

Neutrosophic Sets and Systems

Volume 59 *Neutrosophic Sets and Systems*,
Vol. 59, 2023 - Special Issue on Symbolic
Plithogenic Algebraic Structures

Article 22

10-28-2023

On Clean and Nil-clean Symbolic 2-Plithogenic Rings

P. Prabakaran

Florentin Smarandache

Follow this and additional works at: https://digitalrepository.unm.edu/nss_journal

Recommended Citation

Prabakaran, P. and Florentin Smarandache. "On Clean and Nil-clean Symbolic 2-Plithogenic Rings." *Neutrosophic Sets and Systems* 59, 1 (2023). https://digitalrepository.unm.edu/nss_journal/vol59/iss1/22

This Article is brought to you for free and open access by UNM Digital Repository. It has been accepted for inclusion in *Neutrosophic Sets and Systems* by an authorized editor of UNM Digital Repository. For more information, please contact disc@unm.edu.



On Clean and Nil-clean Symbolic 2-Plithogenic Rings

P. Prabakaran¹, Florentin Smarandache²

¹Department of Mathematics, Bannari Amman Institute of Technology, Sathyamangalam-638401, Tamil Nadu, India; E-mail: prabakaranpvkr@gmail.com

²University of New Mexico, 705 Gurley Ave., Gallup, NM 87301, USA; E-mail: fsmarandache@gmail.com

*Correspondence: prabakaranpvkr@gmail.com

Abstract. A ring is said to be clean if every element of the ring can be written as a sum of an idempotent element and a unit element of the ring and a ring is said to be nil-clean if every element of the ring can be written as a sum of an idempotent element and a nilpotent element of the ring. In this paper, we generalize these arguments to symbolic 2-plithogenic structure. We introduce the structure of clean and nil-clean symbolic 2-plithogenic rings and some of its elementary properties are presented. Also, we have found the equivalence between classical clean(nil-clean) ring R and the corresponding symbolic 2-plithogenic ring $2 - SP_R$.

Keywords: Clean ring; nil-clean ring; symbolic 2-plithogenic ring; clean symbolic 2-plithogenic ring; nil-clean symbolic 2-plithogenic ring.

1. Introduction

The concept of refined neutrosophic structure was studied by many authors in [1–5]. Symbolic plithogenic algebraic structures are introduced by Smarandache, that are very similar to refined neutrosophic structures with some differences in the definition of the multiplication operation [17].

In [14], the algebraic properties of symbolic 2-plithogenic rings generated from the fusion of symbolic plithogenic sets with algebraic rings, and some of the elementary properties and sub-structures of symbolic 2-plithogenic rings such as AH-ideals, AH-homomorphisms, and AHS-isomorphisms are studied. In [7], some more algebraic properties of symbolic 2-plithogenic

P. Prabakaran and Florentin Smarandache, Solution of System of Symbolic 2-Plithogenic Linear Equations using Cramer's Rule

rings are studied. Further, Taffach [15, 16] studied the concepts of symbolic 2-plithogenic vector spaces and modules.

In [8], the concept of symbolic 2-plithogenic matrices with symbolic 2-plithogenic entries, determinants, eigen values and vectors, exponents, and diagonalization are studied. Hamiyet Merkepçi et.al [12], studied the the symbolic 2-plithogenic number theory and integers. Ahmad Khaldi et.al [11], studied the different types of algebraic symbolic 2-plithogenic equations and its solutions.

In [18], H. Suryoto and T. Uidjiani studied the concept of neutrosophic clean ring with many elementary interesting properties. Recently, M. Abobala [6], proved that a neutrosophic ring $R(I)$ is clean if and only if R is clean. Motivated by this works, in this paper we have introduced and studied the notion of clean and nil-clean symbolic 2-plithogenic rings. Also, we proved that a symbolic 2-plithogenic $2 - SP_R$ is clean(nil-clean) if and only if R is clean(nil-clean).

