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## Secure Dominance in Neutrosophic Graphs

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**Abstract:** Secure dominance is a significant proportion of dominance which plays vital role in communication networks. In this article, we present and analyze an idea of the secured neutrosophic dominance and totally neutrosophic dominance number of neutrosophic graphs primarily based on strong arcs and the properties of both notions are studied. The terms 2-neutrosophic dominance number, 2-secured neutrosophic dominance variety, 2-totally neutrosophic dominance, and 2-secured neutrosophic dominance number also are defined. Some of their theoretical properties are investigated.

**Keywords:** Dominance set, total dominance neutrosophic number, Secure dominance neutrosophic number, neutrosophic total secure dominance number.

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### 1. Introduction

In 1965, L.A. Zadeh [28] gave a definition of the word "fuzzy" that deals with uncertainty and fuzzy relatives. Rosenfeld [22] then observed Zadeh's fuzzy functions on fuzzy batches and developed the idea of fuzzy networks with membership value in  $[0,1]$ . K. T. Atanassov [2] expanded the concept of fuzzy networks to intuitionistic fuzzy networks and presented intuitionistic fuzzy relationships with an additional level of indeterminacy. Bipolar fuzzy network is defined by Akram [1] and subjected to a number of procedures. As an extension of the fuzzy network and the intuitionistic fuzzy network, Florentin Smarandache et al. [24, 26, 27] introduced neutrosophic network and single valued neutrosophic network or graphs (SVNG) as a new version of graph notion. Said Broumi et al [5, 6] created the concept of SVNG and studied its additives.

Dominance is the concept that identifies the key nodes in networks that control the entire communication in the networks. Dominance is broadly studied and implemented in graph idea and its extensions. Secure connectivity is crucial in communication networks because it protects against node failure, which could affect the network's stability. Secure dominance is a subset of dominance in which a node's failure is guarded by its adjacent node, securing the network's entire communication. Ore [21] and Berge [4] pioneered the analysis of dominance batches in graphs. Merouane and Chellali [19] proposed the secured dominance batch and the two- dominance batch. A.Somasundaram and S.Somasundaram [25] produced an idea of dominance in fuzzy networks and

acquired numerous bounds for the dominance number. M.G. Karunambigai et al [13] introduced secure dominance in fuzzy networks. We focused on introducing secured neutrosophic dominance and totally secured neutrosophic dominance in neutrosophic networks, prompted via the idea of dominance number and its applicability [3, 9, 10, 11, 13, 17, 18,23]. Secured dominance is extremely important in many fields, including e-commerce, banking, information transfer, telecommunications, and medical diagnosis.

This paper is structured as follows. Section 2 contains preliminary information, and Section 3 defines a secured neutrosophic dominance number, a totally secure neutrosophic dominance number, and a 2-secured neutrosophic dominance number of a fuzzy network, and their bounds has been formulated. The section 4 concludes the paper.

## 2. Preliminaries

**“Definition 2.1 [12]** A pair  $G=(A,B)$  is known as a single valued neutrosophic graph (SVN-graph) with the underlying set  $V$ .

1. The functions  $T_A:V \rightarrow [0,1]$ ,  $I_A:V \rightarrow [0,1]$ , and  $F_A:V \rightarrow [0,1]$ , denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element  $v_i \in V$ , respectively, and  $0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$  for all  $v_i \in V$ .

2. The functions  $T_B:E \subseteq V \times V \rightarrow [0,1]$ ,  $I_B:E \subseteq V \times V \rightarrow [0,1]$ , and  $F_B:E \subseteq V \times V \rightarrow [0,1]$  are defined by  $T_B(v_i, v_j) \leq T_A(v_i) \wedge T_A(v_j)$ ,  $I_B(v_i, v_j) \geq I_A(v_i) \vee I_A(v_j)$  and  $F_B(v_i, v_j) \geq F_A(v_i) \vee F_A(v_j)$  denotes the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge  $(v_i, v_j) \in E$  respectively, where  $0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3$  for all  $(v_i, v_j) \in E$  ( $i, j = 1, 2, \dots, n$ ). We call  $A$  the single valued neutrosophic vertex set of  $V$ ,  $B$  the single valued neutrosophic edge set of  $E$ , respectively.”

