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# On The Dual Symbolic 2-Plithogenic Numbers

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## Abstract:

The objective of this paper is to combine dual numbers with symbolic 2-plithogenic numbers in one algebraic structure called dual symbolic 2-plithogenic real numbers.

Also, many elementary properties of the suggested system such as inverses and idempotents will be handled by many related theorems and examples.

**Keywords:** Symbolic 2-plithogenic number, dual number, dual symbolic 2-plithogenic number.

## Introduction and preliminaries.

Dual numbers are considered as a generalization of real numbers, where they are defined as follows:

$D = \{a + bt; t^2 = 0, a, b \in R\}$ [1]. Dual numbers make together a commutative ring with many interesting properties.

Addition on  $D$  is defined as follows:

$$(a_0 + b_0t) + (a_1 + b_1t) = (a_0 + a_1) + (b_0 + b_1)t$$

Multiplication on  $D$  is defined as follows:

$$(a_0 + b_0t) \cdot (a_1 + b_1t) = (a_0a_1) + (a_0b_1 + b_0a_1)t$$

In [2-4], Smarandache presented symbolic n-plithogenic sets, then they were used in generalizing many famous algebraic structures such as rings, matrices, and other structures [6-11].

We refer to many similar numerical systems that generalize real number, such as neutrosophic numbers, split-complex number, and weak fuzzy numbers [12-18].

We refer to some applications of these generalized number systems in matrix theory and cryptography [19-24].

Through this paper, we use symbolic 2-plithogenic real numbers to build a new generalization of real numbers, and we present some of its elementary algebraic properties.

### Main concepts.

#### Definition.

The set of symbolic 2-plithogenic dual numbers I defined as follows:

$$2 - SP_D = \{(x_0 + x_1t) + (y_0 + y_1t)P_1 + (z_0 + z_1t)P_2; x_i, y_i, z_i \in R, t^2 = 0\}$$

#### Definition.

Addition of  $2 - SP_D$  is defined:

$$\begin{aligned} & [(m_0 + m_1t) + (k_0 + k_1t)P_1 + (s_0 + s_1t)P_2] \\ & \quad + [(n_0 + n_1t) + (l_0 + l_1t)P_1 + (q_0 + q_1t)P_2] \\ & = (m_0 + n_0) + (m_1 + n_1)t + [(k_0 + l_0) + (k_1 + l_1)t]P_1 \\ & \quad + [(s_0 + q_0) + (s_1 + q_1)t]P_2 \end{aligned}$$

$(2 - SP_D, +)$  is an abelian group.

#### Remark.

A symbolic 2-plithogenic dual number  $X = (x_0 + x_1t) + (y_0 + y_1t)P_1 + (z_0 + z_1t)P_2$  can be written:

$$X = (x_0 + y_0P_1 + z_0P_2) + t(x_1 + y_1P_1 + z_1P_2)$$

#### Definition.

Let  $X = (x_0 + x_1P_1 + x_2P_2) + t(\acute{x}_0 + \acute{x}_1P_1 + \acute{x}_2P_2) = M_1 + M_2t$ ,

$Y = (y_0 + y_1P_1 + y_2P_2) + t(\acute{y}_0 + \acute{y}_1P_1 + \acute{y}_2P_2) = N_1 + N_2t \in 2 - SP_D$ ,

then:

Multiplication on  $2 - SP_D$  is defined as follows:

$$X.Y = M_1N_1 + t(M_1N_2 + N_1M_2)$$

**Example.**

Consider  $X = (1 + P_1 + P_2) + t(2 - P_2), Y = P_1 + t(1 - P_1)$ , we have:

$$X + Y = (1 + 2P_1 + P_2) + t(3 - P_1 - P_2)$$

$$X.Y = (1 + P_1 + P_2)P_1 + t[(1 + P_1 + P_2)(1 - P_1) + (2 - P_2)P_1] = (2P_1 + P_2) + t[(1 - P_1 + P_1 - P_1 + P_2 - P_2) + 2P_1 - P_1] = (2P_1 + P_2) + t(1 + P_1 - P_2).$$

**Remark.**

$(2 - SP_D, +, \cdot)$  Is a commutative ring.

