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On Pythagoras Triples in Symbolic 3-Plithogenic Rings

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Abstract:

The objective of this paper is to find necessary and sufficient conditions for a symbolic 3-plithogenic triple

$(t_0 + t_1P_1 + t_2P_2 + t_3P_3, s_0 + s_1P_1 + s_2P_2 + s_3P_3, k_0 + k_1P_1 + k_2P_2 + k_3P_3)$ to be a Pythagoras triple, i.e. to be a solution for the non-linear Diophantine equation

$X^2 + Y^2 = Z^2$. Also, many examples will be illustrated and presented to explain how the theorems work.

Keywords: symbolic 3-plithogenic ring, Pythagoras triple, Pythagoras Diophantine equation

Introduction and Preliminaries.

Symbolic n-plithogenic sets were defined by Smarandache in [1-3], where these sets were used in generalizing classical algebraic structures such as symbolic 2-plithogenic and symbolic 3-plithogenic structures [4-9], with many applications in other fields [10-12].

It is useful to refer that symbolic n-plithogenic algebraic structures are very similar to neutrosophic and refined neutrosophic structures, see [13-21].

In this paper, we continue other efforts to study Pythagoras triples in many different rings [22-25].

We present the concept of Pythagoras triple in a symbolic 3-plithogenic commutative ring with many clear examples that clarify the validity of our work.

Definition.

Let R be a ring, the symbolic 3-plithogenic ring is defined as follows:

$$3 - SP_R = \{a_0 + a_1P_1 + a_2P_2 + a_3P_3; a_i \in R, P_j^2 = P_j, P_i \times P_j = P_{\max(i,j)}\}.$$

Smarandache has defined algebraic operations on $3 - SP_R$ as follows:

Addition:

$$[a_0 + a_1P_1 + a_2P_2 + a_3P_3] + [b_0 + b_1P_1 + b_2P_2 + b_3P_3] = (a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2 + (a_3 + b_3)P_3.$$

Multiplication:

$$[a_0 + a_1P_1 + a_2P_2 + a_3P_3] \cdot [b_0 + b_1P_1 + b_2P_2 + b_3P_3] = a_0b_0 + a_0b_1P_1 + a_0b_2P_2 + a_0b_3P_3 + a_1b_0P_1^2 + a_1b_2P_1P_2 + a_2b_0P_2 + a_2b_1P_1P_2 + a_2b_2P_2^2 + a_1b_3P_3P_1 + a_2b_3P_2P_3 + a_3b_3(P_3)^2 + a_3b_0P_3 + a_3b_1P_3P_1 + a_3b_2P_2P_3 + a_1b_1P_1P_1 = a_0b_0 + (a_0b_1 + a_1b_0 + a_1b_1)P_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)P_2 + (a_0b_3 + a_1b_3 + a_2b_3 + a_3b_3 + a_3b_0 + a_3b_1 + a_3b_2)P_3.$$

Main Discussion

Definition.

Let R be a ring, then (t, s, k) is called a Pythagoras triple if and only if

$$t^2 + s^2 = k^2; t, s, k \in R..$$

Theorem.

Let $T = t_0 + t_1P_1 + t_2P_2 + t_3P_3, S = s_0 + s_1P_1 + s_2P_2 + s_3P_3, K = k_0 + k_1P_1 + k_2P_2 + k_3P_3$ are three arbitrary symbolic 3-plithogenic elements $T, S, K \in 3 - SP_R$, then (T, S, K) are Pythagoras triple in $3 - SP_R$ if and only if:

$$\left\{ \begin{array}{l} (t_0, s_0, k_0), (t_0 + t_1, s_0 + s_1, k_0 + k_1) \text{ are pythagoras triples in } R \\ (t_0 + t_1 + t_2, s_0 + s_1 + s_2, k_0 + k_1 + k_2), (t_0 + t_1 + t_2 + t_3, s_0 + s_1 + s_2 + s_3, k_0 + k_1 + k_2 + k_3) \text{ are pythagoras triples in } R \end{array} \right.$$

Proof.

