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On The Symbolic 2-Plithogenic Number Theory and Integers

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Abstract:

The objective of this paper is to study for the first time the foundational concepts of number theory in 2-plithogenic rings of integers, where concepts such as symbolic 2-plithogenic congruencies, division, semi primes, and greatest common divisors.

In addition, many elementary properties will be discussed in details through many theorems and examples.

Keywords: Symbolic 2-plithogenic integer, symbolic 2-plithogenic division, symbolic 2-plithogenic semi prime.

Introduction and basic concepts

The concept of symbolic plithogenic sets was defined by Smarandache in [13-17,30], and he suggested an algebraic approach of these sets. Laterally, the concept of symbolic 2-plithogenic rings [31]. In general, we can say that symbolic plithogenic structures are very close to neutrosophic algebraic structures with many differences in the definition of multiplication operation [1-10].

Let R be a ring, the symbolic 2-plithogenic ring is defined as follows:

$$2 - SP_R = \{a_0 + a_1P_1 + a_2P_2; a_i \in R, P_j^2 = P_j, P_1 \times P_2 = P_{\max(1,2)} = P_2\}.$$

Smarandache has defined algebraic operations on $2 - SP_R$ as follows:

Addition:

$$[a_0 + a_1P_1 + a_2P_2] + [b_0 + b_1P_1 + b_2P_2] = (a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2.$$

Multiplication:

$$[a_0 + a_1P_1 + a_2P_2] \cdot [b_0 + b_1P_1 + b_2P_2] = a_0b_0 + a_0b_1P_1 + a_0b_2P_2 + a_1b_0P_1^2 + a_1b_2P_1P_2 + a_2b_0P_2 + a_2b_1P_1P_2 + a_2b_2P_2^2 + a_1b_1P_1P_1 = a_0b_0 + (a_0b_1 + a_1b_0 + a_1b_1)P_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)P_2.$$

It is clear that $(2 - SP_R)$ is a ring.

Also, if R is commutative, then $2 - SP_R$ is commutative, and if R has a unity (1), then $2 - SP_R$ has the same unity (1).

If R is a field, then $2 - SP_R$ is called a symbolic 2-plithogenic field.

In this paper, we study the symbolic 2-plithogenic number theoretical concepts according to many points of view, where congruencies, Euclidean division, Euler's function, and greatest common divisors will be presented in terms of theorems. In addition, many examples will be illustrated to explain the novelty of these ideas. In addition, we suggest many future applications of symbolic 2-plithogenic integers in cryptography and public key neutrosophic cryptography.

Main Discussion

Definition.

Let $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2 \in 2 - SP_Z$, we say that $A \setminus B$ if and only if there exists $C \in 2 - SP_Z$ such that $A \times B = C$.

Definition.

Let $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2, C = c_0 + c_1P_1 + c_2P_2$ be three symbolic 2-plithogenic integers, then $A \equiv B \pmod{C}$ if and only if $C \setminus A - B$.

Also, $C = \gcd(A, B)$ if and only if $C \setminus A$ and $C \setminus B$ and for any $D \setminus A, D \setminus B$, then $D \setminus C$.

Definition.

We say that $A \leq B$ if $a_0 \leq b_0, a_0 + a_1 \leq b_0 + b_1, a_0 + a_1 + a_2 \leq b_0 + b_1 + b_2$.

Theorem.

Let $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2, C = c_0 + c_1P_1 + c_2P_2 \in 2 - SP_Z$, then:

- 1). (\leq) is a partial order relation.
- 2). $A \setminus B$ if and only if $a_0 \setminus b_0, a_0 + a_1 \setminus b_0 + b_1, a_0 + a_1 + a_2 \setminus b_0 + b_1 + b_2$.

3). $\gcd(A, B) = C$ if and only if $\gcd(a_0, b_0) = c_0, \gcd(a_0 + a_1, b_0 + b_1) = c_0 + c_1, \gcd(a_0 + a_1 + a_2, b_0 + b_1 + b_2) = c_0 + c_1 + c_2$.

