Neutrosophic Sets and Systems

Volume 56 Article 25

7-29-2023

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NSS

Neutrosophic Sets and Systems, Vol. 56, 2023

University of New Mexico

# Common Fixed Point Theorems for Occasionally Weakly Compatible Mappings in Neutrosophic Cone Metric Spaces

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**Abstract**. The idea of Neutrosophic Cone Metric Space is introduced in this study. In order to illustrate fixed point, the idea of occassionally weakly compatible is also used.

**Keywords:** Fixed point; Neutrosophic Metric Space; Compatible Mappings; Occasionally weakly compatible mapping.

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## 1. **Introduction**

As it lays the groundwork for modern mathematics, classical set theory progresses through numerous extensions. Zadeh [32] first suggested the idea of Fuzzy Sets (FS) in 1965. Fuzzy set theory was crucially applied in all branches of science and engineering. The idea of Fuzzy Meric Space (FMS) was first put forth in 1975 by Kramosil and Michalek[14]. This important characteristic of assigning graded membership polarised the academics, prompting them to develop different analyses and applications for various types of fuzzy metric spaces. George and Veeramani [4] reconstructed FMS using triangular criteria. Following then, other researchers explored the properties of FMS and produced numerous fixed point results.

Intuitionistic Fuzzy Sets, which expanded fuzzy set theory to include the idea of non-membership grade, were introduced by Atanassov [1] in 1983. Since then, a lot of work has been put into coming to new findings and extending existing ideas to the intuitionistic fuzzy environment. Park [17] developed intuitionistic fuzzy metric space (IFMS), and several fixed point findings based on the concept of IFS were published. Alaca et al. [2] and other researchers have developed several fixed point theorems in FMS and IFMS. Tarkan Oner et al. [16] developed the idea of fuzzy cone metric space. By Priyobartal et al. [18], several fixed point outcomes in fuzzy cone metric space were studied. Neutrosophy is an extension of the intuitionistic fuzzy set presented by Florentin Smarandache [20] in 1998. It holds that there exists a continuum-power spectrum of neutralities between a notion and its opponent.

According to this theory, there is a continuum-power spectrum of neutralities that might exist between a concept and its adversary. Neutralities were added to the Intuitionistic Fuzzy Set by neutrosophy, which energised the scientific community, and the field is now thriving with countless investigations, analyses, computing techniques, and applications. Neutrosophic metric space was established by Kirisci et al. [15] in 2019 as an expansion of intuitionistic fuzzy metric space that produces fixed point theorms in complete neutrosophic metric space. In Neutrosophic Metric Spaces(NMS), Sowndrarajan, Jeyaraman, and Florentin Smarandache demonstrated fixed point findings for contraction theorems.

This paper introduces the idea of Neutrosophic Cone Metric Space (NCMS) and explains its key components.. On Neutrosophic Cone Metric Space, the Banach contraction theorem and a few fixed point results are presented and demonstrated. Furthermore, by utilising the idea of occasionally weakly compatible on two self mappings, fixed point results on NCMS have been demonstrated.

## 2. Preliminaries

**Definition 2.1.** Think about a non-empty set that presummably serves as a common fixed point of mappings $\mathfrak{G} : \daleth \times \daleth \to \daleth$ and $\mathfrak{F} : \daleth \to \daleth$ if $\varrho = \mathfrak{F}(\varrho) = \mathfrak{G}(\varrho, \varrho)$.

**Definition 2.2.** If the mappings $\mathfrak{G} : \daleth \times \daleth \to \daleth$ and $\mathfrak{F} : \daleth \to \daleth$ in a nonempty set are considered to be commutative, then $\mathfrak{F}(\mathfrak{G}(\varrho, \varsigma)) = \mathfrak{G}(\mathfrak{F}(\varrho), \mathfrak{F}(\varsigma))$ for all $\varrho, \varsigma \in \daleth$.

**Definition 2.3.** Consider a non-empty set $\daleth$ and $\mathfrak{G}$, $\mathfrak{F}$ self-maps of $\daleth$. $\varrho$ serves as a coincidence point of $\mathfrak{G}$ and $\mathfrak{F}$ if and only if $\mathfrak{G}(\varrho) = \mathfrak{F}(\varrho)$ where $\varrho \in \daleth$. Then $\mathfrak{w} = \mathfrak{G}(\varrho) = \mathfrak{F}(\varrho)$ is referred to as a point of coincidence of $\mathfrak{G}$ and $\mathfrak{F}$.

**Definition 2.4.** Let $\mathfrak{G}$ and $\mathfrak{F}$ represents two set's self- maps of a set $\daleth$. $\mathfrak{G}$ and $\mathfrak{F}$ are referred to be occasionally weakly compatible if and only if a point is made $\varrho \in \daleth$ which is an instance of coincidence point of $\mathfrak{G}$ and $\mathfrak{F}$, where $\mathfrak{G}$ and $\mathfrak{F}$ commute.

**Definition 2.5.** Consider a cone metric space $(\urcorner, d)$ Next, for each $\mathfrak{n}_1 \gg 0$ and $\mathfrak{n}_2 \gg 0$, $\mathfrak{n}_1, \mathfrak{n}_2 \in \mathfrak{E}$, a thing exists $\mathfrak{n} \gg 0$, $\mathfrak{n} \in \mathfrak{E}$ like that $\mathfrak{n} \ll \mathfrak{n}_1$ and $\mathfrak{n} \ll \mathfrak{n}_2$ .

**Lemma 2.6.** *Assume that $\urcorner$ is a collection of $\mathfrak{G}, \mathfrak{F}$'s occasionally weakly compatible self maps. If $\mathfrak{G}$ and $\mathfrak{F}$ share a unique fixed point $\mathfrak{w}$, then $\mathfrak{G}$ and $\mathfrak{F}$ share a special coincidence $\mathfrak{w} = \mathfrak{G}(\varrho) = \mathfrak{F}(\varrho)$.*

**Definition 2.7.** Let $* : [0,1] \times [0,1] \to [0,1]$ where it meets the following requirements be a continuous t-norm [CTN]:

(i) $*$ is commutative and associative,
(ii) $*$ is continuous,
(iii) $\varepsilon_1 * 1 = \varepsilon_1$ for all $\varepsilon_1 \in [0,1]$,
(iv) $\varepsilon_1 * \varepsilon_2 \leq \varepsilon_3 * \varepsilon_4$ whenever $\varepsilon_1 \leq \varepsilon_3$ and $\varepsilon_2 \leq \varepsilon_4$, for every $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4 \in [0,1]$.

**Definition 2.8.** Let $\diamond : [0,1] \times [0,1] \to [0,1]$ where it meets the following requirements be a continuous t-conorm [CTC]:

(i) $\diamond$ is commutative and associative,
(ii) $\diamond$ is continuous,
(iii) $\varepsilon_1 \diamond 0 = \varepsilon_1$ for all $\varepsilon_1 \in [0,1]$,
(iv) $\varepsilon_1 \diamond \varepsilon_2 \leq \varepsilon_3 \diamond \varepsilon_4$ whenever $\varepsilon_1 \leq \varepsilon_3$ and $\varepsilon_2 \leq \varepsilon_4$, for each $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4 \in [0,1]$.

## 3. Neutrosophic Cone Metric Spaces

**Definition 3.1.** It is claimed that a 6-tuple $(\urcorner, \Xi, \Theta, \Upsilon, *, \diamond)$ is a Neutrosophic Cone Metric Space, a cone of E is P, $\urcorner$ can be any non empty set, $*$ be a neutrosophic CTN, $\diamond$ be a neutrosophic CTC and $\Xi, \Theta$ and $\Upsilon$ are neutrosophic sets on $\urcorner^2 \times int(\mathfrak{P})$ where it meets the criteria listed below:
for every $\varrho, \varsigma, \delta, \omega \in \urcorner, \alpha, \mu \in int(\mathfrak{P})$.

(i) $0 \leq \Xi(\varrho, \varsigma, \alpha) \leq 1;\ 0 \leq \Theta(\varrho, \varsigma, \alpha) \leq 1;\ 0 \leq \Upsilon(\varrho, \varsigma, \alpha) \leq 1;$
(ii) $\Xi(\varrho, \varsigma, \alpha) + \Theta(\varrho, \varsigma, \alpha) + \Upsilon(\varrho, \varsigma, \alpha) \leq 3;$
(iii) $\Xi(\varrho, \varsigma, \alpha) > 0;$
(iv) $\Xi(\varrho, \varsigma, \alpha) = 1$ if and only if $\varrho = \varsigma$;
(v) $\Xi(\varrho, \varsigma, \alpha) = \Xi(\varsigma, \varrho, \alpha);$
(vi) $\Xi(\varrho, \varsigma, \alpha) * \Xi(\varsigma, \delta, \mu) \leq \Xi(\varrho, \delta, \alpha + \mu)$, for all $\alpha, \mu > 0$;
(vii) $\Xi(\varrho, \varsigma, \cdot) : int(\mathfrak{P}) \to (0,1]$ is neutrosophic continuous;
(viii) $\lim_{\alpha \to \infty} \Xi(\varrho, \varsigma, \alpha) = 1$ for all $\alpha > 0$;
(ix) $\Theta(\varrho, \varsigma, \alpha) < 1;$
(x) $\Theta(\varrho, \varsigma, \alpha) = 0$ if and only if $\varrho = \varsigma$;
(xi) $\Theta(\varrho, \varsigma, \alpha) = \Theta(\varsigma, \varrho, \alpha);$

(xii) $\Theta(\varrho,\varsigma,\alpha)\diamond\Theta(\varsigma,\delta,\mu)\geq\Theta(\varrho,\delta,\alpha+\mu)$, for all $\alpha,\mu>0$;

(xiii) $\Theta(\varrho,\varsigma,\cdot):int(\mathfrak{P})\to(0,1]$ is neutrosophic continuous;

(xiv) $\lim_{\alpha\to\infty}\Theta(\varrho,\varsigma,\alpha)=0$ for all $\alpha>0$;

(xv) $\Upsilon(\varrho,\varsigma,\alpha)<1$;

(xvi) $\Upsilon(\varrho,\varsigma,\alpha)=0$ if and only if $\varrho=\varsigma$;

(xvii) $\Upsilon(\varrho,\varsigma,\alpha)=\Upsilon(\varsigma,\varrho,\alpha)$;

(xviii) $\Upsilon(\varrho,\varsigma,\alpha)\diamond\Upsilon(\varsigma,\delta,\mu)\geq\Upsilon(\varrho,\delta,\alpha+\mu)$, for all $\alpha,\mu>0$;

(xix) $\Upsilon(\varrho,\varsigma,\cdot):int(\mathfrak{P})\to(0,1]$ is neutrosophic continuous;

(xx) $\lim_{\alpha\to\infty}\Upsilon(\varrho,\varsigma,\alpha)=0$ for all $\alpha>0$;

(xxi) If $\alpha\leq 0$ then $\Xi(\varrho,\varsigma,\alpha)=0;\Theta(\varrho,\varsigma,\alpha)=1;\Upsilon(\varrho,\varsigma,\alpha)=1$.

