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# On The Symbolic 3-Plithogenic Rings and Their Algebraic Properties

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## Abstract:

The aim of this paper is to define and study the concept of symbolic 3-plithogenic rings as a novel extension of classical rings and symbolic 2-plithogenic rings respectively. Also, many related substructures will be presented such as idempotent elements, AH-ideals, AHS-homomorphisms, and kernels.

On the other hand, many examples will be illustrated to show the validity of concepts and theorem.

**Keywords:** Symbolic 3-plithogenic ring, AH-ideal, AH-homomorphism, symbolic plithogenic set

## Introduction

The concept of symbolic neutrosophic algebraic structure has played an important role in the advances of pure algebra and logical algebra. Many interesting structures were defined from this point of view, such as neutrosophic rings, refined neutrosophic rings, neutrosophic spaces, and n-cyclic refined neutrosophic rings [1-5,8-11,13-20].

In [30], Smarandache has presented a novel approach to algebraic structures by using the concept of n-symbolic plithogenic sets, where he defined algebraic operations on these structures and asked many open problems about them.

In [31], the concept of symbolic 2-plithogenic ring was suggested, and concepts such as symbolic 2-plithogenic AH-homomorphisms, ideals, and kernels.

This paper is considered as an additional effort which is dedicated to define a new algebraic structure built over the idea of symbolic n-plithogenic set with algebraic ring in a special case of n=3.

## Main Discussion

### Definition.

Let  $R$  be a ring, the symbolic 3-plithogenic ring is defined as follows:

$$3 - SP_R = \{a_0 + a_1P_1 + a_2P_2 + a_3P_3; a_i \in R, P_j^2 = P_j, P_i \times P_j = P_{\max(i,j)}\}.$$

Smarandache has defined algebraic operations on  $3 - SP_R$  as follows:

Addition:

$$[a_0 + a_1P_1 + a_2P_2 + a_3P_3] + [b_0 + b_1P_1 + b_2P_2 + b_3P_3] = (a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2 + (a_3 + b_3)P_3.$$

Multiplication:

$$[a_0 + a_1P_1 + a_2P_2 + a_3P_3] \cdot [b_0 + b_1P_1 + b_2P_2 + b_3P_3] = a_0b_0 + a_0b_1P_1 + a_0b_2P_2 + a_0b_3P_3 + a_1b_0P_1^2 + a_1b_2P_1P_2 + a_2b_0P_2 + a_2b_1P_1P_2 + a_2b_2P_2^2 + a_1b_3P_3P_1 + a_2b_3P_2P_3 + a_3b_3(P_3)^2 + a_3b_0P_3 + a_3b_1P_3P_1 + a_3b_2P_2P_3 + a_1b_1P_1P_1 = a_0b_0 + (a_0b_1 + a_1b_0 + a_1b_1)P_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)P_2 + (a_0b_3 + a_1b_3 + a_2b_3 + a_3b_3 + a_3b_0 + a_3b_1 + a_3b_2)P_3.$$

It is clear that  $(3 - SP_R)$  is a ring.

Also, if  $R$  is commutative, then  $3 - SP_R$  is commutative, and if  $R$  has a unity (1), then  $3 - SP_R$  has the same unity (1).

### Example.

Consider the ring  $R = Z_5 = \{0,1,2,3,4\}$ , the corresponding  $3 - SP_R$  is:

$$3 - SP_R = \{a + bP_1 + cP_2 + dP_3; a, b, c, d \in Z_5\}.$$

If  $X = 1 + 2P_1 + 3P_2 + P_3, Y = P_1 + 2P_2$ , then:

$$X + Y = 1 + 3P_1 + P_2 + P_3, X - Y = 1 + P_1 + P_2 + P_3, X \cdot Y = P_1 + 2P_2 + 2P_1 + 4P_2 + 3P_2 + 6P_2 + P_3 + 2P_3 = 3P_1 + 3P_2 + 3P_3.$$

### Invertibility.

#### Theorem.

Let  $3 - SP_R$  be a 3-plithogenic symbolic ring, with unity (1).

