

5-1-2023

An Asymmetric Measure of Comparison of Neutrosophic Sets

Surender Singh

Sonam Sharma

Follow this and additional works at: https://digitalrepository.unm.edu/nss_journal

Recommended Citation

Singh, Surender and Sonam Sharma. "An Asymmetric Measure of Comparison of Neutrosophic Sets." *Neutrosophic Sets and Systems* 55, 1 (2023). https://digitalrepository.unm.edu/nss_journal/vol55/iss1/15

This Article is brought to you for free and open access by UNM Digital Repository. It has been accepted for inclusion in *Neutrosophic Sets and Systems* by an authorized editor of UNM Digital Repository. For more information, please contact disc@unm.edu.



An Asymmetric Measure of Comparison of Neutrosophic Sets

Surender Singh^{1,*} and Sonam Sharma²

¹ School of Mathematics, Shri Mata Vaishno Devi University, Katra-182320, Jammu and Kashmir, INDIA
surender.singh@smvdu.ac.in

² School of Mathematics, Shri Mata Vaishno Devi University, Katra-182320, Jammu and Kashmir, INDIA
20dmt001@smvdu.ac.in

* Correspondence: surender.singh@smvdu.ac.in; surender1976@gmail.com

Abstract: The single-valued neutrosophic (SVNS) set is a beneficial and significant tool to deal with uncertainty with the neutrality of truth. This article introduces a non-conventional asymmetric measure of comparison in the single-valued neutrosophic framework. Such a measure is applied to the problems where the conventional symmetric measures of comparison do not produce valid computational results and require a directed closeness or discrimination between two abstractions represented by neutrosophic sets. We prove some properties of the proposed neutrosophic comparison measure and empirically justify its utilization in a problem of strategic decision-making, pattern recognition and medical diagnosis. The assessment of the performance of the proposed measure using “Degree of Confidence” shows the advantage of the proposed measure.

Keywords: Single-valued neutrosophic set (SVNS); asymmetric measure; inaccuracy measure; pattern recognition.

1. Introduction

Many problems concerning decision-making, identification of patterns, machine learning, computer vision, data analytics, etc., predominantly utilize some measure of comparisons. Several studies are available regarding comparison measures in various uncertain and vague settings. The prevalently investigated comparison measures in uncertain environments are divergence, distance, dissimilarity, and similarity measures. One common characteristic of divergence measures, distance measures, dissimilarity measures, and similarity measures in fuzzy and non-standard fuzzy settings is that these are symmetric. But, in specific comparisons, the symmetric comparison is not suitable. For instance, “P is like Q” may be preferred over “Q is like P” or vice-versa. Such situations need an asymmetric or directed comparison measure. This study proposes an asymmetric measure of comparison for single-valued neutrosophic sets.

However, in an uncertain environment due to randomness, an asymmetric measure of comparison of two probability distributions was proposed by Kullback-Leibler [1-2]. These measures found vital application in communication theory and economics. Kerridge inaccuracy is a non-parametric generalization of Shannon’s entropy [3]. Kerridge [4] termed it an ‘inaccuracy measure’ for measuring the inaccuracy between two probability distributions. Moreover, numerous probabilistic information measures were put forward during the second half of the twentieth century.

In 1965, Zadeh [5] coined a new form of uncertainty due to vagueness or linguistic imprecision and developed fuzzy theory. As Shannon’s entropy [3] quantifies the uncertainty due to randomness,

De-Luca and Termini [6] introduced fuzzy entropy to quantify the uncertainty due to vagueness. This pioneered fuzzy entropy is structurally similar to that of Shannon's entropy [3] but practically different. Various extensions of fuzzy theory and information measures in these frameworks have been developed in the last three decades.

Smarandache [7] developed a more advanced notion, "Neutrosophy," to more comprehensively model the vagary of information. The neutrosophic theory reconciles certain pitfalls of the fuzzy approach and its extensions. Wang *et al.* [8] proposed a single-valued neutrosophic set as a subclass of a neutrosophic set. In SVN, the data information is indicated with 3-tuple, i.e., degree of membership, degree of indeterminacy, and degree of non-membership. Hence, the information evaluation in terms of neutrosophic sets seems more suitable for studies concerning decision-making, pattern recognition, clustering analysis, etc. A single-valued neutrosophic set allows us to choose truth membership, false membership, and indeterminacy in an unrestrictive manner in contrast to other fuzzy extensions.

Several neutrosophic information measures have been proposed, such as entropy, similarity, distance, and divergence measures over the years. Some prominent researches are due to Chai *et al.* [9], Wang [10], Biswas, *et al.* [11], Wu *et al.* [12], Aydogdu [13], Bourmi and Smarandache [14], Bourmi and Smarandache [15], Khan *et al.*, [16], Majumdar and Samanta [17], Ye [18-19], Ye and Fu [20], Chakraborty *et al.* [21], Chakraborty *et al.* [22], Haque *et al.* [23], Haque *et al.* [24], and Chakraborty *et al.* [25], Bonissone [26], Eshragh and Mamdani [27], etc. References [28-30] also report the work on developing new similarity/distance measures for fuzzy and SVN.

Motivation and contribution

Recent trends notice that all the existing measures of comparison (distance/similarity/divergence) in the neutrosophic framework are symmetric. But there are practical circumstances where the asymmetric comparison is more suitable. For example, we consider the following two sentences:

- I. Saddam Hussain was like Hitler.
- II. Hitler was like Saddam Hussain.

Off course, sentence-I would be the apparent preference for the comparison, probably due to the genocide instinct of the latter. In such a situation, one concept is the target, another is the base, and the main focus is the target. In sentence-I and -II, Saddam Hussain is the target, and Hitler is the base.

Further, a problem of medical diagnosis, where the symptoms of the patient are compared with symptoms of certain diseases (as established by medical experts), also seems to be better dealt with using asymmetric comparison measures. In such a problem, the patient's symptoms in single-valued neutrosophic representation (P) must be treated as a target, and the pre-assigned symptoms of the disease (Q) may be treated as a base. The direction of comparison in this problem must be $Q \rightarrow P$ instead of $P \rightarrow Q$. Such asymmetric comparisons are unavoidable in any discipline and can essentially need to be investigated using some asymmetric measures of comparison. In view of these facts, natural question arise what is the concept of asymmetric measure of comparison? How to construct an asymmetric measure of comparison in neutrosophic environment? Does such a comparison measure practically valid and effective? Moreover, to best of our knowledge there is no asymmetric

measure of information in the literature concerning neutrosophic information theory. Here-mentioned facts and research gap in neutrosophic information theory motivated us to consider this study.

The novel contribution of this article is as follows.

- We introduce a novel concept of asymmetric comparison measure for SVNSSs and term it a "Single-Valued Neutrosophic Inaccuracy Measure."
- We prove some algebraic properties of the proposed comparison measure for SVNSSs.
- We also deduce a performance index "Degree of Confidence" to examine the performance of various neutrosophic comparison measures in the classification problems
- We also discuss applications of the proposed measure in pattern recognition and medical diagnosis problem.

The remaining paper is structured as follows. Section 2 presents preliminaries. Section 3 introduces inaccuracy measures/asymmetric measure of comparison between SVNSSs. In section 4, we discuss some properties of the proposed measure. Section 5 presents an application of the proposed inaccuracy measure. Section 6 includes the comparative study. Finally, Section 7 concludes the article.

2. Preliminaries

This section considers some notions related to single-valued neutrosophic sets and inaccuracy measures.