4). $A \equiv B \pmod{C}$ if and only if:

$$\begin{cases} a_0 \equiv b_0 \pmod{c_0} \\ a_0 + a_1 \equiv b_0 + b_1 \pmod{c_0 + c_1} \\ a_0 + a_1 + a_2 \equiv b_0 + b_1 + b_2 \pmod{c_0 + c_1 + c_2} \end{cases}$$

Proof.

1). $A \leq A$ that is because $a_0 \leq a_0, a_0 + a_1 \leq a_0 + a_1, a_0 + a_1 + a_2 \leq a_0 + a_1 + a_2$.

If $A \leq B$ and $B \leq A$, then:

$$\begin{cases} a_0 \leq b_0, b_0 \leq a_0, \text{ thus } a_0 = b_0 \\ a_0 + a_1 \leq b_0 + b_1, b_0 + b_1 \leq a_0 + a_1, \text{ thus } a_0 + a_1 = b_0 + b_1, \text{ hence } a_1 = b_1 \\ a_0 + a_1 + a_2 \leq b_0 + b_1 + b_2, b_0 + b_1 + b_2 \leq a_0 + a_1 + a_2, \text{ thus } a_0 + a_1 + a_2 = b_0 + b_1 + b_2, \text{ hence } a_2 = b_2 \end{cases}$$

Hence $A = B$.

If $A \leq B$ and $B \leq C$, then $a_0 \leq b_0 \leq c_0, a_0 + a_1 \leq b_0 + b_1 \leq c_0 + c_1, a_0 + a_1 + a_2 \leq b_0 + b_1 + b_2 \leq c_0 + c_1 + c_2$, thus $A \leq C$.

2). If $A \setminus B$, then there exists C such that $A \cdot C = B$. This equivalent:

$$a_0 c_0 + P_1(a_0 c_1 + a_1 c_0 + a_1 c_1) + P_2(a_0 c_2 + a_2 c_0 + a_2 c_2 + a_1 c_2 + a_2 c_1) = b_0 + b_1 P_1 + b_2 P_2, \quad ,$$

there for:

$$\begin{cases} a_0 c_0 = b_0 \dots (1) \\ a_0 c_1 + a_1 c_0 + a_1 c_1 = b_1 \dots (2) \\ a_0 c_2 + a_2 c_0 + a_2 c_2 + a_1 c_2 + a_2 c_1 = b_2 \dots (3) \end{cases}$$

We add (1) to (2) and (1) to (2) to (3), to get:

$$\begin{cases} a_0 c_0 = b_0 \\ (a_0 + a_1)(c_0 + c_1) = b_0 + b_1 \\ (a_0 + a_1 + a_2)(c_0 + c_1 + c_2) = b_0 + b_1 + b_2 \end{cases}$$

Thus $a_0 \setminus b_0, a_0 + a_1 \setminus b_0 + b_1, a_0 + a_1 + a_2 \setminus b_0 + b_1 + b_2$.

3). Assume that $\gcd(A, B) = C$, then for any $D = d_0 + d_1 P_1 + d_2 P_2 \in 2 - SP_Z$ such that $D \setminus A, D \setminus B$ implies $D \setminus C$.

According to (2), we get $d_0 \setminus c_0, d_0 + d_1 \setminus c_0 + c_1, d_0 + d_1 + d_2 \setminus c_0 + c_1 + c_2$, so that $\gcd(a_0, b_0) = c_0, \gcd(a_0 + a_1, b_0 + b_1) = c_0 + c_1, \gcd(a_0 + a_1 + a_2, b_0 + b_1 + b_2) = c_0 + c_1 + c_2$.

This implies that $\gcd(A, B) = \gcd(a_0, b_0) + P_1[\gcd(a_0 + a_1, b_0 + b_1) - \gcd(a_0, b_0)] + P_2[\gcd(a_0 + a_1 + a_2, b_0 + b_1 + b_2) - \gcd(a_0 + a_1, b_0 + b_1)]$.

