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On Some Algebraic Properties of Symbolic 2-Plithogenic Square Matrices

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Abstract. In this article, the adjoint of symbolic 2-plithogenic square matrices are defined and the inverse of symbolic 2-plithogenic square matrices are studied in terms of symbolic 2-plithogenic determinant and symbolic 2-plithogenic adjoint. We have introduced the concept of symbolic 2-plithogenic characteristic polynomial of symbolic 2-plithogenic square matrices and the symbolic 2-plithogenic version of Cayley-Hamilton theorem. Also, provided enough examples to enhance understanding.

Keywords: Symbolic 2-plithogenic matrix; symbolic 2-plithogenic adjoint; symbolic 2-plithogenic determinant; symbolic 2-plithogenic inverse.

1. Introduction

The concept of refined neutrosophic structure was studied by many authors in [1–4]. Symbolic plithogenic algebraic structures are introduced by Smarandache, that are very similar to refined neutrosophic structures with some differences in the definition of the multiplication operation [15].

In [12], the algebraic properties of symbolic 2-plithogenic rings generated from the fusion of symbolic plithogenic sets with algebraic rings are studied. In [8], some more algebraic properties of symbolic 2-plithogenic rings are studied. Further, Taffach [17, 18] studied the concepts

of symbolic 2-plithogenic vector spaces and modules.

Recently, in [7], the concept of symbolic 2-plithogenic matrices with symbolic 2-plithogenic entries, determinants, eigen values and vectors, exponents, and diagonalization are studied. Hamiyet Merkepci et.al [13], studied the the symbolic 2-plithogenic number theory and integers. Ahmad Khaldi et.al [11], studied the different types of algebraic symbolic 2-plithogenic equations and its solutions.

As a continuation of the previous study of symbolic 2-plithogenic matrices, this work discusses the symbolic 2-plithogenic adjoint, where the inverse of symbolic 2-plithogenic matrices will be defined in terms of the symbolic 2-plithogenic adjoint. We present the symbolic 2-plithogenic characteristic polynomials and the symbolic 2-plithogenic version of the Cayley-Hamilton theorem. Also, we illustrate many examples to clarify the validity of our work.

2. Preliminaries

Definition 2.1. [12] Let R be a ring, the symbolic 2-plithogenic ring is defined as follows:

$$2 - SP_R = \left\{ a_0 + a_1P_1 + a_2P_2; a_i \in R, P_j^2 = P_j, P_1 \times P_2 = P_{\max(1,2)} = P_2 \right\}$$

Smarandache has defined algebraic operations on $2 - SP_R$ as follows:

Addition:

$$[a_0 + a_1P_1 + a_2P_2] + [b_0 + b_1P_1 + b_2P_2] = (a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2$$

Multiplication:

$$[a_0 + a_1P_1 + a_2P_2].[b_0 + b_1P_1 + b_2P_2] = a_0b_0 + a_0b_1P_1 + a_0b_2P_2 + a_1b_0P_1^2 + a_1b_2P_1P_2 + a_2b_0P_2 + a_2b_1P_1P_2 + a_2b_2P_2^2 + a_1b_1P_1P_1 = (a_0b_0) + (a_0b_1 + a_1b_0 + a_1b_1)P_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)P_2.$$

It is clear that $2 - SP_R$ is a ring. If R is a field, then $2 - SP_R$ is called a symbolic 2-plithogenic field. Also, if R is commutative, then $2 - SP_R$ is commutative, and if R has a unity (1), then $2 - SP_R$ has the same unity (1).

Theorem 2.2. [12] Let $2 - SP_R$ be a 2-plithogenic symbolic ring, with unity (1). Let $X = x_0 + x_1P_1 + x_2P_2$ be an arbitrary element, then:

- (1) X is invertible if and only if $x_0, x_0 + x_1, x_0 + x_1 + x_2$ are invertible.
- (2) $X^{-1} = x_0^{-1} + [(x_0 + x_1)^{-1} - x_0^{-1}]P_1 + [(x_0 + x_1 + x_2)^{-1} - (x_0 + x_1)^{-1}]P_2$

Definition 2.3. [7] A symbolic 2-plithogenic real square matrix is a matrix with symbolic 2-plithogenic real entries.