According to [], we have:

$$T^2 = t_0^2 + [(t_0 + t_1)^2 - t_0^2]P_1 + [(t_0 + t_1 + t_2)^2 - (t_0 + t_1)^2]P_2 + [(t_0 + t_1 + t_2 + t_3)^2 - (t_0 + t_1 + t_2)^2]P_3$$

$$S^2 = s_0^2 + [(s_0 + s_1)^2 - s_0^2]P_1 + [(s_0 + s_1 + s_2)^2 - (s_0 + s_1)^2]P_2 + [(s_0 + s_1 + s_2 + s_3)^2 - (s_0 + s_1 + s_2)^2]P_3$$

$$K^2 = k_0^2 + [(k_0 + k_1)^2 - k_0^2]P_1 + [(k_0 + k_1 + k_2)^2 - (k_0 + k_1)^2]P_2 + [(k_0 + k_1 + k_2 + k_3)^2 - (k_0 + k_1 + k_2)^2]P_3$$

The equation $T^2 + S^2 = K^2$ is equivalent to:

$$t_0^2 + s_0^2 = k_0^2 \text{ (equation 1),}$$

$$(t_0 + t_1)^2 + (s_0 + s_1)^2 = (k_0 + k_1)^2 \text{ (equation 2),}$$

$$(t_0 + t_1 + t_2)^2 + (s_0 + s_1 + s_2)^2 = (k_0 + k_1 + k_2)^2 \text{ (equation 3),}$$

$$(t_0 + t_1 + t_2 + t_3)^2 + (s_0 + s_1 + s_2 + s_3)^2 = (k_0 + k_1 + k_2 + k_3)^2 \text{ (equation 4)}$$

Equation (1) implies that (t_0, s_0, k_0) is a Pythagoras triple in R .

Equation (2) implies that $(t_0 + t_1, s_0 + s_1, k_0 + k_1)$ is a Pythagoras triple in R .

Equation (3) implies that $(t_0 + t_1 + t_2, s_0 + s_1 + s_2, k_0 + k_1 + k_2)$ is a Pythagoras triple in R .

Equation (4) implies that $(t_0 + t_1 + t_2 + t_3, s_0 + s_1 + s_2 + s_3, k_0 + k_1 + k_2 + k_3)$ is a Pythagoras triple in R .

Thus, the proof is complete.

Theorem.

Let $(t_0, s_0, k_0), (t_1, s_1, k_1), (t_2, s_2, k_2), (t_3, s_3, k_3)$ be four Pythagoras triples in the ring R , then (T, S, K) Pythagoras triple in $3 - SP_R$, where:

$$T = t_0 + [t_1 - t_0]P_1 + [t_2 - t_1]P_2 + [t_3 - t_2]P_3$$

$$S = s_0 + [s_1 - s_0]P_1 + [s_2 - s_1]P_2 + [s_3 - s_2]P_3$$

$$K = k_0 + [k_1 - k_0]P_1 + [k_2 - k_1]P_2 + [k_3 - k_2]P_3$$

Proof.

We have: $t_0 + (t_1 - t_0) = t_1, t_0 + (t_1 - t_0) + (t_2 - t_1) = t_2, t_0 + (t_1 - t_0) +$

$$(t_2 - t_1) + (t_3 - t_2) = t_3$$

$$s_0 + (s_1 - s_0) = s_1, s_0 + (s_1 - s_0) + (s_2 - s_1)$$

$$= s_2, s_0 + (s_1 - s_0) + (s_2 - s_1) + (s_3 - s_2) = s_3$$

$$k_0 + (k_1 - k_0) = k_1, k_0 + (k_1 - k_0) + (k_2 - k_1) = k_2, k_0 + (k_1 - k_0) + (k_2 - k_1) + (k_3 - k_2) = k_3$$

This implies that (T, S, K) Pythagoras triple in $3 - SP_R$ according to the theorem.

Examples.

