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Nader Mahmoud Taffach

Khadija Ben Othman

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# An Introduction to Symbolic 2-Plithogenic Modules Over Symbolic 2-Plithogenic Rings

[1.] Nader Mahmoud Taffach, [2.]Khadija Ben Othman

[1.] Faculty Of Science, Department Of Mathematics, Idleb University, Syria, ntaffash77@windowslive.com

[2..] Umm Al-Qura Univerity, Mekka, Saudi Arabia, khagijabenothman33@gmail.com

**Abstract:**

The symbolic n-plithogenic sets and algebraic structures are a new branch of pure algebra released as new generalizations of classical algebraic structures.

The main goal of this paper is to define for the first time the concept of symbolic 2-plithogenic module over a symbolic 2-plithogenic ring. Algebraic substructures of symbolic 2-plithogenic modules such as sub-modules, AH-homomorphisms, and algebraic basis.

**Keywords:** 2-plithogenic symbolic set, 2-plithogenic module, 2-plithogenic ring

## Introduction

The concept of symbolic plithogenic sets was defined by Smarandache in [13-17 ,30], and he suggested an algebraic approach of these sets. Laterally, the concept of symbolic 2-plithogenic rings [31], where the concepts such as symbolic AH-ideals, and AH-homomorphisms were presented and discussed.

In general, we can say that symbolic plithogenic structures are very close to neutrosophic algebraic structures with many differences in the definition of multiplication operation [1-10].

Let $R$ be a ring, the symbolic 2-plithogenic ring is defined as follows:

$2-SP_R=\{a_0+a_1P_1+a_2P_2;\ a_i\in R, {P_j}^2=P_j, P_1\times P_2=P_{max(1,2)}=P_2\}$.

Smarandache has defined algebraic operations on $2-SP_R$ as follows:

Addition:

$[a_0 + a_1P_1 + a_2P_2] + [b_0 + b_1P_1 + b_2P_2] = (a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2.$

Multiplication:

$[a_0 + a_1P_1 + a_2P_2].[b_0 + b_1P_1 + b_2P_2] = a_0b_0 + a_0b_1P_1 + a_0b_2P_2 + a_1b_0{P_1}^2 + a_1b_2P_1P_2 + a_2b_0P_2 + a_2b_1P_1P_2 + a_2b_2{P_2}^2 + a_1b_1P_1P_1 = a_0b_0 + (a_0b_1 + a_1b_0 + a_1b_1)P_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)P_2.$

In this paper, we study the symbolic 2-plithogenic modules according to many points of view, where substructures such as AH-submodules, and AH-homomorphisms will be presented in terms of theorems. In addition, many examples will be illustrated to explain the novelty of these ideas.

**Main Discussion**

**Definition.**

Let $M$ be a module over the ring R, let $2 - SP_R$ be the corresponding symbolic 2-plithogenic ring.

$2 - SP_R = \{x + yP_1 + zP_2;\ x, y, z \in R, {P_i}^2 = P_i, P_1P_2 = P_2P_1 = P_2\}.$

We define the symbolic 2-plithogenic module as follows:

$2 - SP_M = M + MP_1 + MP_2 = \{a + bP_1 + cP_2;\ a, b, c \in M\}.$

Operations on $2 - SP_M$ can be defined as follows:

Addition: $(+): 2 - SP_M \to 2 - SP_M$, such that:

$[x_0 + x_1P_1 + x_2P_2] + [y_0 + y_1P_1 + ys_2P_2] = (x_0 + y_0) + (x_1 + y_1)P_1 + (x_2 + y_2)P_2$

Multiplication: $(.): 2 - SP_R \times 2 - SP_M \to 2 - SP_M$, such that:

$[a + bP_1 + cP_2].[x_0 + x_1P_1 + x_2P_2] = ax_0 + (ax_1 + bx_0 + bx_1)P_1 + (ax_2 + bx_2 + cx_0 + cx_1 + cx_2)P_2.$

where $x_i, y_i \in M, a, b, c \in R$

**Theorem.**

Let $(2 - SP_M, +, .)$ Is a module over the ring $2 - SP_R$.

