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On The Symbolic 2-Plithogenic Split-Complex Real Square Matrices and Their Algebraic Properties

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Abstract:

The objective of this paper is to study for the first time the concept of square real matrices with symbolic 2-plithogenic split-complex entries. Many of their algebraic properties will be discussed and handled, where we find the formula of computing inverses, exponents, and powers of these matrices by building a ring isomorphism between the ring of split-complex symbolic 2-plithogenic matrices and the direct product of the symbolic 2-plithogenic matrices with itself. Also, we give the interested reader many related examples to clarify the validity of our work.

Keywords: split-complex number, symbolic 2-plithogenic number, split-complex 2-plithogenic matrix.

Introduction

The concept of symbolic 2-plithogenic rings is considered as a generalization of algebraic rings [1], and these rings were used by many authors to generalize classical algebraic structures such as vector space, modules, and functions into novel symbolic 2-plithogenic and 3-plithogenic versions, see [2-9].

Symbolic 2-plithogenic matrices and other types have been studied in [10], where many results were obtained such as diagonalizations, and eigenvalues.

Symbolic 2-plithogenic split-complex numbers were defined as a combination between split-complex numbers, and symbolic 2-plithogenic numbers, for more details about split-complex structures and their applications, see [11-13]. For similar results about neutrosophic matrices and n-plithogenic matrices check [14-19].

In this paper, we define symbolic 2-plithogenic split-complex matrices, and we present many elementary properties of these matrices, especially those are related to classical matrix theory and applications.

Main Discussion

Definition.

Let $A = (a_{ij})$ be a matrix, it is called symbolic 2-plithogenic split-complex if and only if:

$$a_{ij} = (a_{ij}^{(0)} + a_{ij}^{(1)}P_1 + a_{ij}^{(2)}P_2) + (b_{ij}^{(0)} + b_{ij}^{(1)}P_1 + b_{ij}^{(2)}P_2)J, \quad \text{where}$$

$$a_{ij}^{(k)}, b_{ij}^{(k)} \in R, J^2 = 1, P_i \times P_j = P_{\max(i,j)}, P_i^2 = P_i.$$

Example.

The matrix $A = \begin{pmatrix} 1 + 2P_1 + P_2 + J(1 - P_2) & (P_1 + P_2)J \\ (2 - P_2) + J(3 + P_1 + P_2) & 5 - (P_1 + 4P_2)J \end{pmatrix}$ is a 2×2 symbolic 2-plithogenic split-complex matrix.

$$A = \begin{pmatrix} 1 + 2P_1 + P_2 & 0 \\ 2 - P_2 & 5 \end{pmatrix} + J \begin{pmatrix} 1 - P_2 & P_1 + P_2 \\ 3 + P_1 + P_2 & -(P_1 + 4P_2) \end{pmatrix}$$

Remark.

Any symbolic 2-plithogenic split-complex matrix can be written as follows $A = T + KJ$; T, K are two symbolic 2-plithogenic real matrices.

Also, it can be written as follows:

$$A = (A_0 + A_1P_1 + A_2P_2) + J(B_0 + B_1P_1 + B_2P_2); A_i, B_i \text{ are classical square real matrices.}$$

The matrix presented in the previous example can be written as:

$$\begin{pmatrix} 1 & 0 \\ 2 & 5 \end{pmatrix} + P_1 \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} + P_2 \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix} + J \left[\begin{pmatrix} 1 & 0 \\ 3 & 0 \end{pmatrix} + P_1 \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} + P_2 \begin{pmatrix} -1 & 1 \\ 1 & -4 \end{pmatrix} \right]$$

Remark.

We denote the ring of all symbolic 2-plithogenic split-complex matrices by $2 - SP_{MS}$.

$(2 - SP_{MS}, +, \cdot)$ is a ring.

Definition.

Let $X = M_0 + N_0J, Y = M_1 + N_1J; M_0, N_0, M_1, N_1 \in 2 - P_M$, then:

$$X + Y = (M_0 + M_1) + (N_0 + N_1)J.$$

$$X.Y = (M_0M_1 + N_0N_1) + (M_0N_1 + N_0M_1)J.$$

Example.

