Neutrosophic Sets and Systems

Volume 58 | Article 29

10-5-2023

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Mohammed Abu-Saleem

Omar almallah

Nizar Kh. Al Ouashouh

NSS
Neutrosophic Sets and Systems, Vol. 58, 2023
University of New Mexico

An application of neutrosophic theory on manifolds and their topological transformations

Mohammed Abu-Saleem[1,*], Omar almallah[2] and Nizar Kh. Al Ouashouh[3]
[1]Department of Mathematics, Al-Balqa Applied University, Salt, Jordan.; m_abusaleem@bau.edu.jo
[2]Department of Mathematics, Al-Balqa Applied University, Salt, Jordan.; oamallah@bau.edu.jo
[3]Department of Mathematics, Al-Balqa Applied University, Salt, Jordan.; aloushoush@bau.edu.jo
[*]Correspondence: m_abusaleem@bau.edu.jo

Abstract. This paper presents an investigation into the mathematical concepts of neutrosophic folding and neuretraction on neutrosophic manifolds, specifically focusing on their application in hyperspace. Through the application of specific transformations on a neutrosophic manifold situated in hyperspace, we can obtain neutrosophic manifolds in lower dimensions. Based on our research, we can accurately establish the connection between neutrosophic folding and neuretraction on a neutrosophic manifold. Furthermore, we can determine the relationship between neuretraction and neutrosophic folding.

Keywords: neutrosophic folding; neuretraction; neutrosophic hyperspace; neutrosophic manifold.

1. Introduction

Neutrosophy is a scientific field that combines neutrality and philosophy. Samaransache founded various fields in 1980, such as set theory, probability, and logic, with numerous applications that highlight the deep interaction between mathematics and other scientific disciplines. [19]. The concept of fuzzy sets was introduced by Zadeh as a novel method for elucidating intricate concepts by including the concept of membership. Scholars in the fields of mathematics and computer science developed this theory, which possesses a broad spectrum of expedient applications [22]. Neutrosophy is basically rooted in the fundamental concepts of fuzzy set theory (NS) and intuitionistic fuzzy set theory (IFS) $[8, 10, 13, 22]$. The concept of neutrosophic sets was introduced by Smarandache with the aim of representing uncertain or vague information. This is achieved through the utilization of three distinct functions, namely truth, indeterminacy, and falsity. Unlike other theories, the function of indeterminacy is independent of the functions of truth and falsity [12,18,19]. Smarandache's (NS) theory expanded the scope of (IFS), offering novel perspectives on how to effectively manage uncertainty when making decisions based on personal experience, as stated in reference [20]. The values of the truth, indeterminacy, and falsity functions are within $]^{-}0,\ 1^{+}[$, making it difficult to apply to practical problems [17].

Due to this, Wang created the single-valued neutrosophic sets $(SVNS)$, such that the truth, indeterminacy, and falsity maps are real elements of the $[0,\ 1]$ space [12, 21]. Further investigation on a $(SVNG)$ and a neutrosophic topology was debated in [1, 4, 6, 7, 9, 11, 14, 16]. Additional insights on the applications of homotopy theory were provided in [2, 3]. The paper aims to contribute to the field of mathematics by exploring and providing a deeper understanding of the neutrosophic transformation in the context of neutrosophic manifold theory.

2. Preliminaries

Definition 2.1. [19] Assume that $\mathcal{W}$ is a finite set of objects, and that $(\mathfrak{t})$ stands for a generic component in $\mathcal{W}$. A (NS) E in $\mathcal{W}$ is comprised of three membership functions, a truth-membership function $\upsilon_{\mathrm{E}}(\mathfrak{t})$, an indeterminacy-membership function $\rho_{\mathrm{E}}(\mathfrak{t})$ and a falsity-membership function $\sigma_{\mathrm{E}}(\mathfrak{t})$. Also, $\upsilon_{\mathrm{E}}(\mathfrak{t}),\ \rho_{E}(\mathfrak{t})$ and $\sigma_{E}(\mathfrak{t})$ are the elements of $]^{-}0,\ 1^{+}[$. E can be represented as

E$= \{\mathfrak{t}, (\upsilon_{\mathrm{E}}(\mathfrak{t}),\ \rho_{\mathrm{E}}(\mathfrak{t}),\ \sigma_{\mathrm{E}}(\mathfrak{t})) : \mathfrak{t} \in \mathcal{W},\ \upsilon_{\mathrm{E}}(\mathfrak{t}),\ \rho_{\mathrm{E}}(\mathfrak{t}),\ \sigma_{\mathrm{E}}(\mathfrak{t}) \in\ ]^{-}0,\ 1^{+}[\}$. Indeed, $^{-}0 \leq \upsilon_{\mathrm{E}}(\mathfrak{t}) + \rho_{\mathrm{E}}(\mathfrak{t}) + \sigma_{\mathrm{E}}(\mathfrak{t}) \leq 3^{+}$.