**Proof.**

Let $X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2 \in 2 - SP_M$ , $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2 \in 2 - SP_R$ we have:

1. $X = X, (X + Y) + Z = X + (Y + Z), X + (-X) = -X + X = 0, X + 0 = 0 + X = X$

Also

$A(X+Y) = (a_0 + a_1P_1 + a_2P_2)[(x_0+y_0) + (x_1+y_1)P_1 + (x_2+y_2)P_2]$

$= a_0(x_0+y_0) + \big(a_0(x_1+y_1) + a_1(x_0+y_0) + a_1(x_1+y_1)\big)P_1$

$+ \big(a_0(x_2+y_2) + a_1(x_2+y_2) + a_2(x_0+y_0) + a_2(x_1+y_1) + a_2(x_2+y_2)\big)P_2$

$= A.X + A.Y$

$(A+B)X = [(a_0+b_0) + (a_1+b_1)P_1 + (a_2+b_2)P_2](x_0 + x_1P_1 + x_2P_2)$

$= (a_0+b_0)x_0 + \big((a_0+b_0)x_1 + (a_1+b_1)x_0 + (a_1+b_1)x_1\big)P_1$

$+ \big((a_0+b_0)x_2 + (a_1+b_1)x_2 + (a_2+b_2)x_0 + (a_2+b_2)x_1 + (a_2+b_2)x_2\big)P_2$

$= A.X + B.X$

$(A.B).X = [a_0b_0 + (a_0b_1 + a_1b_0 + a_1b_1)P_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)P_2](x_0 + x_1P_1 + x_2P_2) = a_0b_0x_0 + [a_0b_0x_1 + (a_0b_1 + a_1b_0 + a_1b_1)x_0 + (a_0b_1 + a_1b_0 + a_1b_1)x_1]P_1 + [a_0b_0x_2 + (a_0b_2 + a_2b_0 + a_1b_1)x_2 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)x_0 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)x_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)x_2]P_2 = A(B.X)$.

**Example.**

Let $M = Z^3$ be the module over the ring $R =$.

The corresponding symbolic 2-plithogenic vector space over $2-SP_Z$ is:

$2-SP_{Z^3} = \{(x_0,y_0,z_0) + (x_1,y_1,z_1)P_1 + (x_2,y_2,z_2)P_2; x_i,y_i,z_i \in Z\}$

Consider $X = (1,1,0) + (2,-1,1)P_1 + (0,1,-1)P_2 \in 2-SP_{Z^3}, A = 2 + P_1 + P_2 \in 2-SP_Z$. We have:

$A.X = (2,2,0) + [(4,-2,2) + (1,1,0) + (2,-1,1)]P_1 + [(0,2,2) + (0,1,1) + (1,1,0) + (2,-1,1) + (0,1,1)]P_2 = (2,2,0) + (7,-2,3)P_1 + (3,4,5)P_2$.

**Definition.**

Let $2-SP_M$ be a symbolic 2-plithogenic module over $2-SP_R$, let $M_0, M_1, M_2$ be the three sub-modules of $V$, we define the AH-submodule as follows:

$W = M_0 + M_1P_1 + M_2P_2 = \{x + yP_1 + zP_2;\ x \in M_0, y \in M_1, z \in M_2\}$.

If $M_0 = M_1 = M_2$, then $W$ is called an AHS-sub-module.

**Example.**

Consider $2-SP_{Z^3}$, we have $M_0 = \{(a,0,0);\ a \in R\}, M_1 = \{(0,b,0);\ b \in R\}, M_2 = \{(0,0,c);\ c \in Z\}$ are three sub-modules of $M = Z^3$.

$W = M + M_1P_1 + M_2P_2 = \{(a,0,0) + (0,b,0)P_1 + (0,0,c)P_2;\ a,b,c \in Z\}$ is an AH-submodule of $2-SP_{Z^3}$.

$T = M_1 + MP_1 + M_1P_2 = \{(0,a,0) + (0,b,0)P_1 + (0,c,0)P_2;\ a,b,c \in Z\}$ is an AHS-submodule.

**Theorem.**

Let $2-SP_M$ be a symbolic 2-plithogenic module over $2-SP_R$ , let $W$ be an AHS-submodule of $2-SP_M$, then $W$ is a submodule of $2-SP_M$.

