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Application Of Some Topological Indices In Nover Topologized Graphs

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Abstract: In this paper Nover top Z_1 and Z_2 index and Nover top H – index, Rd – index, GA – index, CON – index are investigated and some of the related theorems are discussed. These indices are also calculated for some specific types of Nover top graphs such as 2 – regular , K_2 and $K_{2,2}$ Nover Top graphs.

Keywords: Nover top Z_1 and Z_2 index , H – index, Rd – index, GA – index, CON – index

1 Introduction

There are many applications of graph theory to a wide variety of subjects which include operation Research, Physics, chemistry, Economics, Genetics, Engineering, computer Science etc., In a classical graph for each vertex or edge there are two possibilities arises that is either in the graph or not in the graph and this will not classical graph model for uncertain problems. Fuzzy set [14] is a generalized version of the classical set in which objects have different membership degrees between zero and one. More work has already been done on fuzzy graphs. Zadeh introduced the degree of membership/Truth(T) in 1965 and defined the fuzzy set. Atanassov [2] introduced the degree of non-memebership/Falsehood(F) in 1983 and defined the intuitionistic fuzzy set. Smarandache [13,14,15,16,18,19] introduced the degree of Indeterminacy/Neutrality(I) as an independent component in 1995 and defined the neutrosophic set on there compenents (T, I, F). Smarandache has introduced in 2020 the n-SuperHyperGraph, with super-vertices [that are groups of vertices] and hyper-edges defined on power-set of power-set... that is the most general form of graph as today, and n-HyperAlgebra. A SuperHyperGraph, is a HyperGraph (where a group of Edges form a HyperEdge) such that a group of vertices are united all together into a SuperVertex like a group of people (=vertices) that are united all together into an organization (=SuperVertex) ;and further on the n-SuperHyperGraph where many groups (=SuperVertices) are united all together to form a group-of-groups (called 2-SuperVertex, or Type-2 SuperVertex), then a group of Type-2 SuperVertices forms a Type-3 SuperVertex, . . . , and so on up to Type-n SuperVertex, for any $n \geq 1$, which better reflects our reality. Later Narmada Devi[5,6,7,8,9,10] worked on new type of neutrosophic over,off graph and minimal domination via neutrosophic over graph and neutrosophic over topologized graph. [20,21,23] A lot of topological indices are available in chemical-graph theory and H. Wiener proposed the first index to estimate the boiling point of alkanes called ‘Wiener index’. Many topological indices exist only in the crisp but it’s new to the Nover graph environment. The main aim of this paper is to define the topological indices in Nover graphs. The various topology indices such as Zagreb index, Randic index, Geometric-arithmetic, Harmonic are

discussed them. Neutrosophic over graphs in addition to the degree of accuracy of each membership function, the degree of its membership is uncertain, as well as its inaccuracy. so in many cases, it may be more logical to use this model than graphs in real-world problems. Since that neutrosophic over graphs are more efficient than fuzzy graphs for modelling real problems. In this paper , we try to calculate some Neutrosophic over topological indices for this type of graphs.

2 Preliminaries

Definition 2.1. [4] A set \mathcal{D} of vertices of \mathcal{G} is said to be a *topologized domination set* \mathcal{D} if \mathcal{G} is a *topologized graph* and every vertex in $\mathcal{V} - \mathcal{D}$ is adjacent to atleast one vertex of in \mathcal{D} .

Definition 2.2. [6,7,9] A Neutrosophic Over set \mathcal{D} is defined as

$\mathcal{D} = (x, < (T(x), I(x), F(x) >), x \in X$ such that there exist some element in \mathcal{D} that have atleast one neutrosophic component that is > 1 and no element has neutrosophic component that are < 0 and $((T(x), I(x), F(x)) \in [0, \Omega]$ where Ω is called Overlimit such that $0 < 1 < \Omega$.

Definition 2.3. [5,6,7] A Nover graph is a pair $\mathcal{G} = (A, B)$ of a crisp graph $\mathcal{G}^* = (\mathcal{V}, \mathcal{E})$ where A is Nvertex over set on \mathcal{V} and B is a Nedge over set on \mathcal{E} such that $\mathcal{T}_{\mathcal{B}}(xy) \leq (T_A(x) \wedge T_A(y)), I_B(xy) \leq (I_A(x) \wedge I_A(y)), F_B(xy) \geq (F_A(x) \vee F_A(y))$.

Definition 2.4. [9,10] A topologized graph is a topological space \mathcal{H} such that

- (i) every singleton is open or closed
- (ii) $\forall h \in \mathcal{H}, |\partial(h)| \leq 2$, since $\partial(h)$ is denoted by the boundary of a point h .

Definition 2.5. [9] A Nover graph $\mathcal{G} = (A, B)$ is called *NOver Top graph* if \mathcal{G}^* satisfy the following condition

- (i) every singleton is open or closed in \mathcal{V} .
- (ii) $\forall f \in \mathcal{F}, |\partial(f)| \leq 2$ where $\partial(f)$ is denoted by the boundary of a point x

Definition 2.6. [9] Let \mathcal{G} be a Nover top graph. Let $x, y \in \mathcal{V}$. Then x dominate y in \mathcal{G} if edge xy is effective edge $T_B(xy) = (T_A(x) \wedge T_A(y)), I_B(xy) = (I_A(x) \wedge I_A(y)), F_B(xy) = (F_A(x) \vee F_A(y))$.

A subset $\mathcal{D}_{\mathcal{N}}$ of \mathcal{V} is called a Nover top dominating set in \mathcal{G} if every vertex $\mathcal{V} \notin \mathcal{D}_{\mathcal{N}}$ there exists $u \in \mathcal{D}_{\mathcal{N}}$ such that u dominates \mathcal{V} .

3 Z_1 and Z_2 index in Nover top graphs

Definition 3.1. Let $\mathcal{G} = (A, B)$ be the Nover top graph with non-empty vertex set. The Z_1 index is denoted by $\mathcal{B}_{\text{Nov}}(\mathcal{G})$ and defined as

$$\mathcal{B}_{\text{Nov}}(\mathcal{G}) = \sum_{i=1}^n (T_A(u_i), I_A(u_i), F_A(u_i)) d_2(u_i), \forall u_i \in \mathcal{V}$$

Definition 3.2. The Z_2 index is denoted by $\mathcal{B}_{\text{Nov}}^*(\mathcal{G})$ and defined as

$$\mathcal{B}_{\text{Nov}}^*(\mathcal{G}) = \frac{1}{2} \sum_{i=1}^n [(T_A(u_i), I_A(u_i), F_A(u_i))d(u_i)] [(T_A(v_j), I_A(v_j), F_A(u_i))d(v_j)],$$

$\forall i \neq j$, and $(u_i, v_j) \in \mathcal{E}$

Example 3.1. Consider the Nover top graph $\mathcal{G} = (A, B)$ as shown in Figure 1.

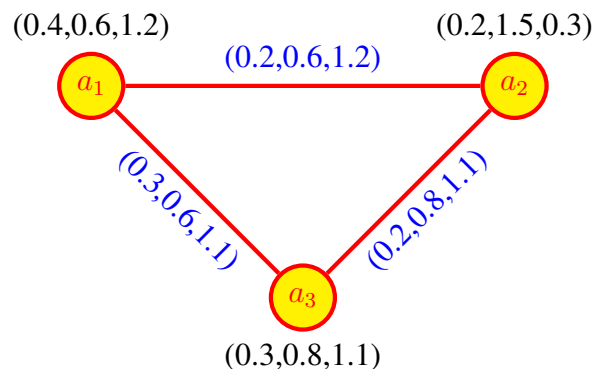


Figure 1: A Nover top graph \mathcal{G}

Let a_1, a_2 and a_3 denote the vertices and $(0.2, 0.6, 1.2), (0.2, 0.8, 1.1), (0.3, 0.6, 1.1)$ denote the edges which are labelled $f_u(0.2, 0.6, 1.2) = (a_1, a_2), f_u(0.2, 0.8, 1.1) = (a_2, a_3), f_u(0.3, 0.6, 1.1) = (a_1, a_3)$.

Let $\mathcal{X} = \{a_1, a_2, a_3, (0.2, 0.6, 1.2), (0.2, 0.8, 1.1), (0.3, 0.6, 1.1)\}$ be a topological space defined by the topology

$$\tau = \{\emptyset, \mathcal{X}, \{a_1\}, \{a_2\}, \{a_3\}, \{a_1, a_2\}, \{a_1, a_3\}, \{a_2, a_3\}\}$$

Here for every $x \in \mathcal{X}$, $\{x\}$ is open.

By the definition of Nover top graph, we have $|\partial(A)| \leq 2$ and $\partial(a_1) = \{a_2, a_3\}, \partial(a_2) = \{a_1, a_3\}, \partial(a_3) = \{a_1, a_2\}$ with $\partial(a_i) = 2$. Hence this graph is Nover top graph.

