Neutrosophic Sets and Systems

Volume 58

10-5-2023

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Neutrosophic $\kappa$-structures in an $AG$-groupoid

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Abstract. An $AG$-groupoid is the midway between commutative semigroup and groupoid. The core structure of Flock theory is an $AG$-groupoid, which focuses on motion replication and distance optimization and has numerous applications in physics and biology. Unfortunately, in many cases, modelling real-world problems in domains like computer science, operations research, artificial intelligence, control engineering, and robotics can be risky. Different theories, such as fuzzy sets, intuitionistic fuzzy sets, probability, soft sets, neutrosophic sets, and others, have been created to deal with similar situations. In this paper, We define the notions of neutrosophic $\kappa$-ideal structures in an $AG$-groupoid and investigate their properties. We also obtain equivalent assertion of neutrosophic $\kappa$-ideals and product of neutrosophic $\kappa$-structures in $AG$-groupoid.

Keywords: $AG$-groupoid; neutrosophic $\kappa$-structures; ideals; neutrosophic $\kappa$-ideals; neutrosophic $\kappa$-interior ideals.

1. Introduction

In [1], Zadeh pioneered the fuzzy set theory to model imprecise ideas in the globe. Atanassov expanded fuzzy set theory principles and termed it Intuitionistic fuzzy set in [2]. In his opinion, there are two types of degrees of freedom in an universe: non-membership in a specific subset and membership in a vague subset. In [3], Rosenfeld proposed the notion of fuzziness in groups and produced a number of results. Recently, several authors studied their research in this field, and similar notions are used in variety of algebraic structures, including semigroups, semiring, ordered semigroups, rings (refer, [4] - [14], [18]- [21]).

To deal with the uncertainty that exists everywhere, Smarandache suggested the notions of neutrosophic sets in [15]. It’s a combination of fuzzy sets and intuitionistic fuzzy sets that’s
been generalized. Neutrosophic sets are defined using these three properties, which include membership functions for truth ($T$), indeterminacy ($I$), and falsity ($F$). These sets can be used in a variety of fields to deal with the difficulties that result from ambiguous data. The relative and absolute membership functions can be distinguished by a neutrosophic set. Smarandache employed neutrosophic sets in non-standard analysis, such as control theory, decision making theory, sports decision (winning/losing/tie), and so on.

In BCK-algebra, Muhiuddin et al. discovered an association between $(\epsilon, \epsilon)$-neutrosophic subalgebra and $(\epsilon, \epsilon)$-neutrosophic ideal in [16], and Muhiuddin et al. created and investigated neutrosophic implicative $\kappa$-ideal in [17]. Additionally, the connection between several neutrosophic implicative $\kappa$-ideals were examined.

In semigroup, neutrosophic $\kappa$-subsemigroup and the $\epsilon$-neutrosophic $\kappa$-subsemigroup were defined and their different features were covered in [18] by Khan et al. We examined the properties of various neutrosophic $\kappa$-structure notions, namely neutrosophic $\kappa$-ideal structures in a semigroup, as inspiration from [18]. A neutrosophic $\kappa$-ideals in a semigroup were suggested by B. Elavarasan et al. in [19] and different features were achieved. The comparable claims for the typical neutrosophic $\kappa$-structure were also given.

Porselvi et al. studied a number of characteristics of the neutrosophic $\kappa$-bi-ideal in a semigroup in [20], and neutrosophic $\kappa$-interior ideal in [21]. We have established equivalent claims for regular semigroup. In [22], Elavarasan et al. presented and studied neutrosophic $\kappa$-filters in semigroups. In [23], Muhiuddin and others proposed the concepts of neutrosophic $\kappa$-structures in ordered semigroup, and examined their properties. Smarandache proposed neutrosophic topologies in [26], Runu Dhar studied compactness and neutrosophic topological space in [27], Sudeep Dey et al. presented neutrosophic composite relation in [28].

We present the ideas of neutrosophic $\kappa$-ideal structures in an AG-groupoid in this paper. We prove that the product of two neutrosophic $\kappa$-right-ideal is a neutrosophic $\kappa$-bi-ideal, and neutrosophic $\kappa$-right-ideal is equivalent to neutrosophic $\kappa$-interior-ideal, under certain condition.

2. Preliminaries

Unless otherwise specified, $\mathcal{M}$ denotes an AG-groupoid throughout this paper. Here is a glossary of the definitions we have already used for your perusal.

For $M_1, M_2 \subseteq \mathcal{M}$, we denote $(M_1) = \{k \in \mathcal{M} : k \leq m \text{ for some } m \in M_1\}$ and $M_1M_2 = \{k_1k_2 : \text{for all } k_1 \in M_1 \text{ and } k_2 \in M_2\}$. Following [24] and [25], an AG-groupoid, $\mathcal{M}$, is a groupoid whose elements hold the left invertive law: $(m_1m_2)k_3 = (k_3m_2)m_1$ for all $m_1, m_2, k_3 \in \mathcal{M}$. An AG-groupoid structure lies between a commutative semigroup and a groupoid. In $\mathcal{M}$, the medial law $(m_1m_2)(k_3k_4) = (m_1k_3)(m_2k_4)$ for all $m_1, m_2, k_3, k_4 \in \mathcal{M}$ holds. If there is
an element $e \in \mathcal{M} \ni em = m \ \forall \ m \in \mathcal{M}$, then $e$ is the left identity. If $\mathcal{M}$ has a right identity, then $\mathcal{M}$ is said to be commutative monoid. If $\mathcal{M}$ is having a left identity, then $(m_1m_2)(k_3k_4) = (k_4k_3)(m_2m_1)$ holds for all $m_1, m_2, k_3, k_4 \in \mathcal{M}$. An element $m \in \mathcal{M}$ is said to be idempotent if $m^2 = m$.

Let $\mathcal{M}$ be an AG-groupoid and $\phi \neq M \subseteq \mathcal{M}$. Then $M$ is called a AG-subgroupoid of $\mathcal{M}$ (see [24]) if $M^2 \subseteq M$. A subset $M \neq \phi$ in $\mathcal{M}$ is called a left(respectively, right) ideal if $M \mathcal{M} \subseteq M$ (respectively, $M \mathcal{M} \subseteq M$), and $M$ is said to be an ideal if it is both a right and a left ideal of $\mathcal{M}$. A subset $M \neq \phi$ in $\mathcal{M}$ is said to be an interior ideal if $(\mathcal{M} \mathcal{M})M \subseteq M$. A subset $M \neq \phi$ in $\mathcal{M}$ is said to be an idempotent if $M \mathcal{M} = M$.

Let $\mathcal{M}$ be an AG-groupoid. Then a function $\nu : \mathcal{M} \to [-1, 0]$ is the $\nu$-function on $\mathcal{M}$, and the set of all the $\nu$-functions is given by $F(\mathcal{M}, [-1, 0])$. A $\nu$-structure is an ordered pair $(\mathcal{M}, h)$ of $\mathcal{M}$ and an $\nu$-function $\nu$ on $\mathcal{M}$.\n
**Definition 2.1.** Let $\mathcal{M}$ be an AG-groupoid. A neutrosophic $\nu$-structure in $\mathcal{M}$ is given in the form:

$$\mathcal{M}_{\nu} := \mathcal{M}_{\nu} = \left\{ (T_{\nu}(k), I_{\nu}(k), F_{\nu}(k)) \mid k \in \mathcal{M} \right\},$$

where $T_{\nu}, I_{\nu}$ and $F_{\nu}$ are the negative truth, negative falsity and negative indeterminacy membership functions respectively in $\mathcal{M}$ ($\nu$-functions). Clearly, $-3 \leq T_{\nu}(m) + I_{\nu}(m) + F_{\nu}(m) \leq 0$ $\forall \ m \in \mathcal{M}$.

Throughout this section, we assume that $\mathcal{M}_{\nu}$ and $\mathcal{M}_{\nu}$ are neutrosophic $\nu$-structures in $\mathcal{M}$, unless otherwise stated.

**Notation 1.** We denote the set of

(i) neutrosophic $\nu$-left ideal by $\mathcal{M}_1$,
(ii) neutrosophic $\nu$-right ideal by $\mathcal{M}_r$,
(iii) neutrosophic $\nu$-ideal by $\mathcal{M}_i$,
(iv) neutrosophic $\nu$-bi-ideal by $\mathcal{M}_b$,
(v) neutrosophic $\nu$-interior ideal by $\mathcal{M}_n$,
(vi) neutrosophic $\nu$-AG-subgroupoid by $\mathcal{M}_s$,
(vii) neutrosophic $\nu$-idempotent by $\mathcal{M}_d$.

**Definition 2.2.** Let $\mathcal{M}_{\nu} \in \mathcal{M}$. Then $\mathcal{M}_{\nu} \subseteq \mathcal{M}_s$ provided the below condition is valid:

$$\forall m_1, m_2 \in \mathcal{M} \left( \begin{array}{c}
T_{\nu}(m_1m_2) \leq T_{\nu}(m_1) \lor T_{\nu}(m_2) \\
I_{\nu}(m_1m_2) \geq I_{\nu}(m_1) \land I_{\nu}(m_2) \\
F_{\nu}(m_1m_2) \leq F_{\nu}(m_1) \lor F_{\nu}(m_2)
\end{array} \right).$$

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Let $\nu, \gamma, \omega \in [-1, 0]$. Consider the sets:
\[
T_{\zeta}^\nu = \{ m_1 \in \mathcal{M} \mid T_{\zeta}(m_1) \leq \nu \}, \\
I_{\zeta}^\gamma = \{ m_1 \in \mathcal{M} \mid I_{\zeta}(m_1) \geq \gamma \}, \\
F_{\zeta}^\omega = \{ m_1 \in \mathcal{M} \mid F_{\zeta}(m_1) \leq \omega \}.
\]

The set $\mathcal{M}_\zeta(\nu, \gamma, \omega) := \{ m_1 \in \mathcal{M} \mid T_{\zeta}(m_1) \leq \nu, I_{\zeta}(m_1) \geq \gamma, F_{\zeta}(m_1) \leq \omega \}$ is known as $(\nu, \gamma, \omega)$-level set on $\mathcal{M}_\zeta$. Obviously, $\mathcal{M}_\zeta(\nu, \gamma, \omega) = T_{\zeta}^\nu \cap I_{\zeta}^\gamma \cap F_{\zeta}^\omega$.