Let  $X = x_0 + x_1P_1 + x_2P_2 + x_3P_3$  be an arbitrary element, then:

1.  $X$  is invertible if and only if  $x_0, x_0 + x_1, x_0 + x_1 + x_2, x_0 + x_1 + x_2 + x_3$  are invertible.
2.  $X^{-1} = x_0^{-1} + [(x_0 + x_1)^{-1} - x_0^{-1}]P_1 + [(x_0 + x_1 + x_2)^{-1} - (x_0 + x_1)^{-1}]P_2 + [(x_0 + x_1 + x_2 + x_3)^{-1} - (x_0 + x_1 + x_2)^{-1}]P_3$ .

**Proof.**

1. Assume that  $X$  is invertible, then there exists  $Y = y_0 + y_1P_1 + y_2P_2 + y_3P_3$  such that  $X.Y = 1$ , hence:

$$\begin{cases} x_0y_3 + x_1y_3 + x_2y_3 + x_3y_3 + x_3y_1 + x_3y_2 + x_3y_0 = 0 & (1) \\ x_0y_0 = 1 & \dots (2) \\ x_0y_1 + x_1y_0 + x_1y_1 = 0 & \dots (3) \\ x_0y_2 + x_2y_0 + x_2y_2 + x_1y_2 + x_2y_1 = 0 & \dots (4), \end{cases}$$

Equation (2), means that  $x_0$  is invertible.

By adding (3) to (2), we get  $(x_0 + x_1)(y_0 + y_1) = 1$ , thus  $x_0 + x_1$  is invertible.

By adding (4) to (3) to (2), we get  $(x_0 + x_1 + x_2)(y_0 + y_1 + y_2) = 1$ , hence  $x_0 + x_1 + x_2$  is invertible.

By adding (1) to (2) to (3) to (4), we get  $(x_0 + x_1 + x_2 + x_3)(y_0 + y_1 + y_2 + y_3) = 1$ , hence  $x_0 + x_1 + x_2 + x_3$  is invertible.

The converse holds by the same.

2. From the previous approach, we can see that:

$y_0 = x_0^{-1}, y_0 + y_1 = (x_0 + x_1)^{-1}, y_0 + y_1 + y_2 = (x_0 + x_1 + x_2)^{-1}, (x_0 + x_1 + x_2 + x_3)^{-1} = y_0 + y_1 + y_2 + y_3$ , then:

$$\begin{aligned} 3. Y &= x_0^{-1} + [(x_0 + x_1)^{-1} - x_0^{-1}]P_1 + [(x_0 + x_1 + x_2)^{-1} - (x_0 + x_1)^{-1}]P_2 + [(x_0 + x_1 + x_2 + x_3)^{-1} - (x_0 + x_1 + x_2)^{-1}]P_3. \\ &= X^{-1}. \end{aligned}$$

**Example.**

Take  $R = Z_5 = \{0,1,2,3,4\}$ ,  $3 - SP_{Z_5}$  is the corresponding symbolic 3-plithogenic ring, consider  $X = 2 + 4P_1 + 2P_2 + P_3 \in 2 - SP_{Z_5}$ , then:

$x_0 = 2$  is invertible with  $x_0^{-1} = 3$ ,  $x_0 + x_1 = 1$  is invertible with  $(x_0 + x_1)^{-1} = 1$ ,  $x_0 + x_1 + x_2 = 3$  is invertible with  $(x_0 + x_1 + x_2)^{-1} = 2$ ,  $x_0 + x_1 + x_2 + x_3 = 4, (x_0 + x_1 + x_2 + x_3)^{-1} = 4$  hence:

$$X^{-1} = 3 + (1 - 3)P_1 + (2 - 1)P_2 + (4 - 2)P_3 = 3 + 3P_1 + P_2 + 2P_3.$$

**Idempotency.**

**Definition.**

Let  $X = a + bP_1 + cP_2 + dP_3 \in 3 - SP_R$ , then  $X$  is idempotent if and only if  $X^2 = X$ .

**Theorem.**

Let  $X = a + bP_1 + cP_2 + dP_3 \in 3 - SP_R$ , then  $X$  is idempotent if and only if  $a, a + b, a + b + c, a + b + c + d$  are idempotent.

