relation between the pole-assignment problem and transmission zeros is also given. A related algorithm was given in [27] to assign $\max(m,p)$ poles. If condition (3.10) fails to hold, then the techniques of this subsection generally allow the assignment of $m+p-1<n$ poles. There are no guarantees however, on the locations of the remaining closed-loop poles, which may often be unstable. A nice geometric framework involving lattices is provided in [28]. It is however difficult to translate that framework into computational techniques.

In the following, we present yet another set of the so-called necessary and sufficient (but non-testable) conditions for pole placement using output feedback. For notational ease assume that B is monic, C is epic. We suppose $p \geq m$; the other case is handled in a similar way. The open-loop input-coupling, output-coupling, and transfer relations are revealed in matrix-fraction description (MFD) form by

$$\begin{aligned}(sI-A)^{-1}B &= N_1(s)D^{-1}(s) && (3.12)\\ C(sI-A)^{-1} &= F^{-1}(s)G_1(s) && (3.13)\\ CN_1(s)D^{-1}(s) &= N(s)D^{-1}(s) && (3.14)\\ F^{-1}(s)G_1(s)B &= F^{-1}(s)G(s) && (3.15)\end{aligned}$$

with (3.12, 3.14) normalized right MFDs (e.g. right coprime, $D(s)$ column-reduced and column-degree ordered), and (3.13, 3.15) normalized left MFDs (e.g. left coprime, $F(s)$ row-reduced and row-degree ordered). The next result was shown in [29].

Theorem 3.1 Let $[N(s),D(s)]$ be a normalized right MFD for $H(s)$. There exists a feedback K that assigns the invariant polynomials if and only if the equation

$$\begin{bmatrix} -Y_m & X_m \\ F(s) & G(s) \end{bmatrix} \begin{bmatrix} -N(s) & X_p \\ D(s) & Y_p \end{bmatrix} = \begin{bmatrix} R_m(s) & 0 \\ 0 & R_p(s) \end{bmatrix}$$

is satisfied for some $R_m(s)$ and $R_p(s)$, both having the desired closed-loop nonunit invariant polynomials. The solution must satisfy the conditions: 1) $[G(s),F(s)]$ left coprime, $F(s)$ row reduced and row-degree ordered and 2) X_m, Y_m, X_p, Y_p constant matrices with X_m and X_p nonsingular. Then the output feedback is given by $K=-X_m^{-1}Y_m=-Y_pX_p^{-1}$ ∎

The condition of the theorem is in terms of coupled Diophantine equations, which should be contrasted with the coupled LMI equations in the previous section.

4 Eigenstructure Assignment With Output Feedback

First, we review eigenstructure assignment by state-variable feedback $u(t)=-Fx(t)+v(t)$. While the pole-placement problem for multivariable systems is fairly complicated, Moore [30] showed that the problem of assigning both eigenvalues and eigenvectors has a straightforward solution. Given a symmetric set of desired closed-loop poles $\{\mu_i\}$, $i=1,\ldots q$, vectors $\{v_i\}$ and $\{u_i\}$ are found such that

$$[\mu_i I - A \quad B]\begin{bmatrix} v_i \\ u_i \end{bmatrix} = 0 \tag{4.16}$$

Then a feedback gain F defined by

$$F[v_1\cdots v_q]=[u_1\cdots u_q] \tag{4.17}$$

results in the closed-loop structure

$$[\mu_i I-(A-BF)]\,v_i=0 \tag{4.18}$$

so that the v_i are assigned as the closed-loop eigenvectors for eigenvalues μ_i. There is a certain freedom in the choice of the v_i, but for a real F to exist they must satisfy 1) $v_i \in (\mu_i I-A)^{-1}R(B)$, 2) $v_i=v_j^*$ when $\mu_i=mu_j^*$, (where "*" means complex conjugation) , and 3) $\{v_i\}$ is a linearly independent set. The integer q may be taken equal to n, but any uncontrollable poles must be included in $\{mu_i\}$, with the associated v_i satisfying $w_i^T v_i \neq 0$, where w_i is the left eigenvector associated with μ_i. Note that we may write (4.16) as the generalized Lyapunov equation