Theorem 2.4. [7] Let $S = S_0 + S_1P_1 + S_2P_2$ be a symbolic 2-plithogenic real square matrix, then

- (1) S is invertible if and only if $S_0, S_0 + S_1, S_0 + S_1 + S_2$ are invertible.
- (2) If S is invertible then

$$S^{-1} = S_0^{-1} + [(S_0 + S_1)^{-1} - S_0^{-1}]P_1 + [(S_0 + S_1 + S_2)^{-1} - (S_0 + S_1)^{-1}]$$

- (3) $S^m = S_0^m + [(S_0 + S_1)^m - S_0^m]P_1 + [(S_0 + S_1 + S_2)^m - (S_0 + S_1)^m]$ for $m \in N$.

Definition 2.5. [7] Let $L = L_0 + L_1P_1 + L_2P_2 \in 2 - SP_M$, we define:

$$\det L = \det(L_0) + [\det(L_0 + L_1) - \det L_0]P_1 + [\det(L_0 + L_1 + L_2) - \det(L_0 + L_1)]P_2.$$

3. Adjoint of Symbolic 2-Plithogenic Square Matrices

We begin this section with the following definition.

Definition 3.1. Let $L = L_0 + L_1P_1 + L_2P_2$ be a symbolic 2-plithogenic square matrix with real entries. The adjoint matrix of L is defined as

$$\text{adj} L = \text{adj} L_0 + [\text{adj}(L_0 + L_1) - \text{adj} L_0]P_1 + [\text{adj}(L_0 + L_1 + L_2) - \text{adj}(L_0 + L_1)]P_2.$$

Example 3.2. Consider the following symbolic 2-plithogenic 2×2 matrix:

$$L = \begin{pmatrix} 2 + P_1 + 3P_2 & 1 - P_1 - P_2 \\ 3 + 4P_1 & 1 + P_2 \end{pmatrix}$$

Here,

$$L_0 = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}, L_0 + L_1 = \begin{pmatrix} 3 & 0 \\ 7 & 1 \end{pmatrix} \text{ and } L_0 + L_1 + L_2 = \begin{pmatrix} 6 & -1 \\ 7 & 2 \end{pmatrix},$$

Then,

$$\text{adj} L_0 = \begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix}, \text{adj}(L_0 + L_1) = \begin{pmatrix} 1 & 0 \\ -7 & 3 \end{pmatrix} \text{ and } \text{adj}(L_0 + L_1 + L_2) = \begin{pmatrix} 2 & 1 \\ -7 & 6 \end{pmatrix}.$$

Therefore,

$$\begin{aligned} \text{adj} L &= \text{adj} L_0 + [\text{adj}(L_0 + L_1) - \text{adj} L_0]P_1 + [\text{adj}(L_0 + L_1 + L_2) - \text{adj}(L_0 + L_1)]P_2 \\ &= \begin{pmatrix} 1 + P_1 & -1 + P_1 + P_2 \\ -3 - 4P_2 & 2 + P_1 + 3P_2 \end{pmatrix} \end{aligned}$$

Example 3.3. Consider the following symbolic 2-plithogenic 3×3 matrix:

$$L = \begin{pmatrix} -3 + P_1 - P_2 & 1 + P_1 & 5 \\ -P_1 + P_2 & 3P_1 & 4P_2 \\ -1 + 2P_1 - P_2 & 5 + 2P_2 & 7 + P_1 + 10P_2 \end{pmatrix}$$

Here,

$$L_0 = \begin{pmatrix} -3 & 1 & 5 \\ 0 & 0 & 0 \\ -1 & 5 & 7 \end{pmatrix}, L_0 + L_1 = \begin{pmatrix} -2 & 2 & 5 \\ -1 & 3 & 0 \\ -1 & 5 & 8 \end{pmatrix} \text{ and } L_0 + L_1 + L_2 = \begin{pmatrix} -3 & 2 & 5 \\ 0 & 3 & 4 \\ 0 & 7 & 18 \end{pmatrix},$$