Take:

$$X = \begin{pmatrix} 1 + P_1 & P_2 \\ P_1 & -P_1 + P_2 \end{pmatrix} + J \begin{pmatrix} 2 + P_1 - P_2 & 3P_1 \\ P_1 - P_2 & 2P_2 \end{pmatrix} = M_0 + N_0J$$

$$Y = \begin{pmatrix} 1 + P_1 - P_2 & 2 - P_2 \\ 1 + P_2 & 1 + P_1 \end{pmatrix} + J \begin{pmatrix} 2 - P_2 & P_1 - P_2 \\ 1 + P_1 & P_2 \end{pmatrix} = M_1 + N_1J$$

$$X + Y = \begin{pmatrix} 2 + 2P_1 - P_2 & 2 - P_1 + P_2 \\ 1 + P_1 + P_2 & 1 + P_2 \end{pmatrix} + J \begin{pmatrix} 4 + P_1 - 2P_2 & 4P_1 - P_2 \\ 1 + 2P_1 - P_2 & 3P_2 \end{pmatrix}$$

$$M_0M_1 = \begin{pmatrix} 1 + P_1 & P_2 \\ P_1 & -P_1 + P_2 \end{pmatrix} \begin{pmatrix} 1 + P_1 - P_2 & 2 - P_2 \\ 1 + P_2 & 1 + P_1 \end{pmatrix} = \begin{pmatrix} 1 + 3P_1 & 2 + 2P_2 \\ P_1 & -P_1 + 2P_2 \end{pmatrix}$$

$$N_0N_1 = \begin{pmatrix} 2 + P_1 - P_2 & 3P_1 \\ P_1 - P_2 & 2P_2 \end{pmatrix} \begin{pmatrix} 2 - P_2 & P_1 - P_2 \\ 1 + P_1 & P_2 \end{pmatrix} = \begin{pmatrix} 4 + 8P_1 - 4P_2 & 3P_1 \\ 2P_1 + 2P_2 & P_1 + P_2 \end{pmatrix}$$

$$M_0N_1 = \begin{pmatrix} 1 + P_1 & P_2 \\ P_1 & -P_1 + P_2 \end{pmatrix} \begin{pmatrix} 2 - P_2 & P_1 - P_2 \\ 1 + P_1 & P_2 \end{pmatrix} = \begin{pmatrix} 2 + 2P_1 & 2P_1 - P_2 \\ P_2 & P_1 - P_2 \end{pmatrix}$$

$$N_0M_1 = \begin{pmatrix} 2 + P_1 - P_2 & 3P_1 \\ P_1 - P_2 & 2P_2 \end{pmatrix} \begin{pmatrix} 1 + P_1 - P_2 & 2 - P_2 \\ 1 + P_2 & 1 + P_1 \end{pmatrix} = \begin{pmatrix} 1 + 7P_1 - P_2 & 4 + 5P_1 - P_2 \\ 2P_1 + 2P_2 & P_1 + 2P_2 \end{pmatrix}$$

So that:

$$X.Y = \begin{pmatrix} 5 + 11P_1 - 4P_2 & 2 + 3P_1 + 2P_2 \\ 3P_1 + 2P_2 & 3P_2 \end{pmatrix} + J \begin{pmatrix} 4 + 9P_1 - P_2 & 2 + 3P_1 + 2P_2 \\ 2P_1 + 3P_2 & 2P_1 + P_2 \end{pmatrix}$$

Theorem.

Let $2 - SP_{MS}$ be the ring of all $n \times n$ symbolic 2-plithogenic split-complex matrices, let $2 - P_M$ be the ring of all $n \times n$ square symbolic 2-plithogenic real matrices, then:

$$f: 2 - SP_{MS} \rightarrow 2 - P_M \times 2 - P_M$$

Such that:

$f(M + NJ) = (M + N, M - N)$; $M, N \in 2 - P_M$ is a ring isomorphism.

Proof.

For $M_0 + N_0J = M_1 + N_1J$, we have:

$$M_0 = M_1, N_0 = N_1, \text{ thus } (M_0 + N_0, M_0 - N_0) = (M_1 + N_1, M_1 - N_1),$$

hence $f(M_0 + N_0J) = f(M_1 + N_1J)$.