Definition 2.2. [21] Assume that $\mathcal{W}$ is a finite set of objects, and that $(\mathfrak{t})$ stands for a generic component in $\mathcal{W}$. A $(SVNS)$ E in $\mathcal{W}$ is comprised of three membership functions, a truth-membership function $\upsilon_{\mathrm{E}}(\mathfrak{t})$, an indeterminacy-membership function $\rho_{\mathrm{E}}(\mathfrak{t})$ and a falsity-membership function $\sigma_{\mathrm{E}}(\mathfrak{t})$. Also each, $\upsilon_{\mathrm{E}}(\mathfrak{t}), \rho_{\mathrm{E}}(\mathfrak{t})$ and $\sigma_{\mathrm{E}}(\mathfrak{t})$ are elements in $]0,\ 1[$. E can be represented as E$= \{\mathfrak{t}, (\upsilon_{\mathrm{E}}(\mathfrak{t}),\ \rho_{\mathrm{E}}(\mathfrak{t}),\ \sigma_{\mathrm{E}}(\mathfrak{t})) : \mathfrak{t} \in \mathcal{W},\ \upsilon_{\mathrm{E}}(\mathfrak{t}),\ \rho_{\mathrm{E}}(\mathfrak{t}),\ \sigma_{\mathrm{E}}(\mathfrak{t}) \in\ ]0,\ 1[\}$. In this approach, $0 \leq \upsilon_{\mathrm{E}}(\mathfrak{t}) + \rho_{\mathrm{E}}(\mathfrak{t}) + \sigma_{\mathrm{E}}(\mathfrak{t}) \leq 3$. In the interest of clarity and concision, we refer to a neutrosophic set $\langle \upsilon_{\mathrm{E}},\ \rho_{\mathrm{E}},\ \sigma_{\mathrm{E}} \rangle$ and $\langle \upsilon_{\mathrm{E}}(\mathfrak{t}),\ \rho_{\mathrm{E}}(\mathfrak{t}),\ \sigma_{\mathrm{E}}(\mathfrak{t}) \rangle$ as ω and $\omega(\mathfrak{t})$ respectively.

Definition 2.3. [15] A topological space that satisfies the T_2 separation axiom and is locally homeomorphic to an open n-dimensional disk U^n is referred to as an n-dimensional manifold.

Definition 2.4. [5] Let X be a topological space and Cbe a subspace of X, where $i : C \to X$ is the inclusion. if there exists a continuous map $r : X \to C$ satisfying the condition $r \circ i = 1|_C$. Then, C is referred to a retract of X. The existence of a map r is denoted as a retraction of X into C.

Theorem 2.5. *[5] The n-dimensional closed disk $L^n = \{z \in \mathcal{R}^n : |z| \leq 1\}$ is a retract of $\mathcal{R}^n$.*

Definition 2.6. [15] Consider two topological spaces X_1 and X_2, and let φ_0 and $\varphi_1$denote continuous mappings from X_1 to X_1. The homotopy between φ_0 and $\varphi_1$is established when a continuous map $\varphi : X_1 \times I \to X_2$ exists, and satisfying the conditions $\varphi(s, 0) = \varphi_0(s)$ and $\varphi(s, 1) = \varphi_1(s)$ for all $s \in X_1$.

3. Neutrosophic manifolds and their transformations

Our study introduces a collection of important concepts that support our paper and enable us to arrive at significant conclusions.

Definition 3.1. A neutrosophic n-dimensional manifold is characterized as a pair $\langle M^n,\ \omega\rangle$ in which, M^n is n -dimensional manifold.

Example 3.2. A neutrosophic Euclidean n-space $\langle \mathcal{R}^n,\ \omega\rangle$ can be regarded as a neutrosophic n-dimensional manifold. Additionally, a neutrosophic unit n-dimensional sphere $\langle S^n,\ \omega\rangle$ can be considered as a neutrosophic n-dimensional manifold.

