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V. Inthumathi

M. Amsaveni

M. Nathibrami

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On Hypersoft Semi-open Sets

V. Inthumathi[1], M. Amsaveni[2,*] and M. Nathibrami[3]

[1] "Associate Professor, Department of Mathematics, Nallamuthu Gounder Mahalingam College, Tamilnadu, India;
inthumathi65@gmail.com

[2] Research Scholar, Department of Mathematics, Nallamuthu Gounder Mahalingam College, Tamilnadu, India;
amsavenim2000@gmail.com

[3] PG Student, Department of Mathematics, Nallamuthu Gounder Mahalingam College, Tamilnadu, India;
nathibrami2000@gmail.com

*Correspondence: amsavenim2000@gmail.com"

Abstract. A generalisation of soft sets called a hypersoft set incorporates a multiargument function. The major goal of this study is to provide appropriate examples for the introduction of hypersoft semi-open sets (SOS) and hypersoft semi-closed sets (SCS). Additionally, we investigate the definition and characteristics of hypersoft semi-open sets in hypersoft topological spaces (TS). The hypersoft semi interior and hypersoft semi closure of the hypersoft set are defined at the end.

Keywords: hypersoft set; hypersoft topology; hypesoft semi-open and closed set; hypersoft semi-interior; hypersoft semi-closure.

1. Introduction

Molodstov [14] established the concept of a soft set in 1999 to handle difficult problems in finance, technical education, and ecological science when no mathematical instruments could effectively address the many types of uncertainty. [13] constructed a number of soft set theory operators and carried out a more thorough conceptual analysis.

Numerous applications of topology, a subfield of mathematics, may be found in the computer and physical sciences. Soft topology is determined on soft sets in two different ways, one by Shabir [20] and the other by Cagman et al. [5].

Soft SOS and soft SCS were first presented in soft TS by Sasikala, V., E. and Sivaraj, D., [19]. Soft semi connected and soft locally semi connected characteristics in soft TS were established by Krishnaveni, J., and Sekar, C., [12].

In 2018, Florentin Smarandache [22] extended the concept of a soft set to a hypersoft set and the hypersoft topology was introduced by Musa and Assad, [15].

The hypersoft sets have been utilised in the Covid-19 Decision Making Model by Inthumathi et al., who cited [9]. Hypersoft subspace topology, hypersoft basis, hypersoft limit point, and hypersoft Hausdorff space were also introduced by Inthumathi et al. in 10. Neutrosophic hypersoft TS and Neutrosophic Semi-open hypersoft sets were produced by Ajay et al. [2] with an illustration to the MAGDM in the Covid-19 Scenario.

When there exist inconclusive data, uncertain functions, or ambiguous sets, Florentin Smarandache [6] recently invented the IndetermHypersoft set as an enhancement of the hypersoft set. He [7] as the company that created the TreeSoft set as an addition to the Multisoft set. It can be seen that the level 2 TreeSoft set resembles the hypersoft set.

The framework of the manuscript is as follows. The preliminary information relevant to this article is provided in section 2. Hypersoft SOS and hypersoft SCS are introduced in section 3 of the paper. We present the idea of hypersoft semi-interior and hypersoft semi-closure in section 4 and demonstrate some of its features.

2. Preliminaries

The preceding definitions are crucial to understanding the content of this manuscript.

Definition 2.1. [14] "Let U be an initial universe and E be a set of parameters. Let P(U) denote the power set of U. The pair (F, E) or simply F_E, is called a soft set over U, where F is a mapping given by $F : E{\to}P(U)$".

Definition 2.2. [22] "Let U be a universe of discourse, P(U) the power set of U and $E_1, E_2, \cdots, E_n$ the pairwise disjoint sets of parameters. Let A_i be the nonempty subset of E_i for each i=1,2,...,n. A hypersoft set can be identified by the pair $(\Omega, A_1 * A_2 * \cdots * A_n)$, where $\Omega : A_1 * A_2 * \cdots * A_n \to P(U)$. For sake of simplicity, we write the symbols $\mathscr{T}$ for $E_1 * E_2 * \cdots * E_n$, P for $A_1 * A_2 * \cdots * A_n$".

Definition 2.3. [1] "Let $(\Omega, \mathscr{Q})$ and $(\mathscr{G}, \mathfrak{R})$ be two hypersoft sets over U. Then union of $(\Omega, \mathscr{Q})$ and $(\mathscr{G}, \mathfrak{R})$ is denoted by $(\mathscr{H}, \mathscr{S}) = (\Omega, \mathscr{Q}) \cup (\mathscr{G}, \mathfrak{R})$ with $\mathscr{S}{=}D_1 * D_2 * \cdots * D_n$, where $D_i{=}Q_i \cup R_i$ for i=1,2,...,n, and $\mathscr{H}$ is defined by

$$\mathscr{H}(\alpha) = \begin{cases} \Omega(\alpha), & if \alpha \in \mathscr{Q} - \mathfrak{R} \\ \mathscr{G}(\alpha), & if \alpha \in \mathfrak{R} - \mathscr{Q} \\ \Omega(\alpha) \cup \mathscr{G}(\alpha), & if \alpha \in \mathscr{Q} \cap \mathfrak{R} \\ 0, & else, \end{cases}$$

where $\alpha = (d_1, d_2, ..., d_n) \in \mathscr{S}$".

Definition 2.4. [1] "Let $(\Omega, \mathscr{Q})$ and $(\mathscr{G}, \mathfrak{R})$ be two hypersoft sets over U. Then intersection of $(\Omega, \mathscr{Q})$ and $(\mathscr{G}, \mathfrak{R})$ is denoted by $(\mathscr{H}, \mathscr{S}) = (\Omega, \mathscr{Q}) \cap (\mathscr{G}, \mathfrak{R})$ with $\mathscr{S}=D_1 * D_2 * \cdots * D_n$, is such that $D_i=Q_i \cap R_i$ for i=1,2,...,n, and $\mathscr{H}$ is defined as $\mathscr{H}(\alpha) = \Omega(\alpha) \cap \mathscr{G}(\alpha)$, where $\alpha = (d_1, d_2, ..., d_n) \in \mathscr{S}$. If D_i is an empty for some i, then $(\Omega, \mathscr{Q}) \cap (\mathscr{G}, \mathfrak{R})$ is defined to be a null hypersoft set".

Definition 2.5. [15] "Let τ be a collection of hypersoft sets over U, then τ is said to be a hypersoft topology over U if

(1) $(\emptyset, \mathsf{P})$ and(Ω, P) belongs to τ,
(2) The intersection of any two hypersoft sets in τ belongs to τ,
(3) The union of any number of a hypersoft sets in τ belongs to τ.
Then $((\Omega, \mathsf{P}), \tau)$ is called a hypersoft TS over U".