4). $A \equiv B \pmod{C}$ if and only if $C \setminus A - B$, thus:

$$c_0 \setminus a_0 - b_0, c_0 + c_1 \setminus (a_0 + a_1) - (b_0 + b_1), c_0 + c_1 + c_2 \setminus (a_0 + a_1 + a_2) - (b_0 + b_1 + b_2)$$

So that:

$$\begin{cases} a_0 \equiv b_0 \pmod{c_0} \\ a_0 + a_1 \equiv b_0 + b_1 \pmod{c_0 + c_1} \\ a_0 + a_1 + a_2 \equiv b_0 + b_1 + b_2 \pmod{c_0 + c_1 + c_2} \end{cases}$$

Theorem.

Let $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2 \in 2 - SP_Z$, then $\gcd(A, B) = 1$ if and only if $\gcd(a_0, b_0) = 1, \gcd(a_0 + a_1, b_0 + b_1) = 1, \gcd(a_0 + a_1 + a_2, b_0 + b_1 + b_2) = 1$.

The proof is clear.

Theorem.

Let $A, B, C, D, E \in 2 - SP_Z$, where:

$A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2, C = c_0 + c_1P_1 + c_2P_2, D = d_0 + d_1P_1 + d_2P_2, E = e_0 + e_1P_1 + e_2P_2; c_i, a_i, b_i, e_i, d_i \in Z$, then:

- 1). If $A \equiv B \pmod{C}, D \equiv E \pmod{C}$, then $A + D \equiv B + E \pmod{C}, A - D \equiv B - E \pmod{C}$.
- 2). $A \cdot D \equiv B \cdot E \pmod{C}$.
- 3). If $\gcd(A, B) = 1$, then:

$$\begin{aligned} A^{-1} \pmod{B} &= a_0^{-1} \pmod{b_0} + P_1[(a_0 + a_1)^{-1} \pmod{b_0 + b_1} - a_0^{-1} \pmod{b_0}] \\ &\quad + P_2[(a_0 + a_1 + a_2)^{-1} \pmod{b_0 + b_1 + b_2} - (a_0 + a_1)^{-1} \pmod{b_0 + b_1}] \end{aligned}$$

Proof.

- 1). Assume that $A \equiv B \pmod{C}, D \equiv E \pmod{C}$, thus:

$$\begin{cases} a_0 \equiv b_0 \pmod{c_0} \\ a_0 + a_1 \equiv b_0 + b_1 \pmod{c_0 + c_1} \\ a_0 + a_1 + a_2 \equiv b_0 + b_1 + b_2 \pmod{c_0 + c_1 + c_2} \end{cases}$$

And

$$\begin{cases} d_0 \equiv e_0 \pmod{c_0} \\ d_0 + d_1 \equiv e_0 + e_1 \pmod{c_0 + c_1} \\ d_0 + d_1 + d_2 \equiv e_0 + e_1 + e_2 \pmod{c_0 + c_1 + c_2} \end{cases}$$

This implies:

$$\begin{cases} a_0 + d_0 \equiv b_0 + e_0 \pmod{c_0} \\ a_0 + a_1 + d_0 + d_1 \equiv b_0 + b_1 + e_0 + e_1 \pmod{c_0 + c_1} \\ a_0 + a_1 + a_2 + d_0 + d_1 + d_2 \equiv b_0 + b_1 + b_2 + e_0 + e_1 + e_2 \pmod{c_0 + c_1 + c_2} \end{cases}$$

So that $A + D \equiv B + E \pmod{C}$.

We can prove that $A - D \equiv B - E \pmod{C}$ by a similar.

- 2). By using a similar discussion, we can write:

$$\begin{cases} a_0d_0 \equiv b_0e_0 \pmod{c_0} \\ (a_0 + a_1)(d_0 + d_1) \equiv (b_0 + b_1)(e_0 + e_1) \pmod{c_0 + c_1} \\ (a_0 + a_1 + a_2)(d_0 + d_1 + d_2) \equiv (b_0 + b_1 + b_2)(e_0 + e_1 + e_2) \pmod{c_0 + c_1 + c_2} \end{cases}$$

Thus $A.D \equiv B.E \pmod{C}$.