Definition 2.1[3]. Let $Y = (y_1, y_2, y_3, \dots, y_n)$ be a random variable associated with an experiment. Let $P = (p_1, p_2, p_3, \dots, p_n)$ be the probability distribution of random variable Y. Shannon's entropy measure is given by

$$H(P) = -\sum_{i=1}^n p_i \log_2 p_i.$$

Definition 2.2[1][2]. Let $Y = (y_1, y_2, y_3, \dots, y_n)$ be a random variable associated with an experiment. Let $P = (p_1, p_2, p_3, \dots, p_n)$ and $Q = (q_1, q_2, q_3, \dots, q_n)$ be two probability distributions. Then the divergence measure between P and Q is given by

$$D(P, Q) = \sum_{i=1}^n p_i \log_2 \frac{p_i}{q_i}.$$

Definition 2.3[4]. Let $P = (p_1, p_2, p_3, \dots, p_n)$ and $Q = (q_1, q_2, q_3, \dots, q_n)$ be two probability distributions. Then inaccuracy of distribution Q with respect to distribution P is given by

$$I(P, Q) = -\sum_{i=1}^n p_i \log_2 q_i.$$

A particular case of a neutrosophic set is a single-valued neutrosophic set which was proposed by Wang *et al.* [8]

Definition 2.5[8]. Let y_i be a generic element of the universal set Y. A truth-membership function characterizes a single-valued neutrosophic set $\rho_A(y_i)$, indeterminacy-membership function $\theta_A(y_i)$ and falsity-membership function $\delta_A(y_i)$. Also, for each $y_i \in Y$, $\rho_A(y_i), \theta_A(y_i), \delta_A(y_i) \in [0, 1]$ with condition $\rho_A(y_i) + \theta_A(y_i) + \delta_A(y_i) \in [0, 3]$.

In other words, a single-valued neutrosophic set A can be denoted by a triplet, *i.e.*,

$$A = \{(\rho_A(y_i), \theta_A(y_i), \delta_A(y_i)) \mid y_i \in Y\}$$

Notation: SVNS (Y) denotes the set of all neutrosophic elements in Y .

Some of the basic and useful operations on SVNS are defined as follows:

Definition 2.6[8]. Let $A = \{(\rho_A(y_i), \theta_A(y_i), \delta_A(y_i)) \mid y_i \in Y\}$ and

$$B = \{(\rho_B(y_i), \theta_B(y_i), \delta_B(y_i)) \mid y_i \in Y\}.$$

be two SVNSs, then the union of A and B is defined as

$$A \cup B = \{< \max. (\rho_A(y_i), \rho_B(y_i)), \min. (\theta_A(y_i), \theta_B(y_i)), \min. (\delta_A(y_i), \delta_B(y_i)) > \mid y_i \in Y\}.$$

Definition 2.7 [8] For two SVNSs, A and B , the intersection of A and B is

$$A \cap B = \{< \min. (\rho_A(y_i), \rho_B(y_i)), \max. (\theta_A(y_i), \theta_B(y_i)), \max. (\delta_A(y_i), \delta_B(y_i)) > \mid y_i \in Y\}.$$

Definition 2.8[8]. Let $A = \{(\rho_A(y_i), \theta_A(y_i), \delta_A(y_i)) \mid y_i \in Y\}$ be an SVNS. Then the complement of A is defined as

$$A^c = \{< 1 - \rho_A(y_i), 1 - \theta_A(y_i), 1 - \delta_A(y_i) > \mid y_i \in Y\}.$$

Definition 2.9[8]. Let $A = \{(\rho_A(y_i), \theta_A(y_i), \delta_A(y_i)) \mid y_i \in Y\}$ and $B = \{(\rho_B(y_i), \theta_B(y_i), \delta_B(y_i)) \mid y_i \in Y\}$ be two SVNSs, then $A \subseteq B$ if

$$\rho_A(y_i) \leq \rho_B(y_i), \theta_A(y_i) \geq \theta_B(y_i), \delta_A(y_i) \geq \delta_B(y_i), \forall y_i \in Y.$$

Hatzimichailidis [31] introduced the notion of Degree of confidence (DoC) in intuitionistic fuzzy environment. The definition of DoC in neutrosophic settings is as follows.

Definition 2.10. Let P_i be an unknown pattern classified to some pattern from the class P_j . Degree of confidence of neutrosophic comparison measure M estimates the confidence level that comparison measure in classifying a pattern P_i to the pattern P_k (belongs to a class of patterns) and it can be computed as

$$DOC = \sum_{j=1, j \neq k}^n |M(P_i, P_k) - M(P_i, P_j)|.$$

The greater the degree of confidence (DOC) for a comparison measure, the more confident the classification result of the measure is.

3. Inaccuracy Measure of a Single-Valued Neutrosophic Set

In this section, we propose an inaccuracy measure of single-valued neutrosophic sets and discuss their properties. Verma and Sharma [32] presented an inaccuracy measure of fuzzy sets as follows:

$$I(A, B) = -\frac{1}{n} \sum_{i=1}^n [\rho_A(y_i) \log(\rho_B(y_i)) + (1 - \rho_A(y_i)) \log(1 - \rho_B(y_i))]. \quad (1)$$

where ρ_A and ρ_B are the membership functions associated with fuzzy sets A and B .

We can write

$$I(A, B) = -\frac{1}{n} \sum_{i=1}^n S(x, y). \quad (2)$$

where, $S(x, y) = -x \log y - (1-x) \log(1-y)$ is called Karridge's inaccuracy function for two events.

Since in a single-valued neutrosophic set, the non-membership and indeterminacy are independent of the membership function; therefore, the inaccuracy function utilized in equation (2) can be modified as

$$S(x, y) = -x_1 \log y_1 - x_2 \log y_2 - x_3 \log y_3, \quad (3)$$

where $x_i, y_i \in [0, 1], i = 1, 2, 3$.

Consequently, the inaccuracy measure for two single-valued neutrosophic sets $A = \{(\rho_A(y_i), \theta_A(y_i), \delta_A(y_i)) | y_i \in Y\}$ and $B = \{(\rho_B(y_i), \theta_B(y_i), \delta_B(y_i)) | y_i \in Y\}$, is defined as follows:

$$I_{SVNS}(A, B) = -\frac{1}{n} \sum_{i=1}^n [\rho_A(y_i) \log(\rho_B(y_i)) + \theta_A(y_i) \log \theta_B(y_i) + \delta_A(y_i) \log \delta_B(y_i)]. \quad (4)$$

with convention $0 \cdot \log 0 = 0$.

Next, we prove some properties of the proposed inaccuracy measure of SVNNSs.

Theorem 3.1. Let $A, B, C \in SVNNS(Y)$, the proposed inaccuracy measure satisfies the following properties:

- $I_{SVNS}(A, B) = 0$ if and only if either $\rho_A(y_i) = \rho_B(y_i) = 0, \theta_A(y_i) = \theta_B(y_i) = 0, \delta_A(y_i) = \delta_B(y_i) = 0$ or $\rho_A(y_i) = \rho_B(y_i) = 1, \theta_A(y_i) = \theta_B(y_i) = 1, \delta_A(y_i) = \delta_B(y_i) = 1$ where $i = 1, 2, 3, \dots, n; \forall A, B, C \in SVNNS(Y)$.
- $I_{SVNS}(A, B \cup C) + I_{SVNS}(A, B \cap C) = I_{SVNS}(A, B) + I_{SVNS}(A, C) \quad \forall A, B, C \in SVNNS(Y)$.
- $I_{SVNS}(A \cup B, C) + I_{SVNS}(A \cap B, C) = (I_{SVNS}(A, C) + I_{SVNS}(B, C)); \quad \forall A, B, C \in SVNNS(Y)$.
- $I_{SVNS}(A \cup B, A \cap B) + I_{SVNS}(A \cap B, A \cup B) = I_{SVNS}(A, B) + I_{SVNS}(B, A); \quad \forall A, B \in SVNNS(Y)$.