Proposition 2.6. *[15] "Let $((\Omega, \mathsf{P}), \tau)$ be a hypersoft space over U. Then*

(1) $(\emptyset, \mathsf{P})$ *and* (Ω, P) *are hypersoft closed sets over U,*
(2) *The union of any two hypersoft closed sets is a hypersoft closed set over U,*
(3) *The intersection of any number of hypersoft closed sets is a hypersoft closed set over U".*

Definition 2.7. [15] "Let $((\Omega, \mathsf{P}), \tau)$ be a hypersoft TS and let $(\Omega, \mathscr{Q})$ be a hypersoft set then

(1) The hypersoft interior of $(\Omega, \mathscr{Q})$ is the hypersoft set
$h\text{-}int(\Omega, \mathscr{Q}) = \bigcup\{(\Omega, \mathfrak{R}) : (\Omega, \mathfrak{R})$ is hypersoft open and $(\Omega, \mathfrak{R}) \subseteq (\Omega, \mathscr{Q})\}$.
(2) The hypersoft closure of $(\Omega, \mathscr{Q})$ is the hypersoft set
$h\text{-}cl(\Omega, \mathscr{Q}) = \bigcap\{(\Omega, \mathfrak{R}) : (\Omega, \mathfrak{R})$ is hypersoft closed and $(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathfrak{R})\}$".

Proposition 2.8. *[15] "Let $((\Omega, \mathsf{P}), \tau)$ be a hypersoft TS and let $(\Omega, \mathscr{Q})$ be a hypersoft set then*

(1) *$h\text{-}cl(\Omega, \mathscr{Q})$ is the smallest hypersoft closed set containing $(\Omega, \mathscr{Q})$.*
(2) *$(\Omega, \mathscr{Q})$ is a hypersoft closed set if and only if $(\Omega, \mathscr{Q})$=$h\text{-}cl(\Omega, \mathscr{Q})$".*

Proposition 2.9. *[15] "Let $((\Omega, \mathsf{P}), \tau)$ be a hypersoft TS and let $(\Omega, \mathscr{Q})$ be a hypersoft set then*

(1) *$h\text{-}int(\Omega, \mathscr{Q})$ is the largest hypersoft open set contained in $(\Omega, \mathscr{Q})$.*
(2) *$(\Omega, \mathscr{Q})$ is a hypersoft open set if and only if $(\Omega, \mathscr{Q})$=$h\text{-}int(\Omega, \mathscr{Q})$".*

Proposition 2.10. *[15] "Let $((\Omega, \mathsf{P}), \tau)$ be a hypersoft TS and let $(\Omega, \mathscr{Q})$,$(\Omega, \mathfrak{R})$ be a hypersoft sets over U. Then*

(1) *$h\text{-}int(h\text{-}int(\Omega, \mathscr{Q}))$=$h\text{-}int((\Omega, \mathscr{Q})$.*
(2) *$(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathfrak{R})$ implies $h\text{-}int(\Omega, \mathscr{Q})$$\subseteq$$h\text{-}int(\Omega, \mathfrak{R})$.*

(3) $h\text{-}int((\Omega,\mathscr{Q})\cup(\Omega,\mathfrak{R}))\subseteq h\text{-}int(\Omega,\mathscr{Q})\cup h\text{-}int(\Omega,\mathfrak{R})$.

(4) $h\text{-}int((\Omega,\mathscr{Q})\cap(\Omega,\mathfrak{R}))=h\text{-}int(\Omega,\mathscr{Q})\cap h\text{-}int(\Omega,\mathfrak{R})$".

Proposition 2.11. *[15] "Let $((\Omega,\mathsf{P}),\tau)$ be a hypersoft TS and let $(\Omega,\mathscr{Q})$,$(\Omega,\mathfrak{R})$ be a hypersoft sets over U. Then*

(1) $(\Omega,\mathscr{Q})\subseteq(\Omega,\mathfrak{R})$ *implies* $h\text{-}cl(\Omega,\mathscr{Q})\subseteq h\text{-}cl(\Omega,\mathfrak{R})$.

(2) $h\text{-}cl((\Omega,\mathscr{Q})\cup(\Omega,\mathfrak{R}))=h\text{-}cl(\Omega,\mathscr{Q})\cup h\text{-}cl(\Omega,\mathfrak{R})$.

(3) $h\text{-}cl((\Omega,\mathscr{Q})\cap(\Omega,\mathfrak{R}))\subseteq h\text{-}cl(\Omega,\mathscr{Q})\cap h\text{-}cl(\Omega,\mathfrak{R})$.

(4) $h\text{-}cl(h\text{-}cl(\Omega,\mathscr{Q}))=h\text{-}cl((\Omega,\mathscr{Q})$".

3. Hypersoft semi-open sets and hypersoft semi-closed sets

In this segment we produce the notion of hypersoft SOS, hypersoft SCS and examine a few of its properties.

Definition 3.1. Let $(\Omega,\mathscr{Q})$ be a hypersoft set of a hypersoft TS $((\Omega,\mathsf{P}),\tau)$. $(\Omega,\mathscr{Q})$ is known as a hypersoft SOS if $(\Omega,\mathscr{Q})\subseteq h\text{-}cl(h\text{-}int(\Omega,\mathscr{Q}))$.

Definition 3.2. A hypersoft set $(\Omega,\mathscr{Q})$ in a hypersoft TS $((\Omega,\mathsf{P}),\tau)$ is called a hypersoft SCS if its relative complement is a hypersoft SOS.

Example 3.3. Let $U=\{h_1,h_2\}$, $Q_1=\{\ell_1,\ell_2\}$, $Q_2=\{\ell_3\}$, $Q_3=\{\ell_4\}$ and let Ω is a function from $\mathsf{P}\to$ P(U). Then the hypersoft sets are classified as follows.

