

9-10-2024

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Recommended Citation

B, Ananda Priya; Gnanachandra P; Saeid Jafari; and Florentin Smarandache. "Neutrosophic Rhotrices for Improved Diagnostic Accuracy through Score Rhotrix Computation." *Neutrosophic Sets and Systems* 73, 1 (2024). https://digitalrepository.unm.edu/nss_journal/vol73/iss1/58

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Neutrosophic Rhotrices for Improved Diagnostic Accuracy through Score Rhotrix Computation

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Abstract. We introduce neutrosophic rhotrices, which serve as a novel extension of neutrosophic matrices. The primary objective is to establish the foundational structure for neutrosophic rhotrices and to define crucial operations that can be performed on them. We explore the concept of neutrosophic rhotrices in depth, outlining the fundamental operations necessary for their effective manipulation. Furthermore, we investigate the essential properties of neutrosophic rhotrices utilizing these newly established operations. In addition, we provide an algorithm designed to enhance decision-making processes in medical diagnostics, supported by an illustrative example to clarify its application.

Keywords: Neutrosophic Rhotrix, Rhotrix, Heart based Multiplication, Trace

In 2003, Ajibade [1] introduced a method for representing arrays of numbers in a rhomboidal shape, which he named rhotrix and developed the structure of an n -dimensional rhotrix to fall between the $(n - 1) \times (n - 1)$ - dimensional matrix and the $n \times n$ - dimensional matrix. Ajibade [10] also proposed the first multiplication method for rhotrices, called heart-based multiplication. In 2004, B. Sani [2] introduced another multiplication method for rhotrices, known as row-column multiplication, which is similar to traditional matrix multiplication. These two methods are the primary techniques for rhotrix multiplication, though various other multiplications can be defined, none of which generate algebraic structures.

Sani [3] also introduced a technique for transforming a rhotrix into a matrix form, referred to as a coupled matrix. In 2008, A. O. Isere [4] expanded on Ajibade's work by introducing

the concept of even-dimensional rhotrix, since Ajibade's rhotrix is always of odd dimension. Additionally, numerous authors have contributed to the development of rhotrix theory, exploring concepts such as rhotrix groups [7, 11], rhotrix rings [5], rhotrix vector spaces [6], and the application of rhotrices in various fields, including cryptography [12–14].

The concept of neutrosophy was introduced by Florentin Smarandache [8, 16] in the 1990s. Neutrosophic logic extends classical and fuzzy logic by considering three components: truth (T), indeterminacy (I), and falsity (F). This triad is utilized to handle real-world problems where information is incomplete, inconsistent, or uncertain.

Kandasamy and Smarandache [17–19] expanded on this concept by introducing neutrosophic algebraic structures, including neutrosophic fields, vector spaces, groups, and rings. In linear algebra, matrices are essential for understanding vector spaces and linear transformations, prompting the creation of neutrosophic matrices. A neutrosophic matrix is an extension of the classical matrix concept, incorporating neutrosophic logic, which deals with indeterminacy. Recently, Mohammad Abobala and et al. [15] investigated the algebraic properties of these matrices, such as diagonalization, invertibility, determinants, and their algebraic representations through linear transformations. Neutrosophic matrices have found applications in various fields, particularly in dealing with uncertain, inconsistent, and incomplete information. Mamoni Dhar [9], introduced neutrosophic soft matrices and also a score matrix addressing patient-symptoms and symptoms-disease neutrosophic soft matrices is also proposed to aid in decision-making. Recently, the authors [20] introduced fuzzy rhotrices and its application in decision making of medical diagnostics has been studied.

We seek to introduce the concept of neutrosophic rhotrices and establish the fundamental operations required for their manipulation. We will define the basic operations and furthermore, conduct an in-depth examination of the fundamental properties of neutrosophic rhotrices, utilizing the newly defined operations to explore their characteristics. Additionally, an algorithm is proposed for medical diagnosis using neutrosophic rhotrices accompanied with an illustrative example.

$$\text{as } (R_1)_N = \left\langle \begin{matrix} & (\mu_R(r_1), \mathcal{I}_R(r_1), \mathcal{F}_R(r_1)) \\ (\mu_R(r_2), \mathcal{I}_R(r_1), \mathcal{F}_R(r_1)) & (\mu_R(r_3), \mathcal{I}_R(r_1), \mathcal{F}_R(r_1)) & (\mu_R(r_4), \mathcal{I}_R(r_1), \mathcal{F}_R(r_1)) \\ & (\mu_R(r_5), \mathcal{I}_R(r_1), \mathcal{F}_R(r_1)) \end{matrix} \right\rangle$$

In simpler terms, it can be written as,

$$(R_1)_N = \left\langle \begin{matrix} & (r_1^T, r_1^I, r_1^F) \\ (r_2^T, r_2^I, r_2^F) & (r_3^T, r_3^I, r_3^F) & (r_4^T, r_4^I, r_4^F) \\ & (r_5^T, r_5^I, r_5^F) \end{matrix} \right\rangle$$

Definition 1.2. We define the **operations on neutrosophic rhotrices** as follows:

(i) The addition of two neutrosophic rhotrices is possible only if they are of the same size.

