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(m,a,n)-Fuzzy Neutrosophic Sets and Their Topological Structure

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Abstract. In this paper the concept of (m, a, n)-fuzzy neutrosophic set is introduced and its basic operations are studied. It is shown by examples that the class of (m, a, n)-fuzzy neutrosophic sets properly contains the class of q-rung orthopair neutrosophic sets. Moreover, a topological structure on (m, a, n)-fuzzy neutrosophic sets is created, and the topological notions such as closure, interior, subspace, connectedness, and separation axioms, are extended to (m, a, n)-fuzzy neutrosophic sets and explore their properties in (m, a, n)-fuzzy neutrosophic topological spaces.

Keywords: Neutrosopic sets; (m,a,n)-fuzzy neutrosophic sets; (m,a,n)-fuzzy neutrosophic topology; (m,a,n)-fuzzy neutrosophic separation axioms;(m,a,n)-fuzzy neutrosophic connectedness;(m,a,n)-fuzzy neutrosophic compactness.

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1. Introduction

The traditional methods relying on crisp sets are unsuitable for decision-making involving incomplete and inconsistent information, given the inherent challenges posed by vagueness, hesitancy, and uncertainties. Consequently, researchers have devised various set-theoretic models to tackle real-life problems characterized by uncertainty. These models aim to manage inconsistent information in a more effective manner. Zadeh [28] in 1965 proposed FSs as a generalization of crisp sets. Since the advent of Zadeh's paper [28] paper, many generalizations of FS such as IFS [3], PyFS [26] FFS [18]and q-ROFS [27]and (m,n)-FS [1] have been introduced by imposing certain conditions on PMD and NMD for each member of universe of discourse. Topological structures on these classes of FSs have been studied by different authors [5, 6, 11, 12, 15, 23]. Smarandache [19]defined neutrosophic set on a non empty set by

Table 1. Abbreviations and their description

Abbreviation	Description	Abbreviation	Description
PMD	Positive membership degree	$\mho_{\mathcal{H}}(v)$	PMD of v to $\mathcal{H}$
IMD	Indeterminacy degree	$\varpi_{\mathcal{H}}(v)$	IMD of v to $\mathcal{H}$
NMD	Negative membership degree	$\Omega_{\mathcal{H}}(v)$	NMD of v to $\mathcal{H}$
FS	Fuzzy set	IFS	Intuitionistic fuzzy set
PyFS	Pythagorean fuzzy set	FFS	Fermatean fuzzy set
q-ROFS	q-rung orthopair fuzzy set	(m,n)-FS	(m,n)-fuzzy set
NS	Neutrosophic set	PyNS	Pythagorean neutrosophic sets
FNS	Fermatean neutrosophic set	q-RONS	q-rung orthopair neutrosophic set
(m,a,n)-FNS	(m,a,n)- fuzzy neutrosophic set	(m,a,n)-FNS($\mathbb{V}$)	Family of (m,a,n)-FNSs on $\mathbb{V}$
(m,a,n)-FNT	(m,a,n)-fuzzy neutrosophic topology	(m,a,n)-FNTS	(m,a,n)-fuzzy neutrosophic topological space
(m,a,n)-FN point	(m,a,n)-fuzzy neutrosophic point	(m,a,n)-FNC($\mathbb{V}$)	Family of (m,a,n)-FN closed sets on $\mathbb{V}$

considering three components, namely PMD, IMD and NMD whose sum lies between 0 and 3. Some more properties of neutrosophic sets are presented by Smarandache [19–21], Salama and Alblowi [16] , Lupiáñez [14],Wang [25]. Smarandache's Neutrosophic concepts have wide range of real time applications for the fields of Information systems, Computer science, Artificial Intelligence, Applied Mathematics and Decision making. In 2008, Lupiáñez [14] introduced the neutrosophic topology as an extension of intuitionistic fuzzy topology. Recent work in neutrosophic topology can be seen in [2, 8, 9, 16, 17] Recently the concepts of PyFS , FFS, and q-ROFS have been extended to neutrosophic environments and studied their topological structures [4, 22, 24]. In this work we introduce the concept of the *(m,a,n)-fuzzy neutrosophic set* focusing on fundamental properties of this kind of set and on *(m,a,n)-fuzzy neutrosophic topological spaces.* The rest of the paper is formulated as follows: Section 2 contains the necessary mathematical background for the understanding of the paper. The concept of the (m,a,n)-fuzzy neutrosophic set is presented in Section 3 together with basic properties of these sets. In Section 4 the classical notion of topological space is extended to *(m,a,n)-fuzzy neutrosophic topological spaces* together with fundamental properties and concepts like ,connectedness and separation axioms in (m,a,n)-fuzzy neutrosophic topological spaces. The paper closes with the final conclusions and some hints for further research included in Section 5.

2. **Preliminaries**

This section contains some basic definitions and preliminary results which will be needed in the sequel. Throughout this paper $\mathbb{V}$ denotes a universe of discourse and $\mathbb{N}$ refers to the set of all natural numbers.

Definition 2.1. Let $\mathbb{V}$ be a non empty set, $\mho_{\mathcal{H}} : \mathbb{V} \to [0,1]$ and $\Omega_{\mathcal{H}} : \mathbb{V} \to [0,1]$. A structure

$$\mathcal{H} = \{< v, \mho_{\mathcal{H}}(v), \Omega_{\mathcal{H}}(v) >: v \in \mathbb{V}\}$$

is called :

(a) IFS [3] in $\mathbb{V}$ if $0 \leq \mho_{\mathcal{H}}(v) + \Omega_{\mathcal{H}}(v) \leq 1, \ \forall \ v \in \mathbb{V}$.
(b) PyFS [26] in $\mathbb{V}$ if $0 \leq \mho^2_{\mathcal{H}}(v) + \Omega^2_{\mathcal{H}}(v) \leq 1, \ \forall \ v \in \mathbb{V}$.
(c) FFS [18] in $\mathbb{V}$ if $0 \leq \mho^3_{\mathcal{H}}(v) + \Omega^3_{\mathcal{H}}(v) \leq 1, \ \forall \ v \in \mathbb{V}$.
(d) q-ROFS [27] in $\mathbb{V}$ if $0 \leq \mho^q_{\mathcal{H}}(v) + \Omega^q_{\mathcal{H}}(v) \leq 1, \ \forall \ v \in \mathbb{V}$ and $q \in \mathbb{N}$.
(e) (m,n)-FS [1] in $\mathbb{V}$ if $0 \leq \mho^m_{\mathcal{H}}(v) + \Omega^n_{\mathcal{H}}(v) \leq 1, \ \forall \ v \in \mathbb{V}$ and $m, n \in \mathbb{N}$.

Remark 2.2. [1] The definition of (m,n)-FS can be reduced to the definition of:

(i) q-ROFS, if m=n=q.
(ii) FFS, if m=n=3.
(iii) PyFS, if m=n=2.
(iv) IFS, if m=n=1.

Remark 2.3. [1] Let $\mathbb{V}$ be a non empty set:

(i) If $m \geq q$ and $n \geq q$ then every q-ROFS is a(m,n)-FS.
(ii) If $m \geq 3$ and $n \geq 3$ then every FFS is a (m,n)-FS.
(iii) If $m \geq 2$ and $n \geq 2$ then every PyFS is a (m,n)-FS.
(iv) Every IFS is an (m,n)-FS.

Definition 2.4. [19] Let $\mathbb{V}$ be a non empty set. A Neutrosophic set (**NS**) $\mathcal{H}$ in $\mathbb{V}$ is a structure

$$\mathcal{H} = \{< v, \mho_{\mathcal{H}}(v), \varpi_{\mathcal{H}}(v), \Omega_{\mathcal{H}}(v) >: v \in \mathbb{V}\}$$

where $\mho_{\mathcal{H}} : \mathbb{V} \to]^-0, 1^+[$, $\varpi_{\mathcal{H}} : \mathbb{V} \to]^-0, 1^+[$, and $\Omega_{\mathcal{H}} : \mathbb{V} \to]^-0, 1^+[$ denotes the PMD, IMD, and NMD of $\mathcal{H}$ which satisfies the condition if $^-0 \leq \mho_{\mathcal{H}}(v) + \varpi_{\mathcal{H}}(v) + \Omega_{\mathcal{H}}(v) \leq 3^+, \ \forall \ v \in \mathbb{V}$.