**“Definition 2.2 [6]** A partial SVN-subgraph of SVN-graph  $G = (A, B)$  is a SVN-graph  $H = (V', E')$  such that  $V' \subseteq V$ , where  $T'_A(v_i) \leq T_A(v_i)$ ,  $I'_A(v_i) \geq I_A(v_i)$ , and  $F'_A(v_i) \geq F_A(v_i)$  for all  $v_i \in V$  and  $E' \subseteq E$ , where  $T'_B(v_i, v_j) \leq T_B(v_i, v_j)$ ,  $I'_B(v_i, v_j) \geq I_B(v_i, v_j)$ ,  $F'_B(v_i, v_j) \geq F_B(v_i, v_j)$  for all  $(v_i, v_j) \in E'$ .”

**“Definition 2.3 [6]** Let  $G = (A, B)$  be a SVNG.  $G$  is said to be a strong SVNG if

$$T_B(u, v) = T_A(u) \wedge T_A(v),$$

$$I_B(u, v) = I_A(u) \vee I_A(v) \text{ and}$$

$$F_B(u, v) = F_A(u) \vee F_A(v) \text{ for every } (u, v) \in E.$$

**“Definition 2.4 [8]** Let  $G = (A, B)$  be a SVNG.  $G$  is said to be a complete SVNG if

$$T_B(u, v) = T_A(u) \wedge T_A(v),$$

$$I_B(u, v) = I_A(u) \vee I_A(v) \text{ and}$$

$$F_B(u, v) = F_A(u) \vee F_A(v) \text{ for every } u, v \in V."$$

**“Definition 2.5 [8]** Let  $G = (A, B)$  be a SVNG.  $\bar{G} = (\bar{A}, \bar{B})$  is the complement of an SVNG if

$$\bar{A} = A \text{ and } \bar{B} \text{ is computed as below.}$$

$$\overline{T_B(u, v)} = T_A(u) \wedge T_A(v) - T_B(u, v),$$

$$\overline{I_B}(u, v) = I_A(u) \vee I_A(v) - I_B(u, v)$$

$$\text{and } \overline{F_B}(u, v) = F_A(u) \vee F_A(v) - F_B(u, v) \text{ for every } (u, v) \in E.$$

Here,  $\overline{T_B}(u, v)$ ,  $\overline{I_B}(u, v)$  and  $\overline{F_B}(u, v)$  denote the true, intermediate, and false membership degree for edge  $(u, v)$  of  $\tilde{G}$ ."

**“Definition 2.6 [16]** Let  $G = (A, B)$  be a SVNG on  $V$ , then the neutrosophic vertex cardinality of  $G$  is defined by”

$$|V| = \sum_{(u,v) \in V} \frac{1 + T_A(u, v) + I_A(u, v) - F_A(u, v)}{2}$$

**“Definition 2.7 [16]** Let  $G = (A, B)$  be a SVNG on  $V$ , then the neutrosophic edge cardinality of  $G$  is defined by”

$$|E| = \sum_{(u,v) \in E} \frac{1 + T_B(u, v) + I_B(u, v) - F_B(u, v)}{2} \quad ,,$$

**“Definition 2.8 [20]** An arc  $(u, v)$  of a SVNG  $G$  is called strong arc if

$$T_B(u, v) = T_A(u) \wedge T_A(v),$$

$$I_B(u, v) = I_A(u) \vee I_A(v) \text{ and}$$

$$F_B(u, v) = F_A(u) \vee F_A(v)”$$

**“Definition 2.9[16]** Let  $G = (A, B)$  be a SVNG on  $V$ . Let  $(u, v) \in V$ , we say that  $u$  dominates  $v$  in  $G$ , if there exist a strong arc between them.”

**“Definition 2.10 [16]** Given  $S \subset V$  is called a dominating set in  $G$  if for every vertex  $v \in V - S$  there exists a vertex  $u \in S$  such that  $u$  dominates  $v$ . for all  $e \in A, u, v \in V$ .”

**“Definition 2.11[16]** A dominance set  $S$  of an Neutrosophic soft graph is said to be minimal dominance set if no proper subset of  $S$  is a dominance set. for all  $e \in A, u, v \in V$ .”

**“Definition 2.13 [14]** Let  $G = (V, E)$  be a fuzzy graph. Let  $u, v \in V$  and we say that  $u$  dominates  $v$  in  $G$  if  $\mu(u, v) = \sigma(u) \vee \sigma(v)$ . A subset  $S$  of  $V$  is called dominance set in  $G$  if for every  $v \in V - S$ , there exists  $u \in S$  such that  $u$  dominates  $v$ . The minimum fuzzy cardinality of a dominating set in  $G$  is called the dominance number of  $G$  and is denoted by  $\gamma(G)$ .”