Then, $(\Xi,\Theta,\Upsilon)$ is referred to as a NCMS on $\urcorner$. The mappings $\Xi,\Theta$ and $\Upsilon$ represents degree of closedness, neturalness and non-closedness between $\varrho$ and $\varsigma$ in relation to $\alpha$ respectively.

**Example 3.2.** Consider a metric space $(\urcorner,d)$. Let $\mathfrak{E}=\mathfrak{R}$ and $\mathfrak{P}=[0,\infty)$. Define $\omega*\sigma=\min\{\omega,\sigma\}$ and $\omega\diamond\sigma=\max\{\omega,\sigma\}$, then every neutrosophic metric spaces became an NCMS.

**Example 3.3.** $\mathfrak{P}$ could be an any cone, $\urcorner=N$. Define $\omega*\sigma=\min\{\omega,\sigma\}$ and $\omega\diamond\sigma=\max\{\omega,\sigma\}$, $\Xi,\Theta,\Upsilon:\urcorner^2\times int(\mathfrak{P})\to[0,1]$ defined by

$$\Xi(\varrho,\varsigma,\alpha)=\begin{cases}\frac{\varrho}{\varsigma}, & \text{if } \varrho\leq\varsigma\\ \frac{\varsigma}{\varrho}, & \text{if } \varsigma\leq\varrho\end{cases}\quad \Theta(\varrho,\varsigma,\alpha)=\begin{cases}\frac{\varsigma-\varrho}{\varsigma}, & \text{if } \varrho\leq\varsigma\\ \frac{\varrho-\varsigma}{\varrho}, & \text{if } \varsigma\leq\varrho\end{cases}\quad \Upsilon(\varrho,\varsigma,\alpha)=\begin{cases}\frac{\varsigma-\varrho}{\varrho}, & \text{if } \varrho\leq\varsigma\\ \frac{\varrho-\varsigma}{\varsigma}, & \text{if } \varsigma\leq\varrho\end{cases}$$

for all $\varrho,\varsigma\in\urcorner$ and $\alpha\gg 0$. Then $(\urcorner,\Xi,\Theta,\Upsilon,*,\diamond)$ be a NCMS.

**Example 3.4.** Assume $\mathfrak{E}=\mathfrak{R}^2$. Then $\mathfrak{P}=\{(\rho_1,\rho_2):\rho_1,\rho_2\geq 0\}\subset\mathfrak{E}$ with normal constant $\mathfrak{P}=1$, let $\mathfrak{P}$ be a normal cone assume $\urcorner=\mathfrak{R}$, $\omega*\sigma=\min\{\omega,\sigma\}$, $\omega\diamond\sigma=\max\{\omega,\sigma\}$ and $\Xi,\Theta,\Upsilon:\urcorner^2\times int(\mathfrak{P})\to[0,1]$ defined by $\Xi(\varrho,\varsigma,\alpha)=\frac{1}{e^{\frac{|\varrho-\varsigma|}{\|\alpha\|}}}$, $\Theta(\varrho,\varsigma,\alpha)=\frac{e^{\frac{|\varrho-\varsigma|}{\|\alpha\|}}-1}{e^{\frac{|\varrho-\varsigma|}{\|\alpha\|}}}$ and $\Upsilon(\varrho,\varsigma,\alpha)=e^{\frac{|\varrho-\varsigma|}{\|\alpha\|}}-1$, for each $\varrho,\varsigma\in\urcorner$ and $\alpha\gg 0$. Then $(\urcorner,\Xi,\Theta,\Upsilon,*,\diamond)$ is a NCMS.

**Definition 3.5.** Consider a NCMS $(\urcorner,\Xi,\Theta,\Upsilon,*,\diamond)$. For $\alpha\gg 0$, the open ball $\mathcal{O}(\varrho,\mathfrak{r},\alpha)$ where $\varrho$ is its center and $\mathfrak{r}\in(0,1)$ is its radius as $\mathcal{O}(\varrho,\mathfrak{r},\alpha)=\{\varsigma\in\urcorner:\Xi(\varrho,\varsigma,\alpha)>1-\mathfrak{r},\Theta(\varrho,\varsigma,\alpha)<\mathfrak{r}$ and $\Upsilon(\varrho,\varsigma,\alpha)<\mathfrak{r}\}$.

**Definition 3.6.** Consider a NCMS $(\urcorner,\Xi,\Theta,\Upsilon,*,\diamond)$. Let $\varrho\in\urcorner$ and $\{\varrho_n\}$ be a sequence in $\urcorner$. Then $\{\varrho_n\}$ suppose converges to $\varrho$ if for any $\alpha\gg 0$ and $\mathfrak{r}\in(0,1)$ are present, a natural integer $n_0$ exists such that $\Xi(\varrho_n,\varrho,\alpha)>1-\mathfrak{r},\Theta(\varrho_n,\varrho,\alpha)<\mathfrak{r}$ and $\Upsilon(\varrho_n,\varrho,\alpha)<\mathfrak{r}$ for all $n>n_0$. Then $\lim_{n\to\infty}\varrho_n=\varrho$ or $\varrho_n\to\varrho$ as $n\to\infty$.

**Definition 3.7.** Consider a NCMS $(\urcorner,\Xi,\Theta,\Upsilon,*,\diamond)$. Let $\varrho\in\urcorner$ and $\{\varrho_n\}$ be a sequence in $\urcorner$. If for any $0<\varepsilon<1$ and any $\alpha\gg 0$ a natural number $n_0$ like that exists and $\Xi(\varrho_n,\varrho_m,\alpha)>$

$1-\varepsilon, \Theta(\varrho_n, \varrho_m, \alpha) < \varepsilon$ and $\Upsilon(\varrho_n, \varrho_m, \alpha) < \varepsilon$ for each $n, m > n_0$ then $\{\varrho_n\}$ referred to as a Cauchy sequence.

**Definition 3.8.** Consider a NCMS $(\daleth, \Xi, \Theta, \Upsilon, *, \diamond)$. One calls $\daleth$ complete if each and every Cauchy sequence converges.

**Definition 3.9.** Consider a NCMS $(\daleth, \Xi, \Theta, \Upsilon, *, \diamond)$. We refer to a subset $\Phi$ of $\daleth$ as $\mathfrak{FC}$ - bounded assuming if $\alpha \gg 0$ and $\mathfrak{r} \in (0,1)$ are present like that $\Xi(\varrho, \varsigma, \alpha) > 1-\mathfrak{r}, \Theta(\varrho, \varsigma, \alpha) < \mathfrak{r}$ and $\Upsilon(\varrho, \varsigma, \alpha) < \mathfrak{r}$ for each $\varrho, \varsigma \in \Phi$.

**Definition 3.10.** Consider a NCMS $(\daleth, \Xi, \Theta, \Upsilon, *, \diamond)$ and $\mathfrak{h} : \daleth \to \daleth$ be a self mapping. Then neutrosophic cone contractive is the name given to $\mathfrak{h}$ and assuming there is $\rho \in (0,1)$ like that $\dfrac{1}{\Xi(\mathfrak{h}(\varrho), \mathfrak{h}(\varsigma), \alpha)} - 1 \leq \rho \left( \dfrac{1}{\Xi(\varrho, \varsigma, \alpha)} - 1 \right), \Theta(\mathfrak{h}(\varrho), \mathfrak{h}(\varsigma), \alpha) \leq \rho\Theta(\varrho, \varsigma, \alpha)$ and $\Upsilon(\mathfrak{h}(\varrho), \mathfrak{h}(\varsigma), \alpha) \leq \rho\Upsilon(\varrho, \varsigma, \alpha)$ for each $\varrho, \varsigma \in \daleth$ and $\alpha \gg 0$. $\rho$ is referred to as the $\mathfrak{h}$ contractive constant.