Proof. a)

Let $I_{SVNS}(A, B) = 0$, then, from Eq. (4), we have

$$-\frac{1}{n} \sum_{i=1}^n [\rho_A(y_i) \log(\rho_B(y_i)) + \theta_A(y_i) \log \theta_B(y_i) + \delta_A(y_i) \log \delta_B(y_i)] = 0,$$

$$[\rho_A(y_i) \log(\rho_B(y_i)) + \theta_A(y_i) \log \theta_B(y_i) + \delta_A(y_i) \log \delta_B(y_i)] = 0 \quad \forall i = 1, 2, 3, \dots, n.$$

The above relation holds, if and only if

$$\rho_A(y_i) = \rho_B(y_i) = 0, \theta_A(y_i) = \theta_B(y_i) = 0, \delta_A(y_i) = \delta_B(y_i) = 0$$

or $\rho_A(y_i) = \rho_B(y_i) = 1, \theta_A(y_i) = \theta_B(y_i) = 1, \delta_A(y_i) = \delta_B(y_i) = 1 \quad \forall i = 1, 2, 3, \dots, n.$

Conversely,

$$\text{Suppose, } \rho_A(y_i) = \rho_B(y_i) = 0, \theta_A(y_i) = \theta_B(y_i) = 0, \delta_A(y_i) = \delta_B(y_i) = 0$$

$$\text{or } \rho_A(y_i) = \rho_B(y_i) = 1, \theta_A(y_i) = \theta_B(y_i) = 1, \delta_A(y_i) = \delta_B(y_i) = 1 \quad \forall i = 1, 2, 3, \dots, n.$$

$$\text{i.e., } [\rho_A(y_i) \log(\rho_B(y_i)) + \theta_A(y_i) \log \theta_B(y_i) + \delta_A(y_i) \log \delta_B(y_i)] = 0$$

or,

$$-\frac{1}{n} \sum_{i=1}^n [\rho_A(y_i) \log(\rho_B(y_i)) + \theta_A(y_i) \log \theta_B(y_i) + \delta_A(y_i) \log \delta_B(y_i)] = 0$$

Which implies that $I_{SVNS}(A, B) = 0$. \square

Proof. b)

For this, we divide the universal set Y into two disjoint subsets, *i.e.*,

$$Y_1 = \{\rho_A(y_i) \geq \rho_B(y_i) \geq \rho_C(y_i); \theta_A(y_i) \leq \theta_B(y_i) \leq \theta_C(y_i); \delta_A(y_i) \leq \delta_B(y_i) \leq \delta_C(y_i) | y_i \in Y\} \quad (5)$$

$$Y_2 = \{\rho_A(y_i) \leq \rho_B(y_i) \leq \rho_C(y_i); \theta_A(y_i) \geq \theta_B(y_i) \geq \theta_C(y_i); \delta_A(y_i) \geq \delta_B(y_i) \geq \delta_C(y_i) | y_i \in Y\} \quad (6)$$

Then by taking L.H.S, we have

$$I_{SVNS}(A, B \cup C) = -\frac{1}{n} \sum_{Y_1} [\rho_A(y_i) \log(\rho_{B \cup C}(y_i)) + \theta_A(y_i) \log \theta_{B \cup C}(y_i) + \delta_A(y_i) \log \delta_{B \cup C}(y_i)] +$$

$$\left(-\frac{1}{n} \sum_{Y_2} [\rho_A(y_i) \log(\rho_{B \cup C}(y_i)) + \theta_A(y_i) \log \theta_{B \cup C}(y_i) + \delta_A(y_i) \log \delta_{B \cup C}(y_i)] \right)$$

Now using (5) and (6), we get

$$I_{SVNS}(A, B \cup C) = -\frac{1}{n} \sum_{Y_1} [\rho_A(y_i) \log(\rho_B(y_i)) + \theta_A(y_i) \log \theta_C(y_i) + \delta_A(y_i) \log \delta_C(y_i)] -$$

$$\frac{1}{n} \sum_{Y_2} [\rho_A(y_i) \log(\rho_C(y_i)) + \theta_A(y_i) \log \theta_B(y_i) + \delta_A(y_i) \log \delta_B(y_i)].$$

Which implies that

$$I_{SVNS}(A, B \cup C) = I_{SVNS}(A, B) + I_{SVNS}(A, C). \quad (7)$$

Now by taking,

$$I_{SVNS}(A, B \cap C) = -\frac{1}{n} \sum_{Y_1} [\rho_A(y_i) \log(\rho_{B \cap C}(y_i)) + \theta_A(y_i) \log \theta_{B \cap C}(y_i) + \delta_A(y_i) \log \delta_{B \cap C}(y_i)]$$

$$-\frac{1}{n} \sum_{Y_2} [\rho_A(y_i) \log(\rho_{B \cap C}(y_i)) + \theta_A(y_i) \log \theta_{B \cap C}(y_i) + \delta_A(y_i) \log \delta_{B \cap C}(y_i)]$$

Now using (5) and (6), we get

$$I_{SVNS}(A, B \cap C) = -\frac{1}{n} \sum_{Y_1} [\rho_A(y_i) \log(\rho_C(y_i)) + \theta_A(y_i) \log \theta_B(y_i) + \delta_A(y_i) \log \delta_B(y_i)]$$

$$-\frac{1}{n} \sum_{Y_2} [\rho_A(y_i) \log(\rho_B(y_i)) + \theta_A(y_i) \log \theta_C(y_i) + \delta_A(y_i) \log \delta_C(y_i)]$$

which implies that

$$I_{SVNS}(A, B \cap C) = I_{SVNS}(A, B) + I_{SVNS}(A, C) \quad (8)$$

Adding (7) and (8), we get

$$I_{SVNS}(A, B \cup C) + I_{SVNS}(A, B \cap C) \leq (I_{SVNS}(A, B) + I_{SVNS}(A, C)). \square$$

Proof. c)

By taking L.H.S, we have

$$I_{SVNS}(A \cup B, C) = -\frac{1}{n} \sum_{Y_1} [\rho_{A \cup B}(y_i) \log(\rho_C(y_i)) + \theta_{A \cup B}(y_i) \log \theta_C(y_i) + \delta_{A \cup B}(y_i) \log \delta_C(y_i)] +$$

$$\left(-\frac{1}{n} \sum_{Y_2} [\rho_{A \cup B}(y_i) \log(\rho_C(y_i)) + \theta_{A \cup B}(y_i) \log \theta_C(y_i) + \delta_{A \cup B}(y_i) \log \delta_C(y_i)] \right).$$

Now using (5) and (6), we get

$$I_{SVNS}(A \cup B, C) = -\frac{1}{n} \sum_{Y_1} [\rho_A(y_i) \log(\rho_C(y_i)) + \theta_B(y_i) \log \theta_C(y_i) + \delta_B(y_i) \log \delta_C(y_i)]$$

$$-\frac{1}{n} \sum_{Y_2} [\rho_B(y_i) \log(\rho_C(y_i)) + \theta_A(y_i) \log \theta_C(y_i) + \delta_A(y_i) \log \delta_C(y_i)].$$

which implies that

$$I_{SVNS}(A \cup B, C) = I_{SVNS}(A, C) + I_{SVNS}(B, C). \quad (9)$$

Again, by taking L.H.S, we have

$$I_{SVNS}(A \cap B, C) = -\frac{1}{n} \sum_{Y_1} [\rho_{A \cap B}(y_i) \log(\rho_C(y_i)) + \theta_{A \cap B}(y_i) \log \theta_C(y_i) + \delta_{A \cap B}(y_i) \log \delta_C(y_i)]$$

$$-\frac{1}{n} \sum_{Y_2} [\rho_{A \cap B}(y_i) \log(\rho_C(y_i)) + \theta_{A \cap B}(y_i) \log \theta_C(y_i) + \delta_{A \cap B}(y_i) \log \delta_C(y_i)].$$

Now using (5) and (6), we get

$$I_{SVNS}(A \cap B, C) = -\frac{1}{n} \sum_{Y_1} [\rho_B(y_i) \log(\rho_C(y_i)) + \theta_A(y_i) \log \theta_C(y_i) + \delta_A(y_i) \log \delta_C(y_i)]$$

$$-\frac{1}{n} \sum_{Y_2} [\rho_A(y_i) \log(\rho_C(y_i)) + \theta_B(y_i) \log \theta_C(y_i) + \delta_B(y_i) \log \delta_C(y_i)].$$

Which implies that

$$I_{SVNS}(A \cap B, C) = I_{SVNS}(A, C) + I_{SVNS}(B, C). \quad (10)$$

Adding (9) and (10), we get

$$I_{SVNS}(A, B \cup C) + I_{SVNS}(A, B \cap C) \leq (I_{SVNS}(A, C) + I_{SVNS}(B, C)). \quad \square$$

Proof. d)

By taking two disjoint subsets of universal set Y , i.e.,

$$Y_1 = \{\rho_A(y_i) \geq \rho_B(y_i); \theta_A(y_i) \leq \theta_B(y_i); \delta_A(y_i) \leq \delta_B(y_i) | y_i \in Y\}, \quad (11)$$

$$Y_2 = \{\rho_A(y_i) \leq \rho_B(y_i); \theta_A(y_i) \geq \theta_B(y_i); \delta_A(y_i) \geq \delta_B(y_i) | y_i \in Y\}. \quad (12)$$