**“Definition 2.14 [15]** Let  $G = (V, E)$  be a fuzzy graph. A dominance set  $S$  of  $V$  is a secure dominance set if for each vertex  $u \in V - S$  is adjacent to a vertex  $v \in S$  such that  $(S - \{v\}) \cup \{u\}$  is dominating set. The secure dominance number of  $G$  is minimum fuzzy cardinality taken over all secure dominance sets of  $G$  and is denoted by  $\gamma_s(G)$ . ”

**“Definition 2.15 [13]** A subset  $S$  of  $V$  is a 2- dominance set in  $G$  if every vertex of  $V - S$  has atleast two neighbour in  $S$ .The 2- dominance number of  $G$  is minimum fuzzy cardinality taken over all 2- dominance sets of  $G$  and is denoted by  $\gamma_2(G)$ .”

**“Theorem 2.16 [13]** Every arc in a complete fuzzy graph is a strong arc.”

Table 1: Some basic notations

Notation	Meaning
$G^* = (V, E)$	Fuzzy Network or Graph
$G = (A, B)$	Neutrosophic Network or Graph
$V$	Vertex batch
$E$	Edge batch
$T_A(v) , I_A(v), F_A(v)$	True, indeterminacy, and falsity membership value of the node $v$ of $G$ .
$T_B(u, v), I_B(u, v), F_B(u, v)$	True, indeterminacy, and falsity membership value of the link $(u, v)$ of $G$ .
$\gamma_{nd}(G)$	Neutrosophic dominance number
$\gamma_{snd}(G)$	Secured neutrosophic dominance number
$\gamma_{tnnd}(G)$	Totally neutrosophic dominance number
$\gamma_{tsnd}(G)$	Totally secure neutrosophic dominance number
$\gamma_{2nd}(G)$	2- neutrosophic dominance number
$\gamma_{2snd}(G)$	2- secured neutrosophic dominance number
$\gamma_{2tnnd}(G)$	2-totally neutrosophic dominance number
$\gamma_{2tsnd}(G)$	2-totally secured neutrosophic dominance number

## 2. Secured Dominance in Neutrosophic Graphs

### Definition 3.1 [16]

Let  $G = (A, B)$  of  $G^* = (V, E)$  be a unique value neutrosophic network. consider a subset  $S$  of  $V$  such that  $u \in S$  dominating  $v$  for every  $v \in V - S$ , then that subset is known to be a neutrosophic dominance batch in  $G$ . The neutrosophic dominance number of  $G$  is given by  $\gamma_{nd}(G)$ . It is the minimal cardinality across all the dominance sets of  $G$ .

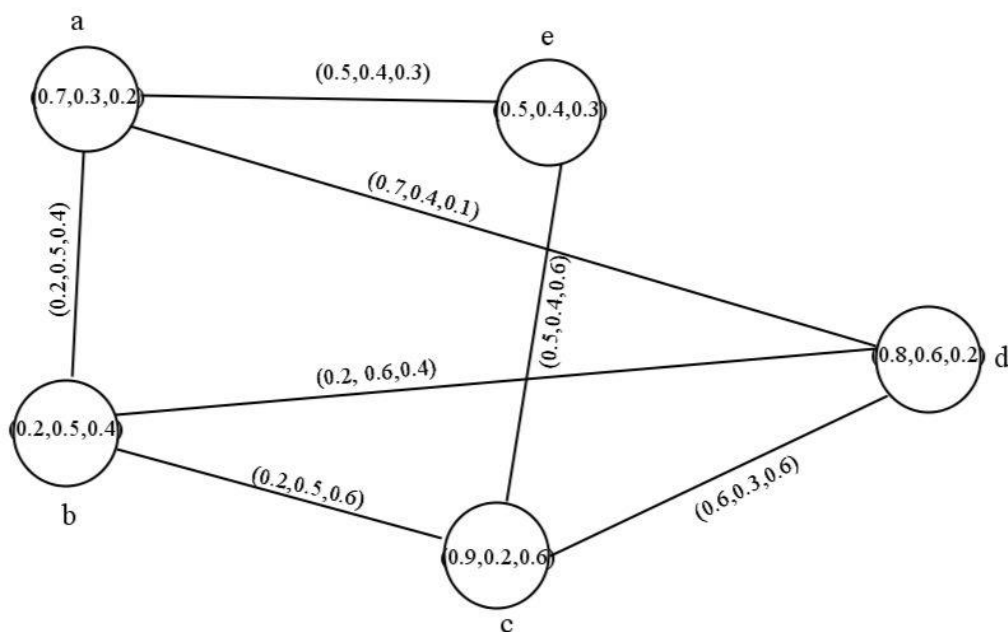


Fig.1 Dominance in a SVNG

Here  $\{a,b\}, \{b,c\}, \{b,e\}, \{a,b,c\}, \{a,b,e\}$  are few dominance batches of  $G$  and  $\gamma_{nd}(G) = 1.4$ .

