

# Neutrosophic Sets and Systems

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Volume 59 *Neutrosophic Sets and Systems*,  
Vol. 59, 2023 - Special Issue on Symbolic  
*Plithogenic Algebraic Structures*

Article 12

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10-28-2023

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### Recommended Citation

Alfahal, Abuobida Mohammed A.; Yaser Ahmad Alhasan; Raja Abdullah Abdulfatah; and Sara Sawalmeh. "On the Conditions for Symbolic 3-Plithogenic Pythagoras Quadruples." *Neutrosophic Sets and Systems* 59, 1 (2023). [https://digitalrepository.unm.edu/nss\\_journal/vol59/iss1/12](https://digitalrepository.unm.edu/nss_journal/vol59/iss1/12)

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# On The Conditions for Symbolic 3-Plithogenic Pythagoras Quadruples

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**Abstract:** The objective of this paper is to find the necessary and sufficient conditions for a symbolic 3-plithogenic quadruple

$(t_0 + t_1P_1 + t_2P_2 + t_3P_3, s_0 + s_1P_1 + s_2P_2 + s_3P_3, k_0 + k_1P_1 + k_2P_2 + k_3P_3, l_0 + l_1P_1 + l_2P_2 + l_3P_3)$  to be a Pythagoras quadruple, i.e. to be a solution for the non-linear Diophantine equation in four variables  $X^2 + Y^2 + Z^2 = T^2$ .

Also, many examples will be illustrated and presented to explain how the theorems work.

**Keywords:** symbolic 3-plithogenic ring, Pythagoras quadruple, Pythagoras Diophantine equation

## Introduction and Preliminaries.

Symbolic n-plithogenic sets were defined by Smarandache in [1-3], where these sets were used in generalizing classical algebraic structures such as symbolic 2-plithogenic and symbolic 3-plithogenic structures [4-9], with many applications in other fields [10-12].

It is useful to refer that symbolic n-plithogenic algebraic structures are very similar to neutrosophic and refined neutrosophic structures, see [13-22].

In this paper, we continue other efforts to study Pythagoras triples in many different rings [23-26].

We present the concept of Pythagoras triple in a symbolic 3-plithogenic commutative ring with many clear examples that clarify the validity of our work.

**Definition.**

Let  $R$  be a ring, the symbolic 3-plithogenic ring is defined as follows:

$$3 - SP_R = \{a_0 + a_1P_1 + a_2P_2 + a_3P_3; a_i \in R, P_j^2 = P_j, P_i \times P_j = P_{\max(i,j)}\}.$$

Smarandache has defined algebraic operations on  $3 - SP_R$  as follows:

Addition:

$$[a_0 + a_1P_1 + a_2P_2 + a_3P_3] + [b_0 + b_1P_1 + b_2P_2 + b_3P_3] = (a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2 + (a_3 + b_3)P_3.$$

Multiplication:

$$[a_0 + a_1P_1 + a_2P_2 + a_3P_3] \cdot [b_0 + b_1P_1 + b_2P_2 + b_3P_3] = a_0b_0 + a_0b_1P_1 + a_0b_2P_2 + a_0b_3P_3 + a_1b_0P_1^2 + a_1b_2P_1P_2 + a_2b_0P_2 + a_2b_1P_1P_2 + a_2b_2P_2^2 + a_1b_3P_3P_1 + a_2b_3P_2P_3 + a_3b_3(P_3)^2 + a_3b_0P_3 + a_3b_1P_3P_1 + a_3b_2P_2P_3 + a_1b_1P_1P_1 = a_0b_0 + (a_0b_1 + a_1b_0 + a_1b_1)P_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)P_2 + (a_0b_3 + a_1b_3 + a_2b_3 + a_3b_3 + a_3b_0 + a_3b_1 + a_3b_2)P_3.$$

**Definition.**

Let  $T = t_0 + t_1P_1 + t_2P_2 + t_3P_3, S = s_0 + s_1P_1 + s_2P_2 + s_3P_3, K = k_0 + k_1P_1 + k_2P_2 + k_3P_3, L = l_0 + l_1P_1 + l_2P_2 + l_3P_3$  be four symbolic 3-plithogenic elements of a symbolic 3-plithogenic commutative ring  $3 - SP_R$ , then  $(T, S, K, L)$  is called a symbolic 3-plithogenic Pythagoras quadruple if and only if  $T^2 + S^2 + K^2 = L^2$ .

**Theorem.**

Let  $T = t_0 + t_1P_1 + t_2P_2 + t_3P_3, S = s_0 + s_1P_1 + s_2P_2 + s_3P_3, K = k_0 + k_1P_1 + k_2P_2 + k_3P_3, L = l_0 + l_1P_1 + l_2P_2 + l_3P_3 \in 3 - SP_R$ , then  $(T, S, K, L)$  is a Pythagoras quadruple if and only if:

$$(t_0, s_0, k_0, l_0), (t_0 + t_1, s_0 + s_1, k_0 + k_1, l_0 + l_1), (t_0 + t_1 + t_2, s_0 + s_1 + s_2, k_0 + k_1 + k_2, l_0 + l_1 + l_2), (t_0 + t_1 + t_2 + t_3, s_0 + s_1 + s_2 + s_3, k_0 + k_1 + k_2 + k_3, l_0 + l_1 + l_2 + l_3) \text{ are four Pythagoras quadruples in } R.$$