The Z_1 index is

$$\begin{aligned} d(a_1) &= (.2 + .3, .6 + .6, 1.2 + 1.1) = (0.5, 1.2, 2.3) \\ d(a_2) &= (.2 + .2, .6 + .8, 1.2 + 1.1) = (0.4, 1.4, 2.3) \\ d(a_3) &= (.2 + .3, .8 + .6, 1.1 + 1.1) = (0.5, 1.4, 2.2) \end{aligned}$$

Now, we have

$$\begin{aligned} d_2(a_1) &= (0.04 + 0.09, 0.36 + 0.36, 1.44 + 1.21) = (0.13, 0.69, 2.65) \\ d_2(a_2) &= (0.04 + 0.04, 0.36 + 0.64, 1.44 + 1.21) = (0.08, 1, 2.65) \\ d_2(a_3) &= (0.04 + 0.09, 0.64 + 0.36, 1.21 + 1.21) = (0.13, 1, 1.42) \end{aligned}$$

$$\mathcal{B}_{\text{Nov}}(\mathcal{G}) = \sum_{i=1}^n (T_A(u_i), I_A(u_i), F_A(u_i))d_2(u_i)$$

$$\begin{aligned}
&= (.4, .6, 1.2)(0.13, 0.69, 2.65) + (.2, 1.5, 0.3)(0.08, 1, 2.65) + (.3, .8, 1.1)(0.13, 1, 1.42) \\
&= (.052 + 0.414 + 3.18) + (.016 + 1.5 + 0.795) + (0.039 + .8 + 1.562) \\
&= 8.358
\end{aligned}$$

Example 3.2. Let \mathcal{G} be a same Nover top graph as defined in example 3.1 Then Z_2 index is

$$\begin{aligned}
\mathcal{B}_{\text{Nov}}^*(\mathcal{G}) &= \frac{1}{2}[(0.4, 0.6, 1.2)(0.5, 1.2, 2.3) \times (0.2, 1.5, 0.3)(0.4, 1.4, 2.3) + \\
&\quad (0.4, 0.6, 1.2)(0.5, 1.2, 2.3) \times (0.3, 0.8, 1.1)(0.5, 1.4, 2.2) + \\
&\quad (0.2, 1.5, 0.3)(0.4, 1.4, 2.3) \times (0.3, 0.8, 1.1)(0.5, 1.4, 2.2)] \\
&= \frac{1}{2}[(0.2 + 0.72 + 2.76) \times (0.08 + 2.1 + 0.69) + \\
&\quad (0.2 + 0.72 + 2.76) \times (0.15 + 1.12 + 2.42) + \\
&\quad (0.08 + 2.1 + 0.69) \times (0.15 + 1.12 + 2.42)] \\
&= \frac{1}{2}[3.68 \times 2.87 + 3.68 \times 3.69 + 2.87 \times 3.69] \\
&= \frac{1}{2}[10.5616 + 13.5792 + 10.5903] \\
&= \frac{1}{2}[34.7311] = 17.3656
\end{aligned}$$

Example 3.3. Consider the Nover top graph $\mathcal{G} = (A, B)$ as shown in Fig. 2

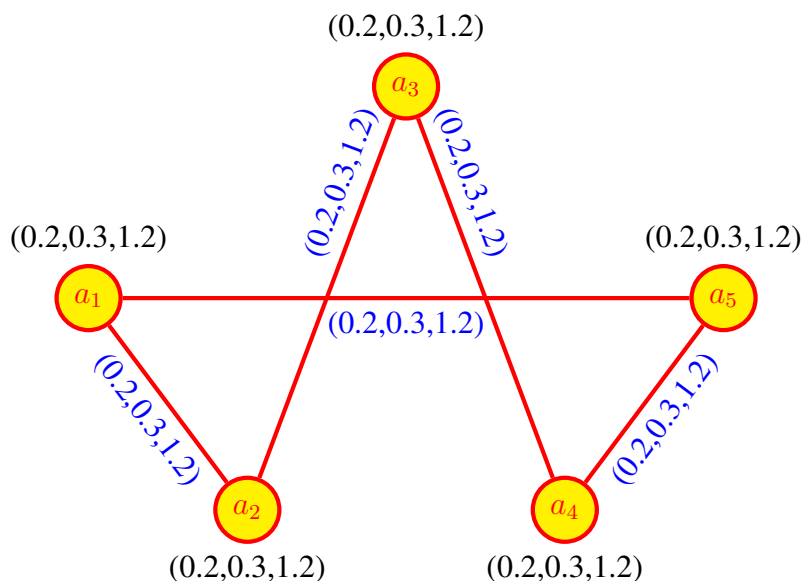


Figure 2: Storng Nover top graph \mathcal{G}

Let a_1, a_2, a_3, a_4 and a_5 denote the vertices and $(0.2, 0.3, 1.2), (0.2, 0.3, 1.2), (0.2, 0.3, 1.2), (0.2, 0.3, 1.2)$ and $(0.2, 0.3, 1.2)$ denote the edges which are labelled $f_u(0.2, 0.3, 1.2) = (a_1, a_2)$, $f_u(0.2, 0.3, 1.2) = (a_2, a_3)$, $f_u(0.2, 0.3, 1.2) = (a_3, a_4)$, $f_u(0.2, 0.3, 1.2) = (a_4, a_5)$, $f_u(0.2, 0.3, 1.2) = (a_1, a_5)$.

Let $\mathcal{X} = \{a_1, a_2, a_3, a_4, a_5, (0.2, 0.3, 1.2), (0.2, 0.3, 1.2), (0.2, 0.3, 1.2), (0.2, 0.3, 1.2), (0.2, 0.3, 1.2)\}$ be a topological space defined by the topology

$$\tau = \left\{ \emptyset, \mathcal{X}, \{a_1\}, \{a_2, a_3\}, \{a_4\}, \{a_5\}, \{a_1, a_2, a_3\}, \{a_1, a_4\}, \{a_1, a_5\}, \{a_1, a_2, a_3, a_4\}, \{a_1, a_2, a_3, a_5\}, \right. \\ \left. \{a_2, a_3, a_4\}, \{a_2, a_3, a_5\}, \{a_2, a_3, a_4, a_5\}, \{a_4, a_5\}, \{a_1, a_4, a_5\}, \{a_1, a_2, a_4, a_5\}, \{a_1, a_3, a_4, a_5\} \right\}$$

Here for every $x \in \mathcal{X}$, $\{x\}$ is open or closed.

By the definition of Nover top graph, we have $|\partial(A)| \leq 2$ and $\partial(a_1) = \{a_2, a_5\}$, $\partial(a_2) = \{a_1, a_3\}$, $\partial(a_3) = \{a_2, a_4\}$, $\partial(a_4) = \{a_3, a_5\}$, $\partial(a_5) = \{a_1, a_4\}$ with $\partial(a_i) = 2$. Hence this graph is Nover top graph.

The Z_1 index is

$$\begin{aligned} d(a_1) &= (0.2 + 0.3, 0.3 + 0.3, 1.2 + 1.2) = (0.4, 0.6, 2.4) \\ d(a_2) &= (0.4, 0.6, 2.4) \\ d(a_3) &= (0.4, 0.6, 2.4) \\ d(a_4) &= (0.4, 0.6, 2.4) \\ d(a_5) &= (0.4, 0.6, 2.4) \end{aligned}$$