Addition of two neutrosophic rhotrices ' A_N ' and ' B_N ' is defined as:

The i^{th} entry of $A_N + B_N = (max\{a_i^T, b_i^T\}, max\{a_i^I, b_i^I\}, min\{a_i^F, b_i^F\})$, for all i .

$$\text{That is, for rhotrices } A_N = \left\langle \begin{matrix} & (a_1^T, a_1^I, a_1^F) \\ (a_2^T, a_2^I, a_2^F) & (a_3^T, a_3^I, a_3^F) & (a_4^T, a_4^I, a_4^F) \\ & (a_5^T, a_5^I, a_5^F) \end{matrix} \right\rangle \text{ of size 3, and } B_N = \left\langle \begin{matrix} & (b_1^T, b_1^I, b_1^F) \\ (b_2^T, b_2^I, b_2^F) & (b_3^T, b_3^I, b_3^F) & (b_4^T, b_4^I, b_4^F) \\ & (b_5^T, b_5^I, b_5^F) \end{matrix} \right\rangle \text{ of size 3,}$$

$$A_N + B_N = \left\langle \begin{matrix} & (max\{a_1^T, b_1^T\}, max\{a_1^I, b_1^I\}, min\{a_1^F, b_1^F\}) \\ (max\{a_2^T, b_2^T\}, max\{a_2^I, b_2^I\}, min\{a_2^F, b_2^F\}) & (max\{a_3^T, b_3^T\}, max\{a_3^I, b_3^I\}, min\{a_3^F, b_3^F\}) & (max\{a_4^T, b_4^T\}, max\{a_4^I, b_4^I\}, min\{a_4^F, b_4^F\}) \\ & (max\{a_5^T, b_5^T\}, max\{a_5^I, b_5^I\}, min\{a_5^F, b_5^F\}) \end{matrix} \right\rangle$$

(ii) The heart based neutrosophic multiplication of two neutrosophic rhotrices is possible only if they are of the same size. Multiplication of two neutrosophic rhotrices ' A ' and ' B ' is defined as:

The i^{th} entry of $A_N \circ B_N = (max\{min(a_i^T, e_b^T), min(b_i^T, e_a^T)\}, max\{min(a_i^I, e_b^I), min(b_i^I, e_a^I)\}, min\{max(a_i^F, e_b^F), max(b_i^F, e_a^F)\})$, for all i , except the heart

Heart of $A_N \circ B_N = (min\{e_a, e_b\}, min\{e_a, e_b\}, max\{e_a, e_b\})$

Definition 1.3 (Trace of Neutrosophic Rhotrix). The trace of a neutrosophic rhotrix of size ' n ' is adding the entries in major vertical axis and it is denoted as $Tr(.)$.

$$\text{That is, for a rhotrix } A_N = \left\langle \begin{matrix} & (a_1^T, a_1^I, a_1^F) \\ (a_2^T, a_2^I, a_2^F) & (a_3^T, a_3^I, a_3^F) & (a_4^T, a_4^I, a_4^F) \\ & (a_5^T, a_5^I, a_5^F) \end{matrix} \right\rangle \text{ of size 3,}$$

$$Tr(A_N) = (max\{a_1^T, a_3^T, a_5^T\}, max\{a_1^I, a_3^I, a_5^I\}, min\{a_1^F, a_3^F, a_5^F\})$$

Theorem 1.4. For any neutrosophic rhotrices with the neutrosophic addition operation, the following axioms holds:

- $(A_N + B_N) + C_N = A_N + (B_N + C_N)$ (Associativity),
- $A_N + O = A_N$, where $O = \begin{pmatrix} (0, 0, 1) \\ (0, 0, 1) & (0, 0, 1) & (0, 0, 1) \\ (0, 0, 1) \end{pmatrix}$ (Additive Identity),
- $A_N + B_N = B_N + A_N$ (Commutativity), where A_N, B_N , and C_N are any neutrosophic rhotrices

Proof. Let A_N, B_N , and C_N of size 3 be any three neutrosophic rhotrices . Then,

$$A_N + (B_N + C_N) = (A_N + B_N) + C_N =$$

$$\begin{pmatrix} (max\{a_1^T, b_1^T, c_1^T\}, max\{a_1^I, b_1^I, c_1^I\}, \\ min\{a_1^F, b_1^F, c_1^F\}) \\ (max\{a_2^T, b_2^T, c_2^T\}, max\{a_2^I, b_2^I, c_2^I\}, \\ min\{a_2^F, b_2^F, c_2^F\}) & (max\{a_3^T, b_3^T, c_3^T\}, max\{a_3^I, b_3^I, c_3^I\}, \\ min\{a_3^F, b_3^F, c_3^F\}) & (max\{a_4^T, b_4^T, c_4^T\}, max\{a_4^I, b_4^I, c_4^I\}, \\ min\{a_4^F, b_4^F, c_4^F\}) \\ (max\{a_5^T, b_5^T, c_5^T\}, max\{a_5^I, b_5^I, c_5^I\}, \\ min\{a_5^F, b_5^F, c_5^F\}) \end{pmatrix}$$