**Definition 3.2**

Let  $G = (A, B)$  of  $G^*$  be a SVNG. Consider a node  $u \in V - S$  is contiguous to any node  $v \in S$  such that  $(S - \{v\}) \cup \{u\}$  is a batch which is again a dominance batch, then the neutrosophic batch  $S$  of vertex  $V$  is said to be a secured neutrosophic dominance batch. Secured neutrosophic dominance number of  $G$  is the number with the lowest vertex cardinality in all secured neutrosophic dominance batches of  $G$ , and it is represented by the symbol  $\gamma_{snd}(G)$ .

From figure 1,  $\{b,e\}, \{a,b,c\}, \{a,b,e\}$  are secure neutrosophic dominance batches of  $G$  and  $\gamma_{snd}(G) = 1.45$ .

**Definition 3.3**

Let  $G = (A, B)$  of  $G^*$  be a connected SVNG.

Consider a subgraph  $\langle S \rangle$  induced by a batch  $S$  of  $V$  is also connected, then the neutrosophic dominance set  $S$  is known to be a totally neutrosophic dominance batch. Totally neutrosophic dominance number of  $G$  is the number with the lowest vertex cardinality among all total neutrosophic dominance batches of  $G$ , and it is represented by the symbol  $\gamma_{tnd}(G)$ .

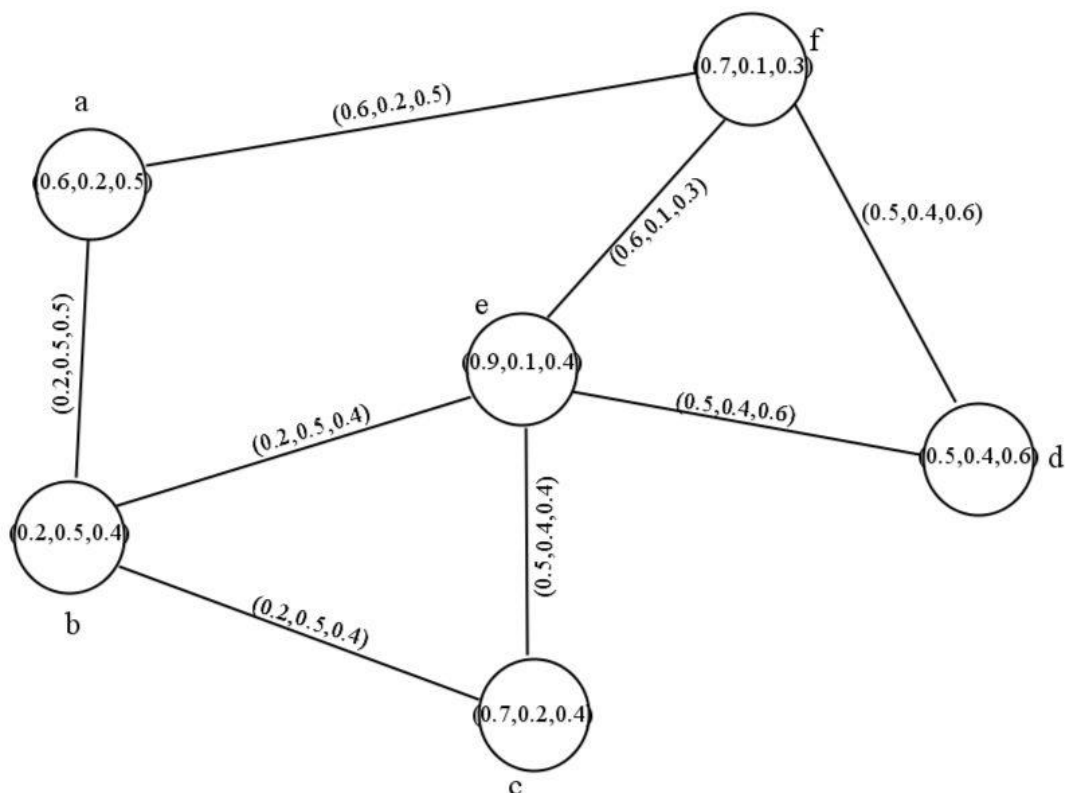


Fig.2 Total neutrosophic dominance in a SVNG

Here  $\{a,b,e\}, \{b,e,d\}, \{a,b,f\}, \{a,b,e,d\}, \{a,b,f,d\}, \{a,d,e,f\}$  are total neutrosophic dominance batches and  $\gamma_{tna}(G) = 2.05$ .

