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New Types of Topologies and Neutrosophic Topologies (Improved Version)

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Abstract: In this paper we recall the six new types of topologies, and their corresponding topological spaces, that we introduced in the last years (2019-20223), such as: NeutroTopology, AntiTopology, Refined Neutrosophic Topology, Refined Neutrosophic Crisp Topology, SuperHyperTopology, and Neutrosophic SuperHyperTopology.

The n^{th} -PowerSets $P^n(H)$ and $P_*^n(H)$, that the SuperHyperTopology and respectively Neutrosophic SuperHyperTopology are based on, better describe our real world, since a system H (that may be a set, company, institution, country, region, etc.) is organized in sub-systems, which in their turn are organized each in sub-sub-systems, and so on.

Keywords: Classical Topology; Topological Space; NeutroTopology; AntiTopology; Refined Neutrosophic Topology; Refined Neutrosophic Crisp Topology; SuperHyperTopology; Neutrosophic SuperHyperTopology.

1. Introduction

We recall the classical definition of Topology, then the procedures of Neutrosophication and respectively AntiSophication of it, that result in adding in two new types of topologies: NeutroTopology and respectively AntiTopology.

Then we define topology on Refined Neutrosophic Set (2013), Refined Neutrosophic Crisp Set [3]. Afterwards, we extend the topology on the framework of SuperHyperAlgebra [6].

The corresponding neutrosophic topological spaces are presented.

This research is an improvement of paper [7].

2. *Classical Topology*

Let \mathcal{U} be a non-empty set, and $P(\mathcal{U})$ the power set of \mathcal{U} .

Let $\tau \subseteq P(\mathcal{U})$ be a family of subsets of \mathcal{U} .

Then τ is called a Classical Topology on \mathcal{U} if it satisfies the following axioms:

(CT-1) ϕ and \mathcal{U} belong to τ .

(CT-2) The intersection of any finite number of elements in τ is in τ .

(CT-3) The union of any finite or infinite number of elements in τ is in τ .

All three axioms are totally (100%) true (or $T = 1, I = 0, F = 0$). We simply call them (classical) *Axioms*.

Then (\mathcal{U}, τ) is called a *Classical Topological Space* on \mathcal{U} .

3. Neutrosophication of the Topological Axioms

Neutrosophication of the topological axioms means that the axioms become partially true, partially indeterminate, and partially false. They are called *NeutroAxioms*.

(NCT-1) Either $\{\phi \notin \tau \text{ and } \mathcal{U} \in \tau\}$, or $\{\phi \in \tau \text{ and } \mathcal{U} \notin \tau\}$.

(NCT-2) There exist a finite number of elements in τ whose intersection belong to τ (degree of truth T); and a finite number of elements in τ whose intersection is indeterminate (degree of indeterminacy I); and a finite number of elements in τ whose intersection does not belong to τ (degree of falsehood F); where $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$ since $(1, 0, 0)$ represents the above Classical Topology, while $(0, 0, 1)$ the below AntiTopology.

(NCT-3) There exist a finite or infinite number of elements in τ whose union belongs to τ (degree of truth T); and a finite or infinite number of elements in τ whose union is indeterminate (degree of indeterminacy I); and a finite or infinite number of elements in τ whose union does not belong to τ (degree of falsehood F); where of course $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$.

4. AntiSophication of the Topological Axioms

AntiSophication of the topological axioms means to negate (anti) the axioms, the axioms become totally (100%) false (or $T = 0, I = 0, F = 1$). They are called *AntiAxioms*.

(ACT-1) $\phi \notin \tau$ and $\mathcal{U} \notin \tau$.

(ACT-2) The intersection of any finite number ($n \geq 2$) of elements in τ is not in τ .

(ACT-3) The union of any finite or infinite number ($n \geq 2$) of elements in τ is not in τ .

5. Neutrosophic Triplets related to Topology

As such, we have a neutrosophic triplet of the form:

$$\langle \text{Axiom}(1, 0, 0), \text{NeutroAxiom}(T, I, F), \text{AntiAxiom}(0, 0, 1) \rangle,$$

where $(T, I, F) \neq (1, 0, 0)$ and $(T, I, F) \neq (0, 0, 1)$.

