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Abstract: Simulation has become a modern-day tool that helps us study many systems that its results could not have been studied or predicted through the work of these systems over time. The simulation process depends on generating a series of random numbers that are subject to a uniform probability distribution on the field [0,1], and then converting these random numbers to random variables that follow the probability distribution in which the system to be simulated, there are several methods that can be used to carry out the conversion.

In previous research, the inverse transformation method has been studied according to the Neutrosophic logic and we have reached Neutrosophic random variables, by using them we get a more accurate simulation of any system we want to simulate. It should be noted that the inverse transformation can be used if the cumulative distribution function has an inverse function, but if the system we want to simulate works according to a probability distribution, and the inverse function of the cumulative distribution function cannot be found, then we use other methods.

In this research, we are examining a study that enables us to generate Neutrosophical random variables that follow probability distributions based on Neutrosophical random numbers that follow for the uniform distribution, using the rejection and acceptance method, which depends on the largest value taken by the probability density function of the distribution in which the system to be simulated operates on its definition.

Keywords: Neutrosophic, Neutrosophical random variables, Neutrosophic logic

Introduction:

In the light of the great development in today's world, the complexity of systems and the significant material and non-material losses that can result from the operation of any system without prior study, there has to be a scientific method that enables us to know the results that we can get when operating systems and helps us to minimize these losses. The simulation process was the modern instrument through which we can predict what results we can get through the functioning of these systems over time. Since the simulation process depends on generating random numbers that follow regular distribution on the field [0, 1] and then converting these numbers into random variables that follow the probable distribution of the system to be simulated In classical logic, many methods were presented by which we were able to obtain the random variables needed for the simulation process, To keep pace with scientific development, the most important of these methods had to be reformulated using the revolutionary logic of the Neutrosophic logic established by the American philosopher and athlete Florentine Smarandache in 1995. So in previous research we prepared a study to generate random Neutrosophic numbers on the field [a, b) Based on the Neutrosophic researchers' findings in the definition of regular distribution and definition of integration according to Neutrosophic logic, and in other research, we converted these Neutrosophic random numbers into Neutrosophic random variables tracking the exponential distribution using the opposite conversion method [1-14].

In this research we will generate Neutrosophic random variables based on previous studies and use the method of rejection and acceptance to convert Neutrosophic random numbers into random variables that follow the probable distribution of the system to be emulated.

Discussion:

During the simulation process, we encounter many systems that operate on probability distributions. The reverse function of the cumulative distribution function cannot be found. Therefore, we cannot use the opposite conversion method that we have formulated according to the Neutrosophic logic. So in this research we will present a study to convert the Neutrosophic random numbers that follow the Neutrosophic regular probability distribution into Neutrosophic random variables.

Method of rejection and acceptance according to classical logic: [15]

We take the distribution of a probability $f(x)$ defined on the field $[a,b]$ with the following relationship:

$$f(x)=\begin{cases} f(x) & ; a \le x \le b \\ 0 & otherwise \end{cases}$$

We assume that the greatest value $f(x)$ takes on your area of definition is M the same as Mode, then the following inequality is achieved:

$$0 \le f(x) \le M \quad ; \quad a \le x \le b$$

Thus:

$$0 \le f(x) \le M \Rightarrow 0 \le \frac{f(x)}{M} \le 1 \quad ; a \le x \le b$$

This means that the composition $\frac{f(x)}{M}$ is valid for comparison with random numbers that follow a uniform distribution on the field [0, 1], because it achieves the following relationship:

$$P\left(R < \frac{f(x)}{M}\right) = \frac{f(x)}{M}$$

We benefit from the above by applying the following algorithm:

1- We generate two random numbers R_1 , R_2 that follow the uniform distribution on the field [0, 1].

We use the mean squared method to generate the two random numbers defined as follows:

To generate random numbers, we apply the following relationship:

$$R_{i+1} = Mid\left[R_i^2\right] \quad ; i = 0,1,2,3,---- \quad (1)$$

Where *Mid* stands for the middle four places of R_i^2 and any fractional $R_\circ$ random number consisting of four places (called the seed) and not containing zero in any of its four places is chosen, to generate two random numbers R_1 , R_2 from the field $[0 , 1]$.

