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A Fresnel Half-Period Zone Plate for Focusing Electromagnetic Energy in the One Meter Wave-Length Region

Henry George Oltman Jr.

OLTMAN — A FRESNEL HALF-PERIOD ZONE PLATE

NAME AND ADDRESS	DATE

A FRESNEL HALF-PERIOD ZONE PLATE FOR FOCUSING ELECTROMAGNETIC ENERGY IN THE ONE METER WAVELENGTH REGION

By

Henry George Oltman, Jr.

A Thesis

In partial fulfillment of the

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I wish to express my appreciation to Professor V. H. Regener for his assistance in performing the experiment.

CHAPTER I

INTRODUCTION

A. Application and History. This thesis describes the experimental work on a scale model of a Fresnel Half-Period Zone Plate for electromagnetic waves in the one-meter region. A possible application for such a zone plate is the focusing of extraterrestrial radiation.

The Fresnel Half-Period Zone Plate has been used at optical wavelengths for forming images. Its use in the radio region is a new application for this device.

When focusing of electromagnetic energy in the radio region is required, refracting lenses become extremely unwieldy, heavy, and expensive. Mirrors in the form of parabolic surfaces have a high gain and good definition but they are bulky, expensive and have limited off-axis resolving power. The Fresnel Zone Plate, consisting of alternately opaque and transparent concentric zones lying in a plane would be easy to construct, light in weight, and inexpensive. It is necessary, however, to determine the effectiveness of the zone plate at these longer wavelengths.

B. The Problem. The purpose of this thesis is a determination of the gain for two zone plates of the same diameter but having different numbers of zones. As the number of zones is increased for the purpose of increasing the gain, the width of the outer zones becomes comparable to the wavelength. Electromagnetic energy incident on these zones

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A. Application and History. [illegible] work on a scale model of a [illegible] electromagnetic waves [illegible] cation for such a zone plate [illegible] radiation.

The Fresnel half-period zone plate has been used [illegible] wavelengths [illegible] new application for this [illegible].

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will be partially reflected, leading to a reduction of the gain. Since there is no theoretical treatment of this effect, it is necessary to perform experiments for the purpose of determining the optimum number of zones on a zone plate of given aperture.

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optimum number of [illegible]

CHAPTER II

THEORY

Theory and past experience predict, (1) that the zone plate should form images analogous to the optical refracting lens, (2) that focal length and gain are determined by the number of zones in a given aperture, (3) and that the resolving power is a function of the aperture dimension and of the wavelength of the radiation.

A. Dimensions of a Zone Plate. The Fresnel Half-Period Zone Plate functions in the following manner. If one considers a plane wave arriving normal to an imaginary plane, then this plane can be divided into alternately opaque and transparent zones of circular shape as shown in Figure 1. The dimensions of these zones affect the electromagnetic energy passing through the transparent zones toward a point located behind the plane and designated the focus, F.

At the focus it is desired to increase the intensity of the electromagnetic radiation. Considering each element of area in the imaginary plane as a secondary source (according to Huygens), one can compute the phase of the radiation arriving at the focus from each element in the plane, since the distances are known. Forgetting the opaque zones for the moment, the radiation from elementary areas lying on circles concentric to the axis of the zone plate will arrive at the focus in the same phase. And, as the circles become larger in radius, the distances to the focus become larger

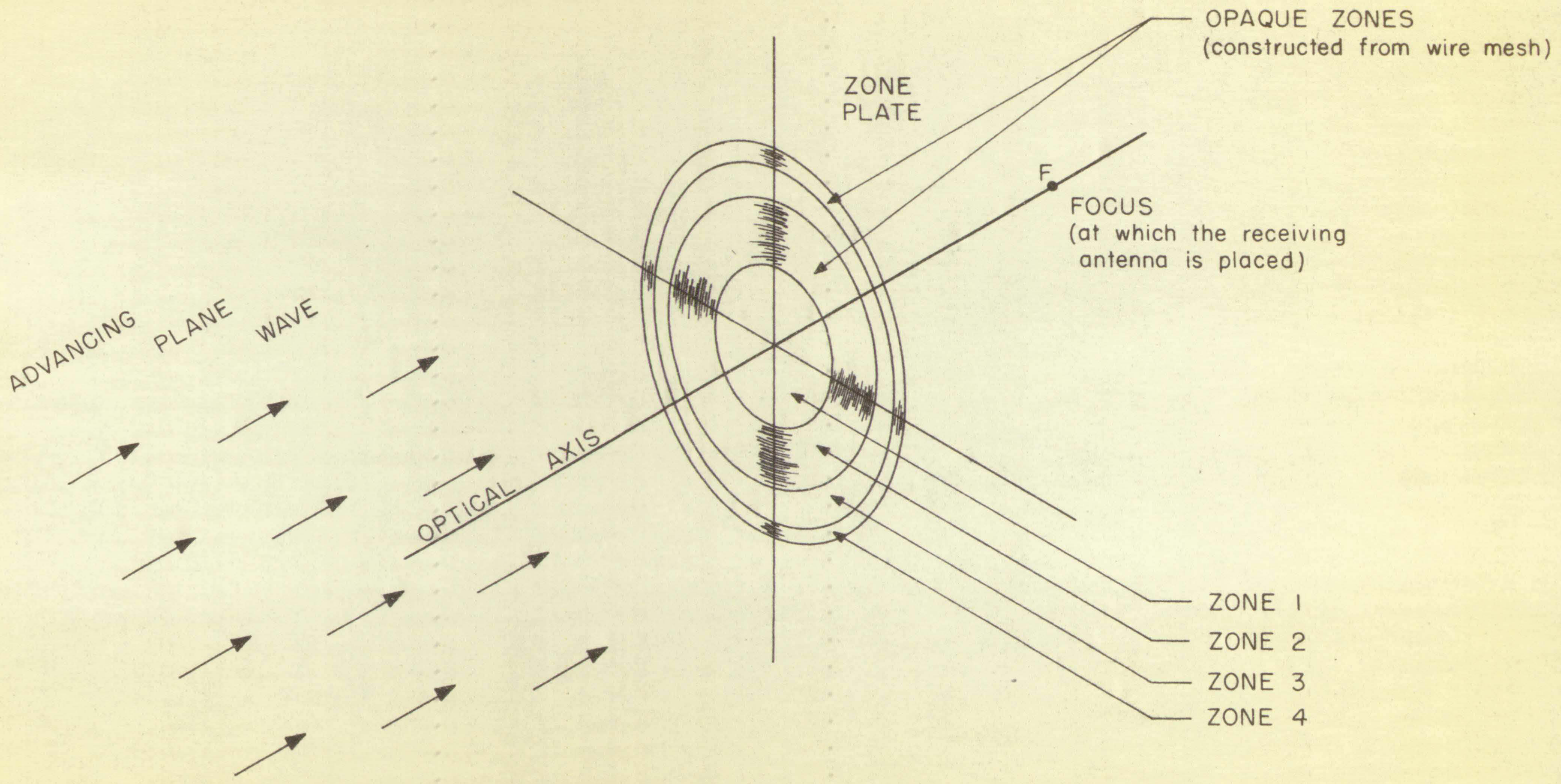

Figure 1

FUNCTION OF A ZONE PLATE

and the phase angles pass through complete cycles. Depending on the phase, the amplitudes of the electromagnetic waves will either add or subtract at the focus.

The first zone of a zone plate is determined so that all the amplitudes arriving at the focus from this zone interfere constructively (see inset of Figure 2). The next zone is constructed similarly but it is obstructed because the resultant amplitude of its radiation would be out of phase with the amplitude from the first zone. The third zone will add and the fourth will subtract. Thus, the third zone is left transparent and the fourth made opaque. Continuing in this manner, the intensity at the focus is increased with an increase in the number of zones.

The dimensions of the zones are determined as illustrated in Figure 2.[1] The distance BF is larger than AF by one-half wavelength. Therefore, when the amplitudes arriving at F from Zone 1 are added as in the inset of Figure 2, a resultant, R_1, will be obtained. The resultant, R_2, is obtained in a similar manner from Zone 2 and can be represented as a summation of the vectors in the lower half of the inset circle, it is directed opposite to R_1.

Using the Pythagorean Theorem,

$$f^2 + X_1^2 = (f + \lambda/2)^2$$

$$f^2 + X_2^2 = (f + \lambda)^2$$

$$\cdot \quad \cdot \quad \cdot$$

$$f^2 + X_1^2 = (f + \lambda i/2)^2 \qquad (1)$$

$$\cdot \quad \cdot \quad \cdot$$

[1] Jenkins and White, Fundamentals of Physical Optics, (New York: McGraw-Hill Book Company, Inc., First Edition, 1937), p. 182.

and the phase angles, and [illegible] the [illegible]. Depending on the phase, the contributions [illegible] the various zones [illegible] add or subtract [illegible].

The light [illegible] amplitudes arriving at the point from [illegible] respectively [illegible]. Theoretically [illegible] arly but it is [illegible] radiation [illegible] radiation would [illegible] than the first zone. The third zone will add [illegible] the third zone [illegible] tinuing in this manner [illegible] zone [illegible] an increase in the number of zones.

The dimensions of the zones are determined as illustrated in Figure 2.[1] The distance [illegible] by successive [illegible]. Therefore, when the [illegible] as in the [illegible] of Figure 2, [illegible] resultant, [illegible] obtained [illegible] be represented as [illegible] the first [illegible].