In the real life applications in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0, 1^+[$. Hence we consider the neutrosophic set which takes the value from the closed interval [0,1] and sum of MD, IMD, and NMD of each element of universe of discourse lies between 0 and 3.

Definition 2.5. Let $\mathbb{V}$ be a non empty set and $\mho_{\mathcal{H}} : \mathbb{V} \to [0, 1]$,$\varpi_{\mathcal{H}} : \mathbb{V} \to [0, 1]$, and $\Omega_{\mathcal{H}} : \mathbb{V} \to [0, 1]$. A structure

$$\mathcal{H} = \{< v, \mho_{\mathcal{H}}(v), \varpi_{\mathcal{H}}(v), \Omega_{\mathcal{H}}(v) >: v \in \mathbb{V}\}$$

is called :

(a) PyNS [4] in $\mathbb{V}$ if $0 \leq \mho^2_{\mathcal{H}}(v) + \Omega^2_{\mathcal{H}}(v) \leq 1$, and $0 \leq \varpi^2_{\mathcal{H}}(v) \leq 1, \ \forall \ v \in \mathbb{V}$.
(b) FNS [22] in $\mathbb{V}$ if $0 \leq \mho^3_{\mathcal{H}}(v) + \Omega^3_{\mathcal{H}}(v) \leq 1$, and $0 \leq \varpi^3_{\mathcal{H}}(v) \leq 1$, $\forall \ v \in \mathbb{V}$.
(c) q-RONS [24] in $\mathbb{V}$ if $0 \leq \mho^q_{\mathcal{H}}(v) + \Omega^q_{\mathcal{H}}(v) \leq 1$, and $0 \leq \varpi^q_{\mathcal{H}}(v) \leq 1$, $\forall \ v \in \mathbb{V}$ and $q \in \mathbb{N}$.

Here $\mho_{\mathcal{H}}(v)$ and $\Omega_{\mathcal{H}}(v)$ are dependent components and $\varpi_{\mathcal{H}}(v)$ is an independent component.

3. (m,a,n)-fuzzy neutrosophic sets

In this section we introduce a new class of orthopair neutrosophic sets which is properly contains the classes of q-RONS, FNS, and PyNS and explores its studied.

Definition 3.1. Let $\mathbb{V}$ be a non empty set. A (m,a,n)-fuzzy neutrosophic set ((m,a,n)-FNS) $\mathcal{H}$ in $\mathbb{V}$ is an object of the form

$$\mathcal{H} = \{< v, \mho_{\mathcal{H}}(v), \varpi_{\mathcal{H}}(v), \Omega_{\mathcal{H}}(v) >: v \in \mathbb{V}\}$$

where $\mho_{\mathcal{H}} : \mathbb{V} \to [0,1]$,$\varpi_{\mathcal{H}} : \mathbb{V} \to [0,1]$, $\Omega_{\mathcal{H}} : \mathbb{V} \to [0,1]$, $0 \leq \mho^m_{\mathcal{H}}(v) + \Omega^n_{\mathcal{H}}(v) \leq 1$, and $0 \leq \varpi^a_{\mathcal{H}}(v) \leq 1$, $\forall\ v \in \mathbb{V}$ and $m, a, n \in \mathbb{N}$. Here $\mho_{\mathcal{H}}(v)$ and $\Omega_{\mathcal{H}}(v)$ are dependent components and $\varpi_{\mathcal{H}}(v)$ is an independent component.

Theorem 3.2. *If $\mathcal{H} = \{< v, \mho_{\mathcal{H}}(v), \varpi_{\mathcal{H}}(v), \Omega_{\mathcal{H}}(v) >: v \in \mathbb{V}\}$ is a (m,a,n)-FNS over $\mathbb{V}$ then $0 \leq \mho^m_{\mathcal{H}}(v) + \varpi^a_{\mathcal{H}}(v) + \Omega^n_{\mathcal{H}}(v) \leq 2$, $\forall\ v \in \mathbb{V}$ and $m, a, n \in \mathbb{N}$.*

For simplicity a (m,a,n)-FNS $\mathcal{H} = \{< v, \mho_{\mathcal{H}}(v), \varpi_{\mathcal{H}}(v), \Omega_{\mathcal{H}}(v) >: v \in \mathbb{V}\}$ over $\mathbb{V}$ will be denoted by $(\mho_{\mathcal{H}}, \varpi_{\mathcal{H}}, \Omega_{\mathcal{H}})$.

Remark 3.3. (m,a,n)-FNS is coincide with:

(i) q-RONS , if m=a=n=q.
(ii) FNS, if m=a=n=3.
(iii) PyNS, if m=a=n=2.

Remark 3.4. Let $\mathbb{V}$ be a non empty set:

(i) If $m \geq q$, $a \geq q$, and $n \geq q$ then every q-RONS is a(m,a,n)-FNS.
(ii) If $m \geq 3$, $a \geq 3$, and $n \geq 3$ then every FNS is a (m,a,n)-FNS.
(iii) If $m \geq 2$, $a \geq 2$, and $n \geq 2$ then every PyNS is a (m,,a,n)-FNS.

Example 3.5. Let $\mathbb{V} = \{v_1, v_2\}$. Then the structure $\mathcal{E} = \{< v_1, 0.8, 0.2, 0.9 >, < v_2, 0, 8, 0.5, 0.7 >\}$ defined over $\mathbb{V}$ is (4,5,6)-FNS but not 4-RONS , and hence not FNS,and PyNS over $\mathbb{V}$.

Definition 3.6. The whole (m,a,n)-FNS and empty (m,a,n)-FNS over universe of discourse $\mathbb{V}$ are defined as follows

$$\tilde{\Phi} = \{< v, 0, 0, 1 >: v \in \mathbb{V}\}.$$

$$\tilde{\mathbb{V}} = \{< v, 1, 1, 0 >: v \in \mathbb{V}\}.$$

Definition 3.7. Let $\mathcal{H} = (\mho_{\mathcal{H}}, \varpi_{\mathcal{H}}, \Omega_{\mathcal{H}})$, $\mathcal{H}_1 = (\mho_{\mathcal{H}_1}, \varpi_{\mathcal{H}_1}, \Omega_{\mathcal{H}_1})$ and $\mathcal{H}_2 = (\mho_{\mathcal{H}_2}, \varpi_{\mathcal{H}_2}, \Omega_{\mathcal{H}_2})$ be three (m,a,n)-FNSs in $\mathbb{V}$. Then the subset, equality, complement, union, and intersection operations over $(m, a, n) - FNS(\mathbb{V})$ are defined as follow:

(a) $\mathcal{H}_1 \subset \mathcal{H}_2 \Leftrightarrow \mho_{\mathcal{H}_1} \leq \mho_{\mathcal{H}_2}$, $\varpi_{\mathcal{H}_1} \geq \varpi_{\mathcal{H}_2}$ and $\Omega_{\mathcal{H}_1} \geq \Omega_{\mathcal{H}_2}$

(b) $\mathcal{H}_1 = \mathcal{H}_2 \Leftrightarrow \mho_{\mathcal{H}_1} = \mho_{\mathcal{H}_2}$, $\varpi_{\mathcal{H}_1} = \varpi_{\mathcal{H}_2}$, and $\Omega_{\mathcal{H}_1} = \Omega_{\mathcal{H}_2}$.

(c) $\mathcal{H}^c = (\Omega_{\mathcal{H}}^{\frac{n}{m}}, 1-\varpi_{\mathcal{H}}, \mho_{\mathcal{H}}^{\frac{m}{n}})$.

