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Abstract: The Laplace distribution, also known as the double exponential distribution, is a continuous probability distribution that is often used for modelling the data having heavy tails. In this paper, we proposed the Neutrosophic Laplace distribution which is the extension of the classical Laplace Distribution. We derived various statistical properties of the Neutrosophic Laplace Distribution such as mean, variance, skewness, rth moment, quartiles, and moment-generating function. The expressions for the estimation of the parameters are also derived using the maximum likelihood function of the distribution. A simulation study has been done to evaluate the performance of estimates. An application of the Neutrosophic Laplace Distribution is discussed to study the daily return of the NIFTY50 from Indian Stock Market. The analysis shows that the neutrosophic Laplace Model is acceptable, effective, and adequate for dealing with uncertainty in an unpredictable context.

Keywords: Neutrosophic Laplace Distribution, Estimation, Indeterminacy, Financial Data Analysis, Simulation, Stock Returns.

1. Introduction

The continuous random variables have commonly been described and analysed by continuous statistical probability models as they provide a framework for understanding the distribution of continuous data and making probabilistic predictions. These models have numerous applications in the fields such as physics and engineering, quality control and process improvement, environmental analysis, financial modeling, market research and consumer behavior, insurance and actuarial science, demography and epidemiology and reliability engineering. The Laplace Distribution (LD) has gained popularity due to its unique properties and its ability to model various phenomena. The LD arises naturally as the distribution of the difference between two independent random variables follows the exponential distribution, which makes it useful for modeling the behavior of certain stochastic processes. Laplace [1] employed this distribution to model the frequency of an error as an exponential function of its magnitude after the sign was ignored. The Laplace model is most well-suited for modelling the data with outliers or heavy-tailed behaviour. The comprehensive reference book [2] provides a detailed treatment of various continuous distributions, including the Laplace distribution. It covers theoretical aspects, properties, and applications. Everitt and Hand [3] explored the mixture models including the Laplace mixture model which is a combination of LDs. It covers estimation techniques and applications in statistical modelling. Rue et al. [4] discussed the use of the Laplace approximation for Bayesian inference in latent Gaussian models. It introduces the Integrated Nested Laplace Approximation (INLA) methodology, which has become popular in Bayesian statistics. Ghosh and Chaudhuri [5] discussed the Bayesian analysis of regression models with Laplace-distributed errors. It discusses the choice of priors, estimation methods and inference in the context of Laplace regression models. One of the reasons for the popularity of the Laplace distribution in research is its

connection to the Laplace transform, which is a mathematical technique used in solving differential equations.
The finance sector plays a vital role in the economy by providing a range of financial services that facilitate the efficient allocation of capital, risk management and economic growth. Uncertainty in financial data refers to the inherent unpredictability and variability observed in financial markets and related economic variables. Fitting uncertain financial data involves developing statistical models or techniques to capture the characteristics and patterns present in such data. This process is essential for understanding and analysing financial variables that exhibit uncertainty such as stock prices, returns, volatility, or option prices. Fuzzy logic, a variant of neutrosophic logic, provides information solely about truth and falsity measures. In contrast, neutrosophic logic, an extension of fuzzy logic, also accounts for the degree of uncertainty. Neutrosophic statistics employ precise numbers to represent data within intervals. Smarandache [6] introduced the concept of neutrosophy to accurately represent and model the inherent indeterminacies present in data. It represents a novel domain in philosophy, serving as an expansion of fuzzy and intuitionistic fuzzy logics [7-11]. Smarandache [12-15] proposed the fundamental principles of neutrosophic sets across multiple domains.

Neutrosophic statistics offers greater flexibility compared to classical statistics. When both data and inference methods are definite, neutrosophic statistics aligns with classical statistics. However, given the prevalence of indeterminate data in real life situations, there is a greater demand for neutrosophic statistical procedures over classical ones. Numerous researchers have introduced highly valuable neutrosophic probability distributions for the analysis of such data sets. Alhasan and Smarandache [16] proposed several distributions under indeterminacy including "neutrosophic Rayleigh distribution, neutrosophic Weibull distribution, neutrosophic five-parameter Weibull distribution, neutrosophic three-parameter Weibull distribution, neutrosophic beta Weibull distribution and neutrosophic inverse Weibull distribution". Aslam [17] introduced the concept of the neutrosophic Raleigh distribution and employed it to model wind speed data. In their work, Alhabib et al. [18] introduced the concept of neutrosophic Uniform, neutrosophic exponential, and neutrosophic Poisson distributions. Khan et al. [19] extended the classical gamma distribution in neutrosophic environment and its application in the complex data analysis.