2. Preliminaries

Definition 2.1. [14] Let R be a ring, the symbolic 2-plithogenic ring is defined as follows:

$$2 - SP_R = \left\{ a_0 + a_1P_1 + a_2P_2; a_i \in R, P_j^2 = P_j, P_1 \times P_2 = P_{\max(1,2)} = P_2 \right\}$$

Smarandache has defined algebraic operations on $2 - SP_R$ as follows:

Addition:

$$[a_0 + a_1P_1 + a_2P_2] + [b_0 + b_1P_1 + b_2P_2] = (a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2$$

Multiplication:

$$[a_0 + a_1P_1 + a_2P_2] \cdot [b_0 + b_1P_1 + b_2P_2] = a_0b_0 + a_0b_1P_1 + a_0b_2P_2 + a_1b_0P_1^2 + a_1b_2P_1P_2 + a_2b_0P_2 + a_2b_1P_1P_2 + a_2b_2P_2^2 + a_1b_1P_1P_1 = (a_0b_0) + (a_0b_1 + a_1b_0 + a_1b_1)P_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)P_2.$$

It is clear that $2 - SP_R$ is a ring. If R is a field, then $2 - SP_R$ is called a symbolic 2-plithogenic field. Also, if R is commutative, then $2 - SP_R$ is commutative, and if R has a unity (1), then $2 - SP_R$ has the same unity (1).

Example 2.2. [14] Consider the ring $R = Z_4 = \{0, 1, 2, 3, 4\}$, the corresponding $2 - SP_R$ is:

$$2 - SP_R = \{a + bP_1 + cP_2; a, b, c \in Z_4\}.$$

If $X = 1 + 2P_1 + 3P_2, Y = P_1 + 2P_2$; then, $X + Y = 1 + 3P_1 + P_2, X - Y = 1 + P_1 + P_2, X \cdot Y = 3P_1 + 3P_2$.

Theorem 2.3. [14] Let $2 - SP_R$ be a 2-plithogenic symbolic ring, with unity (1). Let $X = x_0 + x_1P_1 + x_2P_2$ be an arbitrary element, then:

- (1) X is invertible if and only if $x_0, x_0 + x_1, x_0 + x_1 + x_2$ are invertible.

$$(2) X^{-1} = x_0^{-1} + [(x_0 + x_1)^{-1} - x_0^{-1}]P_1 + [(x_0 + x_1 + x_2)^{-1} - (x_0 + x_1)^{-1}]P_2$$

Definition 2.4. [14] Let $X = a + bP_1 + cP_2 \in 2 - SP_R$, then X is idempotent if and only if $X^2 = X$.

Theorem 2.5. [14] Let $X = a + bP_1 + cP_2 \in 2 - SP_R$, then X is idempotent if and only if $a, a + b, a + b + c$ are idempotent.

Theorem 2.6. [14] Let $2 - SP_R$ be a commutative symbolic 2-plithogenic ring, hence if $X = a + bP_1 + cP_2$, then $X^n = a^n + [(a + b)^n - a^n]P_1 + [(a + b + c)^n - (a + b)^n]P_2$ for every $n \in \mathbb{Z}^+$.

Definition 2.7. [14] X is called nilpotent if there exists $n \in \mathbb{Z}^+$ such that $X^n = 0$.

Theorem 2.8. [14] Let $X = a + bP_1 + cP_2 \in 2 - SP_R$, where R is commutative ring, then X is nilpotent if and only if $a, a + b, a + b + c$ are nilpotent.

3. Clean Symbolic 2-Plithogenic Rings

We begin with the following definition.

Definition 3.1. Let R be any ring, $2 - SP_R$ be its corresponding symbolic 2-plithogenic ring. An element $x \in 2 - SP_R$ is said to be clean if $x = e + u$, where e is an idempotent and u is a unit element of $2 - SP_R$. If, in addition, the existing idempotent e and the unit u are unique, then x is called uniquely clean element.