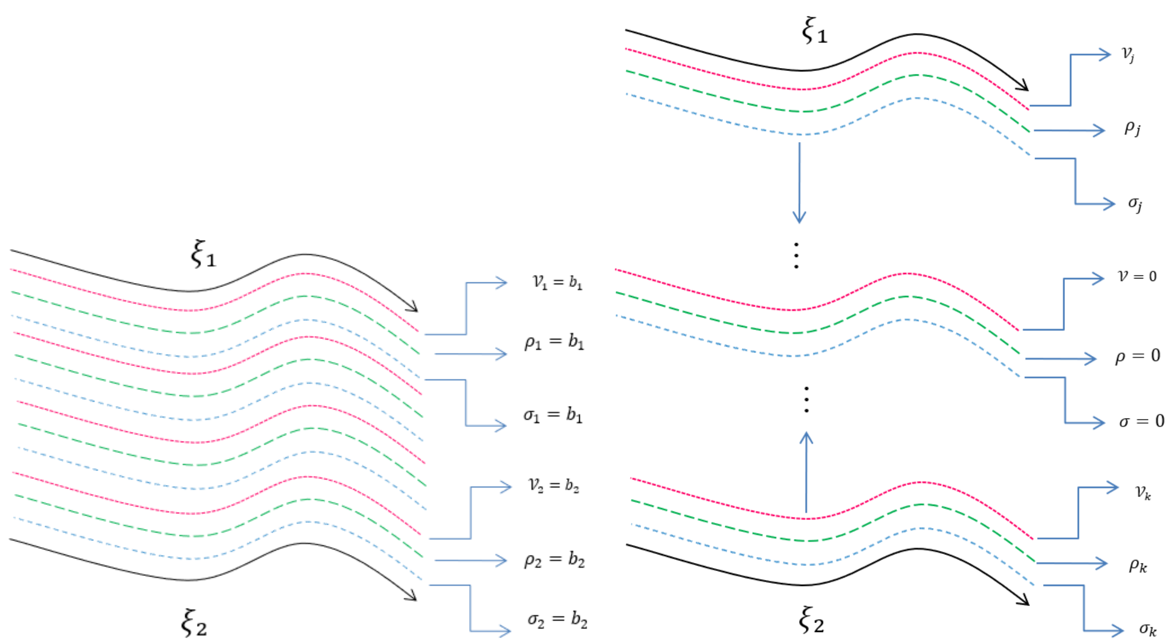
Definition 3.3. The neutrosophic arc $\zeta : [0,\ 1] \to \mathcal{R}^3$ is called a simple neutrosophic arc if, for each $\mathrm{z_j}$, $\mathrm{z_k}\in[0,\ 1]$, $\xi((\mathrm{z_j},\ \omega_\mathrm{j}) \neq \xi((\mathrm{z_k},\ \omega_\mathrm{k})$ whenever $(\mathrm{z_j}\ ,\ \omega_\mathrm{j})\neq(\mathrm{z_k},\ \omega_\mathrm{k})$.

Now, we will delve into the notion of neutrosophy homotopic and describe two types of it.

Definition 3.4. A neutrosophic homotopy is a collection of neutrosophic maps $h_t : \langle M,\ \omega\rangle \to \langle N,\ \omega\rangle$, $t \in [0, 1]$, in which the associated neutrosophic map $\Phi : \langle M,\ \omega\rangle \times [0, 1] \to \langle N,\ \omega\rangle$ given by $\Phi((x,\ \omega), t) = h_t(x,\ \omega)$, and the two neutrosophic maps $h_0,\ h_1 : \langle M,\ \omega\rangle \to \langle N,\ \omega\rangle$ are called neutrosophy homotopic if there is a neutrosophic homotopy h_t that connects them and is represented by $h_0 \approx h_1$.

Theorem 3.5. *Let ξ_1 and ξ_2 be two neutrosophic arcs. Then, there are two types of neutrosophy homotopic arcs.*

Proof. The initial category encompasses a pair of neutrosophic arcs, ξ_1 and ξ_2 with specific values for ω_1 and ω_2 namely, $\omega_1=\mathrm{b}_1$ and $\omega_2=\mathrm{b}_2$ for all points of the arcs as shown in Fig.1a. The second category encompasses a pair of neutrosophic arcs, ξ_1 and ξ_2 with specific values for ω_1 and ω_2, where ξ_1 is a neutrosophic arc that has values for ω_j in the form of $\langle \upsilon_j,\ \rho_j,\ \sigma_j\ \rangle$ and ξ_2 is a neutrosophic arc that has values for ω_k in the form of $\langle \upsilon_k,\ \rho_k,\ \sigma_k\ \rangle$ for which max $\omega_j \longrightarrow 0$ or max $\omega_k \longrightarrow 0$ where, $\omega_j,\ \omega_k \in [0,\ 1]$, as shown in Fig.1b. □



(A) Neutrosophy homotopic of type (I) (B) Neutrosophy homotopic of type (II)

FIGURE 1. neutrosophy homotopic

Definition 3.6. Let $\langle M, \omega \rangle$ be a neutrosophic submanifold with a neutrosophic submanifold $\langle \mathcal{C}, \omega \rangle$, and let us consider the existence of a continuous neutrosophic map $\mathfrak{d} : \langle M, \omega \rangle \longrightarrow \langle \mathcal{C}, \omega \rangle$ for which $\mathfrak{d}(\mathfrak{c}, \omega(\mathfrak{c})) = (\mathfrak{c}\ \omega(\mathfrak{c}))$, $\forall \mathfrak{c} \in \mathcal{C}$. Then, $\mathfrak{d}$ is called neuretraction.

Example 3.7. $\langle S^1, \omega \rangle$ is neuretraction of $\langle R^2 - \{0\}, \omega \rangle$.