Now, we have

$$\begin{aligned} d_2(a_1) &= (0.04 + 0.04, 0.36 + 0.36, 1.44 + 1.44) = (0.08, 0.72, 2.88) \\ d_2(a_2) &= (0.08, 0.72, 2.88) \\ d_2(a_3) &= (0.08, 0.72, 2.88) \\ d_2(a_4) &= (0.08, 0.72, 2.88) \\ d_2(a_5) &= (0.08, 0.72, 2.88) \end{aligned}$$

$$\begin{aligned} \mathcal{B}_{\text{Nov}}(\mathcal{G}) &= \sum_{i=1}^5 (T_A(u_i), I_A(u_i), F_A(u_i)) d_2(u_i) \\ &= (.2, .3, 1.2)(0.08, 0.72, 2.88) + (.2, .3, 1.2)(0.08, 0.72, 2.88) \\ &\quad + (.2, .3, 1.2)(0.08, 0.72, 2.88) + (.2, .3, 1.2)(0.08, 0.72, 2.88) + (.2, .3, 1.2)(0.08, 0.72, 2.88) \\ &= (.016 + 0.216 + 3.456) + (.016 + 0.216 + 3.456) + (.016 + 0.216 + 3.456) \\ &\quad + (.016 + 0.216 + 3.456) + (.016 + 0.216 + 3.456) \\ &= 3.688 + 3.688 + 3.688 + 3.688 + 3.688 \\ &= 18.44 \end{aligned}$$

$$\begin{aligned} \mathcal{B}_{\text{Nov}}^*(\mathcal{G}) &= \frac{1}{2} [(.2, .3, 1.2)(0.4, 0.6, 2.4) \times (.2, .3, 1.2)(0.4, 0.6, 2.4) \\ &\quad + (.2, .3, 1.2)(0.4, 0.6, 2.4) \times (.2, .3, 1.2)(0.4, 0.6, 2.4) \\ &\quad + (.2, .3, 1.2)(0.4, 0.6, 2.4) \times (.2, .3, 1.2)(0.4, 0.6, 2.4) \\ &\quad + (.2, .3, 1.2)(0.4, 0.6, 2.4) \times (.2, .3, 1.2)(0.4, 0.6, 2.4)] \end{aligned}$$

$$\begin{aligned}
& + (.2, .3, 1.2)(0.4, 0.6, 2.4) \times (.2, .3, 1.2)(0.4, 0.6, 2.4)] \\
& = \frac{1}{2}[(0.08 + 0.18 + 2.88) \times (0.08 + 0.18 + 2.88) \\
& \quad + (0.08 + 0.18 + 2.88) \times (0.08 + 0.18 + 2.88) \\
& \quad + (0.08 + 0.18 + 2.88) \times (0.08 + 0.18 + 2.88) \\
& \quad + (0.08 + 0.18 + 2.88) \times (0.08 + 0.18 + 2.88) \\
& \quad + (0.08 + 0.18 + 2.88) \times (0.08 + 0.18 + 2.88)] \\
& = \frac{1}{2}[3.14 \times 3.14 + 3.14 \times 3.14 + 3.14 \times 3.14 + 3.14 \times 3.14 + 3.14 \times 3.14] \\
& = \frac{1}{2}[9.8596 + 9.8596 + 9.8596 + 9.8596 + 9.8596] \\
& = \frac{1}{2}[49.298] = 24.649
\end{aligned}$$

Definition 3.3. Let $\mathcal{G} = (A, B)$ be an neutrosophic over top graph. \mathcal{G} is a regular strong neutrosophic over top graph if it satisfies the following conditions.

$$T_B(a, b) = \min(T_A(a), T_A(b)), I_B(a, b) = \min(I_A(a), I_A(b)), F_B(a, b) = \max(F_A(a), F_A(b))$$

Theorem 3.1. Let \mathcal{G} be the regular Nover top graph. Then, we have

$$\mathcal{B}_{\text{Nov}}(\mathcal{G}) = c^2 \times \sum_{i=1}^n [T_A(u_i) + I_A(u_i)] + c_1^2 \times \sum_{i=1}^n F_A(u_i), \forall u_i \in \mathcal{V}$$

where $\sum_{v \neq u} T_B(v, u) = c$, $\sum_{v \neq u} I_B(v, u) = c$, $\sum_{v \neq u} F_B(v, u) = c_1$.

Proof:

Given the degree of definition of each vertex

$$\begin{aligned}
d(v) &= (d_T(v), d_I(v), d_F(v)) \\
&= \left(\sum_{\substack{v \in \mathcal{V} \\ v \neq u}} T_B(v, u), \sum_{\substack{v \in \mathcal{V} \\ v \neq u}} I_B(v, u), \sum_{\substack{v \in \mathcal{V} \\ v \neq u}} F_B(v, u) \right)
\end{aligned}$$

On the other hand, for regular Nover top graphs, we know that

$$\sum_{v \neq u} T_B(v, u) = c, \sum_{v \neq u} I_B(v, u) = c, \sum_{v \neq u} F_B(v, u) = c_1$$

Therefore,

$$\begin{aligned}
d(v) &= (d_T(v), d_I(v), d_F(v)) \\
&= (c, c, c_1)
\end{aligned}$$

Now, by embedding the formula in the Z_1 index, we will get the desired result. The proof is complete.

Theorem 3.2. Let \mathcal{G} be the regular Nover top graph. Then, we have

$$\mathcal{B}_{\text{Nov}}^*(\mathcal{G}) = \frac{1}{2}c^2 \sum_{i=1}^n [T_A(u_i) + I_A(u_i)][T_A(v_j) + I_A(v_j)] + \frac{1}{2}c_1^2 \sum_{i=1}^n [F_A(u_i)F_A(v_j)],$$

$\forall i \neq j$ and $(u_i, v_j) \in \mathcal{E}$

where $\sum_{v \neq u} T_B(u, v) = c$, $\sum_{v \neq u} I_B(u, v) = c$, $\sum_{v \neq u} F_B(u, v) = c_1$.

Proof: Assume \mathcal{G} is regular Nover top graph, using the Z_2 index formula for \mathcal{G} , we have $\forall i \neq j$ and $(u_i, v_j) \in \mathcal{E}$.

$$\begin{aligned} \mathcal{B}_{\text{Nov}}^*(\mathcal{G}) &= \frac{1}{2} \sum_{i=1}^n [(T_A(u_i), I_A(u_i), I_A(u_i))d(u_i)] [(T_A(v_j), I_A(v_j), F_A(v_j))d(v_j)] \\ &= \frac{1}{2} \sum [(T_A(u_i), I_A(u_i), I_A(u_i))d(d_T(u_i), d_I(u_i), d_F(u_i))] \\ &\quad [(T_A(v_j), I_A(v_j), F_A(v_j))d(d_T(v_j), d_I(v_j), d_F(v_j))] \\ &= \frac{1}{2} \sum [(T_A(u_i), I_A(u_i), I_A(u_i))(c, c, c_1)] [(T_A(v_j), I_A(v_j), F_A(v_j))(c, c, c_1)] \\ &= \frac{1}{2} \sum [cT_A(u_i) + cI_A(u_i) + c_1I_A(u_i)] \times [cT_A(v_j) + cI_A(v_j) + c_1F_A(v_j)] \\ &= \frac{1}{2} \sum c [T_A(u_i) + I_A(u_i)] c_1 [I_A(u_i)] c [T_A(v_j) + I_A(v_j)] c_1 [F_A(v_j)] \\ &= \frac{1}{2}c^2 \sum [T_A(u_i) + I_A(u_i)] [T_A(v_j) + I_A(v_j)] + \frac{1}{2}c_1^2 \sum [I_A(u_i)F_A(v_j)] \end{aligned}$$

The desired result was obtained.

These above two theorems are illustrated the following example.

Example 3.4. Consider the Nover top graph $G = (A, B)$ as shown in Fig

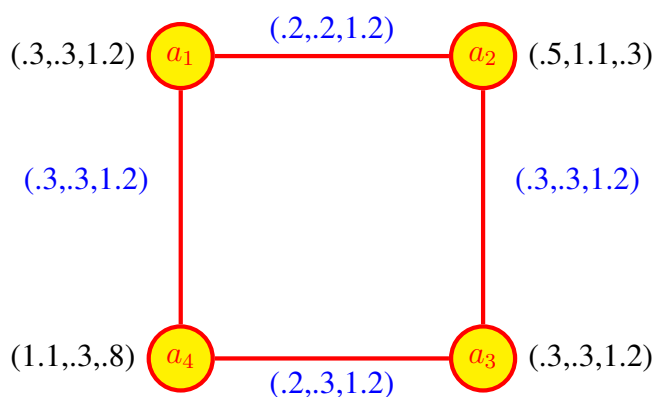


Figure 3: Regular Nover top graph

Let a_1, a_2, a_3 and a_4 denote the vertices and $(.2, .2, 1.2)$, $(.3, .3, 1.2)$, $(.2, .2, 1.2)$ and $(.3, .3, 1.2)$ denote the edges which are labelled $f_u(.2, .2, 1.2) = (a_1, a_2)$, $f_u(.3, .3, 1.2) = (a_2, a_3)$, $f_u(.2, .2, 1.2) = (a_3, a_4)$, $f_u(.3, .3, 1.2) = (a_4, a_1)$.

Let $X = \{a_1, a_2, a_3, a_4, (.2, .2, 1.2), (.3, .3, 1.2), (.2, .2, 1.2), (.3, .3, 1.2)\}$ be a topological space defined by the topology

$$\tau = \left\{ \emptyset, \mathcal{X}, \{a_1\}, \{a_2\}, \{a_3\}, \{a_4\}, \{a_1, a_2\}, \{a_1, a_4\}, \{a_2, a_3\}, \{a_2, a_4\}, \{a_3, a_4\}, \{a_1, a_2, a_3\}, \right. \\ \left. \{a_1, a_2, a_4\}, \{a_2, a_3, a_4\}, \{a_1, a_3, a_4\} \right\}$$

Here for every $x \in \mathcal{X}$, $\{x\}$ is open.