**Invertibility:**

**Theorem.**

Let  $X = (m_0 + m_1P_1 + m_2P_2) + t(n_0 + n_1P_1 + n_2P_2) \in 2 - SP_D$ , then  $X$  is invertible if and only if  $m_0 \neq 0, m_0 + m_1 \neq 0, m_0 + m_1 + m_2 \neq 0$  and:

$$X^{-1} = \frac{1}{X} = \left[ \frac{1}{m_0} + \left( \frac{1}{m_0 + m_1} - \frac{1}{m_0} \right) P_1 + \left( \frac{1}{m_0 + m_1 + m_2} - \frac{1}{m_0 + m_1} \right) P_2 \right] - t \left[ \frac{n_0}{(m_0)^2} + \left( \frac{n_0 + n_1}{(m_0 + m_1)^2} - \frac{n_0}{(m_0)^2} \right) P_1 + \left( \frac{n_0 + n_1 + n_2}{(m_0 + m_1 + m_2)^2} - \frac{n_0 + n_1}{(m_0 + m_1)^2} \right) P_2 \right]$$

**Proof.**

$X$  is invertible if and only if  $\frac{1}{X}$  is defined as follows:

$$\begin{aligned} \frac{1}{X} &= \frac{1}{(m_0 + m_1P_1 + m_2P_2) + t(n_0 + n_1P_1 + n_2P_2)} \\ &= \frac{(m_0 + m_1P_1 + m_2P_2) - t(n_0 + n_1P_1 + n_2P_2)}{[(m_0 + m_1P_1 + m_2P_2) + t(n_0 + n_1P_1 + n_2P_2)][(m_0 + m_1P_1 + m_2P_2) - t(n_0 + n_1P_1 + n_2P_2)]} \\ &= \frac{(m_0 + m_1P_1 + m_2P_2) - t(n_0 + n_1P_1 + n_2P_2)}{(m_0 + m_1P_1 + m_2P_2)^2} \end{aligned}$$

So that  $m_0 + m_1P_1 + m_2P_2$  is invertible in  $2 - SP_R$ .

This is equivalent to  $m_0 \neq 0, m_0 + m_1 \neq 0, m_0 + m_1 + m_2 \neq 0$ .

On the other hand,  $\frac{1}{X} = \frac{1}{m_0 + m_1P_1 + m_2P_2} - t \frac{(n_0 + n_1P_1 + n_2P_2)^2}{(m_0 + m_1P_1 + m_2P_2)^2}$

Put  $Y = \frac{1}{m_0} + \left( \frac{1}{m_0 + m_1} - \frac{1}{m_0} \right) P_1 + \left( \frac{1}{m_0 + m_1 + m_2} - \frac{1}{m_0 + m_1} \right) P_2 - t \left[ \frac{n_0}{(m_0)^2} + \left( \frac{n_0 + n_1}{(m_0 + m_1)^2} - \frac{n_0}{(m_0)^2} \right) P_1 + \left( \frac{n_0 + n_1 + n_2}{(m_0 + m_1 + m_2)^2} - \frac{n_0 + n_1}{(m_0 + m_1)^2} \right) P_2 \right]$