$$VJ-AV=-BU \tag{4.19}$$

with $V=[v_1\cdots v_q]$, $U=[u_1\cdots u_q]$, $J=\mathrm{diag}(\mu_i)$. Then (4.17) reduces to $FV=U$. Turning to the case of output feedback (1.2), Reference [31] assumes that a state-variable feedback F which places both eigenvalues and eigenvectors has been selected by some procedure. Then, a method is given to find an output feedback K that preserves some of the poles of $(A-BF)$. Although eigenvector assignment was not specifically addressed, the technique involves in fact preserving the eigenvectors v_i associated with the modes $\{\mu_i, i=1,...,q\}$. Indeed, although $KC=F$, may have no solution K, the reduced equation $KCV=FV$ may have a solution, so that (4.18) becomes $[\mu_i I-(A-BKC)]\,v_i=0$. In [26], the technique of [30] was extended to output feedback, essentially by replacing (4.17) with $KCV=U$. From that work, it is clear that $\max(m,p)$ poles are assignable by this method. The algorithm given assigns $p-1$ poles, and an additional (interesting but fairly complicated) procedure was given to assign a total of $\min(n,m+p-1)$ poles generically. The case of constrained output feedback (i.e. where some of the entries of K are set to zero) was covered in [32].

A major breakthrough occurred in [33], which we feel has not received the acclaim it deserves. There, some techniques of [20] were extended to show that, in some cases, $m+p$ poles may be assigned. This is a better result than those associated with (3.10). It was obtained by considering the closed-loop right and left eigenstructure. A design example demonstrates the assignment of $m+p$ poles. However, it is not clear in the paper what is actually going on in terms of system structure. A somewhat streamlined description of the main result is as follows. Let the desired closed-loop structure be described by the (possibly non-simple) Jordan matrix J. If there exist a direct sum decomposition $J = J_1 \oplus J_2$ and matrices V_1, W_2, U, and Z such that

$$\begin{aligned} V_1 J_1 - AV_1 &= -BU && (4.20)\\ J_2 W_2^T - W_2^T A &= -Z^T C && (4.21)\\ W_2^T V_1 &= 0 && (4.22) \end{aligned}$$

then $K = U(CV_1)^{-1}$ makes J the Jordan matrix of $(A+BKC)$. Moreover, the right eigenvectors corresponding to the poles in J_1 are the columns of V_1, and the left eigenvectors corresponding to the poles in J_2 are the columns of W_2. It should now be noted that p poles may be placed by using equation of (4.20) (c.f. (4.19)), and possibly m by using the dual relation, equation of (4.21). The construction of the required matrices in (4.20)-(4.22) may be confronted by using the right Nullspace of $[\mu_i I - A \;\; B]$ and the left Nullspace of

$$\begin{bmatrix} \mu_i I - A \\ C \end{bmatrix}$$

with $\{\mu_i\}$ the desired poles [33]. Unfortunately, the proposed solution algorithm is derived from only a sufficient condition, and relies on selecting some vectors to guarantee various conditions, so that some artistic ability and intuition is needed, along with a bit of luck, to apply the technique. In the case where $p+m > n$ a computationally efficient algorithm is proposed in [34] for the solution of the coupled Sylvester equations (4.20)-(4.22).

4.1 Design Approaches and Limitations

Although a given number of poles is generically assignable by the above approaches, nothing is known of the remaining closed-loop poles, which may be unstable. In [35] a technique was given for approximate pole assignment which gives some idea of the location of all of the closed-loop poles. Eigenstructure assignment with output feedback was treated for some special cases in [36, 37]. Note that the condition expressed in terms of (4.20)-(4.22) is sufficient only. A necessary and sufficient condition for eigenstructure assignment using output feedback was also given in [33]; however, it was not used as the basis of any design algorithm.

