

5-1-2023

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### Recommended Citation

Basker, P. and Broumi Said. "On Neutrosophic Homeomorphisms via Neutrosophic Functions." *Neutrosophic Sets and Systems* 55, 1 (2023). [https://digitalrepository.unm.edu/nss\\_journal/vol55/iss1/24](https://digitalrepository.unm.edu/nss_journal/vol55/iss1/24)

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# On Neutrosophic Homeomorphisms via Neutrosophic Functions

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**Abstract.** By using neutrosophic  $m$ -alpha closed sets in neutrosophic topological spaces, we introduce the space known as neutrosophic  $t$ -alpha space in this paper. We also introduce the mappings referred as neutrosophic  $m$ -alpha continuous functions, homeomorphisms, and connectedness, and we research the characterizations and their properties.

**Keywords:** neutrosophic  $t$ -alpha space, neutrosophic  $m$ -alpha closed sets, neutrosophic  $m$ -alpha continuous functions, neutrosophic  $m$ -alpha homeomorphisms and neutrosophic  $m$ -alpha connectedness.

## 1. Introduction

L. A. Zadeh [14] first put forward the idea of fuzzy sets in 1965, C. L. Chang [5] created fuzzy topological spaces in 1968, built around the idea of fuzzy sets, In 1986 [2] K. Atanassov derived the intuitionistic fuzzy sets, In 1997 [6] D.Coker have introduced the intuitionistic fuzzy topological spaces, F. Smarandache [9] proposed A unified field approach in neutrosophic logic in 1999 and analyzed some of its characteristics, F. Smarandache [10] started researching neutrosophy and neutrosophic logic in 2002. A. A. Salama and S. A. Alblowi [8] examined the neutrosophic set and neutrosophic topological spaces in 2012 and mentioned some of their findings. Broumi Said and Florentin Smarandache [4] proposed the intuitionistic neutrosophic soft set concept and derived some results in 2013. Smarandache, Florentin, Said Broumi, Mamoni Dhar, and Pinaki Majumdar [11] brought new intuitionistic fuzzy soft set results and derived some results in 2014. Wadel Faris Al-omeri and Florentin Smarandache [13] suggested a new neutrosophic sets using neutrosophic topological spaces in Wadel Faris Al-omeri and

Florentin Smarandache's article.

In 2017 [1] I.Arokiarani, R.Dhavaseelan, S. Jafari and M.Parimala have derived some new notions and functions in neutrosophic topological spaces, In 2021 [3] P. Basker, Broumi Said have introduced  $N\psi_\alpha^{\#0}$  and  $N\psi_\alpha^{\#1}$ -spaces in neutrosophic topological spaces, In 2021 [7] D. Nagarajan, S. Broumi, F. Smarandache, and J. Kavikumar derived the analysis of neutrosophic multiple regression and have given some properties, In 2021 [12] A. Vadivel1, C. John Sundar derived the neutrosophic  $\delta$ -open maps and neutrosophic  $\delta$ -closed maps

The abbreviations  $NS$  and  $NTS$  refer to the neutrosophic set and neutrosophic topological spaces, respectively, throughout this study.

## 2. Preliminaries

We should review and analyze definitions before we begin our study.

**Definition 2.1.** A  $NS$ ,  $A$  in a  $NTS$  is referred to as a neutrosophic set,  $N\alpha$ -open set ( $N\alpha OS$ ), if  $A$  is a subset of  $Nint(Ncl(Nint(A)))$ . The complement of  $N\alpha OS$  is called  $N\alpha CS$ .

**Definition 2.2.** (a) Assume  $N$  is an  $NTS$  and  $n \in N$ .  $N_1$  is a subset of  $N$  is called as  $N\alpha$ -nbhd of  $n \exists N\alpha$ -open set  $N_2$  such that  $n \in N_2 \subset N_1$ .

The collection of all  $N\alpha$ -nbhd of  $n \in N$  is called  $N\alpha$ -neighbourhood system at  $n$  and shall be denoted by  $NBH_{N\alpha}(n)$ .

(b) Let  $N$  be a  $NTS$  and  $N_1$  be a subset of  $N$ , A subset  $N_2$  of  $N$  is supposed to be  $N\alpha$ -nbhd of  $N_1 \exists N\alpha$ -open set  $M$  such that  $N_1 \in M \subseteq N_2$ .

(c) Let  $N_1$  be a subset of  $N$ . A point  $n_1 \in N_1$  is supposed to be  $N\alpha$ -interior point of  $N_1$ , if  $N_1$  is an  $NBH_{N\alpha}(n_1)$ . The entirety of everything  $N\alpha$ -interior points of  $N_1$  is referred to as an  $N\alpha$ -interior of  $N_1$  and is denoted by  $NBH_{N\alpha}(n_1)$ .

(d)  $N\alpha$ -interior of  $N_1$  is the union of all  $N\alpha OS \subset N_1$  and it is denoted by  $INT_{N\alpha}(N_1)$ .  
 $INT_{N\alpha}(N_1) = \bigcup \{M : M \text{ is } N\alpha OS, M \subseteq N_1\}$ .

(e)  $N\alpha$ -closure of  $N_1$  is the intersection of all  $N\alpha CS \supset N_1$  and it is denoted by  $CL_{N\alpha}(N_1)$ .  
 $CL_{N\alpha}(N_1) = \bigcap \{M : M \text{ is a } N\alpha\text{-closed set and } N_1 \subseteq M\}$ .