Compute the result of  $XY$  as follows:

$$\begin{aligned}
 XY &= (m_0 + m_1P_1 + m_2P_2) \left[ \frac{1}{m_0} + \left( \frac{1}{m_0 + m_1} - \frac{1}{m_0} \right) P_1 \right. \\
 &\quad \left. + \left( \frac{1}{m_0 + m_1 + m_2} - \frac{1}{m_0 + m_1} \right) P_2 \right] \\
 &\quad + t \left[ -(m_0 + m_1P_1 + m_2P_2) \left[ \frac{n_0}{(m_0)^2} + \left( \frac{n_0 + n_1}{(m_0 + m_1)^2} - \frac{n_0}{(m_0)^2} \right) P_1 \right. \right. \\
 &\quad \left. \left. + \left( \frac{n_0 + n_1 + n_2}{(m_0 + m_1 + m_2)^2} - \frac{n_0 + n_1}{(m_0 + m_1)^2} \right) P_2 \right] \right] \\
 &\quad + (n_0 + n_1P_1 + n_2P_2) \left[ \frac{1}{m_0} + \left( \frac{1}{m_0 + m_1} - \frac{1}{m_0} \right) P_1 \right. \\
 &\quad \left. + \left( \frac{1}{m_0 + m_1 + m_2} - \frac{1}{m_0 + m_1} \right) P_2 \right] \\
 &= 1 \\
 &\quad + t \left[ \frac{-n_0}{m_0} + \left( -m_0 \frac{(n_0 + n_1)}{m_0 + m_1} + \frac{n_0}{m_0} - \frac{m_1 n_0}{(m_0)^2} - \frac{m_1(n_0 + n_1)}{(m_0 + m_1)^2} + \frac{m_1 n_0}{(m_0)^2} \right) P_1 \right. \\
 &\quad + \left( -m_0 \frac{(n_0 + n_1 + n_2)}{(m_0 + m_1 + m_2)^2} + \frac{m_0(n_0 + n_1)}{(m_0 + m_1)^2} - m_1 \frac{(n_0 + n_1 + n_2)}{(m_0 + m_1 + m_2)^2} \right. \\
 &\quad \left. + \frac{m_1(n_0 + n_1)}{(m_0 + m_1)^2} - \frac{m_2 n_0}{(m_0)^2} - \frac{m_2(n_0 + n_1)}{(m_0 + m_1)^2} + \frac{m_2 n_0}{(m_0)^2} - m_2 \frac{(n_0 + n_1 + n_2)}{(m_0 + m_1 + m_2)^2} \right. \\
 &\quad \left. + \frac{m_2(n_0 + n_1)}{(m_0 + m_1)^2} \right) P_2 \left] + \frac{n_0}{m_0} + \left( \frac{n_0}{m_0 + m_1} - \frac{n_0}{m_0} + \frac{n_1}{m_0} + \frac{n_1}{m_0 + m_1} - \frac{n_1}{m_0} \right) P_1 \\
 &\quad + \left( \frac{n_0}{m_0 + m_1 + m_2} - \frac{n_0}{m_0 + m_1} + \frac{n_1}{m_0 + m_1 + m_2} - \frac{n_1}{m_0 + m_1} + \frac{n_2}{m_0} \right. \\
 &\quad \left. + \frac{n_2}{m_0 + m_1 + m_2} - \frac{n_2}{m_0} + \frac{n_2}{m_0 + m_1 + m_2} - \frac{n_2}{m_0 + m_1} \right) P_2 = 1
 \end{aligned}$$

So that,  $X^{-1} = \frac{1}{X} = Y$

**Example.**

Take  $X = (1 + P_1 + P_2) + t(2 + P_1 - P_2) \in 2 - SP_D$ :

$$\begin{aligned}
 X^{-1} &= \frac{1}{1} + \left( \frac{1}{2} - 1 \right) P_1 + \left( \frac{1}{3} - \frac{1}{2} \right) P_2 - t \left[ \frac{2}{1} + \left( \frac{3}{4} - \frac{2}{1} \right) P_1 + \left( \frac{2}{9} - \frac{3}{4} \right) P_2 \right] \\
 &= 1 - \frac{1}{2} P_1 - \frac{1}{6} P_2 - t \left( 2 - \frac{5}{4} P_1 - \frac{21}{36} P_2 \right)
 \end{aligned}$$