(d) $\mathcal{H}_1 \cup \mathcal{H}_2 = (max\{\mho_{\mathcal{H}_1}, \mho_{\mathcal{H}_2}\}, max\{\varpi_{\mathcal{H}_1}, \varpi_{\mathcal{H}_2}\}, min\{\Omega_{\mathcal{H}_1}, \Omega_{\mathcal{H}_2}\})$

(e) $\mathcal{H}_1 \cap \mathcal{H}_2 = (min\{\mho_{\mathcal{H}_1}, \mho_{\mathcal{H}_2}\}, min\{\varpi_{\mathcal{H}_1}, \varpi_{\mathcal{H}_2}\}, max\{\Omega_{\mathcal{H}_1}, \Omega_{\mathcal{H}_2}\})$

Example 3.8. Let $\mathbb{V} = \{v_1, v_2, v_3\}$ and (m,a,n)-FNS $\mathcal{E}, \mathcal{F}, \mathcal{G}$ over $\mathbb{V}$ (for m=4,a=5,n=6) bedefined as follows:

$$\mathcal{E} = \{< v_1, 0.8, 0.2, 0.9 >, < v_2, 0, 8, 0.5, 0.7 >, < v_3, 0.6, 0.4, 0.9 >\}.$$

$$\mathcal{F} = \{< v_1, 0.9, 0.3, 0.9 >, < v_2, 0.8, 0.5, 0.6 >, < v_3, 0.7, 0.5, 0.8 >\}.$$

$$\mathcal{G} = \{< v_1, 0.6, 0.5, 0.8 >, < v_2, 0.9, 0.4, 0.9 >, < v_3, 0.5, 0.5, 0.9 >\}.$$

Then,

(i) $\mathcal{E} \subset \mathcal{F}$, but $\mathcal{E} \nsubseteq \mathcal{G}$.

(ii) $\mathcal{E} \cup \mathcal{F} = \{< v_1, 0.9, 0.3, 0.9 >, < v_2, 0.8, 0.5, 0.6 >, < v_3, 0.7, 0.5, 0.8 >\}$.

(iii) $\mathcal{F} \cap \mathcal{G} = \{< v_1, 0.6, 0.3, 0.9 >, < v_2, 0.8, 0.4, 0.9 >, < v_3, 0.5, 0.5, 0.9 >\}$.

(iv) $\mathcal{E}^c = \begin{Bmatrix} < v_1, 0.8538149682, 0.8, 0.8617738754 >, \\ < v_2, 0.5856629186, 0.5, 0.8617738754 >, \\ < v_3, 0.8538149682, 0.6, 0.7113786597 > \end{Bmatrix}$.

Theorem 3.9. *Let* $\mathcal{H} = (\mho_{\mathcal{H}}, \varpi_{\mathcal{H}}, \Omega_{\mathcal{H}}), \mathcal{H}_1 = (\mho_{\mathcal{H}_1}, \varpi_{\mathcal{H}_1}, \Omega_{\mathcal{H}_1}), \mathcal{H}_2 = (\mho_{\mathcal{H}_2}, \varpi_{\mathcal{H}_2}, \Omega_{\mathcal{H}_2}) \in (m,a,n) - FNS(\mathbb{V})$. *Then*

(a) $\mathcal{H}^c \in (m,a,n) - FNS(\mathbb{V})$.

(b) $\mathcal{H}_1 \cup \mathcal{H}_2 \in (m,a,n) - FNS(\mathbb{V})$.

(c) $\mathcal{H}_1 \cap \mathcal{H}_2 \in (m,a,n) - FNS(\mathbb{V})$.

The Proof of Theorem 3.9 is obvious and left to the readers.

Theorem 3.10. *Let* $\mathcal{H}_1 = (\mho_{\mathcal{H}_1}, \varpi_{\mathcal{H}_1}, \Omega_{\mathcal{H}_1}), \mathcal{H}_2 = (\mho_{\mathcal{H}_2}, \varpi_{\mathcal{H}_2}, \Omega_{\mathcal{H}_2}), \mathcal{H}_3 = (\mho_{\mathcal{H}_3}, \varpi_{\mathcal{H}_3}, \Omega_{\mathcal{H}_3}) \in (m,a,n) - FNS(\mathbb{V})$. *Then:*

(a) $\mathcal{H}_1 \cup \mathcal{H}_2 = \mathcal{H}_2 \cup \mathcal{H}_1$.

(b) $\mathcal{H}_1 \cap \mathcal{H}_2 = \mathcal{H}_2 \cap \mathcal{H}_1$.

(c) $(\mathcal{H}_1 \cup \mathcal{H}_2) \cup \mathcal{H}_3 = \mathcal{H}_1 \cup (\mathcal{H}_2 \cup \mathcal{H}_3)$.

(d) $(\mathcal{H}_1 \cap \mathcal{H}_2) \cap \mathcal{H}_3 = \mathcal{H}_1 \cap (\mathcal{H}_2 \cap \mathcal{H}_3)$.

(e) $\mathcal{H}_1 \cup (\mathcal{H}_2 \cap \mathcal{H}_3) = (\mathcal{H}_1 \cup \mathcal{H}_2) \cap (\mathcal{H}_1 \cup \mathcal{H}_3)$

(f) $\mathcal{H}_1 \cap (\mathcal{H}_2 \cup \mathcal{H}_3) = (\mathcal{H}_1 \cap \mathcal{H}_2) \cup (\mathcal{H}_1 \cap \mathcal{H}_3)$

(g) $(\mathcal{H}_1 \cup \mathcal{H}_2)^c = \mathcal{H}_2^c \cap \mathcal{H}_1^c$.

(h) $(\mathcal{H}_1 \cap \mathcal{H}_2)^c = \mathcal{H}_2^c \cup \mathcal{H}_1^c$.

Proof. Follows on similar lines as the proofs of Theorem 1 and Theorem 2 [1] □

Definition 3.11. Let $\{\mathcal{H}_i : i \in \Lambda\}$ be an arbitrary family of (m,a,n)-FNS in $\mathbb{V}$. Then:

(a) $\bigcup \mathcal{H}_i = \{< v, \bigvee \mho_{\mathcal{H}_i}(v),\ \bigvee \varpi_{\mathcal{H}_i}(v), \bigwedge \Omega_{\mathcal{H}_i}(v) >: v \in \mathbb{V}\}$.

(b) $\bigcap \mathcal{H}_i = \{< v, \bigwedge \mho_{\mathcal{H}_i}(v),\ \bigwedge \varpi_{\mathcal{H}_i}(v), \bigvee \Omega_{\mathcal{H}_i}(v) >: v \in \mathbb{V}\}$.

4. (m,a,n)-fuzzy neutrosophic topological spaces

Definition 4.1. A subfamily Γ of $(m, a, n) - FNS(\mathbb{V})$ is called a (m,a,n)-fuzzy neutrosophic topology ((m,a,n)-FNT) on $\mathbb{V}$ if:

(a) $\tilde{\Phi}, \tilde{\mathbb{V}} \in \Gamma$.

(b) $\mathcal{H}_i \in \Gamma,\ \forall\ i \in \Lambda \Rightarrow \cup_{i\in\Lambda}\mathcal{H}_i \in \Gamma$.

(c) $\mathcal{H}_1, \mathcal{H}_2 \in \Gamma \Rightarrow \mathcal{H}_1 \cap \mathcal{H}_2 \in \Gamma$.

If Γ is a (m,a,n)-FNTS on $\mathbb{V}$ then the structure $(\mathbb{V}, \Gamma)$ is called a (m,a,n)-fuzzy neutrosophic topological space ((m,a,n)-FNTS) over $\mathbb{V}$ and the members of Γ are called (m,a,n)-fuzzy neutrosophic open ((m,a,n)-FN open) sets. The complement of a (m,a,n)-FN open set is called (m,a,n)-fuzzy neutrosophic closed((m,a,n)-FN closed).

Example 4.2. Let $\mathbb{V} = \{v_1, v_2, v_3\}$ be the universal set. Consider the following (m,a,n)-FNSs for m=3,a=4, and n=5.

$$\mathcal{G}_1 = \{< v_1, 0.8, 0.4, 0.8 >, < v_2, 0.7, 0.3, 0.9 >, < v_3, 0.9, 0.4, 0.6 >\}.$$

$$\mathcal{G}_2 = \{< v_1, 0.9, 0.4, 0.7 >, < v_2, 0.8, 0.4, 0.7 >, < v_3, 0.9, 0.5, 0.5 >\}.$$

Then $\Gamma = \{\tilde{\Phi}, \tilde{\mathbb{V}}, \mathcal{G}_1, \mathcal{G}_2\}$ is a (2,3,4)-FNT on $\mathbb{V}$.