$(\Omega,\mathsf{P})_1=\{((\ell_1,\ell_3,\ell_4),\emptyset),((\ell_2,\ell_3,\ell_4),\emptyset)\}$,
$(\Omega,\mathsf{P})_2=\{((\ell_1,\ell_3,\ell_4),\emptyset),((\ell_2,\ell_3,\ell_4),\{h_1\})\}$,
$(\Omega,\mathsf{P})_3=\{((\ell_1,\ell_3,\ell_4),\emptyset),((\ell_2,\ell_3,\ell_4),\{h_2\})\}$,
$(\Omega,\mathsf{P})_4=\{((\ell_1,\ell_3,\ell_4),\emptyset),((\ell_2,\ell_3,\ell_4),\{h_1,h_2\})\}$,
$(\Omega,\mathsf{P})_5=\{((\ell_1,\ell_3,\ell_4),\{h_1\}),((\ell_2,\ell_3,\ell_4),\emptyset)\}$,
$(\Omega,\mathsf{P})_6=\{((\ell_1,\ell_3,\ell_4),\{h_1\}),((\ell_2,\ell_3,\ell_4),\{h_1\})\}$,
$(\Omega,\mathsf{P})_7=\{((\ell_1,\ell_3,\ell_4),\{h_1\}),((\ell_2,\ell_3,\ell_4),\{h_2\})\}$,
$(\Omega,\mathsf{P})_8=\{((\ell_1,\ell_3,\ell_4),\{h_1\}),((\ell_2,\ell_3,\ell_4),\{h_1,h_2\})\}$,
$(\Omega,\mathsf{P})_9=\{((\ell_1,\ell_3,\ell_4),\{h_2\}),((\ell_2,\ell_3,\ell_4),\emptyset)\}$,
$(\Omega,\mathsf{P})_{10}=\{((\ell_1,\ell_3,\ell_4),\{h_2\}),((\ell_2,\ell_3,\ell_4),\{h_1\})\}$,
$(\Omega,\mathsf{P})_{11}=\{((\ell_1,\ell_3,\ell_4),\{h_2\}),((\ell_2,\ell_3,\ell_4),\{h_2\})\}$,
$(\Omega,\mathsf{P})_{12}=\{((\ell_1,\ell_3,\ell_4),\{h_2\}),((\ell_2,\ell_3,\ell_4),\{h_1,h_2\})\}$,
$(\Omega,\mathsf{P})_{13}=\{((\ell_1,\ell_3,\ell_4),\{h_1,h_2\}),((\ell_2,\ell_3,\ell_4),\emptyset)\}$,
$(\Omega,\mathsf{P})_{14}=\{((\ell_1,\ell_3,\ell_4),\{h_1,h_2\}),((\ell_2,\ell_3,\ell_4),\{h_1\})\}$,
$(\Omega,\mathsf{P})_{15}=\{((\ell_1,\ell_3,\ell_4),\{h_1,h_2\}),((\ell_2,\ell_3,\ell_4),\{h_2\})\}$,
$(\Omega,\mathsf{P})_{16}=\{((\ell_1,\ell_3,\ell_4),\{h_1,h_2\}),((\ell_2,\ell_3,\ell_4),\{h_1,h_2\})\}$,
$\tau=\{(\Omega,\mathsf{P})_1,(\Omega,\mathsf{P})_5,(\Omega,\mathsf{P})_7,(\Omega,\mathsf{P})_8,(\Omega,\mathsf{P})_{16}\}$.

Then $((\Omega, \mathsf{P}), \tau)$ is a hypersoft TS.
The collection of all hypersoft open sets is
$\{(\Omega, \mathsf{P})_1, (\Omega, \mathsf{P})_5, (\Omega, \mathsf{P})_7, (\Omega, \mathsf{P})_8, (\Omega, \mathsf{P})_{16}\}$.
The set of all hypersoft closed sets is
$\{(\Omega, \mathsf{P})_1, (\Omega, \mathsf{P})_9, (\Omega, \mathsf{P})_{10}, (\Omega, \mathsf{P})_{12}, (\Omega, \mathsf{P})_{16}\}$.

The collection of hypersoft SOS is
$\{(\Omega, \mathsf{P})_1, (\Omega, \mathsf{P})_5, (\Omega, \mathsf{P})_6, (\Omega, \mathsf{P})_7, (\Omega, \mathsf{P})_8, (\Omega, \mathsf{P})_{13}, (\Omega, \mathsf{P})_{14}, (\Omega, \mathsf{P})_{15}, (\Omega, \mathsf{P})_{16}\}$.
The collection of hypersoft SCS is
$\{(\Omega, \mathsf{P})_1, (\Omega, \mathsf{P})_2, (\Omega, \mathsf{P})_3, (\Omega, \mathsf{P})_4, (\Omega, \mathsf{P})_9, (\Omega, \mathsf{P})_{10}, (\Omega, \mathsf{P})_{11}, (\Omega, \mathsf{P})_{12}, (\Omega, \mathsf{P})_{16}\}$.

Theorem 3.4. *Every hypersoft open set in a hypersoft TS $((\Omega, \mathsf{P}), \tau)$ is a hypersoft SOS.*

Proof:
Let $(\Omega, \mathscr{Q})$ be a hypersoft open set. Then $h\text{-}int(\Omega, \mathscr{Q}) = (\Omega, \mathscr{Q})$. we know that, $(\Omega, \mathscr{Q}) \subseteq h\text{-}cl(\Omega, \mathscr{Q})$. Thus $(\Omega, \mathscr{Q}) \subseteq h\text{-}cl(h\text{-}int(\Omega, \mathscr{Q}))$.

The preceding Ex. 3.5 demonstrate that the reverse implification of Thm. 3.4 is not true.

Example 3.5. Consider the hypersoft TS of Ex. 3.3.
Here $(\Omega, \mathsf{P})_6, (\Omega, \mathsf{P})_{13}, (\Omega, \mathsf{P})_{14}, (\Omega, \mathsf{P})_{15}$ are hypersoft semi-open set but not hypersoft open sets, since$(\Omega, \mathsf{P})_6, (\Omega, \mathsf{P})_{13}, (\Omega, \mathsf{P})_{14}, (\Omega, \mathsf{P})_{15} \notin \tau$.

Remark 3.6. $(\emptyset, \mathsf{P})$ and (Ω, P) are always hypersoft SCS and hypersoft SOS.

Proposition 3.7. *A hypersoft set $(\Omega, \mathscr{Q})$ in a hypersoft TS $((\Omega, \mathsf{P}), \tau)$ is a hypersoft SOS iff $\exists$ a hypersoft open set $(\Omega, \mathfrak{R})$ such that $(\Omega, \mathfrak{R}) \subseteq (\Omega, \mathscr{Q}) \subseteq h\text{-}cl(\Omega, \mathfrak{R})$.*

Proof: Assume that $(\Omega, \mathscr{Q}) \subseteq h\text{-}cl(h\text{-}int(\Omega, \mathscr{Q}))$. Then for $(\Omega, \mathfrak{R}) = h\text{-}int(\Omega, \mathscr{Q})$, we have $(\Omega, \mathfrak{R}) \subseteq (\Omega, \mathscr{Q}) \subseteq h\text{-}cl(\Omega, \mathfrak{R})$. Therefore, the condition holds. Conversely, suppose that $(\Omega, \mathfrak{R}) \subseteq (\Omega, \mathscr{Q}) \subseteq h\text{-}cl(\Omega, \mathfrak{R})$ for some hypersoft open set $(\Omega, \mathfrak{R})$. Since $(\Omega, \mathfrak{R}) \subseteq h\text{-}int(\Omega, \mathscr{Q})$, and so $h\text{-}cl(\Omega, \mathfrak{R}) \subseteq h\text{-}cl(h\text{-}int(\Omega, \mathscr{Q}))$. Hence $(\Omega, \mathscr{Q}) \subseteq h\text{-}cl(\Omega, \mathfrak{R}) \subseteq h\text{-}cl(h\text{-}int(\Omega, \mathscr{Q}))$. Hence $(\Omega, \mathscr{Q})$ is hypersoft SOS.

