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Sine Trigonometric Aggregation Operators with Single-Valued Neutrosophic Z-Numbers: Application in Business Site Selection

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Abstract. Business has a massive effect on both people and society as a whole in today's culture. Businesses make significant contributions to humanity's general well-being and progress by promoting economic growth, supporting innovation, producing wealth, and addressing social challenges. However, choosing the right location for a business is a complex task, involving multiple criteria and qualitative and quantitative factors that heavily rely on expert judgement. The researchers propose a unique approach called the SVNZN multi-attributed decision-making method to aid decision-makers in this process. They introduce new operating laws for SVNZNs based on the sine trigonometric (ST) function, known for its periodicity and symmetry over the origin, making it favorable for decision-makers over multi-time phase parameters. Additionally, novel AOs such as SVNZN weighted averaging and geometric operators are defined to combine SVNZNs effectively. Based on these AOs, a decision-making technique for MADM problems is presented, and its applicability is demonstrated through a numerical example of selecting the best location for a business. To validate and enhance the understanding of these proposed techniques, the researchers conducted a comprehensive comparison analysis, including a sensitivity analysis, considering existing literature on MADM difficulties. Figure 1 provides a thorough graphical summary by visually representing all of the contributions and outcomes.

Keywords: Single-valued neutrosophic set, Decision Making, Sine Trigonometric aggregation operator, Z-number.

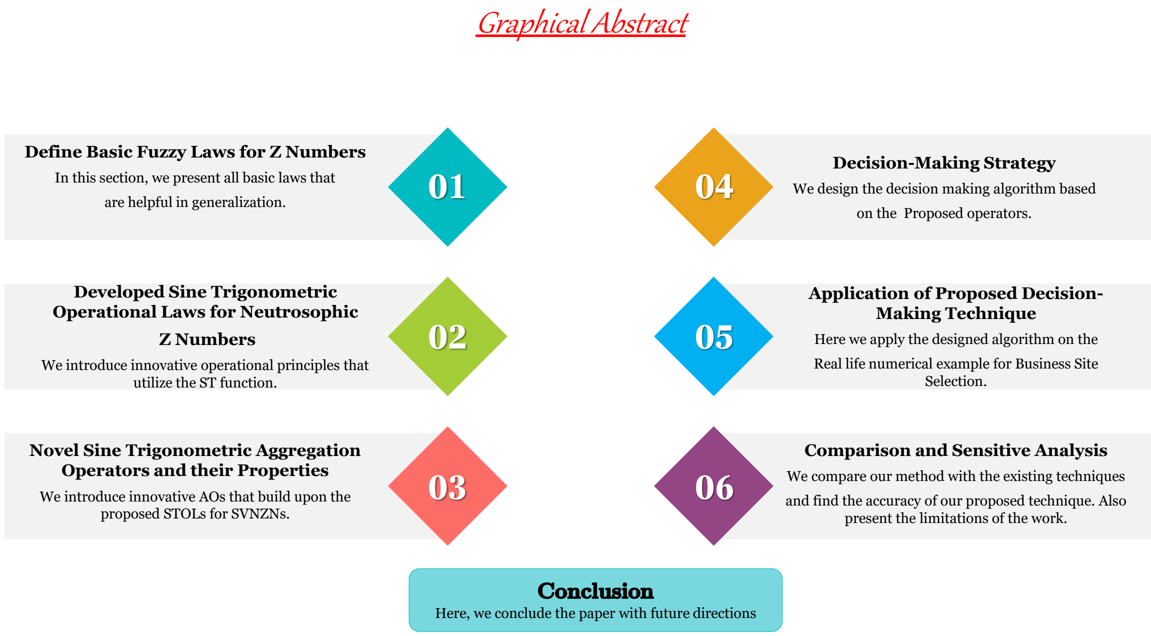

FIGURE 1. Graphical Abstract

We express specific symbols in Table 1, that will be used throughout this work.

TABLE 1. Abbreviations and Descriptions of this Manuscript

Abbreviation	Description	Abbreviation	Description
FS	fuzzy set	ZN	Z-number
FZN	fuzzy ZN	SFZN	spherical FZN
SVNZN	single-valued neutrosophic ZN	AO	aggregation operator
MADM	multi-attribute decision-making	PFZN	picture FZN
ST	sine trigonometric	STAO	ST Aggregation Operator
SVN	single-valued neutrosophic	STOL	ST operational law
DMP	decision-making problem	DMk	decision-maker
WV	weight vector	DM	decision-making

1. Introduction

Research is crucial before deciding on a site for a company, since this choice may affect the organization's long-term performance in significant ways. Before making a final selection, it's important to thoroughly analyze many factors to make sure the selected site can match the organization's goals and operational needs. Numerous important factors must be considered while making decisions; they include, but are not limited to, demographics, infrastructure, and market availability. For instance, several significant factors directly impact a corporation's cost-effectiveness and operational efficiency [2, 3]. The accessibility of trained workers, public transit systems, suppliers, and buyers are all examples of such factors. To guarantee compliance and avoid hazards, more research on legal and regulatory aspects, including zoning limits and tax incentives, is required. Gathering and analyzing as much data as possible is crucial for making decisions; this data must account for market trends, economic patterns, and any hazards that may be present. To gain a variety of opinions and support from everyone involved, it is also important to solicit feedback and participation from stakeholder. According to [4, 5], companies may improve their operational efficiency, promote sustainability, and secure long-term success by thoroughly analyzing these complex challenges and making wise choices.

An additional layer of complexity is added to the decision-making (DM) process when fuzzy set (FS) theory is applied to the process of selecting a location for a company. In the field of FS theory, it is acknowledged that several location criteria may include inherent ambiguities and imprecisions [6]. The application of fuzzy logic assists decision-makers (DMks) in taking into account the complexity of the situation, which is a result of the fact that the variables that exist in the real world are often not binary but rather exist on a spectrum. When it comes to matters like the availability of competent people or a market, for example, there may not be any clear regulations in place. Decision-makers are able to define these features using degrees thanks to FS theory, which enables a representation that is both more realistic and flexible [7] than traditional methods. The provision of a framework for coping with ambiguity is one of the ways in which this strategy improves DM, particularly in the setting of fast-paced commercial environments. The use of FS theory allows companies to make better and more context-aware judgments throughout the site selection process. This helps to guarantee that the selected location is the most suitable for the achievement of their objectives and the fulfillment of their operational requirements [8–10].

In 1965, Zadeh was the first person to present the idea of an FS, which was a generalization of crisp sets. Through the process of providing a membership value between 0 and 1 to each element, FSs provide a unique viewpoint. This value indicates the degree to which the element is associated with the set. The amendment was made in order to provide an achievable solution that could effectively address the inherent ambiguity and uncertainty that is present in reasoning and DM processes [11]. When it comes to processing complicated and imprecise information, embracing FSs enables a more adaptive and intelligent approach. This, in turn, enables a more exact depiction of real-world occurrences and facilitates well-informed decision making in contexts that are unpredictable. The versatility of FSs becomes clear via their wide-ranging applications in many domains, including artificial intelligence [12], control systems [13], pattern recognition [14], decision analysis [15], and decision modeling [16]. Decision modeling is exactly what it sounds like: a strategy. It is a method that is deliberate and planned, and it is used to organize and visualize judgments as well as the qualities that are associated with them. In order to do this, it is required to develop models that include the fundamental aspects of a DM problem (DMP), such as objectives, alternatives, constraints, unpredictability, and interests [17, 18]. The first step in decision modeling is called issue identification, and it involves expressing the choice problem in a clear and simple way while also outlining particular objectives [19]. This phase is when the decision modeling process begins. During this particular phase, it is important to possess a thorough comprehension of the context, to identify the most significant challenges, and to describe the objectives that the process of selecting is intended to achieve [20]. The term "distinctive decision simulation," on the other hand, refers to the use of novel strategies, approaches, or methods within the realm of decision modeling. The process involves experimenting with unique and creative methods of expressing, evaluating, and addressing decision difficulties, often via the use of technological, data analytical, and computational advances. Some examples of inventive document management methods are shown here [21, 22]. Using intelligence-based techniques and predictive algorithms, you may develop decision models that are able to learn from data, make predictions, and optimize the consequences of decisions. Over time, these models are able to autonomously modify and improve themselves based on fresh information and input being received. Leverage the power of big data to provide DM with information. It is important to include vast and complicated data sets from a variety of sources into decision models. Some examples of these sources are social media, sensors, and transaction logs. Obtaining insights and providing support for DM processes may be accomplished via the use of sophisticated data analytic methods like as data mining, predictive modeling, and pattern recognition.

There are two components that make up the representation of a Z-number (ZN), which is represented as $Z = (X, Y)$. These components are described in more detail below. In the case of a real-valued uncertain variable, the first component, which is represented by the letter X, serves as a constraint that determines the range of values that are acceptable with regard to the variable. The second component, which is represented by the letter Y, is accountable for measuring the degree of dependability or confidence that is associated with the information that was provided by the first component. This is the responsible party. Kang et al. [24] were the ones who did the first presentation of fuzzy ZNs (FZNs), while Sari and Kahraman [25] were the ones who developed intuitionistic FZNs. Pythagorean FZNs were presented in [26], and [27] detailed a number of operations via their description.

Each element in an SVN set is given a truth-membership degree denoted by ($\mathfrak{T}$), an indeterminacy-membership degree denoted by ($\mathfrak{I}$), and a falsity-membership degree denoted by ($\mathfrak{F}$), which adds up to 3. The depiction of uncertain and ambiguous features of an element is made easier by these degrees, which also provide a means of accommodating the many degrees of truth and falsehood that are associated with the element. Within the frames of ZNs, FZNs, intuitionistic FZNs, and Pythagorean FZNs, we provide single-valued neutrosophic ZN (SVNZN), a novel notion that enhances the capabilities of these existing frameworks. The formation of SVNZNs involves the combination of two well-known sets, namely SVN sets and ZNs. By combining a number of different frameworks, SVNZNs provide a strong instrument for dealing with the uncertainty and indeterminacy that are present in DM situations. As a result of the introduction of SVNZN, there are now more choices available for dealing with complex DM scenarios that include a variety of different types of uncertainty. This unique idea expands the scope of previously established approaches while simultaneously enhancing the capability to accurately and totally imitate and comprehend occurrences that occur in the actual world. The symbol Z is used to indicate an SVNZN, which is composed of two components: $Z = (X, Y)$ respectively. The first component is made up of three different elements: the truth-membership ($\mathfrak{T}_{\mathfrak{Y}}$), the indeterminacy-membership ($\mathfrak{I}_{\mathfrak{Y}}$), and the falsity-membership ($\mathfrak{F}_{\mathfrak{Y}}$) degree, with the sum of these three components being equal to three. As an alternative, the second component, which is denoted by the symbol Y, is responsible for assessing the level of trustworthiness or dependability that is connected to the information provided by the first component. Additionally, it is composed of three elements, namely ($\mathfrak{T}_{\mathfrak{R}}$), ($\mathfrak{I}_{\mathfrak{R}}$), and ($\mathfrak{F}_{\mathfrak{R}}$), with the sum of these three factors being equal to 3. When used in DM environments where uncertainty and indeterminacy coexist, SVNZNs provide a method that is both comprehensive and rigorous, designed to successfully resolve choice difficulties that occur in the real world. As a consequence of SVNZs, decision makers have access to a more flexible way of presenting and handling complicated data, which ultimately leads to more informed judgments. The procedures and computations that are performed by ZN and SVN sets, which include arithmetic, comparison, and aggregation operations, are comparable to those that are performed by SVNZ. The last point is that SVNZNs provide a flexible framework for dealing with uncertainty and indeterminacy in DM. This framework enables DMks to take into consideration a variety of views and arrive at more robust conclusions.

Motivation and Novelty: It is standard practice to use ST Aggregation Operators (STAOs) in multi-criteria decision making and analysis. As a result of their reliance on the mathematical concept of the sine function, these operators are especially adept at dealing with input that is both perplexing and wrong. STAOs provide a single overall value or preference rating by aggregating individual assessments or criteria values [28]. STAOs have the following important characteristics: STAOs perform a sine change on the input data prior to aggregation. The sine function normalizes values between -1 and 1, allowing for the depiction of imprecise and uncertain data in a continuous and smooth manner. Weighting techniques are used in STAOs to provide relative importance or priority to certain assessments or criteria. This enables DMks to evaluate the relative value of each criterion during the DM process. The weights specify how much each evaluation or criterion effects the overall value or ranking. STAOs offer various AOs to combine the transformed values. These operators include the sine weighted average, sine weighted geometric mean, sine weighted harmonic mean, sine weighted quadratic mean, and other variations etc. Each operator has its own mathematical formula for aggregating the transformed values. The output of STAOs is typically a single aggregated value or a preference ranking of alternatives based on their aggregated scores. The interpretation of the aggregated result depends on the context of the decision problem and the specific requirements of the DMk. STAOs may also handle uncertainty and sensitivity analysis by considering several scenarios or modifications in the given data. Sensitivity analysis assesses the robustness of aggregated data and gives information on the influence of numerous factors on decision making. STAOs are utilized in a wide range of disciplines, such as DM under uncertainty, multi-criteria decision analysis, group DM, and consensus-building techniques. When dealing with ambiguous, vague, or volatile information, STAOs, specifically ST weighted average AOs and ST weighted geometric AOs, provide significant benefits since they effectively capture and represent this complexity. It is important to remember, however, that STAOs are simply one of several AO families used in decision modeling. The most appropriate AO is decided by the specific criteria and features of the decision issue, and various families of operators may be more suited in other cases.

The following justifies the use of ST weighted average AOs and ST weighted geometric AOs on SVNZNs:

- We develop a technique that assigns different criteria different degrees of significance, reflecting their relative relevance in the decision making process, by integrating ST weights. This sensitivity makes sure that important elements are given greater weight, providing a more sophisticated assessment of possible business locations.
- The selected operators are particularly good at managing non-linear connections and interactions between the site selection parameters. The curvature that is created by the ST function has the ability to capture complex and non-linear features that are present in the original data. It is vital to have this insight in order to comprehend how certain factors could interact in non-linearly additive ways, which would result in a more accurate picture of the intricate interactions that influence site suitability.
- By using specified operators, it is possible to efficiently reduce the impact of the extreme values that are included inside the dataset. This is very important since outliers

have the potential to distort the outcomes of choices made when selecting a site for a company. These operators naturally lessen the influence of extreme values, so avoiding outliers from unnecessarily skewing the DM process and encouraging outputs that are reliable and trustworthy.

- The newly implemented aggregation processes are more responsive to even the most minute changes in the variables that are received as input. This is a crucial feature in dynamic business scenarios, where the acceptability of a location may be significantly influenced by even modest changes in the criteria. It is guaranteed that DMks will be able to be more proactive and intelligent in their site selection process if the operators are able to quickly change their judgments in reaction to changing conditions.
- Within the context of site selection, it is usually essential to make compromises between opposing requirements. The operators that were selected provide an approach that is equitable for integrating a large number of criteria, which makes it simpler to evaluate these trade-offs. For instance, the weighted geometric mean takes into consideration a mix of weights and values by default. This encourages a fair compromise between the criteria, as opposed to giving priority to one of the criterion over the others. When it comes to selecting a site for a firm, this makes it feasible for DMks to effectively handle the inherent difficulties of trade-off circumstances.

In light of this, the following are the conclusions of the research:

- The weighted geometric mean and the ST weighted average are two innovative aggregation strategies that we propose. Both of these approaches were built exclusively for statistically significant SVNZNs.
- The incorporation of these additional operators results in the development of a complex decision algorithm that enhances the process of selecting places for commercial ventures.
- Our strategy focuses on SVNZNs in particular, recognizing their special qualities and tackling the difficulties in choosing a site by using designed aggregation operators (AOs).
- We prove the decision algorithm's usefulness in a real-world scenario of business site selection, putting its efficacy in a concrete, applied setting. The system is not merely theoretical.
- Our research leads to the development of a comprehensive framework for DM that makes use of the AOs that we developed. This framework opens the door to the development of more sophisticated and complex business site selection techniques.

This article uses the structure indicated below. In Section 2, fundamental ideas that underpin FS, NS, SVNZNs, and several fundamental operational laws are discussed. We introduce novel

AOs, including Novel Sine Trigonometric Operational Laws, in Section 3. Section 4 introduces new aggregating operators. Key Properties of the Suggested AOs for SVNZNs were also established by Novel Sine Trigonometric Aggregation Operators. In Section 5, a numerical issue solution, numerical illustrations, and a DM method based on the proposed AOs are developed. In Section 6, we compare and contrast a few current practices with suggested ones. In Section 7, we come to a conclusion.

2. Preliminaries and Basic Concepts

In this section, we define some fundamental properties related to our work.

