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# RELATIVE PRICES, TECHNICAL CHANGE, AND ENVIRONMENTAL PRESERVATION

V. KERRY SMITH\*

## INTRODUCTION

Advances in production technology will tend to augment the supply of manmade goods and services while leaving our ability to produce the amenity services furnished by natural environments unaffected.<sup>1</sup> Such natural environments are gifts of nature, depending upon accidents of biological evolution, geomorphology, and ecological succession and are therefore not producible by man. Three recent legislative actions, the Wilderness Act of 1964,<sup>2</sup> the Wild and Scenic Rivers Act of 1968,<sup>3</sup> and the National Environmental Policy Act of 1969<sup>4</sup> have drawn attention to the problems associated with the allocation of such resources to their most appropriate uses.

One use of these resources, for primarily extractive purposes, has apparently experienced sufficiently pervasive improvements in technology so that the supply of these materials (*e.g.*, mineral ores) has been expanding at constant or falling relative supply prices.<sup>5</sup> Alternatively, Krutilla and Cicchetti<sup>6</sup> have recently noted in the evaluation of two incompatible uses of a wilderness area (the Hells Canyon Reach of the Snake River in Idaho and Oregon) that over time the relative value of amenity services provided by the wilderness area in its preserved status is likely to appreciate. Moreover, such an increase in relative value must be taken into account in the analysis of preservation versus development. The specific alternative considered for Hells Canyon is the construction of a hydroelectric facility.

One of the primary components of the Krutilla-Cicchetti analysis is a dynamic demand model which suggests that technical change will cause income growth and this growth will in turn result in relative

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1. In this case I refer to a particular class of amenity services, namely those associated with the recreational use of natural environments.

2. 16 U.S.C. §§ 1131-1136 (1970).

3. 16 U.S.C. §§ 1271-1287 (1970).

4. 42 U.S.C. §§ 4321-4347 (1970).

5. H. Barnett & C. Morse, *Scarcity and Growth* 202-17 (1963).

6. Krutilla & Cicchetti, *Evaluating Benefits of Environmental Resources With Special Application to the Hells Canyon*, 12 Nat. Res. J. 1 (1972).

price appreciation. They note that, given the ability of the Hells Canyon area to supply a fixed amount of amenity services of constant quality, and assuming the growth in incomes, then over time the relative prices of such services will increase. In order to guarantee this behavior they assume that: (1) the present services of the environmental resources have no close substitutes, (2) income and initial price elasticities of demand for such services are larger than for produced goods, and finally (3) the fraction of the budget spent on the environmental service is smaller than for produced goods in general.<sup>7</sup> While the higher income elasticities of demand for amenity services relative to fabricated goods seem reasonable, the larger price elasticities are not as clearly so. Since there may be as many as 2,000 tracts of public lands for which allocation decisions will need to be made in the next ten to fifteen years, there is a need to examine these conditions more closely. Accordingly, the purpose of this paper is to discuss the results of a simple three-good, general equilibrium model which relates the relative prices of any pair of goods to their demand and supply conditions. In so doing, it is shown that the Krutilla-Cicchetti conditions for relative price appreciation are sufficient but not necessary (that is, a less restrictive set would suffice for their results).

More generally, this model has important implications for Baumol's earlier model of unbalanced growth in which he suggested that asymmetric technical change would have specific effects upon resource allocation and relative price change.<sup>8</sup> Baumol's omission of an explicit specification of community demand has precipitated confusion in some later comments on his model. One such example is a paper by Worcester,<sup>9</sup> in which he suggests that Baumol's conclusions can be modified by the specification of a family of community indifference curves rather than through the use of price and income elasticities.<sup>10</sup> However, the choice of income and price elasticities

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7. *Id.* at 1-13.

8. Baumol, *Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis*, 57 *Am. Econ. Rev.* 415 (1967).

9. Worcester, *Macroeconomics of Unbalanced Growth: Comment*, 58 *Am. Econ. Rev.* 886 (1968).

10. Worcester noted:

... his [Baumol's] analysis suggests the less likely of at least two logical conclusions, namely that the production in the less progressive sector will tend to diminish. This may reflect Baumol's concentration on the price and income elasticities of goods in the unprogressive sector and to his failure to introduce a specific community indifference curve. . . . under the technological conditions postulated any of a large family of community indifference curves will eventually place virtually the whole labor force into the unprogressive sector, and it is by no means clear that this family is a less likely eventuality.