By using equations (5), (6) and by taking L.H.S, we have

$$I_{SVNS}(A \cup B, A \cap B)$$

$$= -\frac{1}{n} \sum_{Y_1} [\rho_{A \cup B}(y_i) \log(\rho_{A \cap B}(y_i)) + \theta_{A \cup B}(y_i) \log \theta_{A \cap B}(y_i) + \delta_{A \cup B}(y_i) \log \delta_{A \cap B}(y_i)]$$

$$-\frac{1}{n} \sum_{Y_2} [\rho_{A \cup B}(y_i) \log(\rho_{A \cap B}(y_i)) + \theta_{A \cup B}(y_i) \log \theta_{A \cap B}(y_i) + \delta_{A \cup B}(y_i) \log \delta_{A \cap B}(y_i)].$$

Now using (5) and (6), we get

$$I_{SVNS}(A \cup B, A \cap B) = -\frac{1}{n} \sum_{Y_1} [\rho_A(y_i) \log(\rho_B(y_i)) + \theta_B(y_i) \log \theta_A(y_i) + \delta_B(y_i) \log \delta_A(y_i)] \\ - \frac{1}{n} \sum_{Y_2} [\rho_B(y_i) \log(\rho_A(y_i)) + \theta_A(y_i) \log \theta_B(y_i) + \delta_A(y_i) \log \delta_B(y_i)].$$

Which implies that

$$I_{SVNS}(A \cup B, A \cap B) \\ = -\frac{1}{n} \sum_{Y_1 \cup Y_2} [\rho_A(y_i) \log(\rho_B(y_i)) + \theta_A(y_i) \log \theta_B(y_i) + \delta_A(y_i) \log \delta_B(y_i)] \\ - \frac{1}{n} \sum_{Y_1 \cup Y_2} [\rho_B(y_i) \log(\rho_A(y_i)) + \theta_B(y_i) \log \theta_A(y_i) + \delta_B(y_i) \log \delta_A(y_i)].$$

or

$$I_{SVNS}(A \cup B, A \cap B) = I_{SVNS}(A, B) + I_{SVNS}(B, A). \quad (13)$$

Now,

$$I_{SVNS}(A \cap B, A \cup B) \\ = -\frac{1}{n} \sum_{Y_1} [\rho_{A \cap B}(y_i) \log(\rho_{A \cup B}(y_i)) + \theta_{A \cap B}(y_i) \log \theta_{A \cup B}(y_i) + \delta_{A \cap B}(y_i) \log \delta_{A \cup B}(y_i)] \\ - \frac{1}{n} \sum_{Y_2} [\rho_{A \cap B}(y_i) \log(\rho_{A \cup B}(y_i)) + \theta_{A \cap B}(y_i) \log \theta_{A \cup B}(y_i) + \delta_{A \cap B}(y_i) \log \delta_{A \cup B}(y_i)].$$

Now using (5) and (6), we get

$$I_{SVNS}(A \cap B, A \cup B) \\ = -\frac{1}{n} \sum_{Y_1} [\rho_B(y_i) \log(\rho_A(y_i)) + \theta_A(y_i) \log \theta_B(y_i) + \delta_A(y_i) \log \delta_B(y_i)] \\ - \frac{1}{n} \sum_{Y_2} [\rho_A(y_i) \log(\rho_B(y_i)) + \theta_B(y_i) \log \theta_A(y_i) + \delta_B(y_i) \log \delta_A(y_i)].$$

Which implies that

$$I_{SVNS}(A \cup B, A \cap B) \\ = -\frac{1}{n} \sum_{Y_1 \cup Y_2} [\rho_A(y_i) \log(\rho_B(y_i)) + \theta_A(y_i) \log \theta_B(y_i) + \delta_A(y_i) \log \delta_B(y_i)] \\ - \frac{1}{n} \sum_{Y_1 \cup Y_2} [\rho_B(y_i) \log(\rho_A(y_i)) + \theta_B(y_i) \log \theta_A(y_i) + \delta_B(y_i) \log \delta_A(y_i)].$$

or

$$I_{SVNS}(A \cap B, A \cup B) \leq I_{SVNS}(A, B) + I_{SVNS}(B, A) \quad (14)$$

Adding (13) and (14), we get

$$I_{SVNS}(A \cup B, A \cap B) + I_{SVNS}(A \cap B, A \cup B) \leq (I_{SVNS}(A, B) + I_{SVNS}(B, A)). \square$$

In the next section, we investigate the application of the proposed inaccuracy measure.

4. Applications

In this section, we empirically illustrate the practical application of our proposed asymmetric measure of comparison in strategic decision-making and medical diagnosis.

4.1 Application to strategic decision-making

Let us consider a very pertinent corporate problem in which corporation Y wants to launch one of its five products using five strategies. Let the set of products be $P = \{C_1, C_2, C_3, C_4, C_5\}$, and the set of strategies be $S = \{Z_1, Z_2, Z_3, Z_4, Z_5\}$. Table 1 represents the weights of strategies of corporation Y in terms of memberships $\rho_y(Z_i)$, indeterminacy $\theta_y(Z_i)$ and non-membership value $\delta_y(Z_i)$ where $i = 1, 2, 3, 4, 5$. The weights have been assigned to these strategies because of their feasibility based on certain factors.

Table 1. Weights of strategies of the corporation as Single-Valued neutrosophic number

$(\rho_y(Z_1), \theta_y(Z_1), \delta_y(Z_1))$	$(\rho_y(Z_2), \theta_y(Z_2), \delta_y(Z_2))$	$(\rho_y(Z_3), \theta_y(Z_3), \delta_y(Z_3))$	$(\rho_y(Z_4), \theta_y(Z_4), \delta_y(Z_4))$	$(\rho_y(Z_5), \theta_y(Z_5), \delta_y(Z_5))$
(0.8, 0.2, 0.3)	(0.4, 0.4, 0.2)	(0.5, 0.4, 0.2)	(0.6, 0.2, 0.1)	(0.7, 0.5, 0.3)

It may be noted that $(\rho_y(Z_i), \theta_y(Z_i), \delta_y(Z_i))$ indicates the degree of importance, degree of inconclusiveness, and degree of the unimportance of strategy Z_i to the corporation in its implementation.

Table 2. represent the degree of importance, degree of inconclusiveness, and degree of the unimportance of products $(\rho_{c_j}(Z_i), \theta_{c_j}(Z_i), \delta_{c_j}(Z_i))$ concerning the strategy Z_i , where $j = 1, 2, 3, 4, 5$.

Table 2. Weights of strategies implementation for product launch as Single-Valued neutrosophic number

	$(\rho_{c_1}(Z_1), \theta_{c_1}(Z_1), \delta_{c_1}(Z_1))$	$(\rho_{c_2}(Z_2), \theta_{c_2}(Z_2), \delta_{c_2}(Z_2))$	$(\rho_{c_3}(Z_3), \theta_{c_3}(Z_3), \delta_{c_3}(Z_3))$	$(\rho_{c_4}(Z_4), \theta_{c_4}(Z_4), \delta_{c_4}(Z_4))$	$(\rho_{c_5}(Z_5), \theta_{c_5}(Z_5), \delta_{c_5}(Z_5))$
C_1	(0.5, 0.4, 0.4)	(0.6, 0.3, 0.2)	(0.5, 0.2, 0.1)	(0.6, 0.2, 0.8)	(0.9, 0.2, 0.1)
C_2	(0.9, 0.3, 0.2)	(0.8, 0.7, 0.3)	(0.6, 0.3, 0.2)	(0.3, 0.4, 0.5)	(0.4, 0.2, 0.2)
C_3	(0.6, 0.5, 0.2)	(0.7, 0.6, 0.5)	(0.5, 0.3, 0.3)	(0.2, 0.5, 0.6)	(0.5, 0.4, 0.3)
C_4	(0.9, 0.2, 0.1)	(0.8, 0.8, 0.7)	(0.4, 0.3, 0.2)	(0.8, 0.5, 0.4)	(0.3, 0.4, 0.5)
C_5	(0.7, 0.7, 0.6)	(0.1, 0.5, 0.2)	(0.4, 0.3, 0.7)	(0.6, 0.3, 0.2)	(0.5, 0.4, 0.2)

In an objective of the corporation to launch a suitable product because of the suitability of the five strategies, with the minimum risk (inaccuracy in our case), we use Eq. (4) to compute inaccuracy measures $I_{SVNS}(Y, C_i)$, $i = 1, 2, 3, 4, 5$ using the data of Table 1 and Table 2.