**Proof.**

$$X^2 = X.X = (a + bP_1 + cP_2 + dP_3)(a + bP_1 + cP_2 + dP_3) = a^2 + (ab + ba + b^2)P_1 + (ac + bc + ca + cb + c^2)P_2 + (ad + bd + cd + d.d + da + db + dc)P_3.$$

$$X^2 = X.X \text{ equivalents } \begin{cases} ad + bd + cd + d.d + da + db + dc = 0 & (1) \\ a^2 = a & (2) \\ ab + ba + b^2 = b & (3) \\ ac + bc + ca + cb + c^2 = c & (4) \end{cases}$$

Equation (2) means that  $a$  is idempotent.

By adding (3) to (2), we get  $(a + b)^2 = a + b$ , hence  $a + b$  is idempotent.

By adding (3) to (2) to (4), we get  $(a + b + c)^2 = a + b + c$ , hence  $a + b + c$  is idempotent.

By adding (1) to (2) to (3) to (4), we get  $(a + b + c + d)^2 = a + b + c + d$ , thus  $a + b + c + d$  is idempotent.

Thus the proof is complete.

**Example.**

Take  $R = Z_6 = \{0,1,2,3,4,5\}$ ,  $3 - SP_{Z_6}$  is the corresponding symbolic 3-plithogenic ring, consider  $X = 3 + P_1 + 5P_2 \in 3 - SP_{Z_5}$ , we have:

$$X^2 = 9 + 6P_1 + P_1 + 30P_2 + 25P_2 + 10P_2 = 3 + P_1 + 5P_2 = X.$$

The following theorem clarifies the natural powers in  $2 - SP_R$ .

**Theorem.**

Let  $3 - SP_R$  be a commutative symbolic 3-plithogenic ring, hence if  $X = a + bP_1 + cP_2 + dP_3$ , then  $X^n = a^n + [(a + b)^n - a^n]P_1 + [(a + b + c)^n - (a + b)^n]P_2 + [(a + b + c + d)^n - (a + b + c)^n]P_3$  for every  $n \in Z^+$ .

**Proof.**

For  $n = 1$ , it holds easily. Assume that it is true for  $n = k$ , we prove it for  $n = k + 1$ .

$$X^{k+1} = X.X^k = (a + bP_1 + cP_2 + dP_3)(a^k + [(a + b)^k - a^k]P_1 + [(a + b + c)^k - (a + b)^k]P_2 + [(a + b + c + d)^k - (a + b + c)^k]P_3) = a^{k+1} + [(a + b)^{k+1} - a^{k+1}]P_1 + [(a + b + c)^{k+1} - (a + b + c)^{k+1}]P_2 + [(a + b + c + d)^{k+1} - (a + b + c + d)^{k+1}]P_3$$

So, that proof is complete by induction.

**Example.**

Take  $R = Z$ , the ring of integers. Let  $3 - SP_Z$  be the corresponding symbolic 3-plithogenic ring, hence  $X = 1 + 2P_1 + 3P_2 + P_3, X^3 = 1^3 + P_1[(3)^3 - 1^3] + P_2[(6)^3 - 3^3] + (7^3 - 6^3)P_3 = 1 + 26P_1 + 189P_2 + 127P_3$

**Definition.**

$X$  is called nilpotent if there exists  $n \in Z^+$  such that  $X^n = 0$ .

**Theorem.**

Let  $X \in 3 - SP_R$ , where  $R$  is a commutative ring, then  $X$  is nilpotent if and only if  $a, a + b, a + b + c, a + b + c + d$  are nilpotent.

**Proof.**

$X = a + bP_1 + cP_2 + dP_3$  is nilpotent if and only if there exists  $n \in Z^+$  such that  $X^n = 0$ , hence:

$$\begin{cases} (a + b + c + d)^n - (a + b + c)^n = 0 \\ \quad \quad \quad a^n = 0 \\ \quad \quad (a + b)^n - a^n = 0 \\ (a + b + c)^n - (a + b)^n = 0 \end{cases} \Leftrightarrow \begin{cases} (a + b + c + d)^n = 0 \\ \quad \quad \quad a^n = 0 \\ \quad \quad (a + b)^n = 0 \\ (a + b + c)^n = 0 \end{cases}, \text{ thus the proof is complete.}$$

**Definition.**

Let  $Q_0, Q_{13}, Q_2, Q_3$  be ideals of the ring  $R$ , we define the symbolic 3-plithogenic AH-ideal as follows:

$$Q = Q_0 + Q_1P_1 + Q_2P_2 + Q_3P_3 = \{x_0 + x_1P_1 + x_2P_2 + x_3P_3; x_i \in Q_i\}.$$

If  $Q_0 = Q_1 = Q_2 = Q_3$ , then  $Q$  is called an AHS-ideal.