2- We take one of the two numbers, let it be R_1 and from it a variable suitable for the uniform distribution on the field [a, b], by performing the following conversion:

$$x_1 = (b-a)R_1 + a$$

3- we check whether if R_2 achieves the inequality:

$$R_2 \le \frac{f(x)}{M} \qquad (*)$$

4- if the inequality (*) is achieved We accept that x_1 is subject to the operated distribution in the system which is being simulated.

If the inequality $(*)$ is not achieved, we reject the two numbers R_1 , R_2 and return to the first step, to generate two random numbers again.

Thus, we continue to work and compare as much as we want or as required by the simulation process.

Practical example:

We want to simulate a system that operates according to the probability distribution whose probability density function is given by the following relationship:

$$f(x)=\begin{cases}\frac{1}{16}x & ;\ 2\le x\le 6\\ 0 & ;\ 0therwise\end{cases}$$

We calculate M which is the largest value of $f(x)$ and is equal to:

$$M=\frac{1}{16}\times 6=\frac{3}{8}$$

To perform the simulation, it is necessary to generate random variables that follow this distribution. We will use the rejection and acceptance method to achieve the desired:

1- We use the mean squared method given by the following relationship:

$$R_{i+1}=Mid\left[R_i^2\right]\quad ;i=0,1,2,3,----\qquad (1)$$

Where Mid stands for the middle four places of R_i^2 and any fractional $R_\circ$ random number consisting of four places (called the seed) and not containing zero in any of its four places is chosen, to generate two random numbers R_1 , R_2 from the field $[0\ ,1]$ we take the seed $R_\circ=0,3176$ and from it we get

$$R_2=0.7551,\ R_1=0.0869$$

2- We take one of the two numbers, let it be $R_1=0,0869$ and form it a variable suitable for uniform distribution over the field $[a\ ,b]=[2\ ,6]$ by performing the following transformation:

$$x_1 = (b-a)R_1 + a \quad \Rightarrow$$
$$x_1 = (6-2) \times 0.0869 + 2 = 2.3476$$

We calculate $f(x_1)$:

$$f(x_1) = \frac{1}{16} \times 2.3476 = 0.1467$$

3- We test inequality $(*)$:

$$R_2 \le \frac{f(x_1)}{M} \quad \Rightarrow$$
$$0.7551 \le \frac{0.1467}{0.375} = 0.3912$$

4- We note that the inequality is not achieved and therefore the random variable x_1 does not follow the probability distribution $f(x)$ and therefore we reject the two random numbers R_1 , R_2 and return to (1).

We start again:

1- We generate two random numbers by taking the seed $R_\circ = 0{,}1234$ we get:

$$R_2 = 0.3215 \, , \, R_1 = 0.5227$$

2- We take one of the numbers, let it be $R_1 = 0{,}5227$ and form from it a variable suitable for the uniform distribution on the field $[a \, , b] = [2 \, , 6]$ by doing the following conversion:

$$x_1 = (b-a)R_1 + a \quad \Rightarrow$$
$$x_1 = (6-2) \times 0.5227 + 2 = 4.0908$$

We calculate $f(x_1)$:

$$f(x_1) = \frac{1}{16} \times 4.0908 = 0.2557$$

3- We check the inequality (*):

$$R_2 \le \frac{f(x_1)}{M} \quad \Rightarrow$$
$$0.3215 \le \frac{0.2557}{0.375} = 0.6819$$

4- We see that the inequality is achieved, therefor the random variable x_1 follows the probability distribution $f(x)$, we continue to work and compare as much as we want or as required by the simulation process.

In this research, we formulated the rejection and acceptance method according to the Neutrosophic logic, so the previous algorithm became as follows

1- We generate two random numbers R_1 , R_2 that follow the uniform distribution on the field [0,1].