**Proof.**

We have:

$$T^2 = t_0^2 + [(t_0 + t_1)^2 - t_0^2]P_1 + [(t_0 + t_1 + t_2)^2 - (t_0 + t_1)^2]P_2 + [(t_0 + t_1 + t_2 + t_3)^2 - (t_0 + t_1 + t_2)^2]P_3$$

$$S^2 = s_0^2 + [(s_0 + s_1)^2 - s_0^2]P_1 + [(s_0 + s_1 + s_2)^2 - (s_0 + s_1)^2]P_2 + [(s_0 + s_1 + s_2 + s_3)^2 - (s_0 + s_1 + s_2)^2]P_3$$

$$K^2 = k_0^2 + [(k_0 + k_1)^2 - k_0^2]P_1 + [(k_0 + k_1 + k_2)^2 - (k_0 + k_1)^2]P_2 + [(k_0 + k_1 + k_2 + k_3)^2 - (k_0 + k_1 + k_2)^2]P_3$$

$$L^2 = l_0^2 + [(l_0 + l_1)^2 - l_0^2]P_1 + [(l_0 + l_1 + l_2)^2 - (l_0 + l_1)^2]P_2 + [(l_0 + l_1 + l_2 + l_3)^2 - (l_0 + l_1 + l_2)^2]P_3$$

The equation  $T^2 + S^2 + K^2 = L^2$  is equivalent to:

$$t_0^2 + s_0^2 + k_0^2 = l_0^2 \quad (1)$$

$$(t_0 + t_1)^2 + (s_0 + s_1)^2 + (k_0 + k_1)^2 = (l_0 + l_1)^2 \quad (2)$$

$$(t_0 + t_1 + t_2)^2 + (s_0 + s_1 + s_2)^2 + (k_0 + k_1 + k_2)^2 = (l_0 + l_1 + l_2)^2 \quad (3)$$

$$(t_0 + t_1 + t_2 + t_3)^2 + (s_0 + s_1 + s_2 + s_3)^2 + (k_0 + k_1 + k_2 + k_3)^2 = (l_0 + l_1 + l_2 + l_3)^2 \quad (4)$$

Thus, the proof holds.

**Theorem.**

Let  $(t_0, s_0, k_0, l_0), (t_1, s_1, k_1, l_1), (t_2, s_2, k_2, l_2), (t_3, s_3, k_3, l_3)$  be four Pythagoras quadruples in  $R$ , then the corresponding pythagoras quadruple in  $3 - SP_R$  is  $(T, S, K, L)$ , where:

$$T = t_0 + [t_1 - t_0]P_1 + [t_2 - t_1]P_2 + [t_3 - t_2]P_3$$

$$S = s_0 + [s_1 - s_0]P_1 + [s_2 - s_1]P_2 + [s_3 - s_2]P_3$$

$$K = k_0 + [k_1 - k_0]P_1 + [k_2 - k_1]P_2 + [k_3 - k_2]P_3$$

$$L = l_0 + [l_1 - l_0]P_1 + [l_2 - l_1]P_2 + [l_3 - l_2]P_3$$

**Proof.**

We must compute  $T^2 + S^2 + K^2$ ,

$$\begin{aligned}
T^2 + S^2 + K^2 &= t_0^2 + (t_1^2 - t_0^2)P_1 + (t_2^2 - t_1^2)P_2 + (t_3^2 - t_2^2)P_3 + s_0^2 \\
&\quad + (s_1^2 - s_0^2)P_1 + (s_2^2 - s_1^2)P_2 + (s_3^2 - s_2^2)P_3 + k_0^2 + (k_1^2 - k_0^2)P_1 \\
&\quad + (k_2^2 - k_1^2)P_2 + (k_3^2 - k_2^2)P_3 \\
&= (t_0^2 + s_0^2 + k_0^2) + (t_1^2 + s_1^2 + k_1^2 - t_0^2 - s_0^2 - k_0^2)P_1 \\
&\quad + (t_2^2 + s_2^2 + k_2^2 - t_1^2 - s_1^2 - k_1^2)P_2 \\
&\quad + (t_3^2 + s_3^2 + k_3^2 - t_2^2 - s_2^2 - k_2^2)P_3 \\
&= l_0^2 + (l_1^2 - l_0^2)P_1 + (l_2^2 - l_1^2)P_2 + (l_3^2 - l_2^2)P_3 = L^2
\end{aligned}$$

O that, the proof is complete.

### Example:

We have  $L_1 = (1, -1, i, 1), L_2 = (i, 1, -1, -1), L_3 = (-i, -1, 1, -1), L_4 = (1, -i, -1, -1)$  are four Pythagoras quadruples in  $C$ .