Using the Pythagorean theorem,

$$\rho^2 + d_1^2 = (d_1 + \lambda/2)^2$$

$$\rho^2 = d_1\lambda + \lambda^2/4$$

$$\rho^2 = d_1\lambda + (\lambda/2)^2$$

[1]Jenkins and White, Fundamentals of Physical Optics, [illegible] McGraw-Hill Book Company, Inc., [illegible]

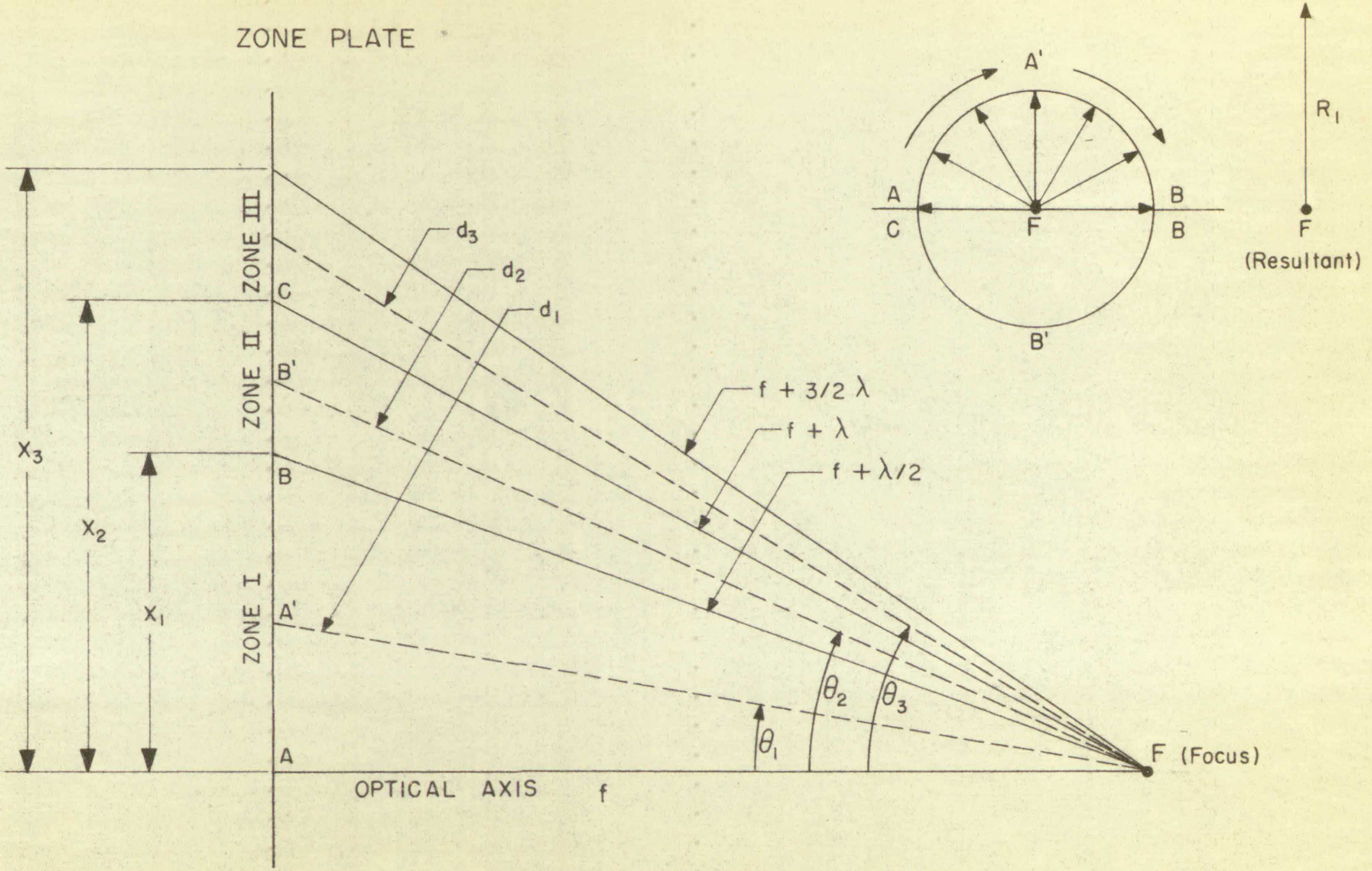

Figure 2

ZONE PLATE DIMENSIONS

where the X's represent the radii of the zones, f the focal length of the zone plate, and λ the wavelength of the incident radiation.

Expanding the last equation and cancelling:

$$X_i^2 = f\lambda i + \frac{\lambda^2 i^2}{4} \tag{2}$$

or

$$X_i = \sqrt{f\lambda i + \frac{\lambda^2 i^2}{4}} \tag{3}$$

This equation determines the radii of the boundaries of the zones in terms of the focal length of the system and the wavelength. It also determines the aperture diameter, D, of a zone plate with n zones:

$$D_n = 2X_n = 2\sqrt{f\lambda n + \frac{\lambda^2 n^2}{4}} \tag{4}$$

B. Gain of a Zone Plate. An expression for the gain of a zone plate can be determined in the following manner.[2]

Consider an infinite plane with a point F, the focus, located behind it and a plane wave incident normal to the plane. The plane is divided into zones as described in Section 2(A), but none of the zones are as yet considered opaque. If the contributions to the amplitude at the focus are designated R_1, R_2, ..., the resultant amplitude at F from all of the zones is:

$$R = R_1 - R_2 + R_3 - R_4 + \ldots. \tag{5}$$

---

[2] Jenkins and White, op cit, pp. 175-182.

where the A's represent the radii of the zones, f the focal length of the zone plate, and λ the wavelength of the incident radiation.

Expanding the first equation and simplifying:

$$A_1^2 - f\lambda = \frac{\lambda^2}{4} \qquad [illegible]$$

or

$$A_1 = \sqrt{f\lambda + \frac{\lambda^2}{4}} \qquad [illegible]$$

This equation determines the radii of the innermost zone in terms of the focal length of the plate and the wavelength. It also determines the maximum number, N, of [illegible] plate with [illegible] zones:

$$[illegible] \qquad (3)$$

B. Gain of a Zone Plate. An expression for the gain of a zone plate can be determined in the following manner.

Consider an infinite plane wave with a point F, the focus, located behind it and a plane zone plate [illegible] in the plane. The plane is divided into zones as described in Section II A, that is, the zones are [illegible] of the [illegible]. The amplitude at the focus are designated U_1, U_2, ... The resultant amplitude at F from all of the zones is:

$$U = U_1 - U_2 + U_3 - U_4 + \ldots. \qquad (4)$$

[2] Jenkins and White, op. cit., pp. 375-378.

The amplitudes with even subscripts are entered negative because the vectors are directed opposite to the amplitude vectors with odd subscripts.

The magnitudes of the amplitude vectors depend on three characteristics of the zones (Figure 2): the total area, A; the distance of the zone from the focus, d; and Fresnel's obliquity factor, $(\cos \Theta)$.[3] Thus, the magnitude of the contribution to the amplitude from zone i is:

$$R_i = C \frac{A_i}{d_i} (\cos \Theta_i), \quad (6)$$

where C is a constant.

The area of zone i is the difference of the areas of the circle i and the circle i-1.

$$A_i = \pi X_i^2 - \pi X_{(i-1)}^2$$

$$= \pi[(f \lambda i + \frac{\lambda^2 i^2}{4}) - (f \lambda [i-1] + \frac{\lambda^2 [i-1]^2}{4})]$$

$$= \pi \lambda [f + (i - \tfrac{1}{2}) \tfrac{\lambda}{2}] \quad (7)$$

The effective distance of zone i to the focus is very nearly equal to the distance from the mid-circle of zone i to the focus (Figure 2).

$$d_i = f + (i - \tfrac{1}{2}) \tfrac{\lambda}{2} \quad (8)$$

Substituting (3) and (4) in (2):

$$R_i = C\pi \lambda (\cos \Theta_i) \quad (9)$$

---

[3]Max Born, <u>Optik</u>, (Ann Arbor, Michigan: Edwards Bros., 1943), p. 146.

The amplitudes of the small [illegible] of the different [illegible]

the various [illegible] odd subscripts.

The magnitudes of the [illegible] actuation of the [illegible] Figure [illegible] of the same form [illegible] and [illegible]

Thus, the amplitudes of [illegible] is:

$$R_i = C\frac{a_i}{d_i}(1 + \cos\theta_i)$$

where C is a constant.

The area of zone i is the difference of the areas of circle i and the circle i-1.

$$a_i = \pi r_i^2 - \pi r_{(i-1)}^2$$

$$= \pi\left[\left(i\lambda f + \frac{i^2\lambda^2}{4}\right) - \left((i-1)\lambda f + \frac{(i-1)^2\lambda^2}{4}\right)\right]$$

$$= \pi\lambda f + \left(i - \frac{1}{2}\right)\lambda^2 \qquad (2)$$

The effective distance of zone i to the focal point [illegible] equal to the distance from the mid-circle of zone i to the focal point:

$$d_i = f + \left(i - \frac{1}{2}\right)\frac{\lambda}{2} \qquad (3)$$

Substituting (2) and (3) in (1):

$$R_i = C\pi\lambda(1 + \cos\theta_i) \qquad (4)$$

---

[1] Max Born, Optik, (Ann Arbor: Edwards Brothers, 1943), p. [illegible]

This shows that the contribution to the amplitude decreases as i becomes larger since Θ increases with i. This decrease is at a decreasing rate, i.e. R_{i+1} is smaller than R_i by a smaller amount than that by which R_i is smaller than R_{i-1}. This fact can be used if Eq. (5) is arranged in the following two ways:

$$R = \frac{R_1}{2} + \left(\frac{R_1}{2} - R_2 + \frac{R_3}{2}\right) + \dots + \frac{R_i}{2}. \quad (10)$$

or

$$R = R_1 - \frac{R_2}{2} - \left(\frac{R_2}{2} - R_3 + \frac{R_4}{2}\right) - \dots - \frac{R_{i-1}}{2} + R_i . \quad (11)$$

Where i is an odd finite number which will be allowed to approach infinity later.

In Eq. 10, each bracketed term is positive and we may write:

$$\frac{R_1}{2} + \frac{R_i}{2} < R, \quad (12)$$

letting the bracketed terms determine the inequality sign. Similarly, each bracketed term in Eq. (11) is positive and we may write:

$$R < R_1 - \frac{R_2}{2} - \frac{R_{i-1}}{2} + R_i, \quad (13)$$

and, upon combining (12) and (13)

$$\frac{R_1}{2} + \frac{R_i}{2} < R < R_1 - \frac{R_2}{2} - \frac{R_{i-1}}{2} + R_i. \quad (14)$$

If f is large compared to the zone widths, there will be very little difference in the amplitudes R_1 and R_2 on one hand and between R_{i-1} and R_i on the other hand. Using this fact in (14):

This shows that [illegible] is [illegible]
becomes larger [illegible]
decreasing [illegible]
then that [illegible]
If Eq. (9) is arranged in [illegible]

[illegible equation]

or

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Where i [illegible]
infinity later.