**Example.**

Let  $R = Z$  be the ring of integers, then  $Q_0 = 2Z, Q_1 = 3Z, Q_2 = 5Z$  are ideals of  $R$ .

$Q = \{2m + 3nP_1 + 5tP_2 + 5sP_3; m, n, t, s \in Z\}$  is an AHS-ideal of  $3 - SP_Z$ .

$M = \{2m + 2nP_1 + 2tP_2 + 2sP_3; m, n, t, s \in Z\}$  is an AHS-ideal of  $3 - SP_Z$ .

**Theorem.**

Let  $Q$  be an AHS-ideal of  $3 - SP_R$ , then  $Q$  is an ideal by the classical meaning.

**Proof.**

$Q$  can be written as  $Q = Q_0 + Q_0P_1 + Q_0P_2 + Q_0P_3$ , where  $Q_0$  is an ideal of  $R$ . It is clear that  $(Q, +)$  is a subgroup of  $(3 - SP_R, +)$ .

Let  $S = s_0 + s_1P_1 + s_2P_2 + s_3P_3 \in 3 - SP_R$ , then if  $X = a + bP_1 + cP_2 + dP_3 \in Q$ , we have:

$$S.X = s_0a + (s_0b + s_1a + s_1b)P_1 + (s_0c + s_1c + s_2a + s_2b + s_2c)P_2 + (s_0d + s_1d + s_2d + s_3d + s_3a + s_3b + s_3c)P_3 \in Q, \text{ that is because:}$$

$s_0a \in Q_0, s_0b + s_1a + s_1b \in Q_0, s_0c + s_1c + s_2a + s_2b + s_2c, s_0d + s_1d + s_2d + s_3d + s_3a + s_3b + s_3c \in Q_0.$

**Definition.**

Let  $R, T$  be two rings,  $3 - SP_R, 3 - SP_T$  are the corresponding symbolic 3-plithogenic rings, let  $f_0, f_1, f_2, f_3: R \rightarrow T$  be four homomorphisms, we define the AH-homomorphism as follows:

$f: 3 - SP_R \rightarrow 3 - SP_T$  such that:

$$f(a + bP_1 + cP_2 + dP_3) = f_0(a) + f_1(b)P_1 + f_2(c)P_2 + f_3(d)P_3$$

If  $f_0 = f_1 = f_2 = f_3$ , then  $f$  is called AHS-homomorphism.

**Remark.**

If  $f_0, f_1, f_2, f_3$  is isomorphisms, then  $f$  is called AH-isomorphism.

**Example.**

Take  $R = Z, T = Z_6, f_0, f_1: R \rightarrow T$  such that:

$f_0(x) = x(mod 6), f_1(2) = 3x(mod 6).$  It is clear that  $f_0, f_1$  are homomorphisms.

We define  $f: 3 - SP_R \rightarrow 3 - SP_T$ , where:

$$f(x + yP_1 + zP_2 + sP_3) = f_0(x) + f_1(y)P_1 + f_2(z)P_2 + f_3(s)P_3 = x(mod 6) + y(mod 6)P_1 + (3z mod 6)P_2 + (3s mod 6)P_3$$

Which is an AH-homomorphism.

**Theorem.**

Let  $f = f_0 + f_1P_1 + f_2P_2 + f_3P_3: 3 - SP_R \rightarrow 3 - SP_T$  be a mapping, then:

1. If  $f$  is an AHS-homomorphism, then  $f$  is a ring homomorphism by the classical meaning.
2. If  $f$  is an AHS-homomorphism, then it is an isomorphism by the classical meaning.

