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A Study of Algebraic Curves in Neutrosophic Real Ring $R(I)$ by Using the One-Dimensional Geometric AH-Isometry

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Abstract: The objective of this paper is to study and define some algebraic curves with neutrosophic variables in neutrosophic real field $R(I)$, where we study what are the relationships between classical algebraic curves and neutrosophic algebraic curves depending on the geometric isometry (AH-Isometry).

Keywords: Neutrosophic real ring $R(I)$, AH-isometry, Neutrosophic algebraic curves.

Introduction

Algebraic Geometry is one of the branches of algebra that deals with the study of geometric shapes through familiar algebraic concepts and theories [1]. There were several approaches to geometry, all of which are usually classified as algebraic geometry, at the end of the nineteenth century. Lazare Carnot (1753-1823) attributed to algebraic geometry which is about algebraic curves and their intersection with the sides of a triangle [2], but this concept developed a lot in the second half of the nineteenth century.

Neutrosophy is a new branch of philosophy concerns with the indeterminacy in all areas of life and science. It has become a useful tool in generalizing many classical systems such as equations [30], number theory [3], and linear spaces [4,5], and ring of matrices [19-31].

Recently, Abobala, and Hatip have presented the concept of one-dimensional AH-isometry to study the correspondence between neutrosophic plane $\boldsymbol{R(I)}$ and the classical module $\boldsymbol{R} \times \boldsymbol{R}$.

In this work, we use the one-dimensional AH-isometry to turn the general case of algebraic curves in real ring $\boldsymbol{R(I)}$ with one variable into two classical algebraic curves so we will go from $\boldsymbol{R(I)}$ space into $\boldsymbol{R} \times \boldsymbol{R}$ space, we study the properties of our algebraic curves then we go back to $\boldsymbol{R(I)}$ space using AH-isometry.

Neutrosophic Functions on $\boldsymbol{R(I)}$.

Definition:

Let $R(I) = \{a + bI\, ; a, b \in R\}$ where $I^2 = I$ be the neutrosophic field of reals. The one-dimensional isometry (AH-Isometry) is defined as follows: [49]

$$T: R(I) \to R \times R$$
$$T(a + bI) = (a, a + b)$$

Definition:

Let $f: R(I) \to R(I); f = f(X)$ and $X = x + yI \in R(I)$ the f is called a neutrosophic real function with one neutrosophic variable.

Example:

Take $f: R(I) \to R(I); f(X) = X^2 + IX + 2I = (x + yI)^2 + I(x + yI) + 2I$

$$= x^2 + I(y^2 + 2xy + x + y + 2)$$

Theorem:

Let $f: R(I) \to R(I)$ be a neutrosophic real function with one variable, $X = x + yI \in R(I)$ then f can be turned into two classical real functions.

Computing Powers in $\boldsymbol{R(I)}$.

To compute such equation: $(a + bI)^{c+dI}\, ; a, b, c, d \in R$ we need the one-dimensional isometry again:

$T[(a + bI)^{c+dI}] = (a, a + b)^{(c,c+d)} = \left(a^c, (a + b)^{c+d}\right)$,

Which means

$(a + bI)^{c+dI} = T^{-1}\left(a^c, (a + b)^{c+d}\right)$,

$= a^c + I[(a + b)^{c+d} - a^c]$.

Theorem:

Let $R(I)$ be the neutrosophic field of reals, we have:

1. $\sin(a+bI) = \sin a + I[\sin(a+b) - \sin a\ ]$
2. $\cos(a+bI) = \cos a + I[\cos(a+b) - \cos a]$
3. $e^{x+Iy} = e^x + I(e^{x+y} - e^x)$

Algebraic Curves In Neutrosophic Real Ring $\boldsymbol{R(I)}$:

Definition : Neutrosophic Strophoide.