In Eq. 10, [illegible]

[illegible equation]

Letting the bracketed terms [illegible] Simil-
arly, each bracketed term [illegible]

[illegible equation]

and, upon [illegible]

[illegible equation]

If f is large compared to [illegible]
difference [illegible]
and R_1 on the other hand. [illegible]

[illegible equation]

$$\frac{R_1}{2} + \frac{R_i}{2} < R < \frac{R_1}{2} + \frac{R_i}{2} . \qquad (15)$$

This equation indicates that

$$R = \frac{R_1}{2} + \frac{R_i}{2} . \qquad (16)$$

Allowing i to approach infinity, we may disregard R_i, since we have an infinite plane and R_i is zero because of $\cos \theta_i = 0$. Thus:

$$R = \frac{R_1}{2} . \qquad (17)$$

However, this resultant amplitude must be equal to the incident amplitude, R_I, since none of the zones are obstructed. Substituting one obtains:

$$R_1 = 2R_I,$$

and the intensity, I, is:

$$I_1 = 4R_I^2 . \qquad (18)$$

Now, if the even zones of subscript less than n are obstructed, allowing the odd zones to contribute to the amplitude at F, one obtains:

$$R = R_1 + R_3 + \ldots + R_n - R_{n+1} + R_{n+2} \ldots \quad (n \text{ odd}) \qquad (19)$$

The above equation represents the amplitude of a zone plate of n zones and $(n - \frac{1}{2})$ determines the effective aperture. The diameter of the last obstructed zone is determined by (n-1), but this aperture

This equation indicates that [illegible]

[illegible]

Allowing [illegible] to [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

an infinitesimal [illegible] and [illegible] [illegible] [illegible] [illegible] [illegible]

$$[illegible] = \frac{[illegible]}{2} \qquad [illegible]$$

However, [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

amplitude. [illegible] [illegible] [illegible] of [illegible] [illegible] [illegible] [illegible] [illegible]

one obtains:

$$[illegible] = [illegible]$$

and the intensity, I, is:

$$I = [illegible] \qquad [illegible]$$

Now, if the [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible],

allowing the [illegible] [illegible] to [illegible] [illegible] [illegible] [illegible] [illegible]

obtains:

$$[illegible] \quad \ldots \quad [illegible]$$

The above equation [illegible] the [illegible] [illegible] [illegible] [illegible]

zones and [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

of the last [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

is not the effective aperture. The n^{th} zone effectively contributes with one half of its area as will be shown. Therefore, the effective aperture is one half zone less than the diameter of the n^{th} zone and is computed with the factor $(n - \frac{1}{2})$.

The terms of subscript n and above add up to $R_n/2$.

$$\sum_{n}^{\infty} (-1)^{n+1} R_n = \frac{R_n}{2} \tag{20}$$

which can be shown by means of a procedure analogous to that used on pp. 9 and 10. If the focal length is much larger than the aperture, one can write in Eq. (9) $\cos \theta_i \approx 1$, and

$$R_1 = R_2 = R_3 = \ldots = R_n \tag{21}$$

so that

$$R = (\tfrac{n}{2})\, R_1 = nR_I \qquad \text{(n odd)} \tag{22}$$

and the intensity, I, is:

$$I = n^2 R_I^2 \qquad \text{(n odd)} \tag{23}$$

The gain, G, of the system is defined as:

$$G = \frac{I}{I_I} = \frac{n^2 R_I^2}{R^2} = n^2, \qquad \text{(n odd)} \tag{24}$$

or

$$G = (2j + 1)^2. \qquad \text{(n odd)} \tag{25}$$

where j is the number of obstructed zones for a zone plate with open center.

is not the effective aperture. [illegible] with one half of its area [illegible]. [illegible] aperture is one half zone plus [illegible] and is computed with [illegible].

The terms of [illegible]

$$\sum_{0}^{n} (-1)^{n} \ldots \quad (21)$$

which can be shown by means of [illegible] on pp. 9 and 10. If the focal length [illegible] ture, one can write [illegible]

$$R_1 = R_2 = R_3 = \ldots \quad (22)$$

so that

$$R = (\tfrac{n}{2}) R_1 = nR_1 \quad (24)$$

and the intensity, I_1 [illegible]

$$I = n^2 R_1^2$$

The gain, G, of the system is defined as

$$G = \frac{I}{I_1} = \ldots \quad (26)$$

or

$$G = (2j + 1)^2 \quad (27)$$

where j is the number [illegible] center.

The above, for n odd, is the approximate solution for the gain when one has a zone plate in which the first zone is a contributing zone. The solution for a zone plate in which the first zone is obstructed differs only slightly from the above. For i even, the terms of Eq. (5) are arranged as follows.

$$R = R_1 - \frac{R_2}{2} - \left(\frac{R_2}{2} - R_3 + \frac{R_4}{2}\right) - \ldots - \frac{R_i}{2} \cdot \quad \text{(i even)} \quad (26)$$

and,

$$R = \frac{R_1}{2} + \left(\frac{R_1}{2} - R_2 + \frac{R_3}{2}\right) + \ldots + \frac{R_{i-1}}{2} - R_i. \quad \text{(i even)} \quad (27)$$

Thus,

$$R_1 - \frac{R_2}{2} - \frac{R_i}{2} > R > \frac{R_1}{2} + \frac{R_{i-1}}{2} - R_i. \qquad \text{(i even)} \quad (28)$$

And, assuming $R_1 = R_2$ and $R_{i-1} = R_i$ we obtain:

$$\frac{R_1}{2} - \frac{R_i}{2} > R > \frac{R_1}{2} - \frac{R_i}{2} .$$

And finally:

$$R = \frac{R_1}{2} - \frac{R_i}{2} \qquad \text{(i even)} \quad (29)$$

which, when R_i is disregarded because of $\cos \theta_i = 0$ as i goes to infinity, becomes:

$$R = \frac{R_1}{2} = R_I, \text{ as before.} \qquad \text{(i even)} \quad (30)$$

This is the same equation for i odd or even, so the even or odd restriction may be dropped for this purpose, and one obtains in general,

The above, [illegible]

when one has [illegible]

some. The [illegible]

corrected [illegible]

of Eq. [illegible]

$$[illegible]$$

and,

$$[illegible]$$

Thus,

$$[illegible]$$

And, assuming [illegible] and [illegible]

$$[illegible]$$

And finally:

$$[illegible] \qquad \text{(Figure [illegible])}$$

which, when [illegible]

infinity, becomes:

$$[illegible] \text{, as before.}$$

This is the same [illegible]

restriction may be [illegible]

general,

$$R_1 = 2R_I \tag{31}$$

Now, if the odd zones of subscript less than n are obstructed, allowing the even zones to contribute to the resultant amplitude, one obtains:

$$R = R_2 + R_4 + R_6 + \ldots R_n - R_{n-1} + R_{n+2} + \ldots . \quad (n \text{ even}) \tag{32}$$

This equation represents the amplitude from a zone plate of n zones. The terms of subscript n and above add up to $R_n/2$, as before, and with the proper restrictions in order to apply Eq. (21) one obtains:

$$R = \frac{(n-1)}{2} R_1 = (n-1)\ R_I. \quad (n \text{ even}) \tag{33}$$

The intensity is

$$I = (n-1)^2\ R_I^2 \quad (n \text{ even}) \tag{34}$$

and the gain is:

$$G = (n-1)^2 \quad (n \text{ even}) \tag{35}$$

or

$$G = (2k-1)^2 \quad (n \text{ even}) \tag{36}$$

where k is the number of obstructed zones for a zone plate with obstructed center.

Similar results are thus obtained for both cases. Close inspection shows that for equivalent outlay of materials in constructing the zone plate, a larger gain can be obtained using a zone plate with a contributing first zone.

$$R_1 = 2R_f \quad (11)$$

Now, if the odd zones of a zone plate are obstructed, allowing the even zones to contribute to the resultant amplitude, we obtain:

$$R = R_2 + R_4 + R_6 + \dots R_n - R_{(n-1)} + R_{n+1} + \dots$$

This equation represents the amplitude from a zone plate [illegible] The terms of subscript n and above add up to $R_n/2$, as before, and [illegible] with the proper restrictions in order to apply eq. (11) [illegible]

$$R = \frac{(n-1)}{2} R_1 = (n-1) R_f \quad (n \text{ odd})$$

The intensity is

$$I = (n-1)^2 \frac{r_1^2}{4} \quad (n \text{ odd})$$

and the gain is:

$$G = (n-1)^2 \quad (n \text{ odd})$$

or

$$G = (2k-1)^2 \quad (n \text{ even})$$

where k is the number of obstructed zones of the zone plate with unobstructed center.

Similar results are thus obtained for both cases. Close inspection shows that for identical conditions in constructing the zone plate, a larger gain can be obtained using a zone plate with a contributing first zone.

The gain of a zone plate is actually less than that indicated by Eq. (25) and Eq. (36) because of the approximation contained in Eq. (21). Even for the second zone plate with the very high aperture/focal length ratio of one, this approximation neglects a decrease of only 10 percent in the contribution of the outermost zone.

C. Resolving Power of a Zone Plate. The resolving power of a zone plate is assumed not to deviate significantly from Rayleigh's Criterion for the angular resolution of a circular aperture.

The angular resolution, θ, for an aperture, D, is given by:

$$\theta \approx 1.22 \frac{\lambda}{D} \text{ radians},$$

where λ is the wavelength of radiation used, 11 cm. for the experiment carried out in this thesis.

D. Frequency Range of a Zone Plate. The usable frequency range of a zone plate can be made quite large with little variation in gain, so long as the position of the focus is appropriately varied with the frequency or wavelength. In fact, Eq. (25) and Eq. (36) for the gain of a zone plate do not contain the wavelength at all. However, this is due to the neglect made in Eq. (21). When a zone plate is used for longer and longer wavelengths, the focal length decreases and cos θ_i can no longer be considered close to unity. This reduces the gain for all zone plates when they are used with an aperture/focal length ratio larger than one. When the focal length is on the order of the aperture diameter or larger, the gain varies only slightly with the wavelength.

The gain of a zone plate is generally less than that indicated by Eq. (25) and Eq. (26) because of the approximation made in Eq. (21). Even for the fixed zone plate with the aperture/focal length ratio of one, this approximation gives an error of only 13 percent in the contribution of the outermost zone.

C. Resolving Power of a Zone Plate. The resolving power of a zone plate is assumed not to deviate significantly from that of a lens. The criterion for the angular resolution of a circular aperture.

The angular resolution, θ, for an aperture, d, is given by

$$\theta \cong 1.22 \frac{\lambda}{d} \text{ radians,}$$

where λ is the wavelength of radiation used, 11 cm. for the experiment carried out in this thesis.