**Natural power.**

**Theorem.**

Let  $X = (m_0 + m_1P_1 + m_2P_2) + t(n_0 + n_1P_1 + n_2P_2) \in 2 - SP_D$ , then:

$$X^n = (m_0)^n + ((m_0 + m_1)^n - (m_0)^n)P_1 + ((m_0 + m_1 + m_2)^n - (m_0 + m_1)^n)P_2 + n(n_0 + n_1P_1 + n_2P_2)[(m_0)^{n-1} + ((m_0 + m_1)^{n-1} - (m_0)^{n-1})P_1 + ((m_0 + m_1 + m_2)^{n-1} - (m_0 + m_1)^{n-1})P_2] \text{ for } n \in N.$$

**Proof.**

Let  $X = A + Bt; A, B \in 2 - SP_D$ , then:

$$A^n = A^n + nA^{n-1}Bt, \text{ we get:}$$

$$A^n = (m_0)^n + ((m_0 + m_1)^n - (m_0)^n)P_1 + ((m_0 + m_1 + m_2)^n - (m_0 + m_1)^n)P_2, \text{ then the proof holds.}$$

**Example.**

$$\text{Take } X = (1 + P_1 + P_2) + t(2 - P_1 + P_2) \in 2 - SP_D$$

$$X^3 = 1 + (8 - 1)P_1 + (1 - 8)P_2 + 3t(2 - P_1 + P_2)[1 + (4 - 1)P_1 + (1 - 4)P_2] = 1 + 7P_1 - 7P_2 + 3t[(2 - P_1 + P_2)(1 + 3P_1 - 3P_2)] = 1 + 7P_1 - 7P_2 + t(6 + 6P_1 - 6P_2).$$

**Idempotency.**

**Definition.**

Let  $X \in 2 - SP_D$ , then  $X$  is called idempotent if and only if  $X^2 = X$ .

**Theorem.**

Let  $X = (m_0 + m_1P_1 + m_2P_2) + t(n_0 + n_1P_1 + n_2P_2) \in 2 - SP_D$ , then  $X$  is called idempotent if and only if:

1.  $m_0 + m_1P_1 + m_2P_2$  is idempotent.
2.  $(n_0 + n_1P_1 + n_2P_2)[2m_0 - 1 + 2m_1P_1 + 2m_2P_2] = 0$

**Proof.**

$X = M + Nt$  is idempotent if and only if:

$$X^2 = X \Rightarrow \begin{cases} M^2 = M \\ 2MN = N \Rightarrow N(2M - 1) = 0 \end{cases}$$

For  $M = m_0 + m_1P_1 + m_2P_2, N = n_0 + n_1P_1 + n_2P_2 \in 2 - SP_R$ .

This implies the proof.

**Remark.**

The idempotency of  $M$  is equivalent to  $m_0^2 = m_0, (m_0 + m_1)^2 = (m_0 + m_1), (m_0 + m_1 + m_2)^2 = (m_0 + m_1 + m_2)$  , hence  $m_0, m_0 + m_1, m_0 + m_1 + m_2 \in \{0,1\}$ .

The equation  $N(2M - 1) = 0$  means that  $N, 2M - 1$  are zero divisors in  $2 - SP_R$ , so they should not invertible in  $2 - SP_R$ .

Now, let's compute the result of  $N(2M - 1) = 0$ .

$N(2M - 1) = (n_0 + n_1P_1 + n_2P_2)(2m_0 - 1 + m_1P_1 + m_2P_2) = 0$ , thus:

$$\begin{cases} n_0(2m_0 - 1) = 0 & (1) \\ (n_0 + n_1)(2m_0 - 1 + 2m_1) = 0 & (2) \\ (n_0 + n_1 + n_2)(2m_0 - 1 + 2m_1 + 2m_2) = 0 & (3) \end{cases}$$

Since  $m_0 \in \{0,1\}$ , then  $2m_0 - 1 \neq 0$  and  $n_0 = 0$ .