The corresponding 3-plithogenic Pythagoras quadruple is  $(T, S, K, L)$ , where:

$$T = 1 + (-1 + i)P_1 - 2iP_2 + (1 + i)P_3$$

$$S = -1 + 2P_1 - 2P_2 + (1 - i)P_3$$

$$K = i + (-1 - i)P_1 + 2P_2 - 2P_3$$

$$L = 1 - 2P_1 + 2P_2 - 2P_3$$

On the other hand, we have:

$$T^2 = 1 - 2iP_1 - 4P_2 + 2iP_3 + 2(-1 + i)P_1 - 4iP_2 + 2(1 + i)P_3 - 4iP_2$$

$$+ 2(-1 + i)(1 + i)P_3 - 4i(1 + i)P_3$$

$$= 1 + (-2i - 2 + 2i)P_1 + (-4 - 4i + 4i + 4)P_2$$

$$+ (2i + 2 + 2i - 4 - 4i + 4)P_3 = 1 - 2P_1 + 2P_3$$

$$S^2 = 1 + 4P_1 + 4P_2 - 2iP_3 - 4P_1 + 4P_2 + 2(-1 + i)P_3 - 8P_2 + 4(1 - i)P_3$$

$$- 4(1 - i)P_3$$

$$= 1 + (4 - 4)P_1 + (4 + 4 - 8)P_2 + (-2i - 2 + 2i + 4 - 4i - 4 + 4i)P_3$$

$$= 1 - 2P_3$$

$$K^2 = -1 + 2P_1$$

$$L^2 = 1 = T^2 + S^2 + K^2$$

### Example.

Consider the following four Pythagoras quadruples in  $Z_2$ :

$$L_1 = (0,0,0,0), L_2 = (1,1,1,1), L_3 = (1,1,0,0), L_4 = (0,0,1,1)$$

For every quadruple  $(L_i, L_j, L_s, L_k); 1 \leq i, j, k \leq 4$ , we can get a symbolic 3-plithogenic pythagoras quadruple.

We will find some symbolic 3-plithogenic Pythagoras quadruple in  $3 - SP_{Z_2}$ .

Let us discuss the following cases:

**case (1).**

$$(L_1, L_1, L_1, L_1): \begin{cases} Y_1 = 0 \\ \dot{Y}_1 = 0 \\ Y_1'' = 0 \\ Y_1''' = 0 \end{cases}$$

**case (2).**

$$(L_1, L_1, L_1, L_2): \begin{cases} Y_2 = P_3 \\ \dot{Y}_2 = P_3 \\ Y_2'' = P_3 \\ Y_2''' = P_3 \end{cases}$$

**case (3).**

$$(L_1, L_1, L_1, L_3): \begin{cases} Y_3 = P_3 \\ \dot{Y}_3 = P_3 \\ Y_3'' = 0 \\ Y_3''' = 0 \end{cases}$$

**case (4).**

$$(L_1, L_1, L_1, L_4): \begin{cases} Y_4 = 0 \\ \dot{Y}_4 = 0 \\ Y_4'' = P_3 \\ Y_4''' = P_3 \end{cases}$$

**case (5).**

$$(L_1, L_1, L_2, L_1): \begin{cases} Y_5 = P_2 + P_3 \\ \dot{Y}_5 = P_2 + P_3 \\ Y_5'' = P_2 + P_3 \\ Y_5''' = P_2 + P_3 \end{cases}$$

**case (6).**

$$(L_1, L_1, L_3, L_1): \begin{cases} Y_6 = P_2 + P_3 \\ \dot{Y}_6 = P_2 + P_3 \\ Y_6'' = P_2 + P_3 \\ Y_6''' = P_2 + P_3 \end{cases}$$

**case (7).**

$$(L_1, L_1, L_4, L_1): \begin{cases} Y_7 = 0 \\ \dot{Y}_7 = 0 \\ Y_7'' = P_2 + P_3 \\ Y_7''' = P_2 + P_3 \end{cases}$$

case (8).

$$(L_1, L_2, L_1, L_1): \begin{cases} Y_8 = P_1 + P_2 \\ \dot{Y}_8 = P_1 + P_2 \\ Y_8'' = P_1 + P_2 \\ Y_8''' = P_1 + P_2 \end{cases}$$

case (9).

$$(L_1, L_3, L_1, L_1): \begin{cases} Y_9 = P_1 + P_2 \\ \dot{Y}_9 = P_1 + P_2 \\ Y_9'' = 0 \\ Y_9''' = 0 \end{cases}$$

case (10).

$$(L_1, L_4, L_1, L_1): \begin{cases} Y_{10} = 0 \\ \dot{Y}_{10} = 0 \\ Y_{10}'' = P_1 + P_2 \\ Y_{10}''' = P_1 + P_2 \end{cases}$$

case (11).

$$(L_2, L_1, L_1, L_1): \begin{cases} Y_{11} = 1 + P_1 \\ \dot{Y}_{11} = 1 + P_1 \\ Y_{11}'' = 1 + P_1 \\ Y_{11}''' = 1 + P_1 \end{cases}$$

case (12).

$$(L_3, L_1, L_1, L_1): \begin{cases} Y_{12} = 1 + P_1 \\ \dot{Y}_{12} = 1 + P_1 \\ Y_{12}'' = 0 \\ Y_{12}''' = 0 \end{cases}$$

case (13).

$$(L_4, L_1, L_1, L_1): \begin{cases} Y_{13} = 0 \\ \dot{Y}_{13} = 0 \\ Y_{13}'' = 1 + P_1 \\ Y_{13}''' = 1 + P_1 \end{cases}$$

case (14).