In this section, we use the notation $U(2 - SP_R)$ to the set of all units in $2 - SP_R$ and $Id(2 - SP_R)$ to the set of all idempotent elements in $2 - SP_R$.

Example 3.2. Consider the symbolic 2-plithogenic ring

$$\begin{aligned} 2 - SP_{Z_2} &= \{a + bP_1 + cP_2; a, b, c \in Z_2\} \\ &= \left\{0, 1, P_1, P_2, P_1 + P_2, 1 + P_1, 1 + P_2, 1 + P_1 + P_2\right\}. \end{aligned}$$

Here, $U(2 - SP_{Z_2}) = 1$ and $Id(2 - SP_{Z_2}) = \{0, 1, P_1, P_2, P_1 + P_2, 1 + P_1, 1 + P_2, 1 + P_1 + P_2\}$. We can easily verify that every element of $2 - SP_{Z_2}$ can be expressed as a sum of an idempotent and a unit in $2 - SP_{Z_2}$. Hence, all the elements in $2 - SP_{Z_2}$ are clean elements. Since 1 is the only unit element in $2 - SP_{Z_2}$, so all the elements in $2 - SP_{Z_2}$ are uniquely clean elements.

Definition 3.3. A symbolic 2-plithogenic ring in which all elements are clean, then the ring is called a clean symbolic 2-plithogenic ring. Furthermore, if each element of the symbolic 2-plithogenic ring is uniquely clean, then the ring is called a uniquely clean symbolic 2-plithogenic ring.

Example 3.4. By the Example 3.2, the ring $2-SP_{Z_2}$ is a uniquely clean symbolic 2-plithogenic ring.

Example 3.5. Consider the symbolic 2-plithogenic ring

$$2 - SP_{Z_3} = \{a + bP_1 + cP_2; a, b, c \in Z_3\}$$

$$= \left\{ \begin{array}{l} 0, 1, 2, P_1, P_2, 2P_1, 2P_2, P_1 + P_2, 2P_1 + 2P_2, P_1 + 2P_2, 2P_1 + P_2, 1 + P_1, \\ 1 + P_2, 1 + P_1 + P_2, 1 + 2P_1, 1 + 2P_2, 1 + 2P_1 + 2P_2, 1 + P_1 + 2P_2, \\ 1 + 2P_1 + P_2, 2 + P_1, 2 + P_2, 2 + P_1 + P_2, 2 + 2P_1, 2 + 2P_2, 2 + 2P_1 + 2P_2, \\ 2 + P_1 + 2P_2, 2 + 2P_1 + P_2 \end{array} \right\}.$$

Here, $U(2 - SP_{Z_3}) = \{1, 2, 1 + P_1, 1 + P_2, 2 + 2P_1, 2 + 2P_2, 1 + P_1 + 2P_2, 2 + 2P_1 + P_2\}$ and $Id(2 - SR_{Z_3}) = \{0, 1, P_1, P_2, P_1 + 2P_2, 1 + 2P_1, 1 + 2P_2, 1 + 2P_1 + P_2\}$. All the elements of $2 - SP_{Z_3}$ are clean elements. Hence $2 - SP_{Z_3}$ is a clean symbolic 2-plithogenic ring. Take $2 + 2P_1 + P_2 \in 2 - SP_{Z_3}$, clearly $2 + 2P_1 + P_2 = (1 + 2P_1 + P_2) + 1$ and also we have $2 + 2P_1 + P_2 = 0 + (2 + 2P_1 + P_2)$. Therefore $2 + 2P_1 + P_2$ is not a uniquely clean element in $2 - SP_{Z_3}$ and hence $2 - SP_{Z_3}$ is not a uniquely clean.