Table 3. Inaccuracy measures between products and strategies

$I_{SVNS}(Y, C_1)$	$I_{SVNS}(Y, C_2)$	$I_{SVNS}(Y, C_3)$	$I_{SVNS}(Y, C_4)$	$I_{SVNS}(Y, C_5)$
1.6421	1.5129	1.4554	1.2930	1.4933

According to the inaccuracy measures presented in Table 3, the product C_4 will be more suitable for launch because of the available strategies.

4.2 Application to Medical Diagnosis

First, we state the problem of medical diagnosis and present it in the framework of a neutrosophic environment.

Medical diagnosis: The process of identifying an actual disease of a patient based on their symptoms is termed a medical diagnosis.

Substantial uncertainties occur in most diagnostic decisions and can be handled using fuzzy methodologies. In medical science, several diseases have many symptoms in common. Therefore, identifying the appropriate illness from which a patient is suffering is difficult for physicians/experts. Let $D = \{D_1, D_2, D_3 \dots D_n\}$ be the set of diseases with several common symptoms and $S = \{r_1, r_2, r_3, \dots, r_n\}$ be the set of symptoms of the patient under investigation. In this scenario, S is an SVN and $D_1, D_2, D_3 \dots D_n$ are also SVNs. We compare each of D_i with S . The patient is diagnosed with a disease D_i with which S is maximum directed closeness.

The flowchart of the process is shown in the figure 1.

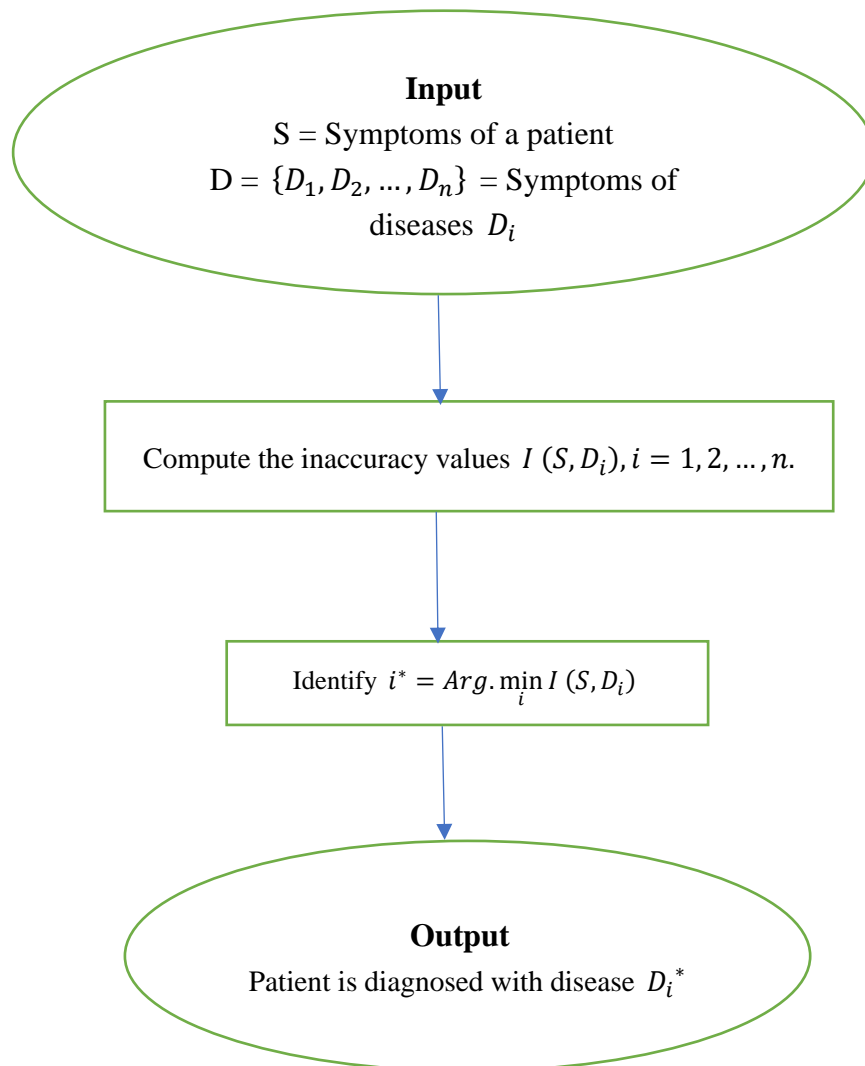


Figure1: Flowchart of Medical Diagnosis using Inaccuracy Measure

We consider the following numerical example to illustrate the procedure.

We present a numerical example to check the impact of the proposed inaccuracy measure.

Numerical Example: We consider a set of three diseases, $D = \{(D_1, \text{viral fever}) (D_2, \text{malaria}) (D_3, \text{typhoid})\}$, each of which has three common symptoms given in set $R = \{(r_1, \text{fever}), (r_2, \text{headache}) (r_3, \text{cough})\}$.

An expert team of doctors in the form of SVN S s assesses the characteristic information of the given diseases. The indicative information of symptoms and diagnosis for patients represented in the form of SVN S s as shown in Table 4:

Table 4. Characteristics information of the diseases described in the form of SVN S s

	r_1	r_2	r_3
D_1	(0.3, 0.2, 0.5)	(0.1, 0.3, 0.7)	(0.4, 0.3, 0.3)
D_2	(0.2, 0.2, 0.6)	(0.1, 0.1, 0.8)	(0.2, 0.3, 0.6)
D_3	(0.2, 0.1, 0.7)	(0.6, 0.3, 0.1)	(0.3, 0.4, 0.3)

The set P_1 represents the symptoms of the patient under investigation as an SVN S .

$$P_1 = \{(r_1, (0.1, 0.2, 0.7))(r_2, (0.8, 0.2, 0.3))(r_3, (0.2, 0.4, 0.4))\}.$$

Our task is to evaluate the closeness of P_1 with D_i using various SVN comparison measures.

To check the effectiveness of the proposed inaccuracy measure, we consider the following similarity/ distance measures for SVN S s.

$$S_1 = 1 - \frac{1}{n} \sum_{i=1}^n \max\{|\rho_A(y_i) - \rho_B(y_i)|, |\theta_A(y_i) - \theta_B(y_i)|, |\delta_A(y_i) - \delta_B(y_i)|\}.$$

(Bourmi and Smarandache [33])

$$S_2 = \frac{\sum_{i=1}^n \{\min(\rho_A(y_i), \rho_B(y_i)) + \min(\theta_A(y_i), \theta_B(y_i)) + \min(\delta_A(y_i), \delta_B(y_i))\}}{\sum_{i=1}^n \{\max(\rho_A(y_i), \rho_B(y_i)) + \max(\theta_A(y_i), \theta_B(y_i)) + \max(\delta_A(y_i), \delta_B(y_i))\}}$$

(Majumdar and Samanta [17])

$$S_3 = \frac{1}{3n} \sum_{i=1}^n \left[\frac{\min(\rho_A(y_i), \rho_B(y_i))}{\max(\rho_A(y_i), \rho_B(y_i))} + \frac{\min(\theta_A(y_i), \theta_B(y_i))}{\max(\theta_A(y_i), \theta_B(y_i))} + \frac{\min(\delta_A(y_i), \delta_B(y_i))}{\max(\delta_A(y_i), \delta_B(y_i))} \right].$$

(Ye and Zhang [34])

$$S_4 = \frac{1}{n} \sum_{i=1}^n \left[1 - \frac{|\rho_A(y_i) - \rho_B(y_i)| + |\theta_A(y_i) - \theta_B(y_i)| + |\delta_A(y_i) - \delta_B(y_i)|}{3} \right].$$