$$(L_2, L_2, L_2, L_2): \begin{cases} Y_{14} = 1 \\ \dot{Y}_{14} = 1 \\ Y_{14}'' = 1 \\ Y_{14}''' = 1 \end{cases}$$

case (15).

$$(L_2, L_2, L_2, L_1): \begin{cases} Y_{15} = 1 + P_3 \\ \dot{Y}_{15} = 1 + P_3 \\ Y_{15}'' = 1 + P_3 \\ Y_{15}''' = 1 + P_3 \end{cases}$$

case (16).

$$(L_2, L_2, L_2, L_3): \begin{cases} Y_{16} = 1 \\ \dot{Y}_{16} = 1 \\ Y_{16}'' = 1 + P_3 \\ Y_{16}''' = 1 + P_3 \end{cases}$$

case (17).

$$(L_2, L_2, L_2, L_4): \begin{cases} Y_{17} = 1 + P_3 \\ \dot{Y}_{17} = 1 + P_3 \\ Y_{17}'' = 1 \\ Y_{17}''' = 1 \end{cases}$$

case (18).

$$(L_2, L_2, L_1, L_2): \begin{cases} Y_{18} = 1 + P_2 + P_3 \\ \dot{Y}_{18} = 1 + P_2 + P_3 \\ Y_{18}'' = 1 + P_2 + P_3 \\ Y_{18}''' = 1 + P_2 + P_3 \end{cases}$$

case (19).

$$(L_2, L_2, L_3, L_2): \begin{cases} Y_{19} = 1 \\ \dot{Y}_{19} = 1 \\ Y_{19}'' = 1 + P_3 \\ Y_{19}''' = 1 + P_3 \end{cases}$$

case (20).

$$(L_2, L_2, L_4, L_2): \begin{cases} Y_{20} = 1 + P_2 + P_3 \\ \dot{Y}_{20} = 1 + P_2 + P_3 \\ Y_{20}'' = 1 \\ Y_{20}''' = 1 \end{cases}$$

case (21).



$$(L_2, L_1, L_2, L_2): \begin{cases} Y_{21} = 1 + P_1 + P_2 \\ Y'_{21} = 1 + P_1 + P_2 \\ Y''_{21} = 1 + P_1 + P_2 \\ Y'''_{21} = 1 + P_1 + P_2 \end{cases}$$

case (22).

$$(L_2, L_3, L_2, L_2): \begin{cases} Y_{22} = 1 \\ Y'_{22} = 1 \\ Y''_{22} = 1 + P_1 + P_2 \\ Y'''_{22} = 1 + P_1 + P_2 \end{cases}$$

case (23).

$$(L_2, L_4, L_2, L_2): \begin{cases} Y_{23} = 1 + P_1 + P_2 \\ Y'_{23} = 1 + P_1 + P_2 \\ Y''_{23} = 1 \\ Y'''_{23} = 1 \end{cases}$$

Permutation (24).

$$(L_1, L_2, L_2, L_2): \begin{cases} Y_{24} = P_1 \\ Y'_{24} = P_1 \\ Y''_{24} = P_1 \\ Y'''_{24} = P_1 \end{cases}$$

case (25).

$$(L_3, L_2, L_2, L_2): \begin{cases} Y_{25} = 1 \\ Y'_{25} = 1 \\ Y''_{25} = P_1 \\ Y'''_{25} = P_1 \end{cases}$$

case (26).

$$(L_4, L_2, L_2, L_2): \begin{cases} Y_{26} = P_1 \\ Y'_{26} = P_1 \\ Y''_{26} = 1 \\ Y'''_{26} = 1 \end{cases}$$

case (27).

$$(L_3, L_3, L_3, L_3): \begin{cases} Y_{27} = 1 \\ Y'_{27} = 1 \\ Y''_{27} = 0 \\ Y'''_{27} = 0 \end{cases}$$

case (28).

$$(L_3, L_3, L_3, L_1): \begin{cases} Y_{28} = 1 + P_3 \\ \check{Y}_{28} = 1 + P_3 \\ Y_{28}'' = 0 \\ Y_{28}''' = 0 \end{cases}$$

**case (29).**

$$(L_3, L_3, L_3, L_2): \begin{cases} Y_{29} = 1 \\ \check{Y}_{29} = 1 \\ Y_{29}'' = P_3 \\ Y_{29}''' = P_3 \end{cases}$$

**case (30).**

$$(L_3, L_3, L_3, L_4): \begin{cases} Y_{30} = 1 + P_3 \\ \check{Y}_{30} = 1 + P_3 \\ Y_{30}'' = P_3 \\ Y_{30}''' = P_3 \end{cases}$$

**case (31).**

$$(L_3, L_3, L_1, L_3): \begin{cases} Y_{31} = 1 + P_3 \\ \check{Y}_{31} = 1 + P_3 \\ Y_{31}'' = P_3 \\ Y_{31}''' = P_3 \end{cases}$$

**case (32).**

$$(L_3, L_3, L_2, L_3): \begin{cases} Y_{32} = 1 \\ \check{Y}_{32} = 1 \\ Y_{32}'' = P_2 + P_3 \\ Y_{32}''' = P_2 + P_3 \end{cases}$$

