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Abstract. Mappings are significant mathematical tools with many applications in our daily lives. The bipolar hypersoft set is one of the effective tools for dealing with ambiguity and vagueness. The purpose of this article is to define mappings between the classes of bipolar hypersoft sets. The notions of bipolar hypersoft image and bipolar hypersoft inverse image of bipolar hypersoft sets are then defined, and some of their properties are studied. Moreover, we discuss the relations between the bipolar hypersoft image and the bipolar hypersoft inverse image of the bipolar hypersoft sets. This proposed work can be extended to *IndetermSoft Set*, *IndetermHyperSoft Set* and *TreeSoft Set* and their corresponding Fuzzy, Intuitionistic Fuzzy, Neutrosophic forms and other Fuzzy-extension.

Keywords: bipolar hypersoft mapping; bipolar hypersoft image; bipolar hypersoft inverse image; bipolar hypersoft set; hypersoft set; soft set

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1. Introduction

In all real-life disciplines, such as environmental science, social science, engineering and economics, there is ambiguity, inaccuracy, and inadequate information. Many researchers have attempted to process such data in the past and present. In 1999, Molodtsov [14] proposed the theory of soft set as a completely flexible mathematical approach to modeling uncertainties. In 2003, Maji et al. [12] developed the theory of soft sets by defining several essential operations like subset, the equal set and the complement of a soft set. Shabir and Naz [25] proposed and studied the concept of bipolar soft sets (a combination of the soft set and the bipolarity structure) and its use in decision-making (2013).

The traditional soft set is built on a determinate function, however there are numerous sources in our world that, due to ignorance or a lack of knowledge, present indeterminate information. Due to the uncertainty in our world, they can be modeled by operators with some degree of uncertainty. As a result, Smarandache [27,28] extended the soft set to hypersoft set in 2018, then both of them to IndetermSoft Set and IndetermHyperSoft Set [29,30] respectively in 2022, and introduced TreeSoft Set [31] as extension of the MultiSoft Set [26]. Several applications are presented for each type of soft set. Musa and Asaad [4,15,16] applied hypersoft set to present some topological concepts such as connectedness and separation axioms.

Defining relations and mappings on soft sets, bipolar soft sets and hypersoft sets was one of the most important steps in the development of these theories. Babitha and Sunil [6] initiated the notion of soft relations and soft functions. Qin et al. [21] introduced the concept of soft relation which is a generalization of soft set relation presented in [6]. They supported their work with an application to information systems. Majumdar and Samanta [13] examined the concept of crisp (soft) set images using soft mappings. Kharal and Ahmad [11] defined the idea of soft class mappings and discussed the characteristics of soft images and soft pre-images. Furthermore, they provided an application of soft mapping in medical diagnosis. Addis et al. [2] has developed a new method to define soft mappings and studying their properties. They used this concept in a new way to study soft homomorphisms and soft homomorphism theorems on groups. They also built a soft mapping to model a symptom–disease relationship in medical diagnosis. The notion of mappings between two collections of bipolar soft sets was introduced by Al-shami [1] and exhaustively studied by Fadel and Dzul-Kifli [8]. Saeed et al. [23, 24] introduced mappings to the hypersoft set environment. They defined hypersoft image and hypersoft pre-image and studied some of their properties. Moreover, the validity and dominance of their suggested technique is demonstrated through practical application and comparative analysis. Other searches for mappings can be seen [3, 5, 7, 9, 10, 22, 32–34].

Musa and Asaad [17], came up with the concept of bipolar hypersoft set as a mixture of hypersoft set and bipolarity structure and is created by looking at not only a collection of carefully chosen parameters, but also a set associated with parameters with opposing meanings known as "not set of parameters". They also presented an application of bipolar hypersoft sets in a decision-making problem [18]. In addition, the authors [19,20] studied the topological structures of bipolar hypersoft sets. Motivated by the interest of researchers for mappings and their applications. We continue to study bipolar hypersoft sets by defining bipolar hypersoft mapping and discuss some of its characteristics.