Definition 2.1. [34] In the context of a predetermined set Υ, a picture FZN (PFZN) set $\eth$ in Υ is viewed as

$$\eth = \left\{ \left\langle \mathfrak{Z}, \left(\mathfrak{T}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right), \left(\mathfrak{I}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right), \left(\mathfrak{F}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right) \right\rangle \mid \mathfrak{Z} \in \Upsilon \right\},$$

for each element $\mathfrak{Z}$ in the set Υ, the memberships of $\mathfrak{Z}$ beneath the PFZN $\eth$ have been classified into positive $\left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right) : \Upsilon \to \phi$, neutral $\left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right) : \Upsilon \to \phi$, and negative $\left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right) : \Upsilon \to \phi$ categories, each associated with a specific degree. These degrees are represented by the unit interval $\phi = [0, 1]$. Furthermore, it is essential to guarantee that the sum of $\left(\mathfrak{T}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right), \left(\mathfrak{I}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)$, and $\left(\mathfrak{F}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)$ for each $\mathfrak{Z}$ in Υ remains within the range of 0 to 1.

Definition 2.2. [30] In the context of a predetermined set Υ, a spherical FZN (SFZN) set $\eth$ in Υ is viewed as

$$\eth = \left\{ \left\langle \mathfrak{Z}, \left(\mathfrak{T}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right), \left(\mathfrak{I}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right), \left(\mathfrak{F}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right) \right\rangle \mid \mathfrak{Z} \in \Upsilon \right\},$$

for each element $\mathfrak{Z}$ in the set Υ, the memberships of $\mathfrak{Z}$ beneath the SFZN $\eth$ have been classified into positive $\left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right) : \Upsilon \to \phi$, neutral $\left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right) : \Upsilon \to \phi$, and negative $\left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right) : \Upsilon \to \phi$ categories, each associated with a specific degree. These degrees are represented by the unit interval $\phi = [0, 1]$. Furthermore, it is essential to guarantee that the sum of $\left(\mathfrak{T}^2_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{T}^2_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right), \left(\mathfrak{I}^2_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{I}^2_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)$, and $\left(\mathfrak{F}^2_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{F}^2_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)$ for each $\mathfrak{Z}$ in Υ remains within the range of 0 to 1.

Definition 2.3. [35] In the context of a predetermined set Υ, a SVNZN set $\eth$ in Υ is viewed as

$$\eth = \left\{ \left\langle \mathfrak{Z}, \left(\mathfrak{T}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right), \left(\mathfrak{I}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right), \left(\mathfrak{F}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right) \right\rangle \mid \mathfrak{Z} \in \Upsilon \right\},$$

for each element $\mathfrak{Z}$ in the set Υ, the memberships of $\mathfrak{Z}$ beneath the SVNZN $\eth$ have been classified into truth $\left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right) : \Upsilon \to \phi$, indeterminacy $\left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right) : \Upsilon \to \phi$, and falsity $\left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right) : \Upsilon \to \phi$ categories, each associated with a specific degree. These degrees are

represented by the unit interval $\phi = [0,1]$. Furthermore, it is essential to guarantee that the sum of $\left(\mathfrak{T}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right), \left(\mathfrak{I}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)$, and $\left(\mathfrak{F}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)$ for each $\mathfrak{Z}$ in $\curlyvee$ remains within the range of 0 to 3.
In brief, the triplet $\eth = \left\{\left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right), \left(\mathfrak{I}_{Vji}, \mathfrak{I}_{Rji}\right), \left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right)\right\}$ SVNZN in the entirety of the attention and the ensemble of SVNZNs signified by SVNZN($\curlyvee$).

Definition 2.4. [35] In the context of a predetermined set $\curlyvee$, $SVNZN(\curlyvee)$ set in a universe set $\curlyvee$ is viewed as:

$$\eth_Z = \left\{\langle \mathfrak{Z},\ \mathfrak{T}(\mathfrak{V},\ \mathfrak{R})(\mathfrak{Z}),\ \mathfrak{I}(\mathfrak{V},\ \mathfrak{R})(\mathfrak{Z}),\ \mathfrak{F}(\mathfrak{V},\ \mathfrak{R})(\mathfrak{Z})\rangle \mid \mathfrak{Z} \in \curlyvee\right\},$$

where $\mathfrak{T}(\mathfrak{V},\ \mathfrak{R})(\mathfrak{Z}) = (\mathfrak{T}_{\mathfrak{V}}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}}(\mathfrak{Z}))$, $\mathfrak{I}(\mathfrak{V},\ \mathfrak{R})(\mathfrak{Z}) = (\mathfrak{I}_{\mathfrak{V}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}}(\mathfrak{Z}))$, $\mathfrak{F}(\mathfrak{V},\ \mathfrak{R})(\mathfrak{Z}) = (\mathfrak{F}_{\mathfrak{V}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}}(\mathfrak{Z})) : \curlyvee \to [0,1]^2$ are the order pairs of truth, indeterminacy and falsity membership, then the component $\mathfrak{R}$ is neutrosophic measures of reliability for $\mathfrak{V}$, along with the sum of $\mathfrak{T}_{\mathfrak{V}}(\mathfrak{Z})$, $\mathfrak{I}_{\mathfrak{V}}(\mathfrak{Z})$, $\mathfrak{F}_{\mathfrak{V}}(\mathfrak{Z})$ remains within the range of 0 to 3, and also sum of $\mathfrak{T}_{\mathfrak{R}}(\mathfrak{Z})$, $\mathfrak{I}_{\mathfrak{R}}(\mathfrak{Z})$, $\mathfrak{F}_{\mathfrak{R}}(\mathfrak{Z})$ remains within the range of 0 to 3. The element $\langle \mathfrak{Z},\ \mathfrak{T}(\mathfrak{V},\ \mathfrak{R})(\mathfrak{Z}),\ \mathfrak{I}(\mathfrak{V},\ \mathfrak{R})(\mathfrak{Z}),\ \mathfrak{F}(\mathfrak{V},\ \mathfrak{R})(\mathfrak{Z})\rangle$ in $\eth_Z$ has been simplified for ease of depiction referred as $\eth_Z = \langle \mathfrak{T}(\mathfrak{V},\ \mathfrak{R}), \mathfrak{I}(\mathfrak{V},\ \mathfrak{R}), \mathfrak{F}(\mathfrak{V},\ \mathfrak{R})\rangle = \langle(\mathfrak{T}_{\mathfrak{V}},\ \mathfrak{T}_{\mathfrak{R}}),\ (\mathfrak{I}_{\mathfrak{V}},\ \mathfrak{I}_{\mathfrak{R}}),\ (\mathfrak{F}_{\mathfrak{V}},\ \mathfrak{F}_{\mathfrak{R}})\rangle$, which is designated as $SVNZN(\curlyvee)$.

Definition 2.5. [35] Let $\eth_1 = \{(\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1}), (\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1}), (\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1})\}$ and $\eth_2 = \{(\mathfrak{T}_{\mathfrak{V}_2}, \mathfrak{T}_{\mathfrak{R}_2}), (\mathfrak{I}_{\mathfrak{V}_2}, \mathfrak{I}_{\mathfrak{R}_2}), (\mathfrak{F}_{\mathfrak{V}_2}, \mathfrak{F}_{\mathfrak{R}_2})\} \in SVNZN(\curlyvee)$. then,

- **(1):** $\eth_1 \subseteq \eth_2$ if and only if $(\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1}) \leq (\mathfrak{T}_{\mathfrak{V}_2}, \mathfrak{T}_{\mathfrak{R}_2})$,$(\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1}) \geq (\mathfrak{I}_{\mathfrak{V}_2}, \mathfrak{I}_{\mathfrak{R}_2})$ and $(\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1}) \geq (\mathfrak{F}_{\mathfrak{V}_2}, \mathfrak{F}_{\mathfrak{R}_2})$ for each $\mathfrak{Z} \in \curlyvee$.
- **(2):** $\eth_1 = \eth_2$ if and only if $\eth_1 \subseteq \eth_2$ and $\eth_2 \subseteq \eth_1$.
- **(3):** $\eth_1 \cap \eth_2 = \left\{\begin{array}{c} \inf\left((\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1}), (\mathfrak{T}_{\mathfrak{V}_2}, \mathfrak{T}_{\mathfrak{R}_2})\right), \sup\left((\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1}), (\mathfrak{I}_{\mathfrak{V}_2}, \mathfrak{I}_{\mathfrak{R}_2})\right), \\ \sup\left((\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1}), (\mathfrak{F}_{\mathfrak{V}_2}, \mathfrak{F}_{\mathfrak{R}_2})\right) \end{array}\right\}$,
- **(4):** $\eth_1 \cup \eth_2 = \left\{\begin{array}{c} \sup\left((\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1}), (\mathfrak{T}_{\mathfrak{V}_2}, \mathfrak{T}_{\mathfrak{R}_2})\right), \inf\left((\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1}), (\mathfrak{I}_{\mathfrak{V}_2}, \mathfrak{I}_{\mathfrak{R}_2})\right), \\ \inf\left((\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1}), (\mathfrak{F}_{\mathfrak{V}_2}, \mathfrak{F}_{\mathfrak{R}_2})\right) \end{array}\right\}$,
- **(5):** $\eth_1^c = \{(\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1}), (\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1}), (\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1})\}$.

Definition 2.6. [35] Let $\eth_1 = \{(\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1}), (\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1}), (\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1})\}$ and $\eth_2 = \{(\mathfrak{T}_{\mathfrak{V}_2}, \mathfrak{T}_{\mathfrak{R}_2}), (\mathfrak{I}_{\mathfrak{V}_2}, \mathfrak{I}_{\mathfrak{R}_2}), (\mathfrak{F}_{\mathfrak{V}_2}, \mathfrak{F}_{\mathfrak{R}_2})\} \in SVNZN(\curlyvee)$ with $\xi > 0$. then,

(1):

$$\eth_1 \otimes \eth_2 = \left\{\begin{array}{c} (\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1})(\mathfrak{T}_{\mathfrak{V}_2}, \mathfrak{T}_{\mathfrak{R}_2}), (\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1}) + (\mathfrak{I}_{\mathfrak{V}_2}, \mathfrak{I}_{\mathfrak{R}_2}) - (\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1}) \cdot (\mathfrak{I}_{\mathfrak{V}_2}, \mathfrak{I}_{\mathfrak{R}_2}), \\ (\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1}) + (\mathfrak{F}_{\mathfrak{V}_2}, \mathfrak{F}_{\mathfrak{R}_2}) - (\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1}) \cdot (\mathfrak{F}_{\mathfrak{V}_2}, \mathfrak{F}_{\mathfrak{R}_2}) \end{array}\right\};$$

(2):

$$\eth_1 \oplus \eth_2 = \left\{\begin{array}{c} (\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1}) + (\mathfrak{T}_{\mathfrak{V}_2}, \mathfrak{T}_{\mathfrak{R}_2}) - (\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1})(\mathfrak{T}_{\mathfrak{V}_2}, \mathfrak{T}_{\mathfrak{R}_2}), (\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1})(\mathfrak{I}_{\mathfrak{V}_2}, \mathfrak{I}_{\mathfrak{R}_2}), \\ (\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1})(\mathfrak{F}_{\mathfrak{V}_2}, \mathfrak{F}_{\mathfrak{R}_2}) \end{array}\right\};$$

(3):

$$(\eth_1)^{\xi} = \left\{ (\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1})^{\xi}, 1-(1-(\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1}))^{\xi}, 1-(1-(\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1}))^{\xi} \right\};$$

(4):

$$\xi \cdot \eth_1 = \left\{ 1-(1-(\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1}))^{\xi}, (\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1})^{\xi}, (\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1})^{\xi} \right\};$$

(5):

$$\xi^{\eth_1} = \left\{ \begin{array}{cc} \left(\xi^{1-\left(\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1}\right)}, 1-\xi^{\left(\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1}\right)}, 1-\xi^{\left(\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1}\right)}\right) & \text{if } \xi \in (0,1) \\ \left(\left(\frac{1}{\xi}\right)^{1-\left(\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1}\right)}, 1-\left(\frac{1}{\xi}\right)^{\left(\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1}\right)}, 1-\left(\frac{1}{\xi}\right)^{\left(\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1}\right)}\right) & \text{if } \xi \geq 1 \end{array} \right\}$$

Definition 2.7. [35] Let $\eth_{\Xi} = \{(\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}}), (\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}}), (\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}})\} \in SVNZN(\Upsilon)$ $(\Xi = 1, 2, 3, ..., n)$. Subsequently, the algebraic averaging AO associated with the set $SVNZN(\Upsilon)$ is identified as $SVNZNWA$ and outlined in the ensuing manner:

$$\begin{aligned} SVNZNWA(\eth_1, \eth_2, \eth_3, ..., \eth_n) &= \sum_{\Xi=1}^{n} \xi_{\Xi} \eth_{\Xi}, \\ &= \left\{ \begin{array}{c} 1-\prod_{\Xi=1}^{n}(1-(\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}}))^{\xi_{\Xi}}, \prod_{\Xi=1}^{n}((\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}}))^{\xi_{\Xi}}, \\ \prod_{\Xi=1}^{n}((\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}}))^{\xi_{\Xi}} \end{array} \right\}. \end{aligned}$$

Here, ξ_{Ξ} (where Ξ ranges from 1 to n) signifies the weights assigned to $\eth_{\Xi}$ (where Ξ ranges from 1 to n), with the stipulation that ξ_{Ξ} is non-negative and the summation of all ξ_{Ξ} values equals 1.

Definition 2.8. [35] Let $\eth_{\Xi} = \{(\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}}), (\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}}), (\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}})\} \in SVNZN(\Upsilon)$ $(\Xi = 1, 2, 3, ..., n)$. Subsequently, the algebraic geometric AO associated with the set $SVNZN(\Upsilon)$ is identified as $SVNZNWG$ and outlined in the ensuing manner:

$$\begin{aligned} SVNZNWG(\eth_1, \eth_2, \eth_3, ..., \eth_n) &= \prod_{\Xi=1}^{n} (\eth_{\Xi})^{\xi_{\Xi}}, \\ &= \left\{ \begin{array}{c} \prod_{\Xi=1}^{n}((\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}}))^{\xi_{\Xi}}, 1-\prod_{\Xi=1}^{n}(1-(\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}}))^{\xi_{\Xi}}, \\ 1-\prod_{\Xi=1}^{n}(1-(\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}}))^{\xi_{\Xi}} \end{array} \right\} \end{aligned}$$

Here, ξ_{Ξ} (where Ξ ranges from 1 to n) signifies the weights assigned to $\eth_{\Xi}$ (where Ξ ranges from 1 to n), with the stipulation that ξ_{Ξ} is non-negative and the summation of all ξ_{Ξ} values equals 1.

3. Novel Sine Trigonometric Operational Laws For SVNZNs

Within this portion, we introduce innovative principles that utilize the ST function within SVNZN settings.

Definition 3.1. Let $\eth = \left\{\left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right), \left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right), \left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right)\right\} \in SVNZN(\Upsilon)$. Subsequently,ST operational laws (STOLs) for SVNZN $\eth$ are outlined below:

$$\sin(\eth) = \left\{ \begin{pmatrix} \mathfrak{Z}, \sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)\right), 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)\right)\right), \\ 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)\right)\right) \end{pmatrix} | \mathfrak{Z} \in \Upsilon \right\}$$

The fact that $\sin(\eth)$ also exhibits the SVNZN property is evident. It is evident that, for every element $\mathfrak{Z}$ within the set Υ, the values representing truth, indeterminacy, and falsity, denoted as $\left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right) : \Upsilon \to \phi$, $\left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right) : \Upsilon \to \phi$, and $\left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right) : \Upsilon \to \phi$ respectively, pertain to the SVNZN set $\eth$. Here, $\phi = [0,1]$ designates the unit interval. Furthermore, it is essential to guarantee that the sum of $\left(\mathfrak{T}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)$, $\left(\mathfrak{I}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)$, and $\left(\mathfrak{F}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)$ for each $\mathfrak{Z}$ in Υ remains within the range of 0 to 3. Moreover, the membership degree of truth

$$\begin{aligned} \sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right)\right) &: \Upsilon \to \phi, \\ \text{for each } \mathfrak{Z} &\in \Upsilon \to \sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)\right) \in [0,1], \end{aligned}$$

membership degree of indeterminacy

$$\begin{aligned} 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right)\right)\right) &: \Upsilon \to \phi, \\ \text{for each } \mathfrak{Z} &\in \Upsilon \to 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)\right)\right) \in [0,1], \end{aligned}$$

and membership degree of falsity

$$\begin{aligned} 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right)\right)\right) &: \Upsilon \to \phi, \\ \text{for each } \mathfrak{Z} &\in \Upsilon \to 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)\right)\right) \in [0,1]. \end{aligned}$$

As such,

$$\sin(\eth) = \left\{ \begin{pmatrix} \mathfrak{Z}, \sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)\right), 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)\right)\right), \\ 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)\right)\right) \end{pmatrix} | \mathfrak{Z} \in \Upsilon \right\}$$

is SVNZN.

Definition 3.2. Let $\eth = \left\{\left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right), \left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right), \left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right)\right\} \in SVNZN(\Upsilon)$. If

$$\sin(\eth) = \left\{ \begin{pmatrix} \mathfrak{Z}, \sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)\right), 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)\right)\right), \\ 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)\right)\right) \end{pmatrix} | \mathfrak{Z} \in \Upsilon \right\}$$

Subsequently, the function sin($\eth$) is referred to as the ST operator, and the outcome of sin($\eth$) is termed the ST-SVNZN (STSVNZN).