Theorem 3.8. *Let $((\Omega, \mathsf{P}), \tau)$ be a hypersoft TS and $\{(\Omega, \mathscr{Q})_\alpha : \alpha \in \Delta\}$ be a set of hypersoft SOS in $((\Omega, \mathsf{P}), \tau)$. Then $\cup_{\alpha \in \Delta}(\Omega, \mathscr{Q})_\alpha$ is also a hypersoft SOS.*

Proof: Let $\{(\Omega, \mathscr{Q})_\alpha : \alpha \in \Delta\}$ be a set of hypersoft SOS in $((\Omega, \mathsf{P}), \tau)$. Then $\forall\ \alpha \in \Delta$, we have a hypersoft open set $(\Omega, \mathfrak{R})_\alpha \subseteq (\Omega, \mathscr{Q})_\alpha$ such that $(\Omega, \mathfrak{R})_\alpha \subseteq (\Omega, \mathscr{Q})_\alpha \subseteq h\text{-}cl(\Omega, \mathfrak{R})_\alpha$. Then $\cup_{\alpha \in \Delta}(\Omega, \mathfrak{R})_\alpha \subseteq \cup_{\alpha \in \Delta}(\Omega, \mathscr{Q})_\alpha \subseteq \cup_{\alpha \in \Delta} h\text{-}cl(\Omega, \mathfrak{R})_\alpha \subseteq$
$h\text{-}cl(\cup_{\alpha \in \Delta}(\Omega, \mathfrak{R})_\alpha)$.

Theorem 3.9. *Every hypersoft closed set in a hypersoft TS* $((\Omega, \mathsf{P}), \tau)$ *is a hypersoft SCS.*

Proof: Let $(\Omega, \mathscr{Q})$ be a hypersoft closed set. Then $h\text{-}cl(\Omega, \mathscr{Q}) = (\Omega, \mathscr{Q})$. we know that, $(\Omega, \mathscr{Q}) \supseteq h\text{-}int(\Omega, \mathscr{Q})$. Thus $(\Omega, \mathscr{Q}) \supseteq h\text{-}int(h\text{-}cl(\Omega, \mathscr{Q}))$.

The opposite simplification of Thm. 3.9 cannot be true, as demonstrated by Ex. 3.10 before it.

Example 3.10. Here $(\Omega, \mathsf{P})_2, (\Omega, \mathsf{P})_3, (\Omega, \mathsf{P})_4$ and $(\Omega, \mathsf{P})_{11}$ are hypersoft SCS but not hypersoft closed sets.

Theorem 3.11. $(\Omega, \mathfrak{R})$ *be a hypersoft semi-closed in a hypersoft TS* $((\Omega, \mathsf{P}), \tau)$ *iff* $h\text{-}int(\Omega, \mathscr{T}) \subseteq (\Omega, \mathfrak{R}) \subseteq (\Omega, \mathscr{T})$ *for some hypersoft closed set* $(\Omega, \mathscr{T})$.

Proof: $(\Omega, \mathfrak{R})$ is hypersoft semi-closed iff $(\Omega, \mathfrak{R})^c$ is hypersoft semi-open iff there is a hypersoft open set $(\Omega, \mathscr{S})$ s.t. $(\Omega, \mathscr{S}) \subseteq (\Omega, \mathfrak{R})^c \subseteq h\text{-}cl(\Omega, \mathscr{S})$, by proposition 3.7 iff there is a hypersoft open set $(\Omega, \mathscr{S})$ s.t. $(h\text{-}cl(\Omega, \mathscr{S}))^c \subseteq (\Omega, \mathfrak{R}) \subseteq (\Omega, \mathscr{S})^c$ iff there is a hypersoft open set $(\Omega, \mathscr{S})$ s.t. $h\text{-}int(\Omega, \mathscr{S})^c \subseteq (\Omega, \mathfrak{R}) \subseteq (\Omega, \mathscr{S})^c$ iff there is a hypersoft closed set $(\Omega, \mathscr{S})$ s.t. $h\text{-}int(\Omega, \mathscr{T}) \subseteq (\Omega, \mathfrak{R}) \subseteq (\Omega, \mathscr{T})$, where $(\Omega, \mathscr{T}) = (\Omega, \mathscr{S})^c$.

Theorem 3.12. *A hypersoft set* $(\Omega, \mathscr{Q})$ *in a hypersoft TS* $((\Omega, \mathsf{P}), \tau)$ *is hypersoft semi-closed iff* $h\text{-}int(h\text{-}cl(\Omega, \mathscr{Q})) \subseteq (\Omega, \mathscr{Q})$.

Proof: $(\Omega, \mathscr{Q})$ is hypersoft semi-closed iff $(\Omega, \mathscr{Q})^c$ is hypersoft semi-open iff $(\Omega, \mathscr{Q})^c \subseteq h\text{-}cl(h\text{-}int(\Omega, \mathscr{Q})^c)$ iff $(\Omega, \mathscr{Q})^c \subseteq h\text{-}cl((h\text{-}cl(\Omega, \mathscr{Q}))^c)$, by definition iff $(\Omega, \mathscr{Q})^c \subseteq (h\text{-}int(h\text{-}cl(\Omega, \mathscr{Q})))^c$, iff $h\text{-}int(h\text{-}cl(\Omega, \mathscr{Q})) \subseteq (\Omega, \mathscr{Q})$.