Lemma 3.6. *Let R be a ring. Then the class of clean symbolic 2-plithogenic rings is closed under homomorphic images.*

Proof. It is clear since the homomorphic image of an idempotent element in a symbolic 2-plithogenic ring is again an idempotent. \square

Theorem 3.7. *Let R be any ring, $2 - SP_R$ be its corresponding symbolic 2-plithogenic ring. $2 - SP_R$ is clean if and only if R is clean.*

Proof. Assume that $2 - SP_R$ is clean. Since R is a homomorphic image of $2 - SP_R$, so R is clean by Lemma 3.6.

Conversely, assume that R is clean, we must prove that $2 - SP_R$ is clean. Let $x = a + bP_1 + cP_2 \in 2 - SP_R$ then $a, a + b, a + b + c \in R$. Since R is clean we have $a = e_1 + u_1, a + b = e_2 + u_2, a + b + c = e_3 + u_3$, where e_i are idempotent elements and u_i are unit elements of R . Now,

$$\begin{aligned} x &= a + bP_1 + cP_2 \\ &= a + [(a + b) - a]P_1 + [(a + b + c) - (a + b)]P_2 \\ &= (e_1 + u_1) + [(e_2 + u_2) - (e_1 + u_1)]P_1 + [(e_3 + u_3) - (e_2 + u_2)]P_2 \\ &= (e_1 + u_1) + [(e_2 - e_1) + (u_2 - u_1)]P_1 + [(e_3 - e_2) + (u_3 - u_2)]P_2 \\ &= [e_1 + (e_2 - e_1)P_1 + (e_3 - e_2)P_2] + [u_1 + (u_2 - u_1)P_1 + (u_3 - u_2)P_2] \\ &= x_1 + x_2. \end{aligned}$$

where, $x_1 = e_1 + (e_2 - e_1)P_1 + (e_3 - e_2)P_2$ and $x_2 = u_1 + (u_2 - u_1)P_1 + (u_3 - u_2)P_2$. By Theorem 2.5, $e_1, e_1 + (e_2 - e_1) = e_2, e_1 + (e_2 - e_1) + (e_3 - e_2) = e_3$ are idempotents in R . Therefore, x_1 is a idempotent element of R . Also, x_2 is a unit element of R by a similar discussion. Hence $2 - SP_R$ is clean. \square

Definition 3.8. Let $2 - SP_R$ be a symbolic 2-plithogenic ring. An idempotent element $e \in 2 - SP_R$ is called a central idempotent if $e.x = x.e$ for every $x \in 2 - SP_R$. The set of all central idempotents of $2 - SP_R$ is denoted by $C(2 - SP_R)$.

Example 3.9. In the symbolic 2-plithogenic ring $2 - SP_{Z_3}$, we have

$$Id(2 - SP_{Z_3}) = \{0, 1, P_1, P_2, P_1 + 2P_2, 1 + 2P_1, 1 + 2P_2, 1 + 2P_1 + P_2\}.$$

As $2 - SP_{Z_3}$ is commutative so all the idempotents of $2 - SP_{Z_3}$ are central. Hence

$$C(2 - SP_{Z_3}) = \{0, 1, P_1, P_2, P_1 + 2P_2, 1 + 2P_1, 1 + 2P_2, 1 + 2P_1 + P_2\} = Id(2 - SP_{Z_3}).$$

Lemma 3.10. *If x is an idempotent element of $2 - SP_R$, then $1 - x$ is also an idempotent element of $2 - SP_R$, where 1 is the unit element of $2 - SP_R$.*

Proof. If x an idempotent element of $2 - SP_R$ then $x^2 = x$. But then, $(1 - x)^2 = 1 - 2x - x^2 = 1 - x$ and so $1 - x$ an idempotent element of $2 - SP_R$. \square

Lemma 3.11. *Let $2 - SP_R$ be a symbolic 2-plithogenic ring with the identity 1 . If $e \in C(2 - SP_R)$ then $1 - e \in C(2 - SP_R)$, where 1 is the unit element of $2 - SP_R$.*