Let $\boldsymbol{Y = y_1 + y_2 I, X = x_1 + x_2 I, A = a_1 + a_2 I \in R(I), a_1, a_2, x_1, x_2, y_1, y_2 \in R}$, then we define a neutroophic strophoide as follows:

$$\boldsymbol{Y^2 = X^2 . \frac{A+X}{A-X}\ ; A > 0}$$

This equation can be written as follows:

$$\boldsymbol{(y_1 + y_2 I)^2 = (x_1 + x_2 I)^2 . \frac{(a_1 + a_2 I) + (x_1 + x_2 I)}{(a_1 + a_2 I) - (x_1 + x_2 I)}\ ; a_1 + a_2 I > 0}$$

Theorem :

Let $Y = y_1 + y_2 I, X = x_1 + x_2 I, A = a_1 + a_2 I \in R(I)$, then if $A = a_1 + a_2 I$ is invertible, the neutrosophic strophoide $(y_1 + y_2 I)^2 = (x_1 + x_2 I)^2 . \frac{(a_1 + a_2 I) + (x_1 + x_2 I)}{(a_1 + a_2 I) - (x_1 + x_2 I)}$ is equivalent to the direct product of two classical strophoide.

Proof. Consider the equation $(y_1 + y_2 I)^2 = (x_1 + x_2 I)^2 . \frac{(a_1 + a_2 I) + (x_1 + x_2 I)}{(a_1 + a_2 I) - (x_1 + x_2 I)}$

Now, we have:

$$y_1^2 + (y_2^2 + 2y_1 y_2)I = [x_1^2 + (x_2^2 + 2x_1 x_2)I] . \frac{(a_1 + x_1) + (a_2 + x_2)I}{(a_1 - x_1) + (a_2 - x_2)I}$$

$$y_1^2 + (y_2^2 + 2y_1 y_2)I$$
$$= [x_1^2 + (x_2^2 + 2x_1 x_2)I] . \left[\frac{(a_1 + x_1)}{(a_1 - x_1)} + \frac{(a_1 - x_1)(a_2 + x_2) - (a_1 + x_1)(a_2 - x_2)}{(a_1 - x_1)[(a_1 + a_2) - (x_1 + x_2)]} I\right]$$

by computing its direct image with AH-isometry, we get:

$$T({y_1}^2+({y_2}^2+2y_1y_2)I)$$
$$=T({x_1}^2+({x_2}^2+2x_1x_2)I)T\left(\frac{(a_1+x_1)}{(a_1-x_1)}\right.$$
$$\left.+\frac{(a_1-x_1)(a_2+x_2)-(a_1+x_1)(a_2-x_2)}{(a_1-x_1)[(a_1+a_2)-(x_1+x_2)]}\ I\right)$$

$$({y_1}^2,{y_1}^2+{y_2}^2+2y_1y_2)$$
$$=({x_1}^2,{x_1}^2+{x_2}^2+2x_1x_2).\left(\frac{(a_1+x_1)}{(a_1-x_1)},\frac{(a_1+x_1)}{(a_1-x_1)}\right.$$
$$\left.+\frac{(a_1-x_1)(a_2+x_2)-(a_1+x_1)(a_2-x_2)}{(a_1-x_1)[(a_1+a_2)-(x_1+x_2)]}\right)$$

Then.

$$({y_1}^2,(y_1+y_2)^2)=({x_1}^2,(x_1+x_2)^2)$$
$$.\left(\frac{(a_1+x_1)}{(a_1-x_1)},\frac{(a_1+x_1)[(a_1+a_2)-(x_1+x_2)]+(a_1-x_1)(a_2+x_2)-(a_1+x_1)(a_2-x_2)}{(a_1-x_1)[(a_1+a_2)-(x_1+x_2)]}\right)$$

$$({y_1}^2,(y_1+y_2)^2)$$
$$=({x_1}^2,(x_1$$
$$+x_2)^2).\left(\frac{(a_1+x_1)}{(a_1-x_1)},\frac{(a_1+x_1)[(a_1+a_2)-(x_1+x_2)-(a_2-x_2)]+(a_1-x_1)(a_2+x_2)}{(a_1-x_1)[(a_1+a_2)-(x_1+x_2)]}\right)$$

$$({y_1}^2,(y_1+y_2)^2)=({x_1}^2,(x_1+x_2)^2).\left(\frac{(a_1+x_1)}{(a_1-x_1)},\frac{(a_1+x_1)[(a_1-x_1)]+(a_1-x_1)(a_2+x_2)}{(a_1-x_1)[(a_1+a_2)-(x_1+x_2)]}\right)$$

$$({y_1}^2,(y_1+y_2)^2)=({x_1}^2,(x_1+x_2)^2).\left(\frac{(a_1+x_1)}{(a_1-x_1)},\frac{(a_1-x_1)(a_1+x_1+a_2+x_2)}{(a_1+x_1)[(a_1+a_2)-(x_1+x_2)]}\right)$$