For $X = M_0 + N_0J, Y = M_1 + N_1J \in 2 - SP_{MS}$,

we have:

$$f(X + Y) = (M_0 + N_0 + M_1 + N_1, M_0 + M_1 - N_0 - N_1) = f(X) + f(Y).$$

$$f(X \cdot Y) = f[(M_0M_1 + N_0N_1) + (M_0N_1 + N_0M_1)J] = [(M_0 + N_0)(M_1 + N_1), (M_0 - N_0)(M_1 - N_1)] = f(X) \cdot f(Y).$$

$$f(X) = 0 \Leftrightarrow \begin{cases} M_0 + N_0 = 0 \\ M_0 - N_0 = 0 \end{cases} \Leftrightarrow M_0 = N_0 = 0$$

Thus, $\ker(f) = \{0\}$.

For any arbitrary element $(M, N) \in 2 - P_M \times 2 - P_M$, there exists $X = \frac{1}{2}(M + N) + \frac{1}{2}(M - N)J \in 2 - P_M$, such that $f(X) = (M, N)$, so that f is a ring isomorphism.

Remark.

The inverse isomorphism is:

$$f^{-1}: 2 - P_M \times 2 - P_M \rightarrow 2 - SP_{MS}; f^{-1}(M, N) = \frac{1}{2}(M + N) + \frac{1}{2}(M - N)J.$$

Example.

Consider:

$$X = \begin{pmatrix} 1 + P_1 + P_2 & 3 - P_2 & 1 + P_1 \\ P_1 - P_2 & 5 + P_1 & P_2 \\ P_1 + P_2 & 2P_1 & -P_2 \end{pmatrix} + J \begin{pmatrix} 1 + P_1 - P_2 & 1 & 1 \\ 4 + 2P_1 - P_2 & P_1 & 2P_2 \\ 2P_1 - P_2 & P_1 & 5P_1 \end{pmatrix} = M + NJ,$$

then:

$$f(X) = \left(\begin{pmatrix} 2 + 2P_1 & 4 - P_2 & 1 + P_1 \\ 4 + 3P_1 - 2P_2 & 5 + 2P_1 & 3P_2 \\ 2P_1 & 3P_1 & 5P_1 - P_2 \end{pmatrix}, \begin{pmatrix} 2P_2 & 2 - P_2 & -1 + P_1 \\ -4 - P_1 & 5 & -P_2 \\ -P_1 + 2P_2 & P_1 & -5P_1 - P_2 \end{pmatrix} \right)$$

Results from the isomorphism.

Let $X = M + NJ \in 2 - SP_M; M = M_0 + M_1P_1 + M_2P_2, N = N_0 + N_1P_1 + N_2P_2 \in 2 - P_M$,

then:

1). X is invertible if and only if $M + N, M - N$ are invertible, which is equivalent to:

$$M_0 + N_0, M_0 - N_0, (M_0 + M_1) + (N_0 + N_1), (M_0 + M_1) - (N_0 + N_1), \\ (M_0 + M_1 + M_2) + (N_0 + N_1 + N_2), (M_0 + M_1 + M_2) + (N_0 + N_1 - N_2),$$

Are invertible matrices.

2). If X is invertible, then:

$$X^{-1} = \frac{1}{2}[(M + N)^{-1} + (M - N)^{-1}] + \frac{1}{2}[(M + N)^{-1} - (M - N)^{-1}]J$$

Where:

$$(M + N)^{-1} = (M_0 + N_0)^{-1} + [(M_0 + M_1 + N_0 + N_1)^{-1} - (M_0 + N_0)^{-1}]P_1 \\ + [(M_0 + M_1 + M_2 + N_0 + N_1 + N_2)^{-1} - (M_0 + M_1 + N_0 + N_1)^{-1}]P_2 \\ (M - N)^{-1} = (M_0 - N_0)^{-1} + [(M_0 + M_1 - N_0 - N_1)^{-1} - (M_0 - N_0)^{-1}]P_1 + \\ [(M_0 + M_1 + M_2 - N_0 - N_1 - N_2)^{-1} - (M_0 + M_1 - N_0 - N_1)^{-1}]P_2.$$