Proof. Assume that $e \in C(2 - SP_R)$. For any $x \in 2 - SP_R$, we have $(1 - e).x = (1.x) - (e.x) = (x.1) - (x.e) = x(1 - e)$. Hence, $1 - e \in C(2 - SP_R)$. \square

Theorem 3.12. *In any symbolic 2-plithogenic ring $2 - SP_R$, every central idempotent is a uniquely clean element.*

Proof. Let $x \in C(2 - SP_R)$. Then we have, $x^2 = x$ and $x = (1 - x) + (2x - 1) = e + u$, where $e = 1 - x$ is an idempotent by Lemma 3.10 and $u = 2x - 1$ is a unit element by Lemma 3.11. Hence x is a clean element. Also, if $x.y = y.x$ we obtain $e + u = (e + u)^2 = e + 2eu + u^2$, so $u = 1 - 2e$. Hence $e = 1 - x$. Thus x is a uniquely clean element. \square

Theorem 3.13. *Every idempotent element in a uniquely clean 2-plithogenic ring is a central idempotent.*

Proof. Assume that $2 - SP_R$ is a uniquely clean 2-plithogenic ring. Let $e \in 2 - SP_R$ be an idempotent element and x be any element of $2 - SP_R$. Now, the element $e + (ex - exe)$ is an idempotent and $1 + (ex - exe)$ is a unit and $[e + (ex - exe)] + 1 = e + [1 + (ex - exe)]$. Since, $2 - SP_R$ is a uniquely clean 2-plithogenic ring we have $e + (ex - exe) = e$. Hence $ex = exe$ and $xe = exe$, so $ex = ex$ as required. \square

Definition 3.14. A symbolic 2-plithogenic ring $2 - SP_R$ is called a boolean symbolic 2-plithogenic ring if $x^2 = x$ for all $x \in 2 - SP_R$.

Example 3.15. In the symbolic 2-plithogenic ring $2 - SR_{Z_2}$, all the elements are idempotent so $2 - SR_{Z_2}$ is a boolean symbolic 2-plithogenic ring

For any boolean symbolic 2-plithogenic ring, we have the following result.

Theorem 3.16. *Every boolean symbolic 2-plithogenic ring is uniquely clean.*

Proof. If $2 - SP_R$ is a boolean symbolic 2-plithogenic ring, then $2 - SP_R = Id(2 - SP_R)$. Since boolean rings are abelian, we have $Id(2 - SP_R) = C(2 - SP_R)$. This implies that, $2 - SP_R = C(2 - SP_R)$. By Theorem 3.12, every element of the ring $2 - SP_R$ are uniquely clean. Hence $2 - SP_R$ is uniquely clean ring. \square

4. Nil-clean Symbolic 2-Plithogenic Rings

We begin with the following definition.

Definition 4.1. Let R be any ring, $2 - SP_R$ be its corresponding symbolic 2-plithogenic ring. An element $x \in 2 - SP_R$ is said to be nil-clean if $x = e + n$, where e is an idempotent and n is a nil-potent element of $2 - SP_R$. If, in addition, the existing idempotent element and nil-potent elements are unique, then x is called uniquely nil-clean element.

Example 4.2. Consider the symbolic 2-plithogenic ring

$$2 - SP_{Z_3} = \{a + bP_1 + cP_2; a, b, c \in Z_3\}$$

$$= \left\{ \begin{array}{l} 0, 1, 2, P_1, P_2, 2P_1, 2P_2, P_1 + P_2, 2P_1 + 2P_2, P_1 + 2P_2, 2P_1 + P_2, 1 + P_1, \\ 1 + P_2, 1 + P_1 + P_2, 1 + 2P_1, 1 + 2P_2, 1 + 2P_1 + 2P_2, 1 + P_1 + 2P_2, \\ 1 + 2P_1 + P_2, 2 + P_1, 2 + P_2, 2 + P_1 + P_2, 2 + 2P_1, 2 + 2P_2, 2 + 2P_1 + 2P_2, \\ 2 + P_1 + 2P_2, 2 + 2P_1 + P_2 \end{array} \right\}.$$