By the definition of Nover top graph, we have $|\partial(A)| \leq 2$ and $\partial(a_1) = \{a_2, a_4\}$, $\partial(a_2) = \{a_1, a_3\}$, $\partial(a_3) = \{a_2, a_4\}$, $\partial(a_4) = \{a_1, a_3\}$ with $\partial(a_i) = 2$. Hence this graph is Nover top graph.

The Z_1 index is

$$d(a_1) = (0.5, 0.5, 2.4)$$

$$d(a_2) = (0.5, 0.5, 2.4)$$

$$d(a_3) = (0.5, 0.5, 2.4)$$

$$d(a_4) = (0.5, 0.5, 2.4)$$

$$\begin{aligned} \mathcal{B}_{\text{Nov}}(\mathcal{G}) &= c^2 \sum_{i=1}^n [T_A(u_i) + I_A(u_i)] + c_1^2 \sum_{i=1}^n [I_A(u_i)] \\ &= (.5)^2[(.3 + .3) + (.5 + 1.1) + (.3 + .3) + (1.1 + .3)] + (2.4)^2[1.2 + .3 + 1.2 + .8] \\ &= (.5)^2[.6 + 1.6 + .6 + 1.4] + (2.4)^2[3.5] \\ &= (.5)^2 \times 4.2 + (2.4)^2 \times 3.5 \\ &= 1.05 + 20.16 \\ &= 21.21 \end{aligned}$$

The Z_2 index is

$$\begin{aligned} \mathcal{B}^*_{\text{Nov}}(\mathcal{G}) &= \frac{1}{2}(.5)^2[.6 \times 1.6 + .6 \times 1.4 + 1.6 \times .6 + .6 \times 1.4] + \\ &\quad \frac{1}{2}(2.4)^2[1.2 \times .3 + 1.2 \times .8 + .3 \times 1.2 + 1.2 \times .8] \\ &= \frac{1}{2}(.5)^2(3.6) + \frac{1}{2}(2.4)^2(2.64) \\ &= \frac{1}{2} \times 0.9 + \frac{1}{2} \times 15.2064 \\ &= 0.45 + 7.6034 \\ &= 8.0532 \end{aligned}$$

4 H-index and Rd-index in Nover top graphs

Definition 4.1. The H-index of Nover top graph \mathcal{G} is defined as

$$H_{\text{Nov}}(\mathcal{G}) = \sum \frac{1}{[A(u_i) \cdot d(u_i)][A(v_j) \cdot d(v_j)]}, u_i, v_j \in \mathcal{E}, i \neq j$$

Definition 4.2. R-index in Nover top graph \mathcal{G} is defined as

$$\text{Rd}_{\text{Nov}}(\mathcal{G}) = \frac{1}{\sum \{[A(u_i)d(u_i)][A(v_j)d(v_j)]\}^{\frac{1}{2}}, u_i, v_j \in \mathcal{E}, \forall i \neq j}$$

Example 4.1. For example (3.1), the H-index of Nover top graph \mathcal{G} is

$$\begin{aligned} \text{H}_{\text{Nov}}(\mathcal{G}) &= \frac{1}{(.4, .6, 1.2)(.5, 1.2, 2.3) + (.2, 1.5, .3)(.4, 1.4, 2.3)} + \\ &\quad \frac{1}{(.4, .6, 1.2)(.5, 1.2, 2.3) + (.3, .8, 1.1)(.5, 1.4, 2.2)} + \\ &\quad \frac{1}{(.2, 1.5, .3)(.4, 1.4, 2.3) + (.3, .8, 1.1)(.5, 1.4, 2.2)} \\ &= \frac{1}{3.68 + 2.87} + \frac{1}{3.68 + 3.69} + \frac{1}{2.87 + 3.69} \\ &= \frac{1}{6.55} + \frac{1}{7.37} + \frac{1}{6.56} \\ &= 0.153 + 0.136 + 0.152 \\ &= 0.441 \end{aligned}$$

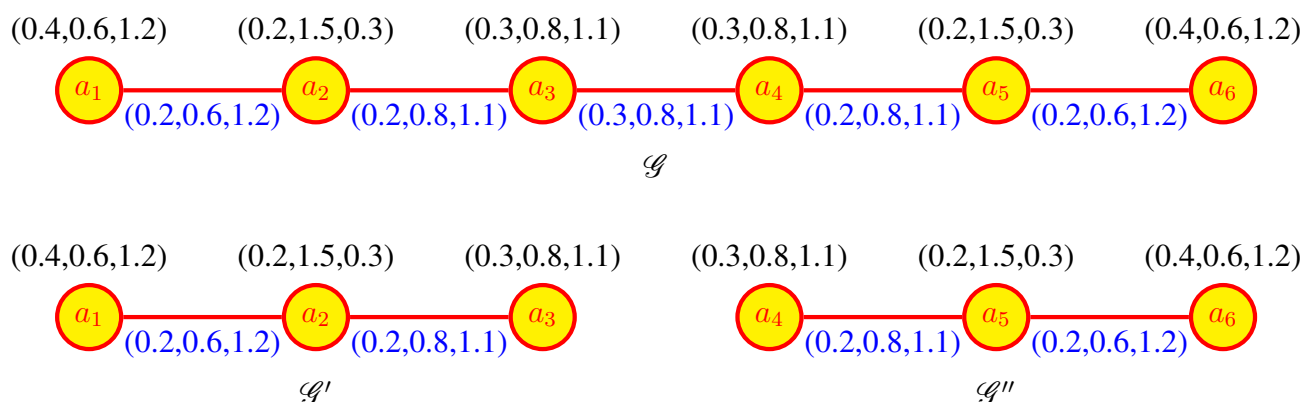
Example 4.2. For example (3.1), the Rd-index of Nover top graph \mathcal{G} is

$$\begin{aligned} \text{Rd}_{\text{Nov}}(\mathcal{G}) &= \frac{1}{\sqrt{(.4, .6, 1.2)(.5, 1.2, 2.3) \times (.2, 1.5, .3)(.4, 1.4, 2.3)}} + \\ &\quad \frac{1}{\sqrt{(.4, .6, 1.2)(.5, 1.2, 2.3) \times (.3, .8, 1.1)(.5, 1.4, 2.2)}} + \\ &\quad \frac{1}{\sqrt{(.2, 1.5, .3)(.4, 1.4, 2.3) \times (.3, .8, 1.1)(.5, 1.4, 2.2)}} \\ &= \frac{1}{\sqrt{10.566}} + \frac{1}{\sqrt{13.5792}} + \frac{1}{\sqrt{10.5903}} \\ &= \frac{1}{3.24986} + \frac{1}{3.68499} + \frac{1}{3.2543} \\ &= 0.0300 + 0.27137 + 0.3073 \\ &= 0.6086 \end{aligned}$$

Definition 4.3. Let \mathcal{G}_1 and \mathcal{G}_2 be any neutrosophic over graphs isomorphism $f : \mathcal{G}_1 \rightarrow \mathcal{G}_2$ is bijective mapping $f : V_1 \rightarrow V_2$ which satisfies the following conditions

- (a) $T_{A_1}(x_1) = T_{A_2}(f(x_1))$, $I_{A_1}(x_1) = I_{A_2}(f(x_1))$ and $F_{A_1}(x_1) = F_{A_2}(f(x_1))$
- (b) $T_{B_1}(x_1, y_1) = T_{B_2}(f(x_1), f(y_1))$, $I_{B_1}(x_1, y_1) = I_{B_2}(f(x_1), f(y_1))$ and $F_{B_1}(x_1, y_1) = F_{B_2}(f(x_1), f(y_1))$ for all $x_1 \in V_1$, $x_1, y_1 \in E_1$

Example 4.3.

Figure 4: \mathcal{G}' and \mathcal{G}'' are isomorphic graphs

Theorem 4.1. Let \mathcal{G} be the connected Nover top $T(u, v)$, $I(u, v)$, $F(u, v)$ graph and (T, I, F) be the true membership, indeterminacy membership and falsity membership value of the chosen edge of \mathcal{G} such that removal of (μ, γ, σ) from \mathcal{G} splits into two Nover top graphs such that whose vertex set satisfies $|V_{\mathcal{G}'}| < |V_{\mathcal{G}''}|$ and therefore its disconnected. Then

- (i) $\mathcal{B}_{Nov}(\mathcal{G}') < \mathcal{B}_{Nov}(\mathcal{G}'')$
- (ii) $\mathcal{B}_{Nov}^*(\mathcal{G}') < \mathcal{B}_{Nov}^*(\mathcal{G}'')$
- (iii) $H_{Nov}(\mathcal{G}') < H_{Nov}(\mathcal{G}'')$
- (iv) $Rd_{Nov}(\mathcal{G}') < Rd_{Nov}(\mathcal{G}'')$

where $|V_{\mathcal{G}'}| < |V_{\mathcal{G}''}|$ denote the cardinality of \mathcal{G}' & \mathcal{G}'' respectively.