Correspondingly, one has:

$$\langle \text{Topology}, \text{NeuroTopology}, \text{AntiTopology} \rangle.$$

Therefore, in general:

(Classical) Topology is a topology that has all axioms totally true. We simply call them *Axioms*.

NeuroTopology is a topology that has at least one *NeutroAxiom* and the others are all *classical Axioms* [therefore, no *AntiAxiom*].

AntiTopology is a topology that has one or more *AntiAxioms*, no matter what the others are (*classical Axioms*, or *NeutroAxioms*).

6. Theorem on the number of Structures/NeuroStructures/AntiStructures

If a Structure has m axioms, with $m \geq 1$, then after NeuroSophication and AntiSophication one obtains 3^m types of structures, categorized as follows:

$$1 \text{ Classical Structure} + (2^m - 1) \text{ NeuroStructures} + (3^m - 2^m) \text{ AntiStructures}.$$

7. Consequence on the number of Topologies/NeuroTopologies/AntiTopologies

As a particular case of the previous theorem, from a Topology which has $m = 3$ axioms, one makes, after NeuroSophication and AntiSophication, $3^3 = 27$ types of structures, as follows: 1 classical Topology, $2^3 - 1 = 7$ NeuroTopologies, and $3^3 - 2^3 = 19$ AntiTopologies.

$$1 \text{ Classical Topology} + 7 \text{ NeuroTopologies} + 19 \text{ AntiTopologies}$$

are presented below:

There is 1 (one) type of Classical Topology, whose axioms are listed below:

1 Classical Topology

$$\begin{pmatrix} CT - 1 \\ CT - 2 \\ CT - 3 \end{pmatrix}$$

8. **Definition of NeuroTopology** [4, 5]

It is a topology that has at least one topological axiom which is partially true, partially indeterminate, and partially false,

or (T, I, F) , where T = True, I = Indeterminacy, F = False,

and no topological axiom is totally false,

in other words: $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$, where $(1, 0, 0)$ represents the classical Topology, while $(0, 0, 1)$ represents the below AntiTopology.

Therefore, the NeutroTopology is a topology in between the classical Topology and the AntiTopology.

There are 7 types of different NeutroTopologies, whose axioms, for each type, are listed below:

7 NeutroTopologies

$$\begin{pmatrix} NCT - 1 \\ CT - 2 \\ CT - 3 \end{pmatrix}, \begin{pmatrix} CT - 1 \\ NCT - 2 \\ CT - 3 \end{pmatrix}, \begin{pmatrix} CT - 1 \\ CT - 2 \\ NCT - 3 \end{pmatrix},$$

$$\begin{pmatrix} NCT - 1 \\ NCT - 2 \\ CT - 3 \end{pmatrix}, \begin{pmatrix} CT - 1 \\ NCT - 2 \\ NCT - 3 \end{pmatrix}, \begin{pmatrix} NCT - 1 \\ CT - 2 \\ NCT - 3 \end{pmatrix},$$

$$\begin{pmatrix} NCT - 1 \\ NCT - 2 \\ NCT - 3 \end{pmatrix}.$$

9. Definition of AntiTopology [4, 5]

It is a topology that has at least one topological axiom that is 100% false $(T, I, F) = (0, 0, 1)$. The NeutroTopology and AntiTopology are particular cases of NeutroAlgebra and AntiAlgebra [4] and, in general, they all are particular cases of the NeutroStructure and AntiStructure respectively, since we consider "Structure" in any field of knowledge [5].