$$3). \det X = \frac{1}{2}[\det(M + N) + \det(M - N)] + \frac{1}{2}[\det(M + N) - \det(M - N)]J$$

$$\det(M + N) = \det(M_0 + N_0) + [\det(M_0 + M_1 + N_0 + N_1) - \det(M_0 + N_0)]P_1 + \\ [\det(M_0 + M_1 + M_2 + N_0 + N_1 + N_2) - \det(M_0 + M_1 + N_0 + N_1)]P_2.$$

$$\det(M - N) = \det(M_0 - N_0) + [\det(M_0 + M_1 - N_0 - N_1) - \det(M_0 - N_0)]P_1 + \\ [\det(M_0 + M_1 + M_2 - N_0 - N_1 - N_2) - \det(M_0 + M_1 - N_0 - N_1)]P_2.$$

$$4). X^n = \frac{1}{2}[(M + N)^n + (M - N)^n] + \frac{1}{2}[(M + N)^n - (M - N)^n]J$$

$$(M + N)^n = (M_0 + N_0)^n + [(M_0 + M_1 + N_0 + N_1)^n - (M_0 + N_0)^n]P_1 + [(M_0 + M_1 + \\ M_2 + N_0 + N_1 + N_2)^n - (M_0 + M_1 + N_0 + N_1)^n]P_2.$$

$$(M - N)^n = (M_0 - N_0)^n + [(M_0 + M_1 - N_0 - N_1)^n - (M_0 - N_0)^n]P_1 + [(M_0 + M_1 + \\ M_2 - N_0 - N_1 - N_2)^n - (M_0 + M_1 - N_0 - N_1)^n]P_2.$$

Example.

Consider the following 2×2 symbolic 2-plithogenic split-complex matrix:

$$X = \begin{pmatrix} 2 + \frac{3}{2}P_1 + 4P_2 & \frac{1}{2} \\ \frac{3}{2}P_1 & \frac{3}{2} + \frac{1}{2}P_1 + \frac{1}{2}P_2 \end{pmatrix} + J \begin{pmatrix} 1 - \frac{7}{2}P_1 + 2P_2 & \frac{1}{2} - P_1 + P_2 \\ \frac{3}{2}P_1 - 3P_2 & \frac{1}{2} - \frac{3}{2}P_1 - \frac{1}{2}P_2 \end{pmatrix} = M +$$

NJ , where:

$$M = \begin{pmatrix} 2 + \frac{3}{2}P_1 + 4P_2 & \frac{1}{2} \\ \frac{3}{2}P_1 & \frac{3}{2} + \frac{1}{2}P_1 + \frac{1}{2}P_2 \end{pmatrix}, N = \begin{pmatrix} 1 - \frac{7}{2}P_1 + 2P_2 & \frac{1}{2} - P_1 + P_2 \\ \frac{3}{2}P_1 - 3P_2 & \frac{1}{2} - \frac{3}{2}P_1 - \frac{1}{2}P_2 \end{pmatrix}$$

$$M + N = \begin{pmatrix} 3 - P_1 + 6P_2 & 1 - P_1 + P_2 \\ 3P_1 - 3P_2 & 2 - P_1 \end{pmatrix}$$

$$M - N = \begin{pmatrix} 1 + 2P_1 + 2P_2 & P_1 - P_2 \\ 3P_2 & 1 + 2P_1 + P_2 \end{pmatrix}$$

We write:

$M + N = K_0 + K_1P_1 + K_2P_2$, $M - N = L_0 + L_1P_1 + L_2P_2$, where:

$$K_0 = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}, K_1 = \begin{pmatrix} -5 & -1 \\ 3 & -1 \end{pmatrix}, K_2 = \begin{pmatrix} 6 & 1 \\ -3 & 0 \end{pmatrix}, K_0 + K_1 = \begin{pmatrix} -2 & 0 \\ 3 & 1 \end{pmatrix}, K_0 + K_1 + K_2 \\ = \begin{pmatrix} 4 & 1 \\ 0 & 1 \end{pmatrix}$$