Then,

$$adjL_0 = \begin{pmatrix} 0 & 18 & 0 \\ 0 & -16 & 0 \\ 0 & 14 & 0 \end{pmatrix}, \quad adj(L_0 + L_1) = \begin{pmatrix} 24 & 9 & -15 \\ 8 & -21 & -5 \\ -8 & 12 & -4 \end{pmatrix} \quad \text{and}$$

$$adj(L_0 + L_1 + L_2) = \begin{pmatrix} 26 & -1 & -7 \\ 0 & -54 & 12 \\ 0 & 21 & 9 \end{pmatrix}.$$

Therefore,

$$\begin{aligned} adjL &= adjL_0 + [adj(L_0 + L_1) - adjL_0]P_1 + [adj(L_0 + L_1 + L_2) - adj(L_0 + L_1)]P_2 \\ &= \begin{pmatrix} 24P_1 + 2P_2 & 18 - 9P_1 - 10P_2 & -15P_1 + 8P_2 \\ 8P_1 - 8P_2 & -16 + 5P_1 + 33P_2 & -5P_1 + 17P_2 \\ -8P_1 + 8P_2 & 14 - 2P_1 + 9P_2 & -4P_1 + 13P_2 \end{pmatrix} \end{aligned}$$

Using the definition of adjoint of symbolic 2-plithogenic matrix we can modify the Theorem 2.4 as follows:

Theorem 3.4. *Let $L = L_0 + L_1P_1 + L_2P_2$ be a symbolic 2-plithogenic square matrix, then L is invertible if and only if $detL_0 \neq 0, det(L_0 + L_1) \neq 0$ and $det(L_0 + L_1 + L_2) \neq 0$ and*

$$L^{-1} = \frac{1}{detL}(adjL).$$

Proof. By Theorem 2.4, L is invertible if and only if $detL_0 \neq 0, det(L_0 + L_1) \neq 0$ and $det(L_0 + L_1 + L_2) \neq 0$.

Also,

$$\begin{aligned} \frac{1}{detL}(adjL) &= \left(\frac{1}{detL_0 + [det(L_0 + L_1) - det(L_0)]P_1 + [det(L_0 + L_1 + L_2) - det(L_0 + L_1)]P_2} \right) \\ &\quad (adjL_0 + [adj(L_0 + L_1) - adjL_0]P_1 + [adj(L_0 + L_1 + L_2) - adj(L_0 + L_1)]P_2) \\ &= \frac{adjL_0}{detL_0} + \left[\frac{adj(L_0 + L_1)}{det(L_0 + L_1)} - \frac{adjL_0}{detL_0} \right] P_1 + \left[\frac{adj(L_0 + L_1 + L_2)}{det(L_0 + L_1 + L_2)} - \frac{adj(L_0 + L_1)}{det(L_0 + L_1)} \right] P_2 \\ &= L_0^{-1} + [(L_0 + L_1)^{-1} - L_0^{-1}] P_1 + [(L_0 + L_1 + L_2)^{-1} - (L_0 + L_1)^{-1}] P_2 \\ &= L^{-1} \end{aligned}$$

Hence the result holds by Theorem 2.4. \square

Example 3.5. Consider the symbolic 2-plithogenic 2×2 matrix

$$L = \begin{pmatrix} 1 + P_1 + P_2 & -1 + P_1 \\ 1 - P_2 & 1 \end{pmatrix}$$

Here, $\det L = 2 + P_2$, and $\text{adj}L = \begin{pmatrix} 1 & 1 - P_1 \\ -1 + P_2 & 1 + P_1 + P_2 \end{pmatrix}$.

