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Abstract: It may be difficult for researchers to memorize or remember the trigonometric ratios of any neutrosophic angle, and this is what prompted us to activate the role of the neutrosophic complex numbers for that. In this paper we presented neutrosophic Euler's formulas and neutrosophic De Moivre's formula. Also, we benefited from that by finding the trigonometric ratios of the multiples of neutrosophic angle in terms of the trigonometric ratios of the neutrosophic angle $(\ddot{\theta}+\ddot{\varphi}I)$ and convert trigonometric ratios from formula $sin^n(\ddot{\theta}+\ddot{\varphi}I)$, or formula $cos^m(\ddot{\theta}+\ddot{\varphi}I)$, into a linear expression for the multiples of the neutrosophic angle $(\ddot{\theta}+\ddot{\varphi}I)$, which made it easier for us to find integrals of the neutrosophic trigonometric functions by other methods.

Keywords: neutrosophic Euler's formulas, neutrosophic complex numbers, neutrosophic De Moivre's formula.

1. Introduction

As Smarandache proposed the Neutrosophic Logic as an alternative to the existing logics to represent a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, and contradiction. Smarandache introduced the concept of neutrosophy as a new school of philosophy [4][8]. He presented the definition of the standard form of neutrosophic real number and conditions for the division of two neutrosophic real numbers to exist [3][5], studying the concept of the Neutrosophic probability [11][6], the Neutrosophic statistics [5], and professor Smarandache entered the concept of preliminary calculus of the differential and integral calculus, where he introduced for the first time the notions of neutrosophic mereo-limit, mereo-continuity, mereoderivative, and mereo-integral [1][8]. Y.Alhasan presented the definition of the concept of neutrosophic complex numbers and its properties including the conjugate of neutrosophic complex number, division of neutrosophic complex numbers, the inverted neutrosophic complex number and the absolute value of a neutrosophic complex number and Theories related to the conjugate of neutrosophic complex numbers, the product of a neutrosophic complex number by its conjugate equals the absolute value of number and he studied the general exponential form of a neutrosophic complex number [2-4]. Madeleine Al- Taha presented results on single valued neutrosophic (weak) polygroups [10]. An algebraic approach to neutrosophic euclidean geometry is presented [7].

Complex numbers play a significant role in daily life because they make it much easier to perform mathematical operations and give us a way to solve equations for which there are no real-number-group solutions. The electrical engineering field makes extensive use of complex numbers to calculate electric voltage and measure alternating current.

Paper is divided into four pieces. provides an introduction in the first portion, which includes a review of neutrosophic science. A few definitions of a neutrosophic complex number are covered in the second section. The third section defined neutrosophic Euler's formulas, neutrosophic De Moivre's formula and discuses applications of neutrosophic complex numbers in triangles. The paper's conclusion is provided in the fourth section.

2. Preliminaries

2.1 The general Trigonometric form of a neutrosophic complex number [4]

Definition 1

The following formula:

$$z = \mathrm{r}\ \ (\cos(\theta + \vartheta I) + sin\ \ (\theta + \vartheta I)\ \ i)$$

is called the general trigonometric form of a neutrosophic complex number

Definition 2 [7]

Let f: $R(I) \rightarrow R(I)$; $f = f(X)$ and $X = x + yI \in R(I)$ the f is called a neutrosophic real function with one neutrosophic variable. a neutrosophic real function $f(X)$ written as follows:

$$f(X) = f(x + yI) = f(x) + I[f(x + y) - f(x)]$$

3. Neutrosophic Euler's formulas

Let:

$$e^{i(\ddot{\theta}+\ddot{\varphi}I)} = cos(\ddot{\theta} + \ddot{\varphi}I) + i\ \ sin(\ddot{\theta} + \ddot{\varphi}I)$$

$$e^{-i(\ddot{\theta}+\ddot{\varphi}I)} = cos(\ddot{\theta} + \ddot{\varphi}I) - i\ \ sin(\ddot{\theta} + \ddot{\varphi}I)$$

by additional:

$$cos(\ddot{\theta} + \ddot{\varphi}I) = \frac{e^{i(\ddot{\theta}+\ddot{\varphi}I)} + e^{-i(\ddot{\theta}+\ddot{\varphi}I)}}{2}$$

by subtraction:

$$sin(\ddot{\theta} + \ddot{\varphi}I) = \frac{e^{i(\ddot{\theta}+\ddot{\varphi}I)} - e^{-i(\ddot{\theta}+\ddot{\varphi}I)}}{2i}$$