There are 19 types of different AntiTopologies, whose axioms, for each type, are listed below:

19 AntiTopologies

$$\begin{pmatrix} ACT - 1 \\ CT - 2 \\ CT - 3 \end{pmatrix}, \begin{pmatrix} CT - 1 \\ ACT - 2 \\ CT - 3 \end{pmatrix}, \begin{pmatrix} CT - 1 \\ CT - 2 \\ ACT - 3 \end{pmatrix},$$

$$\begin{pmatrix} ACT - 1 \\ ACT - 2 \\ CT - 3 \end{pmatrix}, \begin{pmatrix} CT - 1 \\ ACT - 2 \\ ACT - 3 \end{pmatrix}, \begin{pmatrix} ACT - 1 \\ CT - 2 \\ ACT - 3 \end{pmatrix},$$

$$\begin{pmatrix} ACT - 1 \\ NCT - 2 \\ NCT - 3 \end{pmatrix}, \begin{pmatrix} NCT - 1 \\ ACT - 2 \\ NCT - 3 \end{pmatrix}, \begin{pmatrix} NCT - 1 \\ NCT - 2 \\ ACT - 3 \end{pmatrix},$$

$$\begin{pmatrix} ACT - 1 \\ ACT - 2 \\ NCT - 3 \end{pmatrix}, \begin{pmatrix} NCT - 1 \\ ACT - 2 \\ ACT - 3 \end{pmatrix}, \begin{pmatrix} ACT - 1 \\ NCT - 2 \\ ACT - 3 \end{pmatrix},$$

$$\begin{pmatrix} ACT - 1 \\ NCT - 2 \\ CT - 3 \end{pmatrix}, \begin{pmatrix} CT - 1 \\ ACT - 2 \\ NCT - 3 \end{pmatrix}, \begin{pmatrix} NCT - 1 \\ CT - 2 \\ ACT - 3 \end{pmatrix},$$

$$\begin{pmatrix} ACT - 1 \\ CT - 2 \\ NCT - 3 \end{pmatrix}, \begin{pmatrix} CT - 1 \\ NCT - 2 \\ ACT - 3 \end{pmatrix}, \begin{pmatrix} NCT - 1 \\ ACT - 2 \\ CT - 3 \end{pmatrix},$$

$$\begin{pmatrix} ACT - 1 \\ ACT - 2 \\ ACT - 3 \end{pmatrix}.$$

10. Refined Neutrosophic Set

Let U be a universe of discourse, and a non-empty subset R of it,

$$R = \left\{ \begin{array}{l} x \left(T_1(x), T_2(x), \dots, T_p(x) \right); \\ \left(I_1(x), I_2(x), \dots, I_r(x) \right); \\ \left(F_1(x), F_2(x), \dots, F_s(x) \right); \end{array} \right\}$$

with all $T_j, I_k, F_l \in [0,1]$, $1 \leq j \leq p, 1 \leq k \leq r, 1 \leq l \leq s$, and no restriction on their sums

$0 \leq T_m + I_m + F_m \leq 3$, with $1 \leq m \leq \max\{p, r, s\}$, where $p, r, s \geq 0$ are fixed integers, and at least

one of them is ≥ 2 , in order to ensure the refinement (sub-parts) or multiplicity (multi-parts) – depending on the application, of at least one neutrosophic component amongst T (truth), I (indeterminacy), F (falsehood); **and of course** $x \in U$.

By notation we consider that index zero means the empty-set, i.e. $T_0 = I_0 = F_0 = \phi$ (or zero), and

the same for the missing sub-parts (or multi-parts).

For example, the below (2,3,1)-Refined Neutrosophic Set is identical to a (3,3,3)-Refined Neutrosophic Set: $(T_1, T_2; I_1, I_2, I_3; F_1) \equiv (T_1, T_2, 0; I_1, I_2, I_3; F_1, 0, 0)$,

where the missing components T_3 , and F_2, F_3 were replaced each of them by 0 (zero)

R is called a (p, r, s) -refined neutrosophic set { or (p, r, s) -RNT }.

The neutrosophic set has been extended to the Refined Neutrosophic Set (Logic, and Probability) by Smarandache [1] in 2013, where there are multiple parts of the neutrosophic components, as such T was split into subcomponents T_1, T_2, \dots, T_p , and I into I_1, I_2, \dots, I_r , and F into F_1, F_2, \dots, F_s , with $p + r + s = n \geq 2$ and integers $p, r, s \geq 0$ and at least one of them is ≥ 2 in order to ensure the refinement (or multiplicity) of at least one neutrosophic component amongst T, I , and F .