3). Suppose that $\gcd(A, B) = 1$, then $\gcd(a_0, b_0) = \gcd(a_0 + a_1, b_0 + b_1) = \gcd(a_0 + a_1 + a_2, b_0 + b_1 + b_2) = 1$.

We put

$$\begin{aligned} T &= a_0^{-1} \pmod{b_0} + P_1[(a_0 + a_1)^{-1} \pmod{b_0 + b_1} - a_0^{-1} \pmod{b_0}] \\ &\quad + P_2[(a_0 + a_1 + a_2)^{-1} \pmod{b_0 + b_1 + b_2} - (a_0 + a_1)^{-1} \pmod{b_0 + b_1}] \\ A.T &= a_0 a_0^{-1} \pmod{b_0} + P_1[(a_0 + a_1)(a_0 + a_1)^{-1} \pmod{b_0 + b_1} - a_0 a_0^{-1} \pmod{b_0}] \\ &\quad + P_2[(a_0 + a_1 + a_2)(a_0 + a_1 + a_2)^{-1} \pmod{b_0 + b_1 + b_2} \\ &\quad - (a_0 + a_1)(a_0 + a_1)^{-1} \pmod{b_0 + b_1}] = 1 + P_1(1 - 1) + P_2(1 - 1) = 1 \end{aligned}$$

Thus $T = A^{-1}$.

Example:

Consider $A = 5 + 4P_1 + 2P_2, B = 2 + P_1 + P_2, C = 3 + 4P_2$, we have:

$5 \equiv 2 \pmod{3}, 5 + 4 = 9 \equiv (2 + 1) \pmod{3 + 0}, 5 + 4 + 2 = 11 \equiv (2 + 1 + 1) \pmod{3 + 0 + 4}$, thus $A \equiv B \pmod{C}$.

$$\begin{aligned} \gcd(A, B) &= \gcd(5, 2) + P_1[\gcd(9, 3) - \gcd(5, 2)] + P_2[\gcd(11, 4) - \gcd(9, 3)] = 1 + \\ &P_1(3 - 1) + P_2(1 - 3) = 1 + 2P_1 - 2P_2. \end{aligned}$$

Example.

Consider $A = 2 + P_1 + P_2, B = 3 + P_1 + P_2$, it is clear that $\gcd(A, B) = 1$.

$$\begin{aligned} A^{-1} \pmod{B} &= 2^{-1} \pmod{3} + P_1[3^{-1} \pmod{4} - 2^{-1} \pmod{3}] + P_2[4^{-1} \pmod{5} - \\ &3^{-1} \pmod{4}] = 2 + P_1(3 - 2) + P_2(4 - 3) = 2 + P_1 + P_2. \end{aligned}$$

Definition.

Let $A = a_0 + a_1P_1 + a_2P_2 > 0$ be a symbolic 2-plithogenic integer, we define $\varphi_S: 2 - SP_Z \rightarrow 2 - SP_Z$ such that:

$$\varphi_S(A) = \varphi(a_0) + P_1[\varphi(a_0 + a_1) - \varphi(a_0)] + P_2[\varphi(a_0 + a_1 + a_2) - \varphi(a_0 + a_1)].$$

Where φ is the classical phi-Euler's function.

Example.

Take $A = 3 + 5P_1 - P_2, a_0 = 3, a_1 = 5, a_2 = -1$. We have:

$a_0 = 3 > 0, a_0 + a_1 = 8 > 0, a_0 + a_1 + a_2 = 7 > 0$, so that $A > 0$.

$\varphi(a_0) = 2, \varphi(a_0 + a_1) = 4, \varphi(a_0 + a_1 + a_2) = 6$, hence:

$$\varphi_S(A) = 2 + P_1[4 - 2] + P_2[6 - 4] = 2 + 2P_1 + 2P_2.$$

Theorem.