D. Frequency Range of a Zone Plate. The possible frequency range of a zone plate can be made quite large with little variation in gain, as long as the position of the focus is appropriately varied with the frequency or wavelength. In fact, Eq. (25) and Eq. (26) for the gain of a zone plate do not contain the wavelength at all. However, this is due to the neglect made in Eq. (21). When a zone plate is used for longer and longer wavelengths, the focal length increases and it can no longer be considered large compared to λ. This restricts the [illegible] for all zone plates when they are used with an aperture/focal length ratio larger than one, when the focal length or the radius of the aperture diameter [illegible] of the order of magnitude of the wavelength.

CHAPTER III

DESCRIPTION OF THE ZONE PLATE USED

A. Construction. There are two methods of constructing opaque zones. The radiation can be absorbed by a resistive sheet or it can be reflected by a metallic surface. It was decided to reflect the undesired radiation since this was easier to do. One-quarter inch galvanized hardware cloth was used for the reflecting material.

The zones were mounted on tapes stretched over a wooden framework as shown in Figure 3. The framework was 16 feet on a side constructed from 2" x 4" lumber. The tapes were a special packing tape of paper back with strands of fiberglass meshed in it for strength and, especially important for the purpose at hand, for dimensional stability. Some of the tapes served to prevent folding of the frame under wind stress.

The opaque zones were cut from 4 foot rolls of the galvanized hardware cloth. The sectorial cuts were soft soldered along adjacent radii to form a continuous circular zone. This was then laid on the stretched tapes and attached with staples.

Originally, no metal such as nails or screws were used in constructing the framework for the zone plate. The 2" x 4" lumber was notched at the joints, doweled, and glued. Those tapes which were attached to prevent folding of the framework under wind stress, failed to accomplish this during some severe wind storms and it was necessary to reinforce the framework. Additional 2" x 4" lumber was nailed to

CHAPTER FIVE

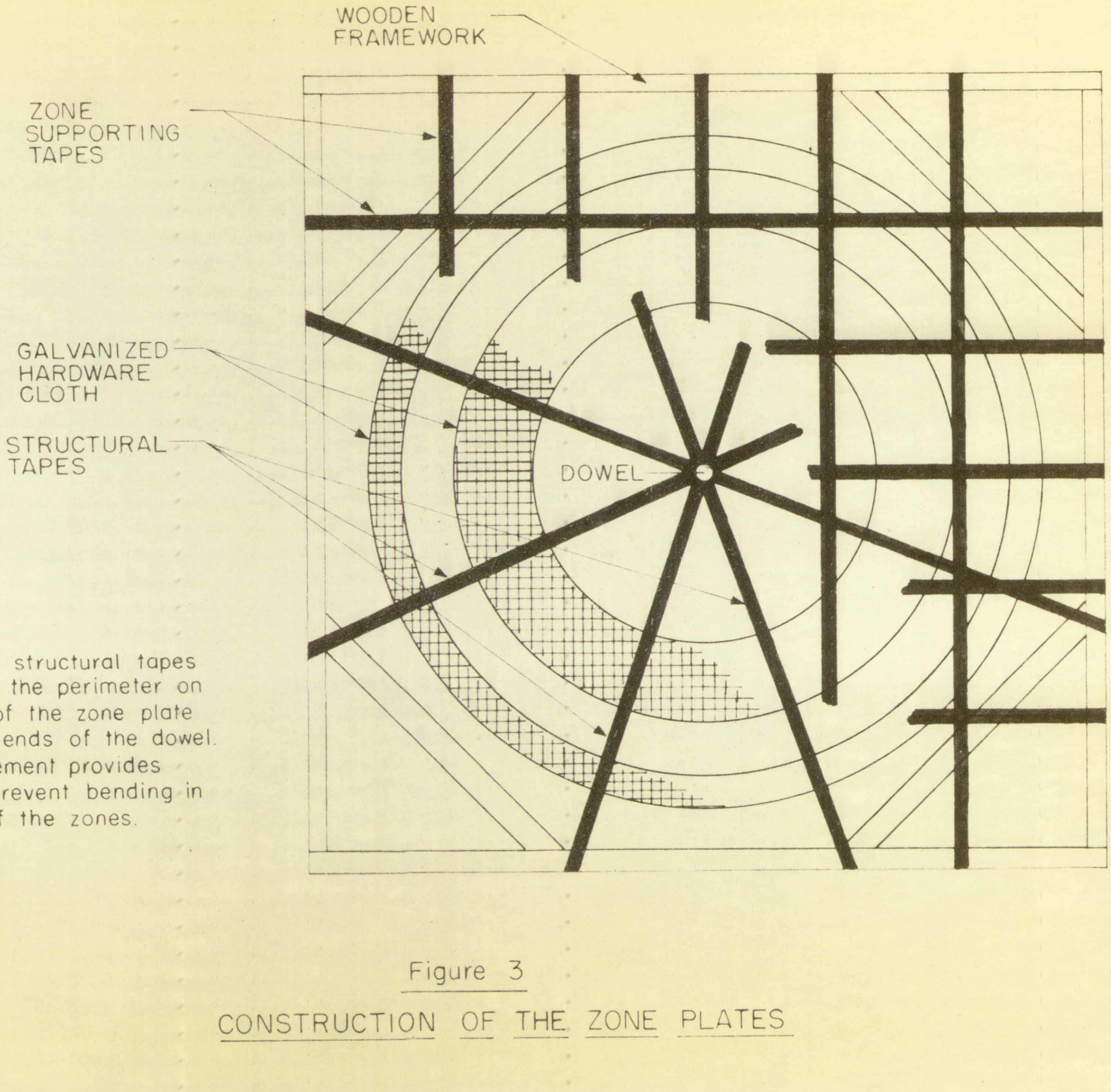
DESCRIPTION OF THE [illegible]

4. Construction. [illegible] methods of construction [illegible] The radiation [illegible] reflected by a metallic surface. [illegible] undesired radiation [illegible] galvanized hardware cloth was used for the [illegible] material.

The mouse were [illegible] work as shown in Figure [illegible] constructed from [illegible] tape of paper [illegible] strength and, especially [illegible] dimensional stability [illegible] of the frame [illegible]

The [illegible] hardware cloth. The material [illegible] jacent [illegible] laid on the [illegible]

Originally, [illegible] structing the [illegible] notched at [illegible] attached to prevent [illegible] to accomplish [illegible] to reinforce [illegible]



NOTE:
Two sets of structural tapes extend from the perimeter on either side of the zone plate to opposite ends of the dowel. This arrangement provides rigidity to prevent bending in the plane of the zones.

Figure 3

CONSTRUCTION OF THE ZONE PLATES

the existing frame members. The nails used were installed so that their lengths were parallel to the direction of propagation of the testing radiation. They thereby presented a minimum effective area to the radiation, and it is assumed that they caused no perturbations of the field at the focus.

The first zone plate constructed had a focal length of ten meters and two obstructing zones. The aperture (maximum diameter of outer obstructing zone) was calculated to be 4.72 meters. (See Figure 4)

The second zone plate had a focal length of five meters and five obstructing zones. The diameter of the outer obstruction zone was 5.07 meters. Both zone plates were designed for a wavelength of 11 cm.

The calculated zone dimensions are given on the following table:

Zone No. (Obstructing)	Zone Plate No. 1 Inner Radius	Zone Plate No. 1 Outer Radius	Zone Plate No. 2 Inner Radius	Zone Plate No. 2 Outer Radius
2	1.05m	1.49m	0.75m	1.05m
4	1.82m	2.11m	1.29m	1.50m
6	-	-	1.68m	1.85m
8	-	-	2.00m	2.14m
10	-	-	2.28m	2.41m

B. Gain and Angular Resolution. Both zone plates had contributing central zones. The gain formula which applied was Eq. (25):

the existing frame members. The units were [illegible] so that their lengths were parallel to the direction of [illegible] of the testing radiation. They [illegible] to the radiation, and it is assumed they have [illegible] of the field at the focus.

The first zone plate so designed had a focal length of [illegible] meters and had 10 [illegible] zones. The [illegible] of outer [illegible] zone [illegible] meters. (See Figure [illegible])

The second zone plate had a focal [illegible] of five meters and five obstructing zones. The diameter of the outer [illegible] was 5.07 meters. Both zone plates were designed for a wavelength of 11 cm.

The calculated zone dimensions are given in the following table:

Zone No. (Obstructing)	[illegible]	[illegible]	[illegible]	[illegible]
2	[illegible]	[illegible]	[illegible]	[illegible]
4	[illegible]	[illegible]	[illegible]	[illegible]
6	[illegible]	[illegible]	[illegible]	[illegible]
8	[illegible]	[illegible]	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]	[illegible]

B. Gain and focusing sensitivity. [illegible]

central zones. [illegible]