$$L_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, L_1 = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}, L_2 = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}, L_0 + L_1 = \begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}, L_0 + L_1 + L_2 = \begin{pmatrix} 5 & 0 \\ 3 & 4 \end{pmatrix}$$

$$\det(M + N) = \det(K_0) + [\det(K_0 + K_1) - \det(K_0)]P_1 + [\det(K_0 + K_1 + K_2) - \det(K_0 + K_1)]P_2 = 6 + (-2 - 6)P_1 + (4 + 2)P_2 = 6 - 8P_1 + 6P_2.$$

$$\det(M - N) = \det(L_0) + [\det(L_0 + L_1) - \det(L_0)]P_1 + [\det(L_0 + L_1 + L_2) - \det(L_0 + L_1)]P_2 = 1 + (9 - 1)P_1 + (20 - 9)P_2 = 1 + 8P_1 + 11P_2.$$

$$\det X = \frac{1}{2}[\det(M + N) + \det(M - N)] + \frac{1}{2}[\det(M + N) - \det(M - N)] \\ = \frac{1}{2}(7 + 17P_2) + \frac{1}{2}J(5 - 16P_1 - 5P_2) \\ = \left(\frac{7}{2} + \frac{17}{2}P_2\right) + J\left(\frac{5}{2} - 8P_1 - \frac{5}{2}P_2\right)$$

now, let's find the inverse of X :

$$K_0^{-1} = -\frac{1}{6}\begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{2} \end{pmatrix}, (K_0 + K_1)^{-1} = -\frac{1}{2}\begin{pmatrix} 1 & 0 \\ -3 & -2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & 0 \\ \frac{3}{2} & 1 \end{pmatrix}$$

$$(K_0 + K_1 + K_2)^{-1} = \frac{1}{4}\begin{pmatrix} 1 & -1 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ 0 & 1 \end{pmatrix}$$

$$L_0^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, (L_0 + L_1)^{-1} = \frac{1}{9}\begin{pmatrix} 3 & -1 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & -\frac{1}{9} \\ 0 & \frac{1}{3} \end{pmatrix}$$

$$(L_0 + L_1 + L_2)^{-1} = \frac{1}{20}\begin{pmatrix} 4 & 0 \\ -3 & 5 \end{pmatrix} = \begin{pmatrix} \frac{1}{5} & 0 \\ -\frac{3}{20} & \frac{1}{4} \end{pmatrix}$$