$$({y_1}^2,(y_1+y_2)^2)=({x_1}^2,(x_1+x_2)^2).\left(\frac{(a_1+x_1)}{(a_1-x_1)},\frac{(a_1+a_2)+(x_1+x_2)}{(a_1+a_2)-(x_1+x_2)}\right)$$
$$({y_1}^2,(y_1+y_2)^2)=\left({x_1}^2\frac{(a_1+x_1)}{(a_1-x_1)},(x_1+x_2)^2\frac{(a_1+a_2)+(x_1+x_2)}{(a_1+a_2)-(x_1+x_2)}\right)$$

So that we have:

$$\begin{cases} \boldsymbol{\Gamma_1: y_1{}^2 = x_1{}^2 \frac{(a_1 + x_1)}{(a_1 - x_1)}; a_1 > 0} \\ \boldsymbol{\Gamma_2: (y_1 + y_2)^2 = (x_1 + x_2)^2 \frac{(a_1 + a_2) + (x_1 + x_2)}{(a_1 + a_2) - (x_1 + x_2)}; (a_1 + a_2) > 0} \end{cases}$$

Remark :

If $a_1 + a_2 I$ is invertible, we can write the equation of neutrosophic strophoide as follows:

$(y_1 + y_2 I)^2 = (x_1 + x_2 I)^2 . \frac{(a_1 + a_2 I) + (x_1 + x_2 I)}{(a_1 + a_2 I) - (x_1 + x_2 I)}$.

Now, we should discuss the cases of non-invertible of $a_1 + a_2 I$.

$a_1 + a_2 I$ is not invertible, then we have cases:

1- $a_1 = 0,\ a_1 + a_2 \neq 0$, this means that the neutrosophic strophoide will be equivalent to direct product of classical strophoide $(y_1 + y_2)^2 = (x_1 + x_2)^2 \frac{(a_1 + a_2) + (x_1 + x_2)}{(a_1 + a_2) - (x_1 + x_2)}; (a_1 + a_2) > 0$ with classical image two line $\begin{cases} y_1 = i\, x_1 \\ y_1 = -i\, x_1 \end{cases}$.

2- $a_1 \neq 0,\ a_1 + a_2 = 0$, this implies that the neutrosophic strophoide will be equivalent to direct product of classical strophoide $y_1{}^2 = x_1{}^2 \frac{(a_1 + x_1)}{(a_1 - x_1)}; a_1 > 0$ with classical image two line $\begin{cases} y_1 = i\,(x_1 + x_2) \\ y_1 = -i\,(x_1 + x_2) \end{cases}$.

3- If $a_1 = 0,\ a_1 + a_2 = 0$, this implies that the neutrosophic strophoide will be equivalent to direct product of classical image two line $\begin{cases} y_1 = i\, x_1 \\ y_1 = -i\, x_1 \end{cases}$ with classical image two line $\begin{cases} y_1 = i\,(x_1 + x_2) \\ y_1 = -i\,(x_1 + x_2) \end{cases}$.

Theorem:

Let Γ_1, Γ_2 are two classical strophoide, then the direct product of Γ_1, Γ_2 is equivalent to the neutrosophic strophoide Γ.

Proof.

Let Γ_1, Γ_2 are two classical strophoide, where:

$$\begin{cases} \Gamma_1: y_1{}^2 = x_1{}^2 . \frac{(a_1 + x_1)}{(a_1 - x_1)}; a_1 > 0 \\ \Gamma_2: y_2{}^2 = x_2{}^2 . \frac{(a_2 + x_2)}{(a_2 - x_2)}; a_2 > 0 \end{cases}$$