**Proof.**

1. Assume that  $f$  is an AHS-homomorphism, then  $f_0 = f_1 = f_2 = f_3$  are homomorphisms.

Let  $X = x_0 + x_1P_1 + x_2P_2 + x_3P_3, Y = y_0 + y_1P_1 + y_2P_2 + y_3P_3 \in 3 - SP_R$ , we have:

$$\begin{aligned} f(X + Y) &= f_0(x_0 + y_0) + f_0(x_1 + y_1)P_1 + f_0(x_2 + y_2)P_2 + f_0(x_3 + y_3)P_3 = f(X) + f(Y) \\ f(X.Y) &= f_0(x_0y_0) + f_0(x_0y_1 + x_1y_0 + x_1y_1)P_1 + f_0(x_0y_2 + x_2y_0 + x_2y_2 + x_2y_1 + x_1y_2)P_2 + \\ &f_0(x_0y_3 + x_1y_3 + x_2y_3 + x_3y_3 + x_3y_1 + x_3y_0 + x_3y_2)P_3 = f_0(x_0)f_0(y_0) + (f_0(x_0)f_0(y_1) + \\ &f_0(x_1)f_0(y_0) + f_0(x_1)f_0(y_1))P_1 + (f_0(x_0)f_0(y_2) + f_0(x_2)f_0(y_0) + f_0(x_2)f_0(y_2) + \\ &f_0(x_2)f_0(y_1) + f_0(x_1)f_0(y_2))P_2 + [(f_0(x_0)f_0(y_3) + f_0(x_1)f_0(y_3) + f_0(x_2)f_0(y_3) + \end{aligned}$$

$$f_0(x_3)f_0(y_3) + f_0(x_3)f_0(y_1) + f_0(x_3)f_0(y_2) + f_0(x_3)f_0(y_0)]P_3 = [f_0(x_0) + f_0(x_1)P_1 + f_0(x_2)P_2 + f_0(x_3)P_3][f_0(y_0) + f_0(y_1)P_1 + f_0(y_2)P_2 + f_0(y_3)P_3] = f(X) + f(Y).$$

So that, the [roof is complete.

2. By a similar discussion of statement 1, we get the proof.

**Definition.**

Let  $f = f_0 + f_1P_1 + f_2P_2 + f_3P_3: 3 - SP_R \rightarrow 3 - SP_T$  be an AH-homomorphism, we define:

1. AH-  $\ker(f) = \ker(f_0) + \ker(f_1)P_1 + \ker(f_2)P_2 + \ker(f_3)P_3 = \{m_0 + m_1P_1 + m_2P_2 + m_3P_3; m_i \in \ker(f_i)\}$ .
2. AH-factor  $3 - SP_R / \text{AH} - \ker(f) = R/\ker(f_0) + R/\ker(f_1)P_1 + R/\ker(f_2)P_2 + R/\ker(f_3)P_3$

If  $f_0 = f_1 = f_2 = f_3$ , then we get an AHS-  $\ker(f)$  and AHS-factor.

**Example.**

Take  $R = Z_{10}$ ,  $f_0: R \rightarrow T$ ,  $f_0(x) = (x \text{ mod } 10)$ ,  $\ker(f_0) = 10Z$ .

The corresponding AHS-homomorphism is  $f = f_0 + f_0P_1 + f_0P_2 + f_0P_3: 3 - SP_R \rightarrow 3 - SP_T$ , such that:

$$\begin{aligned} f(x_0 + x_1P_1 + x_2P_2) &= f_0(x_0) + f_0(x_1)P_1 + f_0(x_2)P_2 + f_0(x_3)P_3 \\ &= (x_0 \text{ mod } 10) + (x_1 \text{ mod } 10)P_1 + (x_2 \text{ mod } 10)P_2 + (x_3 \text{ mod } 10)P_3 \end{aligned}$$

$$\text{AHS-}\ker(f) = 10Z + 10ZP_1 + 10ZP_2 = \{10x + 10yP_1 + 10zP_2 + 10sP_3; x, y, z, s \in Z\}$$

$$\text{AHS-factor} = Z/10Z + Z/10Z P_1 + Z/10Z P_2 + Z/10Z P_3$$

**Definition.**

Let  $(F, +, \cdot)$  be a field, then  $(3 - SP_F, +, \cdot)$  Is called a symbolic 3-plithogenic field.

$(3 - SP_F, +, \cdot)$  Is not a field in the algebraic meaning, that is because  $P_i$  are not invertible, but it is a ring.