We have:

$$\begin{cases} (t_0, s_0, k_0) = (3, 4, 5) \\ (t_1, s_1, k_1) = (6, 8, 10) \\ (t_2, s_2, k_2) = (4, 3, 5) \\ (t_3, s_3, k_3) = (5, 12, 13) \end{cases}$$

Are four Pythagoras triples in Z .

The corresponding symbolic 3-plithogenic Pythagoras triple is (T, S, K) , where:

$$T = 3 + [6 - 3]P_1 + [4 - 6]P_2 + [5 - 4]P_3 = 3 + 3P_1 - 2P_2 + P_3$$

$$S = 4 + [8 - 4]P_1 + [3 - 8]P_2 + [12 - 3]P_3 = 4 + 4P_1 - 5P_2 + 9P_3$$

$$K = 5 + [10 - 5]P_1 + [5 - 10]P_2 + [13 - 5]P_3 = 5 + 5P_1 - 5P_2 + 8P_3$$

Example.

Find all Pythagoras triples in $3 - SP_{Z_2}$, where Z_2 I the ring of integers module 2.

First, we find all Pythagoras triples in Z .

$$L_1 = (0, 0, 0), L_2 = (1, 0, 1), L_3 = (0, 1, 1), L_4 = (1, 1, 0)$$

Remark that for every permutation of the set $\{L_1, L_2, L_3, L_4\}$, we get a different symbolic 3-plithogenic Pythagoras triple.

We discuss all possible cases:

Permutation (1).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_1 = P_1 - P_2 + P_3 = P_1 + P_2 + P_3 \\ \dot{Y}_1 = P_2 \\ Y_1'' = P_1 - P_3 = P_1 + P_3 \end{cases}$$

Permutation (2).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_2 = P_2 \\ \dot{Y}_2 = P_1 - P_2 + P_3 = P_1 + P_2 + P_3 \\ Y_2'' = P_1 - P_3 = P_1 + P_3 \end{cases}$$

Permutation (3).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_3 = P_1 + P_3 \\ \dot{Y}_3 = P_1 + P_2 + P_3 \\ Y_3'' = P_2 \end{cases}$$

Permutation (4).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_4 = P_1 + P_2 + P_3 \\ \dot{Y}_4 = P_1 + P_3 \\ Y_4'' = P_2 \end{cases}$$

Permutation (5).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_5 = P_1 + P_3 \\ \dot{Y}_5 = P_2 \\ Y_5'' = P_1 + P_2 + P_3 \end{cases}$$

Permutation (6).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_6 = P_2 \\ \dot{Y}_6 = P_1 + P_3 \\ Y_6'' = P_1 + P_2 + P_3 \end{cases}$$

Permutation (7).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_7 = 1 + P_2 + P_3 \\ \dot{Y}_7 = P_1 \\ Y_7'' = 1 + P_1 + P_2 + P_3 \end{cases}$$

Permutation (8).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_8 = P_2 \\ \dot{Y}_8 = 1 + P_1 + P_2 \\ Y_8'' = 1 + P_1 \end{cases}$$

Permutation (9).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_9 = 1 + P_1 + P_3 \\ \dot{Y}_9 = P_1 + P_2 + P_3 \\ Y_9'' = 1 + P_2 \end{cases}$$

Permutation (10).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{10} = P_1 + P_2 + P_3 \\ \dot{Y}_{10} = 1 + P_1 + P_3 \\ Y_{10}'' = 1 + P_2 \end{cases}$$

Permutation (11).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{11} = 1 + P_2 \\ \dot{Y}_{11} = 1 + P_1 + P_3 \\ Y_{11}'' = P_1 + P_2 + P_3 \end{cases}$$

Permutation (12).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{12} = 1 + P_1 + P_2 + P_3 \\ \check{Y}_{12} = 1 + P_1 + P_3 \\ Y_{12}'' = P_2 \end{cases}$$

Permutation (13).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{13} = 1 + P_1 + P_2 + P_3 \\ \check{Y}_{13} = 1 + P_2 \\ Y_{13}'' = P_1 + P_3 \end{cases}$$

Permutation (14).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{14} = 1 + P_1 + P_3 \\ \check{Y}_{14} = 1 + P_2 \\ Y_{14}'' = P_1 + P_2 + P_3 \end{cases}$$