*Id.* at 887-8.

and a community indifference mapping *are not* independent considerations. In order to illustrate this point, the present model is expressed in terms of community indifference and production possibility curves. These results are then linked to the implied demand elasticities, since the specification of an indifference mapping whether for the individual or the community determines a specific structure of price, cross, and income elasticities.

The second section of this paper outlines the components of the model, and discusses the implications of asymmetric technical change for relative price behavior. The third section adapts the model to introduce a specific type of intertemporal externalities, irreversibilities, and summarizes their implications for price appreciation. The last section discusses the importance of the analysis for public policy formulation with natural resources.

### ASYMMETRIC TECHNICAL CHANGE AND RELATIVE PRICES

General equilibrium models are typically easy to formulate in concept but operationally difficult to work with. The conclusions reported herein are derived from a three-good, neoclassical model (outlined in the Appendix) in which demand is expressed in terms of community indifference curves (hereinafter CIC) and supply in terms of production transformation frontiers. The use of CIC mapping does not necessarily require that the societal utility function be derived directly from that of the individuals within the community. Rather it is only necessary to assume that the society behaves *as if* the function describes its behavior. Samuelson has noted that these curves: "... give us a 'demand relationship' ... and *essentially nothing more*."<sup>11</sup> For the present model this is sufficient.

The production possibility frontier describes the locus of output combinations available to the society with given technology and resource base. Technical change will be assumed to be given exogenously in this model and to consist of better methods of production and organization which improve the efficiency of all factors. The effects of a changing technology upon the nature of the productive structure and underlying production decisions are left unexplained, subsumed in the specified path of the production possibility curve. In what follows we assume the frontier maintains its conventional contour; the third section will discuss somewhat the effect of irreversibilities upon the shape and also the position of the curve.<sup>12</sup>

11. Samuelson, *Social Indifference Curves*, 70 Q. J. Econ. 1, 4 (1956).

12. An additional issue is the effect of the factor markets upon the transformation locus. The neoclassical model assumes the absence of market imperfections. It is not clear that the production of amenity services can realistically be treated in this fashion. Smith and Krutilla

In order to illustrate the method by which the model is analyzed consider the following graphical example. Figure 1 describes a comparable model with two goods. M is the fabricated or manufactured commodity which is benefited by technical change and F represents the amenity services of a given stock of natural environments. The curves  $I_1$ ,  $I_2$ , and  $I_3$  are excerpts from the community's utility map describing its preferences for F and M. The curves  $TT_1$ ,  $TT_2$ , and  $TT_3$  are three production transformation curves corresponding to different time periods and the resultant change in technology. For example, the shift from  $TT_1$  to  $TT_2$  illustrates the pattern of change such that the production of M, the fabricated good, is progressively favored over time, stretching the terminus of the curve on the M axis.

Points, A, B, and C designate the comparative static equilibrium points. These points indicate the combinations of M and F where the marginal rate of substitution in consumption and marginal rate of transformation from production are equal. Consequently, all resources are fully employed, and there are no gains to be realized by reallocations between the production of M and F. On the demand side there are also no gains to be realized by choosing an alternative combination.

Corresponding to each of these equilibrium points is a commodity mix and price ratio. The price ratio can be determined by examining the slope of tangents to the points A, B, and C. These tangents are given in Figure 1 by  $PP_1$ ,  $PP_2$ , and  $PP_3$  respectively. Our primary interest is in how the slope changes with technical change and a given structure of demand. Thus, for our example the figure makes it clear that the price of M relative to F is declining in moving from  $PP_1$  to  $PP_2$  to  $PP_3$ . However, the rate of price change relative to the rate of technological change is not clear. Thus, there is need to specify a form for the CIC mapping and the production transformation curves

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have recently developed a model under this assumption. However, if we assume that amenity services are produced by combining the direct services furnished by an environmental resource facility and labor (in the form of managerial action), then one might suggest that these environmental services do not exchange in organized markets.

One way of handling this issue is to consider it a factor market imperfection. Johnson has analyzed the effects of such imperfections upon the shape of the transformation locus and found they can introduce nonconvexities. In a more general analysis (*i.e.*, Johnson assumes Cobb-Douglas production functions) Jones found that a small elasticity of substitution between factors in each activity can reduce nonconvexities. Given the nature of the production process for amenity services from natural environments, it is not unreasonable to suggest that there are very limited substitution possibilities between the direct services of environmental resources and labor.