They are neutrosophic Euler's formulas.

3.1 Neutrosophic De Moivre's formula

$$z = \acute{r}e^{i(\ddot{\theta}+\ddot{\varphi}I)}$$

$$\mathrm{z}^{n} = \left(\acute{r}e^{i(\ddot{\theta}+\ddot{\varphi}I)}\right)^{n}$$

$$\acute{r}^{n}e^{i(n\ddot{\theta}+n\ddot{\varphi}I)} = \acute{r}^{n}\ \ cos(n\ddot{\theta} + n\ddot{\varphi}I) + i\ \ \acute{r}^{n}\ \ sin(n\ddot{\theta} + n\ddot{\varphi}I) \qquad ; n \in \mathbb{Z}$$

$$= \acute{r}^n \left[cos(n\ddot{\theta}) + I\left(cos(n\ddot{\theta} + n\ddot{\varphi}) - cos(n\ddot{\theta})\right)\right] + i\ \acute{r}^n \left[sin(n\ddot{\theta}) + I\left(sin(n\ddot{\theta} + n\ddot{\varphi}) - sin(n\ddot{\theta})\right)\right]$$

Example1

$$\left(1 + \left(\frac{\sqrt{2}}{2} - 1\right)I + \left(\frac{\sqrt{2}}{2}\ I\right)i\right)^{24} = \left(e^{i\left(\frac{\pi}{4}I\right)}\right)^{24} = cos24\left(\frac{\pi}{4}I\right) + i\ sin24\left(\frac{\pi}{4}I\right)$$

$$= cos(6\pi I) + i\ sin(6\pi I)$$

$$= cos(0) + I[\cos(0 + (6\pi)) - \cos(0)] + i\ (\sin(0) + I[sin(0 + 6\pi) - \sin(0)])$$

$$= 1 + 0I + 0i = 1$$

Theorem1

Let $(\ddot{\theta} + \ddot{\varphi}I)$ neutrosophic real number, then the solution of the equation:

$$e^{i(\ddot{\theta}+\ddot{\varphi}I)} = e^{i(\ddot{\vartheta}+\ddot{\omega}I)}$$

by unknown $(\ddot{\vartheta} + \ddot{\omega}I)$, is:

$$\{\ddot{\theta} + \ddot{\varphi}I + 2\pi k \quad ; k \in \mathrm{Z}\}$$

Proof:
multiply:

$$e^{i(\ddot{\theta}+\ddot{\varphi}I)} = e^{i(\ddot{\vartheta}+\ddot{\omega}I)}$$

by:

$$e^{-i(\ddot{\vartheta}+\ddot{\omega}I)} \neq 1$$

we find:

$$e^{i(\ddot{\theta}+\ddot{\varphi}I)}e^{-i(\ddot{\vartheta}+\ddot{\omega}I)} = 1$$

$$e^{i(\ddot{\theta}-\ddot{\vartheta}+(\ddot{\omega}-\ddot{\varphi})I)} = 1$$

$$cos(\ddot{\theta} - \ddot{\vartheta} + (\ddot{\omega} - \ddot{\varphi})I) + i\ sin(\ddot{\theta} - \ddot{\vartheta} + (\ddot{\omega} - \ddot{\varphi})I) = 1$$

then:

$$cos(\ddot{\theta} - \ddot{\vartheta} + (\ddot{\omega} - \ddot{\varphi})I) = 1 \qquad and \qquad sin(\ddot{\theta} - \ddot{\vartheta} + (\ddot{\omega} - \ddot{\varphi})I) = 0$$

hence:

$$\ddot{\vartheta} + \ddot{\omega}I = \ddot{\theta} + \ddot{\varphi}I + 2\pi k \quad ; k \in \mathrm{Z}$$