**Definition 3.4**

Consider the connected SVNG  $G = (A, B)$  of  $G^* = (V, E)$ . If the subgraph  $\langle S \rangle$  induced by the neutrosophic set  $S$  of  $V$  is also connected, then the neutrosophic batch  $S$  of  $V$  is known to be a totally secured neutrosophic dominance batch. The term totally secure neutrosophic dominance number of  $G$ , which is abbreviated as  $\gamma_{tsna}(G)$ , refers to the totally secure neutrosophic dominance batch of  $G$  with the least vertex cardinality.

From Figure 2,  $\{a,b,c,e,f\}, \{a,b,c,e,d\}, \{b,c,e,d,f\}, \{a,b,c,d,f\}$  are total secure neutrosophic dominance batches of  $G$  and  $\gamma_{tsna}(G) = 3.45$ .

**Definition 3.5**

Let SVNG be  $G = (A, B)$  of  $G^* = (V, E)$ . If every node of  $V - S$  has at least two neighbours in  $S$ , then the subset  $S$  of  $V$  is a 2-neutrosophic dominance batch in  $G$ . The 2- neutrosophic dominance number of  $G$ , which is shown by the symbol  $\gamma_{2nd}(G)$  is the set of 2- neutrosophic dominance batches with the least vertex cardinality.

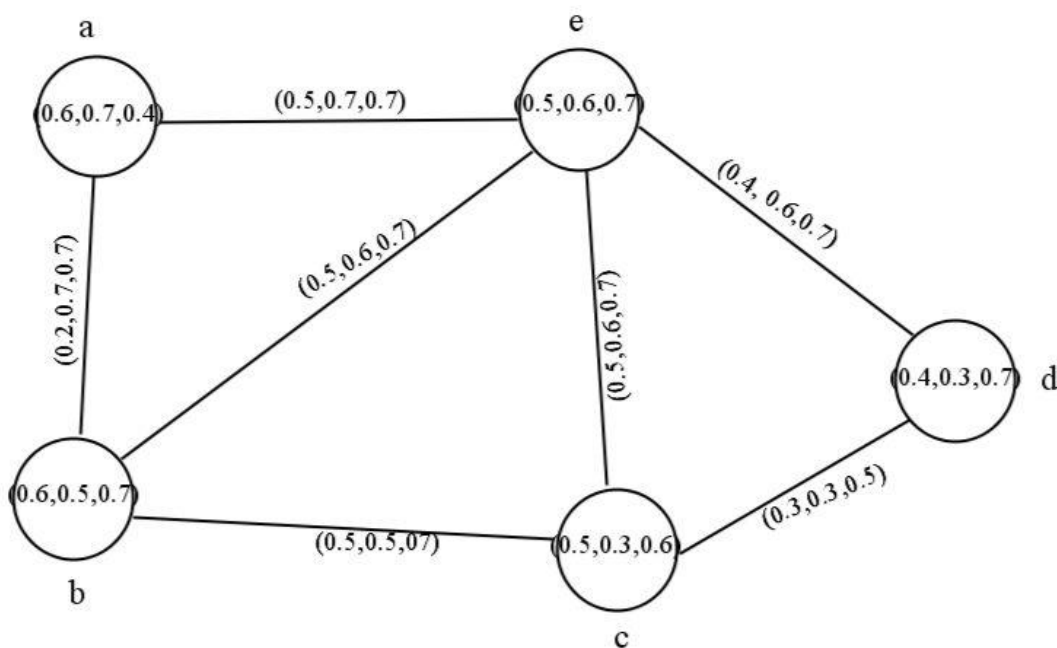


Fig.3 2- neutrosophic dominance in a SVNG

Here {a,b,d},{a,e,d},{a,c,d},{a,b,c,d} are 2- neutrosophic dominance batchs of G and  $\gamma_{2nd}(G) = 2.05$ .