5 LQR With Static Output Feedback

It is desired here to select K to minimize, subject to the constraint (1.3), the performance index

$$J = \int_0^\infty (x^T Q x + u^T R u)dt \tag{5.23}$$

with $Q \geq 0$ and $R > 0$, while stabilizing the closed-loop system. In [38, 39, 40, 41], sufficient conditions for optimality were given as

$$\begin{aligned} 0 &= A_c^T S + SA_c + Q + C^T K^T RKC && (5.24)\\ 0 &= A_c P + PA_c^T + X && (5.25)\\ 0 &= RKCPC^T - B^T SPC^T, && (5.26) \end{aligned}$$

with $X = x(0)x(0)^T$ and $A_c = A - BKC$. Generally, optimal control with reduced information results in such coupled nonlinear matrix equations. If it is desired to eliminate the dependence of (5.24)-(5.26) on the specific initial conditions, then expected values may be taken of the performance index (5.23) so that $X = E\{x(0)x(0)^T\}$ in (5.25). It is generally assumed that $x(0)$ is uniformly distributed on the unit sphere so that $X = I$ [39]. The tracking problem with output feedback was also solved in [42]. The totally singular problem ($R = 0$) was discussed in [43]. In this case (5.24) and (5.25) become the two Lyapunov equations

$$0 = A_c^T S + SA_c + Q; \quad 0 = A_c P + PA_c^T + X \tag{5.27}$$

but (5.26) becomes $0 = B^T SPC^T$, which may not be readily solved for K in an iterative algorithm. In [43], (5.26) was replaced by the condition

$$H(K^*, S^*, P^*) \leq H(K, S^*, P^*) \tag{5.28}$$

with superscript "*" denoting the optimal values, and the solution was carried out numerically. Equations (5.27) are also one formulation of the solution in the case where only the derivatives of the state are weighted, that is when (5.23) is replaced by

$$J = \frac{1}{2}\int_0^\infty \dot{x}^T Q \dot{x} dt. \tag{5.29}$$

In this case, X in (5.27) is replaced by $\dot{X} \equiv \dot{x}(0)\dot{x}(0)^T$. An alternative solution in the case of state derivative weighting is provided, of course, by substituting (1.1) into (5.29) to obtain the performance index with state-input cross- weighting terms,

$$J = \frac{1}{2}\int_0^\infty (x^T Q x + 2x^T W u + u^T R u)dt, \tag{5.30}$$

with $Q = A^T QA, W = A^T QB$, and $R = B^T QB$. By this approach the necessary conditions for solution become

$$\begin{aligned} 0 &= A_c^T S + SA_c + Q + C^T K^T RKC - WKC - C^T K^T W^T\\ 0 &= A_c P + PA_c^T + X\\ 0 &= RKCPC^T - (W + SB)^T SPC^T. \end{aligned}$$

5.1 Design Approaches and Limitations

Algorithms for the solution of (5.24)-(5.26) and their discrete counterparts were proposed in [44, 39, 41, 45, 46]. These algorithms are all iterative in nature. Convergent iterative algorithms for the continuous case were finally presented in 1985 [41, 46]. The algorithm in [41] requires repetitive solution of (5.24) and (5.25) for fixed values of K so that they are considered as two Lyapunov (i.e. linear matrix) equations, and the form $K = R^{-1}B^T SPC^T(CPC^T)^{-1}$ as a candidate for the next choice for K. Compare this expression with that in Section 2.3.2 when $L = 0$. Note however, that it guarantees only a local minimum. Unfortunately, iterative algorithms such as these require the selection of an initial stabilizing gain. A direct procedure for finding such a K is unknown as discussed in section 2.

Conditions for the existence and global uniqueness of solutions to (5.24)-(5.26) such that P and S are positive definite and (1.3) is stable are not known. It has been shown [47] that in the discrete case there exists a gain that minimizes (5.23) locally and also stabilizes the system if $Q \geq 0$, $R > 0$, rank$(C) = p$, $X > 0$, and (A, B, C) is output stabilizable; that is, there exists a K such that A_c is stable. However, there may be more than one local minimum, so that solution of (5.24)-(5.26) may not yield the global minimum. Similar sufficient conditions were given in [41].

6 Conclusion

We hope to have shown by the discussion just completed that the state of affairs in output-feedback design is indeed a marginal one. Various unconnected necessary conditions, sufficient conditions, and ad hoc solution techniques abound. The so-called necessary and sufficient conditions are not testable and as such only succeed in transforming the problem into another unsolved problem or into a numerical search problem with no guarantee of convergence to a solution. A common thread throughout these methods however, is the fact that the problem is equivalent to obtaining the solution of a coupled set of matrix (Lyapunov, Riccati, LMI, Bezout, etc) equations.

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