3.2. Applications of neutrosophic complex numbers in triangles

3.2.1 Finding the trigonometric ratios of the multiples of neutrosophic angle in terms of the trigonometric ratios of the angle neutrosophic $(\ddot{\theta} + \ddot{\varphi}I)$

The trigonometric ratios of angle neutrosophic $\mathbf{2}(\ddot{\theta} + \ddot{\boldsymbol{\varphi}}I)$ in terms of the trigonometric ratios of angle $(\ddot{\theta} + \ddot{\boldsymbol{\varphi}}I)$:

by using De Moivre's formula:

$$\left(cos(\ddot{\theta} + \ddot{\varphi}I) + i\ \sin(\ddot{\theta} + \ddot{\varphi}I)\right)^2 = \cos 2(\ddot{\theta} + \ddot{\varphi}I) + i \sin 2(\ddot{\theta} + \ddot{\varphi}I)$$

$$cos^2(\ddot{\theta} + \ddot{\varphi}I) - sin^2(\ddot{\theta} + \ddot{\varphi}I) + 2i\ cos(\ddot{\theta} + \ddot{\varphi}I)\ sin(\ddot{\theta} + \ddot{\varphi}I) = cos\,2(\ddot{\theta} + \ddot{\varphi}I) + i \sin 2(\ddot{\theta} + \ddot{\varphi}I)$$

by equating the two real parts of both sides of the equality:

$$\cos 2(\ddot{\theta}+\ddot{\varphi}I) = \cos^2(\ddot{\theta}+\ddot{\varphi}I) - \sin^2(\ddot{\theta}+\ddot{\varphi}I)$$

by equating the two imaginary parts on both sides of the equality:

$$\sin 2(\ddot{\theta}+\ddot{\varphi}I) = 2\ \cos(\ddot{\theta}+\ddot{\varphi}I)\ \sin(\ddot{\theta}+\ddot{\varphi}I)$$

where:

$$\cos(\ddot{\theta}+\ddot{\varphi}I) = \cos(\ddot{\theta}) + I\left(\cos(\ddot{\theta}+\ddot{\varphi}) - \cos(\ddot{\theta})\right)$$

$$\sin(\ddot{\theta}+\ddot{\varphi}I) = \sin(\ddot{\theta}) + I\left(\sin(\ddot{\theta}+\ddot{\varphi}) - \sin(\ddot{\theta})\right)$$

to find $\tan 2(\ddot{\theta}+\ddot{\varphi}I)$:

$$\tan 2(\ddot{\theta}+\ddot{\varphi}I) = \frac{\sin 2(\ddot{\theta}+\ddot{\varphi}I)}{\cos 2(\ddot{\theta}+\ddot{\varphi}I)}$$

$$= \frac{2\ \cos(\ddot{\theta}+\ddot{\varphi}I)\ \sin(\ddot{\theta}+\ddot{\varphi}I)}{\cos^2(\ddot{\theta}+\ddot{\varphi}I) - \sin^2(\ddot{\theta}+\ddot{\varphi}I)}$$

$$= \frac{\dfrac{2\ \cos(\ddot{\theta}+\ddot{\varphi}I)\ \sin(\ddot{\theta}+\ddot{\varphi}I)}{\cos^2(\ddot{\theta}+\ddot{\varphi}I)}}{\dfrac{\cos^2(\ddot{\theta}+\ddot{\varphi}I) - \sin^2(\ddot{\theta}+\ddot{\varphi}I)}{\cos^2(\ddot{\theta}+\ddot{\varphi}I)}}$$

$$\Longrightarrow \qquad \tan 2(\ddot{\theta}+\ddot{\varphi}I) = \frac{2\tan(\ddot{\theta}+\ddot{\varphi}I)}{1-\tan(\ddot{\theta}+\ddot{\varphi}I)}$$

where:

$$\tan(\ddot{\theta}+\ddot{\varphi}I) = \tan(\ddot{\theta}) + I\left(\tan(\ddot{\theta}+\ddot{\varphi}) - \tan(\ddot{\theta})\right)$$