Proof:

Let \mathcal{G} be a connected Nover top graph where splitted into two Nover top graph \mathcal{G}' and \mathcal{G}'' by removing the chosen membership value of the edge in \mathcal{G} . We know that \mathcal{G} be the Nover top graph and H be the Nover top sub graph of \mathcal{G} such that $H = \mathcal{G} - u$ then $\mathcal{B}_{Nov}(\mathcal{G} - u) < \mathcal{B}_{Nov}(\mathcal{G})$ and $\mathcal{B}_{Nov}^*(\mathcal{G} - u) < \mathcal{B}_{Nov}^*(\mathcal{G})$.

Therefore, we get, $\mathcal{B}_{Nov}(\mathcal{G}') < \mathcal{B}_{Nov}(\mathcal{G}'') < \mathcal{B}_{Nov}(\mathcal{G})$ which implies that $\mathcal{B}_{Nov}(\mathcal{G}') < \mathcal{B}_{Nov}(\mathcal{G}'')$.

Hence the theorem proved.

The remaining cases are trivially true by the following above method.

5 GA- index in Nover top graphs

Definition 5.1. The GA-index of Nover top graph \mathcal{G} is defined as

$$GA_{Nov}(\mathcal{G}) = \frac{2\{[A(u_i)d(u_i)][A(v_j)d(v_j)]\}^{\frac{1}{2}}}{[A(u_i)d(v_j) + A(v_j)d(v_j)]^{\frac{1}{2}}}$$

Theorem 5.1. Let \mathcal{G} be a Nover top graph with m edges. Then $GA_{Nov}(\mathcal{G}) \leq m$ with equality if and only if every component of \mathcal{G} is regular.

Example 5.1. We have the previous example (3.1)

$$\begin{aligned}
 \text{GA}_{\text{Nov}}(\mathcal{G}) &= \frac{2 \left[\begin{array}{l} (.4, .6, 1.2)(.5, 1.2, 2.3) + (.2, 1.5, .3)(.4, 1.4, 2.3) + \\ (.4, .6, 1.2)(.5, 1.2, 2.3) + (.3, .8, 1.1)(.5, 1.4, 2.3) + \\ (.2, 1.5, .3)(.4, 1.4, 2.3) + (.3, .8, 1.1)(.5, 1.4, 2.2) \end{array} \right]^{\frac{1}{2}}}{\left[\begin{array}{l} (.4, .6, 1.2) + (.5, 1.2, 2.3) + (.2, 1.5, .3) + (.4, 1.4, 2.3) + \\ (.4, .6, 1.2) + (.5, 1.2, 2.3) + (.3, .8, 1.1) + (.5, 1.4, 2.2) + \\ (.2, 1.5, .3) + (.4, 1.4, 2.3) + (.3, .8, 1.1) + (.5, 1.4, 2.2) \end{array} \right]^{\frac{1}{2}}} \\
 &= \frac{2[(3.68 \times 2.87) + (3.68 \times 3.69) + (2.87 \times 3.69)]^{\frac{1}{2}}}{[2.2 + 4 + 2 + 4.1 + 2.2 + 4 + 2.2 + 4.1 + 2 + 4.1 + 2.2 + 4.1]^{\frac{1}{2}}} \\
 &= \frac{2[10.562 + 13.579 + 10.5903]^{\frac{1}{2}}}{[37.2]^{\frac{1}{2}}} \\
 &= \frac{2[34.7313]^{\frac{1}{2}}}{6.099} \\
 &= \frac{2 \times 5.8933}{6.099} \\
 &= 1.9325
 \end{aligned}$$

6 Connectivity Index in Nover top graphs

Definition 6.1. The strength of connectedness between u_i and v_j is defined as

$$\text{CONN}_P(u_i, v_j) = \left(\min_{e \in P_{u_i v_j}} T_B(e), \min_{e \in P_{u_i v_j}} I_B(e), \max_{e \in P_{u_i v_j}} F_B(e) \right)$$

where $P_{u_i v_j}$ is the path between u_i and v_j

$$|\text{CONN}_P(u_i, v_j)| = 2 \left(\min_{e \in P_{u_i v_j}} T_B(e) \right) - \left(\min_{e \in P_{u_i v_j}} I_B(e) \right) - \left(\max_{e \in P_{u_i v_j}} F_B(e) \right)$$

Then $\text{CONN}_P(u_i, v_j) = \max_p \{ |\text{CONN}_P(u_i, v_j)| \}$.

Definition 6.2. The Connectivity index (CI) of \mathcal{G} is defined by

$$\text{CI}_{\text{Nov}}(\mathcal{G}) = \sum_{u_i v_j \in \mathcal{V}} A(u_i) \cdot A(v_j) \times \text{CONN}_{\mathcal{G}}(u_i, v_j)$$

Here $\text{CONN}_{\mathcal{G}}(u_i, v_j)$ is the strength of connectedness between u_i and v_j .

Example 6.1. For example (3.1), the strength of connectedness between a_1 and a_2 from the direct path $p_1 = a_1 a_2$ is $\text{CONN}_{p_1}(a_1, a_2) = (0.2, 0.6, 1.2)$.

From path $p_2 = a_1 a_3 a_2$

$$\text{CONN}_{p_2}(a_1, a_2) = (\min\{0.3, 0.2\}, \min\{0.6, 0.8\}, \max\{1.1, 1.1\})$$

$$= (0.2, 0.6, 1.1)$$

a_1 and a_3 from the direct path $p_1 = a_1 a_3$ is

$$\text{CONN}_{p_1}(a_1, a_3) = (0.3, 0.6, 1.1)$$

From path $p_2 = a_1 a_2 a_3$

$$\begin{aligned}\text{CONN}_{p_2}(a_1, a_3) &= (\min\{0.2, 0.2\}, \min\{0.6, 0.8\}, \max\{1.2, 1.1\}) \\ &= (0.2, 0.6, 1.2)\end{aligned}$$

a_2 and a_3 from the direct path $p_1 = a_2 a_3$ is

$$\text{CONN}_{p_1}(a_2, a_3) = (0.2, 0.8, 1.1)$$

From path $p_2 = a_2 a_1 a_3$

$$\begin{aligned}\text{CONN}_{p_2}(a_2, a_3) &= (\min\{0.2, 0.3\}, \min\{0.6, 0.6\}, \max\{1.2, 1.1\}) \\ &= (0.2, 0.6, 1.2)\end{aligned}$$

Then, we have for a_1 and a_2

$$\begin{aligned}|\text{CONN}_{p_1}(a_1, a_2)| &= 2 \times (0.2) - 0.6 - 1.2 = -1.4 \\ |\text{CONN}_{p_2}(a_1, a_2)| &= 2 \times (0.2) - 0.6 - 1.1 = -1.3\end{aligned}$$

For a_1 and a_3

$$\begin{aligned}|\text{CONN}_{p_1}(a_1, a_3)| &= 2 \times (0.3) - 0.6 - 1.1 = -1.3 \\ |\text{CONN}_{p_2}(a_1, a_3)| &= 2 \times (0.2) - 0.6 - 1.2 = -1.4\end{aligned}$$

For a_2 and a_3

$$\begin{aligned}|\text{CONN}_{p_1}(a_2, a_3)| &= 2 \times (0.2) - 0.8 - 1.1 = -1.5 \\ |\text{CONN}_{p_2}(a_2, a_3)| &= 2 \times (0.2) - 0.6 - 1.2 = -1.4\end{aligned}$$

Since we have

$$\begin{aligned}\text{CONN}_{\mathcal{G}}(a_1, a_2) &= -1.3 \\ \text{CONN}_{\mathcal{G}}(a_1, a_3) &= -1.3 \\ \text{CONN}_{\mathcal{G}}(a_2, a_3) &= -1.4\end{aligned}$$

Then $\text{CI}_{\text{Nov}}(\mathcal{G})$ is calculated as follows.

$$\begin{aligned}\text{CI}_{\text{Nov}}(\mathcal{G}) &= \sum_{u_i v_j \in \mathcal{V}} (T_A(u_i), I_A(u_i), F_A(u_i))(T_A(v_j), I_A(v_j), F_A(v_j)) \times \text{CONN}_{\mathcal{G}}(u_i, v_j) \\ &= (.4, .6, 1.2)(.2, 1.5, .3) \times (-1.3) + (.4, .6, 1.2)(.3, .8, 1.1) \times (-1.3) + \\ &\quad (.2, 1.5, .3)(.3, .8, 1.1) \times (-1.4)\end{aligned}$$

$$\begin{aligned}
&= (.08 + .9 + .36)(-1.3) + (.12 + .48 + 1.32)(-1.3) + (.06 + 1.2 + .33)(-1.4) \\
&= (1.34)(-1.3) + (1.92)(-1.3) + (1.59)(-1.4) \\
&= -1.742 - 2.496 - 2.226 \\
&= -6.464
\end{aligned}$$

Then CI of \mathcal{G} is equal -6.464 , which the negative sign indicates the high level of false and indeterminacy information in the problem.