Definition 4.3. Let $(\mathbb{V}, \Gamma)$ be a (m,a,n)-FNTS with $\Gamma = \{\tilde{\Phi}, \tilde{\mathbb{V}}\}$ then Γ is said to be the indiscrete a (m,a,n)-FNT on $\mathbb{V}$ and $(\mathbb{V}, \Gamma)$ is called the indiscrete (m,a,n)-FNTS. The indiscrete (m,a,n)-FNT is the smallest (m,a,n)-FNT on $\mathbb{V}$.

Definition 4.4. Let $(\mathbb{V}, \Gamma)$ be a (m,a,n)-FNTS with $\Gamma = (m, a, n) - FNS(\mathbb{V})$ then Γ is said to be the discrete (m,a,n)-FNT on $\mathbb{V}$ and $(\mathbb{V}, \Gamma)$ is called the discrete (m,a,n)-FNTS. The discrete (m,a,n)-FNT is the largest (m,a,n)-FNT on $\mathbb{V}$.

Remark 4.5. The union of two (m,a,n)-FNTs over $\mathbb{V}$ is not a (m,a,n)-FNT over $\mathbb{V}$.

Example 4.6. Let $\mathbb{V} = \{v_1, v_2\}$. Consider the following (m,a,n)-FNSs (for m=2,a=3, and n=4)

$$\mathcal{G}_1 = \{< v_1, 0.8, 0.2, 0.4 >, < v_2, 0.6, 0.1, 0.5 >\}$$

$$\mathcal{G}_2 = \{< v_1, 0.7, 0.2, 0.5 >, < v_2, 0.5, 0.1, 0.6 >\}$$

$$\mathcal{H}_1 = \{< v_1, 0.8, 0.4, 0.6 >, < v_2, 0.7, 0.3, 0.7 >\}$$

$$\mathcal{H}_1 = \{< v_1, 0.9, 0.4, 0.5 >, < v_2, 0.8, 0.3, 0.6 >\}.$$

Then $\Gamma_1 = \{\tilde{\Phi}, \tilde{\mathbb{V}}, \mathcal{G}_1, \mathcal{G}_2\}$ and $\Gamma_2 = \{\tilde{\Phi}, \tilde{\mathbb{V}}, \mathcal{H}_1, \mathcal{H}_2\}$ are two (m,a,n)-FNTs over $\mathbb{V}$. Now $\Gamma_1 \cup \Gamma_2 = \{\tilde{\Phi}, \tilde{\mathbb{V}}, \mathcal{G}_1, \mathcal{G}_2, \mathcal{H}_1, \mathcal{H}_2\}$, and $\mathcal{G}_1 \cup \mathcal{H}_1 = \{< v_1, 0.8, 0.4, 0.4 >, < v_2, 0.7, 0.3, 0.5 >\}$. Thus $\mathcal{G}_1, \mathcal{H}_1 \in \Gamma_1 \cup \Gamma_2$, but $\mathcal{G}_1 \cup \mathcal{H}_1 \notin \Gamma_1 \cup \Gamma_2$. Therefore $\Gamma_1 \cup \Gamma_2$ is not a (m,a,n)-FNT on $\mathbb{V}$.

However,

Theorem 4.7. *The intersection of two (m,a,n)-FNTs on $\mathbb{V}$ is also a (m,a,n)-FNT on $\mathbb{V}$.*

Proof. Suppose that Γ_1 and Γ_2 are two (m,a,n)-FNTs on $\mathbb{V}$. Since $\Phi, \mathbb{V} \in \Gamma_1$ and $\Phi, \mathbb{V} \in \Gamma_2$, then $\Phi, \mathbb{V} \in \Gamma_1 \cap \Gamma_2$. Let $\mathcal{G}_1, \mathcal{G}_2 \in \Gamma_1 \cap \Gamma_2 \implies \mathcal{G}_1, \mathcal{G}_2 \in \Gamma_1$ and $(\mathcal{G}_1, \mathcal{G}_2 \in \Gamma_2 \implies \mathcal{G}_1 \cap \mathcal{G}_2 \in \Gamma_1$ and $\mathcal{G}_1 \cap \mathcal{G}_2 \in \Gamma_2 \implies \mathcal{G}_1 \cap \mathcal{G}_2 \in \Gamma_1 \cap \Gamma_2$.

Let $\mathcal{G}_i \in \Gamma_1 \cap \Gamma_2, i \in \Lambda,$ *an index set* $\implies \mathcal{G}_i \in \Gamma_1$ and $\mathcal{G}_i \in \Gamma_2$, $\forall i \in \Lambda \implies \cup_{i\in\Lambda}\mathcal{G}_i \in \Gamma_1$ and $\cup_{i\in\Lambda}\mathcal{G}_i \in \Gamma_2, \implies \cup_{i\in\Lambda}\mathcal{G}_i \in \Gamma_1 \cap \Gamma_2$. Thus $\Gamma_1 \cap \Gamma_2$ satisfies all the requirements to (m,a,n)-FNT on $\mathbb{V}$. □

Definition 4.8. Let $(\mathbb{V}, \Gamma_1)$ and $(\mathbb{V}, \Gamma_2)$ be (m,a,n)-FNTSs. Then:

(a) Γ_2 is called finer than Γ_1 if $\Gamma_2 \subset \Gamma_1$.

(b) Γ_1 is comparable with Γ_2 if either $\Gamma_1 \subset \Gamma_2$ or $\Gamma_2 \subset \Gamma_1$.

Definition 4.9. Let $(\mathbb{V}, \Gamma)$ be a (m,a,n)-FNTS and $\mathcal{H} \in (m, a, n) - FNS(\mathbb{V})$. Then the interior, closure, frontier and exterior of $\mathcal{H}$ denoted respectively by $Int\mathcal{H}$, $Cl(\mathcal{H})$,$Fr(\mathcal{H})$ and $Ext(\mathcal{H})$ are defined as follows:

(a) $Int(\mathcal{H}) = \cup\{\mathcal{K} \in \Gamma : \mathcal{K} \subset \mathcal{H}\}$.

(b) $Cl(\mathcal{H}) = \cap\{\mathcal{F} \in (m, a, n) - FNC(\mathbb{V}) : \mathcal{H} \subset \mathcal{F}\}$.

(c) $Fr(\mathcal{H}) = Cl(\mathcal{H}) \cap Cl(\mathcal{H}^c)$.

(d) $Ext(\mathcal{H}) = Int(\mathcal{H}^c)$.

Theorem 4.10. *Let $(\mathbb{V}, \Gamma)$ be a (m,a,n)-FNTS and $\mathcal{H}, \mathcal{F} \in (m, a, n) - FNS(\mathbb{V})$. Then:*

(a) $Int(\tilde{\Phi}) = \tilde{\Phi}$ *and* $Int(\tilde{\mathbb{V}}) = \tilde{\mathbb{V}}$.

(b) $Int(\mathcal{H}) \subset \mathcal{H}$.

(c) $\mathcal{H} \in \Gamma \Leftrightarrow Int(\mathcal{H}) = \mathcal{H}$.

(d) $Int(Int(\mathcal{H})) = Int(\mathcal{H})$.
(e) $\mathcal{H} \subset \mathcal{F} \Longrightarrow Int(\mathcal{H}) \subset Int(\mathcal{F})$.
(g) $Int(\mathcal{H}) \cup Int(\mathcal{F}) \subset Int(\mathcal{H} \cup \mathcal{F})$.
(h) $Int(\mathcal{H}) \cap Int(\mathcal{F}) = Int(\mathcal{H} \cap \mathcal{F})$.

Proof. Straightforward. □

Theorem 4.11. *Let* $(\mathbb{V}, \Gamma)$ *be a (m,a,n)-FNTS and* $\mathcal{H}, \mathcal{F} \in (m,a,n) - FNS(\mathbb{V})$. *Then:*

(a) $Cl(\tilde{\Phi}) = \tilde{\Phi}$, $Cl(\tilde{\mathbb{V}}) = \tilde{\mathbb{V}}$.
(b) $\mathcal{H} \subset Cl(\mathcal{H})$.
(c) $\mathcal{H} \in (m,a,n) - FNC(\mathbb{V}) \Leftrightarrow Cl(\mathcal{H}) = \mathcal{H}$.
(d) $Cl(Cl(\mathcal{H})) = Cl(\mathcal{H})$.
(e) $\mathcal{H} \subset \mathcal{F} \Longrightarrow Cl(\mathcal{H}) \subset Cl(\mathcal{F})$.
(f) $Cl(\mathcal{H}) \cup Cl(\mathcal{F}) = Cl(\mathcal{H} \cup \mathcal{F})$.
(g) $Cl(\mathcal{H} \cap \mathcal{F}) \subset Cl(\mathcal{H}) \cap Cl(\mathcal{F})$.