The rest of the article is organized in the following order: Section 2 provides an overview of several fundamental concepts that are necessary to understand our research. In section 3, we define the concept of bipolar hypersoft mapping and study its properties. In section 4, we introduce bipolar hypersoft inverse image and related results. We conclude this section by presenting the relationship between the bipolar hypersoft image and the bipolar hypersoft inverse image. Section 5 provide a summary of ongoing work as well as a suggestion for future study.

2. **Preliminaries**

Throughout this work, $\Re$ and $\aleph$ denote the universal sets; $2^{\Re}$ and $2^{\aleph}$ denote the power sets of $\Re$ and $\aleph$, respectively; $\Sigma = \sigma_1 \times \sigma_2 \times \ldots \times \sigma_n$ and $\acute{\Sigma} = \acute{\sigma}_1 \times \acute{\sigma}_2 \times \ldots \times \acute{\sigma}_n$ denote the parameter sets with $\sigma_i \cap \sigma_j = \phi$, $\acute{\sigma}_i \cap \acute{\sigma}_j = \phi$ where $i \neq j$; and $\Lambda, \Delta, \acute{\Lambda}, \acute{\Delta}$ are non-empty sets of parameters where $\Lambda, \Delta \subseteq \Sigma$ and $\acute{\Lambda}, \acute{\Delta} \subseteq \acute{\Sigma}$.

The basic definitions and results introduced in [17] will be collected in this section.

Definition 2.1. A triple $(g, \widehat{g}, \Lambda)$ is called a bipolar hypersoft set over $\Re$, where g and $\widehat{g}$ are mappings given by $g : \Lambda \to 2^{\Re}$ and $\widehat{g} : \neg\Lambda \to 2^{\Re}$ such that $g(\ell) \cap \widehat{g}(\neg\ell) = \phi$ for all $\ell \in \Lambda$.

We represent a bipolar hypersoft set $(g, \widehat{g}, \Lambda)$ as:

$$(g, \widehat{g}, \Lambda) = \{(\ell, g(\ell), \widehat{g}(\neg\ell)) : \ell \in \Lambda \text{ and } g(\ell) \cap \widehat{g}(\neg\ell) = \phi\}.$$

The collection of all bipolar hypersoft sets on $\Re$ (resp., $\aleph$) with the set of parameters Σ (resp., $\acute{\Sigma}$) is denoted by $\Omega_{(\Re,\Sigma)}$ (resp., $\Omega_{(\aleph,\acute{\Sigma})}$).

Definition 2.2. Let $(g, \widehat{g}, \Lambda)$, $(f, \widehat{f}, \Delta) \in \Omega_{(\Re,\Sigma)}$. Then

i. $(g, \widehat{g}, \Lambda)$ is a bipolar hypersoft subset of $(f, \widehat{f}, \Delta)$, denoted by $(g, \widehat{g}, \Lambda) \mathrel{\widetilde{\widetilde{\sqsubseteq}}} (f, \widehat{f}, \Delta)$, if $\Lambda \subseteq \Delta$ and $g(\ell) \subseteq f(\ell)$, $\widehat{f}(\neg\ell) \subseteq \widehat{g}(\neg\ell)$ for all $\ell \in \Lambda$.

ii. $(g, \widehat{g}, \Lambda)$ and $(f, \widehat{f}, \Delta)$ are bipolar hypersoft equal, if $(g, \widehat{g}, \Lambda) \mathrel{\widetilde{\widetilde{\sqsubseteq}}} (f, \widehat{f}, \Delta)$ and $(f, \widehat{f}, \Delta) \mathrel{\widetilde{\widetilde{\sqsubseteq}}} (g, \widehat{g}, \Lambda)$.

iii. If $g(\ell) = \phi$ and $\widehat{g}(\neg\ell) = \Re$ for all $\ell \in \Lambda$, then $(g, \widehat{g}, \Lambda)$ is called a relative null bipolar hypersoft set and denoted by $(\widetilde{\phi}, \widetilde{\Re}, \Lambda)$.

iv. If $g(\ell) = \Re$ and $\widehat{g}(\neg\ell) = \phi$ for all $\ell \in \Lambda$, then $(g, \widehat{g}, \Lambda)$ is called a relative whole bipolar hypersoft set and denoted by $(\widetilde{\Re}, \widetilde{\phi}, \Lambda)$.