$$\begin{aligned}
 (M + N)^{-1} &= (K_0)^{-1} + [(K_0 + K_1)^{-1} - (K_0)^{-1}]P_1 \\
 &\quad + [(K_0 + K_1 + K_2)^{-1} - (K_0 + K_1)^{-1}]P_2 \\
 &= \begin{pmatrix} \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{2} \end{pmatrix} + \begin{pmatrix} \frac{-5}{6} & \frac{1}{6} \\ \frac{3}{2} & \frac{1}{2} \end{pmatrix} P_1 + \begin{pmatrix} \frac{3}{4} & \frac{-1}{4} \\ \frac{-3}{2} & 0 \end{pmatrix} P_2 \\
 &= \begin{pmatrix} \frac{1}{3} - \frac{5}{6}P_1 + \frac{3}{4}P_2 & \frac{1}{6} + \frac{1}{6}P_1 - \frac{1}{4}P_2 \\ \frac{3}{2}P_1 - \frac{3}{2}P_2 & \frac{1}{2} + \frac{1}{2}P_1 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 (M - N)^{-1} &= (L_0)^{-1} + [(L_0 + L_1)^{-1} - (L_0)^{-1}]P_1 + [(L_0 + L_1 + L_2)^{-1} - (L_0 + L_1)^{-1}]P_2 \\
 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \frac{-2}{3} & \frac{-1}{9} \\ 0 & \frac{-2}{3} \end{pmatrix} P_1 + \begin{pmatrix} \frac{-2}{15} & \frac{1}{9} \\ \frac{-3}{20} & \frac{-1}{12} \end{pmatrix} P_2 \\
 &= \begin{pmatrix} 1 - \frac{2}{3}P_1 - \frac{2}{15}P_2 & -\frac{1}{9}P_1 + \frac{1}{9}P_2 \\ -\frac{3}{20}P_2 & 1 - \frac{2}{3}P_1 - \frac{1}{12}P_2 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 X^{-1} &= \frac{1}{2}[(M + N)^{-1} + (M - N)^{-1}] + \frac{1}{2}[(M + N)^{-1} - (M - N)^{-1}]J \\
 &= \frac{1}{2} \begin{pmatrix} \frac{4}{3} - \frac{3}{2}P_1 + \frac{37}{60}P_2 & -\frac{1}{6} + \frac{1}{18}P_1 - \frac{5}{36}P_2 \\ \frac{3}{2}P_1 - \frac{33}{20}P_2 & \frac{3}{2} - \frac{1}{6}P_1 - \frac{1}{12}P_2 \end{pmatrix} \\
 &\quad + \frac{1}{2}J \begin{pmatrix} \frac{-3}{2} - \frac{1}{6}P_1 + \frac{53}{60}P_2 & -\frac{1}{6} + \frac{5}{18}P_1 - \frac{13}{36}P_2 \\ \frac{3}{2}P_1 - \frac{27}{20}P_2 & -\frac{1}{2} + \frac{7}{6}P_1 + \frac{1}{12}P_2 \end{pmatrix} \\
 &= \begin{pmatrix} \frac{2}{3} - \frac{3}{4}P_1 + \frac{37}{120}P_2 & -\frac{1}{12} + \frac{1}{36}P_1 - \frac{5}{72}P_2 \\ \frac{3}{4}P_1 - \frac{33}{40}P_2 & \frac{3}{4} - \frac{1}{12}P_1 - \frac{1}{24}P_2 \end{pmatrix} \\
 &\quad + J \begin{pmatrix} \frac{-1}{3} - \frac{1}{12}P_1 + \frac{53}{120}P_2 & -\frac{1}{12} + \frac{5}{36}P_1 - \frac{13}{72}P_2 \\ \frac{3}{4}P_1 - \frac{27}{40}P_2 & -\frac{1}{4} + \frac{7}{12}P_1 + \frac{1}{24}P_2 \end{pmatrix}
 \end{aligned}$$

As an additional application of the matrix ring isomorphism between $2 - SP_M$ and $2 - P_M \times 2 - P_M$ is to find all possible eigenvalues/vectors for the symbolic 2-plithogenic split-complex matrix.

To find the eigenvalue for a symbolic 2-plithogenic split-complex matrix, we must find all symbolic 2-plithogenic eigen values of $M + N, M - N$.

For the matrix defined in the previous example, we can see:

The eigenvalues of K_0 are $\{3,2\} = Q_0$.

The eigenvalues of $K_0 + K_1$ are $\{-2,1\} = Q_1$.

The eigenvalues of $K_0 + K_1 + K_2$ are $\{4,1\} = Q_2$.

The eigenvalues of $M + N$ are:

$$Q = \{a_0 + (a_1 - a_0)P_1 + (a_2 - a_1)P_2; a_i \in Q_i\} = \{3 - 5P_1 + 6P_2, 3 - 5P_1 + 3P_2, 3 - 2P_1 + 3P_2, 3 - 2P_1, 2 - 4P_1 + 6P_2, 2 - 4P_1 + 3P_2, 2 - P_1 + 3P_2, 2 - P_1\}.$$

The eigenvalue of L_0 is $\{1\} = s_0$.

The eigenvalue of $L_0 + L_1$ is $\{3\} = S_1$.

The eigenvalues of $L_0 + L_1 + L_2$ are $\{5,4\} = S_2$.

The eigenvalues of $M - N$ are:

$$S = \{b_0 + (b_1 - b_0)P_1 + (b_2 - b_1)P_2; b_i \in S_i\} = \{1 + 2P_1 + 2P_2, 1 + 2P_1 + P_2\}$$