(f)  $\bigcap$  of all  $N\alpha$ -open subsets of  $(N, \tau_N)$  containing  $N_1$  is called the  $N\alpha$ -kernel of  $N_1$  (briefly,  $nk_{\#}^{N\alpha}(N_1)$ ).  $nk_{\#}^{N\alpha}(N_1) = \bigcap \{M \in N\alpha(N, \tau_N) : N_1 \subseteq M\}$ .

(g) Let  $n \in N_1$ . Then  $N\alpha$ -kernel of  $n$  is meant to refer to as  $nk_{\#}^{N\alpha}(\{n\}) = \cap\{M \in N\alpha(N, \tau_N) : n \in M\}$ .  $CL_{N\alpha}(N_1) = \cap\{M : N_1 \subset M \in N\alpha(N, \tau_N)\}$ .

### 3. On $t_{\#}^{N\alpha}$ -space via $N\alpha OS$

**Definition 3.1.**  $L$  is  $NS$  in a  $NTS$ ,  $N\alpha^{M\#}CS$  if  $Nint(Ncl(L))$  is a subset of  $Q$ , only when  $L$  is a  $\subset$  of  $Q$  and  $Q$  is  $N\alpha OS$ . The opponent of  $N\alpha^{M\#}CS$  is called an  $N\alpha^{M\#}OS$ .

**Example 3.2.** Here  $N = \{n_1, n_2, n_3\}$  with  $\tau_N = \{0_N, 1_N, O_1, O_2\}$  where

$$O_1 = \langle (\frac{7}{10}, \frac{7}{10}, \frac{5}{10}), (\frac{3}{10}, \frac{8}{10}, 1), (1, \frac{8}{10}, \frac{6}{10}) \rangle,$$

$$O_2 = \langle (\frac{2}{10}, \frac{5}{10}, \frac{9}{10}), (\frac{3}{10}, \frac{7}{10}, 1), (\frac{7}{10}, \frac{6}{10}, 1) \rangle,$$

$$O_3 = \langle (\frac{3}{10}, \frac{3}{10}, \frac{5}{10}), (\frac{7}{10}, \frac{2}{10}, 0), (0, \frac{2}{10}, \frac{4}{10}) \rangle,$$

$$O_4 = \langle (\frac{8}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{7}{10}, \frac{3}{10}, 0), (\frac{3}{10}, \frac{4}{10}, 0) \rangle,$$

$$O_5 = \langle (\frac{4}{10}, \frac{5}{10}, 1), (\frac{2}{10}, \frac{3}{10}, 1), (\frac{5}{10}, \frac{3}{10}, 1) \rangle. \text{ Here the sets } O_3, O_4 \text{ and } O_5 \text{ are the } N\alpha^{M\#}CS.$$

**Definition 3.3.** A  $NTS$  is neutrosophic in nature which is  $t_{\#}^{N\alpha}$ -space if every  $N\alpha^{M\#}CS$  is  $CS$ .

**Theorem 3.4.** For a  $TS$  that is neutrosophic  $(N, \tau_N)$  The criteria listed below are equivalent.

(a)  $(N, \tau_N)$  is  $t_{\#}^{N\alpha}$ -space.

(b) Every singleton  $\{n_1\}$  is either  $N\alpha CS$  (or)  $NclNopen$ .

*Proof.* (a)  $\Rightarrow$  (b) Let  $n_1 \in N$ . Suppose  $\{n_1\}$  is not an  $N\alpha CS$  of  $(N, \tau_N)$ . Then  $N - \{n_1\}$  is not an  $N\alpha OS$ . Thus  $N - \{n_1\}$  is an  $N\alpha CS$  of  $(N, \tau_N)$ . Since  $(N, \tau_N)$  is a  $t_{\#}^{N\alpha}$ -space,  $N - \{n_1\}$  is a  $N\alpha CS$  of  $(N, \tau_N)$ , i.e.,  $\{n_1\}$  is  $N\alpha OS$  of  $(N, \tau_N)$ .

(b)  $\Rightarrow$  (a) Let  $N_1$  be an  $N\alpha^{M\#}CS$  of  $(N, \tau_N)$ . Let  $n_1 \in Nint(Ncl(N_1))$  by (b),  $\{n_1\}$  is either  $N\alpha CS$  (or)  $NclNopen$ .

*Case(i):* Let  $\{n_1\}$  be an  $N\alpha CS$ . If we take the presumption that  $n_1 \notin N_1$ , we would now have  $n_1 \in Nint(Ncl(N_1)) - N_1$  which isn't possible. Hence  $n_1 \in N_1$ .

*Case(ii):* Let  $\{n_1\}$  be a  $NclNopen$ . Since  $n_1 \in Nint(Ncl(N_1))$ , then  $\{n_1\} \cap N_1 \neq \phi_N$ . This demonstrates that  $n_1 \in N_1$ . As a result, in both circumstances, we have  $Nint(Ncl(A)) \subseteq N_1$ .

Trivially  $N_1 \subseteq Nint(Ncl(N_1))$ . Therefore  $N_1 = Nint(Ncl(N_1))$  (or) equivalently  $N_1$  is  $NclNopen$ . Hence  $(N, \tau_N)$  is a  $t_{\#}^{N\alpha}$ -space.

□

**Definition 3.5.** A function  $D : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  is called

(a) an  $N\alpha^{M\#}$ -continuous if  $D^{-1}(Y)$  is  $N\alpha^{M\#}CS$  in  $(N^I, \tau_N^i)$  for every closed set  $Y$  of  $(N^{II}, \tau_N^{ii})$ .