**Lemma 3.11.** *Consider any two points $\varrho, \varsigma \in \daleth$ and $\rho \in (0,1)$ such that $\Xi(\varrho, \varsigma, \rho\alpha) \geq \Xi(\varrho, \varsigma, \alpha), \Theta(\varrho, \varsigma, \rho\alpha) \leq \Theta(\varrho, \varsigma, \alpha)$ and $\Upsilon(\varrho, \varsigma, \rho\alpha) \leq \Upsilon(\varrho, \varsigma, \alpha)$. Then $\varrho = \varsigma$.*

**Theorem 3.12.** *Consider a NCMS $(\daleth, \Xi, \Theta, \Upsilon, *, \diamond)$. Define $\tau = \{\Phi \subseteq \daleth : \varrho \in \Phi$ iff a thing exists $\mathfrak{r} \in (0,1)$ and $\alpha \gg 0$ like that $\mathcal{O}(\varrho, \mathfrak{r}, \alpha) \subset \Phi\}$, which is a topology on $\daleth$.*

**Proof.** Let $\varrho \in \phi$. Hence $\phi = \mathcal{O}(\varrho, \mathfrak{r}, \alpha) \subset \phi$ and $\phi \in \tau$.
Since for any $\varrho \in \daleth$, and $\mathfrak{r} \in (0,1)$, $\alpha \gg 0$ are present then $\mathcal{O}(\varrho, \mathfrak{r}, \alpha) \subset \daleth$, then $\daleth \in \tau$. Let $\Phi, \mathcal{O} \in \tau$ and $\varrho \in \Phi \cap \mathcal{O}$, then $\varrho \in \Phi$ and $\varrho \in \mathcal{O}$ so a thing exists $\alpha_1 \gg 0, \alpha_2 \gg 0$ and $\mathfrak{r}_1, \mathfrak{r}_2 \in (0,1)$ such that $\mathcal{O}(\varrho, \mathfrak{r}_1, \alpha_1) \subset \Phi$ and $\mathcal{O}(\varrho, \mathfrak{r}_2, \alpha_2) \subset \mathcal{O}$.
From Definition (2.5), for $\alpha_1 \gg 0, \alpha_2 \gg 0$, a thing exists $\alpha \gg 0$ such that $\alpha \gg \alpha_1, \alpha \gg \alpha_2$ and take $\mathfrak{r} = \min\{\mathfrak{r}_1, \mathfrak{r}_2\}$. Then $\mathcal{O}(\varrho, \mathfrak{r}, \alpha) \subset \mathcal{O}(\varrho, \mathfrak{r}_1, \alpha_1) \cap \mathcal{O}(\varrho, \mathfrak{r}_2, \alpha_2) \subset \Phi \cap \mathcal{O}$. Hence $\Phi \cap \mathcal{O} \in \tau$. Let $\Phi_j \in \mathfrak{r}$ for every $j \in I$ and $\varrho \in \cup_{j \in I} \Phi_j$. Afterwards, there is $i_0 \in I$ similar to $\varrho \in \Phi_{j_0}$. So, there is $\alpha \gg 0$ and $\mathfrak{r} \in (0,1)$ like that $\mathcal{O}(\varrho, \mathfrak{r}, \alpha) \subset \Phi_{j_0}$. Since $\Phi_{j_0} \subset \cup_{j \in I} \Phi_j, \mathcal{O}(\varrho, \mathfrak{r}, \alpha) \subset \cup_{j \in I} \Phi_j$. Thus $\cup_{j \in I} \Phi_j \in \tau$. Hence, $\tau$ is therefore a topology on $\daleth$.

**Theorem 3.13.** *Consider a NCMS $(\daleth, \Xi, \Theta, \Upsilon, *, \diamond)$. Then $(\daleth, \tau)$ is Hausdorff.*

**Proof.** Consider a NCMS $(\daleth, \Xi, \Theta, \Upsilon, *, \diamond)$. Let $\varrho, \varsigma$ be the two separate points of $\daleth$. Then $0 < \Xi(\varrho, \varsigma, \alpha) < 1, 0 < \Theta(\varrho, \varsigma, \alpha) < 1$ and $0 < \Upsilon(\varrho, \varsigma, \alpha) < 1$. Assume $\Xi(\varrho, \varsigma, \alpha) = \mathfrak{r}_1$, $\Theta(\varrho, \varsigma, \alpha) = \mathfrak{r}_2$ and $\Upsilon(\varrho, \varsigma, \alpha) = \mathfrak{r}_3$ and $\mathfrak{r} = \max\{\mathfrak{r}_1, \mathfrak{r}_2, \mathfrak{r}_3\}$. Then for each $\mathfrak{r}_0 \in (\mathfrak{r}, 1)$, there is $\mathfrak{r}_4$, $\mathfrak{r}_5$ and $\mathfrak{r}_6$ such that $\mathfrak{r}_4 * \mathfrak{r}_4 \geq \mathfrak{r}_0$, $(1-\mathfrak{r}_5) \diamond (1-\mathfrak{r}_5) \leq (1-\mathfrak{r}_0)$ and $(1-\mathfrak{r}_6) \diamond (1-\mathfrak{r}_6) \leq (1-\mathfrak{r}_0)$. Assume $\mathfrak{r}_7 = \max\{\mathfrak{r}_4, \mathfrak{r}_5, \mathfrak{r}_6\}$. Think about open balls $\mathcal{O}(\varrho, 1-\mathfrak{r}_7, \frac{\alpha}{2})$ and $\mathcal{O}(\varsigma, 1-\mathfrak{r}_7, \frac{\alpha}{2})$. Then obviously $\mathcal{O}(\varrho, 1-\mathfrak{r}_7, \frac{\alpha}{2}) \cap \mathcal{O}(\varsigma, 1-\mathfrak{r}_7, \frac{\alpha}{2}) = \emptyset$. Assume that $\mathcal{O}(\varrho, 1-\mathfrak{r}_7, \frac{\alpha}{2}) \cap \mathcal{O}(\varsigma, 1-\mathfrak{r}_7, \frac{\alpha}{2}) \neq \emptyset$.

Then there is $\nu \in \mathcal{O}(\varrho, 1-\mathfrak{r}_7, \frac{\alpha}{2}) \cap \mathcal{O}(\varsigma, 1-\mathfrak{r}_7, \frac{\alpha}{2})$.

$$\begin{aligned}
\mathfrak{r}_1 &= \Xi(\varrho, \varsigma, \alpha) \\
&\geq \Xi(\varrho, \nu, \frac{\alpha}{2}) * \Xi(\nu, \varsigma, \frac{\alpha}{2}) \geq \mathfrak{r}_7 * \mathfrak{r}_7 \geq \mathfrak{r}_4 * \mathfrak{r}_4 \geq \mathfrak{r}_0 > \mathfrak{r}_1, \\
\mathfrak{r}_2 &= \Theta(\varrho, \varsigma, \alpha) \\
&\leq \Theta(\varrho, \nu, \frac{\alpha}{2}) \diamond \Theta(\nu, \varsigma, \frac{\alpha}{2}) \leq (1-\mathfrak{r}_7) \diamond (1-\mathfrak{r}_7) \leq (1-\mathfrak{r}_5) \diamond (1-\mathfrak{r}_5) \leq (1-\mathfrak{r}_0) < \mathfrak{r}_2 \text{ and} \\
\mathfrak{r}_3 &= \Upsilon(\varrho, \varsigma, \alpha) \\
&\leq \Upsilon(\varrho, \nu, \frac{\alpha}{2}) \diamond \Upsilon(\nu, \varsigma, \frac{\alpha}{2}) \leq (1-\mathfrak{r}_7) \diamond (1-\mathfrak{r}_7) \leq (1-\mathfrak{r}_6) \diamond (1-\mathfrak{r}_6) \leq (1-\mathfrak{r}_0) < \mathfrak{r}_3,
\end{aligned}$$

which contradicts itself. Hence, $(\daleth, \Xi, \Theta, \Upsilon, *, \diamond)$ is Hausdorff.

**Theorem 3.14.** *Consider a NCMS $(\daleth, \Xi, \Theta, \Upsilon, *, \diamond)$, $\varrho \in \daleth$ and $(\varrho_n)$ be an $\daleth$ sequence. Then $(\varrho_n)$ converges to $\varrho$ if, then, just $\Xi(\varrho_n, \varrho, \alpha) \to 1, \Theta(\varrho_n, \varrho, \alpha) \to 0$ and $\Upsilon(\varrho_n, \varrho, \alpha) \to 0$ as $n \to \infty$, for every $\alpha \gg 0$.*

**Proof.** Assume that $(\varrho_n) \to \varrho$. Then, for each $\alpha \gg 0$ and $\mathfrak{r} \in (0,1)$, there is a natural number $n_0$ such that $\Xi(\varrho_n, \varrho, \alpha) > 1-\mathfrak{r}, \Theta(\varrho_n, \varrho, \alpha) < \mathfrak{r}$ and $\Upsilon(\varrho_n, \varrho, \alpha) < \mathfrak{r}$, for all $n \gg n_0$. We have $1-\Xi(\varrho_n, \varrho, \alpha) < \mathfrak{r}, \Theta(\varrho_n, \varrho, \alpha) < \mathfrak{r}$ and $\Upsilon(\varrho_n, \varrho, \alpha) < \mathfrak{r}$. Hence $\Xi(\varrho_n, \varrho, \alpha) \to 1, \Theta(\varrho_n, \varrho, \alpha) \to 0$ and $\Upsilon(\varrho_n, \varrho, \alpha) \to 0$ as $n \to \infty$.

However, suppose that $\Xi(\varrho_n, \varrho, \alpha) \to 1$ as $n \to \infty$. Then, there exists a natural integer $n_0$ such that for each $\alpha \gg 0$ and $\mathfrak{r} \in (0,1)$, $1-\Xi(\varrho_n, \varrho, \alpha) < \mathfrak{r}, \Theta(\varrho_n, \varrho, \alpha) < \mathfrak{r}$ and $\Upsilon(\varrho_n, \varrho, \alpha) < \mathfrak{r}$ for each $n \geq n_0$. Hence, $\Xi(\varrho_n, \varrho, \alpha) > 1-\mathfrak{r}, \Theta(\varrho_n, \varrho, \alpha) < \mathfrak{r}$ and $\Upsilon(\varrho_n, \varrho, \alpha) < \mathfrak{r}$ for each $n \geq n_0$. Hence $\varrho_n \to \varrho$ as $n \to \infty$.