See Smith & Krutilla, *Technical Change and Environmental Resources*, 6 Socio-Economic Planning Sciences 125 (1972); Johnson, *Factor Market Distortions and the Shape of the Transformation Curve*, 34 *Econometrica* 686 (1966); Jones, *Distortions in Factor Markets and the General Equilibrium Model of Production*, 79 *J. Pol. Econ.* 437 (1971).

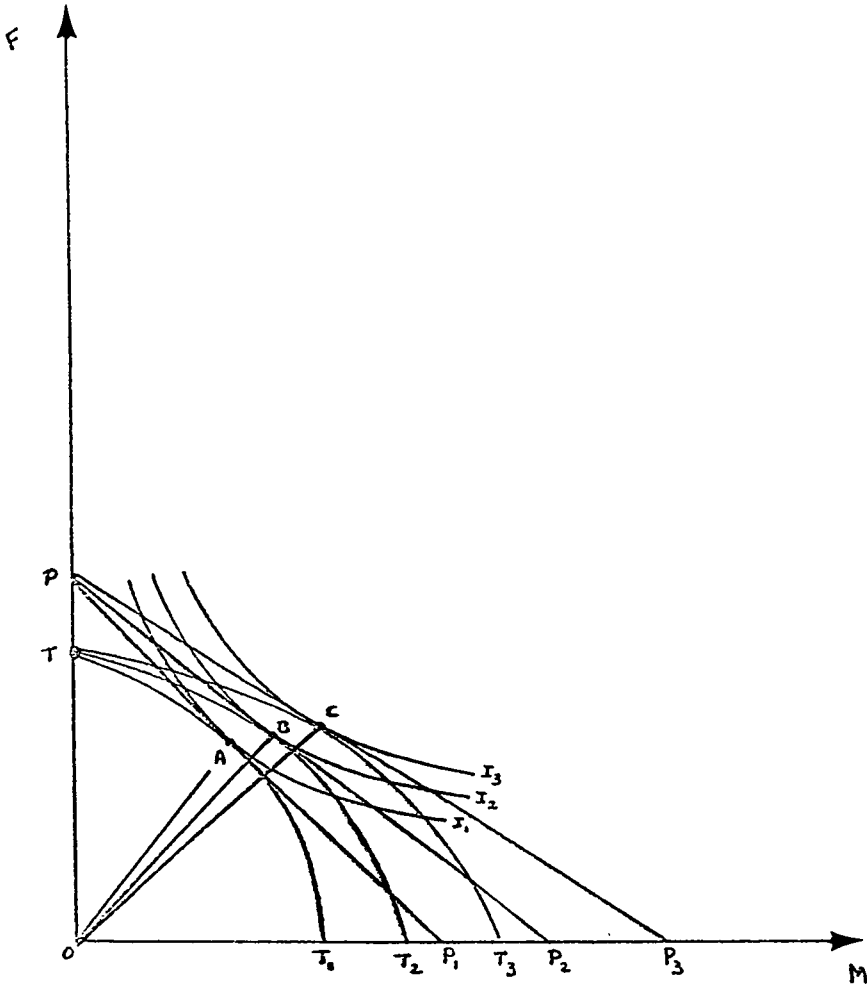


FIGURE 1

and to derive an expression for the equilibrium relative prices, in order to define precisely these interrelationships. The appendix to this paper sets out the formal model for such a derivation.

Figure 2 illustrates the process of asymmetric technical change in a three good model. M and F remain as previously and X represents a third good or service which can be defined to be a substitute for either M or F. Hence the third commodity allows us to illustrate the impact of substitute or compliment goods upon the movement in relative prices. As equation A8 in the Appendix illustrates, there are three important considerations in determining the rate at which the relative price of M will move with respect to that of F. In the absence