Since 0 is a nil-potent element in $2 - SP_{Z_3}$, so the idempotent elements $0, 1, P_1, P_2, P_1 + 2P_2, 1 + 2P_1, 1 + 2P_2, 1 + 2P_1 + P_2$ are nil-clean elements of $2 - SP_{Z_3}$. The only nilpotent elements of $2 - SP_{Z_3}$ is 0, so $0, 1, P_1, P_2, P_1 + 2P_2, 1 + 2P_1, 1 + 2P_2, 1 + 2P_1 + P_2$ are uniquely nil-clean elements of $2 - SP_{Z_3}$.

Definition 4.3. A symbolic 2-plithogenic ring in which all elements are nil-clean, then the ring is called a nil-clean symbolic 2-plithogenic ring. Furthermore, if each element of the symbolic 2-plithogenic ring is uniquely nil-clean, then the ring is called a uniquely nil-clean symbolic 2-plithogenic ring.

Example 4.4. $2 - SP_{Z_2} = \{0, 1, P_1, P_2, P_1 + P_2, 1 + P_1, 1 + P_2, 1 + P_1 + P_2\}$ is a nil-clean symbolic 2-plithogenic ring, that is because all the elements in $2 - SP_{Z_2}$ are idempotents and 0 is a nilpotent element in $2 - SP_{Z_2}$.

Lemma 4.5. *If x is a nilpotent element of $2 - SP_R$, then $1 + x$ is a unit in $2 - SP_R$.*

Proof. If x is a nilpotent element of $2 - SP_R$ then $x^k = 0$ for some $k > 0$. But then, $(1 + x)(1 - x + x^2 - x^3 + \dots + (-1)^{k-1}x^{k-1}) = 1$ and so $1 + x$ is unit in $2 - SP_R$. \square

Theorem 4.6. *Every nil-clean symbolic 2-plithogenic ring is clean symbolic 2-plithogenic ring.*

Proof. Suppose that $2 - SP_R$ is a nil-clean symbolic 2-plithogenic ring, and let $x \in 2 - SP_R$. Then $x - 1$ is an element of $2 - SP_R$ and hence $x - 1 = e + n$, where e is an idempotent element and n is a nilpotent element of $2 - SP_R$.

This implies that, $x = e + (1 + n)$ is a nil-clean element of $2 - SP_R$ because $1 + n$ is a unit element of $2 - SP_R$ by Lemma 4.5. \square

The converse of the Theorem 4.6 is not true. See the following example.

Example 4.7. Consider, the clean symbolic 2-plithogenic ring $2 - SP_{Z_3}$. All the elements of $2 - SP_{Z_3}$ are clean elements. The only nilpotent element of $2 - SP_{Z_3}$ is 0 and $P_1 + P_2$ is not an idempotent element in $2 - SP_{Z_3}$ so it is not nil-clean. Hence $2 - SP_{Z_3}$ is not a nil-clean ring.

Lemma 4.8. *Let R be a ring. Then the class of nil-clean symbolic 2-plithogenic rings is closed under homomorphic images.*

Proof. It is clear since the homomorphic image of a nil-potent element of a symbolic 2-plithogenic rings is again a nil-potent. \square

Theorem 4.9. *Let R be any ring, $2 - SP_R$ be its corresponding symbolic 2-plithogenic ring. $2 - SP_R$ is nil-clean if and only if R is nil-clean.*

Proof. Assume that $2 - SP_R$ is nil-clean. Since R is a homomorphic image of $2 - SP_R$, so R is nil-clean by Lemma 4.8.