(Ali Aydogdu [13])

$$S_5 = 1 - \frac{1}{3n} \sum_{y \in n} \left| (\rho_A^2(y_i) - \rho_B^2(y_i)) - (\theta_A^2(y_i) - \theta_B^2(y_i)) - (\delta_A^2(y_i) - \delta_B^2(y_i)) \right|. \quad (\text{Chai } et \text{ al. [9]})$$

$$S_6 = \frac{1}{n} \sum_{i=1}^n \frac{2(\rho_A(y_i) \cdot \rho_B(y_i) + \theta_A(y_i) \cdot \theta_B(y_i) + \delta_A(y_i) \cdot \delta_B(y_i))}{(\rho_A^2(y_i) + \theta_A^2(y_i) + \delta_A^2(y_i)) + (\rho_B^2(y_i) + \theta_B^2(y_i) + \delta_B^2(y_i))}. \quad (\text{Ye [35]})$$

$$DM_1 = 1 - \frac{1}{n} \sum_{i=1}^n \left[1 - \frac{|\rho_A(y_i) - \rho_B(y_i)| + |\theta_A(y_i) - \theta_B(y_i)| + |\delta_A(y_i) - \delta_B(y_i)|}{3} \right].$$

(Ali Aydogdu [13])

$$DM_2 = \frac{1}{3n} \sum_{y \in n} \left| (\rho_A^2(y_i) - \rho_B^2(y_i)) - (\theta_A^2(y_i) - \theta_B^2(y_i)) - (\delta_A^2(y_i) - \delta_B^2(y_i)) \right|. \quad (\text{Chai } et \text{ al. [9]})$$

$$DM_3 = \frac{1}{n} \sum_{y \in n} (|\rho_A^2(y_i) - \rho_B^2(y_i)| \vee |\theta_A^2(y_i) - \theta_B^2(y_i)| \vee |\delta_A^2(y_i) - \delta_B^2(y_i)|). \quad (\text{Chai } et \text{ al. [9]})$$

Now, compute the similarity/distance measure between patient P_1 and diagnosis D . Similarly, we compute the proposed inaccuracy measure between the patient and the diagnosis. Table 5 shows the result obtained by calculating the different existing and proposed inaccuracy measures.

Table 5: The similarity /distance measures between the symptoms of a patient P_1 and diagnosis D_i

	D_1	D_2	D_3	Ranking
$S_1(P_1, D_i)$	0.5867	0.68	0.8734	$D_3 > D_2 > D_1$
$S_2(P_1, D_i)$	0.5238	0.5609	0.75	$D_3 > D_2 > D_1$
$S_3(P_1, D_i)$	0.0582	0.0623	0.25	$D_3 > D_2 > D_1$
$S_4(P_1, D_i)$	0.3108	0.3108	0.33	$D_3 > D_2 > D_1$
$S_5(P_1, D_i)$	0.8556	0.8389	0.9523	$D_3 > D_1 > D_2$
$S_6(P_1, D_i)$	0.7806	0.7987	0.9598	$D_3 > D_2 > D_1$
$DM_1(P_1, D_i)$	0.6892	0.6892	0.67	$D_3 > D_2 > D_1$
$DM_3(P_1, D_i)$	0.1444	0.1611	0.0477	$D_3 > D_1 > D_2$
$DM_4(P_1, D_i)$	0.33	0.32	0.1266	$D_3 > D_2 > D_1$
$I(D_i, P_1)$	1.7949	1.5818	1.314	$D_3 > D_2 > D_1$
$I(P_1, D_i)$	2.0504	2.0283	1.5870	$D_3 > D_2 > D_1$

Analysis: From the Table 5, we observe that all the comparison measures diagnosing the patient P_1 for Typhoid. Our proposed asymmetric comparison measure from both directions $D_i \rightarrow P_1$ and $P_1 \rightarrow D_i$ also resulting the same diagnosis (refer last two rows of the Table 5). Thus, we conclude that our proposed measure is consistent with existing models. The proposed model is more effective from the following observations.

In the figure 2, the directed comparison $I(D_i, P_1)$ shows the greater discriminating capability within the diseases. Thus, the proposed asymmetric measure is sensitive to the direction of comparison from the view point of the discriminating power. In the considered numerical problem, the diagnostic result due to both directed comparisons ($I(D_i, P_1)$ and $I(P_1, D_i)$) remains same but discriminating power is different.

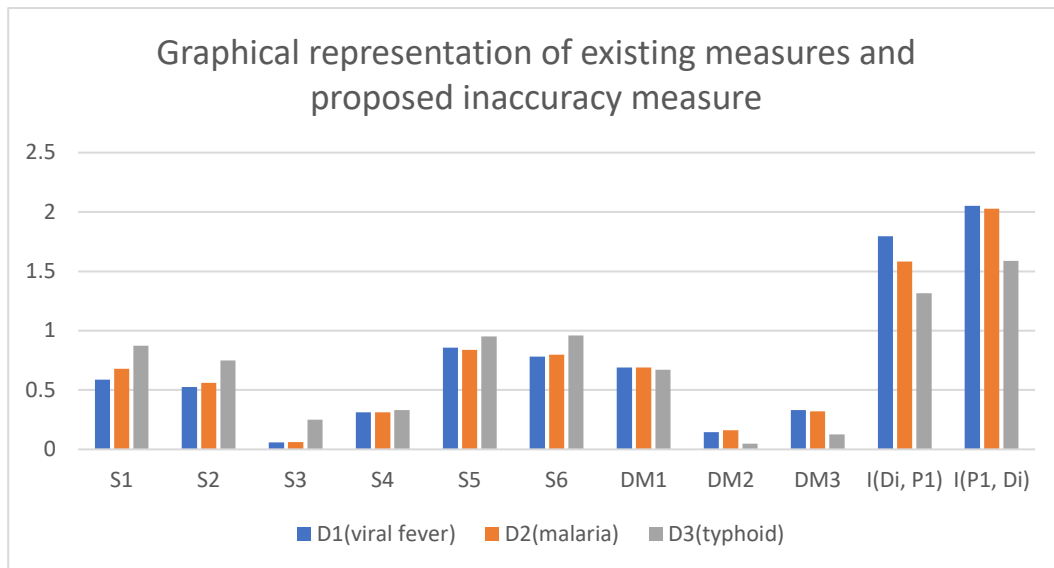


Figure 2: Graphical representation of computed values of closeness due to various measures in Table 5

From the above graph, we see that the diagnosis of patient P_1 is typhoid. It shows that the proposed inaccuracy measure is feasible and effective.

In the next section, we compare some existing divergence measures, similarity measures, and the proposed inaccuracy measure.

5. Comparative Study

To check the superiority of the proposed inaccuracy measure, we consider the numerical example obtained from Thao and Smarandache [36].

Let us suppose, for universal set $U = \{u_1, u_2, u_3, \dots, u_m\}$, there are n patterns in the form of neutrosophic set $\{A_1, A_2, A_3, \dots, A_n\}$. Suppose that we have an unknown sample B. Our goal is to classify sample B into which pattern A_i .

For this, we have to calculate the proposed inaccuracy measure, existing divergence measures, and similarity measures of unknown sample B with each pattern A_i ($n = 1, 2, 3, \dots, n$).

Assume $A_1 = \{(u_1, 0.7, 0.7, 0.2), (u_2, 0.7, 0.8, 0.4), (u_3, 0.6, 0.8, 0.2)\}$.

$A_2 = \{(u_1, 0.5, 0.7, 0.3), (u_2, 0.7, 0.7, 0.5), (u_3, 0.8, 0.6, 0.1)\}$.

$A_3 = \{(u_1, 0.9, 0.5, 0.1), (u_2, 0.7, 0.6, 0.4), (u_3, 0.8, 0.5, 0.2)\}$.

Unknown Sample

$B = \{(u_1, 0.7, 0.8, 0.4), (u_2, 0.8, 0.5, 0.3), (u_3, 0.5, 0.8, 0.5)\}$.