**Definition 2.3.** Let $\mathcal{M}_\zeta \in \mathcal{M}$. Then $\mathcal{M}_\zeta \in \mathcal{M}_i$ provided the below conditions are valid:

(i) $(\forall m_1, m_2 \in \mathcal{M}) \left( T_{\zeta}(m_1 m_2) \leq T_{\zeta}(m_2) \right)$.

(ii) $(\forall m_1, m_2 \in \mathcal{M}) \left( I_{\zeta}(m_1 m_2) \geq I_{\zeta}(m_2) \right)$.

If condition (i) hold, then $\mathcal{M}_\zeta \in \mathcal{M}_i$. If condition (ii) hold, then $\mathcal{M}_\zeta \in \mathcal{M}_f$.

**Definition 2.4.** Let $\mathcal{M}_\zeta \in \mathcal{M}_b$. Then $\mathcal{M}_\zeta \in \mathcal{M}_b$ if the below assertion is valid:

$$(\forall a, k_1, k_2 \in \mathcal{M}) \left( \begin{array}{c}
T_{\zeta}(k_1 a k_2) \leq T_{\zeta}(k_1) \lor T_{\zeta}(k_2) \\
I_{\zeta}(k_1 a k_2) \geq I_{\zeta}(k_1) \land I_{\zeta}(k_2) \\
F_{\zeta}(k_1 a k_2) \leq F_{\zeta}(k_1) \lor F_{\zeta}(k_2)
\end{array} \right).$$

It is obvious that for any $\mathcal{M}_\zeta \in \mathcal{M}_i$, we have $\mathcal{M}_\zeta \in \mathcal{M}_b$. The converse need not be true, as shown by an example.

**Example 2.5.** Suppose $\mathcal{M} := \{ x_1, x_2, x_3, x_4, x_5 \}$. Then $(\mathcal{M}, \cdot)$ is an AG-groupoid as given below:

\[
\begin{array}{c|ccccc}
. & x_1 & x_2 & x_3 & x_4 & x_5 \\
x_1 & x_1 & x_4 & x_1 & x_4 & x_4 \\
x_2 & x_1 & x_2 & x_1 & x_4 & x_4 \\
x_3 & x_1 & x_4 & x_3 & x_4 & x_5 \\
x_4 & x_1 & x_4 & x_1 & x_4 & x_4 \\
x_5 & x_1 & x_4 & x_3 & x_4 & x_5 \\
\end{array}
\]

Let

$\mathcal{M}_\zeta = \left\{ (-0.8, -0.2, -0.6), (-0.5, -0.9, -0.1), (-0.3, -0.1, -0.5), (-0.8, -0.2, -0.6), (-0.2, -0.5, -0.1) \right\}.$

Then $\mathcal{M}_\zeta \in \mathcal{M}_b$, and $\mathcal{M}_\zeta \notin \mathcal{M}_i$ as $T_{\zeta}(x_3 x_5) = -0.2 > T_{\zeta}(x_3)$, $I_{\zeta}(x_3 x_5) = -0.5 < I_{\zeta}(x_3)$, and $F_{\zeta}(x_3 x_5) = -0.1 > F_{\zeta}(x_3)$.

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Definition 2.6. Let $\mathcal{M}_\zeta \in \mathcal{M}$, then $\mathcal{M}_\zeta \in \mathcal{M}_n$ provided the below assertion is valid:

$$
(\forall a, m_1, m_2 \in \mathcal{M}) \left\{ \begin{array}{l}
T_\zeta(m_1 a m_2) \leq T_\zeta(a) \\
I_\zeta(m_1 a m_2) \geq I_\zeta(a) \\
F_\zeta(m_1 a m_2) \leq F_\zeta(a)
\end{array} \right.
$$

It is obvious that for any $\mathcal{M}_\zeta \in \mathcal{M}_i$, we have $\mathcal{M}_\zeta \in \mathcal{M}_n$. The converse is not true, as shown by an example.

Example 2.7. Let $\mathcal{M}$ be the collection of all positive integers with 0 except 1. Then under usual multiplication, $\mathcal{M}$ is an AG-groupoid.

Let

$$
\mathcal{M}_\zeta = \left\{ (-0.8,\ldots,0), (-0.3,\ldots,0), (-0.5,\ldots,0), (-0.2,\ldots,0), (0,\ldots,0), (2,\ldots,2), (5,\ldots,5), (10,\ldots,10), \text{ otherwise} \right\}.
$$

Then $\mathcal{M}_\zeta \in \mathcal{M}_n$, and $\mathcal{M}_\zeta \notin \mathcal{M}$, as $T_\#(2.5) = -0.2 > T_\#(2)$ and $T_\#(2.5) = -0.2 > T_\#(5)$.

Definition 2.8. For any $\mathcal{X} \subseteq \mathcal{M}$, the characteristic neutrosophic $\chi$-structure in $\mathcal{M}$ is referred as

$$
\chi_\mathcal{X}(\mathcal{M}_\zeta) = \frac{\mathcal{M}}{\left( \chi_\mathcal{X}(T)_\zeta, \chi_\mathcal{X}(I)_\zeta, \chi_\mathcal{X}(F)_\zeta \right)}
$$

where

$$
\chi_\mathcal{X}(T)_\zeta : \mathcal{M} \to [-1,0], m_1 \mapsto \begin{cases} -1 & \text{if } m_1 \in \mathcal{X} \\ 0 & \text{otherwise,} \end{cases}
$$

$$
\chi_\mathcal{X}(I)_\zeta : \mathcal{M} \to [-1,0], m_1 \mapsto \begin{cases} 0 & \text{if } m_1 \in \mathcal{X} \\ -1 & \text{otherwise,} \end{cases}
$$

$$
\chi_\mathcal{X}(F)_\zeta : \mathcal{M} \to [-1,0], m_1 \mapsto \begin{cases} -1 & \text{if } m_1 \in \mathcal{X} \\ 0 & \text{otherwise.} \end{cases}
$$

Definition 2.9. Let $\mathcal{M}_\zeta := \frac{\mathcal{M}}{(T_\zeta, I_\zeta, F_\zeta)} \in \mathcal{M}$ and $\mathcal{M}_\zeta := \frac{\mathcal{M}}{(T_\zeta, I_\zeta, F_\zeta)} \in \mathcal{M}$. Then

(i) $\mathcal{M}_\zeta$ is said to be a neutrosophic $\chi$-substructure in $\mathcal{M}_\zeta$, denote by $\mathcal{M}_\zeta \subseteq \mathcal{M}_\zeta$, if $T_\zeta(m_1) \geq T_\zeta(m_1), I_\zeta(m_1) \leq I_\zeta(m_1), F_\zeta(m_1) \geq F_\zeta(m_1)$ for all $m_1 \in \mathcal{M}$.

If $\mathcal{M}_\zeta \subseteq \mathcal{M}_\zeta$ and $\mathcal{M}_\zeta \subseteq \mathcal{M}_\zeta$, then we write $\mathcal{M}_\zeta = \mathcal{M}_\zeta$.

(ii) The union of $\mathcal{M}_\zeta$ and $\mathcal{M}_\zeta$ over $\mathcal{M}$ is described as

$$
\mathcal{M}_\zeta \cup \mathcal{M}_\zeta = \mathcal{M}_\zeta \cup \mathcal{M}_\zeta = (\mathcal{M}; T_\zeta, I_\zeta, F_\zeta),
$$

where $\forall m_1 \in \mathcal{M}$,

$$
(T_\zeta \cup T_\zeta)(m_1) = T_\zeta(m_1) \land T_\zeta(m_1),
$$

$$
(I_\zeta \cup I_\zeta)(m_1) = I_\zeta(m_1) \lor I_\zeta(m_1),
$$

$$
(F_\zeta \cup F_\zeta)(m_1) = F_\zeta(m_1) \land F_\zeta(m_1).
$$

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(iii) The intersection of $\mathcal{M}_\xi$ and $\mathcal{M}_\zeta$ over $\mathcal{M}$ is described as
$$\mathcal{M}_\xi \cap \mathcal{M}_\zeta = \mathcal{M}_{\xi \cap \zeta} = (\mathcal{M}; T_{\xi \cap \zeta}, I_{\xi \cap \zeta}, F_{\xi \cap \zeta}),$$
where $\forall m_1 \in \mathcal{M}, (I_{\xi \cap \zeta}(m_1) = I_{\xi \cap \zeta}(m_1) = I_{\xi \cap \zeta}(m_1), \ldots)
(F_{\xi \cap \zeta}(m_1) = F_{\xi \cap \zeta}(m_1) = F_{\xi \cap \zeta}(m_1)).$

3. Main Results

We present some characteristics of neutrosophic $\kappa$-ideal structures in an $AG$-groupoid $\mathcal{M}$. In $\mathcal{M}$, neutrosophic $\kappa$-ideals are clearly neutrosophic $\kappa$-interior ideals, but the converse is true under certain conditions.