Theorem 3.13. *Let* $((\Omega, \mathsf{P}), \tau)$ *be a hypersoft TS and* $\{(\Omega, \mathscr{Q})_\alpha : \alpha \in \Delta\}$ *be a set of hypersoft SCS in* $((\Omega, \mathsf{P}), \tau)$. *Then* $\cap_{\alpha\in\Delta}(\Omega, \mathscr{Q})_\alpha$ *is also a hypersoft SCS.*

Proof: Let $\{(\Omega, \mathscr{Q})_\alpha : \alpha \in \Delta\}$ be a set of hypersoft SCS in $((\Omega, \mathsf{P}), \tau)$. Then $\forall\ \alpha \in \Delta$, we have a hypersoft soft closed set $(\Omega, \mathfrak{R})_\alpha$ s.t. $h\text{-}int(\Omega, \mathfrak{R})_\alpha \subseteq (\Omega, \mathscr{Q})_\alpha \subseteq (\Omega, \mathfrak{R})_\alpha$. Then $h\text{-}int(\cap_{\alpha\in\Delta}(\Omega, \mathfrak{R}_\alpha)) \subseteq \cap_{\alpha\in\Delta} h\text{-}int(\Omega, \mathfrak{R}_\alpha) \subseteq \cap_{\alpha\in\Delta}(\Omega, \mathscr{Q})_\alpha \subseteq \cap_{\alpha\in\Delta}(\Omega, \mathfrak{R})_\alpha$. Because $\cap_{\alpha\in\Delta}(\Omega, \mathfrak{R})_\alpha = (\Omega, \mathfrak{R})$ is hypersoft closed set by prop 2.6(3), then $\cap_{\alpha\in\Delta}(\Omega, \mathscr{Q})_\alpha$ is hypersoft SCS.

4. Hypersoft semi-interior and hypersoft semi-closure

Definition 4.1. Let $((\Omega, \mathsf{P}), \tau)$ be a hypersoft TS and let $(\Omega, \mathscr{Q})$ be a hypersoft set in (Ω, P).

(1) The hypersoft semi-interior of $(\Omega, \mathscr{Q})$ is the hypersoft set $\bigcup\{(\Omega, \mathfrak{R}) : (\Omega, \mathfrak{R})$ is hypersoft semi-open and $(\Omega, \mathfrak{R}) \subseteq (\Omega, \mathscr{Q})\}$ and it is identified by $h\text{-}sint(\Omega, \mathscr{Q})$.

(2) The hypersoft semi-closure of $(\Omega, \mathscr{Q})$ is the hypersoft set
$\bigcap\{(\Omega, \mathfrak{R}) : (\Omega, \mathfrak{R})$ is hypersoft semi-closed and $(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathfrak{R})\}$ and it is identified by $h\text{-}scl(\Omega, \mathscr{Q})$.
Clearly, $h\text{-}scl(\Omega, \mathscr{Q})$ is the smallest hypersoft SCS containing $(\Omega, \mathscr{Q})$ and $h\text{-}sint(\Omega, \mathscr{Q})$ is the largest hypersoft SOS $\subseteq (\Omega, \mathscr{Q})$. By Thm. 3.8 and 3.13, we have $h\text{-}sint(\Omega, \mathscr{Q})$ is hypersoft SOS and $h\text{-}scl(\Omega, \mathscr{Q})$ is hypersoft SCS.

Example 4.2. Let the hypersoft TS $((\Omega, \mathsf{P}), \tau)$ and the hypersoft set $(\Omega, \mathsf{P})_8$=$\{((\ell_1, \ell_3, \ell_4), \{h_1\}), ((\ell_2, \ell_3, \ell_4), \{h_1, h_2\})\}$ be the same as in Example 3.3, we get $h\text{-}sint(\Omega, \mathscr{Q})_8 = (\Omega, \mathscr{Q})_8$.

Example 4.3. Let the hypersoft TS $((\Omega, \mathsf{P}), \tau)$ and the hypersoft set $(\Omega, \mathsf{P})_{14}$=$\{((\ell_1, \ell_3, \ell_4), \{h_1, h_2\}), ((\ell_2, \ell_3, \ell_4), \{h_1\})\}$ be the same as in Example 3.3, we get $h\text{-}scl(\Omega, \mathscr{Q})_{14} = (\Omega, \mathscr{Q})_{16}$.

Theorem 4.4. *Let $((\Omega, \mathsf{P}), \tau)$ be a hypersoft TS and $(\Omega, \mathscr{Q})$ be a hypersoft set in (Ω, P). Then $h\text{-}int(\Omega, \mathscr{Q}) \subseteq h\text{-}sint(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathscr{Q}) \subseteq h\text{-}scl(\Omega, \mathscr{Q}) \subseteq h\text{-}cl(\Omega, \mathscr{Q})$.*

Proof: The proof follows from the following facts that every hypersoft open set is hypersoft SOS and every hypersoft closed set is hypersoft SCS.

Theorem 4.5. *Let $((\Omega, \mathsf{P}), \tau)$ be a hypersoft TS and let $(\Omega, \mathscr{Q})$ be a hypersoft set in (Ω, P). Then the succeeding conditions holds.*

(1) $(h\text{-}scl(\Omega, \mathscr{Q}))^c$=$h\text{-}sint(\Omega, \mathscr{Q})^c$.
(2) $(h\text{-}sint(\Omega, \mathscr{Q}))^c$=$h\text{-}scl(\Omega, \mathscr{Q})^c$.

Proof:

(1) $(h\text{-}scl(\Omega, \mathscr{Q}))^c$
=$(\bigcap\{(\Omega, \mathfrak{R}) : (\Omega, \mathfrak{R})$ is hypersoft semi-closed and $(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathfrak{R})\})^c$,
=$\bigcup\{(\Omega, \mathfrak{R})^c : (\Omega, \mathfrak{R})$ is hypersoft semi-closed and $(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathfrak{R})\}$,
=$\bigcup\{(\Omega, \mathfrak{R})^c : (\Omega, \mathfrak{R})^c$, is hypersoft semi-open and $(\Omega, \mathfrak{R})^c \subseteq (\Omega, \mathscr{Q})^c\}$,
=$h\text{-}sint(\Omega, \mathscr{Q})^c$.

(2) $(h\text{-}sint(\Omega, \mathscr{Q}))^c$
=$(\bigcup\{(\Omega, \mathfrak{R}) : (\Omega, \mathfrak{R})$ is hypersoft semi-open and $(\Omega, \mathfrak{R}) \subseteq (\Omega, \mathscr{Q})\})^c$,
=$\bigcap\{(\Omega, \mathfrak{R})^c : (\Omega, \mathfrak{R})$ is hypersoft semi-open and $(\Omega, \mathfrak{R}) \subseteq (\Omega, \mathscr{Q})\}$,
=$\bigcup\{(\Omega, \mathfrak{R})^c : (\Omega, \mathfrak{R})^c$ is hypersoft semi-closed and $(\Omega, \mathscr{Q})^c \subseteq (\Omega, \mathfrak{R})^c\}$,
=$h\text{-}scl(\Omega, \mathscr{Q})^c$.