Based on Definition 3.6, we can conclude that any of the following situations qualify as neuretraction:

Definition 3.8. (a) $\mathfrak{d}(\mathfrak{c}, \omega(\mathfrak{c})) = (\mathfrak{c}, \min(\upsilon), \min(\rho), \min(\sigma))$
(b) $\mathfrak{d}(\mathfrak{c}, \omega(\mathfrak{c})) = (\mathfrak{c}, \max(\upsilon), \max(\rho), \max(\sigma))$
(c) $\mathfrak{d}(\mathfrak{c}, \omega(\mathfrak{c})) = (\mathfrak{c}, \omega \in (0, 1))$. Now, for the rest of our discussion, and for simplicity, we shall denote the neutrosophic manifold $\langle M, \omega \rangle$ by the symbol M.

To show that isometry exists on both the upper and lower neutrosophic hypermanifolds, we shall use the potent framework of neutrosophic theory in the concept that follows.

Definition 3.9. A map $\mathcal{F} : \cup M \longrightarrow \cup M$ is said to be an isoneutrosophic folding if $\mathcal{F}(M) = M$ and for each member of the upper neutrosophic hypermanifold $\overline{M}_g$, there is a $\underline{M}_{\mathfrak{g}}$ lower M for which $\overline{\varpi}_{\mathfrak{g}} = \underline{\omega}_{\mathfrak{g}}$ for any corresponding point, i.e., $\overline{\varpi_{\mathfrak{g}}}(\overline{c}) = \underline{\omega}_{\mathfrak{g}}(\underline{c})$ as shown in Fig.2

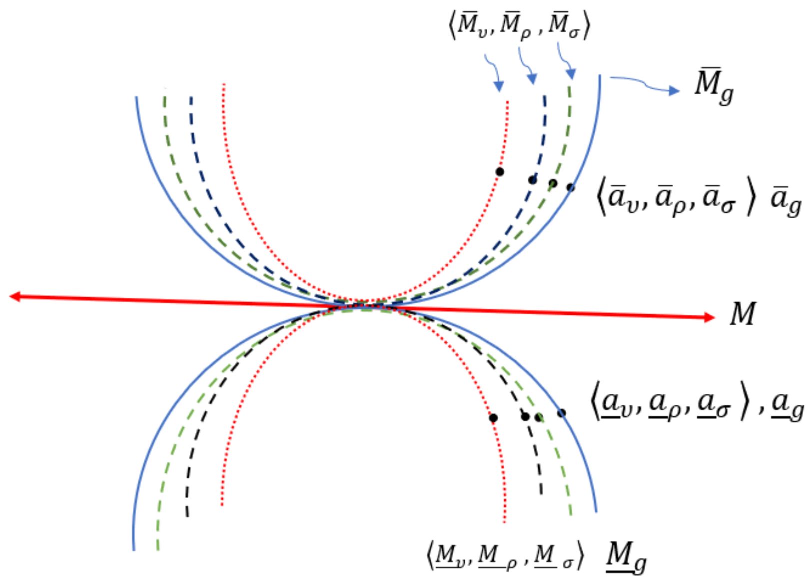


FIGURE 2. Isoneutrosophic folding

Theorem 3.10. *Assuming M is a neutrosophic hyperspace in $\mathcal{R}^{m+1}$. Then, we conclude that there are two types of neutrosophication that coincide in M.*

(a) For every $\mathfrak{c} \in M$, $\omega(\mathfrak{c}) = \langle 1,\ 1,\ 1\rangle$. Geometrically, parallel neutrosophic manifolds, which is known as the "crisp property."

(b) For each distinct point $\mathfrak{c}_{t_1},\ \mathfrak{c}_{t_2} \in M$ and, $\omega(\mathfrak{c}_{t_1}) \neq \omega(\mathfrak{c}_{t_2})$, there is a chain of homeomorphic neutrosophic manifolds connected at a common point.

Proof. (a) In "a crisp property." For all $y_{t_1},\ y_{t_2} \in M$, we have $\omega(y_{t_1}) = \omega(y_{t_2}) = \langle 1,\ 1,\ 1\rangle$ also, all neutrosophic hypermanifolds M_s are parallel, $\forall \mathfrak{c}_s \in \underline{M}_s,\ \omega(\mathfrak{c}_s) < \langle 1,\ 1,\ 1\rangle$ and $\forall \mathfrak{c}_1, \mathfrak{c}_2 \in M_s$, $\omega(\mathfrak{c}_1) = \omega(\mathfrak{c}_2)$, $M_s = \overline{M}_s$ or $M_s = \underline{M}_s$ as shown in Fig.3. In this situation, we can define ω as