Permutation (15).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{15} = 1 + P_1 + P_3 \\ \check{Y}_{15} = 1 + P_1 + P_2 + P_3 \\ Y_{15}'' = P_2 \end{cases}$$

Permutation (16).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{16} = 1 + P_2 \\ \check{Y}_{16} = 1 + P_1 + P_2 + P_3 \\ Y_{16}'' = P_1 + P_3 \end{cases}$$

Permutation (17).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{17} = 1 + P_2 \\ \check{Y}_{17} = P_1 + P_2 + P_3 \\ Y_{17}'' = 1 + P_1 + P_3 \end{cases}$$

Permutation (18).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{18} = 1 + P_2 \\ \check{Y}_{18} = P_1 + P_3 \\ Y_{18}'' = 1 + P_1 + P_2 + P_3 \end{cases}$$

Permutation (19).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{19} = P_1 + P_3 \\ \check{Y}_{19} = 1 + P_1 + P_2 + P_3 \\ Y_{19}'' = 1 + P_2 \end{cases}$$

Permutation (20).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{20} = P_1 + P_3 \\ \check{Y}_{20} = 1 + P_1 + P_2 + P_3 \\ Y_{20}'' = 1 + P_2 \end{cases}$$

Permutation (21).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{21} = P_1 + P_2 + P_3 \\ Y'_{21} = 1 + P_2 + P_3 \\ Y''_{21} = 1 + P_1 \end{cases}$$

Permutation (22).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{22} = 1 + P_1 + P_3 \\ Y'_{22} = P_1 + P_2 + P_3 \\ Y''_{22} = 1 + P_2 \end{cases}$$

Permutation (23).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{23} = P_1 + P_3 \\ Y'_{23} = 1 + P_2 \\ Y''_{23} = 1 + P_1 + P_2 + P_3 \end{cases}$$

Permutation (24).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{24} = P_1 + P_2 + P_3 \\ Y'_{24} = 1 + P_1 + P_3 \\ Y''_{24} = 1 + P_2 \end{cases}$$

Also, other quadraples $(L_i, L_j, L_k, L_s); 1 \leq i, j, k, s \leq 4$ give Pythagoras triples with i, j, k, s are not distinct at all.

We continuo our discussions.

Permutation (25).

$$(L_1, L_1, L_1, L_1): \begin{cases} Y_{25} = (0,0,0) \\ Y'_{25} = (0,0,0) \\ Y''_{25} = (0,0,0) \end{cases}$$

Permutation (26).

$$(L_1, L_1, L_1, L_2): \begin{cases} Y_{26} = P_3 \\ Y'_{26} = 0 \\ Y''_{26} = P_3 \end{cases}$$

Permutation (27).

$$(L_1, L_1, L_1, L_3): \begin{cases} Y_{27} = 0 \\ Y'_{27} = P_3 \\ Y''_{27} = P_3 \end{cases}$$

Permutation (28).

$$(L_1, L_1, L_1, L_4): \begin{cases} Y_{28} = P_3 \\ Y'_{28} = P_3 \\ Y''_{28} = 0 \end{cases}$$

Permutation (29).

$$(L_1, L_2, L_1, L_1): \begin{cases} Y_{29} = P_1 + P_2 \\ Y'_{29} = 0 \\ Y''_{29} = P_1 + P_2 \end{cases}$$

Permutation (30).

$$(L_1, L_3, L_1, L_1): \begin{cases} Y_{30} = 0 \\ Y'_{30} = P_1 + P_2 \\ Y''_{30} = P_1 + P_2 \end{cases}$$

Permutation (31).

$$(L_1, L_4, L_1, L_1): \begin{cases} Y_{31} = P_1 + P_2 \\ Y'_{31} = P_1 + P_2 \\ Y''_{31} = 0 \end{cases}$$

Permutation (32).

$$(L_1, L_1, L_2, L_1): \begin{cases} Y_{32} = P_2 + P_3 \\ Y'_{32} = 0 \\ Y''_{32} = P_2 + P_3 \end{cases}$$