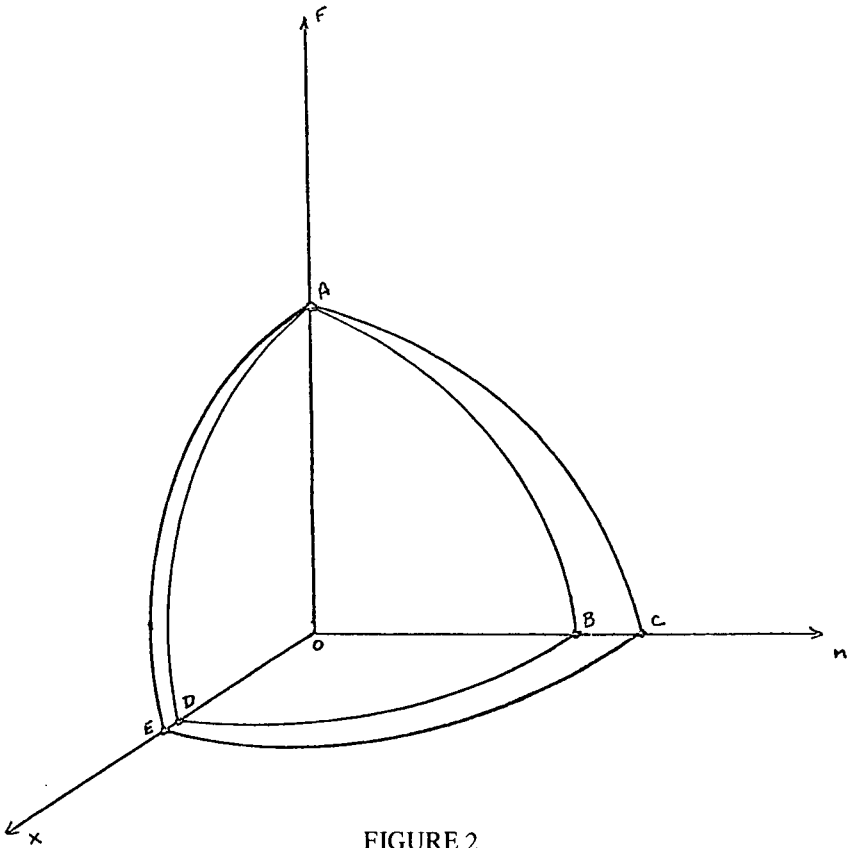


FIGURE 2

of irreversibilities and in the presence of asymmetric technical change, the rate of relative price change is determined by: (1) the income elasticity of demand for amenity services relative to that of fabricated or manufactured goods, (2) the availability of reasonably close substitutes for these amenity services, and (3) the ability of the economy to transform its resources so as to produce alternative commodity mixes.

The first of these conditions is straightforward. It suggests that a relatively larger percent increase in the quantity demanded of amenity services than that for manufactured goods will result from a given increase in income. Hence these amenity services are termed income elastic. The second condition again refers to the character of demand. It indicates that the community feels there are few if any close substitutes for amenity services. It should be noted that this substitution refers to substitution in demand and depends upon

tastes.<sup>13</sup> The last factor refers to the character of the process by which amenity services and manufactured goods are supplied. Movement along a production possibility frontier provides an inventory of the alternative, efficient output combinations available to society. Such movement necessarily defines how readily our resource base may be transformed into one or the other goods which occupy our attention. The production possibility function employed in our model implies that at any level of output of either commodity we can nonetheless get some constant fraction of the amount of the other good given up. Conventional economic analysis suggests that the law of diminishing returns is operative so that as additional amounts of fabricated goods are desired, society must give up ever increasing quantities of amenity services. Unfortunately these statements are not well defined. Thus, our production possibility curve is defined in terms of a partial elasticity of transformation ( $\tau$ ) which measures the severity of the process of diminishing returns.

Thus our results suggest that the Krutilla-Cicchetti conditions on (1) the initial values of price elasticities and (2) the proportion of the budget allocated to each commodity may be dropped without affecting the movement of relative prices. The last condition we derive as important was anticipated in Krutilla's earlier work: "... it is improbable that technology will advance to the point at which the grand geomorphologic wonders could be replicated or extinct species resurrected."<sup>14</sup> Accordingly, for those cases in which one of the goods in our model is the service flow furnished by an irreproducible environmental resource, it is likely that the absolute magnitude of the relevant partial elasticity of transformation will be small, and thus, given equation A8, the rate of relative price change will be accentuated.