For the comparative study, we consider all measures listed in section 4 along with the following existing divergence measures and similarity measures:

$$S_7 = \frac{1}{n} \sum_{i=1}^n \left\{ \frac{(\rho_A^2(y_i) \wedge \rho_B^2(y_i))}{(\rho_A^2(y_i) \vee \rho_B^2(y_i))} + \frac{((1-\theta_A^2(y_i)) \wedge (1-\theta_B^2(y_i)))}{((1-\theta_A^2(y_i)) \vee (1-\theta_B^2(y_i)))} + \frac{((1-\delta_A^2(y_i)) \wedge (1-\delta_B^2(y_i)))}{((1-\delta_A^2(y_i)) \vee (1-\delta_B^2(y_i)))} \right\}. \quad (\text{Chai et al. [9]})$$

$$DM_4(A, B) = \frac{1}{n} \sum_{i=1}^n [D_T^i(A, B) + D_I^i(A, B) + D_F^i(A, B)]. \quad (\text{Thao and Smarandache [36]})$$

$$\text{where, } DM_T^i(A, B) = \rho_A(y_i) \ln \frac{2\rho_A(y_i)}{\rho_A(y_i) + \rho_B(y_i)} + \rho_B(y_i) \ln \frac{2\rho_B(y_i)}{\rho_A(y_i) + \rho_B(y_i)}$$

$$DM_i^i(A, B) = \theta_A(y_i) \ln \frac{2\theta_A(y_i)}{\theta_A(y_i) + \theta_B(y_i)} + \theta_B(y_i) \ln \frac{2\theta_B(y_i)}{\theta_A(y_i) + \theta_B(y_i)}$$

$$D_F^i(A, B) = \delta_A(y_i) \ln \frac{2\delta_A(y_i)}{\delta_A(y_i) + \delta_B(y_i)} + \delta_B(y_i) \ln \frac{2\delta_B(y_i)}{\delta_A(y_i) + \delta_B(y_i)}$$

$$DM_j(A, B) =$$

$$\sum_{i=1}^n 2^\alpha \left[\frac{(\sqrt{\rho_A(y_i)} - \sqrt{\rho_B(y_i)})^{2(\alpha+1)}}{(\rho_A(y_i) + \rho_B(y_i))^\alpha} + \frac{(\sqrt{1 - \rho_A(y_i)} - \sqrt{1 - \rho_B(y_i)})^{2(\alpha+1)}}{(2 - \rho_A(y_i) + \rho_B(y_i))^\alpha} \right] +$$

$$\sum_{i=1}^n 2^\alpha \left[\frac{(\sqrt{\theta_A(y_i)} - \sqrt{\theta_B(y_i)})^{2(\alpha+1)}}{(\theta_A(y_i) + \theta_B(y_i))^\alpha} + \frac{(\sqrt{1 - \theta_A(y_i)} - \sqrt{1 - \theta_B(y_i)})^{2(\alpha+1)}}{(2 - \theta_A(y_i) + \theta_B(y_i))^\alpha} \right] +$$

$$\sum_{i=1}^n 2^\alpha \left[\frac{(\sqrt{\delta_A(y_i)} - \sqrt{\delta_B(y_i)})^{2(\alpha+1)}}{(\delta_A(y_i) + \delta_B(y_i))^\alpha} + \frac{(\sqrt{1 - \delta_A(y_i)} - \sqrt{1 - \delta_B(y_i)})^{2(\alpha+1)}}{(2 - \delta_A(y_i) + \delta_B(y_i))^\alpha} \right]; j = 5, 6. \quad (\text{Guleria et al. [37]})$$

The result obtained by calculating the proposed inaccuracy measure, existing divergence measure, and similarity measure of unknown sample B with each pattern A_i is shown in Table 7.

Table 7. Result of the Existing Similarity Measure and Proposed Divergence Measure, along with the Degree of Confidence

	(A_1, B)	(A_2, B)	(A_3, B)	DOC
S_1	0.7333	0.7333	0.7666	0.0666
S_2	0.7762	0.7036	0.6745	0.1743
S_3	0.7620	0.6781	0.64	0.2059
S_4	0.8666	0.7996	0.7776	0.156
S_5	0.9933	0.9622	0.9555	0.0689
S_6	0.9844	0.9319	0.9203	0.1166
S_7	0.8021	0.7063	0.6993	0.1986
D_1	0.1333	0.2004	0.2224	0.1562
D_2	0.0067	0.0378	0.0445	0.0689
D_3	0.25	0.29	0.2966	0.0866
DM_4	0.1537	0.2674	0.2951	0.25513
$DM_5(\text{when } \alpha = 1)$	0.0352	0.1090	0.1161	0.1547
$DM_6(\text{when } \alpha = 4)$	0.0001	0.0103	0.0032	0.0133
$I_{SVNS}(B_i, A)$	1.4854	1.7288	1.8444	0.6024
$I_{SVNS}(A_i, B)$	1.2092	1.2554	1.1457	0.1732

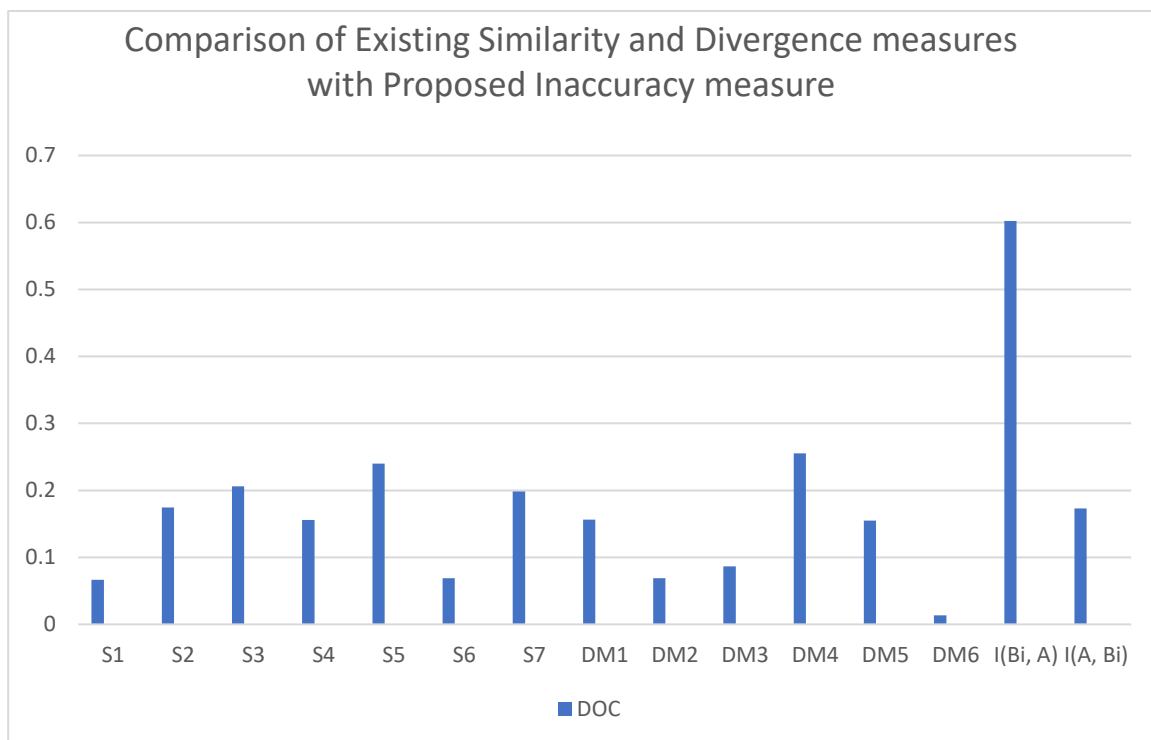


Figure 3: Graphical representation of degree of confidence of various comparison measures

Analysis: The highest value of similarity, the lowest value of divergence/inaccuracy, and the degree of confidence of the existing similarity, divergence measure, and the proposed inaccuracy measure are in bold in Table 7. The computed values of the comparison measures indicate that unknown sample B belongs to the pattern A_1 . Only S_1 shows a different result. Our proposed inaccuracy measure's highest value of DOC, when the direction of comparison is $B_i \rightarrow A$. This justifies its effectiveness over other comparison measures, as illustrated graphically in the figure 3.

6. Conclusion

In this work, we have proposed an inaccuracy measure for SVN S s, to find directed discrimination between two SVN S s and studied some of their mathematical properties. The illustrative numerical problem in a corporate crisis of product launch has shown the applicability of the proposed measure. In addition, the advantage of the proposed measure has been justified by using a performance index DOC and in a medical diagnosis problem. The proposed asymmetric comparison measure may be impactful to the various studies in data science, machine learning and computer vision requiring a directed comparative analysis. The limitation of this article is that all the investigations have been done using hypothetical data. In future, we plan to investigate the applications of the suggested asymmetric comparison metrics in cluster analysis, multiple attribute decision-making, and medical diagnosis using real data sets. However, applying the proposed measures to actual data sets needs an efficient method of converting the crisp data to single valued neutrosophic data set without potential loss of information. Thus, formulating a suitable data conversion process because of the given scenario is also a problem for future investigations. Some recent studies [21-25] investigate the applicability of neutrosophic

methods in various disciplines like decision-making, pattern recognition, inventory management, pollution in megacities, etc. We also plan to explore the relevance of the proposed approach to these disciplines.