Example2

Write the trigonometric ratios of angle neutrosophic $4(\ddot{\theta}+\ddot{\varphi}I)$ in terms of the trigonometric ratios of angle $(\ddot{\theta}+\ddot{\varphi}I)$.

by using De Moivre's formula:

$$\left(\cos(\ddot{\theta}+\ddot{\varphi}I) + i\ \ \sin(\ddot{\theta}+\ddot{\varphi}I)\right)^4 = \cos 4(\ddot{\theta}+\ddot{\varphi}I) + i\sin 4(\ddot{\theta}+\ddot{\varphi}I)$$

$$\cos^4(\ddot{\theta}+\ddot{\varphi}I) + 4i\ \ \cos^3(\ddot{\theta}+\ddot{\varphi}I)\ \sin(\ddot{\theta}+\ddot{\varphi}I) + 6i^2\ \ \cos^2(\ddot{\theta}+\ddot{\varphi}I)\ \sin^2(\ddot{\theta}+\ddot{\varphi}I) + 4i^3\ \ \cos(\ddot{\theta}+\ddot{\varphi}I)\ \sin^3(\ddot{\theta}+\ddot{\varphi}I) + i^4\sin^4(\ddot{\theta}+\ddot{\varphi}I) = \cos 4(\ddot{\theta}+\ddot{\varphi}I) + i\sin 4(\ddot{\theta}+\ddot{\varphi}I)$$

$$\cos^4(\ddot{\theta}+\ddot{\varphi}I) + 4i\ \ \cos^3(\ddot{\theta}+\ddot{\varphi}I)\ \sin(\ddot{\theta}+\ddot{\varphi}I) - 6\ \ \cos^2(\ddot{\theta}+\ddot{\varphi}I)\ \sin^2(\ddot{\theta}+\ddot{\varphi}I) - 4i\ \ \cos(\ddot{\theta}+\ddot{\varphi}I)\ \sin^3(\ddot{\theta}+\ddot{\varphi}I) + \sin^4(\ddot{\theta}+\ddot{\varphi}I) = \cos 4(\ddot{\theta}+\ddot{\varphi}I) + i\sin 4(\ddot{\theta}+\ddot{\varphi}I)$$

$$\left[\cos^4(\ddot{\theta}+\ddot{\varphi}I) - 6\ \ \cos^2(\ddot{\theta}+\ddot{\varphi}I)\ \sin^2(\ddot{\theta}+\ddot{\varphi}I) + \sin^4(\ddot{\theta}+\ddot{\varphi}I)\right] + i\left[4\ \ \cos^3(\ddot{\theta}+\ddot{\varphi}I)\ \sin(\ddot{\theta}+\ddot{\varphi}I) - 4\ \ \cos(\ddot{\theta}+\ddot{\varphi}I)\ \sin^3(\ddot{\theta}+\ddot{\varphi}I)\right] = \cos 4(\ddot{\theta}+\ddot{\varphi}I) + i\sin 4(\ddot{\theta}+\ddot{\varphi}I)$$

by equating the two real parts of both sides of the equality:

$$\cos 4(\ddot{\theta}+\ddot{\varphi}I)=\cos^4(\ddot{\theta}+\ddot{\varphi}I)-6\ \cos^2(\ddot{\theta}+\ddot{\varphi}I)\ \sin^2(\ddot{\theta}+\ddot{\varphi}I)+\sin^4(\ddot{\theta}+\ddot{\varphi}I)$$

by equating the two imaginary parts on both sides of the equality:

$$\sin 4(\ddot{\theta}+\ddot{\varphi}I)=4\ \cos^3(\ddot{\theta}+\ddot{\varphi}I)\ \sin(\ddot{\theta}+\ddot{\varphi}I)-4\ \cos(\ddot{\theta}+\ddot{\varphi}I)\ \sin^3(\ddot{\theta}+\ddot{\varphi}I)$$

where:

$$\cos(\ddot{\theta}+\ddot{\varphi}I)=\cos(\ddot{\theta})+I\left(\cos(\ddot{\theta}+\ddot{\varphi})-\cos(\ddot{\theta})\right)$$

$$\sin(\ddot{\theta}+\ddot{\varphi}I)=\sin(\ddot{\theta})+I\left(\sin(\ddot{\theta}+\ddot{\varphi})-\sin(\ddot{\theta})\right)$$

to find $\tan 4(\ddot{\theta}+\ddot{\varphi}I)$:

$$\tan 4(\ddot{\theta}+\ddot{\varphi}I)=\frac{\sin 4(\ddot{\theta}+\ddot{\varphi}I)}{\cos 4(\ddot{\theta}+\ddot{\varphi}I)}$$

$$=\frac{4\ \cos^3(\ddot{\theta}+\ddot{\varphi}I)\ \sin(\ddot{\theta}+\ddot{\varphi}I)-4\ \cos(\ddot{\theta}+\ddot{\varphi}I)\ \sin^3(\ddot{\theta}+\ddot{\varphi}I)}{\cos^4(\ddot{\theta}+\ddot{\varphi}I)-6\ \cos^2(\ddot{\theta}+\ddot{\varphi}I)\ \sin^2(\ddot{\theta}+\ddot{\varphi}I)+\sin^4(\ddot{\theta}+\ddot{\varphi}I)}$$

$$=\frac{\dfrac{4\ \cos^3(\ddot{\theta}+\ddot{\varphi}I)\ \sin(\ddot{\theta}+\ddot{\varphi}I)-4\ \cos(\ddot{\theta}+\ddot{\varphi}I)\ \sin^3(\ddot{\theta}+\ddot{\varphi}I)}{\cos^4(\ddot{\theta}+\ddot{\varphi}I)}}{\dfrac{\cos^4(\ddot{\theta}+\ddot{\varphi}I)-6\ \cos^2(\ddot{\theta}+\ddot{\varphi}I)\ \sin^2(\ddot{\theta}+\ddot{\varphi}I)+\sin^4(\ddot{\theta}+\ddot{\varphi}I)}{\cos^4(\ddot{\theta}+\ddot{\varphi}I)}}$$

$$\Rightarrow \qquad \tan 4(\ddot{\theta}+\ddot{\varphi}I)=\frac{4\tan(\ddot{\theta}+\ddot{\varphi}I)-4\tan^3(\ddot{\theta}+\ddot{\varphi}I)}{1-6\tan^2(\ddot{\theta}+\ddot{\varphi}I)+\tan^4(\ddot{\theta}+\ddot{\varphi}I)}$$

where:

$$\tan(\ddot{\theta}+\ddot{\varphi}I)=\tan(\ddot{\theta})+I\left(\tan(\ddot{\theta}+\ddot{\varphi})-\tan(\ddot{\theta})\right)$$

3.2.2 Convert trigonometric ratios from Formula $\sin^n(\ddot{\theta}+\ddot{\varphi}I)$, or Formula $\cos^m(\ddot{\theta}+\ddot{\varphi}I)$, into a linear expression for the multiples of the neutrosophic angle $(\ddot{\theta}+\ddot{\varphi}I)$