Now, we take the inverse image of the AH-isometry, we have:

$$T^{-1}({y_1}^2,{y_2}^2)=T^{-1}({x_1}^2,{x_2}^2).T^{-1}\left(\frac{(a_1+x_1)}{(a_1-x_1)},\frac{(a_2+x_2)}{(a_2-x_2)}\right)$$

$${y_1}^2+({y_2}^2-{y_1}^2)I=[{x_1}^2+({x_2}^2-{x_1}^2)I].\left[\frac{(a_1+x_1)}{(a_1-x_1)}+\left(\frac{(a_2+x_2)}{(a_2-x_2)}-\frac{(a_1+x_1)}{(a_1-x_1)}\right)I\right]$$

$$\begin{aligned}{y_1}^2+({y_2}^2-{y_1}^2)I\\ &=[{x_1}^2+({x_2}^2-{x_1}^2)I].\left[\frac{(a_1+x_1)}{(a_1-x_1)}\right.\\ &\left.+\left(\frac{(a_2+x_2)(a_1-x_1)-(a_2-x_2)(a_1+x_1)}{(a_1-x_1)(a_2-x_2)}\right)I\right]\end{aligned}$$

$${y_1}^2+({y_2}^2-{y_1}^2)I=[{x_1}^2+({x_2}^2-{x_1}^2)I].\left[\frac{(a_1+x_1)}{(a_1-x_1)}+\left(\frac{(a_2+x_2)}{(a_2-x_2)}-\frac{(a_1+x_1)}{(a_1-x_1)}\right)I\right]$$

$$\begin{aligned}{y_1}^2+({y_2}^2-{y_1}^2)I\\ &=[{x_1}^2+({x_2}^2-{x_1}^2)I].\left[\frac{(a_1+x_1)}{(a_1-x_1)}\right.\\ &\left.+\left(\frac{(a_2+x_2)}{(a_2-x_2)}-\frac{(a_1+x_1)}{(a_1-x_1)}+\frac{(a_1+x_1)}{(a_2-x_2)}-\frac{(a_1+x_1)}{(a_2-x_2)}\right)I\right]\end{aligned}$$

$$\begin{aligned}{y_1}^2+({y_2}^2-{y_1}^2)I\\ &=[{x_1}^2+({x_2}^2-{x_1}^2)I].\left[\frac{(a_1+x_1)}{(a_1-x_1)}\right.\\ &\left.+\left(\left\{\frac{(a_2+x_2)}{(a_2-x_2)}-\frac{(a_1+x_1)}{(a_2-x_2)}\right\}+\left\{-\frac{(a_1+x_1)}{(a_1-x_1)}+\frac{(a_1+x_1)}{(a_2-x_2)}\right\}\right)I\right]\end{aligned}$$

$$\begin{aligned}{y_1}^2+({y_2}^2-{y_1}^2)I\\ &=[{x_1}^2+({x_2}^2-{x_1}^2)I].\left[\frac{(a_1+x_1)}{(a_1-x_1)}\right.\\ &\left.+\left\{\frac{(a_2+x_2)-(a_1+x_1)}{(a_2-x_2)}-\frac{(a_1+x_1)(a_2-x_2)-(a_1+x_1)(a_1-x_1)}{(a_1-x_1)(a_2-x_2)}\right\}I\right]\end{aligned}$$

$$y_1{}^2 + (y_2{}^2 - y_1{}^2)I$$
$$= [x_1{}^2 + (x_2{}^2 - x_1{}^2)I].\left[\frac{(a_1 + x_1)}{(a_1 - x_1)} + \frac{(a_1 - x_1)[(a_2 + x_2) - (a_1 + x_1)] - (a_1 + x_1)(a_2 - x_2) - (a_1 + x_1)(a_1 - x_1)}{(a_1 - x_1)(a_2 - x_2)}I\right]$$

$$y_1{}^2 + (y_2{}^2 - y_1{}^2)I = [x_1{}^2 + (x_2{}^2 - x_1{}^2)I].$$

$$\left[\frac{(a_1 + x_1)}{(a_1 - x_1)} + \frac{(a_1 - x_1)[(a_2 + x_2) - (a_1 + x_1)] - (a_1 + x_1)[(a_2 - x_2) - (a_1 - x_1)]}{(a_1 - x_1)(a_2 - x_2)}I\right]\frac{(a_1 + x_1)}{(a_1 - x_1)} + \frac{(a_1 - x_1)[(a_2 + x_2) - (a_1 + x_1)] - (a_1 + x_1)[(a_2 - x_2) - (a_1 - x_1)]}{(a_1 - x_1)(a_2 - x_2)}I = \frac{(a_1 + x_1) + [(a_2 + x_2) - (a_1 + x_1)]I}{(a_1 - x_1) + [(a_2 - x_2) - (a_1 - x_1)]I}$$