**case (33).**

$$(L_3, L_3, L_4, L_3): \begin{cases} Y_{33} = 1 + P_2 + P_3 \\ \check{Y}_{33} = 1 + P_2 + P_3 \\ Y_{33}'' = P_2 + P_3 \\ Y_{33}''' = P_2 + P_3 \end{cases}$$

**case (34).**

$$(L_3, L_1, L_3, L_3): \begin{cases} Y_{34} = 1 + P_2 + P_3 \\ \check{Y}_{34} = 1 + P_2 + P_3 \\ Y_{34}'' = 0 \\ Y_{34}''' = 0 \end{cases}$$

**case (35).**

$$(L_3, L_2, L_3, L_3): \begin{cases} Y_{35} = 1 \\ \check{Y}_{35} = 1 \\ Y_{35}'' = P_1 + P_2 \\ Y_{35}''' = P_1 + P_2 \end{cases}$$

case (36).

$$(L_3, L_4, L_3, L_3): \begin{cases} Y_{36} = 1 + P_1 + P_2 \\ \check{Y}_{36} = 1 + P_1 + P_2 \\ Y_{36}'' = P_1 + P_2 \\ Y_{36}''' = P_1 + P_2 \end{cases}$$

case (37).

$$(L_1, L_3, L_3, L_3): \begin{cases} Y_{37} = P_1 \\ \check{Y}_{37} = P_1 \\ Y_{37}'' = 0 \\ Y_{37}''' = 0 \end{cases}$$

case (38).

$$(L_2, L_3, L_3, L_3): \begin{cases} Y_{38} = 1 \\ \check{Y}_{38} = 1 \\ Y_{38}'' = 1 + P_1 \\ Y_{38}''' = 1 + P_1 \end{cases}$$

case (39).

$$(L_4, L_3, L_3, L_3): \begin{cases} Y_{39} = P_1 \\ \check{Y}_{39} = P_1 \\ Y_{39}'' = 1 + P_1 \\ Y_{39}''' = 1 + P_1 \end{cases}$$

case (40).

$$(L_4, L_4, L_4, L_4): \begin{cases} Y_{40} = 0 \\ \check{Y}_{40} = 0 \\ Y_{40}'' = 1 \\ Y_{40}''' = 1 \end{cases}$$

case (41).

$$(L_4, L_4, L_4, L_1): \begin{cases} Y_{41} = 0 \\ \check{Y}_{41} = 0 \\ Y_{41}'' = 1 + P_3 \\ Y_{41}''' = 1 + P_3 \end{cases}$$

case (42).

$$(L_4, L_4, L_4, L_2): \begin{cases} Y_{42} = P_3 \\ \acute{Y}_{42} = P_3 \\ Y_{42}'' = 1 \\ Y_{42}''' = 1 \end{cases}$$

case (43).

$$(L_4, L_4, L_4, L_3): \begin{cases} Y_{43} = P_3 \\ \acute{Y}_{43} = P_3 \\ Y_{43}'' = 1 + P_3 \\ Y_{43}''' = 1 + P_3 \end{cases}$$

case (44).

$$(L_4, L_4, L_1, L_4): \begin{cases} Y_{44} = 0 \\ \acute{Y}_{44} = 0 \\ Y_{44}'' = 1 + P_2 + P_3 \\ Y_{44}''' = 1 + P_2 + P_3 \end{cases}$$

case (45).

$$(L_4, L_4, L_2, L_4): \begin{cases} Y_{45} = P_2 + P_3 \\ \acute{Y}_{45} = P_2 + P_3 \\ Y_{45}'' = 1 \\ Y_{45}''' = 1 \end{cases}$$

case (46).

$$(L_4, L_4, L_3, L_4): \begin{cases} Y_{46} = P_2 + P_3 \\ \acute{Y}_{46} = P_2 + P_3 \\ Y_{46}'' = 1 + P_2 + P_3 \\ Y_{46}''' = 1 + P_2 + P_3 \end{cases}$$

case (47).

$$(L_4, L_1, L_4, L_4): \begin{cases} Y_{47} = 0 \\ \acute{Y}_{47} = 0 \\ Y_{47}'' = 1 + P_1 + P_2 \\ Y_{47}''' = 1 + P_1 + P_2 \end{cases}$$

case (48).

$$(L_4, L_2, L_4, L_4): \begin{cases} Y_{48} = P_1 + P_2 \\ \acute{Y}_{48} = P_1 + P_2 \\ Y_{48}'' = 1 \\ Y_{48}''' = 1 \end{cases}$$

case (49).

$$(L_4, L_3, L_4, L_4): \begin{cases} Y_{49} = P_1 + P_2 \\ \dot{Y}_{49} = P_1 + P_2 \\ Y_{49}'' = 1 + P_1 + P_2 \\ Y_{49}''' = 1 + P_1 + P_2 \end{cases}$$

case (50).

$$(L_1, L_4, L_4, L_4): \begin{cases} Y_{50} = 0 \\ \dot{Y}_{50} = 0 \\ Y_{50}'' = P_1 \\ Y_{50}''' = P_1 \end{cases}$$

case (51).