Suppose $O = \begin{pmatrix} (x_1^T, x_1^I, x_1^F) \\ (x_2^T, x_2^I, x_2^F) & (x_3^T, x_3^I, x_3^F) & (x_4^T, x_4^I, x_4^F) \\ (x_5^T, x_5^I, x_5^F) \end{pmatrix}$. For $A_N + O = A_N$,

$$\begin{pmatrix} (max\{a_1^T, x_1^T\}, max\{a_1^I, x_1^I\}, \\ min\{a_1^F, x_1^F\}) \\ (max\{a_2^T, x_2^T\}, max\{a_2^I, x_2^I\}, \\ min\{a_2^F, x_2^F\}) & (max\{a_3^T, x_3^T\}, max\{a_3^I, x_3^I\}, \\ min\{a_3^F, x_3^F\}) & (max\{a_4^T, x_4^T\}, max\{a_4^I, x_4^I\}, \\ min\{a_4^F, x_4^F\}) \\ (max\{a_5^T, x_5^T\}, max\{a_5^I, x_5^I\}, \\ min\{a_5^F, x_5^F\}) \end{pmatrix}$$

$$= \begin{pmatrix} (a_1^T, a_1^I, a_1^F) \\ (a_2^T, a_2^I, a_2^F) & (a_3^T, a_3^I, a_3^F) & (a_4^T, a_4^I, a_4^F) \\ (a_5^T, a_5^I, a_5^F) \end{pmatrix}$$

Then, $max\{a_i^T, x_i^T\} = a_i^T, max\{a_i^I, x_i^I\} = a_i^I$ and $min\{a_i^F, x_i^F\} = a_i^F$, for all i , which implies $x_i^T = x_i^I = 0, x_i^F = 1$ for all i . Therefore, the additive identity for rhotrix size 3 is

$$\begin{pmatrix} (0, 0, 1) \\ (0, 0, 1) & (0, 0, 1) & (0, 0, 1) \\ (0, 0, 1) \end{pmatrix}$$

In general, the additive identity for rhotrix size 'n' is a rhotrix

with all its entries as $(0, 0, 1)$.

Since each entry of $A_N + B_N = (max\{a_i^T, b_i^T\}, max\{a_i^I, b_i^I\}, min\{a_i^F, b_i^F\})$ and $max\{a_i, b_i\} =$

$\max\{b_i, a_i\}$, and $\min\{a_i, b_i\} = \min\{b_i, a_i\}$ it is obvious that the commutative axiom holds for neutrosophic addition. \square

Theorem 1.5. *The heart based multiplication operation among any neutrosophic rhotrices holds the following axioms:*

- $(A_N \circ B_N) \circ C_N = A_N \circ (B_N \circ C_N)$ (Associativity),
- $A_N \circ I = A_N$, where $I = \begin{pmatrix} (0, 0, 1) & (1, 1, 0) & (0, 0, 1) \\ (0, 0, 1) & & \end{pmatrix}$ (Multiplicative Identity),
- $A_N \circ B_N = B_N \circ A_N$ (Commutativity), where A_N, B_N , and C_N are any neutrosophic rhotrices

Proof. Let us consider any three neutrosophic rhotrices A_N, B_N , and C_N of size 3, and let e_a, e_b , and e_c be the entries of heart of the rhotrices A, B , and C , respectively. Then, the i^{th} entry of $(A_N \circ B_N) \circ C_N$

$$= \begin{cases} \left(\max\{\min(a_i^T, e_b^T, e_c^T), \min(b_i^T, e_b^T, e_c^T), \min(c_i^T, e_b^T, e_c^T)\}, \right. \\ \left. \max\{\min(a_i^I, e_b^I, e_c^I), \min(b_i^I, e_b^I, e_c^I), \min(c_i^I, e_b^I, e_c^I)\}, \right. \\ \left. \min\{\max(a_i^F, e_b^F, e_c^F), \max(b_i^F, e_b^F, e_c^F), \max(c_i^F, e_b^F, e_c^F)\} \right) & , \forall i, \text{ except heart} \\ \left(\min\{e_a^T, e_b^T, e_c^T\}, \min\{e_a^I, e_b^I, e_c^I\}, \max\{e_a^F, e_b^F, e_c^F\} \right) & , \text{ for heart} \end{cases}$$

$$= i^{th} \text{ entry of } A_N \circ (B_N \circ C_N)$$

Suppose $I_N = \begin{pmatrix} (x_1^T, x_1^I, x_1^F) & & \\ (x_2^T, x_2^I, x_2^F) & (x_3^T, x_3^I, x_3^F) & (x_4^T, x_4^I, x_4^F) \\ & (x_5^T, x_5^I, x_5^F) & \end{pmatrix}$

For $A_N \circ I_N = A_N$,

The i^{th} entry of $A_N \circ I_N = \begin{cases} \left(\max\{\min(a_i^T, x_3^T), \min(x_i^T, e_a^T)\}, \right. \\ \left. \max\{\min(a_i^I, x_3^I), \min(x_i^I, e_a^I)\}, \right. \\ \left. \min\{\max(a_i^F, x_3^F), \max(x_i^F, e_a^F)\} \right) & , \forall i, \text{ except the heart} \\ \left(\min\{e_a^T, x_3^T\}, \min\{e_a^I, x_3^I\}, \max\{e_a^F, x_3^F\} \right) & , \text{ for heart} \end{cases}$