$$y_1{}^2 + (y_2{}^2 - y_1{}^2)I = [x_1{}^2 + (x_2{}^2 - x_1{}^2)I].\left[\frac{(a_1 + x_1) + [(a_2 + x_2) - (a_1 + x_1)]I}{(a_1 - x_1) + [(a_2 - x_2) - (a_1 - x_1)]I}\right]$$
$$y_1{}^2 + (y_2{}^2 - y_1{}^2)I = [x_1{}^2 + (x_2{}^2 - x_1{}^2)I].\left[\frac{a_1 + (a_2 - a_1)I + [x_1 + (x_2 - x_1)I]}{a_1 + (a_2 - a_1)I - [x_1 + (x_2 - x_1)I]}\right] \dots (*)$$

We let $X = x_1 + (x_2 - x_1)I, Y = y_1 + (y_2 - y_1)I, A = a_1 + (a_2 - a_1)$, then we can prove that:

$$Y^2 = y_1{}^2 + (y_2{}^2 - y_1{}^2)I, X^2 = x_1{}^2 + (x_2{}^2 - x_1{}^2)$$

Then the equation $(*)$ can be written as follows:

$$\Gamma: Y^2 = X^2.\frac{A + X}{A - X} \quad ; A > 0$$

This equation is a neutrosophic strophoide Γ.

Example:

Let the equation by a neutrosophic strophoide:

$$\Gamma: (y_1 + y_2 I)^2 = (x_1 + x_2 I)^2.\frac{(4 - 2I) + (x_1 + x_2 I)}{(4 - 2I) - (x_1 + x_2 I)}$$

Then, its equation be equivalent to direct product of two classical strophoide:

$$\begin{cases} \Gamma_1: y_1{}^2 = x_1{}^2\left(\frac{4 + x_1}{4 - x_1}\right) \\ \Gamma_2: (y_1 + y_2)^2 = (x_1 + x_2)^2\left(\frac{2 + x_1}{2 - x_1}\right) \end{cases}$$

Example:

Let Γ_1, Γ_2 are two classical strophoide, where:

$$\begin{cases} \Gamma_1 : {y_1}^2 = {x_1}^2 \left(\dfrac{\frac{1}{2} + x_1}{\frac{1}{2} - x_1} \right) \\ \Gamma_2 : {y_2}^2 = {x_2}^2 \left(\dfrac{2 + x_2}{2 - x_2} \right) \end{cases}$$

Then by theorem 6.4 we have.

$$\Gamma : Y^2 = X^2 . \frac{\left(\frac{1}{2} + \frac{3}{2} I\right) + X}{\left(\frac{1}{2} + \frac{3}{2} I\right) - X}$$

.**Definition: Neutrosophic Cycloide.**

Let $\boldsymbol{Y = y_1 + y_2 I, X = x_1 + x_2 I, R = r_1 + r_2 I \,, t = t_1 + t_2 I \in}$ $\boldsymbol{R(I), r_1, r_2, t_1, t_2, x_1, x_2, y_1, y_2 \in R}$, then we define a neutroophic Cycloide as follows:

$$\boldsymbol{X = \mathrm{R}(1 - sint) \ , Y = \mathrm{R}(1 - cost)}$$

This equation can be written as follows:

$$\boldsymbol{x_1 + x_2 I = (r_1 + r_2 I)\big(1 - sin(t_1 + t_2 I)\big) \ , y_1 + y_2 I = (r_1 + r_2 I)\big(1 - cos(t_1 + t_2 I)\big)}$$

Theorem:

Let $\boldsymbol{Y = y_1 + y_2 I, X = x_1 + x_2 I}, R = r_1 + r_2 I \,, t = t_1 + t_2 I \in \boldsymbol{R(I)}$, then if $r_1 + r_2 I$ is invertible, the neutrosophic Cycloide $X = \mathrm{R}(1 - sint) \ , Y = \mathrm{R}(1 - cost)$ is equivalent to the direct product of two classical Cycloide.