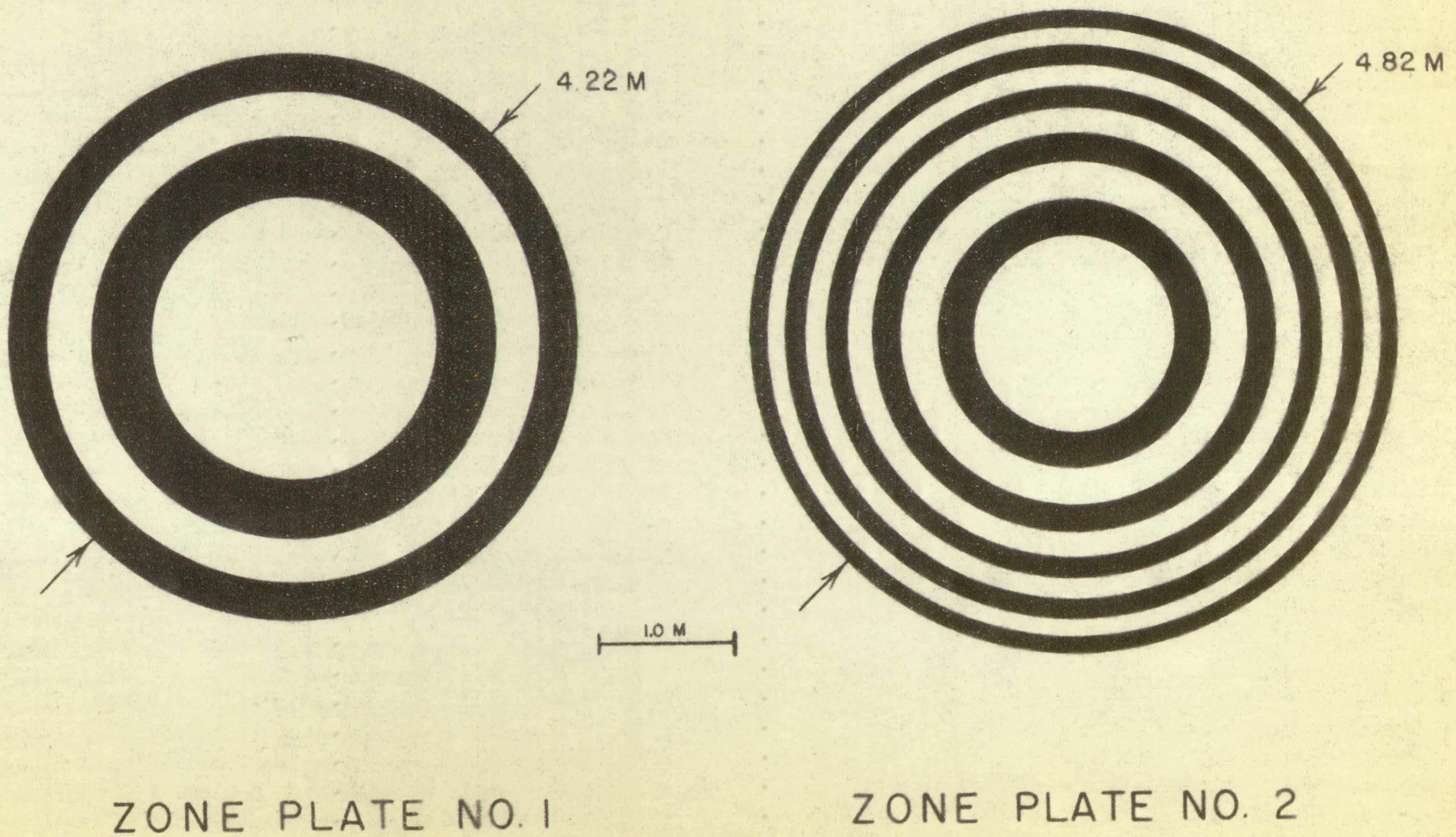


Figure 4

THE ZONE PLATES CONSTRUCTED

$$G = (2j + 1)^2$$

For Zone Plate No. 1, j = 2, and the theoretical gain was therefore:

$$G = 25.$$

For Zone Plate No. 2, j = 5, and the theoretical gain was:

$$G = 121.$$

Using Eq. (37) for the angular resolution

$$\theta \approx 1.22 \frac{\lambda}{D} \text{ radians,}$$

and conservatively entering for D the inside radius of the largest obstructed zone, one obtains for the angular resolution of Zone Plate No. 1:

$$\theta \approx 0.037 \text{ radians} = 2.1 \text{ degrees.}$$

For Zone Plate No. 2, one obtains:

$$\theta \approx 0.029 \text{ radians} = 1.7 \text{ degrees.}$$

CHAPTER IV

EXPERIMENTAL ARRANGEMENT

A. Measurements. It was necessary to determine two characteristics of the zone plate, (1) gain, and (2) the angular resolution of the image, as a function of the position of the source with respect to the axis of the zone plate (Figure 9).

B. Test Range. Figure 5 shows the test range. For the tests of both lenses, the object distance, p, was 27 meters. For test of Lens No. 1, f was 10 meters and q was approximately 16 meters. For test of Lens No. 2, f was 5 meters and q was approximately 6 meters.

The transmitting antenna was located on the roof edge of the Physics Building to allow placement of the receiving antenna carriage on the ground. The zone plate was tilted away from the transmitter so that its plane was normal to the line joining the center of the zone plate and the transmitting antenna.

C. Image. An image of the transmitting antenna at P was formed at Q (Figure 5). The receiving antenna, the vertical tube on the saw-horse type carriage (Figures 5 and 8), was moved horizontally across this image. The receiver output readings, which were proportional to the intensity of the image, were recorded as a function of the horizontal position.

One would expect to find a single large maximum along the optical axis of the zone plate. However, it appeared that over a

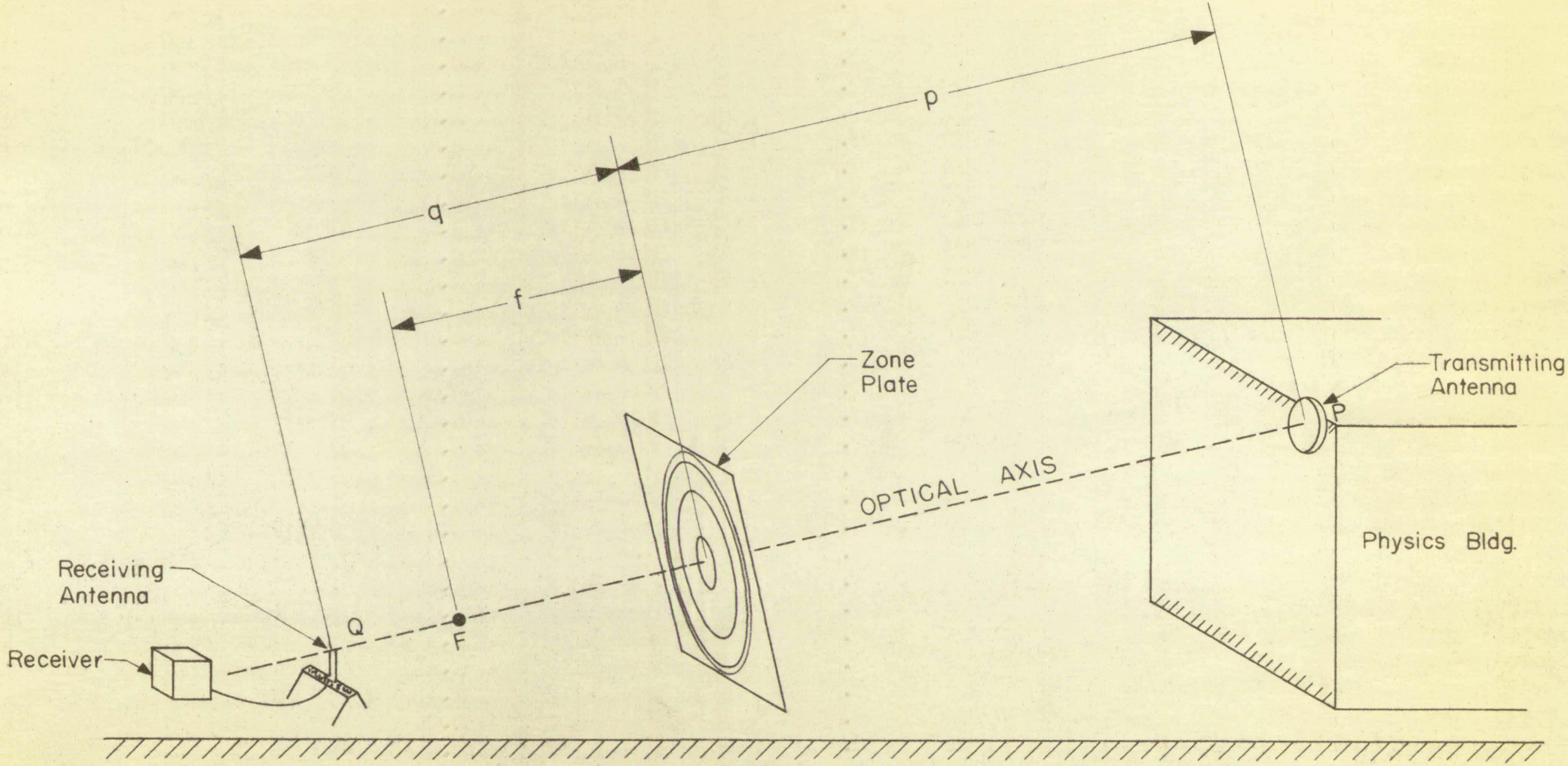


Figure 5

TEST RANGE

large range along the axis, the intensity was high with some superimposed variation.

All of the measurements on the angular resolution of the image were taken at the high intensity region near Q. The above measurements allowed computation of the maximum gain and image width.

D. Field of the Zone Plate. Of prime interest is the useful field of the lens. This is one of the major advantages of the zone plate over a parabolic mirror. Experience with optical lenses leads one to expect broadening of the image with an accompanying decrease in gain as the source and image are moved off the optical axis. The useful field can be defined as twice the included angle between the optical axis and the directions to the source for which the maximum gain has dropped to one-half the maximum obtained when the source is on the optical axis.

To measure the above effects, the zone plate was rotated about a vertical axis instead of moving the transmitting and receiving antennas horizontally off the axis. The intensity vs. position measurements were then repeated.

All of the above measurements were converted from intensity to gain which removed the requirement for absolute calibration of the receiving equipment. An attempt was made to measure the intensity (obtain a receiver output reading) at the plane of the zone plate which would be the reference level in all gain calculations. This was not possible because the signal at the zone plate was less than the noise in the receiver. A value for the intensity at the plane of the zone plate had to be obtained in the following manner.

The receiving antenna was brought close enough to the transmitter antenna so that a receiver scale reading was obtained equal to the maximum intensity reading obtained while probing the image field of the zone plate. The distances from the transmitting antenna to (1) the receiving antenna, and to (2) the zone plate were then measured. And, using the inverse-square law, the intensity reading at the zone plate was calculated.

This determined the gain of any point in the image field, where the intensity reading was equal to the calibration intensity reading, completely independent of the characteristics of the receiving equipment. The gain at all other points, however, will depend on the receiver characteristics. If the receiver output is proportional to power, then the gain at the other points will be equal to the intensity reading divided by the intensity reading at the plane of the zone plate. If the receiver output is not proportional to power, more involved calculations are necessary.

A check was made to assure that the output of the receiver was proportional to the input power (this was expected; see Section 5C). The receiving antenna was brought close enough to the transmitting antenna to obtain a galvanometer deflection near maximum. This distance was measured. The distance was then increased by a factor of $\sqrt{2}$. At this distance, the galvanometer reading should be one half the first reading if the receiver output was proportional to power. This was found to be true. The original distance was then doubled at which position the output reading should be one-fourth the original deflection. This was also verified.

CHAPTER V

TRANSMITTING AND RECEIVING EQUIPMENT

A. Transmitter and Transmitting Antenna. The transmitter was a surplus test oscillator for the SCR-522 Radar Unit. The oscillator tube was a 446B lighthouse tube and was pulse modulated at approximately a 16 kc/s rate. The pulse width was approximately 10 μsec. The frequency of the oscillator was set at 2.72 kmc/s corresponding to a wavelength of 11 cm.

The transmitting antenna consisted of a center-fed dipole mounted in a parabolic reflector ten inches in diameter. The half-power beam width of the antenna was calculated to be 28°.[4] Radiation from the antenna illuminated the aperture of the zone plate at a distance of 27 meters with essentially constant intensity.