Theorem 3.3. *Let* $\eth = \left\{\left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right), \left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right), \left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right)\right\} \in SVNZN(\Upsilon)$. *Then, the result yielded by the operator* sin($\eth$) *possesses the SVNZN property.*

Proof. As $\eth = \left\{\left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right), \left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right), \left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right)\right\}$, that is, $0 \leq \left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right) \leq 1$, $0 \leq \left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right) \leq 1$ and $0 \leq \left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right) \leq 1$. Moreover, $\left(\mathfrak{T}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right) + \left(\mathfrak{I}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right) + \left(\mathfrak{F}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right) \leq 3$, for every $\mathfrak{Z} \in \curlyvee$. In order to demonstrate that sin($\eth$) holds the SVNZN characteristic, two essential conditions are considered:

(1): $\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right)\right)$,$1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right)\right)\right)$ and $1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right)\right)\right) \in [0,1]$

(2): $\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right)\right)+1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right)\right)\right)+1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right)\right)\right) \leq 3$.

As $0 \leq \left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right) \leq 1$ this leads to the inference that $0 \leq \frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right) \leq \frac{\Pi}{2}$. Additionally, it's important to note that the function "sin" is monotonically increasing within the first quadrant; therefore, we have $0 \leq \sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right)\right) \leq 1$.
As $0 \leq \left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right) \leq 1$ this leads to the inference that $0 \leq \frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right)\right) \leq \frac{\Pi}{2}$,
$\Rightarrow 0 \leq \sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right)\right)\right) \leq 1$. Consequently, we obtain $0 \leq 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right)\right)\right) \leq 1$.
Likewise, we acquire $0 \leq 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right)\right)\right) \leq 1$. Hence, part (1) is established.
As $\eth \in SVNZN(\curlyvee) \Rightarrow 0 \leq \left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right), \left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right), \left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right) \leq 1$, and
$\left(\mathfrak{T}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right) + \left(\mathfrak{I}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right) + \left(\mathfrak{F}_{\mathfrak{V}_{ji}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right) \leq 3$, for every $\mathfrak{Z} \in \curlyvee$.
Subsequently, (1) indicates that $0 \leq \sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right)\right), 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right)\right)\right), 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right)\right)\right) \leq 1$. Furthermore, as per Definition 3.1, we possess $0 \leq \sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right)\right)+1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right)\right)\right)+1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right)\right)\right) \leq 3$. Consequently, it can be concluded that sin($\eth$) exhibits the SVNZN property. □

Definition 3.4. Let $\sin(\eth_1) = \left\{\left(\begin{array}{l} \sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1}\right)\right), \\ 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1}\right)\right)\right), \\ 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1}\right)\right)\right) \end{array}\right)\right\}$, and $\sin(\eth_2) = \left\{\left(\begin{array}{l} \sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_2}, \mathfrak{T}_{\mathfrak{R}_2}\right)\right), \\ 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_2}, \mathfrak{I}_{\mathfrak{R}_2}\right)\right)\right), \\ 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_2}, \mathfrak{F}_{\mathfrak{R}_2}\right)\right)\right) \end{array}\right)\right\}$ be two STSVNZNs. Then the operational laws are as follows

(1):

$$\sin(\eth_1) \bigoplus \sin(\eth_2) = \left(\begin{array}{c} 1-\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1}\right)\right)\right)\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_2}, \mathfrak{T}_{\mathfrak{R}_2}\right)\right)\right), \\ \left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1}\right)\right)\right)\right)\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_2}, \mathfrak{I}_{\mathfrak{R}_2}\right)\right)\right)\right), \\ \left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1}\right)\right)\right)\right)\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_2}, \mathfrak{F}_{\mathfrak{R}_2}\right)\right)\right)\right) \end{array}\right),$$

(2):

$$\psi \cdot \sin(\eth_1) = \left(\begin{array}{c} 1-\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1}\right)\right)\right)^{\psi}, \left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1}\right)\right)\right)\right)^{\psi}, \\ \left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1}\right)\right)\right)\right)^{\psi} \end{array}\right),$$

(3):

$$\sin(\eth_1)\bigotimes\sin(\eth_2)=\begin{pmatrix}\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1}\right)\right)\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_2},\mathfrak{T}_{\mathfrak{R}_2}\right)\right),\\ 1-\left(\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1}\right)\right)\right)\right)\left(\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_2},\mathfrak{I}_{\mathfrak{R}_2}\right)\right)\right)\right),\\ 1-\left(\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_1},\mathfrak{F}_{\mathfrak{R}_1}\right)\right)\right)\right)\left(\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_2},\mathfrak{F}_{\mathfrak{R}_2}\right)\right)\right)\right)\end{pmatrix},$$

(4):

$$(\sin(\eth_1))^{\psi}=\begin{pmatrix}\left(\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1}\right)\right)\right)^{\psi},\\ 1-\left(\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1}\right)\right)\right)\right)^{\psi},\\ 1-\left(\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_1},\mathfrak{F}_{\mathfrak{R}_1}\right)\right)\right)\right)^{\psi}\end{pmatrix}.$$

In order to make comparisons between the STSVNZNs, we have introduced the subsequent definitions.

Definition 3.5. Consider $\eth=\left\{\left(\mathfrak{T}_{\mathfrak{V}_{ji}},\mathfrak{T}_{\mathfrak{R}_{ji}}\right),\left(\mathfrak{I}_{\mathfrak{V}_{ji}},\mathfrak{I}_{\mathfrak{R}_{ji}}\right),\left(\mathfrak{F}_{\mathfrak{V}_{ji}},\mathfrak{F}_{\mathfrak{R}_{ji}}\right)\right\}\in SVNZN(\Upsilon)$. In this context, we symbolize and establish the *partial* score and accuracy this way:

(1): $\overline{sc}(\eth)=\left(\mathfrak{T}_{\mathfrak{V}_{ji}},\mathfrak{T}_{\mathfrak{R}_{ji}}\right)-\left(\mathfrak{I}_{\mathfrak{V}_{ji}},\mathfrak{I}_{\mathfrak{R}_{ji}}\right)-\left(\mathfrak{F}_{\mathfrak{V}_{ji}},\mathfrak{F}_{\mathfrak{R}_{ji}}\right)$, and

(2): $\underline{ac}(\eth)=\left(\mathfrak{T}_{\mathfrak{V}_{ji}},\mathfrak{T}_{\mathfrak{R}_{ji}}\right)+\left(\mathfrak{I}_{\mathfrak{V}_{ji}},\mathfrak{I}_{\mathfrak{R}_{ji}}\right)+\left(\mathfrak{F}_{\mathfrak{V}_{ji}},\mathfrak{F}_{\mathfrak{R}_{ji}}\right)$.

Definition 3.6. Let $\eth_1=\{(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1}),(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1}),(\mathfrak{F}_{\mathfrak{V}_1},\mathfrak{F}_{\mathfrak{R}_1})\}$ and $\eth_2=\{(\mathfrak{T}_{\mathfrak{V}_2},\mathfrak{T}_{\mathfrak{R}_2}),(\mathfrak{I}_{\mathfrak{V}_2},\mathfrak{I}_{\mathfrak{R}_2}),(\mathfrak{F}_{\mathfrak{V}_2},\mathfrak{F}_{\mathfrak{R}_2})\}\in SVNZN(\Upsilon)$. Then,

(1): If $\overline{sc}(\eth_1)<\overline{sc}(\eth_2)$, it follows that $\eth_1<\eth_2$.

(2): If $\overline{sc}(\eth_1)>\overline{sc}(\eth_2)$, it follows that $\eth_1>\eth_2$.

(3): If $\overline{sc}(\eth_1)=\overline{sc}(\eth_2)$, then

(a): $\underline{ac}(\eth_1)<\underline{ac}(\eth_2)$, it follows that $\eth_1<\eth_2$,

(b): $\underline{ac}(\eth_1)>\underline{ac}(\eth_2)$, it follows that $\eth_1>\eth_2$,

(c): $\underline{ac}(\eth_1)=\underline{ac}(\eth_2)$, it follows that $\eth_1=\eth_2$.

To compare $SVNZNs$ $\mathfrak{W}_{Z_i}=\langle\mathfrak{T}_i(\mathfrak{V},\ \mathfrak{R}),\ \mathfrak{I}_i(\mathfrak{V},\ \mathfrak{R}),\ \mathfrak{F}_i(\mathfrak{V},\ \mathfrak{R})\rangle=\langle(\mathfrak{T}_{\mathfrak{V}_i},\mathfrak{T}_{\mathfrak{R}_i}),(\mathfrak{I}_{\mathfrak{V}_i},\mathfrak{I}_{\mathfrak{R}_i}),(\mathfrak{F}_{\mathfrak{V}_i},\mathfrak{F}_{\mathfrak{R}_i})\rangle$ $(i=1,2)$, we introduce a score function:

$$Y(\mathfrak{W}_{Z_i})=\frac{2+\mathfrak{T}_{Vi}\mathfrak{T}_{Ri}-\mathfrak{I}_{Vi}\mathfrak{I}_{Ri}-\mathfrak{F}_{Vi}\mathfrak{F}_{Ri}}{3}$$

for $Y(\mathfrak{W}_{Z_i})\in[0,1]$, when $Y(\mathfrak{W}_{Z_1})\geq Y(\mathfrak{W}_{Z_2})$, this leads to the conclusion that the ranking is $\mathfrak{W}_{Z_1}\geq\mathfrak{W}_{Z_2}$.

Example 3.7. Set two $SVNZNs$ as $\mathfrak{W}_{Z_1}=\langle(0.9,\ 0.6),\ (0.6,\ 0.8),\ (0.7,\ 0.9)\rangle$ and $\mathfrak{W}_{Z_2}=\langle(0.8,\ 0.5),\ (0.1,\ 0.4),\ (0.3,\ 0.6)\rangle$. By using score function, we have

$$Y(\mathfrak{W}_{Z_1})=\frac{(2+0.9\times0.6-0.6\times0.8-0.7\times0.9)}{3}=0.477$$

and

$$Y(\mathfrak{W}_{Z_2})=\frac{(2+0.8\times0.5-0.1\times0.4-0.3\times0.6)}{3}=0.727$$

As $Y(\mathfrak{W}_{Z_1}) < Y(\mathfrak{W}_{Z_2})$, it follows that their ranking is $\mathfrak{W}_{Z_1} < \mathfrak{W}_{Z_2}$. Subsequently, we delve into a discussion of fundamental properties of STSVNZNs built upon the introduced STOLs.

Theorem 3.8. *Let* $\eth_1 = \{(\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1}), (\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1}), (\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1})\}$ *and* $\eth_2 = \{(\mathfrak{T}_{\mathfrak{V}_2}, \mathfrak{T}_{\mathfrak{R}_2}), (\mathfrak{I}_{\mathfrak{V}_2}, \mathfrak{I}_{\mathfrak{R}_2}), (\mathfrak{F}_{\mathfrak{V}_2}, \mathfrak{F}_{\mathfrak{R}_2})\} \in SVNZN(\Upsilon)$. *Then,*

(1): $\sin(\eth_1) \bigoplus \sin(\eth_2) = \sin(\eth_2) \bigoplus \sin(\eth_1)$,

(2): $\sin(\eth_1) \bigotimes \sin(\eth_2) = \sin(\eth_2) \bigotimes \sin(\eth_1)$.

Proof. This is evident directly from Definition 3.2. □

Theorem 3.9. *Let* $\eth_\Xi = \{(\mathfrak{T}_{\mathfrak{V}_\Xi}, \mathfrak{T}_{\mathfrak{R}_\Xi}), (\mathfrak{I}_{\mathfrak{V}_\Xi}, \mathfrak{I}_{\mathfrak{R}_\Xi}), (\mathfrak{F}_{\mathfrak{V}_\Xi}, \mathfrak{F}_{\mathfrak{R}_\Xi})\} \in SVNZN(\Upsilon)$ $(\Xi = 1, 2, 3)$. *Then,*

(1): $(\sin(\eth_1) \bigoplus \sin(\eth_2)) \bigoplus \sin(\eth_3) = \sin(\eth_1) \bigoplus (\sin(\eth_2) \bigoplus \sin(\eth_3))$,

(2): $(\sin(\eth_1) \bigotimes \sin(\eth_2)) \bigotimes \sin(\eth_3) = \sin(\eth_1) \bigotimes (\sin(\eth_2) \bigotimes \sin(\eth_3))$.

Proof. This is evident directly from Definition 3.2. □

Theorem 3.10. *Let* $\eth_\Xi = \{(\mathfrak{T}_{\mathfrak{V}_\Xi}, \mathfrak{T}_{\mathfrak{R}_\Xi}), (\mathfrak{I}_{\mathfrak{V}_\Xi}, \mathfrak{I}_{\mathfrak{R}_\Xi}), (\mathfrak{F}_{\mathfrak{V}_\Xi}, \mathfrak{F}_{\mathfrak{R}_\Xi})\} \in SVNZN(\Upsilon)$ $(\Xi = 1, 2)$ *and* $\psi, \psi_1, \psi_2 > 0$. *Then,*

(1): $\psi(\sin(\eth_1) \bigoplus \sin(\eth_2)) = \psi \sin(\eth_1) \bigoplus \psi \sin(\eth_2)$,

(2): $(\sin(\eth_1) \bigotimes \sin(\eth_2))^{\psi} = (\sin(\eth_1))^{\psi} \bigotimes (\sin(\eth_2))^{\psi}$,

(3): $\psi_1 \sin(\eth_1) \bigoplus \psi_2 \sin(\eth_1) = (\psi_1 + \psi_2)\sin(\eth_1)$,

(4): $(\sin(\eth_1))^{\psi_1} \bigotimes (\sin(\eth_1))^{\psi_2} = (\sin(\eth_1))^{\psi_1+\psi_2}$,

(5): $\left((\sin(\eth_1))^{\psi_1}\right)^{\psi_2} = (\sin(\eth_1))^{\psi_1 \cdot \psi_2}$.

Proof.
Let $\eth_\Xi = \{(\mathfrak{T}_{\mathfrak{V}_\Xi}, \mathfrak{T}_{\mathfrak{R}_\Xi}), (\mathfrak{I}_{\mathfrak{V}_\Xi}, \mathfrak{I}_{\mathfrak{R}_\Xi}), (\mathfrak{F}_{\mathfrak{V}_\Xi}, \mathfrak{F}_{\mathfrak{R}_\Xi})\} \in SVNZN(\Upsilon)$ $(\Xi = 1, 2)$ and $\psi, \psi_1, \psi_2 > 0$. Then, by the Definition 3.2, we have $\sin(\eth_1) = \left\{\begin{pmatrix} \sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1})\right), \\ 1 - \sin\left(\frac{\Pi}{2}(1 - (\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1}))\right), \\ 1 - \sin\left(\frac{\Pi}{2}(1 - (\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1}))\right) \end{pmatrix}\right\}$ and $\sin(\eth_2) = \left\{\begin{pmatrix} \sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_2}, \mathfrak{T}_{\mathfrak{R}_2})\right), \\ 1 - \sin\left(\frac{\Pi}{2}(1 - (\mathfrak{I}_{\mathfrak{V}_2}, \mathfrak{I}_{\mathfrak{R}_2}))\right), \\ 1 - \sin\left(\frac{\Pi}{2}(1 - (\mathfrak{F}_{\mathfrak{V}_2}, \mathfrak{F}_{\mathfrak{R}_2}))\right) \end{pmatrix}\right\}$ be two STSVNZNs. Therefore, using the STOLs for SVNZNs, we obtain

$$\sin(\eth_1) \bigoplus \sin(\eth_2) = \begin{pmatrix} 1 - \left(1 - \sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1})\right)\right)\left(1 - \sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_2}, \mathfrak{T}_{\mathfrak{R}_2})\right)\right), \\ \left(1 - \sin\left(\frac{\Pi}{2}(1 - (\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1}))\right)\right)\left(1 - \sin\left(\frac{\Pi}{2}(1 - (\mathfrak{I}_{\mathfrak{V}_2}, \mathfrak{I}_{\mathfrak{R}_2}))\right)\right), \\ \left(1 - \sin\left(\frac{\Pi}{2}(1 - (\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1}))\right)\right)\left(1 - \sin\left(\frac{\Pi}{2}(1 - (\mathfrak{F}_{\mathfrak{V}_2}, \mathfrak{F}_{\mathfrak{R}_2}))\right)\right) \end{pmatrix}.$$

(1): For any $\psi > 0$, the following holds

$$\begin{aligned}
\psi\left(\sin(\eth_1)\bigoplus\sin(\eth_2)\right) &= \begin{pmatrix} 1-\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1}\right)\right)\right)^{\psi}\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_2},\mathfrak{T}_{\mathfrak{R}_2}\right)\right)\right)^{\psi}, \\ \left(\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1}\right)\right)\right)\right)\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_2},\mathfrak{I}_{\mathfrak{R}_2}\right)\right)\right)\right)\right)^{\psi}, \\ \left(\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_1},\mathfrak{F}_{\mathfrak{R}_1}\right)\right)\right)\right)\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_2},\mathfrak{F}_{\mathfrak{R}_2}\right)\right)\right)\right)\right)^{\psi} \end{pmatrix} \\
&= \begin{pmatrix} 1-\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1}\right)\right)\right)^{\psi},\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1}\right)\right)\right)\right)^{\psi}, \\ \left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_1},\mathfrak{F}_{\mathfrak{R}_1}\right)\right)\right)\right)^{\psi} \end{pmatrix} \\
&\quad \bigoplus \begin{pmatrix} 1-\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_2},\mathfrak{T}_{\mathfrak{R}_2}\right)\right)\right)^{\psi},\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_2},\mathfrak{I}_{\mathfrak{R}_2}\right)\right)\right)\right)^{\psi}, \\ \left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_2},\mathfrak{F}_{\mathfrak{R}_2}\right)\right)\right)\right)^{\psi} \end{pmatrix} \\
&= \psi\sin(\eth_1)\bigoplus\psi\sin(\eth_2).
\end{aligned}$$

(2): The proof follows a similar pattern as (1).