Proof. Consider the equation $X = \mathrm{R}(1 - sint) \ , Y = \mathrm{R}(1 - cost)$

Now, we have:

$$x_1 + x_2 I = (r_1 + r_2 I)\left(1 - sin(t_1) - I\big(sin(t_1 + t_2) - sin(t_1)\big)\right)$$

$$y_1 + y_2 I = (r_1 + r_2 I)\left(1 - cos(t_1) - I\big(cos(t_1 + t_2) - cos(t_1)\big)\right)$$

by computing its direct image with AH-isometry, we get:

$$T(x_1 + x_2 I) = T(r_1 + r_2 I).T(1 - sin(t_1) - I[sin(t_1 + t_2) - sin(t_1)])$$

$(x_1, x_1 + x_2) = (r_1, r_1 + r_2).\big(1 - sin(t_1), 1 - sin(t_1 + t_2)\big)$

$$(x_1, x_1 + x_2) = \Big(r_1\big(1 - sin(t_1)\big), (r_1 + r_2)\big(1 - sin(t_1 + t_2)\big)\Big)$$

Then.

$$\begin{cases} x_1 = r_1\big(1 - sin(t_1)\big) \\ x_1 + x_2 = (r_1 + r_2)\big(1 - sin(t_1 + t_2)\big) \end{cases}$$

By a similar, we have.

$$\begin{cases} y_1 = r_1\big(1 - cos(t_1)\big) \\ y_1 + y_2 = (r_1 + r_2)\big(1 - cos(t_1 + t_2)\big) \end{cases}$$

So that we have:

$$\begin{cases} \boldsymbol{\Gamma_1: x_1 = r_1\big(1 - sin(t_1)\big), y_1 = r_1\big(1 - cos(t_1)\big)} \\ \boldsymbol{\Gamma_2: x_1 + x_2 = (r_1 + r_2)\big(1 - sin(t_1 + t_2)\big), y_1 + y_2 = (r_1 + r_2)\big(1 - cos(t_1 + t_2)\big)} \end{cases}$$

Remark:

If $r_1 + r_2 I$ is invertible, we can write the equation of neutrosophic cycloide as follows:

$X = \mathrm{R}(1 - sint)$, $Y = \mathrm{R}(1 - cost)$.

Now, we should discuss the cases of non-invertible of $r_1 + r_2 I$.

The $r_1 + r_2 I$ is not invertible, then we have two cases:

1- $r_1 = 0$, $r_1 + r_2 \neq 0$, this means that the neutrosophic cycloide will be equivalent to direct product of classical cycloide $x_1 + x_2 = (r_1 + r_2)\big(1 - sin(t_1 + t_2)\big), y_1 + y_2 = (r_1 + r_2)\big(1 - cos(t_1 + t_2)\big)$ with the origin point $(0,0)$.

2- $r_1 \neq 0$, $r_1 + r_2 = 0$, this means that the neutrosophic cycloide will be equivalent to direct product of classical cycloide $x_1 = r_1\big(1 - sin(t_1)\big), y_1 = r_1\big(1 - cos(t_1)\big)$ with the origin point $(0,0)$.

3- If $r_1 = 0$, $r_1 + r_2 = 0$, this implies that the neutrosophic cycloide will be equivalent to the origin point $(0,0)$.

Theorem:

Let Γ_1, Γ_2 are two classical cycloide, then the direct product of Γ_1, Γ_2 is equivalent to the neutrosophic cycloide Γ.

Proof.

Let Γ_1, Γ_2 are two classical cycloide, where:

$$\begin{cases}\Gamma_1: x_1 = r_1(1 - sint_1)\,, y_1 = r_1(1 - cost_1)\\ \Gamma_2: x_2 = r_2(1 - sint_2)\,, y_2 = r_2(1 - cost_2)\end{cases}$$

Now, we take the inverse image of the AH-isometry, we have:

$$T^{-1}(x_1, x_2) = T^{-1}(r_1, r_2).T^{-1}\big((1 - sint_1), (1 - sint_2)\big)$$

$$x_1 + (x_2 - x_1)I = [r_1 + (r_2 - r_1)I].\big[1 - sint_1 + \big((1 - sint_2) - (1 - sint_1)\big)I\big]$$

$$x_1 + (x_2 - x_1)I = [r_1 + (r_2 - r_1)I].[1 - sint_1 + (1 - sint_2 - 1 + sint_1)I]$$

$$x_1 + (x_2 - x_1)I = [r_1 + (r_2 - r_1)I].[1 - sint_1 - (sint_2 - sint_1)I]$$

$$x_1 + (x_2 - x_1)I = [r_1 + (r_2 - r_1)I].\big[1 - \big(sint_1 + (sint_2 - sint_1)\big)I\big]$$

$$x_1 + (x_2 - x_1)I = [r_1 + (r_2 - r_1)I].\big[1 - \big(sint_1 + (sin(t_2 - t_1 + t_1) - sint_1)\big)I\big]$$