Proof. Straightforward. □

Theorem 4.12. *Suppose that* $(\mathbb{V}, \Gamma)$ *is a (m,a,n)-FNTS and* $(\mathcal{H})$ *is a (m,a,n)-FNS over* $\mathbb{V}$. *Then*

(a) $(Int(\mathcal{H}))^c = Cl(\mathcal{H}^c)$.
(b) $(Cl(\mathcal{H}))^c = Int(\mathcal{H}^c)$.
(c) $Fr(\mathcal{H}) = Fr(\mathcal{H}^c)$.

Proof. Obvious □

Definition 4.13. Let $(\mathbb{V}, \Gamma)$ be a (m,a,n)-FNT on $\mathbb{V}$ and let $\mathbb{Y} \subset \mathbb{V}$. Then $\Gamma_{\tilde{\mathbb{Y}}} = \{\mathcal{H} \cap \tilde{\mathbb{Y}} : \mathcal{H} \in \Gamma\}$ is called the relative (m,a,n)-FNT on $\mathbb{Y}$. The pair $(\mathbb{Y}, \Gamma_{\mathbb{Y}})$ is known as the (m,a,n)-FN subspace of the (m,a,n)-FNTS $(\mathbb{V}, \Gamma)$.

Example 4.14. Let $\mathbb{V} = \{v_1, v_2, v_3\}$ and $\Gamma = \{\tilde{\Phi}, \tilde{\mathbb{V}}, \mathcal{H}_1, \mathcal{H}_2\}$ where $(\mathcal{H}_1), (\mathcal{H}_2)$ be (m,a,n)-FNS (for m=3 a=4,and n=5) over $\mathbb{V}$, defined as follows:

$$\mathcal{H}_1 = \{< v_1, 0.7, 0.3, 0.8 >< v_2, 0.7, 0.2, 0.9 >, < v_3, 0.6, 0.1, 0.5 >\}$$

$$\mathcal{H}_2 = \{< v_1, 0.8, 0.3, 0.7 >, < v_2, 0.9, 0.2, 0.5 >, < v_3, 0.8, 0.1, 0.4 >\}$$

Then $(\mathbb{V},\Gamma)$ is a (m,a,n)-FNTS. Consider the (m,a,n)-FNS $\mathbb{Y} = \{v_1, v_2\}$. Then $\Gamma_{\mathbb{Y}} = \{\tilde{\Phi}, \tilde{\mathbb{Y}}, (\mathcal{H}_1)_{\mathbb{Y}}, (\mathcal{H}_2)_{\mathbb{Y}}\}$ is a relative (m,a,n)-FNT over $\mathbb{Y}$. Where

$$(\mathcal{H}_1)_Y = \{< v_1, 0.7, 0.3, 0.8 >< v_2, 0.7, 0.2, 0.9 >\}$$

$$(\mathcal{H}_2)_Y = \{< v_1, 0.8, 0.3, 0.7 >, < v_2, 0.9, 0.2, 0.5 >\}$$

Theorem 4.15. *Let $(\mathbb{Y},\Gamma_{\mathbb{Y}})$ be a (m,a,n)-FN-subspace of a (m,a,n)-FNTS$(\mathbb{V},\Gamma)$ and $\mathcal{H} \in (m,a,n) - FN(\mathbb{V})$, then:*

(a) $\mathcal{H} \in \Gamma_{\mathbb{Y}} \Leftrightarrow \mathcal{H} = \mathbb{Y} \cap \mathcal{E}$ *for some* $\mathcal{E} \in \Gamma$.

(b) $\mathcal{H} \in (m,a,n) - FNC(\mathbb{Y}) \Leftrightarrow \mathcal{H} = \tilde{\mathbb{Y}} \cap \mathcal{F}$ *for some* $\mathcal{F} \in (m,a,n) - FNC(\mathbb{V})$.

Theorem 4.16. *Let $(\mathbb{Y},\Gamma_{\mathbb{Y}})$ be a (m,a,n)-FN subspace of a (m,a,n)-FNTS $(\mathbb{V},\Gamma)$ and $\mathcal{H} \in \Gamma_{\mathbb{Y}}$. If $\mathbb{Y} \in \Gamma$ then $\mathcal{H} \in \Gamma$.*

Theorem 4.17. *Let $(\mathbb{Y},\Gamma_{\mathbb{Y}})$ be a (m,a,n)-FN subspace of a (m,a,n)-FNTS $(\mathbb{V},\Gamma)$. Then a (m,a,n)-FNS $\mathcal{H}_{\mathbb{Y}} \in (m,a,n) - FNC(\mathbb{Y}) \Rightarrow \mathcal{H}_{\mathbb{Y}} \in (m,a,n) - FNC(\mathbb{V}) \Leftrightarrow \tilde{\mathbb{Y}} \in (m,a,n) - FNC(\mathbb{V})$.*

Definition 4.18. Let $\zeta, \eta, \varsigma \in [0,1]$. A (m,a,n)-fuzzy neutrosophic point ((m,a,n)-FNP) $v_{(\zeta,\eta,\varsigma)}$ of $\mathbb{V}$ is a (m,a,n)-FNS in $\mathbb{V}$ defined by

$$v_{(\zeta,\eta,\varsigma)}(w) = \begin{cases} (\zeta,\eta,\varsigma) & \text{if } w = v \\ (0,0,1) & \text{if } w \neq v \end{cases}$$

Definition 4.19. Let $v_{(\zeta,\eta,\varsigma)}$ be a (m,a,n)-FNP in $\mathbb{V}$ and $\mathcal{H} = \{< v, \mho_{\mathcal{H}}(v), \varpi_{\mathcal{H}}(v), \Omega_{\mathcal{H}}(v) >: v \in \mathbb{V}\}$ is a (m,a,n)-FNS in $\mathbb{V}$. Then $v_{(\zeta,\eta,\varsigma)} \subseteq \mathcal{H}$ if and only if $\zeta \subseteq \mho_{\mathcal{H}}(v)$, $\eta \subseteq \varpi_{\mathcal{H}}(v)$, and $\varsigma \supseteq \Omega_{\mathcal{H}}(v)$.

Definition 4.20. A (m,a,n)-FNP $v_{(\zeta,\eta,\varsigma)}$ is said to be q-coincident with $\mathcal{H}$, denoted by $v_{(\zeta,\eta,\varsigma)} q\mathcal{H}$ if $v_{(\zeta,\eta,\varsigma)} \nsubseteq \mathcal{H}^c$. If $v_{(\zeta,\eta,\varsigma)}$ is not q-coincident with $\mathcal{H}$, we denote by $\rceil(v_{(\zeta,\eta,\varsigma)} q\mathcal{H})$.

Definition 4.21. Two (m,a,n)-FNSs $\mathcal{H}$ and $\mathcal{G}$ of $\mathbb{V}$ are said to be q-coincident (denoted by $\mathcal{H}q\mathcal{G}$) if $\mathcal{H} \nsubseteq \mathcal{G}^c$.

Lemma 4.22. *If $\mathcal{H}, \mathcal{G} \in (m,a,n) - FN(\mathbb{V})$, then $\rceil(\mathcal{H}q\mathcal{G}) \Leftrightarrow \mathcal{H} \subset \mathcal{G}^c$., where $\rceil(\mathcal{H}q\mathcal{G})$ means $\mathcal{H}$ is not q-coincident with $\mathcal{G}$.*

Definition 4.23. Let $(\mathbb{V},\Gamma)$ a (m,a,n)-FNTS and $\mathcal{H}$ be a(m,a,n)-FNS over $\mathbb{V}$. Then $\mathcal{H}$ is said to be a (m,a,n)-FN neighbourhood of the (m,a,n)-FNP $v_{(\zeta,\eta,\varsigma)}$over $\mathbb{V}$, if $\exists$ (m,a,n)-FNS $\mathcal{G} \in \Gamma$ such that $v_{(\zeta,\eta,\varsigma)} \in \mathcal{G} \subset \mathcal{H}$.