(3): For any $\psi_1, \psi_2 > 0$, we have

$$\psi_1\sin(\eth_1) = \begin{pmatrix} 1-\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1}\right)\right)\right)^{\psi_1},\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1}\right)\right)\right)\right)^{\psi_1}, \\ \left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_1},\mathfrak{F}_{\mathfrak{R}_1}\right)\right)\right)\right)^{\psi_1} \end{pmatrix}$$

and

$$\psi_2\sin(\eth_1) = \begin{pmatrix} 1-\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1}\right)\right)\right)^{\psi_2},\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1}\right)\right)\right)\right)^{\psi_2}, \\ \left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_1},\mathfrak{F}_{\mathfrak{R}_1}\right)\right)\right)\right)^{\psi_2} \end{pmatrix}.$$

Thus, by STOLs for SVNZNs, we get

$$\begin{aligned}
\psi_1\sin(\eth_1)\bigoplus\psi_2\sin(\eth_1) &= \begin{pmatrix} 1-\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1}\right)\right)\right)^{\psi_1},\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1}\right)\right)\right)\right)^{\psi_1}, \\ \left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_1},\mathfrak{F}_{\mathfrak{R}_1}\right)\right)\right)\right)^{\psi_1} \end{pmatrix} \\
&\quad \bigoplus \begin{pmatrix} 1-\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1}\right)\right)\right)^{\psi_2},\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1}\right)\right)\right)\right)^{\psi_2}, \\ \left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_1},\mathfrak{F}_{\mathfrak{R}_1}\right)\right)\right)\right)^{\psi_2} \end{pmatrix} \\
&= \begin{pmatrix} 1-\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1}\right)\right)\right)^{\psi_1+\psi_2},\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1}\right)\right)\right)\right)^{\psi_1+\psi_2}, \\ \left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_1},\mathfrak{F}_{\mathfrak{R}_1}\right)\right)\right)\right)^{\psi_1+\psi_2} \end{pmatrix} \\
&= (\psi_1+\psi_2)\sin(\eth_1)
\end{aligned}$$

The proof of (4), and (5) is similarly as (3). □

Theorem 3.11. *Let* $\eth_\Xi = \{(\mathfrak{T}_{\mathfrak{V}_\Xi},\mathfrak{T}_{\mathfrak{R}_\Xi}),(\mathfrak{I}_{\mathfrak{V}_\Xi},\mathfrak{I}_{\mathfrak{R}_\Xi}),(\mathfrak{F}_{\mathfrak{V}_\Xi},\mathfrak{F}_{\mathfrak{R}_\Xi})\} \in SVNZN(\Upsilon)$ $(\Xi = 1,2)$ *such that* $(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1}) \geq (\mathfrak{T}_{\mathfrak{V}_2},\mathfrak{T}_{\mathfrak{R}_2})$, $(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1}) \leq (\mathfrak{I}_{\mathfrak{V}_2},\mathfrak{I}_{\mathfrak{R}_2})$ *and* $(\mathfrak{F}_{\mathfrak{V}_1},\mathfrak{F}_{\mathfrak{R}_1}) \leq (\mathfrak{F}_{\mathfrak{V}_2},\mathfrak{F}_{\mathfrak{R}_2})$. *Then* $\sin(\eth_1) \geq \sin(\eth_2)$.

Proof. For $\eth_\Xi = \{(\mathfrak{T}_{\mathfrak{V}_\Xi},\mathfrak{T}_{\mathfrak{R}_\Xi}),(\mathfrak{I}_{\mathfrak{V}_\Xi},\mathfrak{I}_{\mathfrak{R}_\Xi}),(\mathfrak{F}_{\mathfrak{V}_\Xi},\mathfrak{F}_{\mathfrak{R}_\Xi})\} \in SVNZN(\Upsilon)$ $(\Xi = 1,2)$, we have $(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1}) \geq (\mathfrak{T}_{\mathfrak{V}_2},\mathfrak{T}_{\mathfrak{R}_2})$. As "sin" is an increasing function in $\left[0,\frac{\Pi}{2}\right]$, thus we have $\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1})\right) \geq \sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_2},\mathfrak{T}_{\mathfrak{R}_2})\right)$. Similarly, we have $(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1}) \leq (\mathfrak{I}_{\mathfrak{V}_2},\mathfrak{I}_{\mathfrak{R}_2})$, which implies that $1-(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1}) \geq 1-(\mathfrak{I}_{\mathfrak{V}_2},\mathfrak{I}_{\mathfrak{R}_2})$. Thus, $\sin\left(\frac{\Pi}{2}\left(1-(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1})\right)\right) \geq \sin\left(\frac{\Pi}{2}\left(1-(\mathfrak{I}_{\mathfrak{V}_2},\mathfrak{I}_{\mathfrak{R}_2})\right)\right)$, which further implies that

$$1-\sin\left(\frac{\Pi}{2}\left(1-(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1})\right)\right) \leq 1-\sin\left(\frac{\Pi}{2}\left(1-(\mathfrak{I}_{\mathfrak{V}_2},\mathfrak{I}_{\mathfrak{R}_2})\right)\right)$$

and similarly we get

$$1-\sin\left(\frac{\Pi}{2}\left(1-(\mathfrak{F}_{\mathfrak{V}_1},\mathfrak{F}_{\mathfrak{R}_1})\right)\right) \leq 1-\sin\left(\frac{\Pi}{2}\left(1-(\mathfrak{F}_{\mathfrak{V}_2},\mathfrak{F}_{\mathfrak{R}_2})\right)\right).$$

Therefore we get

$$\left\{\left(\begin{array}{l}\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1})\right),\\ 1-\sin\left(\frac{\Pi}{2}\left(1-(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1})\right)\right),\\ 1-\sin\left(\frac{\Pi}{2}\left(1-(\mathfrak{F}_{\mathfrak{V}_1},\mathfrak{F}_{\mathfrak{R}_1})\right)\right)\end{array}\right)\right\} \geq \left\{\left(\begin{array}{l}\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_2},\mathfrak{T}_{\mathfrak{R}_2})\right),\\ 1-\sin\left(\frac{\Pi}{2}\left(1-(\mathfrak{I}_{\mathfrak{V}_2},\mathfrak{I}_{\mathfrak{R}_2})\right)\right),\\ 1-\sin\left(\frac{\Pi}{2}\left(1-(\mathfrak{F}_{\mathfrak{V}_2},\mathfrak{F}_{\mathfrak{R}_2})\right)\right)\end{array}\right)\right\}.$$

Hence, according to Definition 3.2, it follows that $\sin(\eth_1) \geq \sin(\eth_2)$. □

4. **Novel Sine Trigonometric Aggregation Operators for SVNZNs**

In this part, we present novel AOs that extend the suggested STOLs for SVNZNs. We define the geometric AOs and weighted averaging that follow.

4.1. *Sine Trigonometric Weighted Averaging AOs for SVNZNs*

Definition 4.1. Let $\eth_\Xi = \{(\mathfrak{T}_{\mathfrak{V}_\Xi}(\mathfrak{z}),\mathfrak{T}_{\mathfrak{R}_\Xi}(\mathfrak{z})),(\mathfrak{I}_{\mathfrak{V}_\Xi}(\mathfrak{z}),\mathfrak{I}_{\mathfrak{R}_\Xi}(\mathfrak{z})),(\mathfrak{F}_{\mathfrak{V}_\Xi}(\mathfrak{z}),\mathfrak{F}_{\mathfrak{R}_\Xi}(\mathfrak{z}))\} \in SVNZN(\Upsilon)$ $(\Xi=1,2,3,...,n)$. Subsequently, the ST weighted averaging AO for $SVNZN(\Upsilon)$ known as ST-SVNZNWA is symbolized and outlined in the subsequent manner:

$$\begin{aligned} ST\text{-}SVNZNWA(\eth_1,\eth_2,...\eth_n) &= \xi_1\sin(\eth_1)\bigoplus\xi_2\sin(\eth_2)\bigoplus...\bigoplus\xi_n\sin(\eth_n)\\ &= \sum_{\Xi=1}^{n}\xi_\Xi\sin(\eth_\Xi). \end{aligned}$$

Here, $\xi_\Xi\,(\Xi=1,2,...,n)$ signifies the weights associated with $\eth_\Xi\,(\Xi=1,2,3,...,n)$, where $\xi_\Xi \geq 0$ and $\sum_{\Xi=1}^{n}\xi_\Xi = 1$.

Theorem 4.2. *Let* $\eth_\Xi = \{(\mathfrak{T}_{\mathfrak{V}_\Xi}(\mathfrak{z}),\mathfrak{T}_{\mathfrak{R}_\Xi}(\mathfrak{z})),(\mathfrak{I}_{\mathfrak{V}_\Xi}(\mathfrak{z}),\mathfrak{I}_{\mathfrak{R}_\Xi}(\mathfrak{z})),(\mathfrak{F}_{\mathfrak{V}_\Xi}(\mathfrak{z}),\mathfrak{F}_{\mathfrak{R}_\Xi}(\mathfrak{z}))\} \in SVNZN(\Upsilon)$ $(\Xi=1,2,3,...,n)$ *and the WV of* $\eth\Xi\,(\Xi=1,2,3,...,n)$ *is represented by* $\xi = (\xi 1,\xi_2,...,\xi_n)^{\mathfrak{T}}$ *with the constraint* $\sum_{\Xi=1}^{n}\xi_\Xi = 1$. *The ST-SVNZNWA operator is a mapping*

$G^n \longrightarrow G$ *that satisfies:*

$$ST\text{-}SVNZNWA\left(\eth_1, \eth_2, ...\eth_n\right) = \sum_{\Xi=1}^{n} \xi_\Xi \sin\left(\eth_\Xi\right) = \begin{pmatrix} 1 - \prod_{\Xi=1}^{n} \left(1 - \sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_\Xi}, \mathfrak{T}_{\mathfrak{R}_\Xi}\right)\right)\right)^{\xi_\Xi}, \\ \prod_{\Xi=1}^{n} \left(1 - \sin\left(\frac{\Pi}{2}\left(1 - \left(\mathfrak{I}_{\mathfrak{V}_\Xi}, \mathfrak{I}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi}, \\ \prod_{\Xi=1}^{n} \left(1 - \sin\left(\frac{\Pi}{2}\left(1 - \left(\mathfrak{F}_{\mathfrak{V}_\Xi}, \mathfrak{F}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi} \end{pmatrix}$$

Proof. We establish the proof of Theorem 4.2 by utilizing mathematical induction on n. For each Ξ,
$\eth_\Xi = \{(\mathfrak{T}_{\mathfrak{V}_\Xi}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_\Xi}(\mathfrak{Z})), (\mathfrak{I}_{\mathfrak{V}_\Xi}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_\Xi}(\mathfrak{Z})), (\mathfrak{F}_{\mathfrak{V}_\Xi}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_\Xi}(\mathfrak{Z}))\} \in SVNZN(\curlyvee)$, which signifies that
$(\mathfrak{T}_{\mathfrak{V}_\Xi}, \mathfrak{T}_{\mathfrak{R}_\Xi}), (\mathfrak{I}_{\mathfrak{V}_\Xi}, \mathfrak{I}_{\mathfrak{R}_\Xi}), (\mathfrak{F}_{\mathfrak{V}_\Xi}, \mathfrak{F}_{\mathfrak{R}_\Xi}) \in [0,1]$ and $(\mathfrak{T}_{\mathfrak{V}_\Xi}, \mathfrak{T}_{\mathfrak{R}_\Xi}) + (\mathfrak{I}_{\mathfrak{V}_\Xi}, \mathfrak{I}_{\mathfrak{R}_\Xi}) + (\mathfrak{F}_{\mathfrak{V}_\Xi}, \mathfrak{F}_{\mathfrak{R}_\Xi}) \leq 3$. Subsequently, the subsequent stages of the mathematical induction process have been carried out.

Step-1: For $n = 2$, we get $ST\text{-}SVNZNWA(\eth_1, \eth_2) = \xi_1 \sin(\eth_1) \bigoplus \xi_2 \sin(\eth_2)$.
Since, by Definition 3.2, $\sin(\eth_1)$ and $\sin(\eth 2)$ are SVNZNs, it follows that $\xi 1 \sin(\eth 1) \bigoplus \xi 2 \sin(\eth_2)$ is also an SVNZN. Furthermore, in the case of $\eth_1$ and $\eth_2$, we have

$$\begin{aligned} ST\text{-}SVNZNWA(\eth_1, \eth_2) &= \xi_1 \sin(\eth_1) \bigoplus \xi_2 \sin(\eth_2) \\ &= \begin{pmatrix} 1 - \left(1 - \sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1}\right)\right)\right)^{\xi_1}, \\ \left(1 - \sin\left(\frac{\Pi}{2}\left(1 - \left(\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1}\right)\right)\right)\right)^{\xi_1}, \\ \left(1 - \sin\left(\frac{\Pi}{2}\left(1 - \left(\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1}\right)\right)\right)\right)^{\xi_1} \end{pmatrix} \\ &\quad \bigoplus \begin{pmatrix} 1 - \left(1 - \sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_2}, \mathfrak{T}_{\mathfrak{R}_2}\right)\right)\right)^{\xi_2}, \\ \left(1 - \sin\left(\frac{\Pi}{2}\left(1 - \left(\mathfrak{I}_{\mathfrak{V}_2}, \mathfrak{I}_{\mathfrak{R}_2}\right)\right)\right)\right)^{\xi_2}, \\ \left(1 - \sin\left(\frac{\Pi}{2}\left(1 - \left(\mathfrak{F}_{\mathfrak{V}_2}, \mathfrak{F}_{\mathfrak{R}_2}\right)\right)\right)\right)^{\xi_2} \end{pmatrix} \\ &= \begin{pmatrix} 1 - \prod_{\Xi=1}^{2} \left(1 - \sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_\Xi}, \mathfrak{T}_{\mathfrak{R}_\Xi}\right)\right)\right)^{\xi_\Xi}, \\ \prod_{\Xi=1}^{2} \left(1 - \sin\left(\frac{\Pi}{2}\left(1 - \left(\mathfrak{I}_{\mathfrak{V}_\Xi}, \mathfrak{I}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi}, \\ \prod_{\Xi=1}^{2} \left(1 - \sin\left(\frac{\Pi}{2}\left(1 - \left(\mathfrak{F}_{\mathfrak{V}_\Xi}, \mathfrak{F}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi} \end{pmatrix} \end{aligned} \tag{1}$$

Step-2: Assume that Equation (1) holds for $n = \kappa$. Consequently, we have:

$$ST\text{-}SVNZNWA\left(\eth_1,\eth_2,...\eth_\kappa\right)=\left(\begin{array}{c}1-\prod_{\Xi=1}^{\kappa}\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_\Xi},\mathfrak{T}_{\mathfrak{R}_\Xi}\right)\right)\right)^{\xi_\Xi},\\ \prod_{\Xi=1}^{\kappa}\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_\Xi},\mathfrak{I}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi},\\ \prod_{\Xi=1}^{\kappa}\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_\Xi},\mathfrak{F}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi}\end{array}\right)$$

Step-3: Our next objective is to demonstrate that Equation (1) holds for $n=\kappa+1$.