We let $X = x_1 + (x_2 - x_1)I, R = r_1 + (r_2 - r_1)I, t = t_1 + (t_2 - t_1)I$, then we can prove that:

$$sint = sin[t_1 + (t_2 - t_1)I] = sint_1 + (sin(t_2 - t_1 + t_1) - sint_1)I$$

Then, we have.

$$X = R.(1 - sint)$$

Now, by the same argument, we have.

$$Y = R.(1 - cost)$$

So.

$$\Gamma: \begin{cases}X = R.(1 - sint)\\ Y = R.(1 - cost)\end{cases}$$

This equation is a neutrosophic cycloid Γ.

Example:

Let the equation by a neutrosophic cycloide:

$$\begin{cases}x_1 + x_2 I = (3 - 2I).[1 - sin(t_1 + t_2 I)]\\ y_1 + y_2 I = (3 - 2I).[1 - cos(t_1 + t_2 I)]\end{cases}$$

Then, its equation be equivalent to direct product of two classical cycloide:

$$\begin{cases} \boldsymbol{\Gamma_1: x_1 = 3(1 - sint_1)\,, y_1 = 3(1 - cost_1)} \\ \boldsymbol{\Gamma_2: x_1 + x_2 = 1 - sin(t_1 + t_2)\,, y_1 + y_2 = 1 - cos(t_1 + t_2)} \end{cases}$$

Example:

Let Γ_1, Γ_2 are two classical cycloide, where:

$$\begin{cases} \Gamma_1: x_1 = 2(1 - sint_1)\,, y_1 = 2(1 - cost_1) \\ \Gamma_2: x_2 = 5(1 - sint_2)\,, y_2 = 5(1 - cost_2) \end{cases}$$

$$\Gamma: \begin{Bmatrix} X = (2 + 3I)(1 - sint) \\ Y = (2 + 3I)(1 - cost) \end{Bmatrix}$$

.**Definition: Neutrosophic Cardioide.**

Let $\boldsymbol{\rho = \rho_1 + \rho_2 I, \theta = \theta_1 + \theta_2 I \in R(I), \rho_1, \rho_2, \theta_1, \theta_2 \in R}$, then we define a neutroophic Cardoide as follows:

$$\boldsymbol{\rho = (1 + cos\theta)}$$

This equation can be written as follows:

$$\boldsymbol{\rho_1 + \rho_2 I = (1 + cos\theta_1) + [cos(\theta_1 + \theta_2) - cos\theta_1]I}$$

Theorem:

Let $\boldsymbol{= \rho_1 + \rho_2 I, \theta = \theta_1 + \theta_2 I \in R(I)}$, then if $\boldsymbol{\theta_1 + \theta_2 I}$ is invertible, the neutrosophic Cardioide

$\boldsymbol{\rho = (1 + cos\theta)}$ is equivalent to the direct product of two classical Cardioide.

Proof. Consider the equation $\rho = (1 + cos\theta)$

Now, we have:

$\rho_1 + \rho_2 I = (1 + cos\theta_1) + [cos(\theta_1 + \theta_2) - cos\theta_1]I$

by computing its direct image with AH-isometry, we get:

$T(\rho_1 + \rho_2 I) = T((1 + cos\theta_1) + [cos(\theta_1 + \theta_2) - cos\theta_1]I)$

$(\rho_1, \rho_1 + \rho_2) = (1 + cos\theta_1, 1 + cos(\theta_1 + \theta_2))$

Then.

$$\begin{cases} \rho_1 = 1 + cos\theta_1 \\ \rho_1 + \rho_2 = 1 + cos(\theta_1 + \theta_2) \end{cases}$$

So that we have:

$$\begin{cases} \boldsymbol{\Gamma_1: \rho_1 = 1 + cos\theta_1} \\ \boldsymbol{\Gamma_2: \rho_1 + \rho_2 = 1 + cos(\theta_1 + \theta_2)} \end{cases}$$

Remark:

If $\theta_1 + \theta_2 I$ is invertible, we can write the equation of neutrosophic Cardioide as follows:

$\rho = (1 + cos\theta)$.