Even more: the subcomponents T_i , I_k , and/or F_i can be countable or uncountable infinite subsets of $[0, 1]$.

This definition also includes the *Refined Fuzzy Set*, when $r = s = 0$ and $p \geq 2$;

and the definition of the *Refined Intuitionistic Fuzzy Set*, when $r = 0$, and either $p \geq 2$ and $s \geq 1$, or $p \geq 1$ and $s \geq 2$.

All other fuzzy extension sets (Pythagorean Fuzzy Set, Spherical Fuzzy Set, Fermatean Fuzzy Set, q-Rung Orthopair Fuzzy Set, etc.) can be refined/multiplied in a similar way.

11. Definition of Refined Neutrosophic Topology

Let \mathcal{U} be a universe of discourse, and $\mathcal{P}(\mathcal{U})$ be the family of all (p, r, s) -refined neutrosophic subsets of \mathcal{U} .

Let $\tau_{RNT} \subseteq \mathcal{P}(\mathcal{U})$ be a family of (p, r, s) -refined neutrosophic subsets of \mathcal{U} .

Then τ_{RNT} is called a *Refined Neutrosophic Topology (RNT)* if it satisfies the axioms:

(RNT-1) ϕ and \mathcal{U} belong to τ_{RNT} ;

(RNT-2) The intersection of any finite number of elements in τ_{RNT} is in τ_{RNT} ;

(RNT-3) The union of any finite or infinite number of elements in τ_{RNT} is in τ_{RNT} .

Then $(\mathcal{U}, \tau_{RNT})$ is called a *Refined Neutrosophic Topological Space* on \mathcal{U} .

The *Refined Neutrosophic Topology* is a topology defined on a *Refined Neutrosophic Set*.

{Similarly, the *Refined Fuzzy Topology* is defined on a *Refined Fuzzy Set*, while the *Refined Intuitionistic Fuzzy Topology* is defined on a *Refined Intuitionistic Fuzzy Set*, etc.

And, as a generalization, on any type of fuzzy extension set [such as: Pythagorean Fuzzy Set, Spherical Fuzzy Set, Fermatean Fuzzy Set, q-Rung Orthopair Fuzzy Set, etc.] one can define a corresponding fuzzy extension topology.}

12. Neutrosophic Crisp Set

The *Neutrosophic Crisp Set* was defined by Salama and Smarandache in 2014 and 2015.

Let X be a non-empty fixed space. And let D be a *Neutrosophic Crisp Set* [2], where $D = \langle A, B, C \rangle$, with A, B, C as subsets of X .

Depending on the intersections and unions between these three sets A, B, C one gets several:

Types of Neutrosophic Crisp Sets [2, 3]

The object having the form $D = \langle A, B, C \rangle$ is called:

(a) A neutrosophic crisp set of Type 1 (NCS-Type1) if it satisfies:

$$A \cap B = B \cap C = C \cap A = \emptyset \text{ (empty set).}$$

(b) A neutrosophic crisp set of Type 2 (NCS-Type2) if it satisfies:

$$A \cap B = B \cap C = C \cap A = \emptyset \text{ and } A \cup B \cup C = X.$$

(c) A neutrosophic crisp set of Type 3 (NCS-Type3) if it satisfies:

$$A \cap B \cap C = \emptyset \text{ and } A \cup B \cup C = X.$$

Of course, more types of Neutrosophic Crisp Sets may be defined by modifying the intersections and unions of the subsets $A, B,$ and $C.$

13. Refined Neutrosophic Crisp Set

The *Refined Neutrosophic Crisp Set* [3] was introduced by Smarandache in 2019, by refining/multiplication of D (and denoting it by $RD =$ Refined D) by refining/multiplication of its sets A, B, C into sub-subsets/multi-sets as follows:

$RD = (A_1, \dots, A_p; B_1, \dots, B_r; C_1, \dots, C_s),$ with $p, r, s \geq 1$ be positive integers and at least one of them be ≥ 2 in order to ensure the refinement/multiplication of at least one component amongs $A, B, C,$ where

$$A = \bigcup_{i=1}^p A_i, B = \bigcup_{j=1}^r B_j, C = \bigcup_{k=1}^s C_k$$

and many types of Refined Neutrosophic Crisp Sets may be defined by modifying the intersections or unions of the subsets/multisets $A_i, B_j, C_k, 1 \leq i \leq p, 1 \leq j \leq r, 1 \leq k \leq s,$

depending on each application.

14. Definition of Refined Neutrosophic Crisp Topology

Let \mathcal{U} be a universe of discourse, and $\mathcal{P}(\mathcal{U})$ be the family of all (p, r, s) -refined neutrosophic crisp subsets of $\mathcal{U}.$

Let $\tau_{RNCT} \subseteq \mathcal{P}(\mathcal{U})$ be a family of (p, r, s) -refined neutrosophic crisp subsets of $\mathcal{U}.$

Then τ_{RNCT} is called a *Refined Neutrosophic Crisp Topology (RNCT)* if it satisfies the axioms:

(RNCT-1) \emptyset and \mathcal{U} belong to $\tau_{RNCT};$

(RNCT-2) The intersection of any finite number of elements in τ_{RNCT} is in τ_{RNCT} ;

(RNCT-3) The union of any finite or infinite number of elements in τ_{RNCT} is in τ_{RNCT} .

Then $(\mathcal{U}, \tau_{RNCT})$ is called a Refined Neutrosophic Crisp Topological Space on \mathcal{U} .

Therefore, the *Refined Neutrosophic Crisp Topology* is a topology defined on the Refined Neutrosophic Crisp Set.

15. SuperHyperOperation

We recall our 2016 concepts of SuperHyperOperation, SuperHyperAxiom, SuperHyperAlgebra, and their corresponding Neutrosophic SuperHyperOperation Neutrosophic SuperHyperAxiom and Neutrosophic SuperHyperAlgebra [6].

Let $P_*^n(H)$ be the n^{th} -powerset of the set H such that none of $P(H), P^2(H), \dots, P^n(H)$ contain the empty set \emptyset .

Also, let $P_n(H)$ be the n^{th} -powerset of the set H such that at least one of the $P(H), P^2(H), \dots, P^n(H)$ contain the empty set \emptyset . For any subset A , we identify $\{A\}$ with A .

The SuperHyperOperations are operations whose codomain is either $P_*^n(H)$ and in this case one has classical-type SuperHyperOperations, or $P_n(H)$ and in this case one has Neutrosophic SuperHyperOperations, for integer $n \geq 2$.

16. The n^{th} -PowerSet better describe our real world

The n^{th} -PowerSets $P^n(H)$ and $P_*^n(H)$, that the SuperHyperTopology and respectively Neutrosophic SuperHyperTopology are based on, better describe our real world, since a system H (that may be a set, company, institution, country, region, etc.) is organized in sub-systems, which in their turn are organized each in sub-sub-systems, and so on.

17. SuperHyperAxiom

A classical-type SuperHyperAxiom or more accurately a (m, n) -SuperHyperAxiom is an axiom based on classical-type SuperHyperOperations.

Similarly, a Neutrosophic SuperHyperAxiom (or Neutrosophic (m, n) -SuperHyperAxiom) is an axiom based on Neutrosophic SuperHyperOperations.

There are:

- Strong SuperHyperAxioms, when the left-hand side is equal to the right-hand side as in non-hyper axioms,
- and Weak SuperHyperAxioms, when the intersection between the left-hand side and the right-hand side is non-empty.

18. SuperHyperAlgebra and SuperHyperStructure

A SuperHyperAlgebra or more accurately $(m-n)$ -SuperHyperAlgebra is an algebra dealing with SuperHyperOperations and SuperHyperAxioms.