More generally these findings have implications for Baumol's model of unbalanced growth. The demand structure (*i.e.*, price, cross, and income elasticities) is directly linked to any specification of a CIC mapping. By assuming a pattern of elasticities, one is, in effect, restricting the admissible CIC mappings. Thus, Worcester's comment is not a legitimate critique. More recently Keren has suggested that Baumol's *Proposition 2* is not generally true.<sup>15</sup> In it Baumol suggests that: "... there is a tendency for the outputs of the non-

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13. For a discussion of the characteristics of wilderness users, see Cicchetti, *A Multivariate Statistical Analysis of Wilderness Users in the United States*, in *Natural Environments: Studies in Theoretical and Applied Analysis* (J. Krutilla ed. 1972).

14. Krutilla, *Conservation Reconsidered*, 57 *Am. Econ. Rev.* 777, 783 (1967).

15. Keren, *Macroeconomics of Unbalanced Growth: Comment*, 62 *Am. Econ. Rev.* 149 (1972).



progressive sector . . . to decline and perhaps, ultimately, to vanish."<sup>16</sup> Keren shows that if we assume each of the two goods has a unitary price elasticity, then the output of the nonprogressive sector will remain constant but *not* vanish. I would suggest that there is a specification of the community's demand structure which will allow the proposition to hold. This point can be made simply with a two-good model similar in format to the three-commodity model previously described. For it, a constant elasticity of substitution utility function and constant elasticity of transformation production possibility curve are assumed.<sup>17</sup> It is clear that the quantity of the nonprogressive output chosen will depend upon the relationship between the two goods in the community's preferences. That is, for our example, if the progressive and nonprogressive goods are substitutes (that is, elasticity of substitution between them exceeds unity,  $\sigma > 1$ ), then the nonprogressive output will approach zero. In the cases where they are independent ( $\sigma = 1$ ) or complementary ( $\sigma < 1$ ), the nonprogressive output will reach a non-zero constant level.<sup>18</sup> Moreover, in the latter cases the price elasticity of demand for the nonprogressive and progressive outputs need not be unity (*i.e.*,

16. Baumol, *supra* note 8, at 418.

17. Alternative assumed utility functions will alter the specific conditions required for the nonprogressive output to remain constant. For example, if the utility function were assumed to be Cobb-Douglas, then the nonprogressive output will approach a constant level with technical change. For a discussion of the relationship between utility functions and the structure of demand, see Smith, *Utility Functions and Demand Structure*, 20 Can. J. Agricultural Econ. 52 (1972).

18. Suppose we postulate the same model mechanism with two goods and our CIC and transformation curves given as:

$$U = (a_1 x_1^{\frac{1+\beta}{\beta}} + a_2 x_2^{\frac{1+\beta}{\beta}})^{\frac{\beta}{1+\beta}}, \beta = -\sigma$$

$$k = \alpha_1 x_1^{1-1/\tau} + \alpha_2 x_2^{1-1/\tau}$$

Solving for the equilibrium commodity mix after equating the marginal rate of substitution between  $x_1$  and  $x_2$  to the marginal rate of transformation, and substituting it back into the transformation function provides:

$$x_2 = \left[ \frac{k}{1 + \left(\frac{a_1}{a_2}\right)^{\gamma(1-1/\tau)} \left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{\tau(\sigma-1)}{\tau-\sigma}}} \right]^{\frac{\tau}{\tau-1}}$$

where  $\gamma = \frac{\tau \cdot \sigma}{\tau - \sigma}$

As  $(\alpha_2/\alpha_1)$  increases, technical change favors  $x_1$  and this equation tells us the conditions under which the equilibrium choice of  $x_2$  will change. I am grateful to Clifford Russell and Karl Göran-Måler for suggesting this line of argument.

$E_{P_i}(x_i) = K_i(1 - \sigma) + \sigma$ , where  $K_i$  = proportion of the budget spent on  $x_i$ ).<sup>19</sup>

### IRREVERSIBILITIES

While most conventional analyses of the implications of externalities have focused upon the contemporaneous effects of one individual's actions upon others, there is nothing inherent in their character to prevent the definition from being broadened to include effects which extend over a number of time periods. Mishan has defined externalities as arising:

... whenever the value of a production function, or a consumption function, depends directly upon the activity of others ... the essential feature of the concept of an external effect is that the effect produced is not a deliberate creation but an *unintended* or *incidental* by-product of some otherwise legitimate activity.<sup>20</sup>

The "others" in Mishan's commentary can be broadened to include not only current consumers and producers but also future and past individuals. The work of Krutilla and Cicchetti suggests that irreversibilities, one member of the class of intertemporal effects, are fundamental to the allocation problem for environmental resources. Since they make no attempt to integrate them explicitly into the analysis of relative price behavior, a simple alteration is proposed to the current model.