Permutation (33).

$$(L_1, L_1, L_3, L_1): \begin{cases} Y_{33} = 0 \\ Y'_{33} = P_2 + P_3 \\ Y''_{33} = P_2 + P_3 \end{cases}$$

Permutation (34).

$$(L_1, L_1, L_4, L_1): \begin{cases} Y_{34} = P_2 + P_3 \\ Y'_{34} = P_2 + P_3 \\ Y''_{34} = 0 \end{cases}$$

Permutation (35).

$$(L_2, L_2, L_2, L_2): \begin{cases} Y_{35} = 1 \\ Y'_{35} = 0 \\ Y''_{35} = 1 \end{cases}$$

Permutation (36).

$$(L_2, L_2, L_2, L_1): \begin{cases} Y_{36} = 1 + P_3 \\ Y'_{36} = 0 \\ Y''_{36} = 1 + P_3 \end{cases}$$

Permutation (37).

$$(L_2, L_2, L_2, L_1): \begin{cases} Y_{37} = 1 + P_3 \\ \check{Y}_{37} = P_3 \\ Y_{37}'' = 1 \end{cases}$$

Permutation (38).

$$(L_2, L_2, L_2, L_4): \begin{cases} Y_{38} = 1 \\ \check{Y}_{38} = P_3 \\ Y_{38}'' = 1 + P_3 \end{cases}$$

Permutation (39).

$$(L_2, L_2, L_1, L_2): \begin{cases} Y_{39} = 1 + P_2 + P_3 \\ \check{Y}_{39} = 0 \\ Y_{39}'' = 1 + P_2 + P_3 \end{cases}$$

Permutation (40).

$$(L_2, L_2, L_3, L_2): \begin{cases} Y_{40} = 1 + P_2 + P_3 \\ \check{Y}_{40} = P_2 + P_3 \\ Y_{40}'' = 1 \end{cases}$$

Permutation (41).

$$(L_2, L_2, L_4, L_2): \begin{cases} Y_{41} = 1 \\ \check{Y}_{41} = P_2 + P_3 \\ Y_{41}'' = 1 + P_2 + P_3 \end{cases}$$

Permutation (42).

$$(L_2, L_1, L_2, L_2): \begin{cases} Y_{42} = 1 + P_1 \\ \check{Y}_{42} = 0 \\ Y_{42}'' = 1 + P_1 \end{cases}$$

Permutation (43).

$$(L_2, L_4, L_2, L_2): \begin{cases} Y_{43} = 1 \\ \check{Y}_{43} = P_1 + P_2 \\ Y_{43}'' = 1 + P_2 + P_1 \end{cases}$$

Permutation (44).

$$(L_2, L_3, L_2, L_2): \begin{cases} Y_{44} = 1 + P_2 + P_3 \\ \check{Y}_{44} = P_1 + P_2 \\ Y_{44}'' = 1 + P_1 + P_3 \end{cases}$$

Permutation (45).

$$(L_3, L_3, L_3, L_3): \begin{cases} Y_{45} = 0 \\ \check{Y}_{45} = 1 \\ Y_{45}'' = 1 \end{cases}$$

Permutation (46).

$$(L_3, L_3, L_3, L_1): \begin{cases} Y_{46} = 0 \\ Y'_{46} = P_3 \\ Y''_{46} = P_3 \end{cases}$$

Permutation (47).

$$(L_3, L_3, L_3, L_2): \begin{cases} Y_{47} = P_3 \\ Y'_{47} = 1 + P_3 \\ Y''_{47} = 1 \end{cases}$$

Permutation (48).

$$(L_3, L_3, L_3, L_4): \begin{cases} Y_{48} = P_3 \\ Y'_{48} = 1 \\ Y''_{48} = 1 + P_3 \end{cases}$$

Permutation (49).

$$(L_3, L_3, L_1, L_3): \begin{cases} Y_{49} = 0 \\ Y'_{49} = 1 + P_2 + P_3 \\ Y''_{49} = 1 + P_2 + P_3 \end{cases}$$