Example3

Write $\cos^3(\ddot{\theta}+\ddot{\varphi}I)$ in the form of the sum of the trigonometric ratios of the multiples of the neutrosophic angle $(\ddot{\theta}+\ddot{\varphi}I)$

by using neutrosophic Euler's formulas:

$$\cos^3(\ddot{\theta}+\ddot{\varphi}I)=\left(\frac{e^{i(\ddot{\theta}+\ddot{\varphi}I)}+e^{-i(\ddot{\theta}+\ddot{\varphi}I)}}{2}\right)^3$$

$$=\frac{1}{8}\left(e^{3i(\ddot{\theta}+\ddot{\varphi}I)}+3e^{2i(\ddot{\theta}+\ddot{\varphi}I)}e^{-i(\ddot{\theta}+\ddot{\varphi}I)}+3e^{i(\ddot{\theta}+\ddot{\varphi}I)}e^{-2i(\ddot{\theta}+\ddot{\varphi}I)}+e^{-3i(\ddot{\theta}+\ddot{\varphi}I)}\right)$$

$$=\frac{1}{8}\left[\left(e^{3i(\ddot{\theta}+\ddot{\varphi}I)}+e^{-3i(\ddot{\theta}+\ddot{\varphi}I)}\right)+3\left(e^{i(\ddot{\theta}+\ddot{\varphi}I)}+e^{-i(\ddot{\theta}+\ddot{\varphi}I)}\right)\right]$$

by using neutrosophic Euler's formulas to return to the trigonometric ratios:

$$cos^3(\ddot{\theta}+\ddot{\varphi}I)=\frac{1}{8}[2cos3(\ddot{\theta}+\ddot{\varphi}I)+6cos(\ddot{\theta}+\ddot{\varphi}I)]$$

$$=\frac{1}{4}cos3(\ddot{\theta}+\ddot{\varphi}I)+\frac{3}{4}cos(\ddot{\theta}+\ddot{\varphi}I)$$

$$=\frac{1}{4}\left[cos(3\ddot{\theta})+I\left(cos(3\ddot{\theta}+3\ddot{\varphi})-cos(3\ddot{\theta})\right)\right]+\frac{3}{4}\left[cos(\ddot{\theta})+I\left(cos(\ddot{\theta}+\ddot{\varphi})-cos(\ddot{\theta})\right)\right]$$

Example4

Write $sin^6(\ddot{\theta}+\ddot{\varphi}I)$ in the form of the sum of the trigonometric ratios of the multiples of the neutrosophic angle $(\ddot{\theta}+\ddot{\varphi}I)$, then find based on what you find:

$$\int sin^6(\ddot{\theta}+\ddot{\varphi}I)\ \ d(\ddot{\theta}+\ddot{\varphi}I)$$

Solution:

by using neutrosophic Euler's formulas:

$$sin^6(\ddot{\theta}+\ddot{\varphi}I)=\left(\frac{e^{i(\ddot{\theta}+\ddot{\varphi}I)}-e^{-i(\ddot{\theta}+\ddot{\varphi}I)}}{2i}\right)^6$$

$$=\frac{-1}{64}\left(e^{i(\ddot{\theta}+\ddot{\varphi}I)}-e^{-i(\ddot{\theta}+\ddot{\varphi}I)}\right)^6$$

$$=\frac{-1}{64}\Big(e^{6i(\ddot{\theta}+\ddot{\varphi}I)}+6e^{5i(\ddot{\theta}+\ddot{\varphi}I)}e^{-i(\ddot{\theta}+\ddot{\varphi}I)}+15e^{4i(\ddot{\theta}+\ddot{\varphi}I)}e^{-2i(\ddot{\theta}+\ddot{\varphi}I)}-20e^{3i(\ddot{\theta}+\ddot{\varphi}I)}e^{-3i(\ddot{\theta}+\ddot{\varphi}I)}$$
$$+15e^{2i(\ddot{\theta}+\ddot{\varphi}I)}e^{-4i(\ddot{\theta}+\ddot{\varphi}I)}-6e^{i(\ddot{\theta}+\ddot{\varphi}I)}e^{-5i(\ddot{\theta}+\ddot{\varphi}I)}e^{-6i(\ddot{\theta}+\ddot{\varphi}I)}\Big)$$