Theorem 4.24. *Let $(\mathbb{V},\Gamma)$ be a (m,a,n)-FNTS . Then a (m,a,n)-FNS $\mathcal{F} \in \Gamma$ if and only if $\forall$, (m,a,n)-FN point $v_{(\zeta,\eta,\varsigma)} \in \mathcal{F}$ $\exists$ $\mathcal{G} \in \Gamma$ such that $v_{(\zeta,\eta,\varsigma)} \in \mathcal{G} \subseteq \mathcal{F}$.*

Definition 4.25. Let $(\mathbb{V},\Gamma)$ be a (m,a,n)-FNTS and $\mathcal{H}$ be a(m,a,n)-FNS over $\mathbb{V}$. Then $\mathcal{H}$ is said to be a (m,a,n)-FN neighbourhood of the(m,a,n)-FNS $\mathcal{H}$, if $\exists$ a $\mathcal{G} \in \Gamma$ such that $\mathcal{H} \in \mathcal{G} \subset \mathcal{H}$.

Theorem 4.26. *Let $(\mathbb{V},\Gamma)$ a (m,a,n)-FNTS. A (m,a,n)-FNS $\mathcal{H} \in \Gamma$ if and only if $\forall$ (m,a,n)-FNS $\mathcal{G}$ such that $\mathcal{G} \subset \mathcal{H}$, $\mathcal{H}$ is a (m,a,n)-FN neighbourhood of $\mathcal{G}$.*

Proof. Suppose that(m,a,n)-FNS $\mathcal{H} \in \Gamma$. Thus for each $\mathcal{G} \subset \mathcal{H}$, $\mathcal{H}$ is a (m,a,n)-FN neighbourhood of $\mathcal{G}$. Conversely suppose that for each $\mathcal{G} \subset \mathcal{H}$, $\mathcal{H}$ is a (m,a,n)-FN neighbourhood of $\mathcal{G}$. Since $\mathcal{H} \subset \mathcal{H}$, $\mathcal{H}$ is (m,a,n)-FN neighbourhood of $\mathcal{H}$ itself. Therefore $\exists$ $\mathcal{F} \in \Gamma$ such that $\mathcal{H} \subset \mathcal{F} \subset \mathcal{H} \Rightarrow \mathcal{H} = \mathcal{F} \Rightarrow \mathcal{H} \in \Gamma$. □

Definition 4.27. Let $(\mathbb{V},\Gamma)$ be a (m,a,n)-FNTS. A sub collection $\mathbb{B}$ of Γ is referred as a (m,a,n)-fuzzy neutrosophic basis((m,a,n)-FN basis) for Γ, if every nonempty (m,a,n)-FN open set is the union of certain members of $\mathbb{B}$.

Theorem 4.28. *Let $(\mathbb{V},\Gamma)$ be a (m,a,n)-FNTS. Let $\mathbb{B} = \{\mathcal{G}_i : i \in \Lambda\}$ be a subcollection of(m,a,n)-FNT Γ. Then $\mathbb{B}$ is a (m,a,n)-FN basis for Γ if and only if for any $\mathcal{F} \in \Gamma$ and a (m,a,n)-FN point $v_{(\zeta,\eta,\varsigma)} \in \mathcal{F}$, there exists a $\mathcal{G}_i \in \mathbb{B}$ for some $i \in \Lambda$, such that $v_{(\zeta,\eta,\varsigma)} \in \mathcal{G}_i \subset \mathcal{F}$.*

Now we extended the concept of connectedness to (m,a,n)-FNSs and explores its study in (m,a,n)-FNTSs.

Definition 4.29. A (m,a,n)-FNTS $(\mathbb{V},\Gamma)$ is said to be (m,a,n)-fuzzy neutrosophic connected ((m,a,n)-FN connected), if $\nexists$ proper (m,a,n)-FN open sets $\mathcal{A}$ and $\mathcal{B}$ in $\mathbb{V}$ such that $\rceil(\mathcal{A}q\mathcal{B})$ and $\rceil(\mathcal{A}^c q\mathcal{B}^c)$.

Example 4.30. Let $\mathbb{V} = \{v_1, v_2\}$ be universe of discourse and the (m,a,n)-FNSs (for m=2, a=3,n=4) $\mathcal{G}$ and $\mathcal{H}$ on $\mathbb{V}$ be defined as follows:

$$\mathcal{G} = \{< v_1, 0.4, 0.3, 0.8 >, < v_1, 0.3, 04, 0.9 >\}$$

$$\mathcal{H} = \{< v_2, 0.6, 0.5, 0.5 >, < v_2, 0.5, 0.7, 0.8 >\}$$

Let $\Gamma = \{\tilde{\Phi}, \tilde{\mathbb{V}}, \mathcal{G}, \mathcal{H}\}$ be a (m,a,n)-FNT on $\mathbb{V}$, then (m,a,n)-FNTS $(\mathbb{V},\Gamma)$ is (m,a,n)-FN connected

Theorem 4.31. *A (m,a,n)-FNTS $(\mathbb{V},\Gamma)$ is (m,a,n)-fuzzy neutrosophic connected if and only if it has no proper (m,a,n)-fuzzy neutrosophic clopen ((m,a,n)-FN closed and (m,a,n)-FN open) set.*

Theorem 4.32. *A (m,a,n)-FNTS $(\mathbb{V},\Gamma)$ is (m,a,n)-fuzzy neutrosophic connected if and only if it has no proper (m,a,n)-FN open sets $\mathcal{G}$ and $\mathcal{H}$ providing that $\mho^m_{\mathcal{G}}(v) = \Omega^n_{\mathcal{H}}(v)$, $\Omega^n_{\mathcal{G}}(v) = \mho^m_{\mathcal{H}}(v)$ and $\varpi_{\mathcal{H}}(v) + \varpi_{\mathcal{H}}(v) = 1$.*

Now we define (m,a,n)-FN separation axioms by using the concept of (m,a,n)-FN point, (m,a,n)-FN open sets and (m,a,n)-FN closed sets.

Definition 4.33. A (m,a,n)-FNTS $(\mathbb{V},\Gamma)$ is said to be (m,a,n)-FN T_0, if for every pair of distinct (m,a,n)-FN points $v_{(\zeta_1,\eta_1,\varsigma_1)}$ and $w_{(\zeta_2,\eta_2,\varsigma_2)}$ over $\mathbb{V}$, $\exists\ \mathcal{H},\mathcal{G}\in\Gamma$ such that $v_{(\zeta_1,\eta_1,\varsigma_1)}\in\mathcal{H}$ but $v_{(\zeta_2,\eta_2,\varsigma_2)}\notin\mathcal{H}$ or $v_{(\zeta_2,\eta_2,\varsigma_2)}\in\mathcal{G}$ but $v_{(\zeta_1,\eta_1,\varsigma_1)}\notin\mathcal{G}$

Example 4.34. All discrete (m,a,n)-FNTSs are (m,a,n)-FN T_0, because for any two distinct (m,a,n)-FN points $v_{(\zeta_1,\eta_1,\varsigma_1)}$ and $w_{(\zeta_2,\eta_2,\varsigma_2)}$ over $\mathbb{V}$, $\exists$ a (m,a,n)-FN open set $\{v_{(\zeta_1,\eta_1,\varsigma_1)}\}$ such that $v_{(\zeta_1,\eta_1,\varsigma_1)}\in\{v_{(\zeta_1,\eta_1,\varsigma_1)}\}$ but $w_{(\zeta_2,\eta_2,\varsigma_2)}\notin\{v_{(\zeta_1,\eta_1,\varsigma_1)}\}$.