However, before proceeding to discuss their implications it is desirable to address the relationship of irreversibilities and the shape of the production possibility frontier. Baumol and Bradford have suggested that sufficiently serious production externalities are likely to distort the contours of the social production curve.<sup>21</sup> The reasoning underlying their statement is straightforward. Consider an economy composed of two firms, each producing a different good. If the production of one of these goods results in the generation of industrial residues or wastes, and these are dispersed in such a way as to affect the other firm, then production externalities are present. Thus at every point between the endpoints of the transformation curve, producing more output by the polluting firm not only requires a larger share of the presumed fixed-factor supplies, but more importantly inhibits the ability of the other firm to produce its output at any

19. See Hicks & Allen, *A Reconsideration of the Theory of Value*, 1 *Economica* 196 (1934).

20. Mishan, *The Postwar Literature on Externalities: An Interpretative Essay*, 9 *J. Econ. Literature* 1, 2 (1971) [emphasis in original].

21. Baumol & Bradford, *Detrimental Externalities and Nonconvexity of the Production Set*, 39 *Economica* 160 (1972).

level of resource usage. Consequently there are more than the usual increasing marginal costs at work with such externalities present.

Irreversibilities affect the frontier in any given period through the available resource base. Thus the tradeoff between manufactured goods and amenity services in period depends upon the production processes for each and the available factor supplies necessary to produce them. Irreversible resource allocations remove some fraction of the stock of environmental resources from effective supply. Hence the dimensions of the Edgeworth-Bowley box are affected in future periods. Smith and Krutilla have demonstrated its affect with Cobb-Douglas production functions.<sup>22</sup> Assuming the process is recursive in that past decisions on the quantity of the manufactured good consumed affect the stock of environmental resources available for the future, then irreversibilities will not distort the shape of the social production curve for any given period. In this case, for a given period a movement along either axis (thereby changing the selected commodity mix) does not affect the production of the other commodity in any way other than the distribution of the fixed stocks of factor inputs. Once a choice from the locus of production possibilities is made, society preempts certain future selections by using resources in an irreversible manner. However, the recursive nature of the allocation prevents distortions to any period's social production set.

Rather than introducing production functions explicitly into the analysis to determine the effects of irreversibilities, an observationally equivalent approach is to postulate that irreversibilities will result in a shifting of the production possibility curve's origin along the vertical axis into the negative quadrants. Technical change favoring the fabricated and third good (X) in our model will offset this effect by shifting the terminus of the production possibility frontier on each of the M and X axes outward. Accordingly, the net observed effect is an increase in the potentially available output levels for these goods and a concomitant increase in the *absolute* scarcity of the amenity services (F). These services do not benefit (by assumption) from technical change and hence suffer the full adverse effects of irreversibilities. In terms of equation A8 in the appendix, it is seen that demand and supply factors again determine the magnitude of the relative price change resulting from irreversibilities.

#### IMPLICATIONS

Conventional benefit-cost analyses have attempted to "adjust" their estimates for potential biases due to inflationary or deflationary

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22. Smith & Krutilla, *supra* note 12, at 130-31.

movements in the general price level.<sup>23</sup> However, changes in the relative prices of goods and services due to non-project related forces such as asymmetric technological change and irreversibilities in the resource allocation have not been taken into account. Krutilla and Cicchetti have argued that there are important reasons why a distinction must be drawn between inflationary movements in the general price level and relative price changes. They note:

The costs of extracting natural resource commodities and their market prices historically were shown to have remained either stable (for some) or actually declined (for others) relative to the price of goods and services in general. Accordingly, since these were the commodities which were being produced, in part, as outputs of public resource development programs, there was in fact an authentic change in the price of outputs of such programs relative to the general price level.<sup>24</sup>

For those cases where public intervention involves a choice between preservation and development of unique natural environments, the movement of relative prices of the incompatible service flows is extremely important to the evaluation of the project. The Krutilla-Cicchetti model provides a means of accounting for their effects in our assessment of the benefits required of the preserved alternative. The research reported in this paper indicates that it is possible to relax some of their assumptions and maintain the postulated movement in relative prices. Moreover, in setting out the direct relationship among technical change, irreversibilities and the rate of relative price change, this analysis broadens the applicability of their model.