Theorem 4.6. *Let $((\Omega, \mathsf{P}), \tau)$ be a hypersoft TS and let $(\Omega, \mathscr{Q})$ and $(\Omega, \mathfrak{R})$ be a hypersoft sets in (Ω, P). Then the preceding condition holds.*

(1) *h-scl*$(\emptyset, \mathsf{P})$=$(\emptyset, \mathsf{P})$ *and h-scl*(Ω, P)=(Ω, P)

(2) $(\Omega, \mathscr{Q})$ *is hypersoft semi-closed set iff* $(\Omega, \mathscr{Q})$=*h-scl*$(\Omega, \mathscr{Q})$.

(3) *h-scl*(*h-scl*$(\Omega, \mathscr{Q})$)=*h-scl*$(\Omega, \mathscr{Q})$.

(4) $(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathfrak{R})$ *implies h-scl*$(\Omega, \mathscr{Q}) \subseteq$ *h-scl*$(\Omega, \mathfrak{R})$.

(5) *h-scl*$((\Omega, \mathscr{Q}) \cap (\Omega, \mathfrak{R})) \subseteq$ *h-scl*$(\Omega, \mathscr{Q}) \cap$ *h-scl*$(\Omega, \mathfrak{R})$.

(6) *h-scl*$((\Omega, \mathscr{Q}) \cup (\Omega, \mathfrak{R})) =$ *h-scl*$(\Omega, \mathscr{Q}) \cup$ *h-scl*$(\Omega, \mathfrak{R})$.

Proof:

(1) The proof is obvious.

(2) If $(\Omega, \mathscr{Q})$ is hypersoft SCS, then $(\Omega, \mathscr{Q})$ is itself a hypersoft SCS in (Ω, P) which $\subset (\Omega, \mathscr{Q})$. So, *h-scl*$(\Omega, \mathscr{Q})$ is the smallest hypersoft SCS $\subset (\Omega, \mathscr{Q})$ and $(\Omega, \mathscr{Q}) =$ *h-scl*$(\Omega, \mathscr{Q})$. Conversely, suppose that $(\Omega, \mathscr{Q}) =$ *h-scl*$(\Omega, \mathscr{Q})$. Since *h-scl*$(\Omega, \mathscr{Q})$ is a hypersoft SCS, so $(\Omega, \mathscr{Q})$ is hypersoft SCS.

(3) Since *h-scl*$(\Omega, \mathscr{Q})$ is a hypersoft SCS therefore by part(2) we obtain
h-scl(*h-scl*$(\Omega, \mathscr{Q})$)=*h-scl*$(\Omega, \mathscr{Q})$.

(4) Suppose that $(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathfrak{R})$. Then every hypersoft semi-closed super set of $(\Omega, \mathfrak{R})$ will also $\subset (\Omega, \mathscr{Q})$. That is every hypersoft semi-closed super set of $(\Omega, \mathfrak{R})$ is also a hypersoft semi-closed super set of $(\Omega, \mathscr{Q})$. Thus *h-scl*$(\Omega, \mathscr{Q}) \subseteq$ *h-scl*$(\Omega, \mathfrak{R})$.

(5) Since $(\Omega, \mathscr{Q}) \cap (\Omega, \mathfrak{R}) \subseteq (\Omega, \mathscr{Q})$ and $(\Omega, \mathscr{Q}) \cap (\Omega, \mathfrak{R}) \subseteq (\Omega, \mathfrak{R})$ and so by part(4)
h-scl$((\Omega, \mathscr{Q}) \cap (\Omega, \mathfrak{R})) \subseteq$ *h-scl*$(\Omega, \mathscr{Q})$ and *h-scl*$((\Omega, \mathscr{Q}) \cap (\Omega, \mathfrak{R})) \subseteq$ *h-scl*$(\Omega, \mathfrak{R})$. Thus
h-scl$((\Omega, \mathscr{Q}) \cap (\Omega, \mathfrak{R})) \subseteq$ *h-scl*$(\Omega, \mathscr{Q}) \cap$ *h-scl*$(\Omega, \mathfrak{R})$.

(6) Since $(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathscr{Q}) \cup (\Omega, \mathfrak{R})$ and $(\Omega, \mathfrak{R}) \subseteq (\Omega, \mathscr{Q}) \cup (\Omega, \mathfrak{R})$. So by part(iv) $(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathfrak{R})$ implies *h-scl*$(\Omega, \mathscr{Q}) \subseteq$ *h-scl*$(\Omega, \mathfrak{R})$. Then *h-scl*$(\Omega, \mathscr{Q}) \subseteq$
h-scl$((\Omega, \mathscr{Q}) \cup (\Omega, \mathfrak{R}))$ and *h-scl*$(\Omega, \mathfrak{R}) \subseteq$ *h-scl*$((\Omega, \mathscr{Q}) \cup (\Omega, \mathfrak{R}))$, which is implies *h-scl*$(\Omega, \mathscr{Q}) \cup$ *h-scl*$(\Omega, \mathfrak{R}) \subseteq$ *h-scl*$((\Omega, \mathscr{Q}) \cup (\Omega, \mathfrak{R}))$. Now, *h-scl*$(\Omega, \mathscr{Q})$, *h-scl*$(\Omega, \mathfrak{R})$ is belong to hypersoft SCS in (Ω, P) which is implies that *h-scl*$(\Omega, \mathscr{Q}) \cup$
h-scl$(\Omega, \mathfrak{R})$ is belong to hypersoft SCS in (Ω, P). Then $(\Omega, \mathscr{Q}) \subseteq$
h-scl$(\Omega, \mathscr{Q})$ and $(\Omega, \mathfrak{R}) \subseteq$ *h-scl*$(\Omega, \mathfrak{R})$ imply $(\Omega, \mathscr{Q}) \cup (\Omega, \mathfrak{R}) \subseteq$ *h-scl*$(\Omega, \mathscr{Q}) \cup$
h-scl$(\Omega, \mathfrak{R})$. That is *h-scl*$(\Omega, \mathscr{Q}) \cup$ *h-scl*$(\Omega, \mathfrak{R})$ is a hypersoft SCS containing $(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathfrak{R})$. Hence *h-scl*$((\Omega, \mathscr{Q}) \cup (\Omega, \mathfrak{R})) \subseteq$ *h-scl*$(\Omega, \mathscr{Q}) \cup$ *h-scl*$(\Omega, \mathfrak{R})$. So,
h-scl$((\Omega, \mathscr{Q}) \cup (\Omega, \mathfrak{R})) =$ *h-scl*$(\Omega, \mathscr{Q}) \cup$ *h-scl*$(\Omega, \mathfrak{R})$.