2- We convert the two numbers R_1 , R_2 into two Neutrosophic random numbers on the field $[0+\delta\ , 1+\delta]$

According to the following formula:

$$NP(R < R_i) = NF(R_i) = R_i - \delta \quad ; \quad \delta \in [0\ , m]$$

Where $\delta \in [0\ , m]$ is the indefinite, and $0 < m < 1$ let's denote them by NR_2, NR_1

We find the cumulative distribution function for the uniform distribution, which is given by the following relationship:

When the indeterminacy is in the upper and lower bounds of the field that means:

$[a_N\ , b_N] = [a+\varepsilon\ , b+\varepsilon]$ where $\varepsilon \in [0\ , n]$ and $a < n < b$ then:

$$NF(x) = \frac{x-a}{b_N - a_N} - i \quad ; \quad i \in \left[0\ , \frac{n}{b_N - a_N}\right]$$

In (1) and (2) we used a study that is mentioned in reference (16).

Then we use the inverse transformation method to find the random variables that follow the uniform distribution on the field $[a_N\ , b_N] = [a+\varepsilon\ , b+\varepsilon]$

We substitute the following relationship:

$$R = F(x) \Rightarrow x = F^{-1}(R)$$

We find:

$$NF(x) = NR \Rightarrow NR = \frac{x-a}{b_N - a_N} - i \Rightarrow$$
$$\frac{x-a}{b_N - a_N} = NR + i \Rightarrow (NR+i)(b_N - a_N) = x - a \Rightarrow$$
$$Nx = (NR+i)(b_N - a_N) + a$$

$$i \in \left[0,\ \frac{n}{b_N - a_N}\right]$$

Accordingly, to generate random variables that follow the Neutrosophic uniform distribution in the case of indeterminacy related to the lower and upper bounds

We use the following relationship:

$$Nx_j = (NR_j + i)(b_N - a_N) + a \quad ; \quad j = 0,1,2,3,4$$

Where $i \in \left[0,\ \frac{n}{b_N - a_N}\right]$ and ε is the indefinite where ε ∈ [0, n] and $a < n < b$

We note that the variables have become neutrosophical values, and therefore the probability density function $f(x)$ becomes a neutrosophical function that we denote by the symbol $f_N(x)$ and it is defined on the field $[a_N, b_N] = [a+\varepsilon\ , b+\varepsilon]$.

We suppose that the largest value that this function takes in its domain is M_N, then the following inequality is achieved:

$$0_N \le f_N(x) \le M_N \ ; \quad a+\varepsilon \le x \le b+\varepsilon$$

Then the following inequality is achieved:

This means that the composition $\frac{f_N(x)}{M_N}$ is valid for comparison with the Neutrosophical random numbers that follow the Neutrosophic uniform distribution on the field $[0+\delta\ , 1+\delta]$ because it achieves the following relationship:

$$NP\left(NR < \frac{f_N(x)}{M_N} \right) = \frac{f_N(x)}{M_N}$$

3- We test whether NR_2 achieves inequality:

$$NR_2 \leq \frac{f_N(Nx_1)}{M_N} \qquad (**)$$

If the inequality (**) is true, then we accept that Nx_1 is subject to the distribution $f_N(x)$ in which the system to be simulated operates.

4- If NR_2 does not achieve the inequality $(**)$, then we reject the two numbers R_1 , R_2 and return to the first step of generating the two random numbers again.

Thus, we continue to work and compare as much as we want or as required by the simulation process.

Note 1: In step (2) we took the indeterminacy on the two terms of the field and used the appropriate relations for that in the process of converting random numbers into Neutrosophical random numbers. Reference [16].

Note 2: When generating the Neutrosophic random variable that follows the uniform distribution on the field, we also took the indeterminacy on the two terms of the field and then applied the inverse transformation method to find the random variable. It should be noted that we follow the same method if the indeterminacy is related to one of the field terms.