Then, $\min\{e_a^T, x_3^T\} = e_a^T$, implies $x_3^T = 1$ and similarly $x_3^I = 1$. Also, $\max\{e_a^F, x_3^F\} = e_a^F$, implies $x_3^F = 0$. Since $x_3^T = 1$ and $\max\{\min(a_i^T, x_3^T), \min(x_i^T, e_a^T)\} = a_i^T, x_i^T = 0$. Similarly, $x_i^I = 0$ and $x_i^F = 1$

Therefore, the additive identity for rhotrix size 3 is $I = \begin{pmatrix} & (0, 0, 1) & \\ (0, 0, 1) & (1, 1, 0) & (0, 0, 1) \\ & (0, 0, 1) & \end{pmatrix}$. In

general, the multiplicative identity for rhotrix size 'n' is a rhotrix of size 'n' with all entries (0,0,1) and heart of the rhotrix as (1,1,0).

Since the heart based rhotrix multiplication is commutative, it is obvious that the commutative axiom holds for heart based neutrosophic multiplication. \square

Theorem 1.6. *Heart based multiplication of neutrosophic rhotrices is distributive with respect to addition of neutrosophic rhotrices. That is, $A_N \circ (B_N + C_N) = A_N \circ B_N + A_N \circ C_N$, where A_N, B_N, C_N are neutrosophic rhotrices.*

Proof. Let $A_N = \begin{pmatrix} & (a_1^T, a_1^I, a_1^F) & \\ (a_2^T, a_2^I, a_2^F) & (e_a^T, e_a^I, e_a^F) & (a_4^T, a_4^I, a_4^F) \\ & (a_5^T, a_5^I, a_5^F) & \end{pmatrix}$,
 $B_N = \begin{pmatrix} & (b_1^T, b_1^I, b_1^F) & \\ (b_2^T, b_2^I, b_2^F) & (e_b^T, e_b^I, e_b^F) & (b_4^T, b_4^I, b_4^F) \\ & (b_5^T, b_5^I, b_5^F) & \end{pmatrix}$ be neutrosophic rhotrices.

Then, the i^{th} entry of

$$\begin{aligned} A_N \circ (B_N + C_N) &= \left(\max\{\min(a_i^T, \max\{e_b^T, e_c^T\}), \min(e_a^T, \max\{b_i^T, c_i^T\})\}, \max\{\min(a_i^I, \max\{e_b^I, e_c^I\}), \right. \\ &\quad \left. \min(e_a^I, \max\{b_i^I, c_i^I\})\}, \min\{\max(a_i^F, \min\{e_b^F, e_c^F\}), \max(a_i^F, \max\{e_b^F, e_c^F\})\} \right) \\ &= \left(\max\{\min(a_i^T, e_b^T, e_c^T), \min(b_i^T, e_a^T, e_c^T), \min(c_i^T, e_a^T, e_b^T)\}, \max\{\min(a_i^I, e_b^I, e_c^I), \right. \\ &\quad \left. \min(b_i^I, e_a^I, e_c^I), \min(c_i^I, e_a^I, e_b^I)\}, \min\{\min(a_i^F, e_b^F, e_c^F), \min(b_i^F, e_a^F, e_c^F), \right. \\ &\quad \left. \min(c_i^F, e_a^F, e_b^F)\} \right) \\ &= i^{th} \text{ entry of } (A_N \circ B_N) + (A_N \circ C_N) \end{aligned}$$

Therefore, $A_N \circ (B_N + C_N) = (A_N \circ B_N) + (A_N \circ C_N)$. This means that, we have proved, for any rhotrices of size 3, distributive property holds. In similar manner, we also can prove, distributive axiom holds for any rhotrices of size 'n'. \square

Theorem 1.7. *For two neutrosophic rhotrices A_N and B_N and any scalar λ such that $0 \leq \lambda \leq 1$, the following properties hold:*

- (i) $Tr(A_N + B_N) = Tr(A_N) + Tr(B_N)$
- (ii) $Tr(\lambda A_N) = \lambda Tr_f(A_N)$
- (iii) $Tr(A_N) = Tr(A_N^T)$

Proof. Consider two neutrosophic rhotrices of size 3,

$$A_N = \left\langle \begin{matrix} (a_1^T, a_1^I, a_1^F) \\ (a_2^T, a_2^I, a_2^F) & (a_3^T, a_3^I, a_3^F) & (a_4^T, a_4^I, a_4^F) \\ (a_5^T, a_5^I, a_5^F) \end{matrix} \right\rangle \text{ and}$$

$$B_N = \left\langle \begin{matrix} (b_1^T, b_1^I, b_1^F) \\ (b_2^T, b_2^I, b_2^F) & (b_3^T, b_3^I, b_3^F) & (b_4^T, b_4^I, b_4^F) \\ (b_5^T, b_5^I, b_5^F) \end{matrix} \right\rangle.$$

(i) Since the maximum operation is associative, we have the following:

$$\begin{aligned} Tr(A_N + B_N) &= (max\{max(a_1^T, b_1^T), max(a_3^T, b_3^T), max(a_5^T, b_5^T)\}, max\{max(a_1^I, b_1^I), max(a_3^I, b_3^I), \\ &max(a_5^I, b_5^I)\}, min\{min(a_1^F, b_1^F), min(a_3^F, b_3^F), min(a_5^F, b_5^F)\}) \\ &= (max\{max(a_1^T, a_3^T, a_5^T), max(b_1^T, b_3^T, b_5^T)\}, max\{max(a_1^I, a_3^I, a_5^I), \\ &max(b_1^I, b_3^I, b_5^I)\}, min\{min(a_1^F, a_3^F, a_5^F), min(b_1^F, b_3^F, b_5^F)\}) \\ &= Tr_f(A_N) + Tr_f(B_N) \end{aligned}$$

Therefore, $Tr(A_N + B_N) = Tr(A_N) + Tr(B_N)$, for any neutrosophic rhotrices of size 'n'.