(b) an  $N\alpha^{M\#}$ -irresolute if  $D^{-1}(Y)$  is  $N\alpha^{M\#}CS$  in  $(N^I, \tau_N^i)$  for every  $N\alpha^{M\#}CS$   $Y$  of  $(N^{II}, \tau_N^{ii})$ .

**Example 3.6.** Let  $N = \{n_1, n_2, n_3\}$  with  $\tau_N = \{0_N, 1_N, \eta_1^{\#}, \eta_2^{\#}, \eta_3^{\#}, \eta_4^{\#}\}$  and  $\delta_N = \{0_N, 1_N, \eta_1^*, \eta_2^*, \eta_3^*, \eta_4^*\}$  where

$$\eta_1^{\#} = \langle (\frac{4}{10}, \frac{4}{10}, \frac{6}{10}), (\frac{5}{10}, \frac{4}{10}, \frac{6}{10}), (\frac{5}{10}, \frac{8}{10}, \frac{7}{10}) \rangle,$$

$$\eta_2^{\#} = \langle (\frac{5}{10}, \frac{7}{10}, \frac{7}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{5}{10}), (\frac{5}{10}, \frac{5}{10}, \frac{4}{10}) \rangle,$$

$$\eta_3^{\#} = \langle (\frac{5}{10}, \frac{7}{10}, \frac{7}{10}), (\frac{5}{10}, \frac{5}{10}, \frac{5}{10}), (\frac{5}{10}, \frac{5}{10}, \frac{4}{10}) \rangle,$$

$$\eta_4^{\#} = \langle (\frac{4}{10}, \frac{4}{10}, \frac{3}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{5}{10}), (\frac{5}{10}, \frac{7}{10}, \frac{7}{10}) \rangle,$$

$$\eta_1^* = \langle (\frac{5}{10}, \frac{7}{10}, \frac{6}{10}), (\frac{4}{10}, \frac{5}{10}, \frac{4}{10}), (\frac{5}{10}, \frac{5}{10}, \frac{6}{10}) \rangle,$$

$$\eta_2^* = \langle (\frac{5}{10}, \frac{5}{10}, \frac{5}{10}), (\frac{4}{10}, \frac{4}{10}, \frac{3}{10}), (\frac{5}{10}, \frac{7}{10}, \frac{6}{10}) \rangle,$$

$$\eta_3^* = \langle (\frac{5}{10}, \frac{5}{10}, \frac{5}{10}), (\frac{4}{10}, \frac{5}{10}, \frac{4}{10}), (\frac{5}{10}, \frac{7}{10}, \frac{6}{10}) \rangle,$$

$$\eta_4^* = \langle (\frac{5}{10}, \frac{7}{10}, \frac{6}{10}), (\frac{4}{10}, \frac{4}{10}, \frac{3}{10}), (\frac{5}{10}, \frac{5}{10}, \frac{6}{10}) \rangle. \text{ Thus, } (N, \tau_N) \text{ and } (N, \delta_N) \text{ are Nutrosophic}$$

Topologies. Define  $\Lambda : (N, \tau_N) \longrightarrow (N, \delta_N)$  as  $\Lambda(n_1) = n_1, \Lambda(n_2) = n_3, \Lambda(n_3) = n_2$ .

Then  $\Lambda$  is  $N\alpha^{M\#}$ -continuous, since  $\Lambda^{-1}(L_{\#})$  is  $N\alpha^{M\#}CS$  in  $(N, \tau_N)$  for every closed set  $L_{\#}$  of  $(N, \delta_N)$  where  $L_{\#} = \langle (\frac{3}{10}, \frac{5}{10}, \frac{5}{10}), (\frac{4}{10}, \frac{4}{10}, \frac{4}{10}), (\frac{5}{10}, \frac{5}{10}, \frac{6}{10}) \rangle$ .

**Proposition 3.7.** If  $D : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  be an  $N\alpha^{M\#}$ -continuous function and  $(N^I, \tau_N^i)$  be a  $t_{\#}^{N\alpha}$ -space,  $D$  is continuous.

*Proof.* Assume  $Y$  to be closed in  $(N^{II}, \tau_N^{ii})$ . As such  $D$  is an  $N\alpha^{M\#}$ -continuous function,  $D^{-1}(Y)$  is an  $N\alpha^{M\#}CS$  in  $(N^I, \tau_N^i)$ . Since  $(N^I, \tau_N^i)$  is a  $t_{\#}^{N\alpha}$ -space,  $D^{-1}(Y)$  is closed set in  $(N^I, \tau_N^i)$ . Hence  $D$  is continuous.

□

**Remark 3.8.** Let  $D : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  be a mapping and  $(N^I, \tau_N^i)$  be a  $t_{\#}^{N\alpha}$ -space, then  $D$  is continuous if one of the following conditions is satisfied.

(a)  $f$  is  $N\alpha^{M\#}$ -continuous.

(b)  $f$  is  $N\alpha^{M\#}$ -irresolute.

**Theorem 3.9.** A map  $D : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  is an  $N\alpha^{M\#}$ -continuous function  $\iff$  every open set's inverted image in  $(N^{II}, \tau_N^{ii})$  are the  $N\alpha^{M\#}$ OS in the  $(N^I, \tau_N^i)$ .