Acknowledgements Authors are highly thankful to the Editor-in-Chief, Associate Editor to value this work and, anonymous reviewers for their constructive and insightful comments for the improvement of the paper.

References

- [1] Kullback, S.; Leibler, R.A. On information and sufficiency. *Annals of Mathematical Statistics*, **1951**, 22 (7), pp. 79-86.
- [2] Kullback, S. Information theory, and statistics. *Dover Publication*, 1968.
- [3] Shannon, C. E. A mathematical theory of communication. *Bell System Technical Journal*, **1948**, 27, pp. 379-423; 623-656.
- [4] Kerridge, D. F. Inaccuracy, and inference. *Journal of Royal Statistical Society*, **1961**, 23, pp. 184-194.
- [5] Zadeh, L. A. Fuzzy sets. In fuzzy sets. *Information Control*, **1965**, 8, pp. 394-432.
- [6] Luca, D.; Termini, S. A definition of non-probabilistic entropy in the setting of fuzzy set Theory. *Information and Control*, **1971**, 20, pp. 301-312.
- [7] Smarandache, F. Neutrosophic Set- A Generalization of the Intuitionistic Fuzzy Set. *International Journal of Pure and Applied Mathematics*, **2005**, 24, pp. 287-297.
- [8] Wang, H.; Smarandache, F.; Zhang, Y.; Sunderraman, R. Single valued neutrosophic sets, *Infinite study*, **2010**.
- [9] Chai, J.S.; Selvachandran, G.; Smarandache, F.; Gerogiannis, V.C.; Son, L.H.; Bui, Q.T.; Vo, B. New similarity measures for single-valued neutrosophic sets with applications in pattern recognition and medical diagnosis problems. *Complex and Intelligent Systems*, **2021**, 7, pp. 703-723.
- [10] Wang, T. Distance of Single-Valued Neutrosophic Set and Its Application in Pattern Recognition. In *Journal of Physics: Conference Series*, **2021**, 2025, 1, p. 012019, DOI:10.1088/1742-6596/2025/1/012019.
- [11] Biswas, P.; Pramanik, S.; Giri, B.C. TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment. *Neural Computing and Applications*, **2016**, 27(3), 2016, pp.727-737.
- [12] Wu, H.; Yuan, Y.; Wei, L.; Pei, L.; On entropy, similarity measure and cross-entropy of single-valued neutrosophic sets and their application in multi-attribute decision making. *Soft Computing*, **2018**, 22, pp. 7367 – 7376.
- [13] Aydogdu, A. On similarity and entropy of single-valued neutrosophic sets. *ICSRs*, **2015** 29,1, pp. 67 – 74.
- [14] Broumi, S.; Smarandache, F. Cosine similarity measure of interval valued neutrosophic sets. *Neutrosophic Sets and Systems*, **2014**, 5, pp. 15–20.
- [15] Broumi, S.; Smarandache, F. New distance and similarity measures of interval neutrosophic sets. In *Neutrosophic Theory and Its Applications. Collected Papers*, **2014**, 1, pp. 249.

- [16] Khan, M.J.; Kumam, P. A New Similarity Measure for Single Valued Neutrosophic Sets. *In International Conference on Intelligent and Fuzzy Systems*, **2021**, pp. 397-404.
- [17] Majumdar, P.; Samanta, S.K. On similarity and entropy of neutrosophic sets. *Journal Intell. Fuzzy Syst.*, **2014**, 26(3), pp.1245–1252.
- [18] Ye, J. Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making. *Journal Intell. Fuzzy Syst.*, **2014**, 26(1), pp. 165–172.
- [19] Ye, J. Improved cosine similarity measures of simplified neutrosophic sets for medical diagnosis. *Artif. Intell. Med.*, **2015**, 63(3), pp.171–179.
- [20] Ye, J.; Fu, J. Multi-period medical diagnosis method using a single-valued neutrosophic similarity measure based on tangent function. *Comput. Methods Programs Biomed*, **2016**, 123, pp. 142–149.
- [21] Chakraborty, A.; Maity, S.; Jain, S.; Mondal, S.P.; Alam, S. Hexagonal fuzzy number and its distinctive representing, ranking, defuzzification technique and application in production inventory management technique. *Granular Computing, Springer*, **2021**, 6(3), pp. 507-521, DOI: 10.1007/s41066-020-00212-8.
- [22] Chakraborty, A.; Banik, B.; Salahshour.; S.; Bromi, S.; Graded mean integral distance measure and VIKOR strategy based MCDM skill in trapezoidal neutrosophic number. *International Journal of Neutrosophic Science*, **2022**, 18 (2), pp. 210-226, DOI: 10.54216/IJNS.180205.
- [23] Haque, T.S.; Alam, S.; Chakraborty, A.; Selection of most effective COVID -19 virus protector using a novel MCGDM technique under linguistic generalized spherical fuzzy environment. *Computational and Applied Mathematics*, **2022**, 44, pp. 84, DOI: 10.1007/s40314-022-01776-8 .
- [24] Haque, T.S.; Chakraborty, A.; Alrabaiah, H.; Alam, S. A new approach to solve MADM problems by Logarithmic operation laws in interval neutrosophic environment. *Granular computing, Springer*, **2021**, DOI: 10.1007/s41066-021-00299-7.
- [25] Haque, T.S.; Chakraborty, A.; Mondal, S.P.; Alam, S. A new exponential operational law for trapezoidal neutrosophic number and pollution in megacities related MCGDM problem. *Journal of Ambient Intelligence and Humanized Computing, Springer*, **2021**, <https://doi.org/10.1007/s12652-021-03223-8>.
- [26] Bonissone, P.P. A pattern recognition approach to the problem of linguistic approximation in system analysis. *In: Proceedings 1200 of the IEEE International Conference On Cybernetics And Society*, **1979**, pp. 793–798.
- [27] Eshragh, F.; Mamdani, EH. A general approach to linguistic approximation. *Int J Man Mach Stud*, **1979**, 11(4), pp. 501–519.
- [28] Shahzadi, G.; Akra, M.; Saeid, A.B. An application of single-valued neutrosophic sets in medical diagnosis. *Neutrosophic Sets and Systems*, **2017**, 18, pp. 80-88.
- [29] Ye, J. Clustering methods using distance-based similarity measures of single-valued neutrosophic sets. *Journal of Intelligent Systems*, **2014**, 23(4), pp. 379-389.
- [30] Qin, K.; Wang, L. New similarity and entropy measures of single-valued neutrosophic sets with applications in multi-attribute decision making. *Soft Computing*, **2020**, 24, pp. 16165-16176.

-
- [31] Hatzimichailidis, A.G.; Papakostas, G.A.; Kaburlasos, V.G. A novel distance measure of intuitionistic fuzzy sets and its application to pattern recognition problems. *International Journal of Intelligent Systems*, **2012**, *27*, pp. 396-409.
- [32] Verma, R.; Sharma, B.D. A new measure of inaccuracy with its application to multicriteria decision making under intuitionistic fuzzy environment. *Journal of Intelligent and Fuzzy Systems*, **2014**, *27*(4), pp. 1811-1824.
- [33] Bourmi, S.; Smarandache, F. Several similarity measures of neutrosophic sets. *Neutrosophic Sets and Systems*, **2013**, *1*(10), 2013, pp. 54-62.
- [34] Ye, J.Z. Single valued neutrosophic similarity measures for multiple attribute decision making. *Neutrosophic Sets and Systems*, **2014**, *2*, 2014, pp. 48-54.
- [35] Ye, J. Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making. *International Journal of Fuzzy Systems*, **2014**, *16* (2), pp. 204-211.
- [36] Thao, N.X.; Smarandache, F. Divergence measure of neutrosophic sets and applications. *Infinite Study*, **2018**.
- [37] Guleria, A.; Srivastava, S.; Bajaj, R.K. On parametric divergence measure of neutrosophic sets with its application in decision-making models. *Infinite Study*, **2019**.

Received: August 09, 2022. Accepted: January 08, 2023