(ii) Now, $Tr(\lambda A_N) = (max\{min(\lambda, a_1^T), min(\lambda, a_3^T), min(\lambda, a_5^T)\}, max\{min(\lambda, a_1^I), min(\lambda, a_3^I), min(\lambda, a_5^I)\}, min\{max(\lambda, a_1^F), max(\lambda, a_3^F), max(\lambda, a_5^F)\}) = (min\{\lambda, max(a_1^T, a_3^T, a_5^T)\}, min\{\lambda, max(a_1^I, a_3^I, a_5^I)\}, max\{\lambda, min(a_1^F, a_3^F, a_5^F)\})$ ($\because (A \wedge B) \vee (A \wedge C) \vee (A \wedge D) = A \wedge (B \vee C \vee D)$, where \wedge and \vee represents *min* and *max* operation resp.) and $\lambda Tr_f(A_f) = (min\{\lambda, max(a_1^T, a_3^T, a_5^T)\}, min\{\lambda, max(a_1^I, a_3^I, a_5^I)\}, max\{\lambda, min(a_1^F, a_3^F, a_5^F)\})$. Therefore, $Tr(\lambda A_N) = \lambda Tr_f(A_N)$.

(iii) Since there is no changes in the major horizontal axis in A_N^T and A_N , $Tr(A_N) = Tr(A_N^T)$.

□

2. Elevating Diagnostic Precision with Neutrosophic Rhotrices and Fuzzy rhotrices

In the context of decision-making, when using matrices for medical diagnosis, if an individual does not exhibit a particular symptom, the value corresponding to that symptom is set to zero, and computations proceed accordingly, accounting for the absence of that symptom. However, when applying rhotrices, individuals can be categorized based on the number of symptoms they exhibit. For example, patients with three symptoms can be grouped together, as can those with two symptoms, which simplifies the overall diagnostic process by creating more manageable groups.

Method 1 (Using Neutrosophic Rhotrices)

In this method, we incorporate neutrosophic rhotrices into medical diagnosis, this framework advances healthcare by effectively managing uncertainty, indeterminacy, and imprecision.

Let S represent the set of symptoms associated with certain diseases, D denote the set of

diseases, and P refer to the set of patients.

Step 1: Construct a symptom-disease neutrosophic rhotrix $A = [a_{i,j}]$ of size n (=number of symptoms and diseases) in the following manner.

Here, $s_i d_j^T$ represents the membership value indicating how much symptom s_i contributes to the occurrence of disease d_j , $s_i d_j^I$ represents the membership value indicating how much symptom s_i contributes to the indeterminacy of disease d_j , and $s_i d_j^F$ represents the membership value indicating how much symptom s_i contributes to the falsity of disease d_j . Thus, the symptom-disease neutrosophic rhotrix A can be formed as

$$\left\langle \begin{matrix} (s_1 d_1^T, s_1 d_1^I, s_1 d_1^F) \\ (s_3 d_1^T, s_3 d_1^I, s_3 d_1^F) & (s_2 d_2^T, s_2 d_2^I, s_2 d_2^F) & (s_1 d_3^T, s_1 d_3^I, s_1 d_3^F) \\ (s_5 d_1^T, s_5 d_1^I, s_5 d_1^F) & (s_4 d_2^T, s_4 d_2^I, s_4 d_2^F) & (s_3 d_3^T, s_3 d_3^I, s_3 d_3^F) & (s_2 d_4^T, s_2 d_4^I, s_2 d_4^F) & (s_1 d_5^T, s_1 d_5^I, s_1 d_5^F) \\ & (s_5 d_3^T, s_5 d_3^I, s_5 d_3^F) & (s_4 d_4^T, s_4 d_4^I, s_4 d_4^F) & (s_3 d_5^T, s_3 d_5^I, s_3 d_5^F) \\ & & (s_5 d_5^T, s_5 d_5^I, s_5 d_5^F) \end{matrix} \right\rangle$$

Step 2: Construct a patient-symptom neutrosophic matrix $B = [b_{i,j}]$ of size n (=number of symptoms and patients) in the following manner.