## 4. Main Results

**Theorem 4.1.** *Consider a complete NCMS $(\daleth, \Xi, \Theta, \Upsilon, *, \diamond)$ in which neutrosophic cone contractive sequences are Cauchy. Let $\mathfrak{F} : \daleth \to \daleth$ be a neutrosophic cone contractive mapping, the contractice constant is $\rho$. Then $\mathfrak{F}$ has a distinct fixed point.*

**Proof.** Consider $\varrho \in \daleth$ and let $\varrho_n = \mathfrak{F}^n(\varrho), n \in \mathbb{N}$. For $\alpha \gg 0$, we have

$$\frac{1}{\Xi(\mathfrak{F}(\varrho), \mathfrak{F}^2(\varrho), \alpha)} - 1 \leq \rho \left( \frac{1}{\Xi(\varrho, \varrho_1, \alpha)} - 1 \right),$$
$$\Theta(\mathfrak{F}(\varrho), \mathfrak{F}^2(\varrho), \alpha) \leq \rho \Theta(\varrho, \varrho_1, \alpha)$$
$$\Upsilon(\mathfrak{F}(\varrho), \mathfrak{F}^2(\varrho), \alpha) \leq \rho \Upsilon(\varrho, \varrho_1, \alpha)$$

and by induction

$$\begin{aligned}
\frac{1}{\Xi(\varrho_{n+1},\varrho_{n+2},\alpha)}-1 &\leq \rho\left(\frac{1}{\Xi(\varrho_n,\varrho_{n+1},\alpha)}-1\right),\\
\Theta(\varrho_{n+1},\varrho_{n+2},\alpha) &\leq \rho\Theta(\varrho_n,\varrho_{n+1},\alpha)\\
\Upsilon(\varrho_{n+1},\varrho_{n+2},\alpha) &\leq \rho\Upsilon(\varrho_n,\varrho_{n+1},\alpha),\ \text{for all } n\in\mathbb{N}
\end{aligned}$$

Then $(\varrho_n)$ is a neutrosophic contractive Cauchy sequence which converges to $\varsigma$ where $\varsigma \in \daleth$. Theorem (3.14), gives us

$$\begin{aligned}
\frac{1}{\Xi(\mathfrak{F}(\varsigma),\mathfrak{F}(\varrho_n),\alpha)}-1 &\leq \rho\left(\frac{1}{\Xi(\varsigma,\varrho_n,\alpha)}-1\right)\to 1,\\
\Theta(\mathfrak{F}(\varsigma),\mathfrak{F}(\varrho_n),\alpha) &\leq \rho\Theta(\varsigma,\varrho_n,\alpha)\to 0 \text{ and}\\
\Upsilon(\mathfrak{F}(\varsigma),\mathfrak{F}(\varrho_n),\alpha) &\leq \rho\Upsilon(\varsigma,\varrho_n,\alpha)\to 0 \text{ as } n\to\infty.
\end{aligned}$$

Then for every $\alpha \gg 0$,
$\lim_{n\to\infty}\Xi(\mathfrak{F}(\varsigma),\mathfrak{F}(\varrho_n),\alpha) = 1$, $\lim_{n\to\infty}\Theta(\mathfrak{F}(\varsigma),\mathfrak{F}(\varrho_n),\alpha) = 0$ and $\lim_{n\to\infty}\Upsilon(\mathfrak{F}(\varsigma),\mathfrak{F}(\varrho_n),\alpha) = 0$ and hence $\lim_{n\to\infty}\mathfrak{F}(\varrho_n) = \mathfrak{F}(\varsigma)$.
Now, we prove uniqueness. Assume $\mathfrak{F}(\nu) = \nu$ for some $\nu \in \mathcal{V}$. For $\alpha \gg 0$, we have

$$\begin{aligned}
\frac{1}{\Xi(\varsigma,\nu,\alpha)}-1 &= \frac{1}{\Xi(\mathfrak{F}(\varsigma),\mathfrak{F}(\nu),\alpha)}-1 \leq \rho\left(\frac{1}{\Xi(\varsigma,\nu,\alpha)}-1\right)\\
&= \rho\left(\frac{1}{\Xi(\mathfrak{F}(\varsigma),\mathfrak{F}(\nu),\alpha)}-1\right) \leq \rho^2\left(\frac{1}{\Xi(\varsigma,\nu,\alpha)}-1\right)\\
&\leq \cdots \leq \rho^n\left(\frac{1}{\Xi(\varsigma,\nu,\alpha)}-1\right)\to 1 \text{ as } n\to\infty,\\
\Theta(\varsigma,\nu,\alpha) &= \Theta(\mathfrak{F}(\varsigma),\mathfrak{F}(\nu),\alpha) \leq \rho\Theta(\varsigma,\nu,\alpha)\\
&= \rho\Theta(\mathfrak{F}(\varsigma),\mathfrak{F}(\nu),\alpha) \leq \rho^2\Theta(\varsigma,\nu,\alpha)\\
&\leq \cdots \leq \rho^n\Theta(\varsigma,\nu,\alpha)\to 0 \text{ as } n\to\infty,\\
\Upsilon(\varsigma,\nu,\alpha) &= \Upsilon(\mathfrak{F}(\varsigma),\mathfrak{F}(\nu),\alpha) \leq \rho\Upsilon(\varsigma,\nu,\alpha)\\
&= \rho\Upsilon(\mathfrak{F}(\varsigma),\mathfrak{F}(\nu),\alpha) \leq \rho^2\Upsilon(\varsigma,\nu,\alpha)\\
&\leq \cdots \leq \rho^n\Upsilon(\varsigma,\nu,\alpha)\to 0 \text{ as } n\to\infty,
\end{aligned}$$

Hence $\Xi(\varsigma,\nu,\alpha) = 1, \Theta(\varsigma,\nu,\alpha) = 0$ and $\Upsilon(\varsigma,\nu,\alpha) = 0$ and $\varsigma = \nu$.

**Theorem 4.2.** *Consider a complete NCMS* $(\daleth,\Xi,\Theta,\Upsilon,*,\diamond)$*, and let* $\mathcal{P},\mathcal{R},\mathcal{Q}$ *and* $\mathcal{S}$ *be self-mappings of* $\daleth$*. Let* $\{\mathcal{P},\mathcal{Q}\}$ *and* $\{\mathcal{R},\mathcal{S}\}$ *be Occasionally Weakly Compatible (OWC) pairings.*

*Assuming there is a* $\rho \in (0,1)$ *such that*

$$\Xi(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)) \geq \min \left\{ \begin{array}{ll} \Xi(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha), & \Xi(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Xi(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha), & \Xi(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha), \\ \Xi(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), \alpha) & \end{array} \right\},$$

$$\Theta(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)) \leq \max \left\{ \begin{array}{ll} \Theta(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha), & \Theta(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Theta(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha), & \Theta(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha), \\ \Theta(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), \alpha) & \end{array} \right\},$$

$$\Upsilon(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)) \leq \max \left\{ \begin{array}{ll} \Upsilon(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha), & \Upsilon(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Upsilon(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha), & \Upsilon(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha), \\ \Upsilon(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), \alpha) & \end{array} \right\} \quad (4.2.1)$$

*for each* $\varrho, \varsigma \in \daleth$ *and for each* $\alpha \gg 0$*, afterward there is a special point* $\omega \in \daleth$ *like that* $\mathcal{P}(\omega) = \mathcal{Q}(\omega) = \omega$ *and a unique point* $\nu \in \daleth$ *such that* $\mathcal{R}(\nu) = \mathcal{S}(\nu) = \nu$*. Moreover* $\nu = \omega$*, hence* $\mathcal{P}, \mathcal{R}, \mathcal{Q}$ *and* $\mathcal{S}$ *have a singular shared fixed point.*

**Proof.** Consider $\{\mathcal{P}, \mathcal{Q}\}$ and $\{\mathcal{R}, \mathcal{S}\}$ which are OWC pairings, consequently points $\varrho, \varsigma \in \daleth$ is such that $\mathcal{P}(\varrho) = \mathcal{Q}(\varrho)$ and $\mathcal{R}(\varsigma) = \mathcal{S}(\varsigma)$. We claim that $\mathcal{P}(\varrho) = \mathcal{R}(\varsigma)$.
By inequality (4.2.1),

$$\begin{aligned} \Xi(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)) &\geq \min \left\{ \begin{array}{ll} \Xi(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha), & \Xi(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Xi(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha), & \Xi(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha), \\ \Xi(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), \alpha) & \end{array} \right\} \\ &= \min \left\{ \begin{array}{ll} \Xi(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), & \Xi(\mathcal{P}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Xi(\mathcal{R}(\varsigma), \mathcal{R}(\varsigma), \alpha), & \Xi(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), \\ \Xi(\mathcal{R}(\varsigma), \mathcal{P}(\varrho), \alpha) & \end{array} \right\} \\ &= \Xi(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), \\ \Theta(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)) &\leq \max \left\{ \begin{array}{ll} \Theta(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha), & \Theta(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Theta(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha), & \Theta(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha), \\ \Theta(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), \alpha) & \end{array} \right\} \\ &= \max \left\{ \begin{array}{ll} \Theta(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), & \Theta(\mathcal{P}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Theta(\mathcal{R}(\varsigma), \mathcal{R}(\varsigma), \alpha), & \Theta(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), \\ \Theta(\mathcal{R}(\varsigma), \mathcal{P}(\varrho), \alpha) & \end{array} \right\} \\ &= \Theta(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha) \\ \Upsilon(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)) &\leq \max \left\{ \begin{array}{ll} \Upsilon(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha), & \Upsilon(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Upsilon(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha), & \Upsilon(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha), \\ \Upsilon(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), \alpha) & \end{array} \right\} \end{aligned}$$

$$= \max \left\{ \begin{array}{ll} \Upsilon(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), & \Upsilon(\mathcal{P}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Upsilon(\mathcal{R}(\varsigma), \mathcal{R}(\varsigma), \alpha), & \Upsilon(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), \\ \Upsilon(\mathcal{R}(\varsigma), \mathcal{P}(\varrho), \alpha) & \end{array} \right\}$$

$$= \Upsilon(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha)$$

By Lemma (3.11), $\mathcal{P}(\varrho) = \mathcal{R}(\varsigma)$, i.e., $\mathcal{P}(\varrho) = \mathcal{Q}(\varrho) = \mathcal{R}(\varsigma) = \mathcal{S}(\varsigma)$.