B. Receiving Antenna. The receiving antenna[5] was a slotted cylinder antenna. A half-wavelength slot was cut longitudinally in a cylinder 0.35 of a wavelength in diameter (Figure 6). The radiation received was transmitted to the receiver by a coaxial cable attached across the slot 1/12 of a wavelength from one end.

The voltage standing wave ratio measured on the antenna at 2.72 kmc was 1.55. This corresponds to a 5 percent loss of power due to mismatch of the antenna and receiver.

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[4] From a Parabolic Antenna Computer based on the work of R. C. Spencer, Mfg. by Perrygraf Corp., Maywood, Ill., 1950.

[5] Very High-Frequency Techniques, H. J. Reich, Editor (New York, McGraw-Hill Book Company, Inc., 1947), Vol. 1, p. 176.

[illegible]

[illegible]

A. <u>Transmitter and Transmitting Antenna</u>. [illegible] test oscillator for the [illegible] a [illegible] kc/s rate. The [illegible] of the oscillator was [illegible] of 11 kc.

The transmitting antenna consisted of [illegible] in a paraboloid reflector [illegible] width of the antenna and [illegible] antenna illuminated [illegible] 27 meters [illegible]

B. <u>Receiving Antenna</u>. The receiving antenna was [illegible] tenna, a half-wavelength [illegible] 0.25 of a wavelength [illegible] was transmitted [illegible] slot 1/18 of [illegible]

The voltage standing wave [illegible] one was 1.5 [illegible] mismatch of the antenna [illegible]

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[1] From a Parabolic [illegible] Mfg. by [illegible] Corp., [illegible]

[2] Very High Frequency [illegible] Hill Book Company, Inc., [illegible]

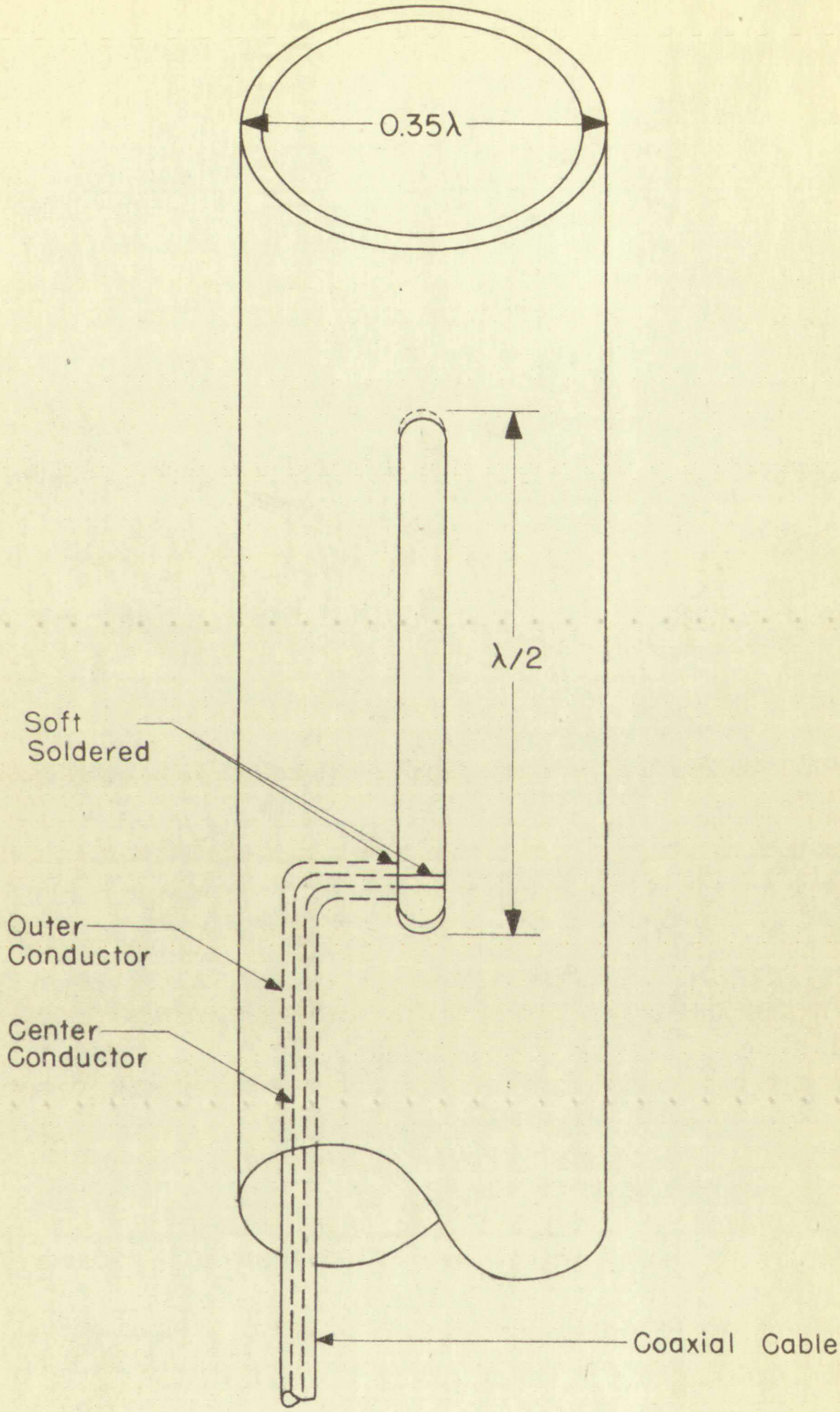


Figure 6

RECEIVING ANTENNA

Figure 7 is the radiation pattern of the receiving antenna. The half-power points of the pattern are 190° apart. The gain was a maximum and was constant over a 90° sector. At the two focal points of the zone plates, at distances of ten and five meters from the plates respectively, the solid angle subtended by the zone plates was covered by the receiving antenna with constant gain. This is desirable because the sensitivity of the receiver for radiation coming from the outer zones would be reduced if the receiving antenna gain fell off in the direction of the outer zones.

The effective receiving area[6] of the antenna will affect our measurements. The effective receiving area is the area through which that amount of power flows which the receiving antenna removes the space. The gain of the receiving antenna, g, is approximately 1.6 as compared to that of an omnidirectional antenna. This is approximately computed by assuming the receiving pattern of Figure 7 to be a surface of revolution about an axis A-A. The gain, g, as a function of orientation, is obtained by comparing the radius-vector, ρ, in a given direction to the radius-vector of a hypothetical omnidirectional antenna pattern with equal integrated response. Thus,

$$g = \frac{4\pi}{\int \rho \, d\Omega} \rho$$

where $d\Omega$ is the element of solid angle.

With this information, the effective receiving area of the antenna can be determined. If the antenna is matched to the receiver,

---

[6] Lovell and Clegg, Radio Astronomy, (New York, John Wiley and Sons, Inc., 1952), p. 47.

Figure 7 is the radiation pattern of the [illegible] antenna. The half-power points of the pattern [illegible] a maximum and was [illegible] ± 9° [illegible] and the first points on the [illegible] plates, [illegible] of the plates respectively, [illegible] and by the area of the [illegible] was covered by the receiving antenna [illegible] from the other [illegible] receiving antenna [illegible] fell off in the direction of the [illegible].

The effective receiving area[6] of the [illegible] measurements. The effective receiving area [illegible] that amount of power [illegible] receiving [illegible] space. The gain of the receiving antenna, g, is approximately [illegible] as compared to that of [illegible] antenna. [illegible] mately [illegible] the receiving pattern [illegible] a surface of revolution about [illegible] of orientation, [illegible] obtained by comparing the radiation [illegible] given direction to the [illegible] antenna pattern [illegible] integrated [illegible]. Thus, [illegible]

[illegible]

where dΩ is the element of solid angle.

[illegible]

[illegible]

[6] Lovell and Clegg, *Radio Astronomy* (New York: John Wiley and Sons, Inc., 1952), p. 47.