Albassam et al. [20] discussed the basic properties of the neutrosophic Weibull distribution and its application in the analysis of the wind speed data and LED manufacturing process. They utilized that the neutrosophic Weibull model is suitable, logical, and efficient when applied within an environment characterized by uncertainty. Sherwan et al. [21] extended the beta distribution under neutrosophic environment and proved the several properties for legitimate the proposed distribution. Jdid et al. [22] developed a mathematical model to minimize the inspection costs and demonstrated the study using both classical and neutrosophic values. Sleem et al. [22] described an integrated framework for assessing customer factors and product requirements in VR Metaverse design by merging CRITIC approach with SVNS. Hezam[23] proposed a strategy for machine tool selection using an innovative hybrid MCDM framework under neutrosophic environment.There is a vast body of literature encompassing various statistical distributions that can be utilized to model different kinds of data.

Classical distributions are only applicable when all data observations are exact in nature. However, real-world data is often imprecise, uncertain and have lack of exactness. The applications of existing classical distributions are not suitable for such cases. By an extensive exploration of the literature, no previous research has focused on examining the properties of the Laplace distribution in the context of uncertainty. To fill the research gap, in this paper, we introduced and analysed several properties of the Laplace distribution under conditions of indeterminacy. The Neutrosophic Laplace Distribution (NLD) is an extension of the Laplace Distribution (LD) that incorporates the concept of neutrosophic logic. The maximum likelihood estimation method has been used to estimate the parameters. The effectiveness of obtained estimators is evaluated through a simulation analysis. An application of the proposed distribution is discussed on the financial data analysis. The neutrosophic Laplace model is anticipated to be more effective in modelling stock return data compared to the traditional Laplace distribution used in classical statistics.

2. Neutrosophic Laplace (double exponential) Distribution

A Neutrosophic continuous random variable $z_n = z_l + z_u i_n$ is said to have Neutrosophic Laplace Distribution if it follows the following probability density function

$$f(z_n;\theta_n,\beta_n) = \begin{cases} \frac{1}{2\beta_l} e^{\left(-\frac{|z_l-\theta_l|}{\beta_l}\right)} + \left\{\frac{1}{2\beta_u} e^{\left(-\frac{|z_u-\theta_u|}{\beta_u}\right)}\right\} i_n, & -\infty < z_n < \infty \; ; -\infty < \theta_n < \infty \; , \beta_n > 0 \\ 0 & \end{cases} \quad \textbf{(1)}$$

Here, $\theta_n = \theta_l + \theta_u i_n$ is the location parameter, $\beta_n = \beta_l + \beta_u i_n$ is the scale parameter where $i_n \in (i_l, i_u)$. The shape of the curve of NLD depends upon the value of β_n .

Suppose that $z_l = z_u = z_n$, the pdf can be written as

$$f(z_n;\theta_n,\beta_n) = \left(\frac{1}{2\beta_n} e^{\left(-\frac{|z_n-\theta_n|}{\beta_n}\right)}\right)(1 + i_n) \quad \textbf{(2)}$$

If $i_l = 0$, the NLD will reduce to the classical Laplace distribution.