$\omega = \langle \upsilon,\ \rho,\ \sigma\rangle$ where

$$\langle \upsilon,\ \rho,\ \sigma\rangle = \left\langle \begin{cases} \frac{1}{1+l_1} & \text{if} \quad l_1 > 0 \\ \frac{1}{1-l_1} & \text{if} \quad l_1 < 0 \end{cases}, \begin{cases} \frac{1}{1+l_2} & \text{if} \quad l_2 > 0 \\ \frac{1}{1-l_2} & \text{if} \quad l_2 < 0 \end{cases}, \begin{cases} \frac{1}{1+l_3} & \text{if} \quad l_3 > 0 \\ \frac{1}{1-l_3} & \text{if} \quad l_3 < 0 \end{cases} \right\rangle,$$

the list $(l_i,\ i = 1,\ 2,\ 3)$ can be represented in Fig.3. Moreover, we have $\omega = \langle 0,\ 0,\ 0\rangle$ whenever $l_i \longrightarrow \pm\infty$. However, this illustrates the degree of neutrosophication in the crisp case of M. In fact, $\forall \gamma$ there is a neutrosophic strip at γ, specifically ζ_γ for which $\omega(\mathfrak{c}) < \langle 1,\ 1,\ 1\rangle$ whenever $\mathfrak{c} \in \zeta_\gamma$ and decreases if $l_i \longrightarrow \pm\infty$.

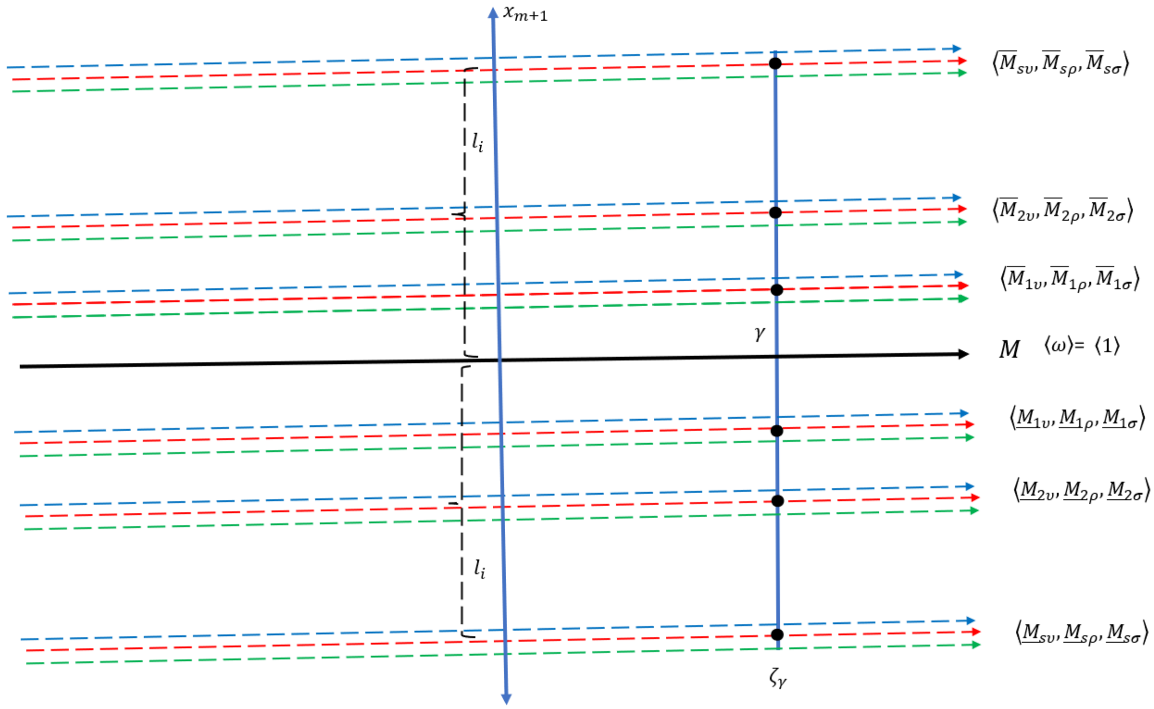


FIGURE 3. parallel hyperspaces and their isoneutrosophic folding

(b) Let M be a neutrosophic hyperspace for which $\omega(\mathfrak{c}_s) \neq \omega(\mathfrak{c}_t)$, $\mathfrak{c}_s \neq \mathfrak{c}_t$ in M, and suppose that $\mathfrak{q}$ is a point at which $\omega(\mathfrak{q}) = (\max \omega_s,\ s \in \mathbb{N})$. For all point $\gamma \in M$, $\exists$ neutrosophic strip ζ_γ such that $\omega(\mathfrak{c}_1) < \omega(\mathfrak{c}_2) < \omega(\gamma)$, whenever $\mathfrak{c}_1,\ \mathfrak{c}_2 \in \zeta_\gamma$. If there is no other common neutrosophic point than $\mathfrak{q}$, then $\exists$ a point $\mathfrak{c}_j \in M_j,\ j = 1,\ 2,\ 3$. For all horizontal neutrosophic strips, q has a maximum value (neutrosophic point) in the neutrosophic strip. □

The sequence of neuretraction within a neutrosophic hyperspace will be inferred from the data that follow.