Practical example:

We want to simulate a system that operates according to the probability distribution whose probability density function is given by the following relationship:

$$f(x) = \begin{cases} \frac{1}{16}x & ; \ 2 \leq x \leq 6 \\ 0 & ; \ 0therwise \end{cases}$$

1- We use the mean squared method given by the following relationship:

$$R_{i+1} = Mid\left[R_i^2\right] \quad ; i = 0,1,2,3,---- \quad (1)$$

We take the seed $R_{\circ} = 0{,}3176$ and from it we form $R_2 = 0.7551\,,\; R_1 = 0.0869$

2- We transform the two numbers $R_1\;,\;R_2$ into two Neutrosophic random numbers on the field

$[0+\delta\;,1+\delta]$ using the following relationship:

$$NR_i = R_i - \delta$$

And in this example we take $\delta = [0\;,\;0.03]$

$$\begin{aligned} NR_i &= R_i - \delta \quad \& \;\; i = 1 \;\Rightarrow \\ NR_1 &= R_1 - \delta = 0.0869 - [0\;,\;0.03] \Rightarrow \\ NR_1 &= [0.0569\;,\;\; 0.0869] \end{aligned}$$

$$\begin{aligned} NR_i &= R_i - \delta \quad \& \quad i = 2 \;\Rightarrow \\ NR_2 &= R_2 - \delta = 0.7551 - [0\;,\;0.03] \Rightarrow \\ NR_2 &= [0.7251\;,\;\; 0.7551] \end{aligned}$$

We take one of the two numbers, let it be $NR_1 = [0.0569\;,\;0.0869]$ and from it we form a

variable suitable for the Neutrosophic uniform distribution over the field

.Here we have taken $\varepsilon = [0\;,\;3]$ $[a_N\;\;,b_N] = [2+[0\;,\;3]\;,\;6+[0\;,\;3]\;] = [\;[2\;,\;5]\;,[6\;,\;9]\;]$

By performing the following conversion:

$$Nx_j = (NR_j + i)(b_N - a_N) + a \quad ; \quad j = 1,2,3,---$$

$$i \in \left[\;0,\;\frac{n}{b_N - a_N}\;\right]$$

First we calculate i :

$$i = \left[\;0,\;\frac{3}{b_N - a_N}\;\right] = \left[\;0\;,\;\frac{3}{[6\;,\;9]-[2\;,\;5]}\;\right] = \left[\;0\;,\frac{3}{4}\;\right] = [0\;,\;0.75]$$

Make up:

$$Nx_1 = (NR_1 + i)(b_N - a_N) + a \Rightarrow$$
$$Nx_1 = \left([0.0569\,,\,0.0869] + [0\,,\,0.75] \right)(4) + 2 \Rightarrow$$
$$Nx_1 = [2.2276\,,\,5.3476]$$

We calculate $f_N(Nx_1)$:

$$f_N(x) = f_N(Nx_1)$$

$$f_N(Nx_1) = \frac{1}{16}[2.2276\,,\,5.3476] = [0.1392\,,\,0.3342]$$

We calculate M_N which is the largest value of $f_N(x)$ and is equal to:

$$M_N = \frac{1}{16} \times [6\,,\,9] = [0.375\,,\,0.5625]$$

3- We test the inequality $(**)$:

$$NR_2 \le \frac{f_N(Nx_1)}{M_N} \quad \Rightarrow [0.7251\,,\,0.7551] \le \frac{[0.1392\,,\,0.3342]}{[0.375\,,\,0.5625]} = [0.3712\ \ ,\ \ 0.5941] \Rightarrow$$
$$[0.7251\,,\,0.7551] \le [0.3712\,,\ \ 0.5941]$$

4- We note that the inequality is not satisfied and therefore we reject the two random numbers

R_2, R_1 and return to step (1)

We start again

1- We use the mean squared method given by the following relationship:

$$R_{i+1} = Mid[R_i^2] \quad ; i = 0,1,2,3,---- \quad (1)$$

We take the seed $R_\circ = 0,1234$ and from it we form $R_2 = 0.3215\,,\,R_1 = 0.5227$