Equation (2) has two possible cases:

$$\begin{cases} n_1 = 0 \\ or \\ 2m_0 + 2m_1 = 1 \end{cases}$$

Equation (3) has two possible cases:

$$\begin{cases} n_1 + n_1 = 0 \\ or \\ 2m_0 + 2m_1 + 2m_2 = 1 \end{cases}$$

We discuss all possible cases.

**Case1.**

$m_0 = 0, m_0 + m_1 = 0, m_0 + m_1 + m_2 = 0$ , thus  $m_1 = m_2 = 0, n_0 = 0, n_1 = 0, n_2 = 0$ , thus  $X = 0$ .

**Case2.**

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{ then } m_1 = 1, m_2 = -1, n_0 = 0, n_1 = n_2 = 0$$

thus  $X = P_1 - P_2$

**Case3.**

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{ then } m_1 = 0, m_2 = 1, n_0 = 0, n_1 = n_2 = 0$$

thus  $X = P_2$

**Case4.**

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{ then } m_1 = 1, m_2 = 0, n_0 = 0, n_1 = n_2 = 0$$

thus  $X = P_1$

**Case5.**

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = 0 \end{cases}, \text{ then } m_1 = -1, m_2 = 0, n_0 = 0, n_1 = n_2 = 0$$

thus  $X = 1 - P_1$

**Case6.**

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 0 \end{cases}, \text{ then } m_1 = 0, m_2 = -1, n_0 = 0, n_1 = n_2 = 0$$

thus  $X = 1 - P_2$

**Case7.**

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{ then } m_1 = -1, m_2 = 1, n_0 = 0, n_1 = n_2 = 0$$

thus  $X = 1 - P_1 + P_2$

**Case8.**

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{ then } m_1 = 0, m_2 = 0, n_0 = 0, n_1 = n_2 = 0$$

thus  $X = 1$

**remark.**

The idempotent in  $2 - SP_D$  are:

$$\{0, 1, 1 - P_1 + P_2, 1 - P_2, 1 - P_1, P_1, P_2, P_1 - P_2\}$$

**Definition.**

Let  $X = M + Nt \in 2 - SP_D$ , we say that  $X$  is 3-potent element if and only if  $X^3 = X$ .

**Remark.**

$X$  is 3-potent if and only if  $X^3 = X$  which is equivalent to:

$$\begin{cases} M^3 = M \\ 3M^2N = N \Rightarrow N(3M^2 - 1) = 0 \end{cases}$$

$M^3 = M$  implies:



$$\begin{cases} m_0^3 = m_0 \\ (m_0 + m_1)^3 = m_0 + m_1 \\ (m_0 + m_1 + m_2)^3 = m_0 + m_1 + m_2 \end{cases}$$

So that  $m_0, m_0 + m_1, m_0 + m_1 + m_2 \in \{0, 1 - 1\}$ .

$N(3M^2 - 1) = 0$  implies:

$$\begin{cases} n_0(3m_0 - 1) = 0 & (1) \\ (n_0 + n_1)(3m_0 - 1 + 3m_1) = 0 & (2) \\ (n_0 + n_1 + n_2)(3m_0 - 1 + 3m_1 + 3m_2) = 0 & (3) \end{cases}$$

Equation (1) means that  $n_0 = 0$ , that is because  $m_0 \neq \frac{1}{3}$ .