**Definition 3.6**

Let SVNG be  $G = (A, B)$  of  $G^* = (V, E)$ . Consider a node  $u \in V - S$  is neighbouring to other node  $v \in S$  such that  $(S - \{v\}) \cup \{u\}$  is a batch which is again a 2-dominance batch, then the 2-neutrosophic batch S of V is known to be a 2-secured neutrosophic dominance batch. The 2-secured neutrosophic dominance number of G, which is represented by the symbol  $\gamma_{2snd}(G)$ , is the batch of 2-secure neutrosophic dominance batchs of G with the least vertex cardinality.

From Figure 3, {a,b,c,d} are 2- secure neutrosophic dominance batchs of G and  $\gamma_{2snd}(G) = 3.45$ .

**Definition 3.7**

Consider the connected SVNG  $G = (A, B)$  of  $G^* = (V, E)$ . Consider a subgraph  $\langle S \rangle$  induced by a subset S is connected, then the subset S is a 2-total neutrosophic dominance batch. The 2-totally neutrosophic dominance number of G, which is shown by the symbol  $\gamma_{2tna}(G)$ , is the batch of 2-totally neutrosophic dominance batchs of G with the least vertex cardinality.



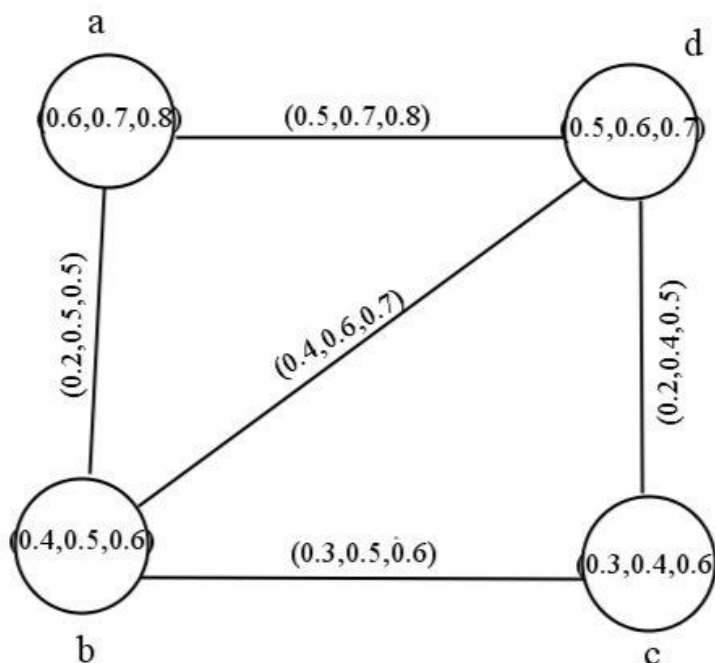


Fig.4 2-Total dominance in a SVNG

Here {a,b,c} and {a,b} are 2- totally neutrosophic dominance batchs and  $\gamma_{2tna}(G) = 1.2$ .

**Definition 3.8**

Consider the connected SVNG  $G = (A, B)$  of  $G^* = (V, E)$ . If the subgraph  $\langle S \rangle$  produced by a neutrosophic set  $S$  of  $V$  is also connected, then the batch  $S$  is a 2-secured dominance and is referred to as a 2-totally secured neutrosophic dominance batch. The 2-totally secured neutrosophic dominance number of  $G$  is represented by the symbol  $\gamma_{2tsna}(G)$  and is the set of 2-totally secured neutrosophic dominance batchs of  $G$  with the least vertex cardinality.

From Figure 4, {a,b,c} are 2-secured total neutrosophic dominance batchs of  $G$  and  $\gamma_{2tsna}(G) = 1.95$ .

**Theorem 3.9** In a complete SVNG  $G$ , each neutrosophic dominance batch is a secure neutrosophic dominance batch.

**Proof:**

If  $G = (A, B)$  is a complete neutrosophic network, then for every  $u, v \in V$  we obtain  $T_B(u, v) = T_A(u) \wedge T_A(v)$ ,  $I_B(u, v) = I_A(u) \vee I_A(v)$  and  $F_B(u, v) = F_A(u) \vee F_A(v)$ .

Here, SVNG  $G = (A, B)$  has only strong arcs.