*Proof. Necessity :* Assume  $D : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  be an  $N\alpha^{M\#}$ -continuous function and  $Z$  be a collection that is open in  $(N^{II}, \tau_N^{ii})$ ,  $N^{II} - Z$  is closed  $(N^{II}, \tau_N^{ii})$ . As such  $D$  is an  $N\alpha^{M\#}$ -continuous function,  $f^{-1}(N^{II} - Z) = N^I - D^{-1}(Z)$  is an  $N\alpha^{M\#}$ CS in  $(N^I, \tau_N^i)$  and hence  $D^{-1}(Z)$  is an  $N\alpha^{M\#}$ OS in  $(N^I, \tau_N^i)$ .

*Sufficiency :* Assume that  $D^{-1}(Y)$  is an  $N\alpha^{M\#}$ OS in  $(N^I, \tau_N^i)$  for each open set  $N^{II}$  in  $(N^{II}, \tau_N^{ii})$ . Assume  $Y$  is a closed set in  $(N^{II}, \tau_N^{ii})$ ,  $N^{II} - Y$  is a set that is open in  $(N^{II}, \tau_N^{ii})$ . By assumption,  $D^{-1}(N^{II} - Y) = N^I - D^{-1}(Y)$  is an  $N\alpha^{M\#}$ OS in  $(N^I, \tau_N^i)$ , which implies that  $D^{-1}(Y)$  is an  $N\alpha^{M\#}$ CS in  $(N^I, \tau_N^i)$ . Hence  $D$  is an  $N\alpha^{M\#}$ -continuous.

□

**Proposition 3.10.** Let  $D_1 : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  be any topological space that is neutrosophic  $(N^{II}, \tau_N^{ii})$  is a  $t_{\#}^{N\alpha}$ -space. If  $D_1 : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  and  $D_2 : (N^{II}, \tau_N^{ii}) \longrightarrow (N^{III}, \tau_N^{iii})$  are  $N\alpha^{M\#}$ -continuous functions, then their composition  $D_2 \circ D_1 : (N^I, \tau_N^i) \longrightarrow (N^{III}, \tau_N^{iii})$  is an  $N\alpha^{M\#}$ -continuous.

*Proof.* Assume  $Y$  is a closed set in  $(N^{III}, \tau_N^{iii})$ . As such  $D_2 : (N^{II}, \tau_N^{ii}) \longrightarrow (N^{III}, \tau_N^{iii})$  is an  $N\alpha^{M\#}$ -continuous function,  $D_2^{-1}(Y)$  is an  $N\alpha^{M\#}$ CS in  $(N^{II}, \tau_N^{ii})$ . Since  $(N^{II}, \tau_N^{ii})$  is a  $t_{\#}^{N\alpha}$ -space,  $D_2^{-1}(Y)$  is a closed set in  $(N^{II}, \tau_N^{ii})$ . Since  $D_1 : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  is an  $N\alpha^{M\#}$ -continuous function,  $D_1^{-1}(D_2^{-1}(Y)) = (D_2 \circ D_1)^{-1}(Y)$  is an  $N\alpha^{M\#}$ CS in  $(N^I, \tau_N^i)$ . Hence  $D_2 \circ D_1 : (N^I, \tau_N^i) \longrightarrow (N^{III}, \tau_N^{iii})$  is an  $N\alpha^{M\#}$ -continuous function.

□

**Definition 3.11.** A map  $D : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  is said to be

(p)  $N\alpha^{M\#}$ -closed map if  $D(Y)$  is  $N\alpha^{M\#}$ -closed in  $(N^{II}, \tau_N^{ii})$  for every NCS  $Y$  of  $(N^I, \tau_N^i)$ .

(q)  $N\alpha^{M\#}$ -open map if  $D(Y)$  is  $N\alpha^{M\#}$ -open in  $(N^{II}, \tau_N^{ii})$  for every NOS  $Y$  of  $(N^I, \tau_N^i)$ .

**Theorem 3.12.** Let  $D_1 : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  and  $D_2 : (N^{II}, \tau_N^{ii}) \longrightarrow (N^{III}, \tau_N^{iii})$  be two mappings and  $(N^{II}, \tau_N^{ii})$  be a  $t_{\#}^{N\alpha}$ -space, then

(a)  $D_2 \circ D_1$  is  $N\alpha^{M\#}$ -continuous, if  $D_1$  and  $D_2$  are  $N\alpha^{M\#}$ -continuous.

(b)  $D_2 \circ D_1$  is  $N\alpha^{M\#}$ -closed, if  $D_1$  and  $D_2$  are  $N\alpha^{M\#}$ -closed.

*Proof.* (a) Let  $Y$  be a NCS of  $(N^{III}, \tau_N^{iii})$ , then  $D_2^{-1}(Y)$  is  $N\alpha^{M\#}$ -closed set in  $(N^{II}, \tau_N^{ii})$ . Since  $(N^{II}, \tau_N^{ii})$  is a  $t_{\#}^{N\alpha}$ -space, then  $D_2^{-1}(Y)$  is a NCS in  $(N^{II}, \tau_N^{ii})$ . But  $D_1$  is  $N\alpha^{M\#}$ -continuous, then  $(D_2 \circ D_1)^{-1}(Y) = D_1^{-1}(D_2^{-1}(Y))$  is  $N\alpha^{M\#}$ -closed in  $(N^I, \tau_N^i)$  this implies that  $(D_2 \circ D_1)$  is  $N\alpha^{M\#}$ -continuous mappings.

(b) The proof is similar.

□

**Remark 3.13.** Let  $D : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  be a mapping from a  $t_{\#}^{N\alpha}$ -space  $(N^I, \tau_N^i)$  into a space  $(N^{II}, \tau_N^{ii})$ , then

(p)  $D_1$  is continuous mapping if,  $D_1$  is  $N\alpha^{M\#}$ -continuous.