The eigen values of the duplet $(M + N, M - N)$ are:

$$\{(3 - 5P_1 + 6P_2, 1 + 2P_1 + 2P_2), (3 - 5P_1 + 6P_2, 1 + 2P_1 + P_2), (3 - 5P_1 + 3P_2, 1 + 2P_1 + P_2), (3 - 5P_1 + 3P_2, 1 + 2P_1 + 2P_2), (3 - 2P_1 + 3P_2, 1 + 2P_1 + 2P_2), (3 - 2P_1 + 3P_2, 1 + 2P_1 + P_2), (2 - P_1, 1 + 2P_1 + 2P_2), (2 - P_1, 1 + 2P_1 + P_2), (2 - 4P_1 + 6P_2, 1 + 2P_1 + 2P_2), (2 - 4P_1 + 6P_2, 1 + 2P_1 + P_2), (2 - 4P_1 + 3P_2, 1 + 2P_1 + 2P_2), (2 - 4P_1 + 3P_2, 1 + 2P_1 + P_2), (2 - P_1 + 3P_2, 1 + 2P_1 + 2P_2), (2 - P_1 + 3P_2, 1 + 2P_1 + P_2), (3 - 2P_1, 1 + 2P_1 + 2P_2), (3 - 2P_1, 1 + 2P_1 + P_2)\}.$$

We put:

$$T_1 = (3 - 5P_1 + 6P_2, 1 + 2P_1 + 2P_2)$$

$$f^{-1}(T_1) = \frac{1}{2}(4 - 3P_1 + 8P_2) + \frac{1}{2}(2 - 7P_1 + 4P_2)J = \left(2 - \frac{3}{2}P_1 + 4P_2\right) + \left(1 - \frac{7}{2}P_1 + 2P_2\right)J.$$

$$T_2 = (3 - 5P_1 + 6P_2, 1 + 2P_1 + P_2)$$

$$f^{-1}(T_2) = \frac{1}{2}(4 - 3P_1 + 7P_2) + \frac{1}{2}(2 - 7P_1 + 5P_2)J = \left(2 - \frac{3}{2}P_1 + \frac{7}{2}P_2\right) + \left(1 - \frac{7}{2}P_1 + \frac{5}{2}P_2\right)J.$$

$$T_3 = (3 - 2P_1 + 3P_2, 1 + 2P_1 + 2P_2)$$

$$f^{-1}(T_3) = \frac{1}{2}(4 + 5P_2) + \frac{1}{2}(2 - 4P_1 + P_2)J = \left(2 + \frac{5}{2}P_2\right) + \left(1 - 2P_1 + \frac{1}{2}P_2\right)J.$$

$$T_4 = (3 - 2P_1 + 3P_2, 1 + 2P_1 + P_2)$$

$$f^{-1}(T_4) = \frac{1}{2}(4 + 4P_2) + \frac{1}{2}(2 - 4P_1 + 2P_2)J = (2 + 2P_2) + (1 - 2P_1 + P_2)J.$$

$$T_5 = (3 - 5P_1 + 3P_2, 1 + 2P_1 + 2P_2)$$

$$f^{-1}(T_5) = \frac{1}{2}(4 - 3P_1 + 5P_2) + \frac{1}{2}(2 - 7P_1 + P_2)J = \left(2 - \frac{3}{2}P_1 + \frac{5}{2}P_2\right) + \left(1 - \frac{7}{2}P_1 + \frac{1}{2}P_2\right)J.$$

$$T_6 = (3 - 5P_1 + 3P_2, 1 + 2P_1 + P_2)$$

$$f^{-1}(T_6) = \frac{1}{2}(4 - 3P_1 + 4P_2) + \frac{1}{2}(2 - 7P_1 + 2P_2)J = \left(2 - \frac{3}{2}P_1 + 2P_2\right) + \left(1 - \frac{7}{2}P_1 + P_2\right)J.$$

$$T_7 = (2 - P_1, 1 + 2P_1 + 2P_2)$$

$$f^{-1}(T_7) = \frac{1}{2}(3 + P_1 + 2P_2) + \frac{1}{2}(1 - 3P_1 - 2P_2)J = \left(\frac{3}{2} + \frac{1}{2}P_1 + P_2\right) + \left(\frac{1}{2} - \frac{3}{2}P_1 - P_2\right)J.$$