Conversely, assume that R is nil-clean, we must prove that $2 - SP_R$ is nil-clean. Let $x = a + bP_1 + cP_2 \in 2 - SP_R$ then $a, a + b, a + b + c \in R$. Since R is nil-clean we have $a = e_1 + n_1, a + b = e_2 + n_2, a + b + c = e_3 + n_3$, where e_i are idempotent elements and n_i are nilpotent elements of R . Now,

$$\begin{aligned} x &= a + bP_1 + cP_2 \\ &= a + [(a + b) - a]P_1 + [(a + b + c) - (a + b)]P_2 \\ &= (e_1 + n_1) + [(e_2 + n_2) - (e_1 + n_1)]P_1 + [(e_3 + n_3) - (e_2 + n_2)]P_2 \\ &= (e_1 + n_1) + [(e_2 - e_1) + (n_2 - n_1)]P_1 + [(e_3 - e_2) + (n_3 - n_2)]P_2 \\ &= [e_1 + (e_2 - e_1)P_1 + (e_3 - e_2)P_2] + [n_1 + (n_2 - n_1)P_1 + (n_3 - n_2)P_2] \\ &= x_1 + x_2. \end{aligned}$$

where, $x_1 = e_1 + (e_2 - e_1)P_1 + (e_3 - e_2)P_2$ and $x_2 = n_1 + (n_2 - n_1)P_1 + (n_3 - n_2)P_2$. By Theorem 2.5, $e_1, e_1 + (e_2 - e_1) = e_2, e_1 + (e_2 - e_1) + (e_3 - e_2) = e_3$ are idempotents in R . Therefore, x_1 is a idempotent element of R . Also, x_2 is a nilpotent element of R by a similar discussion and by Theorem 2.8. Hence $2 - SP_R$ is nil-clean. \square

Theorem 4.10. *If $2 - SP_R$ is a symbolic 2-plithogenic ring, then every central idempotent of $2 - SP_R$ is uniquely nil-clean element.*

Proof. We know that, every idempotent element of $2 - SP_R$ are nil-clean. Let x be a central idempotent element of $2 - SP_R$. Then $x = (1 - x) + (2x - 1)$. Suppose that $x = e + n$, where e is an idempotent and n is a nilpotent element of $2 - SP_R$. Since $nx = xn$, we obtain $e + n = (e + n)^2 = e + 2en + n^2$. So, we have $n = 1 - 2e$ and hence $e = 1 - x$, as required. \square

Lemma 4.11. *Let $2 - SP_R$ be uniquely nil-clean symbolic 2-plithogenic ring. Then all idempotents of $2 - SP_R$ are central.*

Proof. Let $e \in 2 - SP_R$ be an idempotent element and x be any element of $2 - SP_R$. Now, the element $e + ex - exe$ can be written as $e + (ex - exe)$ or $(e + (ex - exe)) + 0$ each time as the sum of an idempotent and a nilpotent element of $2 - SP_R$. Since $2 - SP_R$ is uniquely nil clean, we have $e = e + (ex - exe)$. This implies that $ex - exe = 0$ and so $ex = exe$. In the similar way, we can show that $xe = exe$. Hence $ex = xe$ as required. \square

Theorem 4.12. *Every boolean symbolic 2-plithogenic ring is uniquely nil-clean.*

Proof. If $2 - SP_R$ is a boolean symbolic 2-plithogenic ring, then $2 - SP_R = Id(2 - SP_R)$. Since boolean rings are abelian, we have $Id(2 - SP_R) = C(2 - SP_R)$. This implies that, $2 - SP_R = C(2 - SP_R)$. By Theorem 4.10, every element of the ring $2 - SP_R$ are uniquely nil-clean. Hence $2 - SP_R$ is uniquely nil-clean ring. \square

5. Conclusion

In this article, we have introduced the the new classes of rings called, clean symbolic 2-plithogenic rings and nil-clean symbolic 2-plithogenic rings and we have studied various properties of clean and nil-clean symbolic 2-plithogenic rings with proper examples. Also, we have determined necessary and sufficient condition for a symbolic 2-plithogenic ring to be clean and nil-clean.