(q)  $D_1$  is closed mapping if,  $D_1$  is  $N\alpha^{M\#}$ -closed.

**Theorem 3.14.** Let  $D : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  is surjective closed and  $N\alpha^{M\#}$ -irresolute, then  $(N^{II}, \tau_N^{ii})$   $t_{\#}^{N\alpha}$ -space if  $(N^I, \tau_N^i)$  is also  $t_{\#}^{N\alpha}$ -space.

*Proof.* Let  $Y$  be an  $N\alpha^{M\#}$ -closed subset of  $(N^{II}, \tau_N^{ii})$ . Then  $D_1^{-1}(Y)$  is  $N\alpha^{M\#}$ -closed set in  $(N^I, \tau_N^i)$ . Since,  $(N^I, \tau_N^i)$  is a  $t_{\#}^{N\alpha}$ -space, then  $D_1^{-1}(Y)$  is closed set in  $(N^I, \tau_N^i)$ . Hence,  $Y$  is closed set in  $(N^{II}, \tau_N^{ii})$  and so,  $(N^{II}, \tau_N^{ii})$  is  $t_{\#}^{N\alpha}$ -space.

□

**Proposition 3.15.** If  $D_1 : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  is  $N\alpha^{M\#}$ -closed,  $D_2 : (N^{II}, \tau_N^{ii}) \longrightarrow (N^{III}, \tau_N^{iii})$  is an  $N\alpha^{M\#}$ -closed, and  $(N^{II}, \tau_N^{ii})$  is a  $t_{\#}^{N\alpha}$ -space, then their composition

$D_2 \circ D_1 : (N^I, \tau_N^i) \longrightarrow (N^{III}, \tau_N^{iii})$  is  $N\alpha^{M\#}$ -closed.

*Proof.* Let  $N_1$  be a *NCS* of  $(N^I, \tau_N^i)$ . Then by assumption  $D_1(N_1)$  is  $N\alpha^{M\#}$ -closed in  $(N^{II}, \tau_N^{ii})$ . Since  $(N^{II}, \tau_N^{ii})$  is a  $t_{\#}^{N\alpha}$ -space,  $D_1(N_1)$  is *NCS* in  $(N^{II}, \tau_N^{ii})$  and again by assumption  $D_2(D_1(N_1))$  is  $N\alpha^{M\#}$ -closed in  $(N^{III}, \tau_N^{iii})$ . i.e.,  $(D_2 \circ D_1)(N_1)$  is  $N\alpha^{M\#}$ -closed in  $(N^{III}, \tau_N^{iii})$  and so  $D_2 \circ D_1$  is  $N\alpha^{M\#}$ -closed.

□

**Proposition 3.16.** For any bijection  $D : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  the following statements are equivalent:

(p)  $D^{-1} : (N^{II}, \tau_N^{ii}) \longrightarrow (N^I, \tau_N^i)$  is  $N\alpha^{M\#}$ -continuous.

(q)  $D$  is  $N\alpha^{M\#}$ -open map.

(r)  $D$  is  $N\alpha^{M\#}$ -closed map.

*Proof.* (p)  $\implies$  (q) Let  $U$  be a *NOS* of  $(N^I, \tau_N^i)$ . By assumption,  $(D^{-1})^{-1}(U) = D(U)$  is  $N\alpha^{M\#}$ -open in  $(N^{II}, \tau_N^{ii})$  and so  $D$  is  $N\alpha^{M\#}$ -open.

(q)  $\implies$  (r) Let  $F$  be a *NCS* of  $(N^I, \tau_N^i)$ . Then  $F^c$  is *NOS* in  $(N^I, \tau_N^i)$ . By assumption,  $D(F^c)$  is  $N\alpha^{M\#}$ -open in  $(N^{II}, \tau_N^{ii})$ . That is  $D(F^c) = (D(F))^c$  is  $N\alpha^{M\#}$ -open in  $(N^{II}, \tau_N^{ii})$  and therefore  $D(F)$  is  $N\alpha^{M\#}$ -closed in  $(N^{II}, \tau_N^{ii})$ . Hence  $D$  is  $N\alpha^{M\#}$ -closed.

(r)  $\implies$  (p) Let  $F$  be a *NCS* of  $(N^I, \tau_N^i)$ . By assumption,  $D(F)$  is  $N\alpha^{M\#}$ -closed in  $(N^{II}, \tau_N^{ii})$ . But  $D(F) = (D^{-1})^{-1}(F)$  and therefore  $D^{-1}$  is  $N\alpha^{M\#}$ -continuous.

□

#### 4. On $N\alpha^{M\#}$ -homeomorphisms

**Definition 4.1.** A function  $D : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  is supposed to be an  $N\alpha^{M\#}$ -homeomorphism  $[(hmpm(N, \tau_N))_{N\alpha^{M\#}}]$  if both  $D$  and  $D^{-1}$  are  $N\alpha^{M\#}$ -irresolute.



We are using the entire family of all  $N\alpha^{M\#}$ -homeomorphisms of a  $NTS (N^I, \tau_N^i)$  onto itself by  $N\alpha^{M\#}$ - $H(N, \tau_N)$ .