Theorem 4.35. *Every (m,a,n)-FN subspace of a (m,a,n)-FN T_0 space is (m,a,n)-FN T_0.*

Proof. Let $(\mathbb{Y},\Gamma_{\mathbb{Y}})$ be a (m,a,n)-FN subspace of a (m,a,n)-FN T_0 space $(\mathbb{V},\Gamma)$. Let $v_{(\zeta_1,\eta_1,\varsigma_1)}$ and $w_{(\zeta_2,\eta_2,\varsigma_2)}$ be two distinct (m,a,n)-FN points over $\mathbb{Y}$. Then $v_{(\zeta_1,\eta_1,\varsigma_1)}$ and $w_{(\zeta_2,\eta_2,\varsigma_2)}$ are distinct (m,a,n)-FN points over $\mathbb{V}$. Since $(\mathbb{V},\Gamma)$ is (m,a,n)-FN T_0, $\exists$ a (m,a,n)-FN open set containing one of the (m,a,n)-FN point but not other. Without loss of generality, let $\mathcal{H}\in\Gamma$ such that $v_{(\zeta_1,\eta_1,\varsigma_1)}\in\mathcal{H}$ but $w_{(\zeta_2,\eta_2,\varsigma_2)}\notin\mathcal{H}$. Put $\mathcal{H}_{\mathbb{Y}}=\mathcal{H}\cap\mathbb{Y}$. Then $\mathcal{H}_{\mathbb{Y}}\in\Gamma_{\mathbb{Y}}$ such that $v_{(\zeta_1,\eta_1,\varsigma_1)}\in\mathcal{H}_{\mathbb{Y}}$ but $w_{(\zeta_2,\eta_2,\varsigma_2)}\notin\mathcal{H}_{\mathbb{Y}}$. Hence $(\mathbb{Y},\Gamma_{\mathbb{Y}})$ is (m,a,n)-FN T_0. □

Definition 4.36. A (m,a,n)-FNTS $(\mathbb{V},\Gamma)$ is said to be (m,a,n)-FN T_1, if for every pair of distinct (m,a,n)-FN points $v_{(\zeta_1,\eta_1,\varsigma_1)}$ and $w_{(\zeta_2,\eta_2,\varsigma_2)}$ over $\mathbb{V}$, $\exists\ \mathcal{H},\mathcal{G}\in\Gamma$ such that $v_{(\zeta_1,\eta_1,\varsigma_1)}\in\mathcal{H}$ but $v_{(\zeta_2,\eta_2,\varsigma_2)}\notin\mathcal{H}$ and $v_{(\zeta_2,\eta_2,\varsigma_2)}\in\mathcal{G}$ but $v_{(\zeta_1,\eta_1,\varsigma_1)}\notin\mathcal{G}$

Example 4.37. Every discrete (m,a,n)-FNTS is (m,a,n)-FN T_1, because, for any two distinct (m,a,n)-FN point $v_{(\zeta_1,\eta_1,\varsigma_1)}$ and $w_{(\zeta_2,\eta_2,\varsigma_2)}$ over $\mathbb{V}$, $\exists$ (m,a,n)-FN open sets $\{v_{(\zeta_1,\eta_1,\varsigma_1)}\}$ and $\{w_{(\zeta_2,\eta_2,\varsigma_2)}\}$ such that $v_{(\zeta_1,\eta_1,\varsigma_1)}\in\{v_{(\zeta_1,\eta_1,\varsigma_1)}\}$ but $v_{(\zeta_1,\eta_1,\varsigma_1)}\notin\{w_{(\zeta_2,\eta_2,\varsigma_2)}\}$ and $w_{(\zeta_2,\eta_2,\varsigma_2)}\in\{w_{(\zeta_2,\eta_2,\varsigma_2)}\}$ but $v_{(\zeta_1,\eta_1,\varsigma_1)}\notin\{w_{(\zeta_2,\eta_2,\varsigma_1)}\}$.

Theorem 4.38. *Every (m,a,n)-FN subspace of a (m,a,n)-FN T_1 space is (m,a,n)-FN T_1.*

Definition 4.39. A (m,a,n)-FNTS $(\mathbb{V},\Gamma)$ is said to be (m,a,n)-FN T_2 or (m,a,n)-FN Hausdorff, if for every pair of distinct (m,a,n)-FN points $v_{(\zeta_1,\eta_1,\varsigma_1)}$ and $w_{(\zeta_2,\eta_2,\varsigma_2)}$ over $\mathbb{V}$, $\exists\ \mathcal{H},\mathcal{G}\in\Gamma$ such that $v_{(\zeta_1,\eta_1,\varsigma_1)}\in\mathcal{H}$, $w_{(\zeta_2,\eta_2,\varsigma_2)}\in\mathcal{G}$ and $\mathcal{H}\cap\mathcal{G}=\tilde{\Phi}$.

Theorem 4.40. *A (m,a,n)-FNTS $(\mathbb{V},\Gamma)$ is (m,a,n)-FN T_2 if and only if for any two distinct (m,a,n)-FN points $v_{(\zeta_1,\eta_1,\varsigma_1)}$ and $w_{(\zeta_2,\eta_2,\varsigma_2)}$ over $\mathbb{V}$ $\exists$ $\mathcal{E},\mathcal{F}\in(m,a,n)-FNC(\mathbb{V})$ such that $v_{(\zeta_1,\eta_1,\varsigma_1)}\in\mathcal{E}$ but $w_{(\zeta_2,\eta_2,\varsigma_2)}\notin\mathcal{E}$, $v_{(\zeta_1,\eta_1,\varsigma_1)}\notin\mathcal{F}$ but $w_{(\zeta_2,\eta_2,\varsigma_2)}\in\mathcal{F}$ and $\mathcal{E}\cup\mathcal{F}=\mathbb{V}$.*

Theorem 4.41. *Every (m,a,n)-FN subspace of a (m,a,n)-FN T_2 space is (m,a,n)-FN T_2.*

Definition 4.42. A (m,a,n)-FNTS $(\mathbb{V}, \Gamma)$ is said to be (m,a,n)-FN regular if for every $\mathcal{F} \in (m,a,n){-}FNC(\mathbb{V})$ and every (m,a,n)-FN point $v_{(\zeta,\eta,\varsigma)}$ over $\mathbb{V}$ such that $v_{(\zeta,\eta,\varsigma)} \notin \mathcal{F}$, $\exists\, \mathcal{G}, \mathcal{H} \in \Gamma$ such that $v_{(\zeta,\eta,\varsigma)} \in \mathcal{G}$, $\mathcal{F} \subset \mathcal{H}$ and $\mathcal{G} \cap \mathcal{H} = \tilde{\Phi}$.

Theorem 4.43. *Every (m,a,n)-FN subspace of a (m,a,n)-FN regular space is (m,a,n)-FN regular.*

Definition 4.44. A (m,a,n)-FNTS $(\mathbb{V}, \Gamma)$ is said to be (m,a,n)-FN normal if for every pair $\mathcal{E}, \mathcal{F} \in (m,a,n) - FNC(\mathbb{V})$ such that $\mathcal{E} \cap \mathcal{F} = \tilde{\Phi}$, $\exists\, \mathcal{G}, \mathcal{H} \in \Gamma$ such that $\mathcal{E} \subset \mathcal{G}$, $\mathcal{F} \subset \mathcal{H}$ and $\mathcal{G} \cap \mathcal{H} = \tilde{\Phi}$.

Example 4.45. Let $\mathbb{V} = \{v_1, v_2\}$ and(3,4,5)-FNSs $\mathcal{F}_1$ and $\mathcal{F}_2$ be defined as follows:

$$\mathcal{F}_1 = \{< v_1 0.5, 0.3, 0.4 >, < v_2 0.3, 0.2, 0.5 >\}$$

$$\mathcal{F}_2 = \{< v_1, 0.7, 0.3, 0.2 >, < v_2 0.9, 0.2, 0.1 >\}$$

Let $\Gamma = \{\tilde{\Phi}, \tilde{\mathbb{V}}, \mathcal{F}_1, \mathcal{F}_2\}$ be a (3,4,5)-FNT over $\mathbb{V}$. Then the (3,4,5)-FNTS $(\mathbb{L}, \Gamma_1, \Sigma)$ is (3,4,5)-FN normal.