Let $A = a_0 + a_1P_1 + a_2P_2, M = m_0 + m_1P_1 + m_2P_2 \in 2 - SP_Z$ such that $\gcd(A, M) = 1$, then

$$A^{\varphi_S(M)} \equiv 1 \pmod{M}.$$

Proof.

According to []:

$$A^{\varphi_S(M)} = a_0^{\varphi(m_0)} + P_1[(a_0 + a_1)^{\varphi(m_0+m_1)} - a_0^{\varphi(m_0)}] \\ + P_2[(a_0 + a_1 + a_2)^{\varphi(m_0+m_1+m_2)} - (a_0 + a_1)^{\varphi(m_0+m_1)}]$$

Since $\gcd(A, M) = 1$, then $\gcd(a_0, m_0) = \gcd(a_0 + a_1, m_0 + m_1) = \gcd(a_0 + a_1 + a_2, m_0 + m_1 + m_2) = 1$, so that:

$$\begin{cases} a_0^{\varphi(m_0)} \equiv 1 \pmod{m_0} \\ (a_0 + a_1)^{\varphi(m_0+m_1)} \equiv 1 \pmod{m_0 + m_1} \\ (a_0 + a_1 + a_2)^{\varphi(m_0+m_1+m_2)} \equiv 1 \pmod{m_0 + m_1 + m_2} \end{cases}$$

$$\text{Thus } A^{\varphi_S(M)} \equiv 1 + P_1(1 - 1) + P_2(1 - 1) \pmod{M} \equiv 1 \pmod{M}$$

Example.

Take $A = 2 + 3P_1 - 2P_2, M = 3 + 4P_1 + 4P_2$, we have $\gcd(A, M) = 1$.

$$\varphi_S(M) = 2 + P_1(6 - 2) + P_2(10 - 6) = 2 + 4P_1 + 4P_2$$

$$A^{\varphi_S(M)} = 2^2 + P_1[5^6 - 2^2] + P_2[3^{10} - 5^6]$$

$$2^2 \equiv 1 \pmod{3}, 5^6 \equiv 1 \pmod{7}, 3^{10} \equiv 1 \pmod{11}, \text{ thus } A^{\varphi_S(M)} \equiv 1 \pmod{M}$$

Theorem.

Let $C = \gcd(A, B) \in 2 - SP_Z$, then there exists $M, N \in 2 - SP_Z$ such that $C = MA + NB$.

Proof.

We assume that $C = \gcd(A, B)$, then:

$$\begin{cases} c_0 = \gcd(a_0, b_0) \\ c_1 = \gcd(a_0 + a_1, b_0 + b_1) - \gcd(a_0, b_0) \\ c_2 = \gcd(a_0 + a_1 + a_2, b_0 + b_1 + b_2) - \gcd(a_0 + a_1, b_0 + b_1) \end{cases}$$

So there exists $m_0, n_0, m_1, n_1, m_2, n_2 \in Z$ such that:

$$\begin{cases} c_0 = m_0 a_0 + n_0 b_0 \\ c_0 + c_1 = m_1(a_0 + a_1) + n_1(b_0 + b_1) \\ c_0 + c_1 + c_2 = m_2(a_0 + a_1 + a_2) + n_2(b_0 + b_1 + b_2) \end{cases}$$

We put $M = m_0 + (m_1 - m_0)P_1 + (m_2 - m_1)P_2, N = n_0 + (n_1 - n_0)P_1 + (n_2 - n_1)P_2$, now

let us compute:

$$M.A = [m_0 + (m_1 - m_0)P_1 + (m_2 - m_1)P_2][a_0 + a_1P_1 + a_2P_2]$$

$$M.A = m_0 a_0 + P_1(m_0 a_1 + m_1 a_0 - m_0 a_0 + m_1 a_1 - m_0 a_1) \\ + P_2(m_0 a_2 + m_2 a_0 - m_1 a_0 + m_2 a_1 - m_1 a_1 + m_1 a_2 - m_0 a_2 + m_2 a_2 - m_1 a_2)$$