**Proof.**

Suppose that $W$ is an AHS-submodule, then there exists a submodule $M_0 \le M$, such that $W = M_0 + M_0P_1 + M_0P_2 = \{x + yP_1 + zP_2;\ x,y,z \in M_0\}$.

Let $X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2 \in W$, then:

$X - Y = (x_0 - y_0) + (x_1 - y_1)P_1 + (x_2 - y_2)P_2 \in W$

$\forall\, A = a_0 + a_1P_1 + a_2P_2 \in 2 - SP_R$, then:

$A.X = a_0x_0 + (a_0x_1 + a_1x_0 + a_1x_1)P_1 + (a_0x_2 + a_1x_2 + a_2x_0 + a_2x_1 + a_2x_2)P_2 \in W$, that is because $a_0x_0 \in M_0, a_0x_1 + a_1x_0 + a_1x_1 \in M_0, a_0x_2 + a_1x_2 + a_2x_0 + a_2x_1 + a_2x_2 \in M_0$ , this implies the proof.

**Definition.**

Let $V,W$ be two modules over the ring R. Let $2-SP_V,\ 2-SP_W$ be the corresponding symbolic 2-plithogenic modules over $2-SP_R$.

Let $L_0, L_1, L_2: V \to W$ be three homomorphisms, we define the AH-homomorphism as follows:

$L: 2-SP_V \to 2-SP_W, L = L_0 + L_1P_1 + L_2P_2\ ; L(x + yP_1 + zP_2) = L_0(x) + L_1(y)P_1 + L_2(z)P_2$.

If $L_0 = L_1 = L_2$, then $L$ is called AHS-homomorphism.

**Definition.**

Let $L = L_0 + L_1P_1 + L_2P_2: 2-SP_V \to 2-SP_W$ be an AH-homomorphism, we define:

1. $AH - ker(L) = ker(L_0) + ker(L_1)P_1 + ker(L_2)P_2 = \{x + yP_1 + zP_2\}; x \in ker(L_0), y \in ker(L_1), z \in ker(L_2)$.
2. $AH - Im(L) = Im(L_0) + Im(L_1)P_1 + Im(L_2)P_2 = \{a + bP_1 + cP_2\}; a \in Im(L_0), b \in Im(L_1), c \in Im(L_2)$

If $L$ is AHS-linear homomorphism, then we get $AHS - kernel,\ AHS - Image$.

**Theorem.**

Let $L = L_0 + L_1P_1 + L_2P_2: 2-SP_V \to 2-SP_W$ be an AH-homomorphism, then:

1. $AH - ker(L)$ is AH-submodule of $2-SP_V$.

2. $AH - Im(L)$ is AH-submodule of $2 - SP_W$.

**Proof.**

1. Since $ker(L_0), ker(L_1), ker(L_2)$ are submodules of $V$, then $AH - ker(L)$ is an AH-submodule of $2 - SP_V$.
2. It is holds by the same.

**Remark.**

If $L_0, L_1, L_2$ are isomorphisms, then $ker(L_0) = ker(L_1) = ker(L_2) = \{0\}, Im(L_0) = Im(L_1) = Im(L_2) = W$, thus $AH - ker(L) = \{0\}, AH - Im(L) = 2 - SP_W$.

**Example.**

Take $V = Z^3, W = Z,\ L_0, L_1, L_2: V \to W$ such that:

$L_0(x, y, z) = (x), L_1(x, y, z) = (y), L_2(x, y, z) = (z)$

The corresponding AH-homomorphism is:

$L = L_0 + L_1P_1 + L_2P_2: 2 - SP_{Z^3} \to 2 - SP_Z$:

$L[(x_0, y_0, z_0) + (x_1, y_1, z_1)P_1 + (x_2, y_2, z_2)P_2] = L_0(x_0, y_0, z_0) + L_1(x_1, y_1, z_1)P_1 + L_2(x_2, y_2, z_2)P_2 = (x_0) + (y_1)P_1 + (z_2)P_2$.