#### APPENDIX

The basic components of the model are a community indifference mapping specification and a production possibility curve as given in (A1) and (A2).

$$(A1) \quad U = (a_1 F_t^{-\beta_1} + a_2 M_t^{-\beta_2} + a_3 X_t^{-\beta_3})^{-1/\beta}$$

23. See testimony of Henry P. Caulfield, Jr., Executive Director, Water Resources Council, in *Hearings on Economic Analysis of Public Investment Decisions: Interest Rate Policy and Discounting Analysis Before the Subcomm on Economy of the Joint Economic Comm.*, 90th Cong., 2d Sess. 8 (1968).

24. Krutilla & Cicchetti, *Benefit-Cost Analysis and Technologically Induced Relative Price Changes: The Case of Environmental Irreversibilities*, in Staff of Joint Economic Comm., 92d Cong., 2d Sess., *Benefit Cost Analysis of Federal Programs* 60, 61 (Comm. Print 1973) [footnote in original omitted].

$$(A2) \quad \alpha_{1t} F_t^{1-1/\tau} + \alpha_{2t} M_t^{1-1/\tau} + \alpha_{3t} X_t^{1-1/\tau} = k(1-1/\tau)$$

$\tau$  = partial elasticity of substitution

Technical change is assumed to be moving at a constant rate favoring M and X relative to F. Thus we postulate (A3) and (A4)

$$(A3) \quad \left(\frac{\alpha_2}{\alpha_1}\right)_t = \gamma_0 e^{gt} \quad , \quad g < 0$$

$$(A4) \quad \left(\frac{\alpha_3}{\alpha_1}\right)_t = \lambda_0 e^{rt} \quad , \quad r < 0$$

Solving for the marginal rate of substitution (MRS) between F and M from (A1) and the corresponding marginal rate of transformation (MRT) from (A2), it is possible to equate them and determine the equilibrium price ratio or commodity mix as a function of the parameters of these functions.

Irreversibilities are introduced by replacing (A2) with (A5) and (A6)

$$(A5) \quad \alpha_{1t} (F_t - F_t^*)^{1-1/\tau} + \alpha_{2t} M_t^{1-1/\tau} + \alpha_{3t} X_t^{1-1/\tau} = k(1-1/\tau)$$

$$(A6) \quad F_t^* = f(M_{t-1}, M_{t-2}, \dots, M_{t-n}) \quad , \quad \frac{\partial f}{\partial M_{t-i}} > 0$$

for all i

The rate of change in relative prices may be analyzed by taking the functions derived using the expressions resulting from equating the MRS from (A1) to the MRT from (A5), substituting from (A3) and (A4) and differentiating with respect to time. This process provides an expression for the rate of change in relative prices in terms of the parameters of all these functions. While this expression may be instructive, it is not very useful, since these parameters do not have ready economic interpretation. Using the results from this author's analysis of utility functions (*i.e.*, V. K. Smith, *Utility Functions and Demand Structure*, 20 Can. J. Agricultural Econ. 52 (1972) it is possible to translate into income elasticities. For example we know that the ratio of the income elasticities ( $E_y(\cdot)$ ) for F and M may be expressed as:

$$(A7) \quad \frac{E_y(F)}{E_y(M)} = \frac{1 + \beta_2}{1 + \beta_1}$$

With these relationships it is possible to derive (A8).

$$(A8) \delta = \frac{E_y(F) g}{(E_y(F) - \frac{E_y(M) \cdot \sigma_{FM}}{\tau \sigma_{MX}(1+\beta_3)})} + \frac{E_y(F) - E_y(M)}{\tau \cdot E_y(F) - \frac{E_y(M)\sigma_{FM}}{\sigma_{MX}(1+\beta_3)}} \frac{\dot{F}}{(F_t - F_t^*)}$$

where  $\delta = \left[ \frac{d\left(\frac{P_M}{P_F}\right)}{dt} \Big/ \left(\frac{P_M}{P_F}\right) \right]$

$E_y(M)$  = income elasticity of demand for M

$E_y(F)$  = income elasticity of demand for F

$\sigma_{ij}$  = partial elasticity of substitution between commodities i and j

$\dot{F}$  =  $\frac{dF}{dt}$