Theorem 3.11. *If M is a neutrosophic hyperspace in $\mathcal{R}^{m+1}$, and $\mathfrak{d} : M \longrightarrow \mathcal{C}$, is a neuretraction. Then, there exists a sequence $\langle \mathfrak{d}_i : \cup M \longrightarrow \mathcal{C}_i,\ i = 1, 2, \ldots m \rangle$ of a neuretraction. Also, if we consider $dim\,(\cup M) = dim\ \mathcal{C}_i$, then all $\mathfrak{d}_i$ are special types of isoneutrosophic folding.*

Proof. Let $\mathcal{F}$ be a isoneutrosophic folding of $\cup \underline{M}$ into $\cup \overline{M}$ such that $\mathcal{F}(M) = M$ $\omega(\underline{M}) = \omega(\overline{M})$. Thus, we conclude $\mathcal{F}(\underline{M}) = \overline{M}$ as shown in Fig.4. Now for each neuretraction $\mathfrak{d}$: $M \longrightarrow \mathcal{C}$ (in a case of no common point) we obtain the induced neuretractions $\mathfrak{d}_i : M \longrightarrow \mathcal{C}_i$, $dim\ \mathcal{C}_i = dim M$. But if $\mathfrak{d} : M \longrightarrow p$, there are induced neuretractions $\underline{\mathfrak{d}}_i : \underline{M}_i \longrightarrow \underline{p_i}$ and $\overline{\mathfrak{d}_i} : \overline{M_i} \longrightarrow \overline{p_i}$. However, these neuretractions are not types of neutrosophic folding, because $dim\ \mathcal{C}_i \neq dim M_i$. For example, in Fig.5, $\exists$ an isoneutrosophic folding, whereas there is no isneutrosophic folding as a type of neuretraction in $\mathfrak{d} : \cup M \longrightarrow p$, since $dim\ p \neq dim M_i$. □

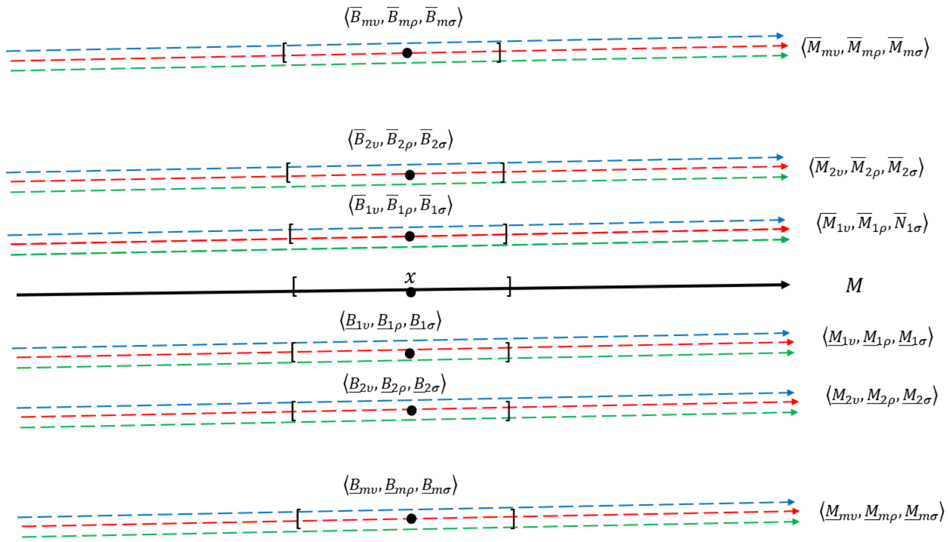


FIGURE 4. Isoneutrosophic folding on parallel hyperspaces

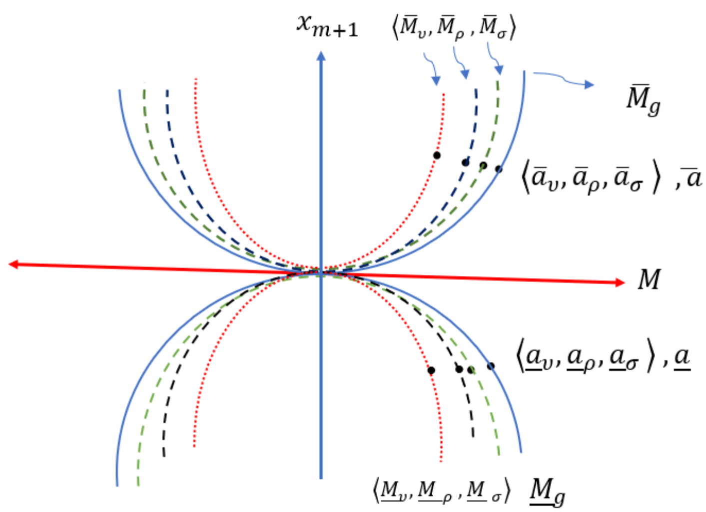


FIGURE 5. Neuretraction on hyperspaces with common point

The advanced results in this study reveal multiple occurrences of neuretractions corresponding to a set of neutrosophic manifolds that exhibit homeomorphism to a set of neutrosophic unit spheres with n dimensions, all of which possess a shared center.