$$\begin{aligned}ST\text{-}SVNZNWA\left(\eth_1,\eth_2,...\eth_{\kappa+1}\right) &= \sum_{\Xi=1}^{\kappa}\xi_\Xi\sin\left(\eth_\Xi\right)\bigoplus\xi_{\kappa+1}\sin\left(\eth_{\kappa+1}\right)\\ &= \left(\begin{array}{c}1-\prod_{\Xi=1}^{\kappa}\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_\Xi},\mathfrak{T}_{\mathfrak{R}_\Xi}\right)\right)\right)^{\xi_\Xi},\\ \prod_{\Xi=1}^{\kappa}\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_\Xi},\mathfrak{I}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi},\\ \prod_{\Xi=1}^{\kappa}\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_\Xi},\mathfrak{F}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi}\end{array}\right)\\ &\quad\bigoplus\left(\begin{array}{c}1-\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_{\kappa+1}},\mathfrak{T}_{\mathfrak{R}_{\kappa+1}}\right)\right)\right)^{\xi_{\kappa+1}},\\ \left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_{\kappa+1}},\mathfrak{I}_{\mathfrak{R}_{\kappa+1}}\right)\right)\right)\right)^{\xi_{\kappa+1}},\\ \left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_{\kappa+1}},\mathfrak{F}_{\mathfrak{R}_{\kappa+1}}\right)\right)\right)\right)^{\xi_{\kappa+1}}\end{array}\right)\\ &= \left(\begin{array}{c}1-\prod_{\Xi=1}^{\kappa+1}\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_\Xi},\mathfrak{T}_{\mathfrak{R}_\Xi}\right)\right)\right)^{\xi_\Xi},\\ \prod_{\Xi=1}^{\kappa+1}\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_\Xi},\mathfrak{I}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi},\\ \prod_{\Xi=1}^{\kappa+1}\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_\Xi},\mathfrak{F}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi}\end{array}\right)\end{aligned}$$

Specifically, for $n=\kappa+1$, we need to establish that Equation (1) remains valid. Hence, it can be concluded that Equation (1) holds for all values of n. □

Example 4.3. Suppose

$$\eth_1=\left\{\left(0.22,0.34\right),\left(0.15,0.57\right),\left(0.66,0.18\right)\right\},$$

$$\eth_2=\left\{\left(0.17,0.63\right),\left(0.52,0.31\right),\left(0.37,0.28\right)\right\},$$

$$\eth_3=\left\{\left(0.71,0.38\right),\left(0.25,0.42\right),\left(0.25,0.67\right)\right\},$$

and

$$\eth_4=\left\{\left(0.32,0.56\right),\left(0.47,0.23\right),\left(0.35,0.41\right)\right\}$$

are the SVNZNs with $\xi=(0.245,0.239,0.254,0.262)^{\mathfrak{T}}$ is the WV. Initially, we determine the $\xi_\Xi=\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_\Xi},\mathfrak{T}_{\mathfrak{R}_\Xi}\right)\right)$ we get

$$\begin{aligned}\xi_1 &= (0.3387,0.5090);\xi_2=(0.2639,0.8358)\\ \xi_3 &= (0.8980,0.5621);\xi_4=(0.4818,0.7705)\end{aligned}$$

As a result, we obtain

$$\begin{aligned}\prod_{\Xi=1}^{4}\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_\Xi},\mathfrak{T}_{\mathfrak{R}_\Xi}\right)\right)\right)^{\xi_\Xi} &= (1-\xi_1)^{0.245}\times(1-\xi_2)^{0.239}\times \\ &\quad (1-\xi_3)^{0.254}\times(1-\xi_4)^{0.262} \\ &= (0.9036,0.8401)\times(0.9294,0.6493)\times \\ &\quad (0.5600,0.8108)\times(0.8418,0.6800) \\ &= (0.3959,0.3007)\end{aligned}$$

Similarly, if $m_\Xi = \sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_\Xi},\mathfrak{I}_{\mathfrak{R}_\Xi}\right)\right)\right)$, we get

$$\begin{aligned} m_1 &= (0.9724,0.6252)\,;m_2=(0.6845,0.8838) \\ m_3 &= (0.9239,0.7902)\,;m_4=(0.7396,0.9354)\end{aligned}$$

As a result, we obtain

$$\begin{aligned}\prod_{\Xi=1}^{4}\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_\Xi},\mathfrak{I}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi} &= (1-m_1)^{0.245}\times(1-m_2)^{0.239}\times \\ &\quad (1-m_3)^{0.254}\times(1-m_4)^{0.262} \\ &= (0.4150,0.7863)\times(0.7590,0.5978)\times \\ &\quad (0.5198,0.6726)\times(0.7029,0.4878) \\ &= (0.1151,0.1542)\end{aligned}$$

Similarly, if $n_\Xi = \sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_\Xi},\mathfrak{F}_{\mathfrak{R}_\Xi}\right)\right)\right)$, we get

$$\begin{aligned} n_1 &= (0.5090,0.9603)\,;n_2=(0.8358,0.9048) \\ n_3 &= (0.9239,0.4955)\,;n_4=(0.8526,0.7997)\end{aligned}$$

As a result, we obtain

$$\begin{aligned}\prod_{\Xi=1}^{4}\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_\Xi},\mathfrak{F}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi} &= (1-n_1)^{0.245}\times(1-n_2)^{0.239}\times \\ &\quad (1-n_3)^{0.254}\times(1-n_4)^{0.262} \\ &= (0.8401,0.4536)\times(0.6493,0.5700)\times \\ &\quad (0.5198,0.8405)\times(0.6055,0.6562) \\ &= (0.1717,0.1426)\end{aligned}$$

Therefore,

$$ST\text{-}SVNZNWA\left(\eth_1,\eth_2,\eth_3,\eth_4\right) = \left(\begin{array}{c} 1-\prod_{\Xi=1}^{4}\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_\Xi},\mathfrak{T}_{\mathfrak{R}_\Xi}\right)\right)\right)^{\xi_\Xi}, \\ \prod_{\Xi=1}^{4}\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_\Xi},\mathfrak{I}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi}, \\ \prod_{\Xi=1}^{4}\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_\Xi},\mathfrak{F}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi} \end{array}\right)$$
$$= \left(1-\left(0.3959,0.3007\right),\left(0.1151,0.1542\right),\left(0.1717,0.1426\right)\right)$$
$$= \left(\left(0.6041,0.6993\right),\left(0.1151,0.1542\right),\left(0.1717,0.1426\right)\right)$$

Moving forward, we outline a series of properties associated with the proposed ST-SVNZNWA AO. Given that these AOs are rooted in the ST function, they exhibit attributes such as idempotency, boundedness, monotonicity, and symmetry.

Theorem 4.4. *(idempotency)*
Let $\eth_\Xi = \{(\mathfrak{T}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{T}_{\mathfrak{R}_\Xi}(\mathfrak{Z})),(\mathfrak{I}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{I}_{\mathfrak{R}_\Xi}(\mathfrak{Z})),(\mathfrak{F}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{F}_{\mathfrak{R}_\Xi}(\mathfrak{Z}))\}$ *belong to* $SVNZN(\Upsilon)\,(\Xi=1,2,3,...,n)$ *where* $\eth_\Xi=\eth$. *Then* $ST\text{-}SVNZNWA\left(\eth_1,\eth_2,...,\eth_n\right) = \sin\left(\eth\right)$.

Proof. Since $\eth_\Xi=\eth$ $(\Xi=1,2,3,...,n)$, we can apply Theorem 4.2 to deduce:

$$ST\text{-}SVNZNWA\left(\eth_1,\eth_2,...,\eth_n\right) = \left(\begin{array}{c} 1-\prod_{\Xi=1}^{n}\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_\Xi},\mathfrak{T}_{\mathfrak{R}_\Xi}\right)\right)\right)^{\xi_\Xi}, \\ \prod_{\Xi=1}^{n}\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_\Xi},\mathfrak{I}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi}, \\ \prod_{\Xi=1}^{n}\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_\Xi},\mathfrak{F}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi} \end{array}\right)$$
$$= \left(\begin{array}{c} 1-\prod_{\Xi=1}^{n}\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_{ji}},\mathfrak{T}_{\mathfrak{R}_{ji}}\right)\right)\right)^{\xi_\Xi}, \\ \prod_{\Xi=1}^{n}\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_{ji}},\mathfrak{I}_{\mathfrak{R}_{ji}}\right)\right)\right)\right)^{\xi_\Xi}, \\ \prod_{\Xi=1}^{n}\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_{ji}},\mathfrak{F}_{\mathfrak{R}_{ji}}\right)\right)\right)\right)^{\xi_\Xi} \end{array}\right)$$
$$= \left(\begin{array}{c} 1-\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_{ji}},\mathfrak{T}_{\mathfrak{R}_{ji}}\right)\right)\right)^{\sum_{\Xi=1}^{n}\xi_\Xi}, \\ \left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_{ji}},\mathfrak{I}_{\mathfrak{R}_{ji}}\right)\right)\right)\right)^{\sum_{\Xi=1}^{n}\xi_\Xi}, \\ \left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_{ji}},\mathfrak{F}_{\mathfrak{R}_{ji}}\right)\right)\right)\right)^{\sum_{\Xi=1}^{n}\xi_\Xi} \end{array}\right)$$
$$= \left(\begin{array}{c} \sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_{ji}},\mathfrak{T}_{\mathfrak{R}_{ji}}\right)\right),1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_{ji}},\mathfrak{I}_{\mathfrak{R}_{ji}}\right)\right)\right), \\ 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_{ji}},\mathfrak{F}_{\mathfrak{R}_{ji}}\right)\right)\right) \end{array}\right)$$
$$= \sin\left(\eth\right)$$

□

Theorem 4.5. *(Boundedness)*
Let $\eth_{\Xi} = \{(\mathfrak{T}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z})), (\mathfrak{I}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z})), (\mathfrak{F}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))\}$,
$\eth_{\Xi}^{-} = \{\min((\mathfrak{T}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))), \max((\mathfrak{I}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))), \max((\mathfrak{F}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z})))\}$ *and* $\eth_{\Xi}^{+} = \{\max((\mathfrak{T}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))), \min((\mathfrak{I}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))), \min((\mathfrak{F}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z})))\} \in SVNZN(\Upsilon)$ $(\Xi = 1, 2, 3, ..., n)$. *Then,* $\sin\left(\eth_{\Xi}^{-}\right) \leq ST\text{-}SVNZNWA(\eth_1, \eth_2, ..., \eth_n) \leq \sin\left(\eth_{\Xi}^{+}\right)$.

Proof. For any value of Ξ, $\min_{\Xi}((\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}})) \leq (\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}}) \leq \min_{\Xi}((\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}}))$, $\min_{\Xi}((\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}})) \leq (\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}}) \leq \min_{\Xi}((\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}}))$ and $\min_{\Xi}((\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}})) \leq (\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}}) \leq \min_{\Xi}((\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}}))$. This implies that $\eth_{\Xi}^{-} \leq \eth_{\Xi} \leq \eth_{\Xi}^{+}$. Suppose that $ST\text{-}SVNZNWA(\eth_1, \eth_2, ...\eth_n) = \sin(\eth_{\Xi}) = \{(\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}}), (\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}}), (\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}})\}$, $\sin\left(\eth_{\Xi}^{-}\right) = \left\{(\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}})^{-}, (\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}})^{-}, (\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}})^{-}\right\}$ and $\sin\left(\eth_{\Xi}^{+}\right) = \left\{(\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}})^{+}, (\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}})^{+}, (\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}})^{+}\right\}$. Then, leveraging the monotonic nature of the sine function, we observe that

$$\begin{aligned}
(\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}}) &= 1 - \prod_{\Xi=1}^{n}\left(1 - \sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}})\right)\right)^{\xi_{\Xi}} \\
&\geq 1 - \prod_{\Xi=1}^{n}\left(1 - \sin\left(\frac{\Pi}{2}\min_{\Xi}((\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}}))\right)\right)^{\xi_{\Xi}} \\
&= \sin\left(\frac{\Pi}{2}\min_{\Xi}((\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}}))\right) = (\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}})^{-}
\end{aligned}$$

and,

$$\begin{aligned}
(\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}}) &= \prod_{\Xi=1}^{n}\left(1 - \sin\left(\frac{\Pi}{2}(1 - (\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}}))\right)\right)^{\xi_{\Xi}} \\
&\geq \prod_{\Xi=1}^{n}\left(1 - \sin\left(\frac{\Pi}{2}(1 - (\min_{\Xi}(\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}})))\right)\right)^{\xi_{\Xi}} \\
&= 1 - \sin\left(\frac{\Pi}{2}(1 - (\min_{\Xi}(\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}})))\right) = (\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}})^{-}
\end{aligned}$$

Similarly,

$$\begin{aligned}
(\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}}) &= \prod_{\Xi=1}^{n}\left(1 - \sin\left(\frac{\Pi}{2}(1 - (\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}}))\right)\right)^{\xi_{\Xi}} \\
&\geq \prod_{\Xi=1}^{n}\left(1 - \sin\left(\frac{\Pi}{2}(1 - (\min_{\Xi}(\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}})))\right)\right)^{\xi_{\Xi}} \\
&= 1 - \sin\left(\frac{\Pi}{2}(1 - (\min_{\Xi}(\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}})))\right) = (\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}})^{-}
\end{aligned}$$

Also, we have

$$\begin{aligned}
\left(\mathfrak{T}_{\mathfrak{V}_\Xi}, \mathfrak{T}_{\mathfrak{R}_\Xi}\right) &= 1-\prod_{\Xi=1}^{n}\left(1-\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_\Xi}, \mathfrak{T}_{\mathfrak{R}_\Xi}\right)\right)\right)^{\xi_\Xi} \\
&\leq 1-\prod_{\Xi=1}^{n}\left(1-\sin\left(\frac{\Pi}{2}\max_\Xi\left(\left(\mathfrak{T}_{\mathfrak{V}_\Xi}, \mathfrak{T}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi} \\
&= \sin\left(\frac{\Pi}{2}\max_\Xi\left(\left(\mathfrak{T}_{\mathfrak{V}_\Xi}, \mathfrak{T}_{\mathfrak{R}_\Xi}\right)\right)\right) = \left(\mathfrak{T}_{\mathfrak{V}_\Xi}, \mathfrak{T}_{\mathfrak{R}_\Xi}\right)^{+}
\end{aligned}$$

and

$$\begin{aligned}
\left(\mathfrak{I}_{\mathfrak{V}_\Xi}, \mathfrak{I}_{\mathfrak{R}_\Xi}\right) &= \prod_{\Xi=1}^{n}\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_\Xi}, \mathfrak{I}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi} \\
&\leq \prod_{\Xi=1}^{n}\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\max_\Xi(\mathfrak{I}_{\mathfrak{V}_\Xi}, \mathfrak{I}_{\mathfrak{R}_\Xi})\right)\right)\right)\right)^{\xi_\Xi} \\
&= 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\max_\Xi(\mathfrak{I}_{\mathfrak{V}_\Xi}, \mathfrak{I}_{\mathfrak{R}_\Xi})\right)\right)\right) = \left(\mathfrak{I}_{\mathfrak{V}_\Xi}, \mathfrak{I}_{\mathfrak{R}_\Xi}\right)^{+}
\end{aligned}$$

Similarly,

$$\begin{aligned}
\left(\mathfrak{F}_{\mathfrak{V}_\Xi}, \mathfrak{F}_{\mathfrak{R}_\Xi}\right) &= \prod_{\Xi=1}^{n}\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_\Xi}, \mathfrak{F}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi} \\
&\leq \prod_{\Xi=1}^{n}\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\max_\Xi\left(\mathfrak{F}_{\mathfrak{V}_\Xi}, \mathfrak{F}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)\right)^{\xi_\Xi} \\
&= 1-\sin\left(\frac{\Pi}{2}\left(1-\left(\max_\Xi\left(\mathfrak{F}_{\mathfrak{V}_\Xi}, \mathfrak{F}_{\mathfrak{R}_\Xi}\right)\right)\right)\right) = \left(\mathfrak{F}_{\mathfrak{V}_\Xi}, \mathfrak{F}_{\mathfrak{R}_\Xi}\right)^{+}
\end{aligned}$$

Based on the score function, we get

$$\begin{aligned}
\overline{sc}\left(\sin\left(\eth_\Xi\right)\right) &= \left(\mathfrak{T}_{\mathfrak{V}_\Xi}, \mathfrak{T}_{\mathfrak{R}_\Xi}\right) - \left(\mathfrak{I}_{\mathfrak{V}_\Xi}, \mathfrak{I}_{\mathfrak{R}_\Xi}\right) - \left(\mathfrak{F}_{\mathfrak{V}_\Xi}, \mathfrak{F}_{\mathfrak{R}_\Xi}\right) \\
&\leq \left(\mathfrak{T}_{\mathfrak{V}_\Xi}, \mathfrak{T}_{\mathfrak{R}_\Xi}\right)^{+} - \left(\mathfrak{I}_{\mathfrak{V}_\Xi}, \mathfrak{I}_{\mathfrak{R}_\Xi}\right)^{-} - \left(\mathfrak{F}_{\mathfrak{V}_\Xi}, \mathfrak{F}_{\mathfrak{R}_\Xi}\right)^{-} = \overline{sc}\left(\sin\left(\eth_\Xi^{+}\right)\right)
\end{aligned}$$

and

$$\begin{aligned}
\overline{sc}\left(\sin\left(\eth_\Xi\right)\right) &= \left(\mathfrak{T}_{\mathfrak{V}_\Xi}, \mathfrak{T}_{\mathfrak{R}_\Xi}\right) - \left(\mathfrak{I}_{\mathfrak{V}_\Xi}, \mathfrak{I}_{\mathfrak{R}_\Xi}\right) - \left(\mathfrak{F}_{\mathfrak{V}_\Xi}, \mathfrak{F}_{\mathfrak{R}_\Xi}\right) \\
&\geq \left(\mathfrak{T}_{\mathfrak{V}_\Xi}, \mathfrak{T}_{\mathfrak{R}_\Xi}\right)^{-} - \left(\mathfrak{I}_{\mathfrak{V}_\Xi}, \mathfrak{I}_{\mathfrak{R}_\Xi}\right)^{+} - \left(\mathfrak{F}_{\mathfrak{V}_\Xi}, \mathfrak{F}_{\mathfrak{R}_\Xi}\right)^{+} = \overline{sc}\left(\sin\left(\eth_\Xi^{-}\right)\right)
\end{aligned}$$

□

Thus, we have $\overline{sc}\left(\sin\left(\eth_\Xi^{-}\right)\right) \leq \overline{sc}\left(\sin\left(\eth_\Xi\right)\right) \leq \overline{sc}\left(\sin\left(\eth_\Xi^{+}\right)\right)$. We will now delve into a discussion of the three cases:

(Case-1): If $\overline{sc}\left(\sin\left(\eth_\Xi^{-}\right)\right) < \overline{sc}\left(\sin\left(\eth_\Xi\right)\right) < \overline{sc}\left(\sin\left(\eth_\Xi^{+}\right)\right)$, then the conclusion remains valid.