**Conclusion**

In this paper, we have defined the concept of 3-plithogenic rings, and we presented many interesting algebraic properties such as invertibility, nilpotency, and idempotency of their elements.

Also, we have presented many related concepts such as AH-ideals, AH-kernels and homomorphisms with their elementary properties in terms of theorems with many clear examples.

In the future, we look for many symbolic 3-plithogenic structures, especially symbolic 3-pithogenic modules, vector spaces, and matrices.



## References

- [1] Sankari, H., and Abobala, M., " AH-Homomorphisms In neutrosophic Rings and Refined Neutrosophic Rings", *Neutrosophic Sets and Systems*, Vol. 38, pp. 101-112, 2020.
- [2] Abobala, M., Bal, M., Aswad, M., "A Short Note On Some Novel Applications of Semi Module Homomorphisms", *International journal of neutrosophic science*, 2022.
- [3] M. Ibrahim. A. Agboola, B.Badmus and S. Akinleye. On refined Neutrosophic Vector Spaces . *International Journal of Neutrosophic Science*, Vol. 7, 2020, pp. 97-109.
- [4] Adeleke, E.O., Agboola, A.A.A.,and Smarandache, F., "Refined Neutrosophic Rings I", *International Journal of Neutrosophic Science*, Vol. 2(2), pp. 77-81. 2020.
- [5] ] Smarandache, F., "  $n$ -Valued Refined Neutrosophic Logic and Its Applications in Physics", *Progress in Physics*, 143-146, Vol. 4, 2013.
- [6] Agboola, A.A.A., Akwu, A.D., and Oyebo, Y.T., " Neutrosophic Groups and Subgroups", *International .J .Math. Combin*, Vol. 3, pp. 1-9. 2012.
- [7] Smarandache, F., "Neutrosophic Set a Generalization of the Intuitionistic Fuzzy Sets", *Inter. J. Pure Appl. Math.*, pp. 287-297. 2005.
- [8] Aswad, F, M., " A Study of Neutrosophic Complex Number and Applications", *Neutrosophic Knowledge*, Vol. 1, 2020.
- [9] Smarandache, F., and Kandasamy, V.W.B., " Finite Neutrosophic Complex Numbers",-Source: arXiv. 2011.
- [10] Smarandache, F., and Ali, M., "Neutrosophic Triplet Group", *Neural. Compute. Appl.* 2019.
- [11] Abobala, M., Hatip, A., Bal,M.," A Study Of Some Neutrosophic Clean Rings", *International journal of neutrosophic science*, 2022.
- [12].Abobala, M., "On Some Special Substructures of Neutrosophic Rings and Their Properties", *International Journal of Neutrosophic Science*", Vol 4 , pp72-81, 2020.
- [13] F. Smarandache, Neutrosophic Quadruple Numbers, Refined Neutrosophic Quadruple Numbers,

---

Absorbance Law, and the Multiplication of Neutrosophic Quadruple Numbers. In *Symbolic Neutrosophic Theory*, Chapter 7, pages 186-193, Europa Nova, Brussels, Belgium, 2015.

[14] F. Smarandache, *Plithogeny, Plithogenic Set, Logic, Probability, and Statistics*, 141 pages, Pons Editions, Brussels, Belgium, 2017. arXiv.org (Cornell University), Computer Science - Artificial Intelligence, 03Bxx:

[15] Florentin Smarandache, *Physical Plithogenic Set*, 71st Annual Gaseous Electronics Conference, Session LW1, Oregon Convention Center Room, Portland, Oregon, USA, November 5–9, 2018.

[16] Florentin Smarandache: *Plithogenic Set, an Extension of Crisp, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Sets – Revisited*, *Neutrosophic Sets and Systems*, vol. 21, 2018, pp. 153-166.

[17] Florentin Smarandache, *Plithogenic Algebraic Structures*. Chapter in “*Nidus idearum Scilogs, V: joining the dots*” (third version), Pons Publishing Brussels, pp. 123-125, 2019.

[18] P. K. Singh, *Data with Turiyam Set for fourth dimension Quantum Information Processing*. *Journal of Neutrosophic and Fuzzy Systems*, Vol. 1, Issue 1, pp. 9-23, 2021.