Theorem 4.46. *Every (m,a,n)-FN closed subspace of a (m,a,n)-FN normal space is (m,a,n)-FN normal.*

Proof. Suppose $(\mathbb{Y}, \Gamma_{\mathbb{Y}}, \Sigma)$ be a (m,a,n)-FN closed subspace of a (m,a,n)-FN normal space $(\mathbb{V}, \Gamma)$. Let $\mathcal{F}_1, \mathcal{F}_2 \in (m,a,n){-}FNC(\mathbb{Y})$ such that $\mathcal{F}_1 \cap \mathcal{F}_2 = \tilde{\Phi}$. Since $\mathbb{Y} \in (m,a,n){-}FNC(\mathbb{V})$, by Theorem 4.17, $\mathcal{F}_1, \mathcal{F}_2 \in (m,a,n){-}FNC(\mathbb{V})$. By (m,a,n)-FN normality of $(\mathbb{V}, \Gamma)$, $\exists\, \mathcal{G}_1, \mathcal{G}_2 \in \Gamma$ such that $\mathcal{F}_1 \subset \mathcal{G}_1$, $\mathcal{F}_2 \subset \mathcal{G}_2$ and $\mathcal{G}_1 \cap \mathcal{G}_2 = \tilde{\Phi}$. Put $(\mathcal{G}_1)_{\mathbb{Y}} = \mathcal{G}_1 \cap \mathbb{Y}$ and $(\mathcal{G}_2)_{\mathbb{Y}} = \mathcal{G}_1 \cap \mathbb{Y}$. Then $(\mathcal{G}_1)_{\mathbb{Y}}, (\mathcal{G}_2)_{\mathbb{Y}} \in \Gamma_{\mathbb{Y}}$. Clearly we have $\mathcal{F}_1 \subset \mathcal{G}_1 \Rightarrow \mathcal{F}_1 \cap \mathbb{Y} \subset \mathcal{G}_1 \cap \mathbb{Y} \Rightarrow \mathcal{F}_1 \subset (\mathcal{G}_1)_{\mathbb{Y}}$ and $\mathcal{F}_2 \subset \mathcal{G}_2 \Rightarrow \mathcal{F}_2 \cap \mathbb{Y} \subset \mathcal{G}_2 \cap \mathbb{Y} \Rightarrow \mathcal{F}_2 \subset (\mathcal{G}_2)_{\mathbb{Y}}$. Moreover, $(\mathcal{G}_1)_{\mathbb{Y}} \cap (\mathcal{G}_2)_{\mathbb{Y}} = (\mathcal{G}_1 \cap \mathcal{G}_2) \cap \mathbb{Y} = \tilde{\Phi} \cap \mathbb{Y} = \tilde{\Phi}$. Hence $(\mathbb{Y}, \Gamma_{\mathbb{Y}})$ is (m,a,n)-FN normal. □

5. **Conclusions**

We introduced the concept of (m,a,n)-fuzzy neutrosophic set as a super class of q-rung orthopair neutrosophic set and we created topological structure on (m,a,n)-fuzzy neutrosophic set. The fundamental topological concepts closure,interior, subspaces,bases, connectedness and separation axioms are extended to (m,a,n)-neutrosophic topological spaces. Examples were also given to illustrate our results. It looks that proper combinations of the theories developed for tackling the existing uncertainty is a promising tool for obtaining better results in a variety of human activities characterized by uncertainty. This is, therefore, a fruitful area for future research.

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References

[1] Al-shami, T.; Mhemdi, A., Generalized frame for orthopair fuzzy sets:(m,n)-fuzzy sets, and their applicationsto multicriteria decision making methods, *Information*, 14(1) (2023): 56. https://doi.org/10.3390/info14010056.

[2] Acikgoz A. ; Esenbel F., Neutrosophic connected topological spaces, Fasciculi Mathematici 66(2023) 5-22. DOI: 10.21008/j.0044-4413.2023.0001.

[3] Atanassov, K., Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986), 87-96.

[4] Broumi, S., Sundareswaran, R., Bakali, Q. A. and Talea, M. (2022). Theory and Applications of Fermatean Neutrosophic Graphs, Neutrosophic Sets and Systems, 50, 248-256.

[5] Chang, C. L., Fuzzy topological spaces, J. Math. Anal. Appl. 24(1968), 182-190.

[6] Coker, D. , a introduction to Intuitionistic fuzzy topological spaces, Fuzzy sets and systems, 88(19997), 81-89.

[7] Das, S., Roy, B.K., Kar, M.B. et al. Neutrosophic fuzzy set and its application in decision making. J Ambient Intell Human Comput 11, 5017–5029 (2020). https://doi.org/10.1007/s12652-020-01808-3

[8] Dhavaseelan R.; Jafari,S., Generalized neutrosophic closed sets, New Trends in Neutrosophic Theory and Applications, Vol.II, Florentin Smarandache, Surapati Pramanik (Editors),Pons Editions Brussels, Belgium, EU 2018 245–258.

[9] Dhavaseelan, R.; Narmada Devi, R. ; Jafari S.,Characterizations of neutrosophic nowhere dense sets,International Journal of Mathematical Archive 9(3)(2018), 1-5

[10] Jansi, R., Mohana, K.,Smarandache, F. , Correlation Measure for Pythagorean Neutrosophic Sets with T and F as Dependent Neutrosophic Components. Neutrosophic Sets and Systems, 30(2019), 202-212.

[11] Ibrahim, H. Z., Fermatean Fuzzy Topological Spaces, *Journal of Applied Mathematics and Informatics*, 40 (1-2),(2022), 85–98. https://doi.org/10.14317/jami.2022.085.

[12] Ibrahim, H. Z., Topological approaches for n,m-Rung orthopair fuzzy sets with applications to diagnosis of learning disability, *Journal of Intelligent and Fuzzy Systems*, (2022), 1-15.https://doi.org/10.3233/JIFS-221528.

[13] Karatas, S. ; Cemil K., Neutrosophic Topology, Neutrosophic Sets and Systems 13(1) (2016). https://digitalrepository.unm.edu/nss_journal/vol13/iss1/12.

[14] Lupiáñez F. G., On neutrosophic topology, The International Journal of Systems and Cybernetics, 37(6)(2008), 797-800.

[15] Olgun,M.; Unver, M. and Yard.,S., Pythagorean Fuzzy Topological Spaces,*Complex and Intelligent Systems* , 5, (2019), 1–7.

[16] Salama A. A. ; Alblowi, S. A., Neutrosophic set and neutrosophic topological spaces, IOSR Journal of Mathematics, Vol. 3(4)(2012), 31-35.

[17] Salama, A. A.; Smarandache F. ; Kroumov, V., Neutrosophic closed set and neutrosophic continuous functions, Neutrosophic Sets and Systems, Vol.4(2014), 4-8.

[18] Senapati, T. and Yager,R. R., Fermatean fuzzy sets, Journal of Ambient Intelligence and Humanized Computing, **11**(2020), 663—674.

[19] Smarandache, F. , A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. American Research Press, Rehoboth, NM, 1999.

[20] Smarandache, F., Neutrosophy and Neutrosophic Logics, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics, University of New Mexico, Gallup, NM 87301, USA(2002).

[21] Smarandache F., Neutrosophic set, a generalisation of the intuitionistic fuzzy sets, Int. J. Pure Appl. Math., (24)(2005), 287-297.

[22] Sweety, C.A.C., Jansi, R., Fermatean Neutrosophic Sets, International Journal of Advanced Research in Computer and Communication Engineering, 10(6)(2021), 23-27.

[23] Turkarslan,E. Unver, M. and Olgun, M., q-Rung orthopair fuzzy topological spaces. *Lobachevskii J. Math.*, 42(2), (2021), 470–478.

[24] Voskoglou, M. Gr., Smarandache, F, and Mohamed,M., q-Rung Neutrosophic Sets and Topological Spaces , Neutrosophic Systems with Applications, 15(2024),58-66.

[25] Wang, H., Smarandanche, F., Zhang, Y., and Sunderraman, R. (2010), Single valued neutrosophic sets. Review of the Air Force Academy(Brasov), 1(16), 10-14.

[26] Yager,R. R., Pythagorean fuzzy subsets , Proceedings of the 2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS), Edmonton, AB, Canada, 24–28 June 2013, 57–61.

[27] Yager, R. R., Generalized orthopair fuzzy sets, IEEE Transactions on Fuzzy Systems **25**(2017),1222–1230.

[28] Zadeh, L. A., Fuzzy sets, Inform. and Control 8(1965), 338-353.

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