Theorem 6.1. Let \mathcal{G} and \mathcal{G}_1 be the two Nover top graphs are isomorphic, then the topological indices values of two Nover top graphs are equal.

Proof: Let $\mathcal{G} = (\mathcal{V}_{\mathcal{G}}, A_{\mathcal{G}}, B_{\mathcal{G}})$ and $\mathcal{G}_1 = (\mathcal{V}_{\mathcal{G}_1}, A_{\mathcal{G}_1}, B_{\mathcal{G}_1})$ be isomorphic Nover top graphs.

Hence there is an identity function

$$\mu_A : A_{\mathcal{G}}(u) \rightarrow A_{\mathcal{G}_1}(u^*) \text{ for all } u \in \mathcal{V}_{\mathcal{G}}, \exists u^* \in \mathcal{V}_{\mathcal{G}_1}$$

as well as

$$\mu_B : B_{\mathcal{G}}(u, v) \rightarrow B_{\mathcal{G}_1}(u^*, v^*),$$

then each vertex of \mathcal{G} corresponds to an vertex in \mathcal{G}_1 , with the same membership value and the same edges.

Hence, the Neutrosophic over top graph structure may differ but collection of vertices and edges are same gives the equal topological indices value.

Theorem 6.2. Let $G = (\mathcal{V}_G, A_G, B_G)$ is a Nover top graph and H is the NOver top subgraph of G , such that H is made by removing edge $uv \in B_G$ from G . Then, we have $CI_{Nov(H)} < CI_{Nov(\mathcal{G})}$ iff uv is a bridge.

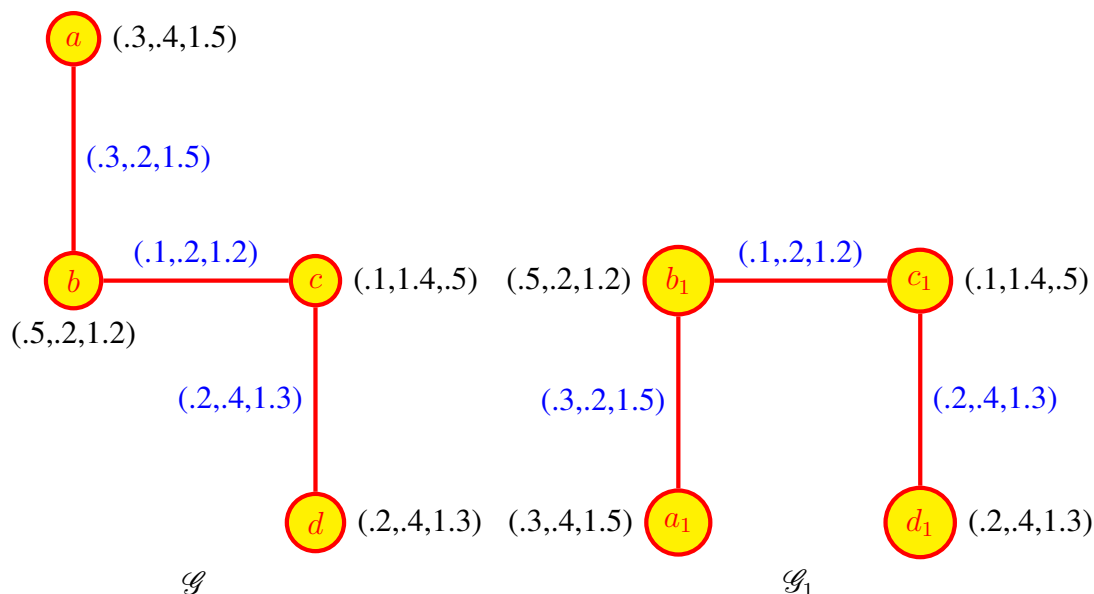
Proof: Now suppose that uv is an edge that has maximum (or) minimum components, so they will have an effect on $CONN_{\mathcal{G}}(u, v)$.

Therefore, by removing edge uv , the value of $CONN_{\mathcal{G}}(u, v)$ will decrease, then we have $CI_{Nov(H)} < CI_{Nov(\mathcal{G})}$.

Since the bridge is called the edge that has its deletion reducing the $CONN_{\mathcal{G}}(u, v)$, however, uv is a bridge.

Conversely, given that uv is a bridge. By the definition of bridge we have, for the edge uv , $CONN_{\mathcal{G}}(u, v) > CONN_{G-uv}(u, v)$, so we conclude that, $CI_{Nov(H)} < CI_{Nov(\mathcal{G})}$.

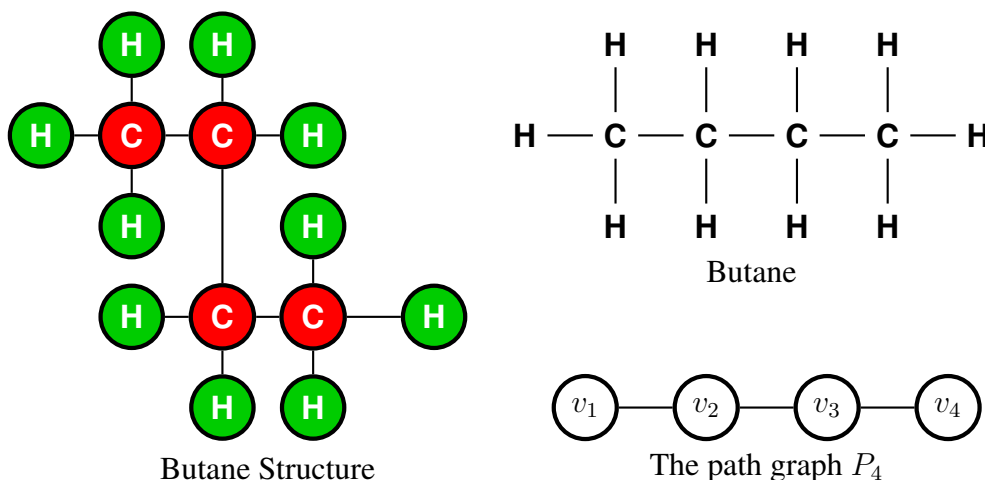
Example 6.2.

Figure 5: \mathcal{G} and \mathcal{G}_1 are isomorphic Nover top graphs

7 Illustration

Wiener was introduced two parameters for the specific purpose of correlating the boiling points of members of the alkane series of molecular structure which satisfied a linear formula $t_B = aW + bP + c$ where t_B is the boiling point of a given alkane, W is the wiener number, P is the polarity number and a, b, c are constants. A topological representing of a molecule structure is called molecule graph which is a collection of points representing the atoms in the molecule and set of lines representing the covalent bonds. A hydrogen-detected graph is a molecular graph in which hydrogen atoms are not considered.

Example 7.1. Consider an alkane series butane where molecular and its hydrogen detected graph are given there.

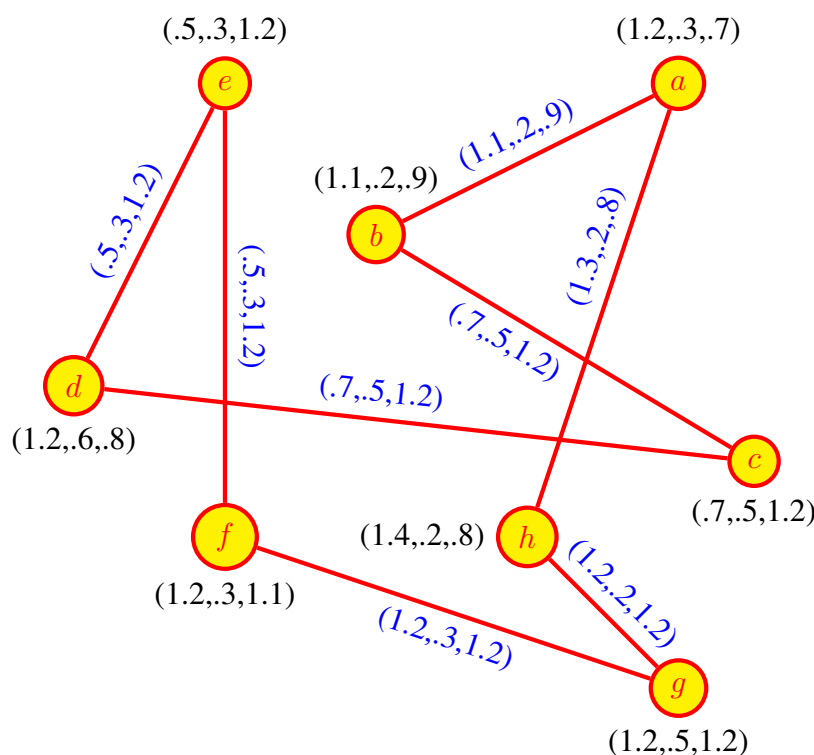


Suppose we can consider hydrogen detected graph P_4 as in Nover top graph structure, we can calculate the Nover top Z_1 index for P_4 . Hence the boiling point of butane in Nover top graph is satisfies linear equation as $t_B = aW + bP + c$. Since butane is non-polar, $b = 0$. so, $t_B = aW + c$. If we can take for particular value of a, b, c and get the boiling point of butane both of them is same.