$$(L_2, L_4, L_4, L_4): \begin{cases} Y_{51} = 1 + P_1 \\ \dot{Y}_{51} = 1 + P_1 \\ Y_{51}'' = 1 \\ Y_{51}''' = 1 \end{cases}$$

case (52).

$$(L_3, L_4, L_4, L_4): \begin{cases} Y_{52} = 1 + P_1 \\ \dot{Y}_{52} = 1 + P_1 \\ Y_{52}'' = P_1 \\ Y_{52}''' = P_1 \end{cases}$$

case (53).

$$(L_1, L_2, L_3, L_4): \begin{cases} Y_{53} = P_1 + P_3 \\ \dot{Y}_{53} = P_1 + P_3 \\ Y_{53}'' = P_1 + P_2 + P_3 \\ Y_{53}''' = P_1 + P_2 + P_3 \end{cases}$$

case (54).

$$(L_1, L_2, L_4, L_3): \begin{cases} Y_{54} = P_1 + P_2 + P_3 \\ \dot{Y}_{54} = P_1 + P_2 + P_3 \\ Y_{54}'' = P_1 + P_2 \\ Y_{54}''' = P_1 + P_2 \end{cases}$$

case (55).

$$(L_1, L_3, L_2, L_4): \begin{cases} Y_{55} = P_1 + P_3 \\ \dot{Y}_{55} = P_1 + P_3 \\ Y_{55}'' = P_2 \\ Y_{55}''' = P_2 \end{cases}$$

case (56).

$$(L_1, L_3, L_4, L_2): \begin{cases} Y_{56} = P_1 + P_2 + P_3 \\ \check{Y}_{56} = P_1 + P_2 + P_3 \\ Y_{56}'' = P_2 \\ Y_{56}''' = P_2 \end{cases}$$

case (57).

$$(L_1, L_4, L_2, L_3): \begin{cases} Y_{57} = P_2 \\ \check{Y}_{57} = P_2 \\ Y_{57}'' = P_1 + P_3 \\ Y_{57}''' = P_1 + P_3 \end{cases}$$

case (58).

$$(L_1, L_4, L_3, L_2): \begin{cases} Y_{58} = P_2 \\ \check{Y}_{58} = P_2 \\ Y_{58}'' = P_1 + P_2 + P_3 \\ Y_{58}''' = P_1 + P_2 + P_3 \end{cases}$$

case (59).

$$(L_2, L_1, L_3, L_4): \begin{cases} Y_{59} = 1 + P_1 + P_2 + P_3 \\ \check{Y}_{59} = 1 + P_1 + P_2 + P_3 \\ Y_{59}'' = 1 + P_1 + P_3 \\ Y_{59}''' = 1 + P_1 + P_3 \end{cases}$$

case (60).

$$(L_2, L_1, L_4, L_3): \begin{cases} Y_{60} = 1 + P_1 + P_3 \\ \check{Y}_{60} = 1 + P_1 + P_3 \\ Y_{60}'' = P_2 + P_3 \\ Y_{60}''' = P_2 + P_3 \end{cases}$$

case (61).

$$(L_3, L_1, L_2, L_4): \begin{cases} Y_{61} = 1 + P_1 + P_2 + P_3 \\ \check{Y}_{61} = 1 + P_1 + P_2 + P_3 \\ Y_{61}'' = P_2 \\ Y_{61}''' = P_2 \end{cases}$$

case (62).

$$(L_3, L_1, L_4, L_2): \begin{cases} Y_{62} = 1 + P_1 + P_3 \\ \check{Y}_{62} = 1 + P_1 + P_3 \\ Y_{62}'' = P_2 \\ Y_{62}''' = P_2 \end{cases}$$

case (63).

$$(L_4, L_1, L_2, L_3): \begin{cases} Y_{63} = P_2 \\ Y'_{63} = P_2 \\ Y_{63}'' = 1 + P_2 + P_3 \\ Y_{63}''' = 1 + P_2 + P_3 \end{cases}$$

case (64).

$$(L_4, L_1, L_3, L_2): \begin{cases} Y_{64} = P_2 \\ Y'_{64} = P_2 \\ Y_{64}'' = 1 + P_3 \\ Y_{64}''' = 1 + P_3 \end{cases}$$

case (65).

$$(L_2, L_3, L_1, L_4): \begin{cases} Y_{65} = 1 + P_2 \\ Y'_{65} = 1 + P_2 \\ Y_{65}'' = 1 + P_3 \\ Y_{65}''' = 1 + P_3 \end{cases}$$

case (66).

$$(L_2, L_3, L_4, L_1): \begin{cases} Y_{66} = 1 + P_2 \\ Y'_{66} = 1 + P_2 \\ Y_{66}'' = 1 + P_1 + P_2 + P_3 \\ Y_{66}''' = 1 + P_1 + P_2 + P_3 \end{cases}$$

case (67).

$$(L_3, L_2, L_1, L_4): \begin{cases} Y_{67} = 1 + P_2 \\ Y'_{67} = 1 + P_2 \\ Y_{67}'' = P_1 + P_2 + P_3 \\ Y_{67}''' = P_1 + P_2 + P_3 \end{cases}$$

case (68).