**Theorem 3.1.** For any $\mathcal{M}$, $(\mathcal{M}_\xi, \circ)$ is an $AG$-groupoid.

**Proof.** It is clear that $(\mathcal{M}_\xi, \circ)$ is closed. Let $\mathcal{M}_\xi, \mathcal{M}_\zeta, \mathcal{M}_\varphi \in \mathcal{M}$. Then for any $t \in \mathcal{M},$

$$(T_{\xi} \circ T_{\zeta}) \circ T_{\varphi}(t) = (T_{\xi} \circ T_{\zeta})(y) \vee T_{\varphi}(z) = \bigwedge \{ \bigwedge \{ T_{\xi}(r) \vee T_{\zeta}(s) \} \vee T_{\varphi}(z) \}$$

$$(I_{\xi} \circ I_{\zeta}) \circ I_{\varphi}(t) = \bigvee \{ (I_{\xi} \circ I_{\zeta})(y) \wedge I_{\varphi}(z) \}$$

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\[ ((F_\xi \circ F_\zeta) \circ F_\vartheta)(t) = \bigwedge_{t=yz} \{ (F_\xi \circ F_\zeta)(y) \lor F_\vartheta(z) \} \]
\[ = \bigwedge_{t=yz} \left\{ \bigwedge_{y=rs} \{ F_\xi(r) \lor F_\zeta(s) \lor F_\vartheta(z) \} \right\} \]
\[ = \bigwedge_{t=(rs)z} \{ F_\xi(r) \lor F_\zeta(s) \lor F_\vartheta(z) \} \]
\[ = \bigwedge_{t=(zs)r} \{ F_\vartheta(z) \lor F_\zeta(s) \lor F_\xi(r) \} \]
\[ = \bigwedge_{t=ur} \{ (F_\vartheta \circ F_\zeta)(u) \lor F_\xi(r) \} \]
\[ = ((F_\vartheta \circ F_\zeta) \circ F_\xi)(t). \]

Therefore \((\mathcal{M}_\xi, \circ)\) is an AG-groupoid. □

**Corollary 3.2.** For any \(\mathcal{M}_\xi, \mathcal{M}_\zeta, \mathcal{M}_\vartheta, \mathcal{M}_\varphi \in \mathcal{M}\), \((\mathcal{M}_\xi \circ \mathcal{M}_\zeta) \circ (\mathcal{M}_\vartheta \circ \mathcal{M}_\varphi) = (\mathcal{M}_\xi \circ \mathcal{M}_\vartheta) \circ (\mathcal{M}_\zeta \circ \mathcal{M}_\varphi)\).

**Proof.** Let \(\mathcal{M}_\xi, \mathcal{M}_\zeta, \mathcal{M}_\vartheta, \mathcal{M}_\varphi \in \mathcal{M}\). Then
\[
(T_\xi \circ T_\zeta) \circ (T_\vartheta \circ T_\varphi) = ((T_\vartheta \circ T_\varphi) \circ T_\zeta) \circ T_\xi = ((T_\zeta \circ T_\vartheta) \circ T_\varphi) \circ T_\xi = (T_\xi \circ T_\vartheta) \circ (T_\zeta \circ T_\varphi),
\]
\[
(F_\xi \circ F_\zeta) \circ (F_\vartheta \circ F_\varphi) = ((F_\vartheta \circ F_\varphi) \circ F_\zeta) \circ F_\xi = ((F_\zeta \circ F_\vartheta) \circ F_\varphi) \circ F_\xi = (F_\xi \circ F_\vartheta) \circ (F_\zeta \circ F_\varphi)
\]
and
\[
(I_\zeta \circ I_\varphi) \circ (I_\vartheta \circ I_\varphi) = ((I_\vartheta \circ I_\varphi) \circ I_\zeta) \circ I_\varphi = ((I_\zeta \circ I_\vartheta) \circ I_\varphi) \circ I_\zeta = (I_\xi \circ I_\vartheta) \circ (I_\zeta \circ I_\varphi).
\]
Hence \((\mathcal{M}_\xi \circ \mathcal{M}_\zeta) \circ (\mathcal{M}_\vartheta \circ \mathcal{M}_\varphi) = (\mathcal{M}_\xi \circ \mathcal{M}_\vartheta) \circ (\mathcal{M}_\zeta \circ \mathcal{M}_\varphi)\). □

**Theorem 3.3.** If \(\mathcal{M}\) has left identity, then for any \(\mathcal{M}_\xi, \mathcal{M}_\zeta, \mathcal{M}_\vartheta, \mathcal{M}_\varphi \in \mathcal{M}\), we have the following:

(i) \(\mathcal{M}_\xi \circ (\mathcal{M}_\zeta \circ \mathcal{M}_\vartheta) = (\mathcal{M}_\xi \circ \mathcal{M}_\zeta) \circ \mathcal{M}_\vartheta\),

(ii) \((\mathcal{M}_\xi \circ \mathcal{M}_\zeta) \circ (\mathcal{M}_\vartheta \circ \mathcal{M}_\varphi) = (\mathcal{M}_\vartheta \circ \mathcal{M}_\varphi) \circ (\mathcal{M}_\zeta \circ \mathcal{M}_\xi)\).

**Proof.** (i) Let \(m \in \mathcal{M}\). If \(m \neq xy\) for any \(x, y \in \mathcal{M}\), then
\[
(T_\xi \circ (T_\zeta \circ T_\vartheta))(m) = 0 = (T_\zeta \circ (T_\xi \circ T_\vartheta))(m),
\]
\[
(I_\zeta \circ (I_\xi \circ I_\varphi))(m) = -1 = (I_\xi \circ (I_\zeta \circ I_\varphi))(m),
\]
\[
(F_\xi \circ (F_\zeta \circ F_\varphi))(m) = 0 = (F_\zeta \circ (F_\xi \circ F_\varphi))(m).
\]
Suppose \(m = yz\) for \(y, z \in \mathcal{M}\). Then

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\( (T_\xi \circ (T_\xi \circ T_\mathcal{R}))(m) = \bigwedge_{m=yz} \{T_\xi(y) \lor (T_\xi \circ T_\mathcal{R})(z)\} \)
\[ = \bigwedge_{m=yz} \{T_\xi(y) \lor \bigwedge_{z=rs} \{T_\xi(r) \lor T_\mathcal{R}(s)\}\} \]
\[ = \bigwedge_{m=y(r)} \{T_\xi(r) \lor T_\xi(y) \lor T_\mathcal{R}(s)\} \]
\[ = \bigwedge_{m=r(y)} \{T_\xi(r) \lor T_\xi(y) \lor T_\mathcal{R}(s)\} \]
\[ = \bigwedge_{m=rp} \{T_\xi(r) \lor (T_\xi \circ T_\mathcal{R})(p)\} \]
\( (I_\xi \circ (I_\xi \circ I_\mathcal{R}))(m) = \bigvee_{m=yz} \{I_\xi(y) \land (I_\xi \circ I_\mathcal{R})(z)\} \)
\[ = \bigvee_{m=yz} \{I_\xi(y) \land \bigvee_{z=rs} \{I_\xi(r) \land I_\mathcal{R}(s)\}\} \]
\[ = \bigvee_{m=y(r)} \{I_\xi(r) \land I_\xi(y) \land I_\mathcal{R}(s)\} \]
\[ = \bigvee_{m=r(y)} \{I_\xi(r) \land I_\xi(y) \land I_\mathcal{R}(s)\} \]
\[ = \bigvee_{m=rp} \{I_\xi(r) \land (I_\xi \circ I_\mathcal{R})(p)\} \]
\( (F_\xi \circ (F_\xi \circ F_\mathcal{R}))(m) = \bigwedge_{m=yz} \{F_\xi(y) \lor (F_\xi \circ F_\mathcal{R})(z)\} \)
\[ = \bigwedge_{m=yz} \{F_\xi(y) \lor \bigvee_{z=rs} \{F_\xi(r) \lor F_\mathcal{R}(s)\}\} \]
\[ = \bigwedge_{m=y(r)} \{F_\xi(r) \lor F_\xi(y) \lor F_\mathcal{R}(s)\} \]
\[ = \bigwedge_{m=r(y)} \{F_\xi(r) \lor F_\xi(y) \lor F_\mathcal{R}(s)\} \]
\[ = \bigwedge_{m=rp} \{F_\xi(r) \lor (F_\xi \circ F_\mathcal{R})(p)\} \]
\[ = (F_\xi \circ (F_\xi \circ F_\mathcal{R}))(m). \]

Therefore \( \mathcal{M}_\xi \circ (\mathcal{M}_\xi \circ \mathcal{M}_\mathcal{R}) = \mathcal{M}_\xi \circ (\mathcal{M}_\xi \circ \mathcal{M}_\mathcal{R}). \)