Theorem 4.7. *Let* $((\Omega, \mathsf{P}), \tau)$ *be a hypersoft TS and let* $(\Omega, \mathscr{Q})$ *and* $(\Omega, \mathfrak{R})$ *be a hypersoft sets in* (Ω, P). *Then the succeeding condition holds.*

(1) *h-sint*$(\emptyset, \mathsf{P})$=$(\emptyset, \mathsf{P})$ *and h-sint*(Ω, P)=(Ω, P).

(2) $(\Omega, \mathscr{Q})$ *is hypersoft SOS iff* $(\Omega, \mathscr{Q})$=*h-sint*$(\Omega, \mathscr{Q})$.

(3) *h-sint*(*h-sint*$(\Omega, \mathscr{Q})$)=*h-sint*$(\Omega, \mathscr{Q})$.

(4) $(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathfrak{R})$ *implies* $h\text{-}sint(\Omega, \mathscr{Q}) \subseteq h\text{-}sint(\Omega, \mathfrak{R})$.

(5) $h\text{-}sint(\Omega, \mathscr{Q}) \cap h\text{-}sint(\Omega, \mathfrak{R}) \subseteq h\text{-}sint((\Omega, \mathscr{Q}) \cap (\Omega, \mathfrak{R}))$.

(6) $h\text{-}sint((\Omega, \mathscr{Q}) \cup (\Omega, \mathfrak{R})) = h\text{-}sint(\Omega, \mathscr{Q}) \cup h\text{-}sint(\Omega, \mathfrak{R})$.

Proof:

(1) The proof is obvious.

(2) If $(\Omega, \mathscr{Q})$ is hypersoft SOS, then $(\Omega, \mathscr{Q})$ is itself a hypersoft SOS in $(\Omega, \mathsf{P}) \subset (\Omega, \mathscr{Q})$. So, $h\text{-}sint(\Omega, \mathscr{Q})$ is the largest hypersoft SOS contained in $(\Omega, \mathscr{Q})$ and $(\Omega, \mathscr{Q}) = h\text{-}sint(\Omega, \mathscr{Q})$. Conversely, suppose that $(\Omega, \mathscr{Q}) = h\text{-}sint(\Omega, \mathscr{Q})$. Since $h\text{-}sint(\Omega, \mathscr{Q})$ is a hypersoft SOS, so $(\Omega, \mathscr{Q})$ is hypersoft semi-open set in (Ω, P).

(3) Since $h\text{-}sint(\Omega, \mathscr{Q})$ is a hypersoft SOS therefore by part(2) we have $h\text{-}sint(h\text{-}sint(\Omega, \mathscr{Q}))=h\text{-}sint(\Omega, \mathscr{Q})$.

(4) Suppose that $(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathfrak{R})$. Since $h\text{-}sint(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathscr{Q}) \subseteq (\Omega, \mathfrak{R})$. $h\text{-}sint(\Omega, \mathscr{Q})$ is a hypersoft semi-open subset of $(\Omega, \mathfrak{R})$, so by defn. of $h\text{-}sint(\Omega, \mathfrak{R})$, $h\text{-}sint(\Omega, \mathscr{Q}) \subseteq h\text{-}sint(\Omega, \mathfrak{R})$.

(5) Since $(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathscr{Q}) \cap ((\Omega, \mathfrak{R})$ and $(\Omega, \mathfrak{R}) \subseteq (\Omega, \mathscr{Q}) \cap ((\Omega, \mathfrak{R})$ and so by part(4), $h\text{-}sint(\Omega, \mathscr{Q}) \subseteq h\text{-}sint((\Omega, \mathscr{Q}) \cap (\Omega, \mathfrak{R}))$ and $h\text{-}sint(\Omega, \mathfrak{R}) \subseteq h\text{-}sint((\Omega, \mathscr{Q}) \cap (\Omega, \mathfrak{R}))$. So that $h\text{-}sint(\Omega, \mathscr{Q}) \cap h\text{-}sint(\Omega, \mathfrak{R}) \subseteq h\text{-}sint((\Omega, \mathscr{Q}) \cap (\Omega, \mathfrak{R}))$, since $h\text{-}sint((\Omega, \mathscr{Q}) \cap (\Omega, \mathfrak{R}))$ is a hypersoft semi-open set.

(6) Since $(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathscr{Q}) \cup (\Omega, \mathfrak{R})$ and $(\Omega, \mathfrak{R}) \subseteq (\Omega, \mathscr{Q}) \cup (\Omega, \mathfrak{R})$ and So by part(4) $(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathfrak{R})$ implies $h\text{-}sint(\Omega, \mathscr{Q}) \subseteq h\text{-}sint(\Omega, \mathfrak{R})$. Then $h\text{-}sint(\Omega, \mathscr{Q}) \subseteq h\text{-}sint((\Omega, \mathscr{Q}) \cup (\Omega, \mathfrak{R}))$ and $h\text{-}sint(\Omega, \mathfrak{R}) \subseteq h\text{-}sint((\Omega, \mathscr{Q}) \cup (\Omega, \mathfrak{R}))$ which is implies $h\text{-}sint(\Omega, \mathscr{Q}) \cup h\text{-}sint(\Omega, \mathfrak{R}) \subseteq h\text{-}sint((\Omega, \mathscr{Q}) \cup (\Omega, \mathfrak{R}))$. Now, $h\text{-}sint(\Omega, \mathscr{Q})$, $h\text{-}sint(\Omega, \mathfrak{R})$ is belong to hypersoft SOS in (Ω, P) which is implies that $h\text{-}sint(\Omega, \mathscr{Q}) \cup h\text{-}sint(\Omega, \mathfrak{R})$ is belong to hypersoft SOS in (Ω, P). Then $(\Omega, \mathscr{Q}) \subseteq h\text{-}sint(\Omega, \mathscr{Q})$ and $(\Omega, \mathfrak{R}) \subseteq h\text{-}sint(\Omega, \mathfrak{R})$ imply $(\Omega, \mathscr{Q}) \cup (\Omega, \mathfrak{R}) \subseteq h\text{-}sint(\Omega, \mathscr{Q}) \cup h\text{-}sint(\Omega, \mathfrak{R})$. That is $h\text{-}sint(\Omega, \mathscr{Q}) \cup h\text{-}sint(\Omega, \mathfrak{R})$. is a hypersoft SOS containing $(\Omega, \mathscr{Q}) \cup (\Omega, \mathfrak{R})$. Hence $h\text{-}sint((\Omega, \mathscr{Q}) \cup (\Omega, \mathfrak{R})) \subseteq h\text{-}sint(\Omega, \mathscr{Q}) \cup h\text{-}sint(\Omega, \mathfrak{R})$. So, $h\text{-}sint((\Omega, \mathscr{Q}) \cup (\Omega, \mathfrak{R}))=h\text{-}sint(\Omega, \mathscr{Q}) \cup h\text{-}sint(\Omega, \mathfrak{R})$.