$$(L_3, L_2, L_4, L_1): \begin{cases} Y_{68} = 1 + P_2 \\ Y'_{68} = 1 + P_2 \\ Y_{68}'' = 1 + P_3 \\ Y_{68}''' = 1 + P_3 \end{cases}$$

case (64).

$$(L_4, L_1, L_3, L_2): \begin{cases} Y_{64} = P_2 \\ Y'_{64} = P_2 \\ Y_{64}'' = P_1 + P_3 \\ Y_{64}''' = P_1 + P_3 \end{cases}$$

case (69).

$$(L_3, L_4, L_1, L_2): \begin{cases} Y_{69} = 1 + P_1 + P_3 \\ \dot{Y}_{69} = 1 + P_1 + P_3 \\ Y_{69}'' = P_1 + P_3 \\ Y_{69}''' = P_1 + P_3 \end{cases}$$

case (70).

$$(L_3, L_4, L_2, L_1): \begin{cases} Y_{70} = 1 + P_1 + P_2 + P_3 \\ \dot{Y}_{70} = 1 + P_1 + P_2 + P_3 \\ Y_{70}'' = P_1 + P_3 \\ Y_{70}''' = P_1 + P_3 \end{cases}$$

case (71).

$$(L_4, L_3, L_1, L_2): \begin{cases} Y_{71} = P_1 + P_2 + P_3 \\ \dot{Y}_{71} = P_1 + P_2 + P_3 \\ Y_{71}'' = 1 + P_3 \\ Y_{71}''' = 1 + P_3 \end{cases}$$

case (72).

$$(L_4, L_1, L_3, L_2): \begin{cases} Y_{72} = P_1 + P_3 \\ \dot{Y}_{72} = P_1 + P_3 \\ Y_{72}'' = 1 + P_1 + P_2 + P_3 \\ Y_{72}''' = 1 + P_1 + P_2 + P_3 \end{cases}$$

case (73).

$$(L_4, L_2, L_1, L_3): \begin{cases} Y_{73} = P_1 + P_2 + P_3 \\ \dot{Y}_{73} = P_1 + P_2 + P_3 \\ Y_{73}'' = 1 + P_2 \\ Y_{73}''' = 1 + P_2 \end{cases}$$

case (74).

$$(L_4, L_1, L_3, L_2): \begin{cases} Y_{74} = P_1 + P_3 \\ \dot{Y}_{74} = P_1 + P_3 \\ Y_{74}'' = 1 + P_2 \\ Y_{74}''' = 1 + P_2 \end{cases}$$

case (75).

$$(L_2, L_4, L_1, L_3): \begin{cases} Y_{75} = 1 + P_1 + P_3 \\ \dot{Y}_{75} = 1 + P_1 + P_3 \\ Y_{75}'' = 1 + P_2 \\ Y_{75}''' = 1 + P_2 \end{cases}$$

case (76).



$$(L_2, L_4, L_3, L_1): \begin{cases} Y_{76} = 1 + P_1 + P_2 + P_3 \\ Y'_{76} = 1 + P_1 + P_2 + P_3 \\ Y''_{76} = 1 + P_2 \\ Y'''_{76} = 1 + P_2 \end{cases}$$

case (77).

$$(L_2, L_2, L_3, L_3): \begin{cases} Y_{77} = 1 \\ Y'_{77} = 1 \\ Y''_{77} = 1 + P_2 \\ Y'''_{77} = 1 + P_2 \end{cases}$$

case (78).

$$(L_2, L_2, L_1, L_1): \begin{cases} Y_{78} = 1 + P_2 \\ Y'_{78} = 1 + P_2 \\ Y''_{78} = 1 + P_2 \\ Y'''_{78} = 1 + P_2 \end{cases}$$

case (79).

$$(L_2, L_2, L_4, L_4): \begin{cases} Y_{79} = 1 + P_2 \\ Y'_{79} = 1 + P_2 \\ Y''_{79} = 1 \\ Y'''_{79} = 1 \end{cases}$$

case (80).

$$(L_1, L_1, L_2, L_2): \begin{cases} Y_{80} = P_2 \\ Y'_{80} = P_2 \\ Y''_{80} = P_2 \\ Y'''_{80} = P_2 \end{cases}$$

## Conclusion.

In this paper, we have studied Pythagoras quadruples in symbolic 3-plithogenic commutative rings, where necessary and sufficient conditions for a symbolic 3-plithogenic quadruple  $(x, y, z, t)$  to be a Pythagoras quadruple.

Also, we have presented some related examples that explain how to find 3-plithogenic quadruples from classical triples.

**Acknowledgments** " This study is supported via funding from Prince sattam bin Abdulaziz University project number (PSAU/2023/R/1445) ".