Funding: This research received no external funding.

References

1. Adeleke, E. O., Agboola, A. A. A., and Smarandache, F., "Refined neutrosophic rings II", International Journal of Neutrosophic Science, vol. 2, pp. 89-94, 2020.
2. Agboola, A. A. A., "On refined neutrosophic algebraic structures", Neutrosophic Sets and Systems, vol. 10, pp. 99?01, 2015.
3. Abobala, M., "On some special elements in neutrosophic rings and refined neutrosophic rings", Journal of New Theory, vol. 33, 2020.
4. Abobala, M., "On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations", Journal Of Mathematics, Hindawi, 2021.
5. Abobala, M., "On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations", Mathematical Problems in Engineering, Hindawi, 2021.
6. Abobala, M., Bal, M., and Hatip, A., "A Study Of Some Neutrosophic Clean Rings", International Journal of Neutrosophic Science (IJNS) Vol. 18, No. 01, pp. 14-19, 2022
7. Albasheer, O., Hajjari, A., and Dalla., R., "On The Symbolic 3-Plithogenic Rings and Their Algebraic Properties", Neutrosophic Sets and Systems, Vol 54, 2023.
8. Abuobida Mohammed A. Alfahal, Yaser Ahmad Alhasan, Raja Abdullah Abdulfatah, Arif Mehmood, Mustafa Talal Kadhim, "On Symbolic 2-Plithogenic Real Matrices and Their Algebraic Properties", International Journal of Neutrosophic Science, Vol. 21, PP. 96-104, 2023.
9. Ali, R., and Hasan, Z., "An Introduction To The Symbolic 3-Plithogenic Vector Spaces", Galoitica Journal Of Mathematical Structures and Applications, vol. 6, 2023.
10. Ben Othman, K., Von Shtawzen, O., Khaldi, A., and Ali, R., "On The Concept Of Symbolic 7-Plithogenic Real Matrices", Pure Mathematics For Theoretical Computer Science, Vol.1, 2023.
11. Khaldi, A., Ben Othman, K., Von Shtawzen, O., Ali, R., and Mosa, S., "On Some Algorithms for Solving Different Types of Symbolic 2-Plithogenic Algebraic Equations", Neutrosophic Sets and Systems, Vol 54, 2023.
12. Merkepçi, H., and Rawashdeh, A., "On The Symbolic 2-Plithogenic Number Theory and Integers", Neutrosophic Sets and Systems, Vol 54, 2023.

13. Merkepci, H., "On Novel Results about the Algebraic Properties of Symbolic 3-Plithogenic and 4-Plithogenic Real Square Matrices", *Symmetry*, MDPI, 2023.
14. Merkepci, H., and Abobala, M., "On The Symbolic 2-Plithogenic Rings", *International Journal of Neutrosophic Science*, 2023.
15. Taffach, N., "An Introduction to Symbolic 2-Plithogenic Vector Spaces Generated from The Fusion of Symbolic Plithogenic Sets and Vector Spaces", *Neutrosophic Sets and Systems*, Vol 54, 2023.
16. Taffach, N., and Ben Othman, K., "An Introduction to Symbolic 2-Plithogenic Modules Over Symbolic 2-Plithogenic Rings", *Neutrosophic Sets and Systems*, Vol 54, 2023.
17. Smarandache, F., "Introduction to the Symbolic Plithogenic Algebraic Structures (revisited)", *Neutrosophic Sets and Systems*, vol. 53, 2023.
18. Suryoto, H., and Uidjiani, T., "On Clean Neutrosophic Rings", *IOP Conference Series, Journal of Physics, Conf. Series* 1217, 2019.
19. Yaser Ahmad Alhasan, "Types of system of the neutrosophic linear equations and Cramer's rule", *Neutrosophic Sets and Systems*, vol. 45, 2021.

Received: 24/5/2023 / **Accepted:** 10/9/2023