v. The complement of $(g, \widehat{g}, \Lambda)$ is a bipolar hypersoft set $(g, \widehat{g}, \Lambda)^c = (g^c, \widehat{g}^c, \Lambda)$ where $g^c(\ell) = \widehat{g}(\neg\ell)$ and $\widehat{g}^c(\neg\ell) = g(\ell)$ for all $\ell \in \Lambda$.

vi. The union of $(g, \widehat{g}, \Lambda)$ and $(f, \widehat{f}, \Delta)$, denoted by $(g, \widehat{g}, \Lambda) \mathrel{\widetilde{\widetilde{\sqcup}}} (f, \widehat{f}, \Delta)$, is a bipolar hypersoft set $(h, \widehat{h}, C)$, where $C = \Lambda \cup \Delta$ and for all $\ell \in C$:

$$h(\ell) = \begin{cases} g(\ell) & \text{if } \ell \in \Lambda \setminus \Delta \\ f(\ell) & \text{if } \ell \in \Delta \setminus \Lambda \\ g(\ell) \cup f(\ell) & \text{if } \ell \in \Lambda \cap \Delta \end{cases}$$

$$\widehat{h}(\neg\ell)=\begin{cases}\widehat{g}(\neg\ell) & \text{if } \neg\ell\in\neg\Lambda\setminus\neg\Delta\\ \widehat{f}(\neg\ell) & \text{if } \neg\ell\in\neg\Delta\setminus\neg\Lambda\\ \widehat{g}(\neg\ell)\cap\widehat{f}(\neg\ell) & \text{if } \neg\ell\in\neg\Lambda\cap\neg\Delta\end{cases}$$

vii. The extended intersection of $(g,\widehat{g},\Lambda)$ and $(f,\widehat{f},\Delta)$, denoted by $(g,\widehat{g},\Lambda)\ \widetilde{\widetilde{\sqcap}}_{\varepsilon}\ (f,\widehat{f},\Delta)$, is a bipolar hypersoft set $(h,\widehat{h},C)$, where $C=\Lambda\cup\Delta$ and for all $\ell\in C$:

$$h(\ell)=\begin{cases}g(\ell) & \text{if } \ell\in\Lambda\setminus\Delta\\ f(\ell) & \text{if } \ell\in\Delta\setminus\Lambda\\ g(\ell)\cap f(\ell) & \text{if } \ell\in\Lambda\cap\Delta\end{cases}$$

$$\widehat{h}(\neg\ell)=\begin{cases}\widehat{g}(\neg\ell) & \text{if } \neg\ell\in\neg\Lambda\setminus\neg\Delta\\ \widehat{f}(\neg\ell) & \text{if } \neg\ell\in\neg\Delta\setminus\neg\Lambda\\ \widehat{g}(\neg\ell)\cup\widehat{f}(\neg\ell) & \text{if } \neg\ell\in\neg\Lambda\cap\neg\Delta\end{cases}$$

viii. The restricted union of $(g,\widehat{g},\Lambda)$ and $(f,\widehat{f},\Delta)$, denoted by $(g,\widehat{g},\Lambda)\ \widetilde{\widetilde{\sqcup}}_{\mathcal{R}}\ (f,\widehat{f},\Delta)$, is a bipolar hypersoft set $(h,\widehat{h},C)$, where $C=\Lambda\cap\Delta$ and for all $\ell\in C$: $h(\ell)=g(\ell)\cup f(\ell)$ and $\widehat{h}(\neg\ell)=\widehat{g}(\neg\ell)\cap\widehat{f}(\neg\ell)$.

ix. The intersection of $(g,\widehat{g},\Lambda)$ and $(f,\widehat{f},\Delta)$, denoted by $(g,\widehat{g},\Lambda)\ \widetilde{\widetilde{\sqcap}}\ (f,\widehat{f},\Delta)$, is a bipolar hypersoft set $(h,\widehat{h},C)$, where $C=\Lambda\cap\Delta$ and for all $\ell\in C$: $h(\ell)=g(\ell)\cap f(\ell)$ and $\widehat{h}(\neg\ell)=\widehat{g}(\neg\ell)\cup\widehat{f}(\neg\ell)$.