(Case-2): If $\overline{sc}\left(\sin\left(\eth_{\Xi}^{+}\right)\right) = \overline{sc}\left(\sin\left(\eth_{\Xi}\right)\right)$, then we have: $(\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}})^{+} - (\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}})^{+} - (\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}})^{+} = (\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}}) - (\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}}) - (\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}})$. This implies that $(\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}})^{+} = (\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}})$, $(\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}})^{+} = (\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}})$, and $(\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}})^{+} = (\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}})$. Consequently, $\underline{ac}\left(\sin\left(\eth_{\Xi}\right)\right) = \underline{ac}\left(\sin\left(\eth_{\Xi}^{+}\right)\right)$.

(Case-3): If $\overline{sc}\left(\sin\left(\eth_{\Xi}\right)\right) = \overline{sc}\left(\sin\left(\eth_{\Xi}^{-}\right)\right)$, then we have: $(\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}}) - (\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}}) - (\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}}) = (\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}})^{-} - (\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}})^{-} - (\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}})^{-}$. This implies that $(\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}}) = (\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}})^{-}$, $(\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}}) = (\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}})^{-}$, and $(\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}}) = (\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}})^{-}$. Consequently, $\underline{ac}\left(\sin\left(\eth_{\Xi}\right)\right) = \underline{ac}\left(\sin\left(\eth_{\Xi}^{-}\right)\right)$. Therefore, we ultimately establish $\sin\left(\eth_{\Xi}^{-}\right) \leq ST\text{-}SVNZNWA\left(\eth_1, \eth_2, ..., \eth_n\right) \leq \sin\left(\eth_{\Xi}^{+}\right)$.

Theorem 4.6. *(Monotonically)*
Let $\eth\Xi = \{(\mathfrak{T}\mathfrak{V}_{\Xi}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z})), (\mathfrak{I}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z})), (\mathfrak{F}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))\}$ *and* $\eth\Xi^{*} = \{(\mathfrak{T}\mathfrak{V}_{\Xi}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))^{*}, (\mathfrak{I}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))^{*}, (\mathfrak{F}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))^{*}\} \in SVNZN(\curlyvee)$ $(\Xi = 1, 2, 3, ..., n)$. *If* $(\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}}) \leq (\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}})^{*}$, $(\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}}) \leq (\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}})^{*}$, *and* $(\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}}) \leq (\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}})^{*}$, *then:*
$ST\text{-}SVNZNWA\left(\eth_1, \eth_2, ..., \eth_n\right) \leq ST\text{-}SVNZNWA\left(\eth_1^{*}, \eth_2^{*}, ..., \eth_n^{*}\right)$.

Proof. Indeed, this conclusion is a direct consequence of Theorem 4.5, and as such, it is not necessary to elaborate further on this point. □

Theorem 4.7. *(Symmetric)*
Let $\eth\Xi = \{(\mathfrak{T}\mathfrak{V}_{\Xi}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z})), (\mathfrak{I}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z})), (\mathfrak{F}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))\}$ *and* $\eth\Xi^{*} = \{(\mathfrak{T}\mathfrak{V}_{\Xi}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))^{*}, (\mathfrak{I}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))^{*}, (\mathfrak{F}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))^{*}\} \in SVNZN(\curlyvee)$ $(\Xi = 1, 2, 3, ..., n)$.
Then, we have: $ST\text{-}SVNZNWA\left(\eth_1, \eth_2, ..., \eth_n\right) = ST - SVNZNWA\left(\eth_1^{*}, \eth_2^{*}, ..., \eth_n^{*}\right)$, *whenever* $\eth_{\Xi}^{*}$ $(\Xi = 1, 2, 3, ..., n)$ *is any version of* $\eth_{\Xi}$ $(\Xi = 1, 2, 3, ..., n)$.

Proof. Indeed, this conclusion is a direct consequence of Theorem 4.5, and as such, it is not necessary to elaborate further on this point. □

4.2. *Sine Trigonometric Weighted Geometric AOs for SVNZNs*

Definition 4.8. Let $\eth_{\Xi} = \{(\mathfrak{T}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z})), (\mathfrak{I}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z})), (\mathfrak{F}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))\} \in SVNZN(\curlyvee)$ $(\Xi = 1, 2, 3, ..., n)$. Then, the ST-SVNZNWG operator for $SVNZN(\curlyvee)$ is designated as the ST weighted geometric AO, and defined in the following manner:

$$\begin{aligned} ST\text{-}SVNZNWG\left(\eth_1, \eth_2, ..., \eth_n\right) &= \left(\sin\left(\eth_1\right)\right)^{\xi_1} \bigotimes \left(\sin\left(\eth_2\right)\right)^{\xi_2} \bigotimes ... \bigotimes \left(\sin\left(\eth_n\right)\right)^{\xi_n} \\ &= \prod_{\Xi=1}^{n} \left(\sin\left(\eth_{\Xi}\right)\right)^{\xi_{\Xi}}. \end{aligned}$$

Here, $\xi_{\Xi}\,(\Xi = 1, 2, ..., n)$ denotes the weights assigned to $\partial\Xi\,(\Xi = 1, 2, 3, ..., n)$, where $\xi\Xi \geq 0$ and the summation over all Ξ values is constrained to be equal to 1.

Theorem

4.9. *Assume that* $\partial\Xi = \{(\mathfrak{T}\mathfrak{V}_{\Xi}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z})), (\mathfrak{I}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z})), (\mathfrak{F}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))\} \in SVNZN(\Upsilon)$ *for* $(\Xi = 1, 2, 3, ..., n)$. *Additionally, the weight vector (WV) associated with each* $\partial\Xi\,(\Xi = 1, 2, 3, ..., n)$ *is denoted by* $\xi = (\xi 1, \xi_2, ..., \xi_n)^{\mathfrak{T}}$, *with the requirement that* $\sum_{\Xi=1}^{n} \xi_{\Xi} = 1$. *The ST-SVNZNWG operator is then a function mapping* G^n *to* G, *defined as follows:*

$$\begin{aligned} ST\text{-}SVNZNWG(\partial_1, \partial_2, ..., \partial_n) &= \prod_{\Xi=1}^{n} (\sin(\partial_{\Xi}))^{\xi_{\Xi}} \\ &= \begin{pmatrix} \prod_{\Xi=1}^{n} \left(\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}})\right)\right)^{\xi_{\Xi}}, \\ 1 - \prod_{\Xi=1}^{n} \left(\sin\left(\frac{\Pi}{2}(1 - (\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}}))\right)\right)^{\xi_{\Xi}}, \\ 1 - \prod_{\Xi=1}^{n} \left(\sin\left(\frac{\Pi}{2}(1 - (\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}}))\right)\right)^{\xi_{\Xi}} \end{pmatrix} \end{aligned} \tag{2}$$

Proof. We establish the proof for Theorem 4.9 through mathematical induction based on n. For each value of Ξ, $\partial\Xi = \{(\mathfrak{T}\mathfrak{V}_{\Xi}(\mathfrak{Z}), \mathfrak{T}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z})), (\mathfrak{I}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{I}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z})), (\mathfrak{F}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}), \mathfrak{F}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))\} \in SVNZN(\Upsilon)$. This implies that $(\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}}), (\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}}), (\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}}) \in [0, 1]$ and $(\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}}) + (\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}}) + (\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}}) \leq 3$. Following this, we proceed with the steps of mathematical induction.

Step-1: When $n = 2$, then the equation becomes as: $ST\text{-}SVNZNWG(\partial_1, \partial_2) = (\sin(\partial_1))^{\xi_1} \bigotimes (\sin(\partial_2))^{\xi_2}$. Since according to Definition 3.2, we know that $\sin(\partial_1)$ and $\sin(\partial_2)$ are both SVNZNs, hence, it follows that $(\sin(\partial 1))^{\xi 1} \bigotimes (\sin(\partial 2))^{\xi 2}$ also exhibits the properties of SVNZN. Moving forward, when considering ∂_1 and ∂_2, we observe that

$$\begin{aligned} ST\text{-}SVNZNWG(\partial_1, \partial_2) &= (\sin(\partial_1))^{\xi_1} \bigotimes (\sin(\partial_2))^{\xi_2} \\ &= \begin{pmatrix} \left(\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_1}, \mathfrak{T}_{\mathfrak{R}_1})\right)\right)^{\xi_1}, \\ 1 - \left(\sin\left(\frac{\Pi}{2}(1 - (\mathfrak{I}_{\mathfrak{V}_1}, \mathfrak{I}_{\mathfrak{R}_1}))\right)\right)^{\xi_1}, \\ 1 - \left(\sin\left(\frac{\Pi}{2}(1 - (\mathfrak{F}_{\mathfrak{V}_1}, \mathfrak{F}_{\mathfrak{R}_1}))\right)\right)^{\xi_1} \end{pmatrix} \\ &\quad \bigotimes \begin{pmatrix} \left(\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_2}, \mathfrak{T}_{\mathfrak{R}_2})\right)\right)^{\xi_2}, \\ 1 - \left(\sin\left(\frac{\Pi}{2}(1 - (\mathfrak{I}_{\mathfrak{V}_2}, \mathfrak{I}_{\mathfrak{R}_2}))\right)\right)^{\xi_2}, \\ 1 - \left(\sin\left(\frac{\Pi}{2}(1 - (\mathfrak{F}_{\mathfrak{V}_2}, \mathfrak{F}_{\mathfrak{R}_2}))\right)\right)^{\xi_2} \end{pmatrix} \\ &= \begin{pmatrix} \prod_{\Xi=1}^{2} \left(\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}})\right)\right)^{\xi_{\Xi}}, \\ 1 - \prod_{\Xi=1}^{2} \left(\sin\left(\frac{\Pi}{2}(1 - (\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}}))\right)\right)^{\xi_{\Xi}}, \\ 1 - \prod_{\Xi=1}^{2} \left(\sin\left(\frac{\Pi}{2}(1 - (\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}}))\right)\right)^{\xi_{\Xi}} \end{pmatrix} \end{aligned}$$

Step-2: Assuming that Equation (2) holds for $n=\kappa$, we can then conclude:

$$ST\text{-}SVNZNWG\left(\eth_1,\eth_2,...,\eth_\kappa\right)=\begin{pmatrix}\prod_{\Xi=1}^{\kappa}\left(\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_\Xi},\mathfrak{T}_{\mathfrak{R}_\Xi}\right)\right)\right)^{\xi_\Xi},\\ 1-\prod_{\Xi=1}^{\kappa}\left(\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_\Xi},\mathfrak{I}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi},\\ 1-\prod_{\Xi=1}^{\kappa}\left(\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_\Xi},\mathfrak{F}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi}\end{pmatrix}$$

Step-3: Our next step is to demonstrate the validity of Equation (2) for $n=\kappa+1$.

$$\begin{aligned}ST\text{-}SVNZNWG\left(\eth_1,\eth_2,...,\eth_{\kappa+1}\right) &= \prod_{\Xi=1}^{\kappa}\left(\sin\left(\eth_\Xi\right)\right)^{\xi_\Xi}\bigotimes\left(\sin\left(\eth_{\kappa+1}\right)\right)^{\xi_{\kappa+1}}\\ &= \begin{pmatrix}\prod_{\Xi=1}^{\kappa}\left(\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_\Xi},\mathfrak{T}_{\mathfrak{R}_\Xi}\right)\right)\right)^{\xi_\Xi},\\ 1-\prod_{\Xi=1}^{\kappa}\left(\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_\Xi},\mathfrak{I}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi},\\ 1-\prod_{\Xi=1}^{\kappa}\left(\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_\Xi},\mathfrak{F}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi}\end{pmatrix}\\ &\quad\bigotimes\begin{pmatrix}\left(\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_{\kappa+1}},\mathfrak{T}_{\mathfrak{R}_{\kappa+1}}\right)\right)\right)^{\xi_{\kappa+1}},\\ 1-\left(\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_{\kappa+1}},\mathfrak{I}_{\mathfrak{R}_{\kappa+1}}\right)\right)\right)\right)^{\xi_{\kappa+1}},\\ 1-\left(\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_{\kappa+1}},\mathfrak{F}_{\mathfrak{R}_{\kappa+1}}\right)\right)\right)\right)^{\xi_{\kappa+1}}\end{pmatrix}\end{aligned}$$

$$=\begin{pmatrix}\prod_{\Xi=1}^{\kappa+1}\left(\sin\left(\frac{\Pi}{2}\left(\mathfrak{T}_{\mathfrak{V}_\Xi},\mathfrak{T}_{\mathfrak{R}_\Xi}\right)\right)\right)^{\xi_\Xi},\\ 1-\prod_{\Xi=1}^{\kappa+1}\left(\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{I}_{\mathfrak{V}_\Xi},\mathfrak{I}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi},\\ 1-\prod_{\Xi=1}^{\kappa+1}\left(\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_\Xi},\mathfrak{F}_{\mathfrak{R}_\Xi}\right)\right)\right)\right)^{\xi_\Xi}\end{pmatrix}$$

In other words, Equation (2) holds true when $n=\kappa+1$.

Consequently, we can conclude that Equation (2) holds for all values of n. □

Theorem 4.10. *(idempotency)*
Let $\eth_\Xi = \{(\mathfrak{T}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{T}_{\mathfrak{R}_\Xi}(\mathfrak{Z})),(\mathfrak{I}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{I}_{\mathfrak{R}_\Xi}(\mathfrak{Z})),(\mathfrak{F}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{F}_{\mathfrak{R}_\Xi}(\mathfrak{Z}))\} \in SVNZN(\Upsilon)$ $(\Xi=1,2,3,...,n)$ *such that* $\eth_\Xi=\eth$. *Then,* $ST\text{-}SVNZNWG\left(\eth_1,\eth_2,...,\eth_n\right)=\sin\left(\eth\right)$.

Theorem 4.11. *(Boundedness)*
Let $\eth_\Xi = \{(\mathfrak{T}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{T}_{\mathfrak{R}_\Xi}(\mathfrak{Z})),(\mathfrak{I}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{I}_{\mathfrak{R}_\Xi}(\mathfrak{Z})),(\mathfrak{F}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{F}_{\mathfrak{R}_\Xi}(\mathfrak{Z}))\}$,
$\eth_\Xi^- = \{\min((\mathfrak{T}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{T}_{\mathfrak{R}_\Xi}(\mathfrak{Z}))),\max((\mathfrak{I}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{I}_{\mathfrak{R}_\Xi}(\mathfrak{Z}))),\max((\mathfrak{F}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{F}_{\mathfrak{R}_\Xi}(\mathfrak{Z})))\}$ *and*
$\eth_\Xi^+ = \{\max((\mathfrak{T}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{T}_{\mathfrak{R}_\Xi}(\mathfrak{Z}))),\min((\mathfrak{I}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{I}_{\mathfrak{R}_\Xi}(\mathfrak{Z}))),\min((\mathfrak{F}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{F}_{\mathfrak{R}_\Xi}(\mathfrak{Z})))\} \in SVNZN(\Upsilon)$
$(\Xi=1,2,3,...,n)$. *Then,* $\sin\left(\eth_\Xi^-\right)\leq ST\text{-}SVNZNWG\left(\eth_1,\eth_2,...,\eth_n\right)\leq\sin\left(\eth_\Xi^+\right)$.