Any neutrosophic dominance batch defined in  $G = (A, B)$  is given by  $S$ . Currently, any vertex  $v \in V - S$  is next to every vertex of  $S$ , and  $(S - \{v\}) \cup \{u\}$  is a neutrosophic dominance batch for all  $u \in S$ . Hence,  $S$  is a secured neutrosophic dominance batch of  $G$ .

**Theorem 3.10**

For a neutrosophic network  $G = (A, B)$ , any 2-secured neutrosophic dominance batch of  $G$  is a secure neutrosophic dominance batch of  $G$ .

**Proof:**

If  $G = (A, B)$  is a neutrosophic network, then let  $S$  be a 2-secured neutrosophic dominance batch of that network. Every node  $u \in V - S$  is then next to a node  $v \in S$  resulting in  $(S - \{v\}) \cup \{u\}$  being a 2-neutrosophic dominance batch. Since  $G$  is a 2-secured neutrosophic dominance batch,  $G$  is a 2- neutrosophic dominance batch, and every 2- neutrosophic dominance batch is a dominance batch. As a result,  $(S - \{v\}) \cup \{u\}$  is a neutrosophic dominance batch since every node  $u \in V - S$  is close to a node  $v \in S$ . As a result,  $S$  is a secured neutrosophic dominance batch of  $G$ .

**Theorem 3.11**

In a SVNG  $G$ , the complement of a neutrosophic dominance batch of  $G$  is a neutrosophic dominance batch.

**Proof:**

Let  $G$  represent a neutrosophic network.

Any subset  $S$  of  $V$  is considered a neutrosophic dominance batch in  $G$ , according to the notion of dominance, if for every  $v \in V - S$ , there is an  $u \in S$  such that  $u$  dominating  $v$ . If  $\bar{S}$  is the complement of  $S$ 's neutrosophic dominance batch, then  $v \in \bar{S}$  and  $u \in V - \bar{S}$  such that  $v$  dominating  $u$ . i.e., for every  $v \in \bar{S}$  and  $u \in V - \bar{S}$  such that  $v$  dominating  $u$  in  $\bar{G}$ . Therefore,  $\bar{S}$  is a neutrosophic dominance batch of  $G$ .

**Theorem 3.12**

In a SVNG  $G$ , the complement of any 2- neutrosophic dominance batch is a 2- neutrosophic dominance batch of  $G$ .

**Proof:**

Let  $G$  represent a neutrosophic network.

According to Theorem 3.11, if  $\bar{S}$  is the complement of  $S$ 's neutrosophic dominance batch, then  $\bar{S}$  is also one of  $G$ 's neutrosophic dominance batches. Every 2-neutrosophic dominance batch of  $G$  is a neutrosophic dominance batch of  $G$ . Therefore, any 2-neutrosophic dominance batch in a SVNG has a complement that is a neutrosophic dominance batch of  $G$ .

**Theorem 3.13**

For any SVNG  $G$ ,  $\gamma_{nd}(G) \leq \gamma_{snd}(G) \leq \gamma_{2snd}(G) \leq \gamma_{2nd}(G)$ .

**Proof:**

Consider  $S$  is a least neutrosophic dominance batch of neutrosophic network  $G$  and  $\gamma_{nd}(G) = k$ . If  $S$  is also a least secure neutrosophic dominance batch of neutrosophic graph  $G$  then  $\gamma_{snd}(G) = k$ . i.e.,  $\gamma_{nd}(G) \leq \gamma_{snd}(G)$ . Suppose  $S$  is not a least secured neutrosophic dominance batch and if  $S'$  is a least secured neutrosophic dominance batch then  $\gamma_{snd}(G) > k$ .

Thus,  $\gamma_{nd}(G) \leq \gamma_{snd}(G)$ . (1)

Let  $D$  be a least 2- neutrosophic dominance batch of neutrosophic graph  $G$  and  $\gamma_{2nd}(G) = l$ . If  $D$  is also a least secure 2- neutrosophic dominance batch of neutrosophic graph  $G$  then  $\gamma_{2snd}(G) =$

$l$ . *i.e.*,  $\gamma_{2snd}(G) \leq \gamma_{2nd}(G)$ . Suppose  $D$  is not a least 2-secured neutrosophic dominance batch and if  $D'$  is a least secured 2- neutrosophic dominance batch then  $\gamma_{2snd}(G) > l$ .

