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1 Introduction

Machine learning research has largely been devoted to binary and multiclass problems relating to data mining, text categorization, and pattern/facial recognition. Recently, popular machine learning algorithms, including support vector machines (SVM), have successfully been applied to wireless communication problems, notably spread spectrum receiver design [1], channel equalization [2]. This paper presents a multiclass implementation of SVMs for direction of arrival (DOA) estimation.

2 Least Squares Support Vector Machines

In a binary classification system the input sequence and a set of training labels are represented as \( \{x_k, y_k\}_{k=1}^{N} \), where \( y_k \in \{-1, 1\} \) represents the classification label of the input vector \( x_k \). If the two classes are linearly separable in the input space then the hyperplane separating the classes is defined as \( wx + b = 0 \), \( w \) is a vector of weights and \( b \) is a bias term. If the input space is projected to a higher dimensional feature space then the hyperplane becomes \( w^T\Phi(x) + b = 0 \), the nonlinear function \( \Phi(·) \) maps the input space to the feature space.

Suykens, et al. [3] introduced a least squares SVM (LS-SVM) which is based on the Vapnik SVM (1).

\[
y(x) = \text{sign} \left( \sum_{k=1}^{K} \alpha_k y_k \Phi(x, x_k) + b \right)
\]

The LS-SVM classifier is generated from the optimization problem:

\[
\min_{w, b, \alpha} L_{LS} (w, b, \alpha) = \frac{1}{2} \|w\|^2 + \frac{1}{2} \sum_{k=1}^{N} \alpha_k^2, \quad \text{with constraints}
\]

\[
y_k \left[ w^T \Phi(x_k) + b \right] \geq 1 - \phi_k, \quad k = 1, \ldots, N,
\]

Misclassifications, due to overlapping distributions, are accounted for with the slack variables \( \phi_k \). \( \psi \) is a regularization parameter that governs the complexity of the SVM, and the margin between the hyperplane and the data points in the feature space is maximized when \( w \) is minimized and the relationship \( \Phi(x, x_k) = 1^T(x, x_k) \) is due to Mercer's Theorem [4]. The Lagrangian of equation (2) is defined as

\[
L_{LS} (w, b, \alpha, \psi) = \sum_{k=1}^{N} \left[ \alpha_k \left[ w^T \Phi(x_k) + b \right] - 1 + \phi_k \right]
\]

where \( \alpha_k \) are Lagrangian multipliers, also known as "support vectors", that can either be positive or negative.

One-vs-one multiclass classification is based on binary LS-SVMs. For \( P \) distinct classes there are \( P(P-1)/2 \) hyperplanes that separate the classes with maximum margin. The hyperplanes with maximum margin are constructed in the LS-SVM training phase. The Decision Directed Acyclic Graph (DAG) is used to construct the decision tree.
(DDAG) is a specific technique for one-vs-one multiclass classification [5]. The technique uses a tree structure to compare the test data to each of the hyperplanes. Through a series of elimination steps the best label is assigned to the input data. The LS-SVM algorithm for DOA estimation is based on the DDAG architecture with each node containing a binary LS-SVM classifier of the $i$th and $j$th classes, see Figure 1.

![DAGSVM for Four Classes](image)

**Figure 1: DAGSVM for Four Classes**

3 SVMs and DOA Estimation

The process of DOA estimation is to monitor the outputs of $D$ antenna elements and predict the angle of arrival of $L$ signals, $L < D$. The output vector for each incident signal from the antenna elements is $\mathbf{a}(t) = [1, e^{-j\theta_1}, \ldots, e^{-j\theta_L}]$, and the vector of incident DOAs is $\theta = [\theta_1, \ldots, \theta_L]$. With a training process the learning algorithms generate DOA estimates based on the responses from the antenna elements, $\mathbf{a}(t)$.

For the LS-SVM based approach to DOA estimation the output of the receiver is used to calculate the sample covariance matrix $\mathbf{R}_{xx}$ of the input data signal $x(t)$: $\mathbf{R}_{xx} = \frac{1}{M} \sum_{k=1}^{M} x(t)_k x(t)_k^H$. The dimension of the observation matrix is $D \times M$, $M$ is ideal sample size (window length), and the dimension of the sample covariance matrix is $D \times D$. The principal eigenvectors, $\mathbf{v}_1, \ldots, \mathbf{v}_P$, are calculated via eigendecomposition (ED) or subspace tracking techniques. Each eigenvector is used to calculate a covariance matrix $\mathbf{R}_{mv}$, $\ldots$, $\mathbf{R}_{mv}$. The LS-SVM DOA estimation algorithm includes preprocessing, training, and testing steps.