Here, $p_i s_j^T$ represents the membership value indicating how much the patient p_i suffers from the symptom s_j , $p_i s_j^I$ represents the membership value indicating how much the patient p_i contributes to the indeterminacy of symptom s_j , and $p_i s_j^F$ represents the membership value indicating how much the patient p_i contributes to the falsity of the symptom s_j . Thus, the symptom-disease neutrosophic rhotrix B can be formed as

$$\left\langle \begin{matrix} (p_1 s_1^T, p_1 s_1^I, p_1 s_1^F) \\ (p_3 s_1^T, p_3 s_1^I, p_3 s_1^F) & (p_2 s_2^T, p_2 s_2^I, p_2 s_2^F) & (p_1 s_3^T, p_1 s_3^I, p_1 s_3^F) \\ (p_5 s_1^T, p_5 s_1^I, p_5 s_1^F) & (p_4 s_2^T, p_4 s_2^I, p_4 s_2^F) & (p_3 s_3^T, p_3 s_3^I, p_3 s_3^F) & (p_2 s_4^T, p_2 s_4^I, p_2 s_4^F) & (p_1 s_5^T, p_1 s_5^I, p_1 s_5^F) \\ & (p_5 s_3^T, p_5 s_3^I, p_5 s_3^F) & (p_4 s_4^T, p_4 s_4^I, p_4 s_4^F) & (p_3 s_5^T, p_3 s_5^I, p_3 s_5^F) \\ & & (p_5 s_5^T, p_5 s_5^I, p_5 s_5^F) \end{matrix} \right\rangle$$

Step 3: Compute the rhotrix C by performing heart-based neutrosophic multiplication between rhotrices A and B , where the operation \circ denotes the heart based neutrosophic multiplication.

Step 4: Construct the complement rhotrices A^c and B^c for rhotrices A and B respectively, both having the same dimensions. Afterward, calculate the composition rhotrix D by applying the heart-based neutrosophic multiplication \circ to the complement matrices A^c and B^c .

Step 5: Transform the rhotrix A and B to A_S and B_S by adding the truth and indeterminacy values, then subtracting the falsity value for each neutrosophic number in the rhotrix A and

B.

Step 6: Derive the score rhotrix M by applying the minimum operator, denoted as $(-)$, to the rhotrices A_S and B_S . This results in $M = A_S(-)B_S$.

This will be used to analyze and interpret the relationships or differences between the elements under study.

Illustrative Analysis: Assume there are five symptoms and five diseases. Out of these, three symptoms (s_1, s_3, s_5) are associated with three specific diseases (d_1, d_3, d_5) , while the remaining two symptoms (s_2, s_4) correspond to two other diseases (d_2, d_4) .

Let us, for illustration, take symptom disease neutrosophic rhotrix as

$$A = \left\langle \begin{array}{ccccc} & & (0.2, 0.4, 0.6) & & \\ & (0.5, 0.1, 0.3) & (0.2, 0.5, 0.2) & (0.4, 0.4, 0.1) & \\ (0.9, 0.6, 0.3) & (0.2, 0.9, 0.1) & (0.1, 0.7, 0.4) & (0.2, 0.6, 0.3) & (0.1, 0.5, 0.4) \\ & (0.3, 0.4, 0.8) & (0.7, 0.9, 0.1) & (0.5, 0.5, 0.5) & \\ & & (0.1, 0.4, 0.1) & & \end{array} \right\rangle$$

Consider a scenario with five symptoms and five diseases. Three patients (p_1, p_3, p_5) experience three symptoms (s_1, s_3, s_5) , while the other two patients (p_2, p_4) exhibit the remaining two symptoms (s_2, s_4) .

Let us, for illustration, take the patient symptom neutrosophic rhotrix as

$$B = \left\langle \begin{array}{ccccc} & & (0.3, 0.4, 0.1) & & \\ & (0.2, 0.1, 0.2) & (0.1, 0.9, 0.4) & (0.7, 0.3, 0.1) & \\ (0.4, 0.3, 0.3) & (0.7, 0.1, 0.3) & (0.4, 0.1, 0.3) & (0.7, 0.7, 0.7) & (0.4, 0.2, 0.7) \\ & (0.4, 0.4, 0.1) & (0.3, 0.4, 0.7) & (0.9, 0.9, 0.9) & \\ & & (0.4, 0.4, 0.2) & & \end{array} \right\rangle$$

For the above illustration, we heart based multiply the two rhotrices A and B and get

$$C = \left\langle \begin{array}{ccccc} & & (0.2, 0.4, 0.4) & & \\ & (0.4, 0.1, 0.3) & (0.2, 0.7, 0.3) & (0.4, 0.3, 0.3) & \\ (0.4, 0.3, 0.3) & (0.2, 0.1, 0.3) & (0.1, 0.1, 0.3) & (0.2, 0.7, 0.3) & (0.1, 0.2, 0.4) \\ & (0.3, 0.4, 0.4) & (0.4, 0.4, 0.3) & (0.4, 0.7, 0.5) & \\ & & (0.1, 0.4, 0.3) & & \end{array} \right\rangle$$