Again, a Neutrosophic SuperHyperAlgebra (or Neutrosophic (m, n)-SuperHyperAlgebra) is an algebra dealing with Neutrosophic SuperHyperOperations and Neutrosophic SuperHyperOperations.

In general, we have SuperHyperStructures {or (m-n)-SuperHyperStructures}, and corresponding Neutrosophic SuperHyperStructures.

For example, there are SuperHyperGrupoid, SuperHyperSemigroup, SuperHyperGroup, SuperHyperRing, SuperHyperVectorSpace, etc.

19. Distinction between SuperHyperAlgebra vs. Neutrosophic SuperHyperAlgebra

- i. If none of the power sets $P^k(H)$, $1 \leq k \leq n$, do not include the empty set ϕ , then one has a classical-type SuperHyperAlgebra;
- ii. If at least one power set, $P^k(H)$, $1 \leq k \leq n$, includes the empty set ϕ , then one has a Neutrosophic SuperHyperAlgebra.

20. Definition of SuperHyperTopology (SHT) [6]

It is a topology designed on the n^{th} -PowerSet of a given non-empty set H , that excludes the empty-set, denoted as $P_*(H)$, built as follows:

$P_*(H)$ is the first powerset of the set H , and the index $*$ means without the empty-set (\emptyset);

$P_*^2(H) = P_*(P_*(H))$ is the second powerset of H (or the powerset of the powerset of H), without the empty-sets; and so on,
the n -th powerset of H ,

$P_*^n(H) = P_*(P_*^{n-1}(H)) = \underbrace{P_*(P_*(\dots P_*(H)\dots))}_n$, where P_* is repeated n time ($n \geq 2$), and without the empty-sets.

Let consider τ_{SHT} be a family of subsets of $P_*^n(H)$.

Then τ_{SHT} is called a Neutrosophic SuperHyperTopology on $P_*^n(H)$, if it satisfies the following axioms:

(SHT-1) ϕ and $P_*^n(H)$ belong to τ_{SHT} .

(SHT-2) The intersection of any finite number of elements in τ_{SHT} is in τ_{SHT} .

(SHT-3) The union of any finite or infinite number of elements in τ_{SHT} is in τ_{SHT} .

Then $(P_*^n(H), \tau_{SHT})$ is called a SuperHyperTopological Space on $P_*^n(H)$.

21. Definition of Neutrosophic SuperHyperTopology (NSHT) [6]

It is, similarly, a topology designed on the n -th PowerSet of a given non-empty set H , but includes the empty-sets [that represent indeterminacies] too.

As such, in the above formulas, $P_*(H)$ that excludes the empty-set, is replaced by $P(H)$ that includes the empty-set.

$P(H)$ is the first powerset of the set H , including the empty-set (\emptyset);

$P^2(H) = P(P(H))$ is the second powerset of H (or the powerset of the powerset of H), that

includes the empty-sets;

and so on, the n -th powerset of H ,

$$P^n(H) = P(P^{n-1}(H)) = \underbrace{P(P(\dots P(H)\dots))}_n$$

where P is repeated n times ($n \geq 2$), and includes the empty-sets (\emptyset).

Let consider τ_{NSHT} be a family of subsets of $P^n(H)$.

Then τ_{NSHT} is called a Neutrosophic SuperHyperTopology on $P^n(H)$, if it satisfies the following axioms:

(NSHT-1) \emptyset and $P^n(H)$ belong to τ_{NSHT} .

(NSHT-2) The intersection of any finite number of elements in τ_{NSHT} is in τ_{NSHT} .

(NSHT-3) The union of any finite or infinite number of elements in τ_{NSHT} is in τ_{NSHT} .

Then $(P^n(H), \tau_{NSHT})$ is called a Neutrosophic SuperHyperTopological Space on $P^n(H)$.

22. Conclusion

These six new types of topologies, and their corresponding topological space, were introduced by Smarandache in 2019-2023, but they have not yet been much studied and applied, except the NeuroTopologies and AntiTopologies which got some attention from researchers.

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