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(ii) Let \( m \in \mathcal{M} \). If \( m \neq xy \) for any \( x, y \in \mathcal{M} \), then
\[
((T_\xi \circ T_\xi) \circ (T_\mathcal{A} \circ T_\mathcal{A}))(m) = 1 = ((T_\mathcal{A} \circ T_\mathcal{A}) \circ (T_\xi \circ T_\xi))(m).
\]
Suppose \( m = yz \) for any \( y, z \in \mathcal{M} \). Then
\[
((T_\xi \circ T_\xi) \circ (T_\mathcal{A} \circ T_\mathcal{A}))(m) = \bigwedge_{y=yz} \{((T_\xi \circ T_\xi)(y) \lor (T_\mathcal{A} \circ T_\mathcal{A})(z))
\]
\[=
\bigwedge_{y=yz} \{\bigwedge_{y=yz} \{T_\xi(p) \lor T_\xi(q)\} \lor \bigwedge_{z=rps} \{T_\mathcal{A}(r) \lor T_\mathcal{A}(s)\}\}
\]
\[=
\bigwedge_{m=yz} \{T_\xi(p) \lor T_\xi(q)\} \lor \bigwedge_{z=rps} \{T_\mathcal{A}(r) \lor T_\mathcal{A}(s)\}
\]
\[=
\bigwedge_{m=yz} \{T_\mathcal{A}(s) \lor T_\mathcal{A}(r) \lor T_\xi(q) \lor T_\xi(p)\}
\]
\[=
\bigwedge_{m=yz} \{(T_\mathcal{A} \circ T_\mathcal{A})(v) \lor (T_\xi \circ T_\xi)(w)\}
\]
\[=
((T_\mathcal{A} \circ T_\mathcal{A}) \circ (T_\xi \circ T_\xi))(m),
\]
\[
((I_\xi \circ I_\xi) \circ (I_\mathcal{A} \circ I_\mathcal{A}))(m) = \bigvee_{m=yz} \{(I_\xi \circ I_\xi)(y) \land (I_\mathcal{A} \circ I_\mathcal{A})(z)\}
\]
\[=
\bigvee_{m=yz} \{\bigvee_{m=yz} \{I_\xi(p) \land I_\xi(q)\} \land \bigvee_{z=rps} \{I_\mathcal{A}(r) \land I_\mathcal{A}(s)\}\}
\]
\[=
\bigvee_{m=yz} \{I_\xi(p) \land I_\xi(q) \land I_\mathcal{A}(r) \land I_\mathcal{A}(s)\}
\]
\[=
\bigvee_{m=yz} \{I_\mathcal{A}(s) \land I_\mathcal{A}(r) \land I_\xi(q) \land I_\xi(p)\}
\]
\[=
\bigvee_{m=yz} \{(I_\mathcal{A} \circ I_\mathcal{A})(v) \land (I_\xi \circ I_\xi)(w)\}
\]
\[=
((I_\mathcal{A} \circ I_\mathcal{A}) \circ (I_\xi \circ I_\xi))(m),
\]
\[
((F_\xi \circ F_\xi) \circ (F_\mathcal{A} \circ F_\mathcal{A}))(m) = \bigwedge_{m=yz} \{(F_\xi \circ F_\xi)(y) \lor (F_\mathcal{A} \circ F_\mathcal{A})(z)\}
\]
\[=
\bigwedge_{m=yz} \{\bigwedge_{m=yz} \{F_\xi(p) \lor F_\xi(q)\} \lor \bigwedge_{z=rps} \{F_\mathcal{A}(r) \lor F_\mathcal{A}(s)\}\}
\]
\[=
\bigwedge_{m=yz} \{F_\xi(p) \lor F_\xi(q) \lor F_\mathcal{A}(r) \lor F_\mathcal{A}(s)\}
\]
\[=
\bigwedge_{m=yz} \{F_\mathcal{A}(s) \lor F_\mathcal{A}(r) \lor F_\xi(q) \lor F_\xi(p)\}
\]
\[=
\bigwedge_{m=yz} \{F_\mathcal{A}(s) \lor F_\mathcal{A}(r) \lor F_\xi(q) \lor F_\xi(p)\}
\]
\[=
((F_\mathcal{A} \circ F_\mathcal{A}) \circ (F_\xi \circ F_\xi))(m),
\]

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\[= \bigwedge_{m=vw} \{(F_\varphi \circ F_{\varphi})(v) \lor (F_\zeta \circ F_\zeta)(w)\}\]

\[= ((F_\varphi \circ F_{\varphi}) \circ (F_\zeta \circ F_\zeta))(m).\]

Therefore \( (M_\zeta \circ M_\zeta) \circ (M_\varphi \circ M_\varphi) = (M_\varphi \circ M_\varphi) \circ (M_\zeta \circ M_\zeta). \]

**Theorem 3.4.** Let \( M_\zeta \in M \). Then the listed conditions hold:

(i) \( M_\zeta \in M_s \iff M_\zeta \circ M_\zeta \subseteq M_\zeta \).

(ii) \( M_\zeta \in M_l \iff \chi_{\varphi}(M_\zeta) \circ M_\zeta \subseteq M_\zeta \) for any \( M_\zeta \in M \).

(iii) \( M_\zeta \in M_r \iff M_\zeta \circ \chi_{\zeta}(M_\zeta) \subseteq M_\zeta \) for any \( M_\zeta \in M \).

(iv) \( M_\zeta \in M_i \iff \chi_{\varphi}(M_\zeta) \circ M_\zeta \subseteq M_\zeta \) and \( M_\zeta \circ \chi_{\zeta}(M_\zeta) \subseteq M_\zeta \) for any \( M_\zeta \in M \).

**Proof.**

(i) Assume \( M_\zeta \in M_s \). Now, for any \( k \in M \),

\[(T_\zeta \circ T_\zeta)(k) = \bigwedge_{k_1k_2} \{T_\zeta(k_1) \lor T_\zeta(k_2)\} \geq \bigwedge_{k_1k_2} T_\zeta(k_1k_2) = T_\zeta(k),\]

\[(I_\zeta \circ I_\zeta)(k) = \bigvee_{k_1k_2} \{I_\zeta(k_1) \land I_\zeta(k_2)\} \leq \bigvee_{k_1k_2} I_\zeta(k_1k_2) = I_\zeta(k),\]

\[(F_\zeta \circ F_\zeta)(k) = \bigwedge_{k_1k_2} \{F_\zeta(k_1) \lor F_\zeta(k_2)\} \geq \bigwedge_{k_1k_2} F_\zeta(k_1k_2) = F_\zeta(k).\]

So \( M_\zeta \circ M_\zeta \subseteq M_\zeta \).

Conversely, assume \( M_\zeta \circ M_\zeta \subseteq M_\zeta \). Now, for any \( k_1, k_2 \in M \),

\[T_\zeta(k_1k_2) \leq (T_\zeta \circ T_\zeta)(k_1k_2) = \bigwedge_{k_1k_2} \{T_\zeta(k_1) \lor T_\zeta(k_2)\} \leq T_\zeta(k_1) \lor T_\zeta(k_2),\]

\[I_\zeta(k_1k_2) \geq (I_\zeta \circ I_\zeta)(k_1k_2) = \bigvee_{k_1k_2} \{I_\zeta(k_1) \land I_\zeta(k_2)\} \geq I_\zeta(k_1) \land I_\zeta(k_2),\]

\[F_\zeta(k_1k_2) \leq (F_\zeta \circ F_\zeta)(k_1k_2) = \bigwedge_{k_1k_2} \{F_\zeta(k_1) \lor F_\zeta(k_2)\} \leq F_\zeta(k_1) \lor F_\zeta(k_2).\]

So \( M_\zeta \in M_s \).

(ii) Assuming \( M_\zeta \in M_l \). Now for any \( M_\zeta \in M \) and \( k \in M \),

\[\chi_{\varphi}(T_\zeta \circ T_\zeta)(k) = \bigwedge_{k_1k_2} \{\chi_{\varphi}(T_\zeta)(k_1) \lor T_\zeta(k_2)\} \]

\[= \bigwedge_{k_1k_2} T_\zeta(k_2) \]

\[\geq T_\zeta(k_1k_2) \]

\[= T_\zeta(k),\]
\[ (\chi_{\mathcal{M}}(I) \circ I_{\xi})(k) = \bigvee_{k=k_{1}k_{2}} \{ \chi_{\mathcal{M}}(I)(k_{1}) \land I_{\xi}(k_{2}) \} \]
\[ = \bigvee_{k=k_{1}k_{2}} I_{\xi}(k_{2}) \]
\[ \leq I_{\xi}(k_{1}k_{2}) \]
\[ = I_{\xi}(k), \]

\[ (\chi_{\mathcal{M}}(F) \circ F_{\xi})(k) = \bigwedge_{k=k_{1}k_{2}} \{ \chi_{\mathcal{M}}(F)(k_{1}) \lor F_{\xi}(k_{2}) \} \]
\[ = \bigwedge_{k=k_{1}k_{2}} F_{\xi}(k_{2}) \]
\[ \geq F_{\xi}(k_{1}k_{2}) \]
\[ = F_{\xi}(k). \]

Therefore \( \chi_{\mathcal{M}}(\mathcal{M}_{\xi}) \circ \mathcal{M}_{\xi} \subseteq \mathcal{M}_{\xi} \).

Conversely, suppose \( \chi_{\mathcal{M}}(\mathcal{M}_{\xi}) \circ \mathcal{M}_{\xi} \subseteq \mathcal{M}_{\xi} \) for any \( \mathcal{M}_{\xi} \in \mathcal{M} \). Now for any \( k_{1}, k_{2} \in \mathcal{M} \),

\[ T_{\xi}(k_{1}k_{2}) \leq (\chi_{\mathcal{M}}(T) \circ T_{\xi})(k_{1}k_{2}) \]
\[ = \bigwedge_{k=k_{1}k_{2}} \{ \chi_{\mathcal{M}}(T)(k_{1}) \lor T_{\xi}(k_{2}) \} \]
\[ \leq \chi_{\mathcal{M}}(T)(k_{1}) \lor T_{\xi}(k_{2}) \]
\[ = T_{\xi}(k_{2}), \]

\[ I_{\xi}(k_{1}k_{2}) \geq (\chi_{\mathcal{M}}(T) \circ I_{\xi})(k_{1}k_{2}) \]
\[ = \bigvee_{k=k_{1}k_{2}} \{ \chi_{\mathcal{M}}(T)(k_{1}) \land I_{\xi}(k_{2}) \} \]
\[ \geq \chi_{\mathcal{M}}(T)(k_{1}) \land I_{\xi}(k_{2}) \]
\[ = I_{\xi}(k_{2}), \]

\[ F_{\xi}(k_{1}k_{2}) \leq (\chi_{\mathcal{M}}(F) \circ F_{\xi})(k_{1}k_{2}) \]
\[ = \bigwedge_{k=k_{1}k_{2}} \{ \chi_{\mathcal{M}}(F)(k_{1}) \lor F_{\xi}(k_{2}) \} \]
\[ \leq \chi_{\mathcal{M}}(F)(k_{1}) \lor F_{\xi}(k_{2}) \]
\[ = F_{\xi}(k_{2}). \]

Hence \( \mathcal{M}_{\xi} \in \mathcal{M}_{1} \).