Proposition 2.3. *Let* $(g,\widehat{g},\Lambda)$, $(f,\widehat{f},\Lambda)\in\Omega_{(\Re,\Sigma)}$. *Then*

i. $((g,\widehat{g},\Lambda)^c)^c=(g,\widehat{g},\Lambda)$.

ii. *If* $(g,\widehat{g},\Lambda)\ \widetilde{\widetilde{\sqsubseteq}}\ (f,\widehat{f},\Lambda)$, *then* $(f,\widehat{f},\Lambda)^c\ \widetilde{\widetilde{\sqsubseteq}}\ (g,\widehat{g},\Lambda)^c$.

iii. $(\widetilde{\phi},\widetilde{\Re},\Lambda)\ \widetilde{\widetilde{\sqsubseteq}}\ (g,\widehat{g},\Lambda)\ \widetilde{\widetilde{\sqcap}}\ (g,\widehat{g},\Lambda)^c\ \widetilde{\widetilde{\sqsubseteq}}\ (g,\widehat{g},\Lambda)\ \widetilde{\widetilde{\sqcup}}\ (g,\widehat{g},\Lambda)^c\ \widetilde{\widetilde{\sqsubseteq}}\ (\widetilde{\Re},\widetilde{\phi},\Lambda)$.

iv. $(g,\widehat{g},\Lambda)\ \widetilde{\widetilde{\sqcup}}\ (f,\widehat{f},\Lambda)=(g,\widehat{g},\Lambda)\ \widetilde{\widetilde{\sqcup}}_{\mathcal{R}}\ (f,\widehat{f},\Lambda)$.

v. $(g,\widehat{g},\Lambda)\ \widetilde{\widetilde{\sqcap}}\ (f,\widehat{f},\Lambda)=(g,\widehat{g},\Lambda)\ \widetilde{\widetilde{\sqcap}}_{\varepsilon}\ (f,\widehat{f},\Lambda)$.

Proposition 2.4. *Let* $(g,\widehat{g},\Lambda)$, $(f,\widehat{f},\Delta)\in\Omega_{(\Re,\Sigma)}$. *Then*

i. $((g,\widehat{g},\Lambda)\ \widetilde{\widetilde{\sqcup}}\ (f,\widehat{f},\Delta))^c=(g,\widehat{g},\Lambda)^c\ \widetilde{\widetilde{\sqcap}}\ (f,\widehat{f},\Delta)^c$.

ii. $((g,\widehat{g},\Lambda)\ \widetilde{\widetilde{\sqcap}}\ (f,\widehat{f},\Delta))^c=(g,\widehat{g},\Lambda)^c\ \widetilde{\widetilde{\sqcup}}\ (f,\widehat{f},B)^c$.

3. Bipolar Hypersoft Mappings

In this section, we study mappings between families of bipolar hypersoft sets with different universes and sets of parameters. In addition, illustrative examples are offered to help understand the main results.

Definition 3.1. Let $\gamma : \Re \to \aleph$ be an injective mapping. Let $\delta : \Sigma \to \acute{\Sigma}$ and $\lambda : \neg\Sigma \to \neg\acute{\Sigma}$ be two mappings such that $\lambda(\neg\ell) = \neg\delta(\ell)$ for all $\neg\ell \in \neg\Sigma$. Then a bipolar hypersoft mapping $\Psi_{\gamma\delta\lambda} : \Omega_{(\Re,\Sigma)} \to \Omega_{(\aleph,\acute{\Sigma})}$ is defined as: for any bipolar hypersoft set $(g,\widehat{g},\Lambda) \in \Omega_{(\Re,\Sigma)}$, the image of $(g,\widehat{g},\Lambda)$ under $\Psi_{\gamma\delta\lambda}$, $\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda)) = (\Psi_{\gamma\delta\lambda}(g), \Psi_{\gamma\delta\lambda}(\widehat{g}), \acute{\Sigma})$ is a bipolar hypersoft set in $\Omega_{(\aleph,\acute{\Sigma})}$ given as, for all $\acute{\ell} \in \acute{\Sigma}$:

$$\Psi_{\gamma\delta\lambda}(g)(\acute{\ell}) = \begin{cases} \gamma\left(\bigcup_{\ell\in\delta^{-1}(\acute{\ell})\cap\Lambda} g(\ell)\right), & \text{if } \delta^{-1}(\acute{\ell}) \cap \Lambda \neq \phi \\ \phi, & \text{otherwise} \end{cases}$$

$$\Psi_{\gamma\delta\lambda}(\widehat{g})(\neg\acute{\ell}) = \begin{cases} \gamma\left(\bigcap_{\neg\ell\in\lambda^{-1}(\neg\acute{\ell})\cap\neg\Lambda} \widehat{g}(\neg\ell)\right), & \text{if } \lambda^{-1}(\neg\acute{\ell}) \cap \neg\Lambda \neq \phi \\ \aleph, & \text{otherwise} \end{cases}$$

Example 3.2. Let $\Re = \{r_1, r_2, r_3\}$ and $\aleph = \{\eta_1, \eta_2, \eta_3, \eta_4\}$ be two sets, $\sigma_1 = \{\ell_1, \ell_2, \ell_3, \ell_4\}, \sigma_2 = \{\ell_5\}, \sigma_3 = \{\ell_6\}$, and $\acute{\sigma_1} = \{\acute{\ell_1}, \acute{\ell_2}, \acute{\ell_3}, \acute{\ell_4}\}, \acute{\sigma_2} = \{\acute{\ell_5}\}, \acute{\sigma_3} = \{\acute{\ell_6}\}$ be sets of parameters, $\gamma : \Re \to \aleph$ be a mapping defined as $\gamma(r_i) = \eta_i$ for $i = 1, 2, 3$, the mapping $\delta : \Sigma \to \acute{\Sigma}$ be defined as $\delta((\ell_1, \ell_5, \ell_6)) = \delta((\ell_2, \ell_5, \ell_6)) = (\acute{\ell_1}, \acute{\ell_5}, \acute{\ell_6})$, $\delta((\ell_3, \ell_5, \ell_6)) = (\acute{\ell_3}, \acute{\ell_5}, \acute{\ell_6})$, $\delta((\ell_4, \ell_5, \ell_6)) = (\acute{\ell_4}, \acute{\ell_5}, \acute{\ell_6})$, the mapping $\lambda : \neg\Sigma \to \neg\acute{\Sigma}$ be defined as $\lambda(\neg\ell_i) = \neg\delta(\ell_i)$ for $i = 1, 2, 3$, and $\Psi_{\gamma\delta\lambda} : \Omega_{(\Re,\Sigma)} \to \Omega_{(\aleph,\acute{\Sigma})}$ be a bipolar hypersoft mapping. Let $\Lambda_1 = \{\ell_1, \ell_2, \ell_3\}, \Lambda_2 = \{\ell_5\}, \Lambda_3 = \{\ell_6\}$ and $(g,\widehat{g},\Lambda) = \{((\ell_1, \ell_5, \ell_6), \{r_1\}, \{r_2\}), ((\ell_2, \ell_5, \ell_6), \{r_3\}, \{r_1, r_2\}), ((\ell_3, \ell_5, \ell_6), \{r_3\}, \{r_1\})\}$. Then, the bipolar hypersoft image of $(g,\widehat{g},\Lambda)$:

Since $\delta(\Lambda) = \delta(\{(\ell_1, \ell_5, \ell_6), (\ell_2, \ell_5, \ell_6), (\ell_3, \ell_5, \ell_6)\}) = \{(\acute{\ell_1}, \acute{\ell_5}, \acute{\ell_6}), (\acute{\ell_3}, \acute{\ell_5}, \acute{\ell_6})\}$, then for $(\acute{\ell_1}, \acute{\ell_5}, \acute{\ell_6}) : \delta^{-1}((\acute{\ell_1}, \acute{\ell_5}, \acute{\ell_6})) \cap \Lambda = \{(\ell_1, \ell_5, \ell_6), (\ell_2, \ell_5, \ell_6)\} \cap \{(\ell_1, \ell_5, \ell_6), (\ell_2, \ell_5, \ell_6), (\ell_3, \ell_5, \ell_6)\}$ $= \{(\ell_1, \ell_5, \ell_6), (\ell_2, \ell_5, \ell_6)\}$. We have