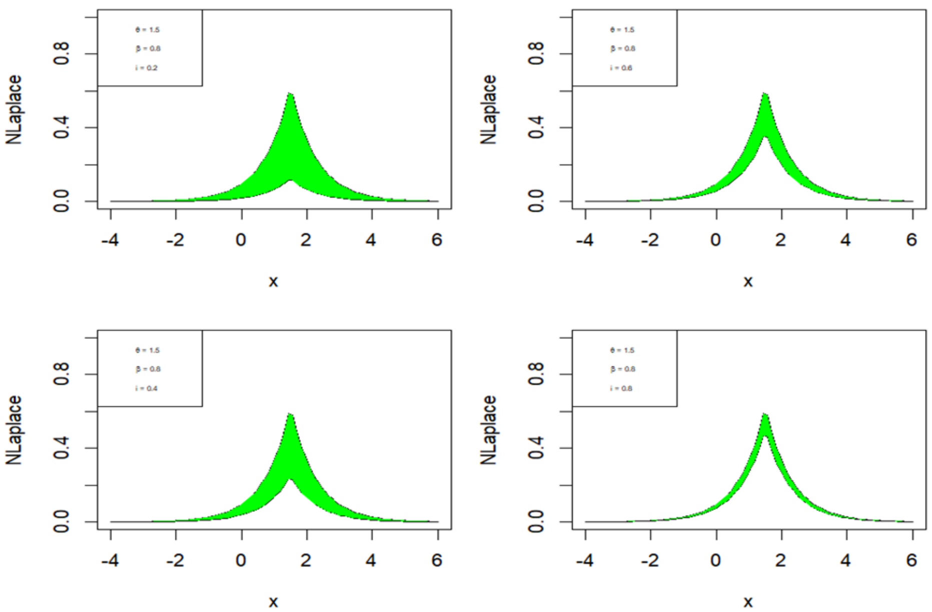

Figure. 1: The pdf graph representing the distribution of NLD with different level of indeterminant scale and shape parameters

The cumulative distribution function (cdf) is

$$F(z_n) = \begin{cases} \frac{1}{2} e^{\left(\frac{z_n-\theta_n}{\beta_n}\right)}(1 + i_n) \;, & z_n < \theta_n \\ 1 - \frac{1}{2} e^{\left(-\frac{(z_n-\theta_n)}{\beta_n}\right)}(1 + i_n) & , z_n \geq \theta_n \end{cases} \quad \textbf{(3)}$$

Where $\theta_n = \theta_l + \theta_u i_n$ is the location parameter, $\beta_n = \beta_l + \beta_u i_n$ is the scale parameter where $i_n \in (i_l, i_u)$.

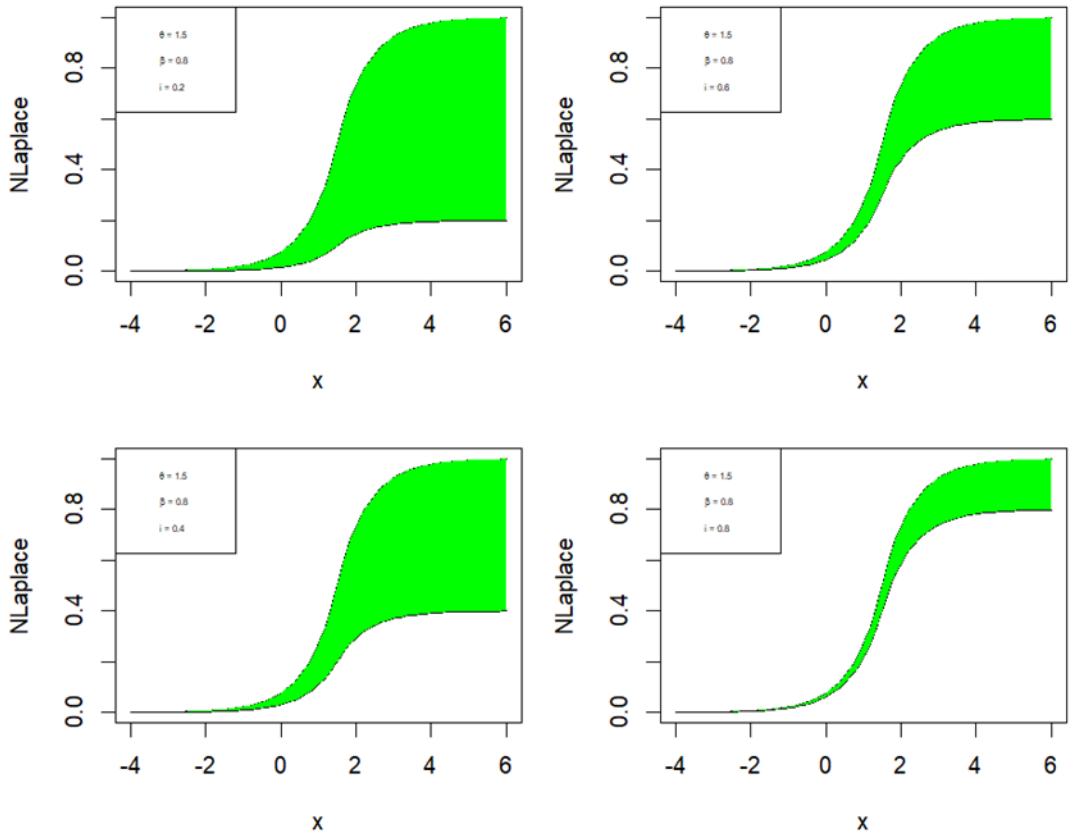

Fig. 2: The cdf plot representing the distribution of NLD with different level of indeterminant scale and shape parameters