$$T_8 = (2 - P_1, 1 + 2P_1 + P_2)$$

$$f^{-1}(T_8) = \frac{1}{2}(3 + P_1 + P_2) + \frac{1}{2}(1 - 3P_1 - P_2)J = \left(\frac{3}{2} + \frac{1}{2}P_1 + \frac{1}{2}P_2\right) + \left(\frac{1}{2} - \frac{3}{2}P_1 - \frac{1}{2}P_2\right)J.$$

$$T_9 = (2 - 4P_1 + 6P_2, 1 + 2P_1 + 2P_2)$$

$$f^{-1}(T_9) = \frac{1}{2}(3 - 2P_1 + 8P_2) + \frac{1}{2}(1 - 6P_1 + 4P_2)J = \left(\frac{3}{2} - P_1 + 4P_2\right) + \left(\frac{1}{2} - 3P_1 + 2P_2\right)J.$$

$$T_{10} = (2 - 4P_1 + 6P_2, 1 + 2P_1 + P_2)$$

$$f^{-1}(T_{10}) = \frac{1}{2}(3 - 2P_1 + 7P_2) + \frac{1}{2}(1 - 6P_1 + 5P_2)J = \left(\frac{3}{2} - P_1 + \frac{7}{2}P_2\right) + \left(\frac{1}{2} - 3P_1 + \frac{5}{2}P_2\right)J.$$

$$T_{11} = (2 - 4P_1 + 6P_2, 1 + 2P_1 + 2P_2)$$

$$f^{-1}(T_{11}) = \frac{1}{2}(3 - 2P_1 + 5P_2) + \frac{1}{2}(1 - 6P_1 + P_2)J = \left(\frac{3}{2} - P_1 + \frac{5}{2}P_2\right) + \left(\frac{1}{2} - 3P_1 + \frac{1}{2}P_2\right)J.$$

$$T_{12} = (2 - 4P_1 + 3P_2, 1 + 2P_1 + P_2)$$

$$f^{-1}(T_{12}) = \frac{1}{2}(3 - 2P_1 + 4P_2) + \frac{1}{2}(1 - 6P_1 + 2P_2)J = \left(\frac{3}{2} - P_1 + 2P_2\right) + \left(\frac{1}{2} - 3P_1 + P_2\right)J.$$

$$T_{13} = (2 - P_1 + 3P_2, 1 + 2P_1 + 2P_2)$$

$$f^{-1}(T_{13}) = \frac{1}{2}(3 + P_1 + 5P_2) + \frac{1}{2}(1 - 3P_1 + P_2)J = \left(\frac{3}{2} + \frac{1}{2}P_1 + \frac{5}{2}P_2\right) + \left(\frac{1}{2} - \frac{3}{2}P_1 + \frac{1}{2}P_2\right)J.$$

$$T_{14} = (2 - P_1 + 3P_2, 1 + 2P_1 + P_2)$$

$$f^{-1}(T_{14}) = \frac{1}{2}(3 + P_1 + 4P_2) + \frac{1}{2}(1 - 3P_1 + 2P_2)J = \left(\frac{3}{2} + \frac{1}{2}P_1 + 2P_2\right) + \left(\frac{1}{2} - \frac{3}{2}P_1 + P_2\right)J.$$

$$T_{15} = (3 - 2P_1, 1 + 2P_1 + 2P_2)$$

$$f^{-1}(T_{15}) = \frac{1}{2}(4 + 2P_2) + \frac{1}{2}(2 - 4P_1 - 2P_2)J = (2 + P_2) + (1 - 2P_1 - P_2)J.$$

$$T_{16} = (3 - 2P_1, 1 + 2P_1 + P_2)$$

$$f^{-1}(T_{16}) = \frac{1}{2}(4 + P_2) + \frac{1}{2}(2 - 4P_1 - P_2)J = \left(2 + \frac{1}{2}P_2\right) + \left(1 - 2P_1 - \frac{1}{2}P_2\right)J.$$

Conclusion

In this paper, we studied for the first time the concept of square real matrices with symbolic 2-plithogenic split-complex entries, where we find the formula of computing inverses, exponents, and powers of these matrices by building a ring isomorphism between the ring of split-complex symbolic 2-plithogenic matrices and the direct product of the symbolic 2-plithogenic matrices with itself. Also, we give the interested reader many related examples to clarify the validity of our work.

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