By

using,

$$\begin{aligned}
 A^c = & \left\langle \begin{array}{cccccc} & & (0.8, 0.6, 0.4) & & & \\ & (0.5, 0.9, 0.7) & (0.8, 0.5, 0.8) & (0.6, 0.6, 0.9) & & \\ (0.1, 0.4, 0.7) & (0.8, 0.1, 0.9) & (0.9, 0.3, 0.6) & (0.8, 0.4, 0.7) & (0.9, 0.5, 0.6) & \\ & (0.7, 0.6, 0.2) & (0.3, 0.1, 0.9) & (0.5, 0.5, 0.5) & & \\ & (0.9, 0.6, 0.9) & & & & \\ & (0.7, 0.6, 0.9) & & & & \\ & (0.8, 0.9, 0.8) & (0.9, 0.1, 0.6) & (0.3, 0.7, 0.9) & & \\ (0.6, 0.7, 0.7) & (0.3, 0.9, 0.7) & (0.6, 0.9, 0.7) & (0.3, 0.3, 0.3) & (0.6, 0.8, 0.3) & \\ & (0.6, 0.6, 0.9) & (0.7, 0.6, 0.3) & (0.1, 0.1, 0.1) & & \\ & (0.6, 0.6, 0.8) & & & & \\ & (0.7, 0.6, 0.7) & & & & \\ & (0.8, 0.9, 0.7) & (0.9, 0.5, 0.6) & (0.6, 0.6, 0.9) & & \\ (0.6, 0.4, 0.7) & (0.6, 0.3, 0.7) & (0.6, 0.3, 0.7) & (0.6, 0.4, 0.6) & (0.6, 0.5, 0.6) & \\ & (0.6, 0.6, 0.7) & (0.7, 0.3, 0.6) & (0.5, 0.5, 0.6) & & \\ & (0.6, 0.6, 0.8) & & & & \end{array} \right\rangle \text{ and} \\
 B^c = & \left\langle \begin{array}{cccccc} & & (0.8, 0.9, 0.8) & (0.9, 0.1, 0.6) & (0.3, 0.7, 0.9) & \\ & (0.6, 0.7, 0.7) & (0.3, 0.9, 0.7) & (0.6, 0.9, 0.7) & (0.3, 0.3, 0.3) & (0.6, 0.8, 0.3) \\ & (0.6, 0.6, 0.9) & (0.7, 0.6, 0.3) & (0.1, 0.1, 0.1) & & \\ & (0.6, 0.6, 0.8) & & & & \\ & (0.7, 0.6, 0.7) & & & & \\ & (0.8, 0.9, 0.7) & (0.9, 0.5, 0.6) & (0.6, 0.6, 0.9) & & \\ (0.6, 0.4, 0.7) & (0.6, 0.3, 0.7) & (0.6, 0.3, 0.7) & (0.6, 0.4, 0.6) & (0.6, 0.5, 0.6) & \\ & (0.6, 0.6, 0.7) & (0.7, 0.3, 0.6) & (0.5, 0.5, 0.6) & & \\ & (0.6, 0.6, 0.8) & & & & \end{array} \right\rangle, \text{ we get } D =
 \end{aligned}$$

Then, by the aforementioned algorithm, we transform A to

$$\begin{aligned}
 A_S = & \left\langle \begin{array}{ccccc} & & 0.2 & & \\ & & 0.2 & 0.6 & 0.4 \\ 0.4 & 0 & -0.1 & 0.6 & -0.1 \\ & & 0.3 & 0.5 & 0.6 \\ & & 0.2 & & \\ & & 0.6 & & \\ & & 1 & 0.8 & 0.3 \\ 0.3 & 0.2 & 0.2 & 0.4 & 0.5 \\ & & 0.5 & 0.4 & 0.4 \\ & & 0.4 & & \end{array} \right\rangle \\
 \text{and } B \text{ to } B_S = & \left\langle \begin{array}{ccccc} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{array} \right\rangle.
 \end{aligned}$$

Then, the score rhotrix thus obtained is, $\left\langle \begin{array}{ccccc} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{array} \right\rangle$

We can observe that, p_1 is affected by d_3 , p_2 is affected by d_4 , p_3 is affected by d_5 , p_4 is affected by d_4 and p_5 is affected by d_1 , by finding maximum of each row.

Method 2 (Using Fuzzy Rhotrices)

In optimizing diagnostics with fuzzy rhotrices, let S represent the set of symptoms associated with certain diseases, D denote the set of diseases, and P refer to the set of patients.

Step 1: Construct a symptom-disease fuzzy rhotrix $A = [a_{ij}]$ of size n (=number of symptoms and diseases) in the following manner, based on the following example. Consider there are 5 symptoms and 5 diseases, in which 3 symptoms(s_1, s_3, s_5) can indicate 3 diseases (d_1, d_3, d_5) and another 2 symptoms (s_2, s_4) indicate another 2 diseases (d_2, d_4).Here, $s_i d_j$ represents the membership value indicating how much symptom s_i contributes to the occurrence of disease d_j .

Then

the symptom-disease fuzzy rhotrix A can be formed as $A = \left\langle \begin{matrix} & s_1 d_1 & & & \\ & s_3 d_1 & s_2 d_2 & s_1 d_3 & \\ s_5 d_1 & s_4 d_2 & s_3 d_3 & s_2 d_4 & s_1 d_5 \\ & s_5 d_3 & s_4 d_4 & s_3 d_5 & \\ & & & & s_5 d_5 \end{matrix} \right\rangle$

Suppose, for illustration, $A = \left\langle \begin{matrix} & & & & 0.2 \\ & 0.5 & 0.7 & 0.4 & \\ 0.6 & 0.1 & 0.9 & 0.6 & 0.4 \\ & 0.2 & 0.1 & 0.4 & \\ & & & & 0.5 \end{matrix} \right\rangle$

Step 2: Construct a patient-symptom fuzzy matrix $B = [b_{ij}]$ of size n (=number of symptoms and patients) in the following manner, based on the following example. Consider there are 5 symptoms and 5 diseases, in which 3 patients (p_1, p_3, p_5) has 3 symptoms(s_1, s_3, s_5) and another 2 patients (p_2, p_4) indicate another 2 symptoms (s_2, s_4).