3. Statistical Properties

Theorem 1: *Suppose 'z' is a random variable that conforms to the NLD. In that case, the neutrosophic rth moment can be expressed as follows:*

$$E(z_n^r) = \mu_r' = \frac{1}{2}\sum_{k=0}^{r}\binom{r}{k}\beta_n^k\theta_n^{r-k}\{1+(-1)^k\}k!\,(1+i_n), \quad \forall \; k = 0,1,2\ldots r \tag{4}$$

where μ_r' *is neutrosophic* r^{th} *moment of NLD.*

Proof: We know that **(5)**

$$E(z_n^r) = \int_{-\infty}^{\infty} z_n^r f(z_n) dz_n$$

$$E(z_n^r) = \int_{-\infty}^{\infty} z_n^r \left(\frac{1}{2\beta_n} e^{\left(-\frac{|z_n-\theta_n|}{\beta_n}\right)}\right)(1+i_n)dz_n$$

Putting $y = \frac{|z_n-\theta_n|}{\beta_n}$, we get

$$E(z_n^r) = \frac{1}{2}\int_{-\infty}^{\infty}(y\beta_n+\theta_n)^r\left(e^{-|y|}\right)(1+i_n)dy$$

$$E(z_n^r) = \frac{1}{2}\int_{-\infty}^{\infty}\left(\sum_{k=0}^{r}\binom{r}{k}(y\beta_n)^k(\theta_n)^{r-k}\right)\left(e^{-|y|}\right)(1+i_n)dy$$

We get the following expression after some algebraic simplification.

$$E(z_n^r) = \mu_r' = \frac{1}{2}\sum_{k=0}^{r}\binom{r}{k}\beta_n^k\theta_n^{r-k}\{1+(-1)^k\}k!\,(1+i_n) \quad \textbf{(6)}$$

The first four moments of the NLD is given by:

$$\mu_1' = E(z_n) = \theta_n(1+i_n) \quad \textbf{(7)}$$

$$\mu_2' = E(z_n^2) = (\theta_n^2 + 2\beta_n^2)(1+i_n) \quad \textbf{(8)}$$

$$\mu_3' = E(z_n^3) = (\theta_n^3 + 6\theta_n\beta_n^2)(1+i_n) \quad \textbf{(9)}$$

$$\mu_4' = E(z_n^4) = (\theta_n^4 + 12\theta_n^2\beta_n^2 + 24\beta_n^4)(1+i_n) \quad \textbf{(10)}$$

The NLD's mean, variance, skewness, and kurtoses are expressed as:

$$\text{Neutrosphic Mean} = \theta_n(1+i_n) \quad \textbf{(11)}$$

$$\text{Neutrosphic Variance} = (\theta_n^2 + 2\beta_n^2)(1+i_n) - \big(\theta_n(1+i_n)\big)^2 \quad \textbf{(12)}$$

$$\text{Skewness} = \frac{\mu_3^2}{\mu_2^3} = \frac{\mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3}{\mu_2' - {\mu_1'}^2} = 0 \quad \textbf{(13)}$$

$$\text{Kurtosis} = \frac{\mu_4}{\mu_2^2} = 6 \quad \textbf{(14)}$$

Theorem 2: *The first, second and third quartile of the NLD is given by* $Q_{n_1} = \theta_n + \beta_n.\log_e 0.5\,(1+i_n)^{-1}$ *,* $Q_{n_2} = \theta_n(1+i_n)$ *,* $Q_{n_3} = \theta_n - \beta_n.\log_e 0.25\,(1+i_n)^{-1}$ *respectively.*

Proof: We know that, $F(Q_{n_i}) = P(z_n \le Q_{n_i}) = \frac{i}{4}$ where $i = 1,2,3$