Supposing there is a point $\nu$ that is different $\mathcal{P}(\nu) = \mathcal{Q}(\nu)$ then by (4.2.1), we have $\mathcal{P}(\nu) = \mathcal{Q}(\nu) = \mathcal{R}(\varsigma) = \mathcal{S}(\varsigma)$, so $\mathcal{P}(\varrho) = \mathcal{P}(\nu)$ and $\omega = \mathcal{P}(\varrho) = \mathcal{Q}(\varrho)$ is the special place where $\mathcal{P}$ and $\mathcal{Q}$ coincide.

By Lemma (3.11), the only fixed point between $\mathcal{P}$ and $\mathcal{Q}$ is $\omega$. Likewise, there is a special point $\nu \in \daleth$ like that $\nu = \mathcal{R}(\nu) = \mathcal{S}(\nu)$. Assume that $\omega \neq \nu$, we have

$$\begin{aligned}
\Xi(\omega, \nu, \rho(\alpha)) &= \Xi(\mathcal{P}(\omega), \mathcal{R}(\nu), \rho(\alpha)) \\
&\geq \min \left\{ \begin{array}{ll} \Xi(\mathcal{Q}(\omega), \mathcal{S}(\nu), \alpha), & \Xi(\mathcal{Q}(\omega), \mathcal{P}(\nu), \alpha), \\ \Xi(\mathcal{R}(\nu), \mathcal{S}(\nu), \alpha), & \Xi(\mathcal{P}(\omega), \mathcal{S}(\nu), \alpha), \\ \Xi(\mathcal{R}(\nu), \mathcal{Q}(\omega), \alpha) & \end{array} \right\} \\
&= \min \Big\{ \Xi(\omega, \nu, \alpha), \Xi(\omega, \nu, \alpha), \Xi(\nu, \nu, \alpha), \Xi(\omega, \nu, \alpha), \Xi(\nu, \omega, \alpha) \Big\} \\
&= \Xi(\omega, \nu, \alpha), \\
\Theta(\omega, \nu, \rho(\alpha)) &= \Theta(\mathcal{P}(\omega), \mathcal{R}(\nu), \rho(\alpha)) \\
&\leq \max \left\{ \begin{array}{ll} \Theta(\mathcal{Q}(\omega), \mathcal{S}(\nu), \alpha), & \Theta(\mathcal{Q}(\omega), \mathcal{P}(\nu), \alpha), \\ \Theta(\mathcal{R}(\nu), \mathcal{S}(\nu), \alpha), & \Theta(\mathcal{P}(\omega), \mathcal{S}(\nu), \alpha), \\ \Theta(\mathcal{R}(\nu), \mathcal{Q}(\omega), \alpha) & \end{array} \right\} \\
&= \max \Big\{ \Theta(\omega, \nu, \alpha), \Theta(\omega, \nu, \alpha), \Theta(\nu, \nu, \alpha), \Theta(\omega, \nu, \alpha), \Theta(\nu, \omega, \alpha) \Big\} \\
&= \Theta(\omega, \nu, \alpha), \\
\Upsilon(\omega, \nu, \rho(\alpha)) &= \Upsilon(\mathcal{P}(\omega), \mathcal{R}(\nu), \rho(\alpha)) \\
&\leq \max \left\{ \begin{array}{ll} \Upsilon(\mathcal{Q}(\omega), \mathcal{S}(\nu), \alpha), & \Upsilon(\mathcal{Q}(\omega), \mathcal{P}(\nu), \alpha), \\ \Upsilon(\mathcal{R}(\nu), \mathcal{S}(\nu), \alpha), & \Upsilon(\mathcal{P}(\omega), \mathcal{S}(\nu), \alpha), \\ \Upsilon(\mathcal{R}(\nu), \mathcal{Q}(\omega), \alpha) & \end{array} \right\} \\
&= \max \Big\{ \Upsilon(\omega, \nu, \alpha), \Upsilon(\omega, \nu, \alpha), \Upsilon(\nu, \nu, \alpha), \Upsilon(\omega, \nu, \alpha), \Upsilon(\nu, \omega, \alpha) \Big\} \\
&= \Upsilon(\omega, \nu, \alpha).
\end{aligned}$$

Hence, we have $\nu = \omega$ by Lemma (3.11), a common fixed point of $\mathcal{P}, \mathcal{R}, \mathcal{Q}$ and $\mathcal{S}$ is $\nu$. (4.2.1) states that the fixed point's uniqueness is true.

**Theorem 4.3.** *Consider a complete NCMS* $(\daleth, \Xi, \Theta, \Upsilon, *, \diamond)$ *and let* $\mathcal{P}, \mathcal{R}, \mathcal{Q}$ *and* $\mathcal{S}$ *be self-mappings of* $\daleth$*. Let* $\{\mathcal{P}, \mathcal{Q}\}$ *and* $\{\mathcal{R}, \mathcal{S}\}$ *be OWC pairings. If* $\rho \in (0,1)$ *exists in a way that*

$$\Xi(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)) \geq \chi\left[\min\left\{\begin{array}{ll} \Xi(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha), & \Xi(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Xi(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha), & \Xi(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha), \\ \Xi(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), \alpha) & \end{array}\right\}\right],$$

$$\Theta(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)) \leq \psi\left[\max\left\{\begin{array}{ll} \Theta(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha), & \Theta(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Theta(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha), & \Theta(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha), \\ \Theta(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), \alpha) & \end{array}\right\}\right],$$

$$\Upsilon(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)) \leq \phi\left[\max\left\{\begin{array}{ll} \Upsilon(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha), & \Upsilon(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Upsilon(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha), & \Upsilon(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha), \\ \Upsilon(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), \alpha) & \end{array}\right\}\right],$$

*for each* $\varrho, \varsigma \in \daleth$ *and* $\chi, \psi, \phi : [0,1] \to [0,1]$*, such that* $\chi(\alpha) > \alpha, \psi(\alpha) < \alpha, \phi(\alpha) < \alpha$ *for all* $0 \ll \alpha < 1$*, therefore*$\mathcal{P}, \mathcal{R}, \mathcal{Q}$ *and* $\mathcal{S}$ *have a special shared fixed point.*

**Proof.** Theorem (4.2) leads to the theorem's proof.

**Theorem 4.4.** *Consider a complete NCMS* $(\daleth, \Xi, \Theta, \Upsilon, *, \diamond)$ *and let* $\mathcal{P}, \mathcal{R}, \mathcal{Q}$ *and* $\mathcal{S}$ *be self-mappings of* $\daleth$*. Let* $\{\mathcal{P}, \mathcal{Q}\}$ *and* $\{\mathcal{R}, \mathcal{S}\}$ *be OWC pairings. If there is a* $\rho \in (0,1)$ *such that*

$$\Xi(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)) \geq \chi\left\{\begin{array}{ll} \Xi(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha), & \Xi(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Xi(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha), & \Xi(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha), \\ \Xi(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), \alpha) & \end{array}\right\},$$

$$\Theta(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)) \leq \psi\left\{\begin{array}{ll} \Theta(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha), & \Theta(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Theta(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha), & \Theta(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha), \\ \Theta(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), \alpha) & \end{array}\right\},$$

$$\Upsilon(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)) \leq \phi\left\{\begin{array}{ll} \Upsilon(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha), & \Upsilon(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Upsilon(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha), & \Upsilon(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha), \\ \Upsilon(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), \alpha) & \end{array}\right\} \quad (4.4.1)$$

*for each* $\varrho, \varsigma \in \daleth$ *and* $\chi, \psi, \phi : [0,1]^5 \to [0,1]$*, such that* $\chi(\alpha, 1, 1, \alpha, \alpha) > \alpha, \psi(\alpha, 0, 0, \alpha, \alpha) < \alpha, \phi(\alpha, 0, 0, \alpha, \alpha) < \alpha$ *for all* $0 \ll \alpha < 1$*, then* $\mathcal{P}, \mathcal{R}, \mathcal{Q}$ *and* $\mathcal{S}$ *have a special shared fixed point.*

**Proof.** Consider $\{\mathcal{P}, \mathcal{Q}\}$ and $\{\mathcal{R}, \mathcal{S}\}$ which are OWC pairings, there are points $\varrho, \varsigma \in \daleth$ such that $\mathcal{P}(\varrho) = \mathcal{Q}(\varrho)$ and $\mathcal{R}(\varsigma) = \mathcal{S}(\varsigma)$.