- **Preprocessing for SVM Training**
  1) Generate the $D \times N$ training vectors for the $P$ SVM classes, $D$ is the number of antenna elements, $N$ is the number of input data samples. 2) Generate the $P$ sample covariance matrices, $\mathbf{C}$ with $M$ samples from the $D \times N$ data vector. 3) Calculate the signal eigenvector, $\mathbf{S}$, from each of the $P$ sample covariance matrices. 4) Calculate the $D \times 1$ projection vectors, $\mathbf{C}$, $\mathbf{S}$, for each of the $P$ classes. 5) Store the projection vectors for the training phase and the eigenvectors for the testing phase.

- **LS-SVM Training**
  1) With the $P$ projection vectors train the $P(P-1)/2$ nodes with the one-vs-one LS-SVM algorithm. 2) Store the LS-SVM variables, $a_0$ and $b$ from equation (1).

- **Preprocessing for SVM Testing**
  1) Acquire $D \times N$ input signal from antenna array. 2) Generate the sample covariance matrix with $M$ samples from the $D \times N$ data vector. 3) Calculate the eigenvectors for the signal subspace and the noise subspace. 4) Generate the covariance matrices for each eigenvector.

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3.1 Simulation

Model that includes amplitude and simulations. This variable in Section 3 has the capability in DOAs includes the transmitted noise phaseshift and modeled delay for each signal path, real and imaginary for the antenna array includes eight elements and the winning and modeled as a channel. With this constraint the machine learning algorithm must be a multiclass system and able to process multiple labels. The machine learning algorithm must generate multiclass labels, $y_i \in \chi$, where $\chi \subset [-90,90]$ is a set of real numbers that represent an appropriate range of expected DOA values, and multiple labels $y_i, i = 1, \ldots, L$, for $L$ dominant signal paths. If antenna sectoring is used in the cellular system the multiclass labels are from the set $\chi \subset [S]$, where $S$ is field of view for the $i$th sector. Multilabel classification is possible with the LS-SVM DDAG model presented in Section 3. The LS-SVM algorithm for DOA estimation assigns DOA labels to each eigenvector in the signal subspace. By repeating the DDAG cycle for each eigenvector the multiclass algorithm has the capability of assigning multiple labels to the input signal.

3.2 Multilabel Capability for Multiple DOAs

In DOA estimation for cellular systems there can be multiple DOAs for a given signal. This results from multipath effects induced by the environment. The machine learning system must be able to discriminate between a small number of independent DOAs that include signal components with similar tone delays. With this constraint the machine learning algorithm must be a multiclass system and able to process multiple labels. The machine learning algorithm must generate multiclass labels, $y_i \in \chi$, where $\chi \subset [-90,90]$ is a set of real numbers that represent an appropriate range of expected DOA values, and multiple labels $y_i, i = 1, \ldots, L$, for $L$ dominant signal paths. If antenna sectoring is used in the cellular system the multiclass labels are from the set $\chi \subset [S]$, where $S$ is field of view for the $i$th sector. Multilabel classification is possible with the LS-SVM DDAG model presented in Section 3. The LS-SVM algorithm for DOA estimation assigns DOA labels to each eigenvector in the signal subspace. By repeating the DDAG cycle for each eigenvector the multiclass algorithm has the capability of assigning multiple labels to the input signal.
4 Conclusion

In this paper we presented a multiclass LS-SVM architecture for DOA estimation as applied to a CDMA cellular system. Simulation results show a high degree of accuracy, as related to the DOA classes and prove that the LS-SVM DDAG system has a wide range of performance capabilities. The broad range of our research in machine learning based DOA estimation includes multilabel and multiclass classification, classification accuracy, error control and validation, kernel selection, estimation of signal subspace dimension, and overall performance characterization of the LS-SVM DDAG DOA estimation algorithm.

References