Theorem 4.12. *(Monotonically)*
Let $\eth_\Xi = \{(\mathfrak{T}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{T}_{\mathfrak{R}_\Xi}(\mathfrak{Z})),(\mathfrak{I}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{I}_{\mathfrak{R}_\Xi}(\mathfrak{Z})),(\mathfrak{F}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{F}_{\mathfrak{R}_\Xi}(\mathfrak{Z}))\}$,
$\eth_\Xi^* = \{(\mathfrak{T}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{T}_{\mathfrak{R}_\Xi}(\mathfrak{Z}))^*,(\mathfrak{I}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{I}_{\mathfrak{R}_\Xi}(\mathfrak{Z}))^*,(\mathfrak{F}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{F}_{\mathfrak{R}_\Xi}(\mathfrak{Z}))^*\} \in SVNZN(\Upsilon)$ $(\Xi=1,2,3,...,n)$.
If $(\mathfrak{T}_{\mathfrak{V}_\Xi},\mathfrak{T}_{\mathfrak{R}_\Xi})\leq(\mathfrak{T}_{\mathfrak{V}_\Xi},\mathfrak{T}_{\mathfrak{R}_\Xi})^*$, $(\mathfrak{I}_{\mathfrak{V}_\Xi},\mathfrak{I}_{\mathfrak{R}_\Xi})\leq(\mathfrak{I}_{\mathfrak{V}_\Xi},\mathfrak{I}_{\mathfrak{R}_\Xi})^*$ *and* $(\mathfrak{F}_{\mathfrak{V}_\Xi},\mathfrak{F}_{\mathfrak{R}_\Xi})\leq(\mathfrak{F}_{\mathfrak{V}_\Xi},\mathfrak{F}_{\mathfrak{R}_\Xi})^*$, *then* $ST\text{-}SVNZNWG\left(\eth_1,\eth_2,...,\eth_n\right)\leq ST\text{-}SVNZNWG\left(\eth_1^*,\eth_2^*,...,\eth_n^*\right)$.

Theorem 4.13. *(Symmetric)*
Let $\eth_{\Xi}=\{(\mathfrak{T}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}),\mathfrak{T}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z})),(\mathfrak{I}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}),\mathfrak{I}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z})),(\mathfrak{F}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}),\mathfrak{F}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))\}$, $\eth^*_{\Xi}=\{(\mathfrak{T}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}),\mathfrak{T}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))^*,(\mathfrak{I}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}),\mathfrak{I}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))^*,(\mathfrak{F}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}),\mathfrak{F}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))^*\}\in SVNZN(\Upsilon)$ $(\Xi=1,2,3,...,n)$. *Then* $ST\text{-}SVNZNWG(\eth_1,\eth_2,...,\eth_n)=ST\text{-}SVNZNWG(\eth^*_1,\eth^*_2,...,\eth^*_n)$, *whenever* $\eth^*_{\Xi}$ $(\Xi=1,2,3,...,n)$ *is any of* $\eth_{\Xi}$ $(\Xi=1,2,3,...,n)$.

Proof. Proofs of the above theorems, Theorem 4.10–4.13 follow from Theorems 4.4–4.7 likewise. □

4.3. *Fundamental Properties of the Proposed AOs for SVNZNs*

In this section, we have delved into different connections among the suggested AOs and analyzed some of their essential characteristics.

Theorem 4.14. *Let* $\eth_{\Xi}=\{(\mathfrak{T}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}),\mathfrak{T}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z})),(\mathfrak{I}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}),\mathfrak{I}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z})),(\mathfrak{F}_{\mathfrak{V}_{\Xi}}(\mathfrak{Z}),\mathfrak{F}_{\mathfrak{R}_{\Xi}}(\mathfrak{Z}))\}\in SVNZN(\Upsilon)$ $(\Xi=1,2)$. *Then,*

$$\sin(\eth_1)\bigoplus\sin(\eth_2)\geq\sin(\eth_1)\bigotimes\sin(\eth_2).$$

Proof. Given that $\eth_{\Xi}\in SVNZN(\Upsilon)$ $(\Xi=1,2)$, we can apply Definition 3.4 to obtain:

$$\sin(\eth_1)\bigoplus\sin(\eth_2)=\begin{pmatrix}1-\left(1-\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1})\right)\right)\left(1-\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_2},\mathfrak{T}_{\mathfrak{R}_2})\right)\right),\\ \left(1-\sin\left(\frac{\Pi}{2}(1-(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1}))\right)\right)\left(1-\sin\left(\frac{\Pi}{2}(1-(\mathfrak{I}_{\mathfrak{V}_2},\mathfrak{I}_{\mathfrak{R}_2}))\right)\right),\\ \left(1-\sin\left(\frac{\Pi}{2}(1-(\mathfrak{F}_{\mathfrak{V}_1},\mathfrak{F}_{\mathfrak{R}_1}))\right)\right)\left(1-\sin\left(\frac{\Pi}{2}(1-(\mathfrak{F}_{\mathfrak{V}_2},\mathfrak{F}_{\mathfrak{R}_2}))\right)\right)\end{pmatrix}$$

and

$$\sin(\eth_1)\bigotimes\sin(\eth_2)=\begin{pmatrix}\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1})\right)\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_2},\mathfrak{T}_{\mathfrak{R}_2})\right),\\ 1-\left(\sin\left(\frac{\Pi}{2}(1-(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1}))\right)\right)\left(\sin\left(\frac{\Pi}{2}(1-(\mathfrak{I}_{\mathfrak{V}_2},\mathfrak{I}_{\mathfrak{R}_2}))\right)\right),\\ 1-\left(\sin\left(\frac{\Pi}{2}(1-(\mathfrak{F}_{\mathfrak{V}_1},\mathfrak{F}_{\mathfrak{R}_1}))\right)\right)\left(\sin\left(\frac{\Pi}{2}(1-(\mathfrak{F}_{\mathfrak{V}_2},\mathfrak{F}_{\mathfrak{R}_2}))\right)\right)\end{pmatrix}$$

For any pair of non-negative real numbers ξ and m, we know that their arithmetic mean is greater than or equal to their geometric mean, expressed as $\frac{\xi+m}{2}\geq\sqrt{lm}$. This inequality can be rearranged as $\xi+m-2\sqrt{lm}\geq 0$, which further simplifies to $1-\sqrt{1-\xi}\sqrt{1-m}\geq lm$.
Hence, by considering $\xi=\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1})\right)$ and $m=\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_2},\mathfrak{T}_{\mathfrak{R}_2})\right)$, we obtain
$1-\left(1-\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1})\right)\right)\left(1-\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_2},\mathfrak{T}_{\mathfrak{R}_2})\right)\right)\geq\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1})\right)\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_2},\mathfrak{T}_{\mathfrak{R}_2})\right)$,
which leads to
$1-\left(1-\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1})\right)\right)\left(1-\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_2},\mathfrak{T}_{\mathfrak{R}_2})\right)\right)\geq\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_1},\mathfrak{T}_{\mathfrak{R}_1})\right)\sin\left(\frac{\Pi}{2}(\mathfrak{T}_{\mathfrak{V}_2},\mathfrak{T}_{\mathfrak{R}_2})\right)$.
Likewise, we obtain
$\left(1-\sin\left(\frac{\Pi}{2}(1-(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1}))\right)\right)\left(1-\sin\left(\frac{\Pi}{2}(1-(\mathfrak{I}_{\mathfrak{V}_2},\mathfrak{I}_{\mathfrak{R}_2}))\right)\right)\leq$
$1-\left(\sin\left(\frac{\Pi}{2}(1-(\mathfrak{I}_{\mathfrak{V}_1},\mathfrak{I}_{\mathfrak{R}_1}))\right)\right)\left(\sin\left(\frac{\Pi}{2}(1-(\mathfrak{I}_{\mathfrak{V}_2},\mathfrak{I}_{\mathfrak{R}_2}))\right)\right)$
and

$\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_1},\mathfrak{F}_{\mathfrak{R}_1}\right)\right)\right)\right)\left(1-\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_2},\mathfrak{F}_{\mathfrak{R}_2}\right)\right)\right)\right) \leq$
$1-\left(\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_1},\mathfrak{F}_{\mathfrak{R}_1}\right)\right)\right)\right)\left(\sin\left(\frac{\Pi}{2}\left(1-\left(\mathfrak{F}_{\mathfrak{V}_2},\mathfrak{F}_{\mathfrak{R}_2}\right)\right)\right)\right)$. Therefore,

$$\sin(\eth_1)\bigoplus\sin(\eth_2)\geq\sin(\eth_1)\bigotimes\sin(\eth_2).$$

□

Theorem 4.15. *Let* $\eth=\left\{\left(\mathfrak{T}_{\mathfrak{V}_{ji}}(\mathfrak{Z}),\mathfrak{T}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right),\left(\mathfrak{I}_{\mathfrak{V}_{ji}}(\mathfrak{Z}),\mathfrak{I}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right),\left(\mathfrak{F}_{\mathfrak{V}_{ji}}(\mathfrak{Z}),\mathfrak{F}_{\mathfrak{R}_{ji}}(\mathfrak{Z})\right)\right\}\in SVNZN(\Upsilon)$ *and* $\psi\geq 0$ *be any real number, then*

- **(1):** $\psi\sin(\eth)\geq(\sin(\eth))^{\psi}$ *if and only if* $\psi\geq 1$,
- **(2):** $\psi\sin(\eth)\leq(\sin(\eth))^{\psi}$ *if and only if* $0<\psi\leq 1$.

Proof. This can be deduced from Theorem 4.14 in a similar manner. □

Lemma 4.16. *For* $\xi_\Xi\geq 0$ *and* $m_\Xi\geq 0$, *then we have* $\prod_{\Xi=1}^{n}(\xi_\Xi)^{m_\Xi}\leq\sum_{\Xi=1}^{n}m_\Xi\xi_\Xi$ *and if* $\xi_1=\xi_2=...=\xi_n$ *then equality holds.*

Lemma 4.17. *Let* $0\leq\xi$, $m\leq 1$, *and* $0\leq x\leq 1$, *then* $0\leq lx+m(1-x)\leq 1$.

Lemma 4.18. *Let* $0\leq\xi$, $m\leq 1$, *then* $\sqrt{1-(1-\xi^2)(1-m^2)}\geq lm$.

Theorem 4.19. *Let* $\eth_\Xi=\{(\mathfrak{T}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{T}_{\mathfrak{R}_\Xi}(\mathfrak{Z})),(\mathfrak{I}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{I}_{\mathfrak{R}_\Xi}(\mathfrak{Z})),(\mathfrak{F}_{\mathfrak{V}_\Xi}(\mathfrak{Z}),\mathfrak{F}_{\mathfrak{R}_\Xi}(\mathfrak{Z}))\}\in SVNZN(\Upsilon)$ $(\Xi=1,2,3,...,n)$. *Then,*

$$ST\text{-}SVNZNWA(\eth_1,\eth_2,...,\eth_n)\geq ST\text{-}SVNZNWG(\eth_1,\eth_2,...,\eth_n),$$

where equality holds if and only if $\eth_1=\eth_2=...=\eth_n$.

Proof. Likewise, it derives from Theorem 4.14. □

5. Decision-Making Strategy

This section presents a DM methodology, along with an illustrative example, designed to address DMPs in the context of the SVNZN framework. Aspects related to multi-attribute DM (MADM) can be effectively showcased through the utilization of a decision matrix structure, where columns signify attributes and rows pertain to alternatives. For a given decision matrix $D_{n\times m}$, we consider a set of n alternatives: $\{\Upsilon_1,\Upsilon_2,\Upsilon_3,...,\Upsilon_n\}$, and m attributes: $\{t_1,t_2,t_3,...,t_m\}$. The undetermined WV associated with the m attributes is signified as $W=\{\kappa_1,\kappa_2,\kappa_3,...,\kappa_m\}$, subject to the constraint that $\xi_\Xi\in[0,1]$ and $\sum_{\Xi=1}^{m}\xi_\Xi=1$. Let's designate the SVN decision matrix as $D=(\eth_{ji})_{n\times m}=\left\langle\left(\mathfrak{T}_{\mathfrak{V}_{ji}},\mathfrak{T}_{\mathfrak{R}_{ji}}\right),\left(\mathfrak{I}_{\mathfrak{V}_{ji}},\mathfrak{I}_{\mathfrak{R}_{ji}}\right),\left(\mathfrak{F}_{\mathfrak{V}_{ji}},\mathfrak{F}_{\mathfrak{R}_{ji}}\right)\right\rangle_{n\times m}$, where $\left(\mathfrak{T}_{\mathfrak{V}_{ji}},\mathfrak{T}_{\mathfrak{R}_{ji}}\right)$ signifies the truth degree of the alternative satisfying the criteria t_j assessed by DMk, $\left(\mathfrak{I}_{\mathfrak{V}_{ji}},\mathfrak{I}_{\mathfrak{R}_{ji}}\right)$ represents the degree of the alternative's indeterminacy with respect to the criteria t_j evaluated by DMk, and $\left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right)$ denotes the degree of the alternative not meeting the criteria t_j considered by DMk. The algorithm encompasses the following steps:

Step-1: Compile the assessments of each alternative into the decision matrix $D^{(k)} = \left(\eth_{ji}^{(k)}\right)_{n\times m}$ using the SVNZN information.

Step-2: Form the normalized decision matrix $P = (p_{ji})$ from $D = (\eth_{ji})$, where p_{ji} is computed as follows:

$$p_{ji} = \left\{ \begin{array}{l} \left(\left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right), \left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right), \left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right)\right) \text{ in case if the criteria are of the benefit type} \\ \left(\left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right), \left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right), \left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right)\right) \text{ in case if the criteria are of the cost type} \end{array} \right\} \tag{3}$$

Step-3: Compute the collective information derived from DMk's input using either the SVNZNWA/SVNZNWG operator:

$$SVNZNWA\left(\eth_1, \eth_2, ..., \eth_n\right) = \left\{ \begin{array}{c} 1 - \prod_{\Xi=1}^{n} \left(1 - \left(\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}}\right)\right)^{\xi_\Xi}, \prod_{\Xi=1}^{n} \left(\left(\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}}\right)\right)^{\xi_\Xi}, \\ \prod_{\Xi=1}^{n} \left(\left(\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}}\right)\right)^{\xi_\Xi} \end{array} \right\}$$

or

$$SVNZNWG\left(\eth_1, \eth_2, ..., \eth_n\right) = \left\{ \begin{array}{c} \prod_{\Xi=1}^{n} \left(\left(\mathfrak{T}_{\mathfrak{V}_{\Xi}}, \mathfrak{T}_{\mathfrak{R}_{\Xi}}\right)\right)^{\xi_\Xi}, 1 - \prod_{\Xi=1}^{n} \left(1 - \left(\mathfrak{I}_{\mathfrak{V}_{\Xi}}, \mathfrak{I}_{\mathfrak{R}_{\Xi}}\right)\right)^{\xi_\Xi}, \\ 1 - \prod_{\Xi=1}^{n} \left(1 - \left(\mathfrak{F}_{\mathfrak{V}_{\Xi}}, \mathfrak{F}_{\mathfrak{R}_{\Xi}}\right)\right)^{\xi_\Xi} \end{array} \right\}$$

Step-4: If the attribute weights are pre-determined, they should be employed. However, if they are not known, they can be calculated using the entropy measure concept. In this context, the entropy-based information for criteria t_j is determined as follows:

$$E_j\left(\eth\right) = \frac{1}{\left(\sqrt{2}-1\right)m} \sum_{i=1}^{m} \left[\begin{array}{c} \sin\left(\frac{\Pi}{4}\left(1 + \left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right) - \left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right) - \left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right)\right)\right) + \\ \sin\left(\frac{\Pi}{4}\left(1 - \left(\mathfrak{T}_{\mathfrak{V}_{ji}}, \mathfrak{T}_{\mathfrak{R}_{ji}}\right) + \left(\mathfrak{I}_{\mathfrak{V}_{ji}}, \mathfrak{I}_{\mathfrak{R}_{ji}}\right) + \left(\mathfrak{F}_{\mathfrak{V}_{ji}}, \mathfrak{F}_{\mathfrak{R}_{ji}}\right)\right)\right) - 1 \end{array} \right].$$

Here, the term $\frac{1}{\left(\sqrt{2}-1\right)m}$ serves as a constant to ensure that $0 \leq E_j\left(\eth\right) \leq 1$.

Step-5: By employing the suggested STAOs and attribute WV, the combined SVN information for each alternative within the set $\{\curlyvee_1, \curlyvee_2, \curlyvee_3, ..., \curlyvee_n\}$ is acquired.

Step-6: Compute the score values $\overline{sc}\left(\eth\right)$ for the aggregated SVN numbers and arrange them in descending order of their score values. If two sets $\eth_1$ and $\eth_2$ yield identical score values, proceed to determine the accuracy degrees $\underline{ac}\left(\eth_1\right)$ and $\underline{ac}\left(\eth_2\right)$ for each set respectively. Subsequently, rank $\eth_1$ and $\eth_2$ based on the highest accuracy degree.

Step-7: Choose the optimal alternative based on either the highest score value or the maximum accuracy degree.

5.1. *Application of Proposed Decision-Making Technique*

Forecasting in business is an activity that is very valuable to both the strategic planning and DM processes that are carried out inside firms. It has a tremendous impact, both in terms of the day-to-day operations of the business as well as the results that it produces. It is crucial to have the ability to successfully anticipate market trends, the requirements of clients, and the achievement of success by a company. It is of the utmost importance for the process of DM due to the fact that it enables administration to make informed judgements about production, stock management, resource allocation, promotion, and development. An instance of the approach for making judgements that has been presented is first demonstrated with the help of a numerical application concerning the forecasting of a firm's selection problem in this section of the article. The first example that will be given is this one. In order to emphasize the characteristics and benefits offered by the given AOs, a comparison is made between the STAOs that have been delivered and the SVNZN AOs that are already in use. This is done in order to showcase the attributes and benefits of the AOs that are currently being provided.