The proof of (iii) and (iv) is left to the reader. \( \Box \)

**Lemma 3.5.** (i) If \( \mathcal{M}_{\xi}, \mathcal{M}_{\xi} \in \mathcal{M}_{s} \), then \( \mathcal{M}_{\xi} \cap \mathcal{M}_{\xi} \in \mathcal{M}_{s} \).

(ii) If \( \mathcal{M}_{\xi}, \mathcal{M}_{\xi} \in \mathcal{M}_{1} \), then \( \mathcal{M}_{\xi} \cap \mathcal{M}_{\xi} \in \mathcal{M}_{1} \).

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(iii) If $\mathcal{M}_+, \mathcal{M}_- \in \mathcal{M}_r$, then $\mathcal{M}_+ \cap \mathcal{M}_- \in \mathcal{M}_r$.

(iv) If $\mathcal{M}_+, \mathcal{M}_- \in \mathcal{M}_l$, then $\mathcal{M}_+ \cap \mathcal{M}_- \in \mathcal{M}_l$.

Proof. (i) Let $\mathcal{M}_+$ and $\mathcal{M}_-$ be two neutrosophic $\kappa$-$AG$-subgroupoids in $\mathcal{M}$. Now for $k_1, k_2 \in \mathcal{M}$,

$$(T\chi \cap T\eta)(k_1k_2) = T\chi(k_1k_2) \vee T\eta(k_1k_2)$$

$$\leq (T\chi(k_1) \vee T\eta(k_2)) \vee (T\chi(k_1) \vee T\eta(k_2))$$

$$= (T\chi(k_1) \vee T\eta(k_1)) \vee (T\chi(k_2) \vee T\eta(k_2))$$

$$= (T\chi \cap T\eta)(k_1) \vee (T\chi \cap T\eta)(k_2),$$

$$(I\chi \cap I\eta)(k_1k_2) = I\chi(k_1k_2) \wedge I\eta(k_1k_2)$$

$$\geq (I\chi(k_1) \wedge I\chi(k_2)) \wedge (I\chi(k_1) \wedge I\chi(k_2))$$

$$= (I\chi(k_1) \wedge I\chi(k_2)) \wedge (I\chi(k_2) \wedge I\chi(k_2))$$

$$= (I\chi \cap I\chi)(k_1) \wedge (I\chi \cap I\chi)(k_2),$$

$$(F\chi \cap F\eta)(k_1k_2) = F\chi(k_1k_2) \vee F\eta(k_1k_2)$$

$$\leq (F\chi(k_1) \vee F\eta(k_2)) \vee (F\chi(k_1) \vee F\eta(k_2))$$

$$= (F\chi(k_1) \vee F\chi(k_1)) \vee (F\chi(k_2) \vee F\chi(k_2))$$

$$= (F\chi \cap F\chi)(k_1) \vee (F\chi \cap F\chi)(k_2).$$

So $\mathcal{M}_+ \cap \mathcal{M}_- \in \mathcal{M}_b$.

(ii) Let $\mathcal{M}_+, \mathcal{M}_- \in \mathcal{M}_l$. Now for any $k_1, k_2 \in \mathcal{M}$,

$$(T\chi \cap T\eta)(k_1k_2) = T\chi(k_1k_2) \vee T\eta(k_1k_2) \leq T\chi(k_2) \vee T\eta(k_2) = (T\chi \cap T\eta)(k_2),$$

$$(I\chi \cap I\eta)(k_1k_2) = I\chi(k_1k_2) \wedge I\chi(k_1k_2) \geq I\chi(k_2) \wedge I\chi(k_2) = (I\chi \cap I\chi)(k_2),$$

$$(F\chi \cap F\eta)(k_1k_2) = F\chi(k_1k_2) \vee F\eta(k_1k_2) \leq F\chi(k_2) \vee F\eta(k_2) = (F\chi \cap F\chi)(k_2).$$

So $\mathcal{M}_+ \cap \mathcal{M}_- \in \mathcal{M}_l$.

The proof of (iii) and (iv) is left to the reader. \(\square\)

**Lemma 3.6.** If $\mathcal{M}$ is having left identity $e$, then $\kappa_\mathcal{M}(\mathcal{M}_\pm) = \kappa_\mathcal{M}(\mathcal{M}_\pm) \odot \kappa_\mathcal{M}(\mathcal{M}_\pm)$ for any $\mathcal{M}_\pm \in \mathcal{M}$.

Proof. Let $k_1 \in \mathcal{M}$. Then $k_1 = ek_1$. Now,

$$(\kappa_\mathcal{M}(T)_\chi \circ \kappa_\mathcal{M}(T)_\eta)(k_1) = \bigwedge_{k_1=x_1x_2} \{\chi_\mathcal{M}(T)_\chi(x_1) \vee \chi_\mathcal{M}(T)_\eta(x_2)\} \leq \chi_\mathcal{M}(T)_\chi(e) \vee \chi_\mathcal{M}(T)_\eta(k_1) = 0$$

which implies $(\kappa_\mathcal{M}(T)_\chi \circ \kappa_\mathcal{M}(T)_\eta)(k_1) = 0 = \kappa_\mathcal{M}(T)_\chi(k_1)$.

$$(\kappa_\mathcal{M}(I)_\chi \circ \kappa_\mathcal{M}(I)_\eta)(k_1) = \bigvee_{k_1=x_1x_2} \{\chi_\mathcal{M}(I)_\chi(x_1) \wedge \chi_\mathcal{M}(I)_\eta(x_2)\} \geq \chi_\mathcal{M}(I)_\chi(e) \wedge \chi_\mathcal{M}(I)_\eta(k_1) = -1$$

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which implies \((\chi_{\mathcal{M}}(I)\circ \chi_{\mathcal{M}}(I))(k_1) = -1 = \chi_{\mathcal{M}}(I)(k_1)\).

\[(\chi_{\mathcal{M}}(F)\circ \chi_{\mathcal{M}}(F))(k_1) = \bigwedge_{k_1=x_1\times x_2} \{\chi_{\mathcal{M}}(F)(x_1) \vee \chi_{\mathcal{M}}(F)(x_2)\} \leq \chi_{\mathcal{M}}(F)(e) \vee \chi_{\mathcal{M}}(F)(k_1) = 0\]

which implies \((\chi_{\mathcal{M}}(F)\circ \chi_{\mathcal{M}}(F))(k_1) = 0 = \chi_{\mathcal{M}}(F)(k_1)\).

Therefore \(\chi_{\mathcal{M}}(\mathcal{M}_\xi) = \chi_{\mathcal{M}}(\mathcal{M}_\zeta) \circ \chi_{\mathcal{M}}(\mathcal{M}_\xi).\)

**Lemma 3.7.** If \(\mathcal{M}\) has left identity \(e\), then for any \(\mathcal{M}_\zeta \in \mathcal{M}\), we have \(\chi_{\mathcal{M}}(\mathcal{M}_\zeta) \circ \mathcal{M}_\xi = \mathcal{M}_\xi\) for every \(\mathcal{M}_\xi \in \mathcal{M}_1\).

**Proof.** Let \(k_1 \in \mathcal{M}\). Then \(k_1 = ek_1\). Now,

\[(\chi_{\mathcal{M}}(T)\circ (T))(k_1) = \bigwedge_{k_1=x_1\times x_2} \{\chi_{\mathcal{M}}(T)(x_1) \vee (T)(x_2)\} \leq \chi_{\mathcal{M}}(T)(e) \vee (T)(k_1) = (T)(k_1),\]

\[(\chi_{\mathcal{M}}(I)\circ (I))(k_1) = \bigvee_{k_1=x_1\times x_2} \{\chi_{\mathcal{M}}(I)(x_1) \wedge (I)(x_2)\} \geq \chi_{\mathcal{M}}(I)(e) \wedge (I)(k_1) = (I)(k_1),\]

\[(\chi_{\mathcal{M}}(F)\circ (F))(k_1) = \bigwedge_{k_1=x_1\times x_2} \{\chi_{\mathcal{M}}(F)(x_1) \vee (F)(x_2)\} \leq \chi_{\mathcal{M}}(F)(e) \vee (F)(k_1) = (F)(k_1).\]

So \(\mathcal{M}_\xi \subseteq \chi_{\mathcal{M}}(\mathcal{M}_\zeta) \circ \mathcal{M}_\xi\). By Theorem 3.4, \(\chi_{\mathcal{M}}(\mathcal{M}_\zeta) \circ \mathcal{M}_\xi \leq \mathcal{M}_\xi\) and hence \(\chi_{\mathcal{M}}(\mathcal{M}_\zeta) \circ \mathcal{M}_\xi = \mathcal{M}_\xi\). \(\square\)

**Proposition 3.8.** Suppose \(\mathcal{M}\) is having left identity. If \(\mathcal{M}_\xi, \mathcal{M}_\zeta \in \mathcal{M}_1\), then for any \(\mathcal{M}_\xi, \mathcal{M}_\eta \in \mathcal{M}\), \(\mathcal{M}_\xi \circ \mathcal{M}_\eta = \mathcal{M}_\zeta \circ \mathcal{M}_\eta\) implies \(\mathcal{M}_\xi \circ \mathcal{M}_\eta = \mathcal{M}_\xi \circ \mathcal{M}_\eta\).