8 Application

Most of the administration in the society, they depends the supporters than on themselves. The application of dominating set and connecting index of Nover Top graph is expressed as a following example. An office decide to appoint a head under 8 employees. The employees are assumed to be a, b, c, d, e, f, g, h . In this Nover Top graph, the employees and strength between them are considered as vertex and edges. The true membership, indeterminacy and false membership is taken as a vertex. The true membership value is based on employees talent, work experience and salary basis. The indeterminacy membership function are considered as the persons with ability and skill but do not work in the suitable task. The false membership function is considered as the lack of compatibility between educational major and occupation, lack of ability, non-skill and health issue.

Consider the following graphical structure



Let a, b, c, d, e, f, g and h denote the vertices and $(1.1, 0.2, 0.9)$, $(0.7, 0.5, 1.2)$, $(0.5, 0.3, 1.2)$, $(0.5, 0.3, 1.2)$, $(1.2, 0.3, 1.2)$, $(1.2, 0.2, 0.8)$, and $(1.3, 0.2, 0.8)$ denote the edges and there is a strong relationship between them.

Let $\mathcal{V} = \{a, b, c, d, e, f, g, h, (1.1, 0.2, 0.9), (0.7, 0.5, 1.2), (0.5, 0.3, 1.2), (0.5, 0.3, 1.2), (1.2, 0.3, 1.2), (1.2, 0.2, 0.8), (1.3, 0.2, 0.8)\}$ be a topology

$\tau = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \{f\}, \{g\}, \{h\}, \{a, b\}, \{a, d\}\{b, c\}, \{e, f\}, \{f, g\}, \{g, h\}, \dots$
 $\{a, b, c, d\}, \{a, b, d, f\}, \{e, f, g, h\}, \dots \{a, b, c, d, e, f, g\}, \}$.

Here for every $x \in X$, $\{x\}$ is open or closed. By the definition of Nover top graph, we have $|\partial(A)| \leq 2$ and $\partial(a) = (b, h)$, $\partial(b) = (a, c)$, $\partial(c) = (b, d)$, $\partial(d) = (e, c)$, $\partial(e) = (d, f)$, $\partial(f) = (e, g)$, $\partial(g) = (f, h)$, $\partial(h) = (a, g)$ with $\partial(a_i) = 2$. Hence this graph is Nover top graph.

$$\begin{aligned} T_A(a) &= \min[T_B(a, b), T_B(a, h)] = \min[1.1, 1.1] = 1.1 \\ I_A(a) &= \min[I_B(a, b), I_B(a, h)] = \min[0.2, 0.2] = 0.2 \\ F_A(a) &= \max[F_B(a, b), F_B(a, h)] = \max[0.9, 0.8] = 0.9 \\ T_A(b) &= \min[T_B(b, a), T_B(b, c)] = \min[1.1, 0.7] = 0.7 \\ I_A(b) &= \min[I_B(b, a), I_B(b, c)] = \min[0.2, 0.5] = 0.2 \\ F_A(b) &= \max[F_B(b, a), F_B(b, c)] = \max[0.9, 1.2] = 1.2 \\ T_A(c) &= \min[T_B(c, b), T_B(c, d)] = \min[0.7, 0.7] = 0.7 \\ I_A(c) &= \min[I_B(c, b), I_B(c, d)] = \min[0.2, 0.5] = 0.2 \\ F_A(c) &= \max[F_B(c, b), F_B(c, d)] = \max[1.3, 1.3] = 1.3 \\ T_A(d) &= \min[T_B(d, c), T_B(d, e)] = \min[0.1, 0.1] = 0.1 \\ I_A(d) &= \min[I_B(d, c), I_B(d, e)] = \min[0.7, 0.7] = 0.7 \\ F_A(d) &= \max[F_B(d, c), F_B(d, e)] = \max[1.2, 1.2] = 1.2 \\ T_A(e) &= \min[T_B(e, d), T_B(e, f)] = \min[0.5, 0.5] = 0.5 \\ I_A(e) &= \min[I_B(e, d), I_B(e, f)] = \min[0.3, 0.3] = 0.3 \\ F_A(e) &= \max[F_B(e, d), F_B(e, f)] = \max[1.2, 1.2] = 1.2 \\ T_A(f) &= \min[T_B(f, e), T_B(f, g)] = \min[0.5, 1.2] = 0.5 \\ I_A(f) &= \min[I_B(f, e), I_B(f, g)] = \min[0.3, 0.3] = 0.3 \\ F_A(f) &= \max[F_B(f, e), F_B(f, g)] = \max[1.3, 1.3] = 1.3 \\ T_A(g) &= \min[T_B(g, h), T_B(g, f)] = \min[1.2, 1.2] = 1.2 \\ I_A(g) &= \min[I_B(g, h), I_B(g, f)] = \min[0.2, 0.3] = 0.2 \\ F_A(g) &= \max[F_B(g, h), F_B(g, f)] = \max[1.2, 1.2] = 1.2 \\ T_A(h) &= \min[T_B(h, a), T_B(h, g)] = \min[1.3, 1.2] = 1.2 \\ I_A(h) &= \min[I_B(h, a), I_B(h, g)] = \min[0.2, 0.2] = 0.2 \\ F_A(h) &= \max[F_B(h, a), F_B(h, g)] = \max[0.8, 1.2] = 1.2 \end{aligned}$$

Here a dominates b because

$$\begin{aligned} T_B(ab) &\leq T_A(a) \wedge T_A(b), 1.1 \leq 1.1 \wedge 0.7 \\ I_B(ab) &\leq I_A(a) \wedge I_A(b), 0.2 \leq 0.2 \wedge 0.2 \\ F_B(ab) &\geq F_A(a) \vee F_A(b), 0.9 \geq 0.9 \vee 0.9 \end{aligned}$$

Here b dominates c because

$$\begin{aligned} T_B(bc) &\leq T_A(b) \wedge T_A(c), 0.7 \leq 0.7 \wedge 0.7 \\ I_B(bc) &\leq I_A(b) \wedge I_A(c), 0.2 \leq 0.2 \wedge 0.2 \end{aligned}$$

$$F_B(bc) \geq F_A(b) \vee T_B(c), 1.2 \geq 0.9 \vee 1.2$$

Here c dominates d because

$$T_B(cd) \leq T_A(c) \wedge T_A(d), 0.7 \leq 0.7 \wedge 0.5$$

$$I_B(cd) \leq I_B(c) \wedge T_B(d), 0.5 \leq 0.2 \wedge 0.3$$

$$F_B(cd) \geq F_A(c) \vee T_B(d), 1.2 \geq 1.2 \vee 1.2$$

Here d dominates e because

$$T_B(de) \leq T_A(d) \wedge T_A(e), 0.5 \leq 0.5 \wedge 0.0.5$$

$$I_B(de) \leq I_B(d) \wedge T_B(e), 0.3 \leq 0.3 \wedge 0.3$$

$$F_B(de) \geq F_A(d) \vee T_B(e), 1.2 \geq 1.2 \vee 1.2$$

Here e dominates f because

$$T_B(ef) \leq T_A(e) \wedge T_A(f), 0.5 \leq 0.5 \wedge 0.5$$

$$I_B(ef) \leq I_B(e) \wedge I_B(f), 0.3 \leq 0.3 \wedge 0.3$$

$$F_B(ef) \geq F_A(e) \vee F_B(f), 1.2 \geq 1.2 \vee 1.2$$

Here f dominates g because

$$T_B(fg) \leq T_A(f) \wedge T_A(g), 1.2 \leq 0.5 \wedge 1.2$$

$$I_B(fg) \leq I_B(f) \wedge I_B(g), 0.3 \leq 0.3 \wedge 0.2$$

$$F_B(fg) \geq F_A(f) \vee F_B(g), 1.2 \geq 1.2 \vee 1.2$$

Here g dominates h because

$$T_B(gh) \leq T_A(g) \wedge T_A(h), 1.2 \geq 1.2 \vee 1.2$$

$$I_B(gh) \leq I_B(g) \wedge I_B(h), 0.2 \leq 0.2 \wedge 0.2$$

$$F_B(gh) \geq F_A(g) \vee F_B(h), 1.2 \geq 1.2 \vee 1.2$$

Here h dominates a because

$$T_B(ha) \leq T_A(a) \wedge T_A(h), 1.3 \geq 1.1 \vee 1.2$$

$$I_B(ha) \leq I_B(a) \wedge I_B(h), 0.2 \leq 0.2 \wedge 0.2$$

$$F_B(ha) \geq F_A(a) \vee F_B(h), 0.8 \geq 0.9 \vee 1.2$$

$V = \{a, b, c, d, e, f, g, h\}$, $D_N = \{c, e, h\}$ and $V - D_n = \{a, b, d, f, g\}$, $|D_N| = 3$.
Now we obtain the connectivity index for all paths