Equation (2) means that:

$$\begin{cases} n_0 + n_1 = 0 \\ \text{or} \\ 3m_0 + 3m_1 = 1 \end{cases}, \text{ since } m_0 + m_1 \text{ is integer, then } n_1 = 0.$$

Equation (3) means that:

$$\begin{cases} n_0 + n_1 + n_2 = 0 \\ \text{or} \\ 3m_0 + 3m_1 + 3m_2 = 1 \end{cases}, \text{ since } m_0 + m_1 + m_2 \text{ is integer, then } n_2 = 0.$$

This implies that:

**Case1.**

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = 0 \end{cases}, \text{ then } X = 0.$$

**Case2.**

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 0 \end{cases}, \text{ then } X = P_1 - P_2$$

**Case3.**

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = -1 \\ m_0 + m_1 + m_2 = 0 \end{cases}, \text{ then } X = -P_1 + P_2$$

**Case4.**

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{ then } X = P_2$$

**Case5.**

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = -1 \end{cases}, \text{ then } X = -P_2$$

**Case6.**

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{ then } X = P_1$$

**Case7.**

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = -1 \end{cases}, \text{ then } X = P_1 - 2P_2$$

**Case8.**

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = -1 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{ then } X = -P_1 + 2P_2$$

**Case9.**

$$\begin{cases} m_0 = 0 \\ m_0 + m_1 = -1 \\ m_0 + m_1 + m_2 = -1 \end{cases}, \text{ then } X = -P_1$$

**Case10.**

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = 0 \end{cases}, \text{ then } X = 1 - P_1$$

**Case11.**

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 0 \end{cases}, \text{ then } X = 1 - P_2$$

**Case12.**

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = -1 \\ m_0 + m_1 + m_2 = 0 \end{cases}, \text{ then } X = 1 - 2P_1 + P_2$$

**Case13.**

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{ then } X = 1 - P_1 + P_2$$

**Case14.**

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = -1 \end{cases}, \text{ then } X = 1 - P_1 - P_2$$

**Case15.**

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{ then } X = 1$$

**Case16.**

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = -1 \end{cases}, \text{ then } X = 1 - 2P_2$$

**Case17.**

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = -1 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{ then } X = 1 - 2P_1 - 2P_2$$

**Case18.**

$$\begin{cases} m_0 = 1 \\ m_0 + m_1 = -1 \\ m_0 + m_1 + m_2 = -1 \end{cases}, \text{ then } X = 1 - 2P_1$$

**Case19.**

$$\begin{cases} m_0 = -1 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = 0 \end{cases}, \text{ then } X = -1 + P_1$$

**Case20.**

$$\begin{cases} m_0 = -1 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 0 \end{cases}, \text{ then } X = -1 + 2P_1 - P_2$$

**Case21.**

$$\begin{cases} m_0 = -1 \\ m_0 + m_1 = -1 \\ m_0 + m_1 + m_2 = 0 \end{cases}, \text{ then } X = -1 + P_2$$

**Case22.**

$$\begin{cases} m_0 = -1 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{ then } X = -1 + P_1 + P_2$$

**Case23.**

$$\begin{cases} m_0 = -1 \\ m_0 + m_1 = 0 \\ m_0 + m_1 + m_2 = -1 \end{cases}, \text{ then } X = -1 + P_1 - P_2$$

**Case24.**

$$\begin{cases} m_0 = -1 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{ then } X = -1 + 2P_1$$

**Case25.**

$$\begin{cases} m_0 = -1 \\ m_0 + m_1 = -1 \\ m_0 + m_1 + m_2 = -1 \end{cases}, \text{ then } X = -1$$

**Case26.**

$$\begin{cases} m_0 = -1 \\ m_0 + m_1 = 1 \\ m_0 + m_1 + m_2 = -1 \end{cases}, \text{ then } X = -1 + 2P_1 - 2P_2$$

**Case27.**

$$\begin{cases} m_0 = -1 \\ m_0 + m_1 = -1 \\ m_0 + m_1 + m_2 = 1 \end{cases}, \text{ then } X = -1 + 2P_2$$

## Conclusion

In this paper, we have studied for the first time the combination of symbolic 2-plithogenic numbers with dual numbers. The novel algebraic structure generated by them is called dual symbolic 2-plithogenic numbers.

We have determined the invertibility condition and the formula of the inverse for dual symbolic 2-plithogenic numbers. Also, all idempotent elements in the ring of dual symbolic 2-plithogenic numbers were presented and computed.

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