Hence,

$$\begin{aligned} L^{-1} &= \frac{1}{\det L}(\text{adj}L) \\ &= \frac{1}{2 + P_2} \begin{pmatrix} 1 & 1 - P_1 \\ -1 + P_2 & 1 + P_1 + P_2 \end{pmatrix} \\ &= \left(\frac{1}{2} - \frac{1}{6}P_2\right) \begin{pmatrix} 1 & 1 - P_1 \\ -1 + P_2 & 1 + P_1 + P_2 \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2} - \frac{1}{6}P_2 & \frac{1}{2} - \frac{1}{2}P_1 \\ -\frac{1}{2} + \frac{1}{2}P_2 & \frac{1}{2} + \frac{1}{2}P_1 \end{pmatrix} \end{aligned}$$

Example 3.6. Consider the symbolic 2-plithogenic 3×3 matrix

$$L = \begin{pmatrix} 1 + P_1 & 1 - P_1 & 1 + P_1 - P_2 \\ 1 + P_2 & -1 + P_1 + P_2 & 2 + P_1 \\ 1 - P_1 + P_2 & -1 + P_2 & 1 + P_1 \end{pmatrix}$$

Here, $\det L = 2 + 2P_1 - P_2$, and

$$\text{adj}(L) = \begin{pmatrix} 1 + 2P_1 - P_2 & -2 + 2P_2 & 3 - 3P_1 - P_2 \\ 1 - 3P_1 + P_2 & 4P_1 - P_2 & -1 - 3P_1 \\ -P_1 & 2 - 2P_2 & -2 + 2P_1 + 2P_2 \end{pmatrix}$$

$$\begin{aligned} L^{-1} &= \frac{1}{\det L}(\text{adj}L) \\ &= \frac{1}{2 + 2P_1 - P_2} \begin{pmatrix} 1 + 2P_1 - P_2 & -2 + 2P_2 & 3 - 3P_1 - P_2 \\ 1 - 3P_1 + P_2 & 4P_1 - P_2 & -1 - 3P_1 \\ -P_1 & 2 - 2P_2 & -2 + 2P_1 + 2P_2 \end{pmatrix} \\ &= \left(\frac{1}{2} - \frac{1}{4}P_1 + \frac{1}{12}P_2\right) \begin{pmatrix} 1 + 2P_1 - P_2 & -2 + 2P_2 & 3 - 3P_1 - P_2 \\ 1 - 3P_1 + P_2 & 4P_1 - P_2 & -1 - 3P_1 \\ -P_1 & 2 - 2P_2 & -2 + 2P_1 + 2P_2 \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2} + \frac{1}{4}P_1 - \frac{1}{12}P_2 & -1 + P_2 & \frac{3}{2} - \frac{3}{2}P_1 - \frac{1}{3}P_2 \\ \frac{1}{2} - P_1 + \frac{1}{6}P_2 & P_1 + \frac{8}{3}P_2 & -\frac{1}{2} - \frac{1}{2}P_1 - \frac{1}{3}P_2 \\ -\frac{1}{4}P_1 - \frac{1}{12}P_2 & 1 - \frac{1}{2}P_1 - \frac{1}{2}P_2 & -1 + P_1 + \frac{2}{3}P_2 \end{pmatrix} \end{aligned}$$

Remark 3.7. If X is a invertible symbolic 2-plithogenic square matrix and X^{-1} is its inverse, then $\text{adj}X = \det X \cdot X^{-1}$.

Theorem 3.8. Let $X = A + BP_1 + CP_2$ and $Y = M + NP_1 + SP_2$ be two symbolic 2-plithogenic invertible square matrices. Then XY is also invertible and $(XY)^{-1} = Y^{-1}X^{-1}$.

Proof. By Theorem 3.4, if X is invertible then

$$\det(A) \neq 0, \det(A + B) \neq 0 \text{ and } \det(A + B + C) \neq 0.$$

Similarly, if Y is invertible then

$$\det M \neq 0, \det(M + N) \neq 0 \text{ and } \det(M + N + S) \neq 0.$$

This implies that,

$$\begin{aligned} \det(AM) &= \det A \det M \neq 0 \\ \det[(A + B)(M + N)] &= \det(A + B) \det(M + N) \neq 0 \\ \det[(A + B + C)(M + N + S)] &= \det(A + B + C) \det(M + N + S) \neq 0. \end{aligned}$$