First quartile $(Q_{n_1} < \theta_n)$ is give by

$$F(Q_{n_1}) = \frac{1}{2}e^{\left(\frac{Q_{n_1}-\theta_n}{\beta_n}\right)}(1+i_n) = \frac{1}{4}$$

$$e^{\left(\frac{Q_{n_1}-\theta_n}{\beta_n}\right)}(1+i_n) = \frac{1}{2}$$

$$\left(\frac{Q_{n_1}-\theta_n}{\beta_n}\right)(1+i_n) = \log_e 0.5$$

$$Q_{n_1} = \theta_n + \beta_n.\log_e 0.5\,(1+i_n)^{-1} \quad \textbf{(15)}$$

Second quartile $(Q_{n_2} \ge \theta_n)$ is give by

$$F(Q_{n_2}) = 1 - \frac{1}{2}e^{\left(-\frac{(Q_{n_2}-\theta_n)}{\beta_n}\right)}(1+i_n) = \frac{2}{4}$$

$$Q_{n_2} = \theta_n(1+i_n) \quad \textbf{(16)}$$

Second quartile $(Q_{n_3} \ge \theta_n)$ is give by

$$F(Q_{n_3}) = 1 - \frac{1}{2}e^{\left(-\frac{(Q_{n_3}-\theta_n)}{\beta_n}\right)}(1+i_n) = \frac{3}{4}$$

$$e^{\left(-\frac{(Q_{n_2}-\theta_n)}{\beta_n}\right)}(1+i_n) = \frac{1}{4}$$

$$\left(\frac{Q_{n_3}-\theta_n}{\beta_n}\right)(1+i_n) = \log_e 0.25$$

$$Q_{n_3} = \theta_n - \beta_n.\log_e 0.25\,(1+i_n)^{-1} \quad \textbf{(17)}$$

where $\theta_n$is the location parameter, β_n is the scale parameter and $i_n \in (i_l, i_u\ )\ is\ indeterminancy.$

Theorem 3: *The moment generating function of NLD is*

$M_{z_n}(t) = \frac{e^{\theta_n t}}{(1-\beta_n^2 t^2)} \qquad for\ \ |t| < \frac{1}{\beta_n}$

Where $M_{z_n}(t)$ *= moment generating function*

Proof : We know that

$\mathrm{M_{z_n}(t)} = \int_{-\infty}^{\infty} \mathrm{e^{tz_n} f(z_n) dz_n}$ **(18)**

$$\mathrm{M_{z_n}(t)} = \int_{-\infty}^{\infty} \mathrm{e^{t(z_n-\theta_n+\theta_n)}} \left(\frac{1}{2\beta_n} \mathrm{e}^{\left(-\frac{|z_n-\theta_n|}{\beta_n}\right)}\right)(1+\mathrm{i_n})\mathrm{dz_n}$$

$$\mathrm{M_{z_n}(t)} = \frac{\mathrm{e^{t\theta_n}}}{2\beta_n}\int_{-\infty}^{\infty} \mathrm{e^{t(z_n-\theta_n)}} \left(\mathrm{e}^{\left(-\frac{|z_n-\theta_n|}{\beta_n}\right)}\right)(1+\mathrm{i_n})\mathrm{dz_n}$$

Let $(\mathrm{z_n} - \theta_n) = \mathrm{u}$ such that $\mathrm{z_n} = \mathrm{du}$

$$\mathrm{M_{z_n}(t)} = \frac{\mathrm{e^{t\theta_n}}}{2\beta_n}\int_{-\infty}^{\infty} \mathrm{e^{tu}} \mathrm{e}^{\left(-\frac{|u|}{\beta_n}\right)}(1+\mathrm{i_n})\mathrm{du}$$

$$\mathrm{M_{z_n}(t)} = \frac{\mathrm{e^{t\theta_n}}}{2\beta_n}\int_{-\infty}^{0} \mathrm{e}^{\left(t+\frac{1}{\beta_n}\right)u}(1+\mathrm{i_n})\mathrm{du} + \int_{0}^{\infty} \mathrm{e}^{-\left(t-\frac{1}{\beta_n}\right)u}(1+\mathrm{i_n})\mathrm{du}$$

$$\mathrm{M_{z_n}(t)} = \frac{\mathrm{e^{t\theta_n}}}{2\beta_n}\left\{\left[\frac{\beta_n}{1+t} - 0\right] + \left[0 - \frac{\beta_n}{1-t}\right]\right\}(1+\mathrm{i_n})$$

$\mathrm{M_{z_n}(t)} = \frac{\mathrm{e^{t\theta_n}(1+i_n)}}{1-\beta_n^2 t^2} \qquad \text{for } |t| < \frac{1}{\beta_n} \ , 0 \le \mathrm{i_n} \le 1$ **(19)**