**Example 4.2.** Let  $M^{N^1} = \{\alpha, \beta\}$ ,  $M^{N^2} = \{\gamma, \delta\}$ ,  $O_1^\# = \langle (\frac{2}{10}, \frac{6}{10}, \frac{3}{10}), (\frac{3}{10}, \frac{6}{10}, \frac{4}{10}), (\frac{3}{10}, \frac{7}{10}, \frac{4}{10}) \rangle$ ,  $O_2^\# = \langle (\frac{4}{10}, \frac{6}{10}, \frac{5}{10}), (\frac{5}{10}, \frac{6}{10}, \frac{6}{10}), (\frac{5}{10}, \frac{7}{10}, \frac{6}{10}) \rangle$ . Then  $\tau_{E1} = \{0_E, 1_E, O_1^\#\}$  and  $\tau_{E2} = \{0_E, 1_E, O_2^\#\}$  are neutrosophic topologies on  $M^{N^1}$  and  $M^{N^2}$  respectively. Define a bijective mapping  $F_{Nf\#} = (M^{N^1}, \tau_{E1}) \rightarrow (M^{N^2}, \tau_{E2})$  by  $F_{Nf\#}(\alpha) = \gamma$  and  $F_{Nf\#}(\beta) = \delta$ . Then  $F_{Nf\#}$  is a  $N\alpha^{M\#}$ -irresolute  $F_{Nf\#}^{-1}$  is also a  $N\alpha^{M\#}$ -irresolute. Therefore the bijection function  $F_{Nf\#}$  is a  $(hmpm(N, \tau_N))_{N\alpha^{M\#}}$ .

**Proposition 4.3.** Let  $D_1 : (N^I, \tau_N^i) \rightarrow (N^{II}, \tau_N^{ii})$  and  $D_2 : (N^{II}, \tau_N^{ii}) \rightarrow (N^{III}, \tau_N^{iii})$  are  $(hmpm(N, \tau_N))_{N\alpha^{M\#}}$ , then their composition

$D_2 \circ D_1 : (N^I, \tau_N^i) \rightarrow (N^{III}, \tau_N^{iii})$  is also  $(hmpm(N, \tau_N))_{N\alpha^{M\#}}$ .

*Proof.* Let  $J$  be an  $N\alpha^{M\#}OS$  in  $(N^{III}, \tau_N^{iii})$ . Since  $D_2$  is  $N\alpha^{M\#}$ -irresolute,  $D_2^{-1}(J)$  is  $N\alpha^{M\#}OS$  in  $(N^{II}, \tau_N^{ii})$ . Since  $D_1$  is  $N\alpha^{M\#}$ -irresolute,  $D_1^{-1}(D_2^{-1}(Y)) = (D_2 \circ D_1)^{-1}(Y)$  is  $N\alpha^{M\#}OS$  in  $(N^I, \tau_N^i)$ . Therefore  $D_2 \circ D_1$  is  $N\alpha^{M\#}$ -irresolute.

Also for an  $N\alpha^{M\#}OS, G$  in  $(N^I, \tau_N^i)$ , we have  $(D_2 \circ D_1)(G) = D_2(D_1(G)) = D_2(W)$ , where  $W = D_1(G)$ . By hypothesis,  $D_1(G)$  is  $N\alpha^{M\#}OS$  in  $(N^{II}, \tau_N^{ii})$  and so again by hypothesis,  $D_2(D_1(G))$  is an  $N\alpha^{M\#}OS$  in  $(N^{III}, \tau_N^{iii})$ . That is  $(D_2 \circ D_1)(G)$  is an  $N\alpha^{M\#}OS$  in  $(N^{III}, \tau_N^{iii})$  and therefore  $(D_2 \circ D_1)^{-1}$  is  $N\alpha^{M\#}$ -irresolute. Also  $D_2 \circ D_1$  is a bijection. Hence  $D_2 \circ D_1$  is  $(hmpm(N, \tau_N))_{N\alpha^{M\#}}$ .

□

**Theorem 4.4.** The set  $N\alpha^{M\#}$ - $H(N, \tau_N)$  is a subset of the map composition.

*Proof.* Establish a binary operation  $* : N\alpha^{M\#}$ - $H(N, \tau_N) \times N\alpha^{M\#}$ - $H(N, \tau_N) \rightarrow N\alpha^{M\#}$ - $H(N, \tau_N)$  by  $D_1 * D_2 = D_2 \circ D_1$  for all  $D_1, D_2 \in N\alpha^{M\#}$ - $H(N, \tau_N)$  and  $circ$  is the standard map composition operation.  $D_2 \circ D_1 \in N\alpha^{M\#}$ - $H(N, \tau_N)$ .

We notice that maps are made up of associative elements, and the identity map is no exception  $I : (N, \tau_N) \rightarrow (N, \tau_N)$  belonging to  $N\alpha^{M\#}$ - $H(N, \tau_N)$  identity element as a distinguishing feature. If  $D_1 \in N\alpha^{M\#}$ - $H(N, \tau_N)$ , then  $D_1^{-1} \in N\alpha^{M\#}$ - $H(N, \tau_N)$  such that  $D_1 \circ D_1^{-1} = D_1^{-1} \circ D_1 = I$ . As a result, there is an inverse for each element of  $N\alpha^{M\#}$ - $H(N, \tau_N)$ .

Consequently  $N\alpha^{M\#}\text{-}H(N, \tau_N), \circ$  is a network of under the operation on map composition.