5.1.1. *Practical Case Study*

f_1 **Efficient Resource Allocation, Cost Reduction, and Risk Mitigation:** When businesses are able to accurately forecast future demand, they are in a better position to utilize their available resources in an effective way, which not only helps them save costs but also helps them avoid risks. One further advantage is that this assists cut down on expenditures. They are able to adapt the levels of production, inventory, and people needs to fit the predicted demand, which lowers the risk of either overstocking or running out of supplies. This is because they are able to alter the levels of production, inventory, and manpower requirements. When businesses have precise estimates, they are better able to optimize their supply chains and manufacturing processes, which, in turn, results in less waste and less expenses that aren't necessary. This can be a win-win situation for everyone involved. This has the potential to be a win-win circumstance for all parties concerned. It helps to avoid having an excess inventory, which may lead to charges associated with storage and holding, and it reduces the frequency of urgent orders, which may contribute to higher production costs. Both of these factors may lead to higher overall costs. Both of these considerations have the potential to drive up total expenditures. The practise of forecasting future business activity may be of assistance to firms in spotting prospective hazards and ambiguities, which, in turn, enables these companies to design strategies to cope with unanticipated results. If companies are aware of the many difficulties that might be thrown their way, they may be better prepared to cope with unanticipated occurrences such as swings in the economy, disruptions in the market, and other unexpected occurrences.

f_2 **Strategic Planning, Enhanced Budgeting, and Competitive Advantage:** The process of forecasting must act as the foundation for the planning process in order for it to be effective when it comes to long-term strategy planning. Not only is it possible to enhance one's finances via accurate forecasting, but it may also provide one an advantage over their competitors. It offers aid in the process of setting goals that are attainable, defining objectives, and devising strategies that can be implemented in order to accomplish targets for growth and profitability. Due to the fact that it gives estimates of both revenue and expenditures, realistic forecasting makes it feasible to construct budgets that are more realistic. This makes it possible to distribute resources in a more effective way and helps firms to more effectively integrate the financial procedures they utilize with the larger corporate goals. In addition, this makes it possible to deploy resources in a more effective manner. If a company is able to precisely forecast what will occur in the future and react swiftly to changing circumstances in the market, they will have a considerable advantage over their rivals and will be able to more effectively compete. When companies have the ability to anticipate the needs of their customers and the trends in the market, they are in a better position to tailor the goods and services they provide in order to fulfil the particular demands of their customers.

f_3 **Customer Satisfaction and Investor Confidence:** An growth in both the amount of confidence maintained by investors and the degree to which consumers are happy with the product or service offered. An accurate forecast provides a regular supply of goods or services, which in turn leads in improved levels of customer satisfaction. Predictions may be made using historical data or by using predictive models. Customers who are pleased with the products or services they acquire are more likely to continue their patronage of the business and to suggest it to their friends and family, all of which contribute to the sustained prosperity of the enterprize. The capacity to create accurate forecasts inspires higher confidence among investors because it demonstrates an acute awareness and understanding of the mechanics of the market. In other words, it demonstrates that the investor is well-informed. This is due to the fact that it reveals to everyone that the investor has a solid grasp of the dynamics involved. This may be successful in persuading a greater number of investors and other stakeholders to support the firm's aims of development and expansion, which may be advantageous to the company.

The practise of business forecasting, in general, is advantageous to companies because it assists them in adapting to the ever-changing circumstances of the market, in making choices based on credible information, and in maximizing their operations in order to achieve long-term development and success. It makes it possible for organizations to construct their future in a way that is proactive and to effectively react to both opportunities and challenges in a positive manner.

When it comes to dealing with the placement of all renewable resources for the purpose of developing the most accurate business projections, the subject of making the appropriate choice is always one that is crucial for business. In order to find a solution to this issue, industry experts and those in charge of making decisions need to take into account as many qualitative and quantitative factors as they possibly can. A DM procedure that takes into consideration a wide range of factors is often used when one is tasked with selecting the most suitable location at which to build a new location of an existing firm. This is because the task is considered to be of especially high importance. The business industry is one of the most productive and ecologically friendly kinds of business. It also provides a significant contribution to the development of a nation.

The areas under examination must have been selected by the knowledgeable specialists after engaging in professional discussion with one another. The viewpoint of the person responsible for making the choice as well as the available research were used to compile a list of all of the factors that had a role in the selection of the location. The individuals in charge of making decisions need to collect and consider all of the available information in order to choose the most suitable place or location. We pick a case study for this selection issue and place it in a typical frame. In this example, there are four possible sites, which we will refer to as $\mathfrak{W}_1$, $\mathfrak{W}_2$, $\mathfrak{W}_3$, and $\mathfrak{W}_4$, and all of them will be taken into account while attempting to solve the problem. These websites have been scrutinized in a methodical manner with regard to the three primary characteristics, which are referred to above as f_1, f_2, and f_3 respectively. When the number of characteristics is raised, it is reasonable to anticipate an improvement in the solution. The issue of picking the best feasible location for a company from the set of possibilities that are now accessible is being mathematically and critically addressed within the context of the SVNZN environment, taking into account the expert's or DMk's viewpoint as well as the weights of the criteria. They are unable to supply the whole choice information because of the fuzziness and doubt that exists inside their brain, and the information on the assessment can be found in Table 2, which can be seen below. During this assessment, the expert was requested to utilize SVN information, with attribute weights set as $(0.33, 0.35, 0.32)^{\mathfrak{T}}$.

Step-1: Table 2 reveals the information handed over by the expert.

Step-2: As per the expert's input, attributes f_1 and f_3 are categorized as benefit types, while f_2 is a cost attribute. The normalized matrix computed using equation 3 yields the following results, which are displayed in Table 3.

Step-3: There is no need to estimate the aggregation decision matrix in this real-world case study because just one analyst DMk is involved.

Step-4: WV is a well-known criterion:

$$\kappa = \{\kappa_1 = 0.33, \kappa_2 = 0.35, \kappa_3 = 0.32\}$$

TABLE 2. Information result of the expert

	f_1	f_2	f_3
$\mathfrak{W}_1$	$\left\langle \begin{array}{c}(0.6, 0.8),\\(0.2, 0.3),\\(0.1, 0.5)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.1, 0.3),\\(0.2, 0.6),\\(0.7, 0.8)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.1, 0.3),\\(0.5, 0.6),\\(0.2, 0.9)\end{array}\right\rangle$
$\mathfrak{W}_2$	$\left\langle \begin{array}{c}(0.8, 0.7),\\(0.1, 0.8),\\(0.2, 0.6)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.5, 0.4),\\(0.3, 0.2),\\(0.7, 0.6)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.4, 0.2),\\(0.1, 0.5),\\(0.7, 0.6)\end{array}\right\rangle$
$\mathfrak{W}_3$	$\left\langle \begin{array}{c}(0.4, 0.6),\\(0.2, 0.5),\\(0.9, 0.4)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.4, 0.1),\\(0.5, 0.6),\\(0.2, 0.4)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.4, 0.3),\\(0.5, 0.1),\\(0.2, 0.1)\end{array}\right\rangle$
$\mathfrak{W}_4$	$\left\langle \begin{array}{c}(0.1, 0.5),\\(0.2, 0.3),\\(0.7, 0.5)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.3, 0.1),\\(0.2, 0.9),\\(0.6, 0.2)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.1, 0.5),\\(0.4, 0.1),\\(0.1, 0.2)\end{array}\right\rangle$

TABLE 3. Normalized matrix

	f_1	f_2	f_3
$\mathfrak{W}_1$	$\left\langle \begin{array}{c}(0.6, 0.8),\\(0.2, 0.3),\\(0.1, 0.5)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.7, 0.8),\\(0.2, 0.6),\\(0.1, 0.3)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.1, 0.3),\\(0.5, 0.6),\\(0.2, 0.9)\end{array}\right\rangle$
$\mathfrak{W}_2$	$\left\langle \begin{array}{c}(0.8, 0.7),\\(0.1, 0.8),\\(0.2, 0.6)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.7, 0.6),\\(0.3, 0.2),\\(0.5, 0.4)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.4, 0.2),\\(0.1, 0.5),\\(0.7, 0.6)\end{array}\right\rangle$
$\mathfrak{W}_3$	$\left\langle \begin{array}{c}(0.4, 0.6),\\(0.2, 0.5),\\(0.9, 0.4)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.2, 0.4),\\(0.5, 0.6),\\(0.4, 0.1)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.4, 0.3),\\(0.5, 0.1),\\(0.2, 0.1)\end{array}\right\rangle$
$\mathfrak{W}_4$	$\left\langle \begin{array}{c}(0.1, 0.5),\\(0.2, 0.3),\\(0.7, 0.5)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.6, 0.2),\\(0.2, 0.9),\\(0.3, 0.1)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.1, 0.5),\\(0.4, 0.1),\\(0.1, 0.2)\end{array}\right\rangle$

Step-5: By utilizing the proposed STAOs and the provided WV, the collective SVNZN information for each alternative is obtained and presented in Table 4.

Step-6: Calculate the score values for each aggregated SVNZN information of every alternative, as demonstrated in Table 5.

Step-7: As shown in Table 6, select the best option based on the greatest score value. Our goal in our case study is to use three factors to help us choose the best location for the company. Following the application of the planned algorithm stages, the aggregate

Table 4. aggregated SVNZN information of each alternative

	$ST\text{-}SVNZNWA$	$ST\text{-}SVNZNWG$
$\mathfrak{W}_1$	$\left\langle \begin{array}{c}(0.748, 0.894),\\(0.086, 0.266),\\(0.019, 0.291)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.495, 0.751),\\(0.135, 0.326),\\(0.024, 0.527)\end{array}\right\rangle$
$\mathfrak{W}_2$	$\left\langle \begin{array}{c}(0.872, 0.760),\\(0.026, 0.208),\\(0.198, 0.315)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.797, 0.614),\\(0.047, 0.403),\\(0.323, 0.343)\end{array}\right\rangle$
$\mathfrak{W}_3$	$\left\langle \begin{array}{c}(0.506, 0.650),\\(0.162, 0.120),\\(0.202, 0.030)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.469, 0.601),\\(0.220, 0.262),\\(0.505, 0.075)\end{array}\right\rangle$
$\mathfrak{W}_4$	$\left\langle \begin{array}{c}(0.498, 0.604),\\(0.075, 0.111),\\(0.092, 0.054)\end{array}\right\rangle$	$\left\langle \begin{array}{c}(0.278, 0.529),\\(0.096, 0.499),\\(0.263, 0.126)\end{array}\right\rangle$

Table 5. The aggregated SVNZN information of each alternative's score value

	$Y(\mathfrak{W}_1)$	$Y(\mathfrak{W}_2)$	$Y(\mathfrak{W}_3)$	$Y(\mathfrak{W}_4)$
$ST\text{-}SVNZNWA$	0.880	0.865	0.768	0.762
$ST\text{-}SVNZNWG$	0.787	0.772	0.729	0.689

Table 6. Optimal alternative based on the highest score value

	Score Ranking	Best Alternatives
$ST\text{-}SVNZNWA$	$Y(\mathfrak{W}_1) > Y(\mathfrak{W}_2) > Y(\mathfrak{W}_3) > Y(\mathfrak{W}_4)$	$\mathfrak{W}_1$
$ST\text{-}SVNZNWG$	$Y(\mathfrak{W}_2) > Y(\mathfrak{W}_1) > Y(\mathfrak{W}_3) > Y(\mathfrak{W}_4)$	$\mathfrak{W}_1$

data based on the innovative ST operational principles is presented as an SVNZN set. Drawing conclusions from the computational method described above, we find that $\mathfrak{W}_2$ is the optimal choice among the alternatives; as such, it is strongly advised to use it for the necessary work or plan.

6. Comparison Analysis

The feasibility of the proposed process, its aggregation's adaptability to specific inputs and outcomes, the influence of scoring functions, analysis of sensitivity, supremacy, and, lastly, a comparison of the proposed technique with current methods are all covered in this part. The recommended approach was accurate and appropriate for a wide range of input data types. The approach that was developed worked well for managing uncertainty. It included

Z numbers and STAO-based SVNS spaces. We may effectively use our approach in a wide range of circumstances by expanding the distance among the pleasure and displeasure classes by altering the real-world importance of particular parameters. We came across a variety of elements and parameters for input in multiple MADM problems that were appropriate for the given situation. The suggested SVNZNs were easy to grasp, basic, and versatile enough to fit a wide range of needs. We saw in Table 7, that every one of our suggested aggregating operators generated the same outcomes, demonstrating accuracy and strength. This essay's goal was to show, through a comparative analysis with a few current approaches, the superiority and reliability of our original study. We compared our results with neutrosophic ZNs (NZN) weighted arithmetic averaging (NZNWAA) and NZN weighted geometric averaging (NZNWGA) operators [35], NZN AczelAlsina weighted arithmetic averaging (NZNAAWAA) and NZN AczelAlsina weighted geometric averaging (NZNAAWGA) operators [32] and linguistic neutrosophic ZN (LNZN) weighted arithmetic mean (LNZNWAM) and LNZN weighted geometric mean (LNZNWGM) operators [33], and the work that is connected to decision making difficulties in [34–36] as well as the great work that is important to SVN structure in [37] is really remarkable.

Table 7. Comparison Analysis

	Score Ranking	Best Alternatives
$ST\text{-}NZNWAA$	$Y(\mathfrak{W}_1) > Y(\mathfrak{W}_2) > Y(\mathfrak{W}_4) > Y(\mathfrak{W}_3)$	$\mathfrak{W}_1$
$ST\text{-}NZNWGA$	$Y(\mathfrak{W}_1) > Y(\mathfrak{W}_2) > Y(\mathfrak{W}_3) > Y(\mathfrak{W}_4)$	$\mathfrak{W}_1$
$ST\text{-}NZNAAWAA$	$Y(\mathfrak{W}_2) > Y(\mathfrak{W}_1) > Y(\mathfrak{W}_3) > Y(\mathfrak{W}_4)$	$\mathfrak{W}_2$
$ST\text{-}NZNAAWGA$	$Y(\mathfrak{W}_1) > Y(\mathfrak{W}_2) > Y(\mathfrak{W}_3) > Y(\mathfrak{W}_4)$	$\mathfrak{W}_1$
$ST\text{-}LNZNWAM$	$Y(\mathfrak{W}_1) > Y(\mathfrak{W}_2) > Y(\mathfrak{W}_3) > Y(\mathfrak{W}_4)$	$\mathfrak{W}_1$
$ST\text{-}LNZNWGM$	$Y(\mathfrak{W}_2) > Y(\mathfrak{W}_1) > Y(\mathfrak{W}_3) > Y(\mathfrak{W}_4)$	$\mathfrak{W}_2$
$ST\text{-}SVNZNWA$	$Y(\mathfrak{W}_1) > Y(\mathfrak{W}_2) > Y(\mathfrak{W}_3) > Y(\mathfrak{W}_4)$	$\mathfrak{W}_1$
$ST\text{-}SVNZNWG$	$Y(\mathfrak{W}_1) > Y(\mathfrak{W}_2) > Y(\mathfrak{W}_3) > Y(\mathfrak{W}_4)$	$\mathfrak{W}_1$

7. Conclusion

The rapid process of industrialization has led to a significant expansion of the global business landscape. In this research manuscript, our goal is to introduce a novel method for selecting business locations. To achieve this, we propose the utilization of fresh operational laws based on the ST function under SVNZNs, which we refer to as Z-STOLs. Below is a more detailed discussion of these strategies' benefits.

- The important features of the SVNZNs and their operational characteristics, such as boundedness, monotonicity, commutativity, and idempotency, are covered first.
- Next, the ideas for developing specialized AOs such ST ZN SVNZN-weighted AOs and ST ZN SVNZN-ordered weighted averaging/geometric AOs are formed. We investigate the underlying links between these newly introduced aggregation operations in depth.
- Then we developed a new MADM algorithm for dealing with DM situations in which preferences are evaluated using SVNZNs. This enables us to apply the proposed legislation to Decision-Making Problems correctly.
- Our research findings highlight the high efficacy of using SVNZ information measures in handling ambiguity in DM issues. We employ a real-world case to assess the efficacy of our suggested strategy of site selection for a business, subjecting it to thorough scrutiny to ascertain its superiority and viability.
- At the end, we conduct a comparative analysis with several previously published studies to further validate its efficiency. The approach presented in this study holds significant promise for application in various domains, including medical diagnostics, green supplier selection, and more.
- Future studies on two-sided combining making choices with multi-granular and unfinished criteria weight information, widespread agreement accomplishing with uncooperative behavioral DMPs, and personalized individual uniformity control consensus problems could make use of the suggested AOs. This examination of the constraints imposed by proposed AOs is independent of the levels of involvement, abstention, and non-membership. On this side of the intended AOs, an innovative hybrid structure of interactive, prioritized AOs is being implemented.
- In future studies, we aim to extend this approach to address other ambiguous domains, such as interval-value SVNZNs and probabilistic linguistic term sets. The versatility of our proposed approach makes it a valuable tool for DMks across different industries.

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