4. Parameter estimation and simulation

The maximum likelihood approach can be utilized to measure the parameters of the NLD. The likelihood function can be expressed as follows:

$$\prod_{k=1}^{n} f(z_{kn}) = \prod_{i=1}^{N}\left[\frac{1}{2\beta_n} e^{\left(-\frac{|z_{kn}-\theta_n|}{\beta_n}\right)} + \left\{\frac{1}{2\beta_n} e^{\left(-\frac{|z_{kn}-\theta_n|}{\beta_n}\right)}\right\} i_n\right] \tag{20}$$

The log likelihood function is given by

$$L(\theta_n, \beta_n, i_n) = \prod_{i=1}^{N} \log\left[\frac{1}{2\beta_n} e^{\left(-\frac{|z_{kn}-\theta_n|}{\beta_n}\right)}\right](1+i_n)$$

$$L(\theta_n, \beta_n, i_n) = log\left(\left(\frac{1}{2\beta_n}\right)^N \exp\left(-\frac{\sum_{k=1}^{N}|z_{kn}-\theta_n|}{\beta_n}\right)\right)(1+i_n)^N$$

$$L(\theta_n, \beta_n, i_n) = -Nlog(2) - Nlog(\beta_n) - \frac{\sum_{k=1}^{N}|z_{kn}-\theta_n|}{\beta_n} + N(1+i_n)$$

By differentiating the $L(\theta_n, \beta_n, i_n)$ w.r.t. the parameters

We have

$$\frac{\partial L}{\partial \beta_n} = -\frac{N}{\beta_n} + \frac{\sum_{k=1}^{N}|z_{kn}-\theta_n|}{\beta_n^2} \tag{21}$$

$$\frac{\partial L}{\partial \theta_n} = \frac{\sum_{k=1}^{N}(z_{kn}-\theta_n)}{\beta_n |z_{kn}-\theta_n|} \tag{22}$$

The derived MLE of θ_n and β_n are $\widetilde{z_{kn}}$ i.e., median of the k neutrosophic observations and $\frac{1}{N}\sum_{k=1}^{N}|z_{kn}-\theta_n|$ respectively. Here, $\theta_n$is the location parameter, β_n is the scale parameter and $i_n \in (i_l, i_u \ )$ is indeterminacy.

Now, we presented a simulation analysis to assess the accuracy of the estimates. To conduct the simulation, we generate N=10000 random samples from the NLD with varying sizes, namely n = 30, 50, 100, 200, 300 and 1000.The Table 1 shows the average estimates (AEs) and mean square errors (MSEs) of $\widehat{\theta_n}$ and $\widehat{\beta_n}$. R studio software (version 2023.03.1+446) is used to generate the numerical findings.

Table 1. Results obtained from simulating the NLD estimates

Sample	**Actual Value**			**Average estimates**		**Mean square Error**	
n	$\boldsymbol{\theta_n}$	$\boldsymbol{\beta_n}$	$\boldsymbol{i_n}$	$\widehat{\boldsymbol{\theta_n}}$	$\widehat{\boldsymbol{\beta_n}}$	$\widehat{\boldsymbol{\theta_n}}$	$\widehat{\boldsymbol{\beta_n}}$
30	0.4	0.8	0	0.6392	1.0102	0.0612	0.1844
50				0.3733	0.9445	0.0350	0.1336
100				0.2972	0.8075	0.0259	0.0807
200				0.3961	0.8056	0.0105	0.0577
30	0.6	1.0	0.2	0.8319	1.6266	0.1958	0.2970
50				0.7526	1.0042	0.0732	0.1420
100				0.7295	1.0023	0.0959	0.1023
300				0.7100	1.0013	0.0380	0.065
30	1.0	1.0	0.3	1.4783	1.3674	0.1834	0.1789
50				1.3877	0.1797	0.1001	0.1668
100				1.2427	1.0713	0.0809	0.1471
300				1.1765	1.0652	0.0568	0.0753
30	3.0	5.0	0.5	6.5462	8.8494	0.6397	1.6157
50				6.1257	7.9343	0.5596	1.1221
100				6.1292	7.2815	0.4966	0.7282
300				4.1914	7.0772	0.2884	0.4086
1000				3.1789	7.0376	0.1965	0.2873

The simulation finding in table 1 shows that as sample size increases, the difference between the actual and estimated scale and shape parameters decreases i.e. (average bias reduces). It indicates that the compatibility between practice and theory improves as the sample size increases and the mean square errors of the estimators decreases. The resulting estimators are clearly asymptotically consistent and the MLE of the parameters performs worthily and provides asymptotically exact and correct results.