Permutation (50).

$$(L_3, L_3, L_2, L_3): \begin{cases} Y_{50} = P_2 + P_3 \\ Y'_{50} = 1 + P_2 + P_3 \\ Y''_{50} = 1 \end{cases}$$

Permutation (51).

$$(L_3, L_3, L_4, L_3): \begin{cases} Y_{51} = P_2 + P_3 \\ Y'_{51} = 1 \\ Y''_{51} = 1 + P_2 + P_3 \end{cases}$$

Permutation (52).

$$(L_3, L_1, L_3, L_3): \begin{cases} Y_{52} = 0 \\ Y'_{52} = 1 + P_1 + P_2 \\ Y''_{52} = 1 + P_1 + P_2 \end{cases}$$

Permutation (53).

$$(L_3, L_2, L_3, L_3): \begin{cases} Y_{53} = P_1 + P_2 \\ Y'_{53} = 1 + P_1 + P_2 \\ Y''_{53} = 1 \end{cases}$$

Permutation (54).

$$(L_3, L_4, L_3, L_3): \begin{cases} Y_{54} = P_1 + P_2 \\ Y'_{54} = 1 \\ Y_{54}'' = 1 + P_1 + P_2 \end{cases}$$

Permutation (55).

$$(L_4, L_4, L_4, L_4): \begin{cases} Y_{55} = 1 \\ Y'_{55} = 1 \\ Y_{55}'' = 0 \end{cases}$$

Permutation (56).

$$(L_4, L_4, L_4, L_1): \begin{cases} Y_{56} = 1 + P_3 \\ Y'_{56} = 1 + P_3 \\ Y_{56}'' = 0 \end{cases}$$

Permutation (57).

$$(L_4, L_4, L_4, L_2): \begin{cases} Y_{57} = 1 \\ Y'_{57} = 1 + P_3 \\ Y_{57}'' = P_3 \end{cases}$$

Permutation (58).

$$(L_4, L_4, L_4, L_3): \begin{cases} Y_{58} = 1 + P_3 \\ Y'_{58} = 1 \\ Y_{58}'' = P_3 \end{cases}$$

Permutation (59).

$$(L_4, L_4, L_1, L_4): \begin{cases} Y_{59} = 1 + P_2 + P_3 \\ Y'_{59} = 1 + P_2 + P_3 \\ Y_{59}'' = 0 \end{cases}$$

Permutation (60).

$$(L_4, L_4, L_2, L_4): \begin{cases} Y_{60} = 1 \\ Y'_{60} = 1 + P_2 + P_3 \\ Y_{60}'' = P_2 + P_3 \end{cases}$$

Permutation (61).

$$(L_4, L_4, L_3, L_4): \begin{cases} Y_{61} = 1 + P_2 + P_3 \\ Y'_{61} = 1 \\ Y_{61}'' = P_2 + P_3 \end{cases}$$

Permutation (62).

$$(L_4, L_1, L_4, L_4): \begin{cases} Y_{62} = 1 + P_1 + P_2 \\ Y'_{62} = 1 + P_1 + P_2 \\ Y_{62}'' = 0 \end{cases}$$

Permutation (63).

$$(L_4, L_2, L_4, L_4): \begin{cases} Y_{63} = 1 \\ Y'_{63} = 1 + P_1 + P_2 \\ Y''_{63} = P_1 + P_2 \end{cases}$$

Permutation (64).

$$(L_4, L_3, L_4, L_4): \begin{cases} Y_{64} = 1 + P_1 + P_2 \\ Y'_{64} = 1 \\ Y''_{64} = P_1 + P_2 \end{cases}$$

By continuing this argument, we can get all Pythagoras triples in $3 - SP_{Z_2}$

Conclusion.

In this paper, we have studied Pythagoras triples in symbolic 3-plithogenic commutative rings, where necessary and sufficient conditions for a symbolic 3-plithogenic triple (x, y, z) to be a Pythagoras triple.

Also, we have presented some related examples that explain how to find 3-plithogenic triples from classical triples.

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