□

**Proposition 4.5.** *Let  $J : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  be an  $N\alpha^{M\#}$ -homeomorphism,  $J$  causes the group to become isomorphic  $N\alpha^{M\#}\text{-}H(N^I, \tau_N^i)$  onto  $N\alpha^{M\#}\text{-}H(N^{II}, \tau_N^{ii})$ .*

*Proof.* Making use of the map  $J$ , We construct a map  $\Psi_J : N\alpha^{M\#}\text{-}H(N^I, \tau_N^i) \longrightarrow N\alpha^{M\#}\text{-}H(N^{II}, \tau_N^{ii})$  by  $\Psi_J(F) = J \circ F \circ J^{-1}$  for every  $F \in N\alpha^{M\#}\text{-}H(N^I, \tau_N^i)$ . Then  $\Psi_J$  is a bijection. Further, for all  $h_1, h_2 \in N\alpha^{M\#}\text{-}H(N^I, \tau_N^i)$ ,  $\Psi_J(F_1 \circ F_2) = J \circ (F_1 \circ F_2) \circ J^{-1} = (J \circ F_1 \circ J^{-1}) \circ (J \circ F_2 \circ J^{-1}) = \Psi_J(F_1) \circ \Psi_J(F_2)$ .

Therefore,  $\Psi_J$  It is an isomorphism caused by a homeomorphism by  $J$ .

□

## 5. On $N\alpha^{M\#}$ -connectedness

**Definition 5.1.** A  $NTS(N, \tau_N)$  is noted to be  $N\alpha^{M\#}$ -connected if  $N$  can't be characterized as a non-empty union of two distinct elements  $N\alpha^{M\#}OS$ . A subset of  $N$  is  $N\alpha^{M\#}$ -connected if any of this  $N\alpha^{M\#}$ -connected as a subspace.

**Theorem 5.2.** *For a  $NTS(N, \tau_N)$ , the following are better compared.*

(a)  $(N, \tau_N)$  is  $N\alpha^{M\#}$ -connected.

(b)  $(N, \tau_N)$  and  $\phi_N$  seem to be the only subsets of  $(N, \tau_N)$  both of which are  $N\alpha^{M\#}$ -open and  $N\alpha^{M\#}$ -closed.

(c) Each  $N\alpha^{M\#}$ -continuous map of  $(N^I, \tau_N^i)$  into a discrete space  $(N^{II}, \tau_N^{ii})$  the map is constant if there are at least two points.

*Proof.* (a)  $\implies$  (b): Suppose  $(N^I, \tau_N^i)$  is  $N\alpha^{M\#}$ -connected. Let  $S$  be both a valid subset  $N\alpha^{M\#}OS$  and  $N\alpha^{M\#}CS$  in  $(N^I, \tau_N^i)$ . Its complement  $N/S$  is also  $N\alpha^{M\#}$ -open and  $N\alpha^{M\#}$ -closed.  $N = S \cup (N/S)$ , a non-empty union that is disjointed  $N\alpha^{M\#}$ -open sets that are incompatible (a). Therefore  $S = \phi$  or  $N$ .

(b)  $\implies$  (a): Suppose that  $N = I_1 \cup I_2$  where  $I_1$  and  $I_2$  are disjoint non-empty  $N\alpha^{M\#}$ -open subsets of  $(N^I, \tau_N^i)$ . Then  $I_1$  is both  $N\alpha^{M\#}$ -open and  $N\alpha^{M\#}$ -closed. By assumption  $I_1 = \phi$  or  $N$ . Therefore  $N$  is  $N\alpha^{M\#}$ -connected.

(b)  $\implies$  (c): Let  $D : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  be an  $N\alpha^{M\#}$ -continuous map. Then  $(N^I, \tau_N^i)$  is covered by  $N\alpha^{M\#}$ -open and  $N\alpha^{M\#}$ -closed covering  $\{D^{-1}(n_{ii}) : n_{ii} \in N_{ii}\}$ . By assumption  $D^{-1}(n_{ii}) = \phi_N$  or  $N$  for each  $n_{ii} \in N_{ii}$ . If  $D^{-1}(n_{ii}) = \phi$  for all  $n_{ii} \in N_{ii}$ , then  $D$  a map that isn't a map. Then  $\exists$  a point  $n_{ii} \in N_{ii}$  such that  $D^{-1}(n_{ii}) \neq \phi_N$  and hence  $D^{-1}(n_{ii}) = N$ . This shows that  $D$  is a constant map.

(c)  $\implies$  (b): Let  $S$  be both  $N\alpha^{M\#}$ -open and  $N\alpha^{M\#}$ -closed in  $N$ . Suppose  $S \neq \phi$ . Let  $D : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  be an  $N\alpha^{M\#}$ -continuous map defined by  $D(S) = n_{ii}$  and  $D(S^c) = \{\omega\}$  for a few key reasons  $n_{ii}$  and  $\omega$  in  $(N^{II}, \tau_N^{ii})$ . By assumption  $D$  is a constant map. Therefore we have  $S = N$ .

□

**Theorem 5.3.** *Every  $N\alpha^{M\#}$ -Space that is linked is connected.*

*Proof.* Let  $(N^I, \tau_N^i)$  be  $N\alpha^{M\#}$ -linked(connected). Suppose  $N$  is not connected. There is then a suitable non-empty subset.  $B$  of  $(N^I, \tau_N^i)$  which has both an open and a closed sets in  $(N^I, \tau_N^i)$ . Since every closed set is  $N\alpha^{M\#}$ -closed,  $B$  is a proper non empty subset of  $(N^I, \tau_N^i)$  as well as  $N\alpha^{M\#}OS$  and  $N\alpha^{M\#}CS$  in  $(N^I, \tau_N^i)$ ,  $(N^I, \tau_N^i)$  is not  $N\alpha^{M\#}$ -connected. This proves the theorem.