Theorem 3.12. *Suppose that M is a neutrosophic manifold of dimension m, which is homeomorphic to a neutrosophic unit sphere, with $\omega(y_i) = 1$ for each $y_j \in \mathcal{S}^m$, else $\omega = \langle \upsilon,\ \rho,\ \sigma \rangle$ where*

$$\langle \upsilon,\ \rho,\ \sigma \rangle = \left\langle \begin{cases} l_1, & 0 < l_1 < 1 \\ \frac{1}{l_1}, & l_1 > 1 \end{cases}, \begin{cases} l_2, & 0 < l_2 < 1 \\ \frac{1}{l_2}, & l_2 > 1 \end{cases}, \begin{cases} l_3, & 0 < l_3 < 1 \\ \frac{1}{l_3}, & l_3 > 1 \end{cases} \right\rangle \text{ where, } y_j \in$$

$\cup\mathcal{S}_j^m$ as a union of m-dimensional neutrosophic spheres with a common center and let $H = \{(y,\ \omega) : |y| \le 1\}$ be an n-dimensional neutrosophic closed ball. Then, for every neuretraction of $(H - \mathfrak{q})$ onto $\mathcal{S}^{m-1}$ there are induced neuretractions of $H_j - \mathfrak{q}$ onto $\mathcal{S}_j^{m-1}$. Moreover, under the condition $\mathcal{F}(\mathcal{S}^m) = \mathcal{S}^m$, we get an isoneutrosophic folding $\mathcal{F} : \overline{\mathcal{S}}\,_j^m \longrightarrow \underline{\mathcal{S}}_j^m$.

Proof. Assume M is a neutrosophic manifold, $\mathcal{S}^n$ is a neutrosophic unit sphere, and M is homeomorphic to $\mathcal{S}^n$ as shown in Fig.6. If there is a neutrosophic sphere $\mathcal{S}^m$ inside the neutrosophic system, say $\underline{\mathcal{S}}_j^m$ (Neutrosophication will be reduced, $\omega = \langle \upsilon,\ \rho,\ \sigma\rangle \longrightarrow \langle 0,\ 0,\ 0\rangle$ if $l_i \longrightarrow 0$ and $\omega = \langle \upsilon,\ \rho,\ \sigma\rangle \longrightarrow \langle 0,\ 0,\ 0\rangle$ if $l_i \longrightarrow \infty$ for $i = 1, 2, 3$. Indeed, for all neutrosophic points (c, $\omega = \langle 1,\ 1,\ 1\rangle$) $\exists$ (a neutrosophic) strip of neutrosophic points $(\overline{c}_j,\ \overline{\omega}_j < \langle 1,\ 1,\ 1\rangle) \in \overline{\mathcal{S}}\,_j^m$, and $(\underline{c}_j,\ \underline{\omega}_j < \langle 1,\ 1,\ 1\rangle) \in \underline{\mathcal{S}}_j^m$. However, for the isoneutrosophic folding $\mathcal{F} : \overline{\mathcal{S}}\,_j^m \longrightarrow \underline{\mathcal{S}}_j^m$, in which $\overline{\omega_j} = \underline{\omega_j}$ there is an induced isoneutrosophic folding $\overline{\mathcal{F}} : \overline{H}_j \longrightarrow \underline{H}_j$, as well as neuretractions $\overline{\mathfrak{d}}_j : (\overline{H}_j - \mathfrak{q}) \longrightarrow \overline{\mathcal{S}}\,_j^{m-1}$ and $\underline{\mathfrak{d}}_j : (\underline{H}_j - \mathfrak{q}) \longrightarrow \underline{\mathcal{S}}_j^{m-1}$. □

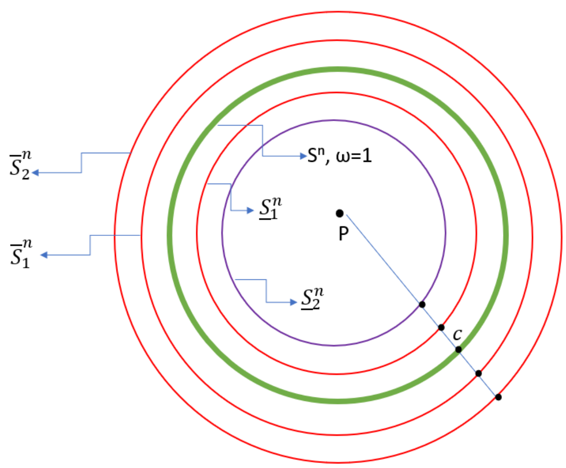


FIGURE 6. Neuretraction and neutrosophic folding on a spheres

Theorem 3.13. *Suppose that $\mathcal{N}$ is a neutrosophic manifold with $\mathfrak{d} : \mathcal{N} \longrightarrow \mathcal{M}$ is a neuretraction, then the geometric neuretraction $\mathfrak{d}_{\mathfrak{g}}$ induces a neuretractions $\mathfrak{d}_\upsilon,\ \mathfrak{d}_\rho,\ \mathfrak{d}_\sigma$. On the other hand, the converse is not true.*