$$M.A = m_0 a_0 + P_1(m_1 a_0 + m_1 a_1 - m_0 a_0) \\ + P_2(m_2 a_0 - m_1 a_0 + m_2 a_1 - m_1 a_1 + m_1 a_2 + m_2 a_2)$$

$$N.B = n_0 b_0 + P_1(n_1 b_0 + n_1 b_1 - n_0 b_0) + P_2(n_2 b_0 - n_1 b_0 + n_2 b_1 - n_1 b_1 + n_1 b_2 + n_2 b_2)$$

$$\begin{aligned}
MA + NB &= (m_0a_0 + n_0b_0) + P_1[m_1(a_0 + a_1) + n_1(b_0 + b_1) - n_0b_0 - m_0a_0] \\
&\quad + P_2[m_2(a_0 + a_1 + a_2) + n_2(b_0 + b_1 + b_2) - m_1(a_0 + a_1) - n_1(b_0 + b_1)] \\
&= c_0 + c_1P_1 + c_2P_2 = C
\end{aligned}$$

Example.

Consider $A = 3 + 2P_1 + P_2, B = 3 + P_1 + 3P_2$, we have:

$$a_0 = 3, a_1 = 2, a_2 = 1, b_0 = 3, b_1 = 1, b_2 = 3.$$

$$\begin{aligned}
gcd(a_0, b_0) &= 3, gcd(a_0 + a_1, b_0 + b_1) = gcd(5, 4) = 1, gcd(a_0 + a_1 + a_2, b_0 + b_1 + b_2) \\
&= gcd(6, 7) = 1
\end{aligned}$$

$$\text{Thus } gcd(A, B) = 3 + (1 - 3)P_1 + (1 - 1)P_2 = 3 - 2P_1.$$

On the other hand, we have:

$$\begin{cases}
3 = 1.3 + 0.3 \text{ hence } m_0 = 1, n_0 = 0 \\
1 = 1.5 - 1.4 \text{ hence } m_1 = 1, n_1 = -1 \\
1 = -1.6 + 1.7 \text{ hence } m_2 = -1, n_2 = 1
\end{cases}$$

$$\text{Thus } M = 1 + (1 - 3)P_1 + (-1 - 1)P_2 = 1 - 2P_2, N = 0 + (-1 - 0)P_1 + (1 + 1)P_2 = -P_1 + 2P_2$$

We can see that:

$$\begin{aligned}
MA + NB &= (1 - 2P_2)(3 + 2P_1 + P_2) + (-P_1 + 2P_2)(3 + P_1 + 3P_2) \\
&= 3 + 2P_1 + P_2 - 6P_2 - 4P_2 - 2P_2 - 3P_1 - P_1 - 3P_1 + 6P_2 + 2P_2 + 6P_2 \\
&= 3 - 2P_1 = C = gcd(A, B)
\end{aligned}$$

Definition.

Let $S = s_0 + s_1P_1 + s_2P_2 \in 2 - SP_Z$, we say that S is a 2-plithogenic semi prime if $s_0, s_0 + s_1, s_0 + s_1 + s_2$ are primes.

Example.

The 2-plithogenic integer $S = 2 + P_1 + 2P_2$ is a semi prime, that is because $s_0 = 2, s_0 + s_1 = 3, s_0 + s_1 + s_2 = 5$ are primes.

Application In Future Studies

Symbolic 2-plithogenic number theory as a new research direction maybe very useful branches of knowledge.

We suggest the following research points that symbolic 2-plithogenic integers may have a very big effect on it.

1-). How can we use symbolic 2-plithogenic integers in the improvement of crypto-systems [39-41], for example:

a). How can we build a 2-plithogenic version of RSA algorithm.

b). How can we build a 2-plithogenic version of Diffie-Hellman key exchange algorithm.

- c). How can we build a 2-plithogenic version of EL-Gamal algorithm for cryptography.
- 2-). How can we solve non-linear symbolic 2-plithogenic Diophantine equations and congruencies.

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