We show that $\mathcal{P}(\varrho) = \mathcal{R}(\varsigma)$. By inequality (4.4.1), we have

$$\begin{aligned}
\Xi(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)) &\geq \chi \left\{ \begin{array}{ll} \Xi(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha), & \Xi(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Xi(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha), & \Xi(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha), \\ \Xi(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), \alpha) & \end{array} \right\} \\
&= \chi \left\{ \begin{array}{ll} \Xi(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), & \Xi(\mathcal{P}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Xi(\mathcal{R}(\varsigma), \mathcal{R}(\varsigma), \alpha), & \Xi(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), \\ \Xi(\mathcal{R}(\varsigma), \mathcal{P}(\varrho), \alpha) & \end{array} \right\} \\
&= \chi \Big\{ \Xi(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), 1, 1, \Xi(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), \Xi(\mathcal{R}(\varsigma), \mathcal{P}(\varrho), \alpha) \Big\} \\
&\geq \Xi(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), \\
\Theta(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)) &\leq \psi \left\{ \begin{array}{ll} \Theta(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha), & \Theta(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Theta(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha), & \Theta(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha), \\ \Theta(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), \alpha) & \end{array} \right\} \\
&= \psi \left\{ \begin{array}{ll} \Theta(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), & \Theta(\mathcal{P}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Theta(\mathcal{R}(\varsigma), \mathcal{R}(\varsigma), \alpha), & \Theta(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), \\ \Theta(\mathcal{R}(\varsigma), \mathcal{P}(\varrho), \alpha) & \end{array} \right\} \\
&= \psi \Big\{ \Theta(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), 0, 0, \Theta(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), \Theta(\mathcal{R}(\varsigma), \mathcal{P}(\varrho), \alpha) \Big\} \\
&\leq \Theta(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), \\
\Upsilon(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)) &\leq \phi \left\{ \begin{array}{ll} \Upsilon(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha), & \Upsilon(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Upsilon(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha), & \Upsilon(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha), \\ \Upsilon(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), \alpha) & \end{array} \right\} \\
&= \phi \left\{ \begin{array}{ll} \Upsilon(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), & \Upsilon(\mathcal{P}(\varrho), \mathcal{P}(\varrho), \alpha), \\ \Upsilon(\mathcal{R}(\varsigma), \mathcal{R}(\varsigma), \alpha), & \Upsilon(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), \\ \Upsilon(\mathcal{R}(\varsigma), \mathcal{P}(\varrho), \alpha) & \end{array} \right\} \\
&= \phi \Big\{ \Upsilon(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), 0, 0, \Upsilon(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha), \Upsilon(\mathcal{R}(\varsigma), \mathcal{P}(\varrho), \alpha) \Big\} \\
&\leq \Upsilon(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha),
\end{aligned}$$

which is a contradiction, hence $\mathcal{P}(\varrho) = \mathcal{R}(\varsigma)$. That is $\mathcal{P}(\varrho) = \mathcal{Q}(\varrho) = \mathcal{R}(\varsigma) = \mathcal{S}(\varsigma)$.
Assume that there is a point $\nu$ such that $\mathcal{P}(\nu) = \mathcal{Q}(\nu)$, then by (4.4.1) $\mathcal{P}(\nu) = \mathcal{Q}(\nu) = \mathcal{R}(\varsigma) = \mathcal{S}(\varsigma)$, so $\mathcal{P}(\varrho) = \mathcal{P}(\nu)$ and $\omega = \mathcal{P}(\varrho) = \mathcal{S}(\varrho)$ is the special place where $\mathcal{P}$ and $\mathcal{Q}$ coincide. From Lemma (2.6), $\omega$ is the sole fixed point that connects $\mathcal{P}$ and $\mathcal{Q}$. Likewise, there is a speical point $\nu \in \daleth$ like that $\nu = \mathcal{R}(\nu) = \mathcal{S}(\nu)$. Thus a common fixed point between $\mathcal{P}, \mathcal{R}, \mathcal{Q}$ and $\mathcal{S}$ is $\nu$. (4.4.1) states that the fixed point's uniqueness holds.

**Theorem 4.5.** *Consider a complete NCMS* $(\daleth, \Xi, \Theta, \Upsilon, *, \diamond)$ *and let* $\mathcal{P}, \mathcal{R}, \mathcal{Q}$ *and* $\mathcal{S}$ *be self - mappings of* $\daleth$. *Consider* $\{\mathcal{P}, \mathcal{Q}\}$ *and* $\{\mathcal{R}, \mathcal{S}\}$ *which are OWC pairings. If point* $\rho \in (0,1)$

*exists, then for every* $\varrho, \varsigma \in \daleth$ *and* $\alpha \gg 0$ *satisfying*

$$\Xi\big(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)\big) \geq \Xi(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) * \Xi(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) * \Xi(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha) * \Xi(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha)$$
$$\Theta\big(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)\big) \leq \Theta(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) \diamond \Theta(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) \diamond \Theta(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha) \diamond \Theta(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha)$$
$$\Upsilon\big(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)\big) \leq \Upsilon(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) \diamond \Upsilon(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) \diamond \Upsilon(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha) \diamond \Upsilon(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha) \quad (4.5.1)$$

*then* $\mathcal{P}, \mathcal{R}, \mathcal{Q}$ *and* $\mathcal{S}$ *have a special shared fixed point.*

**Proof.** Consider $\{\mathcal{P}, \mathcal{Q}\}$ and $\{\mathcal{R}, \mathcal{S}\}$ which are OWC pairings. There are points $\varrho, \varsigma \in \daleth$ such that $\mathcal{P}(\varrho) = \mathcal{Q}(\varrho)$ and $\mathcal{R}(\varsigma) = \mathcal{S}(\varsigma)$. We claim that $\mathcal{P}(\varrho) = \mathcal{R}(\varsigma)$.
By inequality (4.5.1), we have

$$\begin{aligned}
\Xi\big(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)\big) &\geq \Xi(\mathcal{Q}(\varrho), \mathcal{R}(\varsigma), \alpha) * \Xi(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) * \Xi(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha) * \Xi(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha) \\
&= \Xi(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha) * \Xi(\mathcal{P}(\varrho), \mathcal{P}(\varrho), \alpha) * \Xi(\mathcal{R}(\varsigma), \mathcal{R}(\varsigma), \alpha) * \Xi(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha) \\
&\geq \Xi(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha) * 1 * 1 * \Xi(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha) \\
&= \Xi(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha) \\
\Theta\big(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)\big) &\leq \Theta(\mathcal{Q}(\varrho), \mathcal{R}(\varsigma), \alpha) \diamond \Theta(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) \diamond \Theta(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha) \diamond \Theta(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha) \\
&= \Theta(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha) \diamond \Theta(\mathcal{P}(\varrho), \mathcal{P}(\varrho), \alpha) \diamond \Theta(\mathcal{R}(\varsigma), \mathcal{R}(\varsigma), \alpha) \diamond \Theta(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha) \\
&\leq \Theta(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha) \diamond 0 \diamond 0 \diamond \Theta(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha) \\
&\leq \Theta(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha) \\
\Upsilon\big(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)\big) &\leq \Upsilon(\mathcal{Q}(\varrho), \mathcal{R}(\varsigma), \alpha) \diamond \Upsilon(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) \diamond \Upsilon(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha) \diamond \Upsilon(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha) \\
&= \Upsilon(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha) \diamond \Upsilon(\mathcal{P}(\varrho), \mathcal{P}(\varrho), \alpha) \diamond \Upsilon(\mathcal{R}(\varsigma), \mathcal{R}(\varsigma), \alpha) \diamond \Upsilon(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha) \\
&\leq \Upsilon(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha) \diamond 0 \diamond 0 \diamond \Upsilon(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha) \\
&\leq \Upsilon(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \alpha)
\end{aligned}$$

By Lemma (3.11), we have $\mathcal{P}(\varrho) = \mathcal{R}(\varsigma)$ i.e., $\mathcal{P}(\varrho) = \mathcal{Q}(\varrho) = \mathcal{R}(\varsigma) = \mathcal{S}(\varsigma)$. Assume that there is a another point $\nu$ such that $\mathcal{P}(\nu) = \mathcal{R}(\nu)$ then by (4.5.1), we have $\mathcal{P}(\nu) = \mathcal{Q}(\nu) = \mathcal{R}(\varsigma) = \mathcal{S}(\varsigma)$, so $\mathcal{P}(\varrho) = \mathcal{P}(\nu)$ and $\omega = \mathcal{P}(\varrho) = \mathcal{Q}(\varrho)$ is the one and only place where $\mathcal{P}$ and $\mathcal{Q}$ coincide. Similarly, there is a special aspect $\omega \in \daleth$ like that $\omega = \mathcal{R}(\omega) = \mathcal{S}(\omega)$.
The common fixed point between $\mathcal{P}, \mathcal{R}, \mathcal{Q}$ and $\mathcal{S}$ is $\omega$.