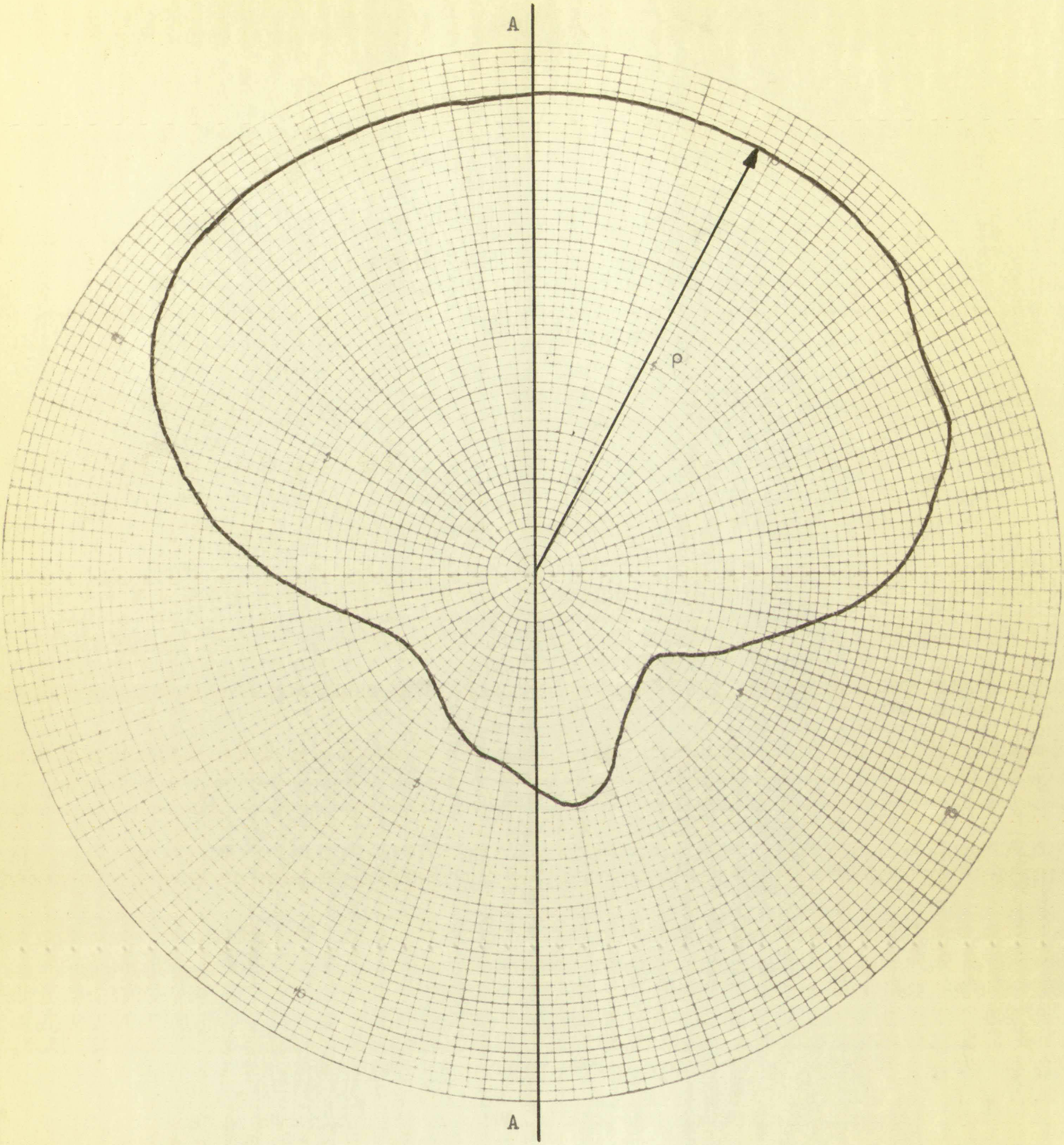


Figure 7

RECEIVING ANTENNA PATTERN

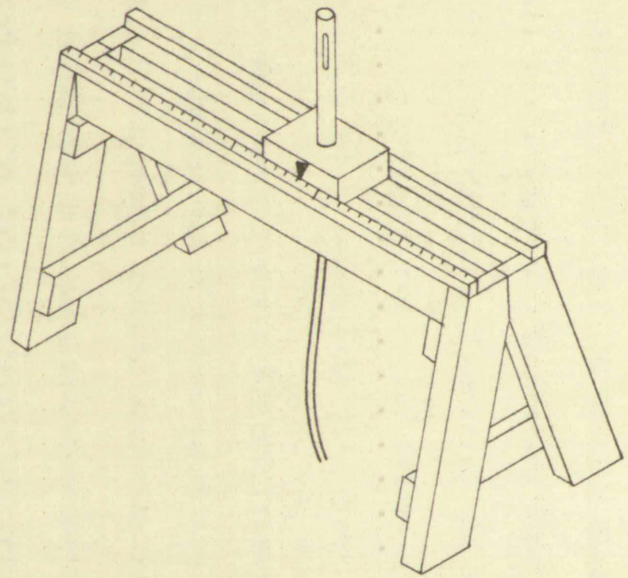


Figure 8

RECEIVING ANTENNA CARRIAGE
WITH ANTENNA AND COAXIAL CABLE

then the effective receiving area of the antenna is given by:

$$S = \frac{g\lambda^2}{2\pi} .$$

For $g = 1.6$ and $\lambda = 11$ cm, $S = 30.8\ cm^2$. This is equivalent to a circle 6.2 cm in diameter. The antenna, then, effectively removes all of the incident radiation in a 6.2 cm circle, absorbs one half and re-radiates the remainder. The intensity plotted at a point x, is then roughly the average of the intensity over the distance (x-3) cm to (x+3) cm. In testing Zone Plate No. 1, this is equivalent to 0.2 degrees. In testing Zone Plate No. 2, this is equivalent to 0.6 degrees. Therefore, the intensity received by the antenna is averaged over a 0.2 degree angle with Zone Plate No. 1, and over a 0.6 degree angle with Zone Plate No. 2.

C. Receiver. The receiver was a SCR-522 frequency meter which was converted to allow the use of a more sensitive galvanometer. The instrument was originally used to determine frequency by tuning a series resonant cavity for a minimum galvanometer reading. For our purpose it was necessary to tune the cavity off resonance to obtain a maximum reading.

The input circuit consisted of the cavity in parallel with a diode detector. This was followed by an amplifier designed to amplify the 16 KC pulse frequency of the 2.72 kmc transmitter. The output of the amplifier was rectified by a 1N34 crystal diode. A Rubicon sensitive galvanometer indicated the current output of the rectifier. The crystal diode is a device which, when a potential difference is

then the effective resolving power [illegible] is given by

$$ [illegible] $$

For [illegible] and λ = [illegible]. This is equivalent to a circle [illegible] in the [illegible]. The [illegible], moves all of the [illegible] in a [illegible] half and [illegible] point x, is [illegible] the [illegible] tance x-[illegible] [illegible] equivalent to [illegible] degrees [illegible] equivalent to [illegible] the antenna [illegible] and over [illegible].

C. Receiver. The [illegible] verted to allow [illegible] galvanometer. [illegible] strument was originally [illegible] frequency [illegible] resonant cavity for a [illegible]. [illegible] it was necessary to tune [illegible] for maximum reading.

The [illegible] in the cavity [illegible] diode detector. This was [illegible] the 15 KC pulse frequency of the [illegible] of the amplifier [illegible]. [illegible] sensitive galvanometer indicated [illegible]. The crystal [illegible]

applied, produces a current that is proportional to the square of the potential difference. Therefore, the current which flowed through the galvanometer was proportional to the power incident on the antenna.

applied, produces a current that is proportional to the magnitude of the potential difference. Therefore, the current which flows through the galvanometer and tongue is equal to the potentialities of the tongue.

CHAPTER VI

DATA

A. Parameters Used to Describe the Data. Figure 9 shows the parameters used to describe the data. ϕ is the angle between the optical axis of the zone plate and the direction of the source. θ is the half-angle of the image subtended at the center of the zone plate.

Figure 10 shows three gain curves on each of the two zone plates tested. The curves are for various angles between the source and optical axis, and their comparison gives an indication of the useful field of the lens. The data are given as gain vs. the half-angle, θ.

B. Zone Plate No. 1. The maximum gain for Zone Plate No. 1 was approximately 25. This figure is not accurate, but it is estimated to be within 15 percent of the actual value. The theoretical gain is also 25. This maximum gain is obtained when the source is located on the optical axis. With the source removed 14° ($\phi = 14^\circ$) from the optical axis, the maximum gain drops to 23; and with the source removed 28° the maximum gain is 18. The angular resolution of the images (to the half gain point) were all approximately 1.4°. This compares with the theoretical value of 2.1° (Section 3B).

The information given by the extremes of the curves is influenced by the noise level (shown at $G = 7$). It would be expected that the curves would continue toward zero gain at a decreasing rate. There should be side lobes, but these were masked by the noise level.