For example, take $X = (1,9,8) + (9,10,-9)P_1 + (3,2,1)P_2$, then:

$L(X) = 1 + (10)P_1 + P_2$.

$$\begin{cases} ker(L_0) = \{ (0, y_0, z_0);\ y_0, z_0 \in Z\} \\ ker(L_1) = \{(x_1, 0, z_1);\ x_1, z_1 \in Z\} \\ ker(L_2) = \{(x_2, y_2, 0);\ x_2, y_2 \in Z\} \\ AH - ker(L) = \{(0, y_0, z_0) + (x_1, 0, z_1)P_1 + (x_2, y_2, 0)P_2; y_0, z_0, x_1, z_1, x_2, y_2 \in Z\} \end{cases}$$

Also,

$$\begin{cases} Im(L_0) = Z \\ Im(L_1) = Z \\ Im(L_2) = Z \\ AH - Im(L) = Z + ZP_1 + ZP_2 = 2 - SP_W \end{cases}$$

**Theorem.**

Let $L = f + fP_1 + fP_2: 2 - SP_V \to 2 - SP_W$ be an AHS-homomorphism, then $L$ is a module homomorphism.

**Proof.**

Let $X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2 \in 2 - SP_V$, then:

$L(X+Y)=f(x_0+y_0)+f(x_1+y_1)P_1+f(x_2+y_2)P_2$

$=[f(x_0)+f(x_1)P_1+f(x_2)P_2]+[f(y_0)+f(y_1)P_1+f(y_2)P_2]=L(X)+L(Y)$

Let $A=a_0+a_1P_1+a_2P_2\in 2-SP_F$, then:

$L(A.X)=f(a_0x_0)+f(a_0x_1+a_1x_0+a_1x_1)P_1+f(a_0x_2+a_2x_0+a_2x_2+a_1x_2+a_2x_1)P_2$

$=a_0f(x_0)+\big(a_0f(x_1)+a_1f(x_0)+a_1f(x_1)\big)P_1$

$+\big(a_0f(x_2)+a_2f(x_0)+a_2f(x_2)+a_1f(x_2)+a_2f(x_1)\big)P_2$

$=[a_0+a_1P_1+a_2P_2].[f(x_0)+f(x_1)P_1+f(x_2)P_2]=A.L(X)$

Thus, $L$ is a module homomorphism.

**The algebraic relations between symbolic 2-plithogenic modules and neutrosophic modules .**

**Theorem.**

Let $M$ be a module over the ring R, consider $M(I)=M+MI=\{x+yI;x,y\in M\}$ is the corresponding neutrosophic module over the neutrosophic ring $R(I)=\{a+bI;a,b\in R\}$.

$M(I_1,I_2)=M+MI_1+MI_2=\{x+yI_1+zI_2;x,y,z\in M\}$ is the corresponding refined neutrosophic module over the refined neutrosophic ring $R(I_1,I_2)=\{a+bI_1+cI_2;a,b,c\in R\}$.

$2-SP_M=M+MP_1+MP_2=\{x+yP_1+zP_2;x,y,z\in M\}$ is the corresponding symbolic 2-plithogenic module over $2-SP_R$, then:

1. $2-SP_M$ is semi homomorphic to $M(I)$.
2. $2-SP_M$ is semi isomorphic to $M(I_1,I_2)$.

**Proof.**

1. We define $f:2-SP_M\to M(I), g:2-SP_R\to R(I)$ such that:

$f(x+yP_1+zP_2)=x+yI;x,y,z\in M$

$g(a+bP_1+cP_2)=a+bI;a,b,c\in R$

We have the following:

$g$ is a ring homomorphism, that is because:

$A=a_0+a_1P_1+a_2P_2, B=b_0+b_1P_1+b_2P_2;\ a_i,b_i\in R$, then:

If $A=B$, then $a_i=b_i$ for all $i$, thus $a_0+a_1I=b_0+b_1I, i.e.\ \ g(A)=g(B)$.

$g(A+B)=g[(a_0+b_0)+(a_1+b_1)P_1+(a_2+b_2)P_2]=a_0+b_0+(a_1+b_1)I=g(A)+g(B)$.