## References

- [1] F. Smarandache, *Plithogeny, Plithogenic Set, Logic, Probability, and Statistics*, 141 pages, Pons Editions, Brussels, Belgium, 2017. arXiv.org (Cornell University), Computer Science - Artificial Intelligence, 03Bxx:
- [2] Florentin Smarandache, *Physical Plithogenic Set*, 71st Annual Gaseous Electronics Conference, Session LW1, Oregon Convention Center Room, Portland, Oregon, USA, November 5–9, 2018.
- [3] Florentin Smarandache: *Plithogenic Set, an Extension of Crisp, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Sets – Revisited*, *Neutrosophic Sets and Systems*, vol. 21, 2018, pp. 153-166.
- [4] M. B. Zeina, N. Altounji, M. Abobala, and Y. Karmouta, "Introduction to Symbolic 2-Plithogenic Probability Theory," *Galoitica: Journal of Mathematical Structures and Applications*, vol. 7, no. 1, 2023.
- [5] Nader Mahmoud Taffach , Ahmed Hatip., "A Review on Symbolic 2-Plithogenic Algebraic Structures " *Galoitica Journal Of Mathematical Structures and Applications*, Vol.5, 2023.
- [6] Nader Mahmoud Taffach , Ahmed Hatip.," A Brief Review on The Symbolic 2-Plithogenic Number Theory and Algebraic Equations ", *Galoitica Journal Of Mathematical Structures and Applications*, Vol.5, 2023.
- [7] Merkepci, H., and Abobala, M., " On The Symbolic 2-Plithogenic Rings", *International Journal of Neutrosophic Science*, 2023.
- [8] Albasheer, O., Hajjari., A., and Dalla., R., " On The Symbolic 3-Plithogenic Rings and Their Algebraic Properties", *Neutrosophic Sets and Systems*, Vol 54, 2023.
- [9] Ben Othman, K., "On Some Algorithms For Solving Symbolic 3-Plithogenic Equations", *Neoma Journal Of Mathematics and Computer Science*, 2023.
- [10] Ali, R., and Hasan, Z., "An Introduction To The Symbolic 3-Plithogenic Vector Spaces", *Galoitica Journal Of Mathematical Structures and Applications*, vol. 6, 2023.

- [11] Rawashdeh, A., "An Introduction To The Symbolic 3-plithogenic Number Theory", Neoma Journal Of Mathematics and Computer Science, 2023.
- [12] Alhasan, Y., Alfahal, A., Abdulfatah, R., Ali, R., and Aljibawi, M., " On A Novel Security Algorithm For The Encryption Of  $3 \times 3$  Fuzzy Matrices With Rational Entries Based On The Symbolic 2-Plithogenic Integers And El-Gamal Algorithm", International Journal of Neutrosophic Science, 2023.
- [13] Merkepci, M., Abobala, M., and Allouf, A., " The Applications of Fusion Neutrosophic Number Theory in Public Key Cryptography and the Improvement of RSA Algorithm ", Fusion: Practice and Applications, 2023.
- [14] Merkepci, M., and Abobala, M., " Security Model for Encrypting Uncertain Rational Data Units Based on Refined Neutrosophic Integers Fusion and El Gamal Algorithm ", Fusion: Practice and Applications, 2023.
- [15] Abobala, M., and Allouf, A., " On A Novel Security Scheme for The Encryption and Decryption Of  $2 \times 2$  Fuzzy Matrices with Rational Entries Based on The Algebra of Neutrosophic Integers and El-Gamal Crypto-System", Neutrosophic Sets and Systems, vol.54, 2023.
- [16] Abobala, M., " $n$ -Cyclic Refined Neutrosophic Algebraic Systems Of Sub-Indeterminacies, An Application To Rings and Modules", International Journal of Neutrosophic Science, Vol. 12, pp. 81-95 . 2020.
- [17] Ben Othman, K., Von Shtawzen, O., Khaldi, A., and Ali, R., "On The Concept Of Symbolic 7-Plithogenic Real Matrices", Pure Mathematics For Theoretical Computer Science, Vol.1, 2023.
- [18] Ben Othman, K., Von Shtawzen, O., Khaldi, A., and Ali, R., "On The Symbolic 8-Plithogenic Matrices", Pure Mathematics For Theoretical Computer Science, Vol.1, 2023.
- [19] Mohamed, M., "Modified Approach for Optimization of Real-Life Transportation Environment: Suggested Modifications", American Journal of Business and Operations research, 2021.

- [20] Nabeeh, N., Alshaimaa, A., and Tantawy, A., "A Neutrosophic Proposed Model For Evaluation Blockchain Technology in Secure Enterprise Distributed Applications", Journal of Cybersecurity and Information Management, 2023.
- [21] Abualkishik, A., Almajed, R., Thompson, W., "Improving The Performance of Fog-assisted Internet of Things Networks Using Bipolar Trapezoidal Neutrosophic Sets", Journal of Wireless and Ad Hoc Communication, 2023.
- [22] R. A. Trivedi, S. A. Bhanotar, Pythagorean Triplets - Views, Analysis and Classification, IOSR J. Math., 11 (2015).
- [23] J. Rukavicka, Dickson's method for generating Pythagorean triples revisited, Eur. J. Pure Appl. Math., 6 (2013).
- [24] T. Roy and F. J. Sonia, A Direct Method To Generate Pythagorean Triples And Its Generalization To Pythagorean Quadruples And n-tuples, , Jadavpur University, India (2010)
- [25] Paul Oliverio, " Self-Generating Pythagorean Quadruples And TV-Tuples", Los Angeles, (1993).

Received 3/7/2023, Accepted 4/10/2023