**Theorem 4.6.** *Consider a complete NCMS* $(\daleth, \Xi, \Theta, \Upsilon, *, \diamond)$ *and let* $\mathcal{P}, \mathcal{R}, \mathcal{Q}$ *and* $\mathcal{S}$ *be self-mappings of* $\daleth$*. Let* $\{\mathcal{P}, \mathcal{Q}\}$ *and* $\{\mathcal{R}, \mathcal{S}\}$ *be OWC pairings. If there is a* $\rho \in (0, 1)$ *for each*

*$\varrho, \varsigma \in \urcorner$ and $\alpha \gg 0$ satisfying*

$$\begin{aligned}\Xi\big(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)\big) \geq\ & \Xi(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) * \Xi(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) * \Xi(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha) * \Xi(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), 2\alpha) * \\ & \Xi(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha) \\ \Theta\big(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)\big) \leq\ & \Theta(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) \diamond \Theta(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) \diamond \Theta(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha) \diamond \Theta(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), 2\alpha) \diamond \\ & \Theta(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha) \\ \Upsilon\big(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)\big) \leq\ & \Upsilon(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) \diamond \Upsilon(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) \diamond \Upsilon(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha) \diamond \Upsilon(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), 2\alpha) \diamond \\ & \Upsilon(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha)\end{aligned}$$

*then there is a singular common fixed point between $\mathcal{P}, \mathcal{R}, \mathcal{Q}$ and $\mathcal{S}$.*

**Proof.** We have

$$\begin{aligned}\Xi\big(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)\big) \geq\ & \Xi(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) * \Xi(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) * \Xi(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha) * \Xi(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), 2\alpha) * \\ & \Xi(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha) \\ \geq\ & \Xi(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) * \Xi(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) * \Xi(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha) * \Xi(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) * \\ & \Xi(\mathcal{S}(\varsigma), \mathcal{R}(\varsigma), \alpha) * \Xi(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha) \\ \geq\ & \Xi(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) * \Xi(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) * \Xi(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha) * \Xi(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha) \\ \Theta\big(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)\big) \leq\ & \Theta(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) \diamond \Theta(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) \diamond \Theta(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha) \diamond \Theta(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), 2\alpha) \diamond \\ & \Theta(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha) \\ \leq\ & \Theta(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) \diamond \Theta(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) \diamond \Theta(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha) \diamond \Theta(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) \diamond \\ & \Theta(\mathcal{S}(\varsigma), \mathcal{R}(\varsigma), \alpha) \diamond \Theta(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha) \\ \leq\ & \Theta(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) \diamond \Theta(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) \diamond \Theta(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha) \diamond \Theta(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha) \\ \Upsilon\big(\mathcal{P}(\varrho), \mathcal{R}(\varsigma), \rho(\alpha)\big) \leq\ & \Upsilon(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) \diamond \Upsilon(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) \diamond \Upsilon(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha) \diamond \Upsilon(\mathcal{R}(\varsigma), \mathcal{Q}(\varrho), 2\alpha) \diamond \\ & \Upsilon(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha) \\ \leq\ & \Upsilon(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) \diamond \Upsilon(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) \diamond \Upsilon(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha) \diamond \Upsilon(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) \diamond \\ & \Upsilon(\mathcal{S}(\varsigma), \mathcal{R}(\varsigma), \alpha) \diamond \Upsilon(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha) \\ \leq\ & \Upsilon(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) \diamond \Upsilon(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) \diamond \Upsilon(\mathcal{R}(\varsigma), \mathcal{S}(\varsigma), \alpha) \diamond \Upsilon(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha)\end{aligned}$$

and hence from Theorem (4.5) there is a shared fixed point for $\mathcal{P}, \mathcal{R}, \mathcal{Q}$ and $\mathcal{S}$.

**Corollary 4.7.** *Consider a complete NCMS $(\urcorner, \Xi, \Theta, \Upsilon, *, \diamond)$. Then continuous self-mappings $\mathcal{Q}$ and $\mathcal{S}$ of $\urcorner$ possess a shared fixed point in $\urcorner$ if and only if a self - mapping $\mathcal{P}$ of $\urcorner$ exists so that that the aforementioned requirements are met.*

*(1) $\mathcal{P}\urcorner \subset \mathcal{S}\urcorner \cap \mathcal{Q}\urcorner$*

*(2) the pairs $\{\mathcal{P}, \mathcal{Q}\}$ and $\{\mathcal{P}, \mathcal{S}\}$ are weakly compatible,*

*(3) a point has been made $\rho \in (0,1)$ such that for every $\varrho, \varsigma \in \daleth$ and $\alpha \gg 0$,*

$$\Xi\big(\mathcal{P}(\varrho), \mathcal{P}(\varsigma), \rho(\alpha)\big) \geq \Xi(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) * \Xi(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) * \Xi(\mathcal{P}(\varsigma), \mathcal{S}(\varsigma), \alpha) * \Xi(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha),$$

$$\Theta\big(\mathcal{P}(\varrho), \mathcal{P}(\varsigma), \rho(\alpha)\big) \leq \Theta(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) \diamond \Theta(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) \diamond \Theta(\mathcal{P}(\varsigma), \mathcal{S}(\varsigma), \alpha) \diamond \Theta(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha),$$

$$\Upsilon\big(\mathcal{P}(\varrho), \mathcal{P}(\varsigma), \rho(\alpha)\big) \leq \Upsilon(\mathcal{Q}(\varrho), \mathcal{S}(\varsigma), \alpha) \diamond \Upsilon(\mathcal{P}(\varrho), \mathcal{Q}(\varrho), \alpha) \diamond \Upsilon(\mathcal{P}(\varsigma), \mathcal{S}(\varsigma), \alpha) \diamond \Upsilon(\mathcal{P}(\varrho), \mathcal{S}(\varsigma), \alpha)$$

*Fixed point in common between $\mathcal{P}, \mathcal{Q}$ and $\mathcal{S}$ is distinct.*

**Proof.** Since compatibility also implies OWC , Theorem (4.6) leads to the conclusion.

**Theorem 4.8.** *Consider a complete NCMS $(\daleth, \Xi, \Theta, \Upsilon, *, \diamond)$ and let $\mathcal{P}$ and $\mathcal{Q}$ be self -mappings of $\daleth$. Let the $\mathcal{P}$ and $\mathcal{Q}$ are OWC . If there is a point $\rho \in (0,1)$ for every $\varrho, \varsigma \in \daleth$ and $\alpha \gg 0$*

$$\Xi\big(\mathcal{Q}(\varrho), \mathcal{Q}(\varsigma), \rho(\alpha)\big) \geq \alpha\Xi(\mathcal{P}(\varrho), \mathcal{P}(\varsigma), \alpha) + \beta \min\Big\{\Xi(\mathcal{P}(\varrho), \mathcal{P}(\varsigma), \alpha), \Xi(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \Xi(\mathcal{Q}(\varsigma), \mathcal{P}(\varsigma), \alpha)\Big\},$$

$$\Theta\big(\mathcal{Q}(\varrho), \mathcal{Q}(\varsigma), \rho(\alpha)\big) \leq \alpha\Theta(\mathcal{P}(\varrho), \mathcal{P}(\varsigma), \alpha) + \beta \max\Big\{\Theta(\mathcal{P}(\varrho), \mathcal{P}(\varsigma), \alpha), \Theta(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \Theta(\mathcal{Q}(\varsigma), \mathcal{P}(\varsigma), \alpha)\Big\},$$

$$\Upsilon\big(\mathcal{Q}(\varrho), \mathcal{Q}(\varsigma), \rho(\alpha)\big) \leq \alpha\Upsilon(\mathcal{P}(\varrho), \mathcal{P}(\varsigma), \alpha) + \beta \max\Big\{\Upsilon(\mathcal{P}(\varrho), \mathcal{P}(\varsigma), \alpha), \Upsilon(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \Upsilon(\mathcal{Q}(\varsigma), \mathcal{P}(\varsigma), \alpha)\Big\} \tag{4.8.1}$$

*for all $\varrho, \varsigma \in \daleth$, where $\alpha, \beta > 0, \alpha + \beta > 1$. $\mathcal{P}$ and $\mathcal{Q}$ share a distinct common fixed point.*

**Proof.** Consider $\{\mathcal{P}, \mathcal{Q}\}$ which are OWC pair, so that there is a point $\varrho \in \daleth$ such that $\mathcal{P}(\varrho) = \mathcal{Q}(\varrho)$. Consider the possibiity of another point $\varsigma \in \daleth$ for which $\mathcal{P}(\varsigma) = \mathcal{Q}(\varsigma)$. We claim that $\mathcal{Q}(\varrho) = \mathcal{Q}(\varsigma)$. By inequality (4.8.1), we have