$g(A.B)=g[a_0b_0+(a_0b_1+a_1b_0+a_1b_1)P_1+(a_0b_2+a_1b_2+a_2b+a_2b_1+a_2b_2)P_2]=$
$a_0b_0+(a_0b_1+a_1b_0+a_1b_1)I=(a_0+a_1I)(b_0+b_1I)=g(A).g(B)$.

On the other hand, $f$ is well defined, that is because:

If $X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2$, then $x_i = y_i$ for all $i$, hence $a_0 + a_1I = b_0 + b_1I$, thus $f(X) = f(Y)$.

$f$ preserves addition, that is because:

For $X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2$, we have:

$f(X + Y) = f[(x_0 + y_0) + (x_1 + y_1)P_1 + (x_2 + y_2)P_2] = x_0 + y_0 + (x_1 + y_1)I = f(X) + f(Y)$.

$f$ preserves multiplication, that is because:

For $A = a_0 + a_1P_1 + a_2P_2 \in 2 - SP_M$, we have:

$f(A.X) = a_0x_0 + (a_0x_1 + a_1x_0 + a_1x_1)I = (a_0 + a_1I)(x_0 + x_1I) = g(A).f(X)$

Thus $f$ is a semi module homomorphism.

We define $f: 2 - SP_M \to M(I_1, I_2)$, $g: 2 - SP_R \to M(I_1, I_2)$, where $f(x + yP_1 + zP_2) = x + zI_1 + yI_2$, and $g(a + bP_1 + cP_2) = a + cI_1 + bI_2; x, y, z \in M, a, b, c \in R$.

$(g)$ is well defined, that is because:

If $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2$, then:

$a_0 = a_1, b_0 = b_1, c_0 = c_1$, hence: $a_0 + c_0I_1 + b_0I_2 = a_1 + c_1I_1 + b_1I_2$, so that $g(A) = g(B)$.

$(f)$ is well defined by a similar discussion.

$(g)$ is one-to-one mapping, that is because:

$ker(g) = \{a + bP_1 + cP_2; g(a + bP_1 + cP_2) = 0\} = O$

$Im(g) = \{a + cI_1 + bI_2; g(a + bP_1 + cP_2) \in R(I_1, I_2);\ \exists A \in 2 - SP_R, g(A) = a + cI_1 + bI_2\} = R(I_1, I_2)$.

$(f)$ is one-to-one mapping, it can be proved by the same.

$(g)$ and $(f)$ preserve addition, that is because:

Consider $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2 \in 2 - SP_R$, $X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2 \in 2 - SP_M$, then:

$$g(A + B) = g[(a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2] = a_0 + b_0 + (a_1 + b_1)I_1 + (a_2 + b_2)I_2$$
$$= g(A) + g(B)$$

$f(X + Y) = f(X) + f(Y)$ by a similar discussion.

$(g)$ preserves multiplication, that is because:

$g(A.B) = a_0b_0 + (a_0b_2 + a_2b_0 + a_2b_2 + a_1b_2 + a_2b_1)I_1 + (a_0b_1 + a_1b_0 + a_1b_1)I_2 = g(A).g(B)$.

$(f)$ is semi module homomorphism, that is because:

$$f(A.X) = a_0x_0 + (a_0x_2 + a_2x_0 + a_2x_2 + a_1x_2 + a_2x_1)I_1 + (a_0x_1 + a_1x_0 + a_1x_1)I_2$$
$$= (a_0 + a_1I_1 + a_2I_2)(x_0 + x_2I_1 + x_1I_2) = g(A).f(X)$$

**The basis of a symbolic 2-plithogenic module:**

**Theorem.**

Let $T = \{t_1, \dots, t_n\}$ be a basis of the module $V$ over the ring R, then the set:

$T_P = \{t_i + (t_j - t_i)P_1 + (t_k - t_j)P_2; 1 \le i,j,k \le n\}$ is a basis of $2 - SP_V$.

**Proof.**

Let $X = x_0 + x_1P_1 + x_2P_2 \in 2 - SP_M, x_0, x_1, x_2 \in M$.