Theorem 4.8. *Let $((\Omega, \mathsf{P}), \tau)$ be a hypersoft TS and let $(\Omega, \mathscr{Q})$ be a hypersoft set in (Ω, P). Then the preceding conditions holds.*

(1) $h\text{-}scl(h\text{-}cl(\Omega, \mathscr{Q})) = h\text{-}cl(h\text{-}scl(\Omega, \mathscr{Q})) = h\text{-}cl(\Omega, \mathscr{Q})$.

(2) $h\text{-}sint(h\text{-}int(\Omega, \mathscr{Q})) = h\text{-}int(h\text{-}sint(\Omega, \mathscr{Q})) = h\text{-}int(\Omega, \mathscr{Q})$.

Proof:

(1) Let $h\text{-}cl(\Omega, \mathscr{Q})$ is hypersoft closed set, then $h\text{-}cl(\Omega, \mathscr{Q})$ is hyersoft SCS by Thm. 3.9. So we can get $h\text{-}scl(h\text{-}cl(\Omega, \mathscr{Q})) = h\text{-}cl(\Omega, \mathscr{Q})$ by Theorem 4.6(2). By Thm. 4.4, we have $(\Omega, \mathscr{Q}) \subseteq h\text{-}scl(\Omega, \mathscr{Q}) \subseteq h\text{-}cl(\Omega, \mathscr{Q})$, then we can get
$h\text{-}cl(\Omega, \mathscr{Q}) \subseteq h\text{-}cl(h\text{-}scl(\Omega, \mathscr{Q})) \subseteq h\text{-}cl(\Omega, \mathscr{Q})$ and so $h\text{-}cl(h\text{-}scl(\Omega, \mathscr{Q})) = h\text{-}cl(\Omega, \mathscr{Q})$. This completes the proof.

(2) Let $((\Omega, \mathsf{P}), \tau)$ be a hypersoft TS and because $h\text{-}int(\Omega, \mathscr{Q})$ is hypersoft open set, we have $h\text{-}int(\Omega, \mathscr{Q})$ is hypersoft SOS by Thm. 3.4. So we can get $h\text{-}sint(h\text{-}int(\Omega, \mathscr{Q})) = h\text{-}int(\Omega, \mathscr{Q})$ by Thm. 4.7(2). By Thm. 4.4, we have $h\text{-}int(\Omega, \mathscr{Q}) \subseteq h\text{-}sint(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathscr{Q})$, then we can get $h\text{-}int(\Omega, \mathscr{Q}) \subseteq$
$h\text{-}int(h\text{-}sint(\Omega, \mathscr{Q})) \subseteq h\text{-}int(\Omega, \mathscr{Q})$ and so $h\text{-}int(h\text{-}sint(\Omega, \mathscr{Q})) = h\text{-}int(\Omega, \mathscr{Q})$.

Theorem 4.9. *Let $((\Omega, \mathsf{P}), \tau)$ be a hypersoft TS and let $(\Omega, \mathscr{Q})$ be a hypersoft set in (Ω, P). Then the succeeding are equivalent.*

(1) *$(\Omega, \mathscr{Q})$ is hypersoft SCS.*
(2) $h\text{-}int(h\text{-}cl(\Omega, \mathscr{Q})) \subseteq (\Omega, \mathscr{Q})$.
(3) $h\text{-}cl(h\text{-}int((\Omega, \mathscr{Q})^c) \supseteq (\Omega, \mathscr{Q})^c$.
(4) *$(\Omega, \mathscr{Q})^c$ is hypersoft SOS.*

Proof:
(1)⇒(2): If $(\Omega, \mathscr{Q})$ is hypersoft SCS, then $\exists$ hypersoft closed set $(\Omega, \mathfrak{R})$ s.t. $h\text{-}int(\Omega, \mathfrak{R}) \subseteq (\Omega, \mathscr{Q}) \subseteq (\Omega, \mathfrak{R}) \Rightarrow h\text{-}int(\Omega, \mathfrak{R}) \subseteq (\Omega, \mathscr{Q}) \subseteq h\text{-}cl(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathfrak{R})$. By the property of interior, we get $h\text{-}int(h\text{-}cl(\Omega, \mathscr{Q})) \subseteq h\text{-}int(\Omega, \mathfrak{R}) \subseteq (\Omega, \mathscr{Q})$.
(2)⇒(3): $h\text{-}int(h\text{-}cl(\Omega, \mathscr{Q})) \subseteq (\Omega, \mathscr{Q}) \Rightarrow (\Omega, \mathscr{Q})^c \subseteq h\text{-}int(h\text{-}cl(\Omega.\mathscr{Q}))^c=$
$h\text{-}cl(h\text{-}int(\Omega, \mathscr{Q})^c) \supseteq (\Omega, \mathscr{Q})^c$.

(3)⇒(4): $(\Omega, \mathfrak{R}) = h\text{-}int((\Omega, \mathscr{Q})^c)$ is an hypersoft open set s.t. $h\text{-}int((\Omega, \mathscr{Q})^c) \subseteq (\Omega, \mathscr{Q})^c \subseteq h\text{-}cl(h\text{-}int((\Omega, \mathscr{Q})^c)$, hence $(\Omega, \mathscr{Q})^c$ is hypersoft SOS.
(4)⇒(1): As $(\Omega, \mathscr{Q})^c$ is hypersoft SOS, $\exists$ an hypersoft open set $(\Omega, \mathfrak{R})$ s.t. $(\Omega, \mathfrak{R}) \subseteq (\Omega, \mathscr{Q})^c \subseteq h\text{-}cl(\Omega, \mathfrak{R}) \Rightarrow (\Omega, \mathfrak{R})^c$ is a hypersoft closed set such that $(\Omega, \mathscr{Q}) \subseteq (\Omega, \mathfrak{R})^c$ and $(\Omega, \mathscr{Q})^c \subseteq h\text{-}cl(\Omega, \mathfrak{R}) \Rightarrow h\text{-}int(\Omega, \mathfrak{R})^c \subseteq (\Omega, \mathscr{Q})$. Hence $(\Omega, \mathscr{Q})$ is hypersoft SCS.

5. Conclusion

We have introduced hypersoft semi-open sets in hypersoft TS which are identified over an initial universe with a fixed set of parameters. We then define hypersoft semi-interior and hypersoft semi-closure with suitable example. The concept of open sets produced in this work may be developed to α-open hypersoft sets and β-open hypersoft sets. Based on the works of [6] and [7], our future research may be on IndetermHypersoft semi-open sets and on TreeSoft semi-open sets.

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