**Proof.** Since \(\mathcal{M}_\xi, \mathcal{M}_\zeta \in \mathcal{M}_1\), we have by Lemma 3.7, \(\chi_{\mathcal{M}}(\mathcal{M}_\xi) \circ \mathcal{M}_\xi = \mathcal{M}_\xi\) and \(\chi_{\mathcal{M}}(\mathcal{M}_\eta) \circ \mathcal{M}_\eta = \mathcal{M}_\eta\) for \(\mathcal{M}_\xi, \mathcal{M}_\eta \in \mathcal{M}\). Now, for any \(\mathcal{M}_\eta \in \mathcal{M}\), \(\mathcal{M}_\xi \circ \mathcal{M}_\eta = (\chi_{\mathcal{M}}(\mathcal{M}_\xi) \circ \mathcal{M}_\eta) \circ \mathcal{M}_\eta = (\mathcal{M}_\xi \circ \mathcal{M}_\eta) \circ \chi_{\mathcal{M}}(\mathcal{M}_\eta) = (\mathcal{M}_\xi \circ \mathcal{M}_\eta) \circ \mathcal{M}_\eta = \mathcal{M}_\xi \circ \mathcal{M}_\eta = \mathcal{M}_\xi \circ \mathcal{M}_\eta = \mathcal{M}_\xi \circ \mathcal{M}_\eta = \mathcal{M}_\xi \circ \mathcal{M}_\eta.\) \(\square\)

**Corollary 3.9.** For any \(\mathcal{M}_\xi, \mathcal{M}_\zeta, \mathcal{M}_\eta \in \mathcal{M}\), the listed claims are equivalent:

(i) \((\mathcal{M}_\xi \circ \mathcal{M}_\zeta) \circ \mathcal{M}_\eta = \mathcal{M}_\xi \circ (\mathcal{M}_\zeta \circ \mathcal{M}_\eta)\),

(ii) \((\mathcal{M}_\xi \circ \mathcal{M}_\zeta) \circ \mathcal{M}_\eta = \mathcal{M}_\xi \circ (\mathcal{M}_\zeta \circ \mathcal{M}_\eta)\).

**Proposition 3.10.** Let \(\mathcal{M}_\xi \in \mathcal{M}_1\). If \(\mathcal{M}_\xi \in \mathcal{M}_d\), then \(\mathcal{M}_\xi \in \mathcal{M}_1\).

**Proof.** Let \(\mathcal{M}_\xi \in \mathcal{M}_1\) and \(\mathcal{M}_\xi \in \mathcal{M}_d\). Then for any \(\mathcal{M}_\zeta \in \mathcal{M}\), \(\mathcal{M}_\xi \circ \chi_{\mathcal{M}}(\mathcal{M}_\zeta) = (\mathcal{M}_\xi \circ \mathcal{M}_\zeta) \circ \mathcal{M}_\xi \subseteq \mathcal{M}_\xi \circ \mathcal{M}_\zeta = \mathcal{M}_\xi\), so \(\mathcal{M}_\xi \in \mathcal{M}_1\). \(\square\)

**Remark 3.11.** If \(\mathcal{M}\) has left identity, then \(\mathcal{M}_1 = \mathcal{M}_d\).

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Theorem 3.12. Suppose \( M \) has left identity and \( M_\zeta \in M \). Then the listed claims holds:

(i) \( \chi_\hat{M}(M_\zeta) \circ M_\zeta \in M \) for \( M_\zeta \in M \),

(ii) Every \( M_\zeta \in M_1 \) commutes with \( M_\zeta \).

Proof. (i) It is clear from Corollary 3.2 and Lemma 3.6.

(ii) Let \( M_\zeta, M_\xi \in M \). Now, \( M_\zeta \circ M_\zeta = (M_\zeta \circ M_\zeta) \circ M_\zeta = (M_\zeta \circ M_\zeta) \circ M_\zeta \subseteq (M_\zeta \circ \chi_\hat{M}(M_\zeta)) \circ M_\zeta \subseteq M_\zeta \circ M_\zeta \). Also, \( M_\zeta \circ M_\zeta = M_\zeta \circ (M_\zeta \circ M_\zeta) \subseteq M_\zeta \circ (M_\zeta \circ M_\zeta) \subseteq M_\zeta \circ (M_\zeta \circ M_\zeta) \subseteq M_\zeta \circ M_\zeta \). □

Lemma 3.13. If \( M \) has left identity and \( M_\zeta \in M_1 \), then \( M_\zeta \in M_1 \).

Proof. Let \( M_\zeta \in M_1 \). Then for \( M_\zeta \in M \), \( M_\zeta \circ \chi_\hat{M}(M_\zeta) \subseteq M_\zeta \). By Lemma 3.6, \( \chi_\hat{M}(M_\zeta) \circ M_\zeta = (\chi_\hat{M}(M_\zeta) \circ \chi_\hat{M}(M_\zeta)) \circ M_\zeta = (M_\zeta \circ \chi_\hat{M}(M_\zeta) \circ \chi_\hat{M}(M_\zeta)) \subseteq M_\zeta \circ \chi_\hat{M}(M_\zeta) \subseteq M_\zeta \). So \( M_\zeta \in M_1 \) and hence \( M_\zeta \in M_1 \).

Remark 3.14. Suppose \( M \) has left identity. If \( M_\zeta \in M_1 \), then \( M_\zeta \cup (\chi_\hat{M}(M_\zeta) \circ M_\zeta) \) and \( M_\zeta \cup (M_\zeta \circ M_\zeta) \) are neutrosophic \( \zeta \)-ideals for \( M_\zeta \in M \).

Theorem 3.15. Suppose \( M_\zeta \in M_1 \) with left identity. Then \( M_\zeta \cup (M_\zeta \circ \chi_\hat{M}(M_\zeta)) \) and \( M_\zeta \cup (M_\zeta \circ M_\zeta) \) are neutrosophic \( \zeta \)-ideals for \( M_\zeta \in M \).

Proof. Now,

\[
(M_\zeta \cup (M_\zeta \circ \chi_\hat{M}(M_\zeta))) \circ \chi_\hat{M}(M_\zeta) = (M_\zeta \circ \chi_\hat{M}(M_\zeta)) \cup ((M_\zeta \circ \chi_\hat{M}(M_\zeta)) \circ \chi_\hat{M}(M_\zeta))
\]

\[
= (M_\zeta \circ \chi_\hat{M}(M_\zeta)) \cup (\chi_\hat{M}(M_\zeta) \circ \chi_\hat{M}(M_\zeta)) \circ M_\zeta
\]

\[
= (M_\zeta \circ \chi_\hat{M}(M_\zeta)) \cup (\chi_\hat{M}(M_\zeta) \circ M_\zeta)
\]

\[
= (M_\zeta \circ \chi_\hat{M}(M_\zeta)) \cup M_\zeta
\]

\[
= M_\zeta \cup (M_\zeta \circ \chi_\hat{M}(M_\zeta))
\]

Thus \( M_\zeta \cup (M_\zeta \circ \chi_\hat{M}(M_\zeta)) \in M_1 \), and hence \( M_\zeta \cup (M_\zeta \circ \chi_\hat{M}(M_\zeta)) \in M_1 \) by Lemma 3.13. Now, for any \( M_\zeta \in M \),

\[
(M_\zeta \cup (M_\zeta \circ M_\zeta)) \circ \chi_\hat{M}(M_\zeta) = (M_\zeta \circ \chi_\hat{M}(M_\zeta)) \cup ((M_\zeta \circ M_\zeta) \circ \chi_\hat{M}(M_\zeta))
\]

\[
= (M_\zeta \circ \chi_\hat{M}(M_\zeta)) \cup ((M_\zeta \circ M_\zeta) \circ M_\zeta)
\]

\[
\subseteq (M_\zeta \circ \chi_\hat{M}(M_\zeta)) \cup (M_\zeta \circ M_\zeta)
\]

\[
= (M_\zeta \circ M_\zeta) \cup (M_\zeta \circ M_\zeta)
\]

\[
\subseteq (M_\zeta \circ M_\zeta) \cup M_\zeta
\]

\[
= M_\zeta \cup (M_\zeta \circ M_\zeta).
\]

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Thus $M_\xi \cup (M_\xi \circ M_\xi) \in M_r$ and so $M_\xi \cup (M_\xi \circ M_\xi) \in M_s$ by Lemma 3.13.

\textbf{Theorem 3.16.} Suppose $\phi \neq U \subseteq M$. Then the below claims are equivalent:

(i) $U$ is bi-ideal,

(ii) For any $M_\xi \in M$, $\chi_U(M_\xi) \in M_b$.

\textbf{Proof.} This is similar to Theorem 3.1 in [20].

\textbf{Lemma 3.17.} Let $M_\xi \in M_s$. Then the listed claims are equivalent:

(i) $M_\xi \in M_b$,

(ii) $(M_\xi \circ \chi_M(M_\xi)) \circ M_\xi \subseteq M_\xi$ for any $M_\zeta \in M$.