$x_0 = \sum_{i=1}^{n} \alpha_i t_i,\ x_0 + x_1 = \sum_{j=1}^{n} \beta_j t_j,\ x_0 + x_1 + x_2 = \sum_{k=1}^{n} \gamma_k t_k\,; \alpha_i, \beta_j, \gamma_k \in R.$

We put $A_{i,j,k} = \alpha_i + (\beta_j - \alpha_i)P_1 + (\gamma_k - \beta_j)P_2; 1 \le i,j,k \le n$

$T_{i,j,k} = t_i + (t_j - t_i)P_1 + (t_k - t_j)P_2; 1 \le i,j,k \le n$

$$\sum_{i,j,k=1}^{n} A_{i,j,k}\, T_{i,j,k}$$
$$= \sum_{i=1}^{n} [\alpha_i t_i + [\beta_j t_j - \beta_j t_i - \alpha_i t_j + \alpha_i t_i + \beta_j t_i - \alpha_i t_i + \alpha_i t_j - \alpha_i t_i]P_1$$
$$+ [\alpha_i t_k - \alpha_i t_j + \gamma_k t_i - \beta_j t_i - \gamma_k t_j + \gamma_k t_i - \beta_j t_j + \beta_j t_i + \gamma_k t_k - \gamma_k t_j - \beta_j t_k$$
$$+ -\beta_j t_j + \beta_j t_k - \beta_j t_j - \alpha_i t_k + \alpha_i t_j]P_2]$$
$$\sum_{i=1}^{n} \alpha_i t_i + P_1\left[\sum_{j=1}^{n} \beta_j t_j - \sum_{i=1}^{n} \alpha_i t_i\right] + P_2\left[\sum_{k=1}^{n} \gamma_k t_k - \sum_{j=1}^{n} \beta_j t_j\right]$$
$$= x_0 + P_1[x_0 + x_1 - x_0] + P_2[x_0 + x_1 + x_2 - (x_0 + x_1)] = x_0 + x_1P_1 + x_2P_2$$
$$= X$$

Thus $T$ generates $2 - SP_M$.

On the other hand, $T$ is linearly independent, that is because:

If $\sum_{i,j,k=1}^{n} A_{i,j,k}\,.X = 0$, then:

$\sum_{i=1}^{n} \alpha_i t_i = 0, \sum_{j=1}^{n} \beta_j t_j = 0, \sum_{k=1}^{n} \gamma_k t_k = 0$ , hence$\alpha_i = \beta_j = \gamma_k = 0$ for all $i,j,k$, thus $A_{i,j,k} =$ 0.

This implies that $T$ is a basis of $2 - SP_M$.

**Example.**

Find a basis of $2 - SP_{Z^2}$.

**Solution.**

First of all, we have $\{u_1 = (1,0), u_2 = (0,1)\}$ is a basis of $Z^2$.

The corresponding basis of $2-SP_{Z^2}$ is:

$T=\{T_1,T_2,T_3,T_4,T_5,T_6,T_7,T_8\}$ such that:

$T_1=(1,0), T_2=(0,1), T_3=u_1+(u_2-u_1)P_1+(u_2-u_2)P_2=(1,0)+(-1,1)P_1$

$T_4=u_1+(u_2-u_1)P_1+(u_1-u_2)P_2=(1,0)+(-1,1)P_1+(1,-1)P_2$

$T_5=u_2+(u_2-u_1)P_1+(u_1-u_1)P_2=(0,1)+(1,-1)P_1$

$T_6=u_2+(u_2-u_1)P_1+(u_2-u_1)P_2=(0,1)+(1,-1)P_1+(-1,1)P_2$

$T_7=u_1+(u_1-u_1)P_1+(u_2-u_1)P_2=(1,0)+(-1,1)P_2$

$T_8=u_2+(u_2-u_2)P_1+(u_1-u_2)P_2=(0,1)+(1,-1)P_2$

**Remark.**

$$dim\,(2-SP_M)=(dimM)^3$$

**Conclusion**

In this paper we have defined the concept of symbolic 2-plithogenic modules over a symbolic 2-plithogenic ring, where we have presented some of their elementary properties such as basis, homomorphisms, and AH-submodules. On the other hand, we have suggested many examples to clarify the validity of our work.

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