2- We transform the two numbers $R_1\,,\,R_2$ into two Neutrosophic random numbers on the field $[0+\delta\,,\,1+\delta]$ using the following relationship:

$$NR_i = R_i - \delta$$

We take it in this example $\delta = [0\,,\,0.03]$

$$NR_i = R_i - \delta \quad \& \quad i = 1 \Rightarrow$$
$$NR_1 = R_1 - \delta = 0.5227 - [0\ ,0.03] \Rightarrow$$
$$NR_1 = [0.5227\ ,\ 0.4927]$$

$$NR_i = R_i - \delta \quad \& \quad i = 2 \Rightarrow$$
$$NR_2 = R_2 - \delta = 0.3215 - [0\ ,0.03] \Rightarrow$$
$$NR_2 = [0.3215\ ,\ 0.2915]$$

We take one of the two numbers, let it be $NR_1 = [0.5227, 0.4927]$, and from it form a variable suitable for the Neutrosophic uniform distribution over the field

$[a_N\ , b_N] = [2 + [0\ ,3]\ , 6 + [0\ ,3]\ ] = [\ [2\ ,5], [6\ ,9]\ ]$. Here we have taken $\varepsilon = [0\ ,3]$

By performing the following conversion**:**

$$Nx_j = (NR_j + i)(b_N - a_N) + a \quad ; \quad j = 1,2,3,---$$

$$i \in \left[0,\ \frac{n}{b_N - a_N}\right]$$

First, we calculate i **we find:**

$$i = \left[0,\ \frac{3}{b_N - a_N}\right] = \left[0\ ,\frac{3}{[6\ ,9] - [2\ ,5]}\right] = \left[0\ ,\frac{3}{4}\right] = [0\ ,0.75]$$

Make up:

$$Nx_1 = (NR_1 + i)(b_N - a_N) + a \Rightarrow$$
$$Nx_1 = (\ [0.5227\ ,0.4927] + [0\ ,0.75]\ )(4) + 2 \Rightarrow$$
$$Nx_1 = [4.0908\ ,5.4927]$$

We calculate $f_N(Nx_1)$**:**

$$f_N(x) = f_N(Nx_1)$$
$$f_N(Nx_1) = \frac{1}{16}[4.0908\ ,6.9708] = [0.2557\ ,0.3106]$$

3- We calculate M_N which is the largest value of $f_N(x)$ and is equal to:

$$M_N = \frac{1}{16} \times [6\ ,9] = [0.375\ ,0.5625]$$

4- We test the inequality $(**)$:

$$NR_2 \le \frac{f_N(Nx_1)}{M_N} \Rightarrow [0.3215\,,\,0.2915] \le \frac{[0.2557\,,\,0.3106]}{[0.375\,,\,0.5625]} = [0.5522\,,\,0.6819] \Rightarrow$$
$$[\,0.2915\,,\,0.3215\,] \le [0.5522\,,\,0.6819]$$

We note that the inequality is achieved, and therefore we accept Nx_1 a Neutrosophical variable that follows the Neutrosophical probability distribution defined as follows:

$$f_N(x) = \begin{cases} \frac{1}{16}x & ;\ 2+\varepsilon \le x \le 6+\varepsilon \\ 0_N & ;\ 0therwise \end{cases}$$

Where $\varepsilon = [0\,,\,3]$.

We continue in the same way until we obtain the required number of Neutrosophic random variables needed for the simulation process.

Conclusion and Results:

From the previous study we note that we do not need the reverse function of the cumulative distribution function of the probable distribution of the system imposed when using the Neutrosophic rejection and acceptance method and therefore it is a suitable method to generate the Neutrosophic random variables needed for the simulation process when obtaining the inverse function of the cumulative distribution function is difficult or not possible and using Neutrosophic variables we get more accurate and appropriate simulation results for changes that can occur during the work of the system to be simulated.

We look forward in the near future to preparing studies in which we use the method of refusal of acceptance to generate random variables following popular and widely used potential distributions in applied fields such as beta distribution and other distributions.

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