\textbf{Proof.} Assume $M_\xi \in M_b$ and let $k_1 \in M$. Suppose $\exists x_1, x_2 \in M \ni k_1 = x_1x_2$. Then

\[
(((T)_\xi \circ \chi_M(T)_\zeta) \circ (T)_\xi)(k_1) = \bigwedge_{k_1 = x_1x_2} \{(T)_\zeta(x_1) \lor (T)_\zeta(x_2)\}
\]

\[
= \bigwedge_{k_1 = x_1x_2} \{ \bigwedge_{x_3 = x_4x_3} \{(T)_\zeta(x_3) \lor \chi_M(T)_\zeta(x_4)\} \lor (T)_\zeta(x_2)\}
\]

\[
= \bigwedge_{k_1 = x_3x_4x_2} \{(T)_\zeta(x_3) \lor (T)_\zeta(x_2)\}
\]

\[
= \bigwedge_{k_1 = x_3x_4x_2} \{(T)_\zeta(x_3 \lor (T)_\zeta(x_2)\}
\]

\[
= \bigwedge_{k_1 = x_3x_4x_2} \{(T)_\zeta(x_3) \lor (T)_\zeta(x_2)\}
\]

\[
= \bigwedge_{k_1 = x_3x_4x_2} \{(T)_\zeta(x_3x_4x_2)\}
\]

\[
= (T)_\xi(k_1),
\]
\[
((I)_\xi \circ \chi_\mathcal{M}(I)_\zeta)(k_1) = \bigvee_{k_1=x_1 x_2} \{((I)_\xi \circ \chi_\mathcal{M}(I)_\zeta)(x_1) \wedge (I)_\xi(x_2)\} = \bigvee_{k_1=x_1 x_2} \{ \bigvee_{x_1=x_1 x_4} \{((I)_\xi(x_3) \wedge (I)_\xi(x_4)) \wedge (I)_\xi(x_2)\} = \bigvee_{k_1=x_3 x_4 x_2} \{((I)_\xi(x_3) \wedge (I)_\xi(x_2)\} \}
\]
\[
((F)_\xi \circ \chi_\mathcal{M}(F)_\zeta)(k_1) = \bigwedge_{k_1=x_1 x_2} \{((F)_\xi \circ \chi_\mathcal{M}(F)_\zeta)(x_1) \vee (F)_\xi(x_2)\} = \bigwedge_{k_1=x_1 x_2} \{ \bigwedge_{x_1=x_1 x_4} \{((F)_\xi(x_3) \vee (F)_\xi(x_4)) \vee (F)_\xi(x_2)\} = \bigwedge_{k_1=x_3 x_4 x_2} \{((F)_\xi(x_3) \vee (F)_\xi(x_2)\} \}
\]

Suppose there is no \(x_1, x_2 \in \mathcal{M} \ni k_1 = x_1 x_2\). Then
\[
((T)_\xi \circ \chi_\mathcal{M}(T)_\zeta)(T)_\xi)(k_1) = 0 \geq (T)_\xi(k_1), \\
((I)_\xi \circ \chi_\mathcal{M}(I)_\zeta)(I)_\xi(k_1) = 0 \leq (I)_\xi(k_1), \\
((F)_\xi \circ \chi_\mathcal{M}(F)_\zeta)(F)_\xi(k_1) = 0 \geq (F)_\xi(k_1). 
\]

Therefore \((\mathcal{M}_\xi \circ \chi_\mathcal{M}(\mathcal{M}_\zeta)) \circ \mathcal{M}_\xi \subseteq \mathcal{M}_\xi\) for any \(\mathcal{M}_\zeta \in \mathcal{M}\).

Conversely, assume \((\mathcal{M}_\xi \circ \chi_\mathcal{M}(\mathcal{M}_\zeta)) \circ \mathcal{M}_\xi \subseteq \mathcal{M}_\xi\) for any \(\mathcal{M}_\zeta \in \mathcal{M}\). Let \(x_1, x_2 \in \mathcal{M}\). Then
\[
(T)_\xi(x_1 x_2) \leq ((T)_\xi \circ (T)_\xi)(x_1 x_2) \leq (T)_\xi(x_1) \vee (T)_\xi(x_2), \\
(I)_\xi(x_1 x_2) \geq ((I)_\xi \circ (I)_\xi)(x_1 x_2) \geq (I)_\xi(x_1) \wedge (I)_\xi(x_2), \\
(F)_\xi(x_1 x_2) \leq ((F)_\xi \circ (F)_\xi)(x_1 x_2) \leq (F)_\xi(x_1) \vee (F)_\xi(x_2). 
\]

So \(\mathcal{M}_\xi \in \mathcal{M}_s\).

Let \(x_1, x_2, x_3 \in \mathcal{M}\). Then
\[
(T)_\xi(x_1 x_2 x_3) \leq (((T)_\xi \circ \chi_\mathcal{M}(T)_\zeta)(T)_\xi)(x_1 x_2 x_3) \leq ((T)_\xi \circ \chi_\mathcal{M}(T)_\zeta)(x_1 x_2) \vee (T)_\xi(x_3) \leq \{(T)_\xi(x_1) \vee \chi_\mathcal{M}(T)_\xi(x_2)\} \vee (T)_\xi(x_3) = (T)_\xi(x_1) \vee (T)_\xi(x_3), \\
(I)_\xi(x_1 x_2 x_3) \geq (((I)_\xi \circ \chi_\mathcal{M}(I)_\zeta)(I)_\xi)(x_1 x_2 x_3) \geq ((I)_\xi \circ \chi_\mathcal{M}(I)_\zeta)(x_1 x_2) \wedge (I)_\xi(x_3) \geq \{(I)_\xi(x_1) \wedge (I)_\xi(x_2) \wedge (I)_\xi(x_3)\}. 
\]
\[ \chi_{\mathcal{A}}(I)(x_2) \} \land (I)(x_3) = (I)(x_1) \land (I)(x_3), \]
\[ (F)(x_1x_2x_3) \leq (((F)(x_2) \lor \chi_{\mathcal{A}}(F)(x_1x_2x_3) \leq ((F)(x_1 \lor \chi_{\mathcal{A}}(F)(x_1x_2x_3) \leq \{ (F)(x_1) \lor \chi_{\mathcal{A}}(F)(x_2) \} \lor (F)(x_3) = (F)(x_1) \lor (F)(x_3). \]
Therefore \( M_\xi \in M_b. \)

**Lemma 3.18.** Suppose \( M_\xi, M_\zeta \in M_r \) having left identity. Then \( M_\xi \circ M_\zeta \in M_b \) and \( M_\zeta \circ M_\xi \in M_b. \)

**Proof.** By Corollary 3.2, \( (M_\xi \circ M_\zeta) \circ (M_\xi \circ M_\zeta) = (M_\xi \circ M_\zeta) \circ (M_\zeta \circ M_\xi) \subseteq M_\xi \circ M_\zeta. \)
Hence \( M_\xi \circ M_\zeta \subseteq M. \) Now, by Corollary 3.2 and Lemma 3.6, for any \( M \subseteq M. \)
\[ ((M_\xi \circ M_\zeta) \circ \chi_{M_\zeta}(M_\zeta)) \circ (M_\xi \circ M_\zeta) = ((M_\xi \circ M_\zeta) \circ (\chi_{M_\zeta}(M_\zeta) \circ \chi_{M_\zeta}(M_\zeta))) \circ (M_\xi \circ M_\zeta) \]
\[ = ((M_\xi \circ \chi_{M_\zeta}(M_\zeta)) \circ (M_\zeta \circ \chi_{M_\zeta}(M_\zeta))) \circ (M_\xi \circ M_\zeta) \]
\[ \subseteq (M_\xi \circ M_\zeta) \circ (M_\zeta \circ M_\xi) \]
\[ \subseteq M_\xi \circ M_\zeta. \]
By Lemma 3.17, \( M_\zeta \circ M_\zeta \subseteq M_b. \) Similarly, \( M_\zeta \circ M_\xi \subseteq M_b. \)

**Lemma 3.19.** Let \( M_\xi, M_\zeta \in M_b. \) Then \( M_\xi \cap M_\zeta \in M_b. \)

**Proof.** Let \( M_\xi, M_\zeta \in M_b \) and \( k_1, k_2, a \in M. \) Then
\[ (T_\xi \cap T_\zeta)(k_1ak_2) = T_\xi(k_1ak_2) \lor T_\zeta(k_1ak_2) \]
\[ \leq (T_\xi(k_1) \lor T_\zeta(k_1)) \lor (T_\zeta(k_2)) \]
\[ = (T_\xi \cap T_\zeta)(k_1) \lor (T_\zeta(k_2)), \]
\[ (I_\xi \cap I_\zeta)(k_1ak_2) = I_\xi(k_1ak_2) \land I_\zeta(k_1ak_2) \]
\[ \geq (I_\xi(k_1) \land I_\xi(k_2))) \land (I_\zeta(k_1) \land I_\zeta(k_2)) \]
\[ = (I_\xi \cap I_\zeta)(k_1) \land (I_\zeta(k_2)), \]
\[ (F_\xi \cap F_\zeta)(k_1ak_2) = F_\xi(k_1ak_2) \lor F_\zeta(k_1ak_2) \]
\[ \leq (F_\xi(k_1) \lor F_\zeta(k_1)) \lor (F_\zeta(k_2)) \]
\[ = (F_\xi \cap F_\zeta)(k_1) \lor (F_\xi \cap F_\zeta)(k_2) \]
Hence \( M_\xi \cap M_\zeta \subseteq M_b. \)

**Theorem 3.20.** Let \( M_\xi \in M_n. \) Then \( M_\xi \in M_n \) if and only if \( \chi_{\mathcal{A}}(M_\zeta) \circ M_\xi \circ \chi_{\mathcal{A}}(M_\zeta) \subseteq M_\xi \) for any \( M_\zeta \in M. \)

**Proof.** This is similar to Theorem 3.18 in [21].

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Proposition 3.21. If every neutrosophic $\mathcal{I}$-left ideal is neutrosophic $\mathcal{I}$-idempotent in $\mathcal{M}$, then the following statements hold:

(i) $\mathcal{M}_\xi \in \mathcal{M}_b$,

(ii) $\mathcal{M}_\xi \in \mathcal{M}_n$.