Now,

$$\det(XY) = \det(AM) + [\det((A + B)(M + N))]P_1 + [\det((A + B + C)(M + N + S))]P_2 \neq 0$$

and hence XY is invertible. Also by associativity of matrix multiplication, we have

$$\begin{aligned} (XY)(Y^{-1}X^{-1}) &= X(YY^{-1})X^{-1} = XX^{-1} = U_{n \times n} \\ (Y^{-1}X^{-1})(XY) &= Y^{-1}(X^{-1}X)Y = Y^{-1}Y = U_{n \times n}. \end{aligned}$$

Thus, $(MN)^{-1} = N^{-1}M^{-1}$. \square

Theorem 3.9. *Let X and Y be two $m \times m$ symbolic 2-plithogenic invertible matrices. Then the following properties holds.*

- (1) $\det(\text{adj} X) = (\det X)^{m-1}$.
- (2) $\text{adj}(XY) = \text{adj} X \text{adj} Y$.
- (3) $\text{adj}(X^k) = (\text{adj} X)^k$ for any positive integer k .
- (4) $\text{adj}(X^T) = (\text{adj} X)^T$.
- (5) $\text{adj}(\text{adj} X) = (\det X)^{m-2} X$

Proof. We can prove this results based on the properties adjoint of classical matrices. \square

4. Characteristic Polynomial of Symbolic 2-Plithogenic Square Matrices

We begin this section with the following definition.

Definition 4.1. Let $L = L_0 + L_1P_1 + L_2P_2$ be a symbolic 2-plithogenic $n \times n$ square matrix with real entries. The characteristic polynomial of L is defined as

$$\phi(\lambda) = \alpha(\lambda) + [\beta(\lambda) - \alpha(\lambda)] P_1 + [\gamma(\lambda) - \beta(\lambda)] P_2$$

where,

$$\alpha(\lambda) = \det(L_0 - \lambda U_{n \times n})$$

$$\beta(\lambda) = \det(L_0 + L_1 - \lambda U_{n \times n}) - \det(L_0 - \lambda U_{n \times n})$$

$$\gamma(\lambda) = \det(L_0 + L_1 + L_2 - \lambda U_{n \times n}) - \det(L_0 + L_1 - \lambda U_{n \times n}).$$

Example 4.2. Consider the following symbolic 2-plithogenic 2×2 matrix:

$$L = \begin{pmatrix} 2 + P_1 + 3P_2 & 1 - P_1 - P_2 \\ 3 + 4P_1 & 1 + P_2 \end{pmatrix}$$

with

$$L_0 = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}, L_0 + L_1 = \begin{pmatrix} 3 & 0 \\ 7 & 1 \end{pmatrix} \text{ and } L_0 + L_1 + L_2 = \begin{pmatrix} 6 & -1 \\ 7 & 2 \end{pmatrix}.$$

Here,

$$\alpha(\lambda) = \det(L_0 - \lambda U_{n \times n}) = \begin{vmatrix} 2 - \lambda & 1 \\ 3 & 1 - \lambda \end{vmatrix} = \lambda^2 - 3\lambda - 1.$$

$$\begin{aligned} \beta(\lambda) &= \det(L_0 + L_1 - \lambda U_{n \times n}) - \det(L_0 - \lambda U_{n \times n}) \\ &= \begin{vmatrix} 3 - \lambda & 0 \\ 7 & 1 - \lambda \end{vmatrix} = \lambda^2 - 4\lambda + 3. \end{aligned}$$

$$\begin{aligned} \gamma(\lambda) &= \det(L_0 + L_1 + L_2 - \lambda U_{n \times n}) - \det(L_0 + L_1 - \lambda U_{n \times n}) \\ &= \begin{vmatrix} 6 - \lambda & -1 \\ 7 & 2 - \lambda \end{vmatrix} = \lambda^2 - 8\lambda - 19. \end{aligned}$$