□

**Theorem 5.4.** *If  $J : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  is an  $N\alpha^{M\#}$ -continuous and  $N$  is  $N\alpha^{M\#}$ -connected, then  $(N^{II}, \tau_N^{ii})$  is linked.*

*Proof.* Presume that  $(N^{II}, \tau_N^{ii})$  is not linked. Let  $N^{ii} = V_1 \cup V_2$  where  $V_1$  and  $V_2$  are disjoint non-empty  $OS$  in  $(N^{II}, \tau_N^{ii})$ . As such  $J$  is  $N\alpha^{M\#}$ -continuous and onto,  $N = J^{-1}(V_1) \cup J^{-1}(V_2)$  where  $J^{-1}(V_1)$  and  $J^{-1}(V_2)$  are disjoint non-empty  $N\alpha^{M\#}$ -open sets in  $(N^I, \tau_N^i)$ .

This is diametrically opposed to the fact that  $(N^I, \tau_N^i)$  is  $N\alpha^{M\#}$ -connected. Furthermore  $N^{ii}$  is connected.

□

**Theorem 5.5.** *If  $J : (N^I, \tau_N^i) \longrightarrow (N^{II}, \tau_N^{ii})$  is an  $N\alpha^{M\#}$ -irresolute and  $(N^I, \tau_N^i)$  is  $N\alpha^{M\#}$ -connected, then  $(N^{II}, \tau_N^{ii})$  is  $N\alpha^{M\#}$ -connected.*

*Proof.* Suppose that  $(N^{II}, \tau_N^{ii})$  is not  $N\alpha^{M\#}$ -connected. Let  $N^{ii} = V_1 \cup V_2$  where  $V_1$  and  $V_2$  are disjoint non-empty  $N\alpha^{M\#}$ -open sets in  $(N^{II}, \tau_N^{ii})$ . Since  $J$  is  $N\alpha^{M\#}$ -irresolute and onto,  $N = j^{-1}(V_1) \cup j^{-1}(V_2)$  where  $J^{-1}(V_1)$  and  $J^{-1}(V_2)$  are disjoint non-empty  $N\alpha^{M\#}$ -open sets in  $(N^I, \tau_N^i)$ .

This contradicts the fact that  $(N^I, \tau_N^i)$  is  $N\alpha^{M\#}$ -connected. Hence  $(N^{II}, \tau_N^{ii})$  is  $N\alpha^{M\#}$ -connected.

□

**Theorem 5.6.** *Suppose that  $(N^I, \tau_N^i)$  is  $t_{\#}^{N\alpha}$ -space then  $(N^I, \tau_N^i)$  is connected  $\iff N\alpha^{M\#}$ -connected.*

*Proof.* Suppose that  $(N^I, \tau_N^i)$  is connected. Then  $(N^I, \tau_N^i)$  disjoint union of two non-empty proper subsets of the set cannot be expressed in  $(N^I, \tau_N^i)$ . Suppose  $(N^I, \tau_N^i)$  is not a  $N\alpha^{M\#}$ -connected space. Let  $V_1$  and  $V_2$  be any two  $N\alpha^{M\#}$ -open subsets of  $(N^I, \tau_N^i)$  such that  $N^{ii} = V_1 \cup V_2$ , where  $V_1 \cap V_2 = \phi_N$  and  $V_1 \subset N, V_2 \subset N$ . Since  $(N^I, \tau_N^i)$  is  $t_{\#}^{N\alpha}$ -space and  $V_1, V_2$  are  $N\alpha^{M\#}$ -open.  $V_1, V_2$  are open subsets of  $(N^I, \tau_N^i)$ , which contradicts that  $(N^I, \tau_N^i)$  is connected. Therefore  $(N^I, \tau_N^i)$  is  $N\alpha^{M\#}$ -connected.

Conversely, every open set is  $N\alpha^{M\#}$ -open. Therefore every  $N\alpha^{M\#}$ -connected space is connected.

□

**Theorem 5.7.** *If the  $N\alpha^{M\#}$ -open sets  $Z_1$  and  $Z_2$  form a separation of  $(N^I, \tau_N^i)$  and if  $(N^{II}, \tau_N^{ii})$  is  $N\alpha^{M\#}$ -connected subspace of  $(N^I, \tau_N^i)$ , then  $(N^{II}, \tau_N^{ii})$  lies entirely within  $Z_1$  or  $Z_2$ .*

*Proof.* Since  $Z_1$  and  $Z_2$  are both  $N\alpha^{M\#}$ -open in  $(N^I, \tau_N^i)$ , the sets  $Z_1 \cap N^{ii}$  and  $Z_2 \cap N^{ii}$  are  $N\alpha^{M\#}$ -open in  $(N^{II}, \tau_N^{ii})$ . These two sets are incompatible, thus their union is impossible in  $(N^{II}, \tau_N^{ii})$ . They would represent a separation if they were both non-empty  $(N^{II}, \tau_N^{ii})$ .

Therefore, one of them is empty. Hence  $(N^{II}, \tau_N^{ii})$  must lie entirely in  $Z_1$  or in  $Z_2$ .

□

## 6. Conclusion

The notions of  $N\alpha^{M\#}CS$  in neutrosophic topological spaces have been discussed in this research study. We have also introduced the neutrosophic  $t_{\#}^{N\alpha}$ -space in this paper. The mappings known as neutrosophic  $N\alpha^{M\#}$ -continuous functions,  $N\alpha^{M\#}$ -irresolute functions, homeomorphisms and connectedness have also been introduced and investigate their characterizations and distinguishing features.

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Received: August 16, 2022. Accepted: January 12, 2023