Now, we should discuss the cases of non-invertible of $\theta_1 + \theta_2 I$.

The $\theta_1 + \theta_2 I$ is not invertible, then we have two cases:

1- $\theta_1 = 0,\ \theta_1 + \theta_2 \neq 0$, this means that the neutrosophic Cardioide will be equivalent to direct product of classical Cardioide $(\rho_1 + \rho_2) = 1 + cos(\theta_1 + \theta_2)$ with the classical circle $\rho_1 = 2$.

2- $\theta_1 \neq 0,\ \theta_1 + \theta_2 = 0$, this means that the neutrosophic Cardioide will be equivalent to direct product of classical Cardioide $\rho_1 = 1 + cos(\theta_1)$ with the classical circle $(\rho_1 + \rho_2) = 2$.

3- If $\theta_1 = 0,\ \theta_1 + \theta_2 = 0$ this means that the neutrosophic Cardioide will be equivalent to direct product of classical circle $(\rho_1 + \rho_2) = 2$ with the classical circle $\rho_1 = 2$.

Theorem:

Let Γ_1, Γ_2 are two classical Cardioide, then the direct product of Γ_1, Γ_2 is equivalent to the neutrosophic Cardioide Γ.

Proof.

Let Γ_1, Γ_2 are two classical Cardioide, where:

$$\begin{cases} \Gamma_1: \rho_1 = 1 + cos\theta_1 \\ \Gamma_2: \rho_2 = 1 + cos\theta_2 \end{cases}$$

Now, we take the inverse image of the AH-isometry, we have:

$$T^{-1}(\rho_1, \rho_2) = T^{-1}(1 + cos\theta_1, 1 + cos\theta_2)$$

$$\rho_1 + (\rho_2 - \rho_1)I = [1 + cos\theta_1 + (1 + cos\theta_2 - (1 + cos\theta_1))I]$$

$$\rho_1 + (\rho_2 - \rho_1)I = [1 + cos\theta_1 + (cos\theta_2 - cos\theta_1)I]$$

$$\rho_1 + (\rho_2 - \rho_1)I = 1 + [cos\theta_1 + (cos(\theta_1 + [\theta_2 - \theta_1]) - cos\theta_1)I]$$

We let $\rho = \rho_1 + (\rho_2 - \rho_1)I, \theta = \theta_1 + (\theta_2 - \theta_1)I$, then we can prove that:

$$cos\theta = cos(\theta_1 + [\theta_2 - \theta_1]I) = cos\theta_1 + (cos(\theta_1 + [\theta_2 - \theta_1]) - cos\theta_1)I$$

Then, we have.

$$\rho_1 + (\rho_2 - \rho_1)I = 1 + cos(\theta_1 + [\theta_2 - \theta_1]I)$$

So.

$$\Gamma: \rho = 1 + cos\theta$$

This equation is a neutrosophic Cardioide Γ.

Example:

Let the equation by a neutrosophic Cardioide:

$$\rho_1 + \rho_2 I = 1 + cos\left(\frac{\pi}{3} + \frac{\pi}{4}I\right)$$

Then, its equation be equivalent to direct product of two classical Cardioide:

$$\begin{cases} \boldsymbol{\Gamma_1: \rho_1 = 1 + cos\left(\frac{\pi}{3}\right)} \\ \boldsymbol{\Gamma_2: \rho_1 + \rho_2 = 1 + cos\left(\frac{7\pi}{12}\right)} \end{cases}$$

Example:

Let Γ_1, Γ_2 are two classical Cardioide, where:

$$\begin{cases} \Gamma_1: \rho_1 = 1 + cos\left(\frac{\pi}{4}\right) \\ \Gamma_2: \rho_2 = 1 + cos\left(\frac{\pi}{2}\right) \end{cases}$$

$$\Gamma: \left\{\rho = 1 + cos\left(\frac{\pi}{4} + \frac{\pi}{4}I\right)\right.$$

Conclusions

In this paper we have studied some concepts of neutrosophic real analysis depending on the one-dimensional AH-isometry. We have provided a strict definition of some algebraic curves in neutrosophic real ring $\boldsymbol{R(I)}$, and we study the properties of this curves, and we proved some theorems for this curves, also, we find relationships between a classical algerbraic curves and neutrosophic algebraic curves.

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