Hence the characteristic polynomial of L is

$$\begin{aligned} \phi(\lambda) &= \lambda^2 - 3\lambda - 1 + [(\lambda^2 - 4\lambda + 3) - (\lambda^2 - 3\lambda - 1)]P_1 + [(\lambda^2 - 8\lambda - 19) - (\lambda^2 - 4\lambda + 3)]P_2 \\ &= \lambda^2 - 3\lambda - 1 + (-\lambda + 4)P_1 + (-4\lambda + 16)P_2. \end{aligned}$$

Example 4.3. Consider the symbolic 2-plithogenic 3×3 matrix

$$L = \begin{pmatrix} 1 + P_1 & 1 - P_1 & 1 + P_1 - P_2 \\ 1 + P_2 & -1 + P_1 + P_2 & 2 + P_1 \\ 1 - P_1 + P_2 & -1 + P_2 & 1 + P_1 \end{pmatrix}$$

with

$$L_0 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 1 & -1 & 1 \end{pmatrix}, L_0 + L_1 = \begin{pmatrix} 2 & 0 & 2 \\ 1 & 0 & 3 \\ 0 & -1 & 2 \end{pmatrix}, \text{ and } L_0 + L_1 + L_2 = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & 0 & 2 \end{pmatrix}.$$

Here,

$$\alpha(\lambda) = \det(L_0 - \lambda U_{n \times n}) = \det \begin{pmatrix} 1 - \lambda & 1 & 1 \\ 1 & -1 - \lambda & 2 \\ 1 & -1 & 1 - \lambda \end{pmatrix} = -\lambda^3 + \lambda^2 + \lambda + 2.$$

$$\begin{aligned} \beta(\lambda) &= \det(L_0 + L_1 - \lambda U_{n \times n}) - \det(L_0 - \lambda U_{n \times n}) \\ &= \det \begin{pmatrix} 2 - \lambda & 0 & 2 \\ 1 & -\lambda & 3 \\ 0 & -1 & 2 - \lambda \end{pmatrix} \\ &= -\lambda^3 + 4\lambda^2 - 7\lambda + 4. \end{aligned}$$

$$\begin{aligned} \gamma(\lambda) &= \det(L_0 + L_1 + L_2 - \lambda U_{n \times n}) - \det(L_0 + L_1 - \lambda U_{n \times n}) \\ &= \det \begin{pmatrix} 2 - \lambda & 0 & 1 \\ 2 & 1 - \lambda & 3 \\ 1 & 0 & 2 - \lambda \end{pmatrix} \\ &= -\lambda^3 + 5\lambda^2 - 7\lambda + 3. \end{aligned}$$

Hence the characteristic polynomial of L is

$$\begin{aligned} \phi(\lambda) &= -\lambda^3 + \lambda^2 + \lambda + 2 + [(-\lambda^3 + 4\lambda^2 - 7\lambda + 4) - (-\lambda^3 + \lambda^2 + \lambda + 2)]P_1 \\ &\quad + [(-\lambda^3 + 5\lambda^2 - 7\lambda + 3) - (-\lambda^3 + 4\lambda^2 - 7\lambda + 4)]P_2 \\ &= -\lambda^3 + \lambda^2 + \lambda + 2 + (3\lambda^2 - 8\lambda + 2)P_1 + (\lambda^2 - 1)P_2. \end{aligned}$$

Theorem 4.4 (Symbolic 2-plithogenic Cayely-Hamilton Theorem). *Every symbolic 2-plithogenic square matrix satisfies its characteristic polynomial.*

Proof. We can prove this result based on the Cayely-Hamilton theorem for classical matrices.

□

Example 4.5. Consider the symbolic 2-plithogenic 2×2 matrix given in Example 4.2

$$L = \begin{pmatrix} 1 + P_1 + P_2 & -1 + P_1 \\ 1 - P_2 & 1 \end{pmatrix}$$

The characteristic polynomial of L is $\phi(\lambda) = \lambda^2 - 3\lambda - 1 + (-\lambda + 4)P_1 + (-4\lambda + 16)P_2$. This implies that,

$$\begin{aligned} \phi(L) &= L^2 - 3L - 1 + (-L + 4)P_1 + (-4L + 16)P_2 \\ &= \begin{pmatrix} -P_1 + 11P_2 & -5P_2 \\ 7P_1 + 28P_2 & -3P_1 - 7P_2 \end{pmatrix} + \begin{pmatrix} P_1 - 3P_2 & P_2 \\ -7P_1 & 3P_1 - P_2 \end{pmatrix} + \begin{pmatrix} -8P_2 & 4P_2 \\ -28P_1 & 8P_2 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{aligned}$$

Hence, $\phi(L) = 0$.