$$|\text{CONN}_{p_1}(a, b)| = 2 \times (1.1) - 0.5 - 0.9 = 0.8$$

$$|\text{CONN}_{p_1}(b, c)| = 2 \times (0.7) - 0.2 - 1.2 = 0$$

$$|\text{CONN}_{p_1}(c, d)| = 2 \times (0.7) - 0.5 - 1.2 = -0.3$$

$$\begin{aligned}
|\text{CONN}_{p_1}(d, e)| &= 2 \times (0.5) - 0.3 - 1.2 = -0.5 \\
|\text{CONN}_{p_1}(e, f)| &= 2 \times (0.5) - 0.3 - 1.2 = -0.5 \\
|\text{CONN}_{p_1}(f, g)| &= 2 \times (1.2) - 0.3 - 1.1 = 1 \\
|\text{CONN}_{p_1}(g, h)| &= 2 \times (1.2) - 0.2 - 1.1 = 1.1 \\
|\text{CONN}_{p_1}(h, a)| &= 2 \times (1.3) - 0.2 - 0.8 = 1.6
\end{aligned}$$

Also, If needed, we can calculate the connectivity index for indirect relationship for Novertop graph.

Then $\text{CI}_{\text{Nov}}(\mathcal{G})$ we have,

$$\begin{aligned}
\text{CI}_{\text{Nov}}(\mathcal{G}) &= \sum_{u_i v_j \in \mathcal{V}} (T_A(u_i), I_A(u_i), F_A(u_i))(T_A(v_j), I_A(v_j), F_A(v_j)) \times \text{CONN}_G(u_i, v_j) \\
&= (1.3, 0.3, 0.7)(1.1, 0.2, 0.9) \times (0.8) + (1.3, 0.3, 0.2)(1.4, 0.2, 0.8) \times (1.6) + \\
&\quad (1.1, 0.2, 0.9)(0.7, 0.5, 1.2) \times (-0.3) + (0.7, 0.5, 1.2)(1.2, 0.6, 0.8) \times (-0.3) + \\
&\quad (1.2, 0.6, 0.8)(0.5, 0.3, 1.2) \times (-0.4) + (0.5, 0.3, 1.2)(1.2, 0.3, 1.1) \times (-0.4) + \\
&\quad (1.2, 0.5, 1.1)(1.4, 0.2, 0.8) \times (1) \\
&= (2.12)(0.8) + (2.23)(1.6) + (2.03)(-0.3) + (2.1)(-0.3) + (1.74)(-0.4) + \\
&\quad (2.82)(-0.4) + (2.94)(1) \\
&= 4.478
\end{aligned}$$

Then connectivity index of G is equal 4.478, which the positive sign indicates the high level of true information in the problem.

Hence the employee (h) a relationship is high and good. so "h" is the most dominating person. so we can select h is the head of the office.

9 Conclusion

In this paper has focussed an some topological indices for Neutrosophic over topologized graphs by using strong domination. It is an easy way to calculate the connectivity index. so we use the new method that is strong domination to find and calculate the connectivity index. The some topological indices for some standard neutrosophic over topologized graphs such as $2 - regular$, K_2 and $K_{2,2}$ are given.

References

- [1] Akram Muhammad, single-Valued Neutrosophic Graphs, Springer Nature singapore Pte Ltd. 2018
- [2] Atanassov, K.T.(1983). Intuitionistic fuzzy set. In Proceedings of the VII ITKP Session, Sofia, Bulgaria, 9 June 1983 (Deposited in Central Sci.-Techn. Library of Bulg. Acad. Of Sci., 1697/84) (in Bulgaria).
- [3] A Nagoorgani and VT Chandrasekaran. Domination in fuzzy graph. Advances in fuzzy sets and system I (1), pages 17–26, 2006.

- [4] O Ore. Theory of graphs, american mathematical society colloquium publications. Amer. Math. Soc., Providence, 38, 1962
- [5] R Narmada Devi, N Kalaivani, S Broumi, and KA Venkatesan. Characterizations of Strong and Balanced Neutrosophic Complex Graphs. Infinite Study, 2018.
- [6] R Narmada Devi. Neutrosophic complex N-continuity. Infinite Study, 2017.
- [7] Narmada Devi.R And Dhavaseelan.R "New type of Neutrosophic Off grpahs", Advanced in Mathematics Scientific journal9(2020), No:3,1331-1338.
- [8] R.Narmada Devi "Minimal domination via neutrosophic over graphs" AIP Conference Proceedings 2277, 100019 (2020); <https://doi.org/10.1063/5.0025568> Published Online: 06 November 2020
- [9] R.Narmada Devi and G.Muthumari, Properties on Topologized Domination in Neutrosophic Graphs, Neutrosophic Sets and Systems, Vol. 47, 2021, pp. 511-519, DOI: 10.5281/zenodo.5775172
- [10] R.Narmada Devi and G.Muthumari, View On Neutrosophic Over Topologized Domination Graphs, Neutrosophic Sets and Systems, Vol. 47, 2021, pp. 520-532, DOI: 10.5281/zenodo.5775174
- [11] Masoud Ghods, Zahra Rostami, Introduction to topological indices in Neutrosophic Graphs" Neutrosophic Sets and systems, Vol.35,2020.
- [12] Masoud Ghods, Zahra Rostami "Wiener index and applications in the Neutrosophic graphs" Neutrosophic Sets and Systems, Vol. 46, 2021
- [13] Said Broumi, Mohamed Talea, Assia Bakali, and Florentin Smarandache. Single valued neutrosophic graphs. Journal of New theory, (10):86–101, 2016.
- [14] Said Broumi, Mahamed Talea, Assia Bakali, and Florentin Smarandache, "Single valued neutrosophic graphs", Laboratory of Information processing, Faculty of science Ben M'Sik, University Hassen II, B.P. 7955, Sidi Othman, Casablanca, Morocco, Ecole Royale Navale-Boulevard Sour Jdid, B.P 16303 Casablanca-Morocco, Department of Mathematics, University of New Mexico, 705 curley Avenue, Gallup, NM 87301, USA.
- [15] Florentin Smarandache, Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-) HyperAlgebra, Neutrosophic Sets and Systems, Vol. 33, 290-296, 2020;
- [16] Florentin Smarandache. A unifying field in logics: Neutrosophic logic. In Philosophy, pages 1–141. American Research Press, 1999.
- [17] Smarandache .F: Neutrosophic Over set, Neutrosophic under set, Neutrosophic off set, Pons Editions, Brussels, 2016.
- [18] A Somasundaram and S Somasundaram. Domination in fuzzy graphs–i. Pattern Recognition Letters, 19(9):787–791, 1998.

- [19] Smarandache, F. A Unifying Field in Logic: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics; Infinite Study; Modern Science Publisher: New York, NY, USA, 2005.
- [20] Shriram Kalathian, Sujatha Ramalingam , Sundareswaran Raman and Narasimman srinivasan "Some topological indices in fuzzy graphs" 2020 Journal of Intelligent and Fuzzy Systems 39(5):6033-6046 DOI:10.3233/JIFS-189077
- [21] Simon Mukwembi Farai Nyabadza A new model for predicting boiling points of alkanes DOI: 10.21203/rs.3.rs-812041/v, 18 August, 2021.
- [22] Lotfi A Zadeh. Fuzzy sets. Information and control, 8(3):338–353, 1965.
- [23] Zeeshan Saleem Mufti , Rukhshanda Anjum , Ayesha Abbas, Shahbaz Ali , Muhammad Afzal, and Ashraful Alam 4 Hindawi "Computation of Vertex Degree-Based Molecular Descriptors of Hydrocarbon Structure" Journal of Chemistry Volume 2022, Article ID 3621403, 15 pages <https://doi.org/10.1155/2022/3621403>

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