Thus,  $\gamma_{2snd}(G) \leq \gamma_{2nd}(G)$ . (2)

Let  $Q$  be a least secured neutrosophic dominance batch of neutrosophic graph  $G$  and  $\gamma_{snd}(G) = n$ . Every 2-secured neutrosophic dominance batch is a secured neutrosophic dominance batch of  $G$ . If  $Q$  is also a least secured 2- dominance batch of neutrosophic graph  $G$  then  $\gamma_{2snd}(G) = n$ . *i.e.*,  $\gamma_{snd}(G) \leq \gamma_{2snd}(G)$ . Suppose  $Q$  is not a least 2-secured neutrosophic dominance batch and if  $Q'$  is a least secured 2- neutrosophic dominance batch then  $\gamma_{2snd}(G) > n$ . Thus,  $\gamma_{snd}(G) \leq \gamma_{2snd}(G)$ . (3)

From (1), (2), (3) we get,  $\gamma_{nd}(G) \leq \gamma_{snd}(G) \leq \gamma_{2snd}(G) \leq \gamma_{2nd}(G)$ .

### Theorem 3.14

For any SVNG  $G$ ,  $\gamma_{tnd}(G) \leq \gamma_{tsnd}(G) \leq \gamma_{2tsnd}(G) \leq \gamma_{2tnd}(G)$ .

#### Proof:

Consider  $S$  is a least totally neutrosophic dominance batch of neutrosophic network  $G$  and  $\gamma_{tnd}(G) = k$ . If  $S$  is also a least totally secured neutrosophic dominance batch of neutrosophic graph  $G$  then  $\gamma_{tsnd}(G) = k$ . *i.e.*,  $\gamma_{tnd}(G) \leq \gamma_{tsnd}(G)$ . Suppose  $S$  fails to be a least secured totally neutrosophic dominance batch and consider  $S'$  is a least totally secured neutrosophic dominance batch then  $\gamma_{tsnd}(G) > k$ .

Thus,  $\gamma_{tnd}(G) \leq \gamma_{tsnd}(G)$ . (1)

Consider  $D$  is a least 2-totally neutrosophic dominance batch of neutrosophic graph  $G$  and  $\gamma_{2tnd}(G) = l$ . If  $D$  is also a least 2- totally secured neutrosophic dominance batch of neutrosophic graph  $G$  then  $\gamma_{2tsnd}(G) = l$ . *i.e.*,  $\gamma_{2tsnd}(G) \leq \gamma_{2tnd}(G)$ . Suppose  $D$  is not a least 2-totally secured neutrosophic dominance batch and if  $D'$  is a least 2- totally secured neutrosophic dominance batch then  $\gamma_{2tsnd}(G) > l$ .

Thus,  $\gamma_{2tsnd}(G) \leq \gamma_{2tnd}(G)$ . (2)

Let  $Q$  be a least totally secured neutrosophic dominance batch of neutrosophic graph  $G$  and  $\gamma_{tsnd}(G) = n$ . Every 2- totally secured neutrosophic dominance batch is a totally secure neutrosophic dominance batch of  $G$ . If  $Q$  is also a least 2- totally secure neutrosophic dominance batch of neutrosophic graph  $G$  then  $\gamma_{2tsnd}(G) = n$ . *i.e.*,  $\gamma_{tsnd}(G) \leq \gamma_{2tsnd}(G)$ . Suppose  $Q$  is not a least 2-totally secured neutrosophic dominance batch and if  $Q'$  is a least 2-total secured neutrosophic dominance batch then  $\gamma_{2tsnd}(G) > n$ .

Thus,  $\gamma_{tsnd}(G) \leq \gamma_{2tsnd}(G)$ . (3)

From (1), (2), (3) we get,  $\gamma_{tnd}(G) \leq \gamma_{tsnd}(G) \leq \gamma_{2tsnd}(G) \leq \gamma_{2tnd}(G)$ .

## 4. Conclusion

Neutrosophic network theory is being extensively used in a wide range of scientific and technological domains, including cognitive field, genetic methods, optimisation techniques, clustering, medical

treatments, and decision trees. A neutrosophic network is initiated by Florentin Smarandache from neutrosophic groups. When compared to other generic and fuzzy analogs, neutrosophic analogs give the system greater accuracy, adaptable, and capable. However, the independence of indeterminacy-membership occasionally allows real data to be unbounded. The idea of secure neutrosophic network dominance is developed in this work, and we intend to keep developing the application that would secure social network connectivity in the neutrosophic environment.

### **“Compliance with Ethical Standards**

#### **Conflict of Interest**

The authors declare that they do not have any financial or associative interest indicating a conflict of interest in about submitted work.”

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