Remark 4.6. If L is a invertible symbolic 2-plithogenic matrix, then using Cayely-Hamilton theorem we can compute the inverse of L . See the following example.

Example 4.7. Consider the symbolic 2-plithogenic 2×2 matrix

$$L = \begin{pmatrix} 1 + P_1 + P_2 & -1 + P_1 \\ 1 - P_2 & 1 \end{pmatrix}$$

with

$$L_0 = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, L_0 + L_1 = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}, \text{ and } L_0 + L_1 + L_2 = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}.$$

Here,

$$\begin{aligned} \alpha(\lambda) &= \det(L_0 - \lambda U_{n \times n}) = \begin{vmatrix} 1 - \lambda & -1 \\ 1 & 1 - \lambda \end{vmatrix} = \lambda^2 - 2\lambda + 2. \\ \beta(\lambda) &= \det(L_0 + L_1 - \lambda U_{n \times n}) - \det(L_0 - \lambda U_{n \times n}) \\ &= \begin{vmatrix} 2 - \lambda & 0 \\ 1 & 1 - \lambda \end{vmatrix} = \lambda^2 - 3\lambda + 2. \\ \gamma(\lambda) &= \det(L_0 + L_1 + L_2 - \lambda U_{n \times n}) - \det(L_0 + L_1 - \lambda U_{n \times n}) \\ &= \begin{vmatrix} 3 - \lambda & 0 \\ 0 & 1 - \lambda \end{vmatrix} = \lambda^2 - 4\lambda + 3. \end{aligned}$$

Hence the characteristic polynomial of L is

$$\begin{aligned} \phi(\lambda) &= \alpha(\lambda) + [\beta(\lambda) - \alpha(\lambda)] P_1 + [\gamma(\lambda) - \beta(\lambda)] P_2 \\ &= \lambda^2 - 2\lambda + 2 - \lambda P_1 + (-\lambda + 1) P_2. \end{aligned}$$

Now, by Cayely-Hamilton theorem we have $\phi(\lambda) = 0$, we have,

$$\begin{aligned} L^2 - 2L + 2 - LP_1 + (-L + 1)P_2 &= 0 \\ (2 + P_2)LL^{-1} &= -L^2 + 2L + LP_1 + LP_2. \end{aligned}$$

This implies that,

$$\begin{aligned} L^{-1} &= \frac{1}{2 + P_2} [-L + (2 + P_1 + P_2)U_{n \times n}] \\ &= \frac{1}{2 + P_2} \begin{pmatrix} 1 & 1 - P_1 \\ -1 + P_2 & 1 + P_1 + P_2 \end{pmatrix} \\ &= \left(\frac{1}{2} - \frac{1}{6} P_2 \right) \begin{pmatrix} 1 & 1 - P_1 \\ -1 + P_2 & 1 + P_1 + P_2 \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2} - \frac{1}{6} P_2 & \frac{1}{2} - \frac{1}{2} P_1 \\ -\frac{1}{2} + \frac{1}{2} P_2 & \frac{1}{2} + \frac{1}{2} P_1 \end{pmatrix} \end{aligned}$$

5. Conclusion

In this work, the adjoint of symbolic 2-plithogenic square matrices was defined and the inverse of invertible symbolic 2-plithogenic square matrices was studied in terms of symbolic 2-plithogenic adjoint and symbolic 2-plithogenic determinant. Also, we have presented the concept of the characteristic polynomial of symbolic 2-plithogenic matrices and we have proved the symbolic 2-plithogenic version of Cayley-Hamilton theorem with many examples that clarify the validity of this work.

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