$$\begin{aligned}
\Xi\big(\mathcal{Q}(\varrho), \mathcal{Q}(\varsigma), \rho(\alpha)\big) &\geq \alpha\Xi(\mathcal{P}(\varrho), \mathcal{P}(\varsigma), \alpha) + \beta \min\Big\{\Xi(\mathcal{P}(\varrho), \mathcal{P}(\varsigma), \alpha), \Xi(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \Xi(\mathcal{Q}(\varsigma), \mathcal{P}(\varsigma), \alpha)\Big\} \\
&= \alpha\Xi(\mathcal{Q}(\varrho), \mathcal{Q}(\varsigma), \alpha) + \beta \min\Big\{\Xi(\mathcal{Q}(\varrho), \mathcal{Q}(\varsigma), \alpha), \Xi(\mathcal{Q}(\varrho), \mathcal{Q}(\varrho), \alpha), \Xi(\mathcal{Q}(\varsigma), \mathcal{Q}(\varsigma), \alpha)\Big\} \\
&= (\alpha + \beta)\Xi(\mathcal{Q}(\varrho), \mathcal{Q}(\varsigma), \alpha), \\
\Theta\big(\mathcal{Q}(\varrho), \mathcal{Q}(\varsigma), \rho(\alpha)\big) &\leq \alpha\Theta(\mathcal{P}(\varrho), \mathcal{P}(\varsigma), \alpha) + \beta \max\Big\{\Theta(\mathcal{P}(\varrho), \mathcal{P}(\varsigma), \alpha), \Theta(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \Theta(\mathcal{Q}(\varsigma), \mathcal{P}(\varsigma), \alpha)\Big\} \\
&= \alpha\Theta(\mathcal{Q}(\varrho), \mathcal{Q}(\varsigma), \alpha) + \beta \max\Big\{\Theta(\mathcal{Q}(\varrho), \mathcal{Q}(\varsigma), \alpha), \Theta(\mathcal{Q}(\varrho), \mathcal{Q}(\varrho), \alpha), \Theta(\mathcal{Q}(\varsigma), \mathcal{Q}(\varsigma), \alpha)\Big\} \\
&= (\alpha + \beta)\Theta(\mathcal{Q}(\varrho), \mathcal{Q}(\varsigma), \alpha), \\
\Upsilon\big(\mathcal{Q}(\varrho), \mathcal{Q}(\varsigma), \rho(\alpha)\big) &\leq \alpha\Upsilon(\mathcal{P}(\varrho), \mathcal{P}(\varsigma), \alpha) + \beta \max\Big\{\Upsilon(\mathcal{P}(\varrho), \mathcal{P}(\varsigma), \alpha), \Upsilon(\mathcal{Q}(\varrho), \mathcal{P}(\varrho), \alpha), \Upsilon(\mathcal{Q}(\varsigma), \mathcal{P}(\varsigma), \alpha)\Big\} \\
&= \alpha\Upsilon(\mathcal{Q}(\varrho), \mathcal{Q}(\varsigma), \alpha) + \beta \max\Big\{\Upsilon(\mathcal{Q}(\varrho), \mathcal{Q}(\varsigma), \alpha), \Upsilon(\mathcal{Q}(\varrho), \mathcal{Q}(\varrho), \alpha), \Upsilon(\mathcal{Q}(\varsigma), \mathcal{Q}(\varsigma), \alpha)\Big\} \\
&= (\alpha + \beta)\Upsilon(\mathcal{Q}(\varrho), \mathcal{Q}(\varsigma), \alpha)
\end{aligned}$$

a contradiction, since $(\alpha + \beta) > 1$. Therefore $\mathcal{Q}(\varrho) = \mathcal{Q}(\varsigma)$. Therefore $\mathcal{P}(\varrho) = \mathcal{P}(\varsigma)$ and $\mathcal{P}(\varrho)$ is unique. $\mathcal{P}$ and $\mathcal{Q}$ have a distinct fixed point from lemma (2.6).

**Example 4.9.** Let $\Sigma = [0,1]$ and let $\Xi(\varrho, \varsigma, \alpha) = \frac{\alpha}{\alpha + |\varrho - \varsigma|}$, $\Theta(\varrho, \varsigma, \alpha) = \frac{|\varrho - \varsigma|}{\alpha + |\varrho - \varsigma|}$ and $\Upsilon(\varrho, \varsigma, \alpha) = \frac{|\varrho - \varsigma|}{\alpha}$ are neutrosophic metric on $\Sigma$. Define self mappings $\mathcal{P}$ and $\mathcal{Q}$ on $\Sigma$ as follows $\mathcal{P}(\varrho) = \frac{1-\varrho}{3}$ and $\mathcal{Q}(\varrho) = \frac{\sqrt{5-4(1-2\varrho)^2}-1}{4}$. Clearly $\mathcal{P}$ and $\mathcal{Q}$ are OWC maps. Also, $\mathcal{P}$ and $\mathcal{Q}$ are satisfy all the conditions of Theorem 4.8. The self maps $\mathcal{P}$ and $\mathcal{Q}$ have coincidence points $\varrho = 1, \frac{1}{4}$ and the common fixed point $\varrho = \frac{1}{4}$.

## 5. Conclusion

Using the idea of contractive conditions and OWC , we have demonstrated that there is a common fixed point for four self mappings, three self mappings and two self mappings in a complete NCMS.

## References


1. Atanassov.K, Intuitionistic Fuzzy Sets, Fuzzy Sets and System, 1986, 20, 87 - 96.
2. Alaca, C., Turkoglu D., Yildiz C. Fixed points in intuitionistic fuzzy metric spaces, Chaos Solitons and Fractals , 2006, 29, 1073 - 1078.
3. Basset, M., Gamal, A., Son, L. H., and Smarandache, F., A Bipolar Neutrosophic Multi Criteria Decision Making Framework for Professional Selection. Applied Sciences, 10,4, 2020.
4. George A, Veeramani P. On some results in fuzzy metric spaces. Fuzzy Sets and System., 1994, 64, 395 - 399.
5. Huang. L and Zhang. X, Cone metric spaces and fixed point theorems of contractive mappings, J. Math. Anal. Appl., 332(2007), 1468 - 1476.
6. Hussain A, Al Sulami H, Ishtiaq U, Some New Aspects in the Intuitionistic Fuzzy and Neutrosophic Fixed Point Theory, Journal of Function Spaces, 2022.
7. Ishtiaq U, Hussain A, Al Sulami H, Certain new aspects in fuzzy fixed point theory, AIMS Mathematics 7 (5), 8558-8573,2021.
8. Ishtiaq U, Javed K, Uddin F, Sen M, Ahmed K, Ali Mu, Fixed point results in orthogonal neutrosophic metric spaces, Complexity 2021.
9. Jeyaraman. M, Jeyanthi. V, Mangayarkkarasi. A.N, Smarandache. F, Some New Structures in Neutrosophic Metric Spaces, Neutrosophic sets and systems, 2021, 42, 49 - 64.
10. Jeyaraman M, Suganthi M, and Wasfi Shatanawi, Common Fixed Point Theorems in Intuitionistic Generalized Fuzzy Cone Metric Spaces, Mathematics 2020, 8, 1212.
11. Jeyaraman M and Sowndrarajan S, Some common fixed point theorems for $(\phi - \psi)$-weak contractions in intuitionistic generalized fuzzy cone metric spaces, Malaya Journal of Matematik, Vol. S, No. 1, 154-159, 2019.
12. Jeyaraman. M. Suganthi. M, Common Fixed Point Theorems in M-Fuzzy Cone Metric Spaces, Results in Nonlinear Analysis, 4, 1, 33-46, 2021.
13. Jungck. G and Rhoades. B. E, Fixed Point Theorems for Occasionally Weakly compatible Mappings, Fixed Point Theory, 7(2006), 287 - 296.
14. Kramosil O, Michalek J. Fuzzy metric and statistical metric spaces. Kybernetika, 1975, 11, 326 - 334.
15. Kirisci, M., Simsek, N. Neutrosophic metric spaces. Mathematical Sciences. 2020,14, 241 - 248.
16. Oner.T, Kandemir.M.B and Tanay.B, Fuzzy cone metric spaces, J. Nonlinear Sci. and Appl., 8(2015), 610 - 616.
17. Park J.H. Intuitionistic fuzzy metric spaces. Chaos Solitons and Fractals, 2004, 22, 1039 - 1046.
18. Priyobartal.N, Yumnam Rohen and Upadhyay. B. B, Some fixed point results in fuzzy cone metric spaces, International Journal of Pure and Applied Mathematics, 109(3)(2016), 573 - 582.

19. Schweizer.B, Sklar.A, Statistical metric spaces, Pacific Journal of Mathematics, 10(1), 313 - 334, 1960. https://doi.org/pjm/1103038644
20. Smarandache, F. A Unifying Field in Logics, Neutrosophy: Neutrosophic Probability, Set and Logic; American Research Press: Reheboth, MA, USA, 1998.
21. Smarandache, F. Neutrosophy: Neutrosophic Probability, Set and Logic; ProQuest Information and Learning: Ann Arbor, MI, USA, 1998; pp. 105
22. Smarandache, F. Neutrosophic set, a generalisation of the intuitionistic fuzzy sets, International Journal of Pure and Applied Mathematics. 2005, 24, 287 - 297.
23. Sowndrarajan. S, Jeyaraman. M. Smarandache, F., Fixed Point theorems in neutrosophic metric spaces, Neutrosophic Sets and Systems, 2020, 36, 251 - 268.
24. Sowndrarajan. S, Jeyaraman. M., Fixed point Results for Contraction Theorems in Neutrosophic Metric Spaces, Neutrosophic sets and systems, 2020, 36, 308 - 318.
25. Suganthi. M, Jeyaraman. M., A Generalized Neutrosophic Metric Space and Coupled Coincidence point Results, Neutrosophic sets and systems, 2021, 42, 253 - 269.
26. Simsek N, Kirisci M. Fixed Point Theorems in Neutrosophic Metric Spaces. Sigma Journal of Engineering and Natural Sciences., 2019, 10(2), 221 - 230.
27. Turkoglu D., Alaca C., Cho Y.J., Yildiz C. Common fixed point theorems in intuitionistic fuzzy metric spaces, J Appl Math Comput, 2006, 22, 411 - 424.
28. Turkoglu. D and Abuloha.M, Cone metric spaces and fixed point theorems in diametrically contractive mappings, Acta Math. Sin., 26(2010), 489 - 496.
29. Vishal Gupta, Saini R. K, A Kanwar, Some coupled fixed point results on modified intuitionistic fuzzy metric spaces and application to integral type contraction, Iranian Journal of Fuzzy Systems, 14(5), 123-137, 2017.
30. Vishal Gupta, N Mani, Existence and Uniqueness of Fixed Point in Fuzzy Metric Spaces and its Applications, Advances in Intelligent Systems and Computing, 236, 217-224, 2014, Springer, Berlin.
31. Wasfi Shatanawi, V Gupta, A Kanwar, New results on modified intuitionistic generalized fuzzy metric spaces by employing E.A property and common E.A property for coupled maps, Journal of Intelligent & Fuzzy Systems, 38(3), 3003-3010, 2020.
32. Zadeh L.A. Fuzzy sets. Information and Control, 1965, 8, 338 - 353.

Received: March 19, 2023. Accepted: July 15, 2023