$$=\frac{-1}{64}\left[\left(e^{6i(\ddot{\theta}+\ddot{\varphi}I)}+e^{-6i(\ddot{\theta}+\ddot{\varphi}I)}\right)-6\left(e^{4i(\ddot{\theta}+\ddot{\varphi}I)}+e^{4i(\ddot{\theta}+\ddot{\varphi}I)}\right)+15\left(e^{2i(\ddot{\theta}+\ddot{\varphi}I)}+e^{-2i(\ddot{\theta}+\ddot{\varphi}I)}\right)-20\right]$$

by using neutrosophic Euler's formulas to return to the trigonometric ratios:

$$cos^3(\ddot{\theta}+\ddot{\varphi}I)=\frac{-1}{64}[2cos6(\ddot{\theta}+\ddot{\varphi}I)-12\,cos\,4(\ddot{\theta}+\ddot{\varphi}I)+30\,cos\,2(\ddot{\theta}+\ddot{\varphi}I)-20]$$
$$\Rightarrow\qquad cos^3(\ddot{\theta}+\ddot{\varphi}I)=\frac{-1}{32}[cos6(\ddot{\theta}+\ddot{\varphi}I)-6\,cos\,4(\ddot{\theta}+\ddot{\varphi}I)+15\,cos\,2(\ddot{\theta}+\ddot{\varphi}I)-10]$$

$$=\frac{-1}{32}\Big(\left[cos(6\ddot{\theta})+I\left(cos(6\ddot{\theta}+6\ddot{\varphi})-cos(6\ddot{\theta})\right)\right]+6\left[cos(4\ddot{\theta})+I\left(cos(4\ddot{\theta}+4\ddot{\varphi})-cos(4\ddot{\theta})\right)\right]$$
$$+15\left[cos(2\ddot{\theta})+I\left(cos(2\ddot{\theta}+2\ddot{\varphi})-cos(2\ddot{\theta})\right)\right]-10\Big)$$

to find:

$$\int sin^6(\ddot{\theta}+\ddot{\varphi}I)\ \ d(\ddot{\theta}+\ddot{\varphi}I)$$
$$=\int\frac{-1}{32}[cos6(\ddot{\theta}+\ddot{\varphi}I)-6\,cos\,4(\ddot{\theta}+\ddot{\varphi}I)+15\,cos\,2(\ddot{\theta}+\ddot{\varphi}I)-10]\,d(\ddot{\theta}+\ddot{\varphi}I)$$

$$= \frac{-1}{32}\left[\frac{1}{6} sin\, 6(\ddot{\theta} + \ddot{\varphi}I) - \frac{3}{2} sin\, 4(\ddot{\theta} + \ddot{\varphi}I) + \frac{15}{2} sin\, 2(\ddot{\theta} + \ddot{\varphi}I) - 10(\ddot{\theta} + \ddot{\varphi}I)\right] + a + bI$$

$$= \frac{-1}{32}\Bigg(\frac{1}{6}\left[sin(6\ddot{\theta}) + I\left(sin(6\ddot{\theta} + 6\ddot{\varphi}) - sin(6\ddot{\theta})\right)\right] - \frac{3}{2}\left[sin(4\ddot{\theta}) + I\left(sin(4\ddot{\theta} + 4\ddot{\varphi}) - sin(4\ddot{\theta})\right)\right]$$
$$+ \frac{15}{2}\left[sin(2\ddot{\theta}) + I\left(sin(2\ddot{\theta} + 2\ddot{\varphi}) - sin(2\ddot{\theta})\right)\right] - 10(\ddot{\theta} + \ddot{\varphi}I)\Bigg) + a + bI$$

4. Conclusions

The importance of this paper comes from the fact that we were able to find the formula of neutrosophic Euler's formulas and neutrosophic De Moivre's formula according to an accurate scientific method, which facilitated finding applications of neutrosophic complex numbers in triangles, and access to easy ways to calculate trigonometric integrals. One of the research most important on neutrosophic complex numbers is this paper.

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