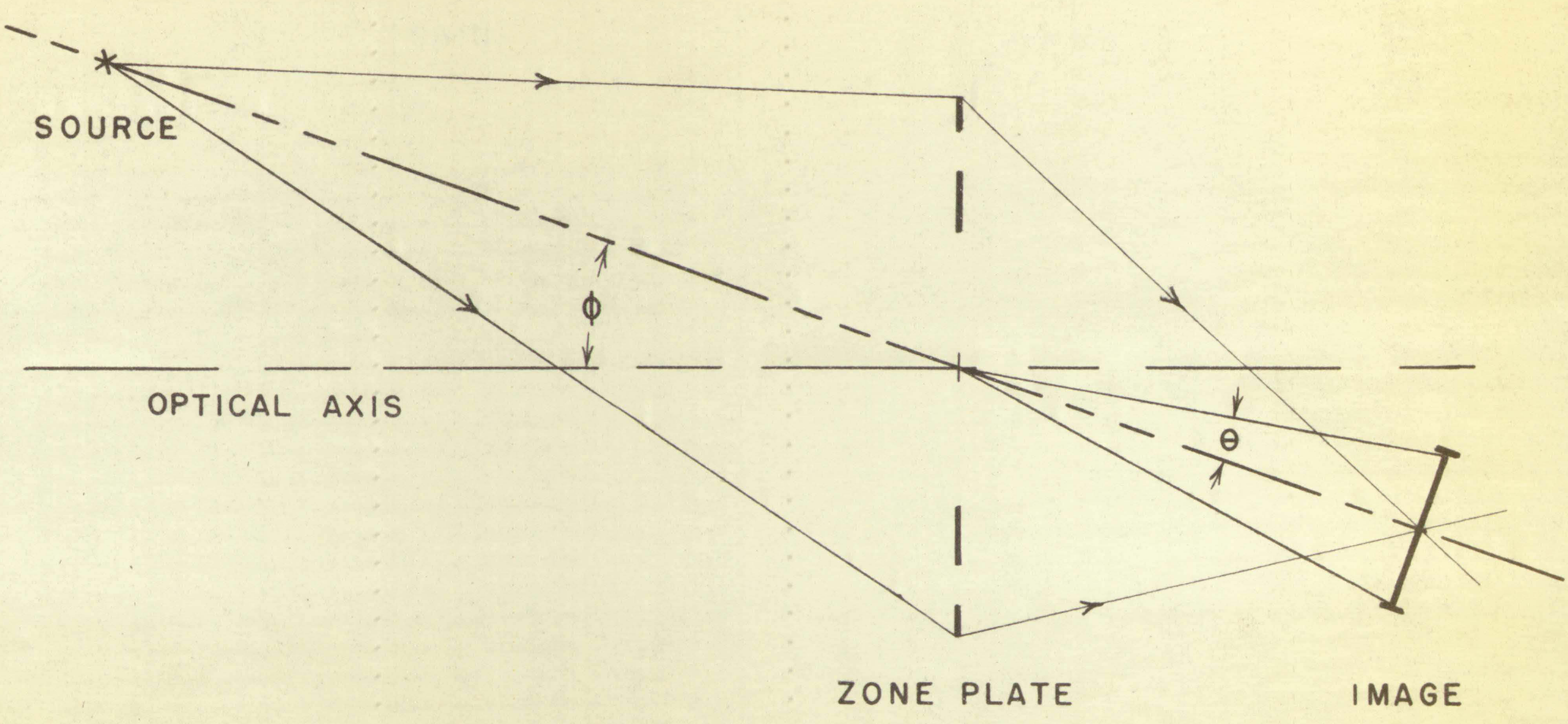
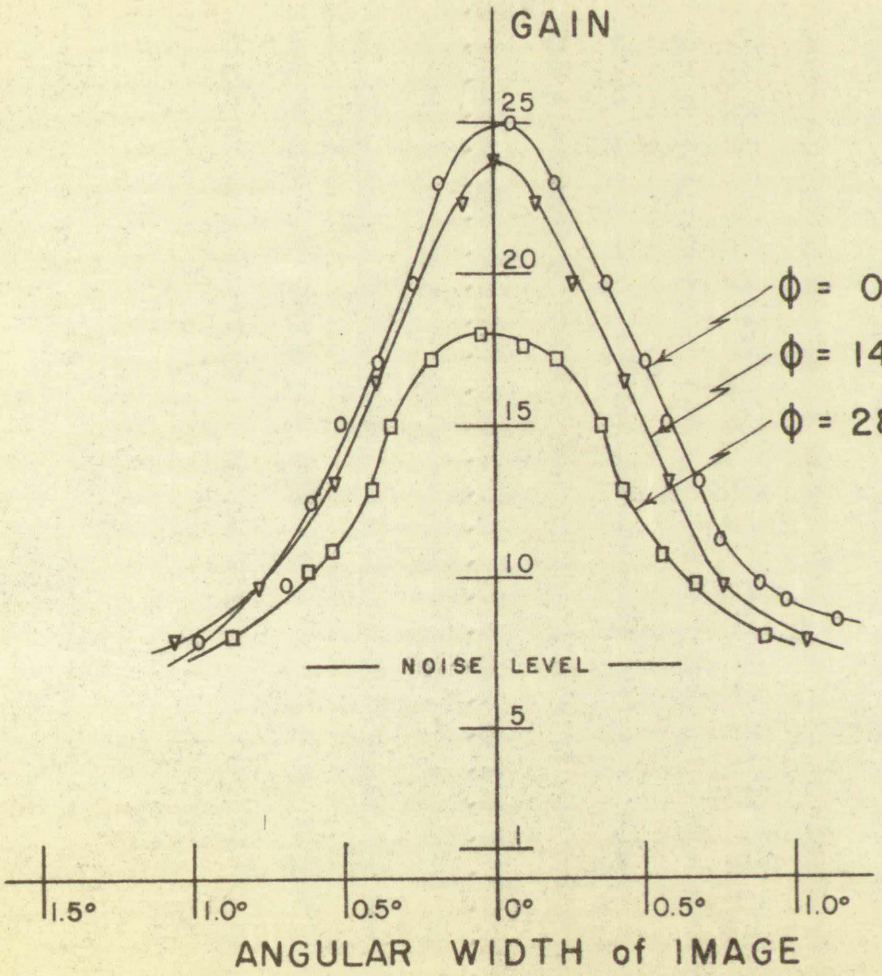
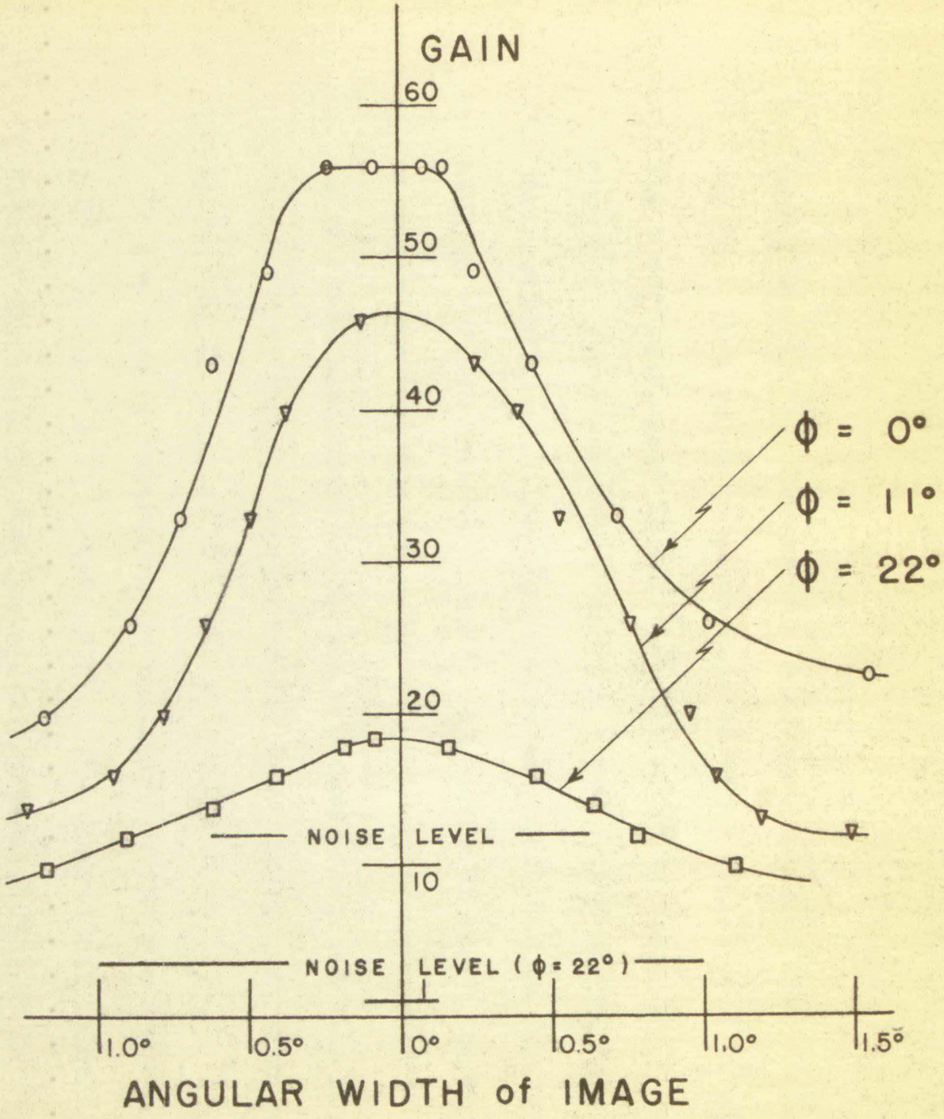


Figure 9

EXPERIMENTAL ARRANGEMENT



ZONE PLATE NO. 1



ZONE PLATE NO. 2

Figure 10

DATA

C. Zone Plate No. 2. The maximum gain obtained with Zone Plate No. 2 was 56. This value is an average of three values; 51, 55 and 61. The theoretical gain is 121. A gain of 56 is slightly larger than the theoretical value obtained for a zone plate having only three obstructing zones. The discrepancy is attributed to reduced transmission through the outer zones, the widths of which are approximately a wavelength (Section 1B). The relative ineffectiveness of these outer zones also shows up in the broadened image as will be described below. With the source removed 11° from the optical axis, the gain is 46. At an angle of 22°, the gain dropped to 18.

The angular resolution of the image of the $\phi = 0$ curve is about 1.8°, for the $\phi = 11^{\circ}$ curve, 1.5°. The theoretical value is 1.7°. Because the second curve has a smaller half-width, it is suggested that some perturbation, not accounted for, enlarged the first curve, particularly on the right side. The flat top of the first curve is also difficult to explain, but is possibly due to the finite size of the source. There is no evidence of limiting in the receiving circuit since larger deflections were observed during calibration measurements. The measurements are affected by the effective receiving area of the receiving antenna (Section 5B). The image of the source will be broadened because of the averaging effect of the receiving area. As described, this averaging is over a 0.2° angle for Zone Plate No. 1 and over a 0.6° angle for Zone Plate No. 2. The larger averaging angle of Zone Plate No. 2 possibly accounts for the broader image.

C. Zone Plate No. 2. [illegible] was 56. [illegible] The theoretical gain is [illegible] the theoretical value [illegible] obstructing zones. [illegible] mission through [illegible] a wavelength (Section 15). [illegible] outer zones also [illegible] below. With the [illegible] is 16. At an angle of [illegible].

The angular resolution of [illegible] 1.8°; for the [illegible] curve, [illegible]. Because the second curve has a smaller half-width, [illegible] that some perturbation, [illegible] particularly on the right side. [illegible] also difficult to explain, but [illegible] the source. There is [illegible] since [illegible] The measurements [illegible] receiving [illegible] (Section [illegible]) [illegible] broadened because of [illegible] of the receiving area. As described, [illegible] and over a [illegible] angle of Zone Plate No. 2 [illegible]

CHAPTER VII

CONCLUSIONS

A. Comparison of Zone Plates. Each zone plate has advantages over the other. Zone Plate No. 2 has slightly more than twice the gain of Zone Plate No. 1. However, the angular resolution of the image formed by Zone Plate No. 1 was appreciably better than that of Zone Plate No. 2.

It can be deduced by considering the above, that the gain of a zone plate with a five-meter aperture cannot be increased much more, if any. This is because the effective aperture is apparently less for Zone Plate No. 2 than for Zone Plate No. 1. The reasons are that the width of Image No. 2 is greater than Image No. 1 and the gain is so much smaller than theoretical. This means that the outer zones are not contributing or, more probably, contributing much less than they should. By increasing the number of zones on the aperture to increase the gain, the zones would have to be made narrower, and the effective aperture would be decreased more. It is doubtful whether the gain would be appreciably increased.

B. Comparison of a Zone Plate with a Parabola. Zone Plate No. 1 has an angular resolution slightly better than that of a parabola of equivalent aperture.[7] The parabola having a half-power beam width of 1.57°, Zone Plate No. 1 having 1.4°, and Zone Plate No. 2 having 1.8°.

The parabola has a decided advantage, however, in gain: 11,000 compared with 25 for Zone Plate No. 1 and 56 for Zone Plate No. 2.

[7] Parabolic Antenna Computer, op cit.

However, a zone plate can be made much larger in diameter, thereby increasing the gain at a cost which is insignificant compared to that of a parabola with its supporting and rotating mechanism.

The zone plate has an important advantage in size of the useful field. The high gain of a parabola deteriorates very rapidly as the source is moved off the optical axis. The gain of Zone Plate No. 1 would be halved if the source were located in the neighborhood of 35° off the optical axis.

C. Application to Reception of Celestial Noise. For applications to reception of celestial radio noise, all three of the factors mentioned above (7B) are important. A high gain antenna system is required because of the low power involved. The narrower the image formed by a receiving system, the greater the amount of information that will be obtained. Finally, scanning of the skies is greatly simplified if the receiving system has a large field.

A zone plate can be made quite large if the application is for celestial observations since the plane of the zone plate can be stretched horizontally over the ground. It is necessary, however, to support it above the ground a distance roughly equivalent to the focal length. This can be done easily on towers, or between adjacent tall buildings, or over natural canyons when they are narrow and deep enough.

The receiving equipment could be mounted on a mobile table, or specially built track to move the antenna so as to keep a particular celestial source in focus.

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The receiving equipment could be mounted on a mobile truck or specially built track to move the antenna so as to keep a particular celestial source in focus.

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