Here, $p_i s_j$ represents the membership value indicating how much a patient p_i have affected by the symptom s_j .

Then

the patient-symptom fuzzy rhotrix can be formed as $B = \left\langle \begin{matrix} & & & & p_1 s_1 \\ & p_3 s_1 & p_2 s_2 & p_1 s_3 & \\ p_5 s_1 & p_4 s_2 & p_3 s_3 & p_2 s_4 & p_1 s_5 \\ & p_5 s_3 & p_4 s_4 & p_3 s_5 & \\ & & & & p_5 s_5 \end{matrix} \right\rangle$

Suppose, for illustration, $B = \left\langle \begin{matrix} & & & & 0.5 \\ & 0.7 & 0.9 & 0.8 & \\ 0.2 & 0.5 & 0.9 & 0.2 & 0.3 \\ & 0.4 & 0.5 & 0.1 & \\ & & & & 0.2 \end{matrix} \right\rangle$

Step 3: Evaluate $C = A \circ B$, where \circ is heart based fuzzy multiplication

$$C = \begin{pmatrix} 0.5 & & & & \\ & 0.7 & 0.9 & 0.8 & \\ & 0.6 & 0.5 & 0.9 & 0.6 & 0.4 \\ & & 0.4 & 0.5 & 0.4 & \\ & & & & & 0.5 \end{pmatrix}$$

Step 4: Build the complement matrix A^c and B^c of A and B of size n and form the composition matrix D by computing $A^c \circ B^c$.

$$D = \begin{pmatrix} 0.1 & & & & \\ & 0.1 & 0.1 & 0.1 & \\ & 0.1 & 0.1 & 0.1 & 0.1 & \\ & & 0.1 & 0.1 & 0.1 & \\ & & & & & 0.1 \end{pmatrix}$$

Step 5: Compute $M = C(-)D$, where (-) denotes min operator Then, M for the above illustration is

$$M = \begin{pmatrix} 0.4 & & & & \\ & 0.6 & 0.8 & 0.7 & \\ & 0.5 & 0.4 & 0.8 & 0.5 & 0.3 \\ & & 0.3 & 0.4 & 0.3 & \\ & & & & & 0.4 \end{pmatrix}$$

Step 6: Calculating the relativity values and form the comparison matrix

$$Comparison\ Matrix = \begin{pmatrix} 0 & & & & \\ & -0.143 & 0 & 0.143 & \\ & 0.4 & -0.2 & 0 & 0.2 & -0.4 \\ & & 0 & 0 & 0 & \\ & & & & & 0 \end{pmatrix}$$

We can observe that, p_1 is affected by d_3 , p_2 is affected by d_4 , p_3 is affected by d_3 and d_5 , p_4 is affected by d_4 and p_5 is affected by d_1 , by finding maximum of each row.

Comparative Analysis: The method applied for medical diagnostics using fuzzy rhotrices, observe that it only deals with truthfulness and do not explicitly address indeterminacy, which limits their ability to manage situations where medical data is incomplete or contradictory. But, our current algorithm uses neutrosophic rhotrices which are designed to handle three types of uncertainties: truth, indeterminacy, and falsity. In medical diagnosis, this means that a symptom can be partially present (truth), uncertain (indeterminacy), or absent (falsity), offering a richer framework for capturing ambiguity in symptoms.

Neutrosophic rhotrices, due to their sophisticated structure, provide significant advantages in decision-making processes, especially in situations where data can be organized into distinct categories or groups. For instance, in the field of medical diagnostics, neutrosophic rhotrices can facilitate the classification of patients based on their symptoms. This grouping allows healthcare professionals to make more precise and personalized decisions regarding diagnosis and treatment.

Compared to neutrosophic matrices, computations in neutrosophic rhotrices are much simpler. Rhotrices reduce computational complexity by handling multi-dimensional data in a more streamlined manner, enabling faster execution of operations like matrix multiplication and inversion. This efficiency is particularly valuable when dealing with large datasets in medical diagnosis.

In summary, the use of neutrosophic rhotrices in medical diagnostics not only streamlines the decision-making process but also promotes a more informed and personalized approach to patient care.

Conclusion

We introduced neutrosophic rhotrices, a novel extension of neutrosophic matrices, designed to effectively manage the complexities of uncertainty, indeterminacy, and falsity in decision-making processes, particularly in the context of medical diagnostics. By extending the traditional matrix structure with an additional dimension, neutrosophic rhotrices provide a more flexible and comprehensive framework for representing and processing medical data where ambiguity is inherent. Further, we explored the basic properties of neutrosophic rhotrices, such as their algebraic properties and their computational simplicity compared to neutrosophic matrices. Additionally, we compared the medical diagnostic method using neutrosophic rhotrices with that of fuzzy rhotrices and found that neutrosophic rhotrices provide significantly better results in the decision-making process.

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Received: July 19, 2024. Accepted: September 07, 2024