Proof. Let us consider $\mathfrak{d} : \mathcal{N} \longrightarrow \mathcal{M}$ as a neuretraction, such that $\mathcal{N} = \langle \mathcal{N}_{\mathfrak{g}}, \mathcal{N}_\upsilon,\ \mathcal{N}_\rho, \mathcal{N}_\sigma\rangle$ *and* $\mathcal{M} \subseteq \mathcal{N}$. Now, consider the geometric neuretraction of $\mathfrak{d}_{\mathfrak{g}} : \mathcal{N}_{\mathfrak{g}} \longrightarrow \mathcal{M}_{\mathfrak{g}}$ of $\mathcal{N}_{\mathfrak{g}}$ into $\mathcal{M}_{\mathfrak{g}}$, then we get the induced neuretractions $\mathfrak{d}_\upsilon : \mathcal{N}_\upsilon \longrightarrow \mathcal{M}_\upsilon$, $\mathfrak{d}_\rho : \mathcal{N}_\rho \longrightarrow \mathcal{M}_\rho$, $\mathfrak{d}_\sigma : \mathcal{N}_\sigma \longrightarrow \mathcal{M}_\sigma$ as shown in Fig.7. On the other hand, consider the neuretractions $\mathfrak{d}_\upsilon : \mathcal{N}_\upsilon \longrightarrow \mathcal{M}_\upsilon$, $\mathfrak{d}_\rho : \mathcal{N}_\rho \longrightarrow \mathcal{M}_\rho$, $\mathfrak{d}_\sigma : \mathcal{N}_\sigma \longrightarrow \mathcal{M}_\sigma$ as the identity neuretractions for all membership degrees, which have no impact on the geometric manifold $\mathcal{N}_{\mathfrak{g}}$ as shown in Fig.8 .

□

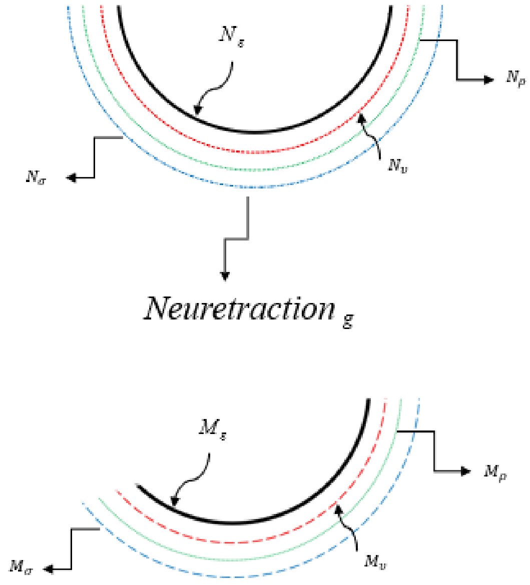


FIGURE 7. A neuretraction of type (I)

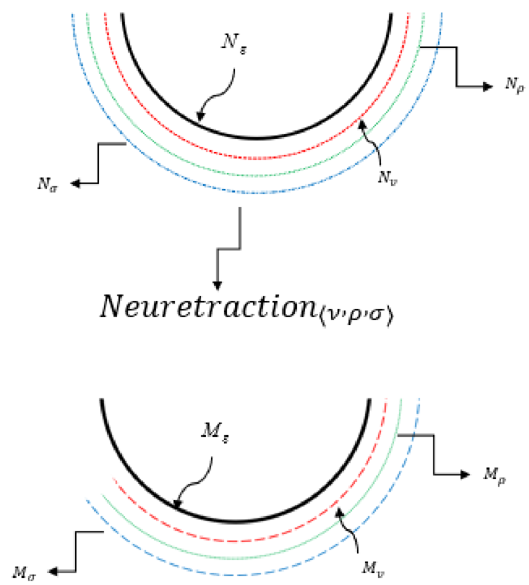


FIGURE 8. A neuretraction of type (II)

4. Conclusion

The present study aimed to develop a theoretical basis for neuretraction on a neutrosophic manifold. The neutrosophic folding and neuretraction on a neutrosophic manifold are achieved geometrically and topologically. The sequence of neuretractions in a neutrosophic hyperspace is obtained. The relationship between some types of transformations is deduced. An area that necessitates additional investigation pertains to the establishment and exploration of a fitting notion of neutrosophic homotopy groups, in conjunction with a thorough examination of their consequent neutrosophic homomorphism.

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Received: June 5, 2023. Accepted: Sep 27, 2023