Proof. (i) Assume $\mathcal{M}_\xi \in \mathcal{M}$. Then $\mathcal{M}_\xi \circ \mathcal{M}_\xi = \mathcal{M}_\xi$. By Corollary 3.2, for any $\mathcal{M}_\zeta \in \mathcal{M}$,

$$(\mathcal{M}_\xi \circ \mathcal{M}_\zeta) \circ \mathcal{M}_\xi = (\mathcal{M}_\xi \circ \mathcal{M}_\zeta) \circ (\mathcal{M}_\xi \circ \mathcal{M}_\zeta) = (\mathcal{M}_\xi \circ \mathcal{M}_\xi) \circ (\mathcal{M}_\xi \circ \mathcal{M}_\zeta) \subseteq \mathcal{M}_\zeta \circ \mathcal{M}_\xi = \mathcal{M}_\xi.$$

Hence $\mathcal{M}_\xi \in \mathcal{M}_b$.

(ii) For any $\mathcal{M}_\zeta \in \mathcal{M}$, $\mathcal{M}_\zeta \circ \mathcal{M}_\xi \circ \mathcal{M}_\zeta \subseteq \mathcal{M}_\xi$, hence $\mathcal{M}_\xi \in \mathcal{M}_n$. $\square$

Lemma 3.22. Suppose $\mathcal{M}$ is having left identity $e$. Then the listed claims are equivalent:

(i) $\mathcal{M}_\xi \in \mathcal{M}_r$,

(ii) $\mathcal{M}_\xi \in \mathcal{M}_n$.

Proof. Let $r \in \mathcal{M}$. Then $er = r$.

(i) $\Rightarrow$ (ii) Assume $\mathcal{M}_\xi \in \mathcal{M}_r$ and $x_1, x_2, x_3 \in \mathcal{M}$. Then

$$(T)\xi((x_1 x_2) x_3) \leq (T)\xi((x_1 x_2) x_2) = (T)\xi((e x_1) x_2) = (T)\xi((x_2 x_1) e) \leq (T)\xi((x_2 x_1) x_2),$$

$$(I)\xi((x_1 x_2) x_3) \geq (I)\xi((x_1 x_2) x_2) = (I)\xi((e x_1) x_2) = (I)\xi((x_2 x_1) e) \geq (I)\xi((x_2 x_1) x_2),$$

$$(F)\xi((x_1 x_2) x_3) \leq (F)\xi((e x_1) x_2) = (F)\xi((x_2 x_1) e) \leq (F)\xi((x_2 x_1) x_2) \leq (F)\xi((x_2 x_1) x_2).$$

So $\mathcal{M}_\xi \in \mathcal{M}_n$.

(ii) $\Rightarrow$ (i) Let $\mathcal{M}_\xi \in \mathcal{M}_n$. For any $x_1, x_3 \in \mathcal{M}$, we can have $(T)\xi(x_1 x_3) = (T)\xi((e x_1) x_3) \leq (T)\xi(x_1), (I)\xi((e x_1) x_3) \geq (I)\xi(x_1), (F)\xi((e x_1) x_3) \leq (F)\xi(x_1)$. So $\mathcal{M}_\xi \in \mathcal{M}_r$. $\square$

Lemma 3.23. Let $\mathcal{M}_\xi \in \mathcal{M}_l$ such that $e \in \mathcal{M}$ as left identity. If $\mathcal{M}_\xi \in \mathcal{M}_n$, then $\mathcal{M}_\xi \in \mathcal{M}_b$.

Proof. Since $\mathcal{M}_\xi \in \mathcal{M}_l$, $(T)\xi(x_1 x_2) \leq (T)\xi(x_2), (I)\xi(x_1 x_2) \geq (I)\xi(x_2)$ and $(F)\xi(x_1 x_2) \leq (F)\xi(x_2)$ for any $x_1, x_2 \in \mathcal{M}$. As $e \in \mathcal{M}$, $ex_1 = x_1 \forall x_1 \in \mathcal{M}$. Now for any $x_1, x_2 \in \mathcal{M}$,

$$(T)\xi(x_1 x_2) = (T)\xi((e x_1) x_2) \leq (T)\xi(x_1), (I)\xi(x_1 x_2) = (I)\xi((e x_1) x_2) \geq (I)\xi(x_1), (F)\xi(x_1 x_2) = (F)\xi((e x_1) x_2) \leq (F)\xi(x_1) \bigwedge (I)\xi(x_1 x_2), (F)\xi(x_1 x_2) \leq (F)\xi(x_1) \bigvee (F)\xi(x_2).$$

So $\mathcal{M}_\xi \in \mathcal{M}_s$.

For any $x_1, x_2, x_3 \in \mathcal{M}$,

$$(T)\xi((x_1 x_2) x_3) = (T)\xi((x_1 (e x_2)) x_3) \leq (T)\xi((e x_2) x_3) \leq (T)\xi(x_2) = (T)\xi((e x_2) x_3) \leq (T)\xi(x_2),$$

$$(I)\xi((x_1 x_2) x_3) = (I)\xi((x_1 (e x_2)) x_3) \leq (I)\xi((e x_2) x_3) \geq (I)\xi(x_2) = (I)\xi((e x_2) x_3) \geq (I)\xi(x_2),$$

$$(F)\xi((x_1 x_2) x_3) = (F)\xi((x_1 (e x_2)) x_3) = (F)\xi((e x_2) x_3) \leq (F)\xi(x_2) = (F)\xi((e x_2) x_3) \leq (F)\xi(x_2).$$

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Also, \((T)\xi((x_1x_2)x_3) = (T)\xi((x_3x_2)x_1) = (T)\xi((x_3(e_3x_2))x_1) = (T)\xi((e(x_3x_2))x_1) \leq (T)\xi((x_3x_2) = (T)\xi((e_3x_2)\leq (T)\xi(x_3))\). Hence \((T)\xi((x_1x_2)x_3) \leq (T)\xi(x_1) \vee (T)\xi(x_3)). Now, \((I)\xi((x_1x_2)x_3) = (I)\xi((x_3x_2)x_1) = (I)\xi((x_3(e_3x_2))x_1) = (I)\xi((e(x_3x_2))x_1) \geq (I)\xi(x_3) = (I)\xi((e_3x_2) \geq (I)\xi(x_3)). Hence \((I)\xi((x_1x_2)x_3) \geq (I)\xi(x_1) \wedge (I)\xi(x_3)). Now, \((F)\xi((x_1x_2)x_3) = (F)\xi((x_3x_2)x_1) = (F)\xi((e(x_3x_2))x_1) \leq (F)\xi(x_3) = (F)\xi((e_3x_2) \leq (F)\xi(x_3)). Hence \((F)\xi((x_1x_2)x_3) \leq (F)\xi(x_1) \vee (F)\xi(x_3)). Therefore \(M_\xi \in M_b). □

Proposition 3.24. Let \(e \in M\) be identity. If \(M_\xi \in M_i\) (resp., \(M_\xi \in M_r\), \(M_\xi \in M_i\)), then \(M_\xi \circ M_\xi \in M_i\).

Proof. Since \(M_\xi \in M_i\), then for \(M_\xi \in M\), by Theorem 3.4, \(\chi_M(\xi) \circ M_\xi \subseteq M_\xi\). By Lemma 3.6 and Corollary 3.2, \(\chi_M(\xi) \circ (M_\xi \circ M_\xi) = (\chi_M(\xi) \circ \chi_M(\xi)) \circ (M_\xi \circ M_\xi) = (\chi_M(\xi) \circ M_\xi) \circ (\chi_M(\xi) \circ M_\xi) \subseteq M_\xi \circ M_\xi\). Also by Theorem 3.1, \((M_\xi \circ M_\xi) \circ \chi_M(\xi) = (\chi_M(\xi) \circ M_\xi) \circ M_\xi \subseteq M_\xi \circ M_\xi\). Thus \(M_\xi \circ M_\xi \in M_i\). □

Corollary 3.25. Let \(M_\xi \in M\) has identity. If \(M_\xi \in M_i\), then \(M_\xi \circ M_\xi \in M_b\) and \(M_\xi \circ M_\xi \in M_n\).

Proof. By Proposition 3.24, \(M_\xi \circ M_\xi \in M_i\). Now by Lemmas 3.22 and 3.23, \(M_\xi \circ M_\xi \in M_b\) and \(M_\xi \circ M_\xi \in M_n\). □

Theorem 3.26. If \(M_\xi \in M_i\), then \(M_\xi \in M_b\) and \(M_\xi \in M_n\).

Proof. Let \(M_\xi \in M_i\). Then \(M_\xi\) is neutrosophic \(\alpha\)-AG-subgroupoid since \(M_\xi \circ M_\xi \subseteq \chi_M(\xi) \circ M_\xi \subseteq M_\xi\). Now, \((\chi_M(\xi) \circ M_\xi) \circ \chi_M(\xi) \subseteq M_\xi \circ \chi_M(\xi) \subseteq M_\xi \circ M_\xi \subseteq M_\xi \circ M_\xi \subseteq M_\xi \) which imply \(M_\xi \in M_b\) and \(M_\xi \in M_n\). □

4. Conclusion

We presented the ideas of neutrosophic \(\alpha\)-ideal structures in an \(AG\)-groupoid and proved that the product of two neutrosophic \(\alpha\)-right-ideal is a neutrosophic \(\alpha\)-bi-ideal, and neutrosophic \(\alpha\)-right-ideal is equivalent to neutrosophic \(\alpha\)-interior-ideal, under certain condition. In future, we will define neutrosophic \(\alpha\)-structures over an ordered \(AG\)-groupoid and investigate the features of an ordered \(AG\)-groupoid using the results in an \(AG\)-groupoid.

Conflict of interest:

The authors declare that they have no conflict of interest.

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Acknowledgement:

The authors express their gratitude to the referees for valuable comments and suggestions which improve the article.

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Received: June 6, 2023. Accepted: Oct 1, 2023