5. Application and Comparative Analysis

The Laplace Distribution is commonly used in finance to model asset returns. However, the financial data often exhibit indeterminacy and fat tails which means extreme events occur more frequently. The Neutrosophic Laplace Distribution's characteristics of handling indeterminacy and heavy tails in data makes it a suitable choice for modelling. Stock returns are influenced by various factors such as economic conditions, market sentiment, company-specific news, geopolitical events and investor behaviour which introduce uncertainty into the stock market. We have considered the data of daily returns (in %) of NIFTY50 from Indian Stock market. The dataset contains 827 observations i.e. (from 01-01-2020 to 28-03-2023). However, due to uncertainties and incomplete information, we applied the NLD to capture the indeterminacy and ambiguity associated with the returns. We also compared the fitness of distribution of the returns using the LD and NLD. The statistical summary of the data is given in table 2.

Table 2. The descriptive statistics of the daily returns of NIFTY50

Min	1st Q	Median	Mean	3rd Q	MAD	Var	Skewness	Kurtosis	Max
-6.818	-0.571	-0.0068	-0.0795	0.4508	0.718	1.1703	0.725	13.27992	9.306

The Bayesian Information Criterion (BIC) and Akaike Information Criterion (AIC) are model selection criteria used to compare the goodness of fit and complexity of different statistical models, including the Laplace distribution. Both AIC and BIC are calculated based on the likelihood function and the number of parameters in the model. The formulas for AIC and BIC are as follows:

$$AIC = -2 * logL + 2k$$

$$BIC = -2 * logL + klog(N)$$

Where:

$logL$: The logarithm of the likelihood functions of the model.

k: The number of parameters in the model.

n: The sample size.

Lower values of AIC and BIC indicate a better balance between model fit and complexity. The model with the lower AIC or BIC is generally preferred as it suggests a better trade-off between goodness of fit and model complexity. It is important to note that the values of AIC and BIC are not specific to the LD but can be applied to compare the models in general. The loglikelihood estimated value along with AIC and BIC corresponding to indeterminacy value (i_n) is given in table 3.

Table 3: The MLE, AIC and BIC measure of the daily returns of NIFTY50

	i_n	$logL$	**AIC**	**BIC**
LD	0	-1122.67	2249.34	2258.78
NLD	0.1	-1043.85	2091.70	2101.13
	0.2	-971.89	1947.78	1957.22
	0.3	-905.69	1815.39	1824.82
	0.4	-844.41	1692.81	1702.25
	0.5	-787.35	1578.70	1588.14
	0.6	-733.98	1471.95	1481.39
	0.7	-683.84	1371.68	1381.12
	0.8	-636.57	1277.14	1286.58
	0.9	-591.86	1187.71	1197.15
	1	-549.44	1102.87	1112.31

The goodness-of-fit metrics and MLEs for the classical LD and the NLD with varying indeterminacy parameter values are shown in table 3. In terms of goodness of fit, the neutrosophic Laplace distribution exceeds the standard Laplace distribution. The indeterminacy parameter is found to have a considerable impact on fitting quality. The AIC and BIC along with log likelihood values decreases as change in the value of indeterminacy parameter. The NLD fits better in the daily return of the financial data of NIFTY50 as compare to the classical LD.

6. Conclusion

Here, a neutrosophic Laplace distribution has been introduced as a generalization of the classical Laplace distribution by considering the interval form of data commonly encountered in real-life scenarios. We investigated various properties of the proposed distribution such as rth moment, mean, variance, skewness, kurtosis, first four moments, moment generating function and quartiles. The maximum likelihood estimation approach is employed to estimate the parameters and the performance of these estimators is evaluated via a simulation study. An application of the proposed distribution is discussed on the financial data analysis. From the comparative analysis, the indeterminacy parameter is found to have a considerable impact on fitting quality. The AIC and BIC along with log likelihood values decreases as change in the value of indeterminacy parameter. Therefore, it is concluded that the Neutrosophic Laplace Distribution fits better in the daily return of the financial data of NIFTY50 as compared to the classical Laplace Distribution. In future, this work could be extended for some other continuous distributions and mixture distributions.

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