[19] Khaldi, A., " A Study On Split-Complex Vector Spaces", *Neoma Journal Of Mathematics and Computer Science*, 2023.

[20] Ahmad, K., " On Some Split-Complex Diophantine Equations", *Neoma Journal Of Mathematics and Computer Science*, 2023.

[21]. Ali, R., " On The Weak Fuzzy Complex Inner Products On Weak Fuzzy Complex Vector Spaces", *Neoma Journal Of Mathematics and Computer Science*, 2023.

[22] A. Alrida Basher, Katy D. Ahmad, Rosina Ali, *An Introduction to the Symbolic Turiyam Groups and AH-Substructures*, *Journal of Neutrosophic and Fuzzy Systems*, Vol. 03, No. 02, pp. 43-52, 2022.

[23] P. K. Singh, *On the Symbolic Turiyam Rings*, *Journal of Neutrosophic and Fuzzy Systems*, Vol. 1, No. 2, pp. 80-88, 2022.

- 
- [24] T.Chalapathi and L. Madhavi, "Neutrosophic Boolean Rings", *Neutrosophic Sets and Systems*, Vol. 33, pp. 57-66, 2020.
- [25] G. Shahzadi, M. Akram and A. B. Saeid, "An Application of Single-Valued Neutrosophic Sets in Medical Diagnosis," *Neutrosophic Sets and Systems*, vol. 18, pp. 80-88, 2017.
- [26] Hatip, A., and Olgun, N., " On Refined Neutrosophic R-Module", *International Journal of Neutrosophic Science*, Vol. 7, pp.87-96, 2020.
- [27] J. Anuradha and V. S, "Neutrosophic Fuzzy Hierarchical Clustering for Dengue Analysis in Sri Lanka," *Neutrosophic Sets and Systems*, vol. 31, pp. 179-199, 2020.
- [28] Celik, M., and Olgun, N., " An Introduction To Neutrosophic Real Banach And Hillbert Spaces", *Galoitica Journal Of Mathematical Structures And Applications*, 2022.
- [29] Celik, M., and Olgun, N., " On The Classification Of Neutrosophic Complex Inner Product Spaces", *Galoitica Journal Of Mathematical Structures And Applications*, 2022.
- [30] Smarandache, F., " Introduction to the Symbolic Plithogenic Algebraic Structures (revisited)", *Neutrosophic Sets and Systems*, vol. 53, 2023.
- [31] Merkepçi, H., and Abobala, M., " On The Symbolic 2-Plithogenic Rings", *International Journal of Neutrosophic Science*, 2023.
- [32] Abobala, M., and Hatip, A., "An Algebraic Approach To Neutrosophic Euclidean Geometry", *Neutrosophic Sets and Systems*, Vol. 43, 2021.
- [33] Abobala, M., "On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations", *Mathematical Problems in Engineering*, Hindawi, 2021
- [34] Agboola, A.A.A., Akinola, A.D., and Oyebola, O.Y., " Neutrosophic Rings I" , *International J.Mathcombin*, Vol 4,pp 1-14. 2011
- [35] Smarandache, F., and Ali, M., "Neutrosophic Triplet Group", *Neural. Compute. Appl.* 2019.
- [36] Chellamani, P., and Ajay, D., "Pythagorean neutrosophic Fuzzy Graphs", *International Journal of Neutrosophic Science*, Vol. 11, 2021.

- [37] Giorgio, N, Mehmood, A., and Broumi, S., " Single Valued neutrosophic Filter", International Journal of Neutrosophic Science, Vol. 6, 2020.
- [38] Abobala, M., "On The Characterization of Maximal and Minimal Ideals In Several Neutrosophic Rings", Neutrosophic sets and systems, Vol. 45, 2021.
- [39] Abobala, M., "A Study Of Nil Ideals and Kothe's Conjecture In Neutrosophic Rings", International Journal of Mathematics and Mathematical Sciences, hindawi, 2021
- [40] Abobala, M., "Neutrosophic Real Inner Product Spaces", Neutrosophic Sets and Systems, Vol. 43, 2021.
- [41] Hatip, A., " On Intuitionistic Fuzzy Subgroups of (M-N) Type and Their Algebraic Properties", Galoitica Journal Of Mathematical Structures and Applications, Vol.4, 2023.

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