$\Psi_{\gamma\delta\lambda}(g)(\acute{\ell_1}, \acute{\ell_5}, \acute{\ell_6}) = \gamma\left(\bigcup_{\ell\in\delta^{-1}(\acute{\ell_1},\acute{\ell_5},\acute{\ell_6})\cap\Lambda} g(\ell)\right) = \gamma\left(g(\ell_1, \ell_5, \ell_6) \cup g(\ell_2, \ell_5, \ell_6)\right) = \gamma\left(\{r_1\} \cup \{r_3\}\right) = \gamma\left(\{r_1, r_3\}\right) = \{\eta_1, \eta_3\}$.

Also, $\lambda(\neg\Lambda) = \{\neg(\acute{\ell_1}, \acute{\ell_5}, \acute{\ell_6}), \neg(\acute{\ell_3}, \acute{\ell_5}, \acute{\ell_6})\}$, then for $\neg(\acute{\ell_1}, \acute{\ell_5}, \acute{\ell_6}) : \lambda^{-1}(\neg(\acute{\ell_1}, \acute{\ell_5}, \acute{\ell_6})) \cap \neg\Lambda = \{\neg(\ell_1, \ell_5, \ell_6), \neg(\ell_2, \ell_5, \ell_6)\}$. We have

$\Psi_{\gamma\delta\lambda}(\widehat{g})(\neg(\acute{\ell_1}, \acute{\ell_5}, \acute{\ell_6})) = \gamma\left(\bigcap_{\neg\ell\in\lambda^{-1}(\neg(\acute{\ell_1},\acute{\ell_5},\acute{\ell_6}))\cap\neg\Lambda} \widehat{g}(\neg\ell)\right) = \gamma(\widehat{g}(\neg(\ell_1, \ell_5, \ell_6)) \cap \widehat{g}(\neg(\ell_2, \ell_5, \ell_6))) = \gamma\left(\{r_2\} \cap \{r_1, r_2\}\right) = \gamma\left(\{r_2\}\right) = \{\eta_2\}$

Then, $\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda))((\acute{\ell_1}, \acute{\ell_5}, \acute{\ell_6})) = ((\acute{\ell_1}, \acute{\ell_5}, \acute{\ell_6}), \{\eta_1, \eta_3\}, \{\eta_2\})$.

Now, for $(\acute{\ell_3}, \acute{\ell_5}, \acute{\ell_6}) : \delta^{-1}((\acute{\ell_3}, \acute{\ell_5}, \acute{\ell_6})) \cap \Lambda = \{(\ell_3, \ell_5, \ell_6)\} \cap \{(\ell_1, \ell_5, \ell_6), (\ell_2, \ell_5, \ell_6), (\ell_3, \ell_5, \ell_6)\}$ $= \{(\ell_3, \ell_5, \ell_6)\}$. We have

$$\Psi_{\gamma\delta\lambda}(g)((\acute{\ell_3}, \acute{\ell_5}, \acute{\ell_6})) = \gamma\left(\bigcup_{\ell \in \delta^{-1}((\acute{\ell_3}, \acute{\ell_5}, \acute{\ell_6})) \cap \Lambda} g(\ell)\right) = \gamma\left(g(\ell_3, \ell_5, \ell_6)\right) = \gamma\left(\{r_3\}\right) = \{\eta_3\}.$$

Also, for $\neg(\acute{\ell_3}, \acute{\ell_5}, \acute{\ell_6}) : \lambda^{-1}(\neg(\acute{\ell_3}, \acute{\ell_5}, \acute{\ell_6})) \cap \neg\Lambda = \{\neg(\ell_3, \ell_5, \ell_6)\}$. We have

$$\Psi_{\gamma\delta\lambda}(\widehat{g})(\neg(\acute{\ell_3}, \acute{\ell_5}, \acute{\ell_6})) = \gamma\left(\bigcap_{\neg\ell \in \lambda^{-1}(\neg(\acute{\ell_3}, \acute{\ell_5}, \acute{\ell_6})) \cap \neg\Lambda} \widehat{g}(\neg\ell)\right) = \gamma\left(\widehat{g}(\neg(\ell_3, \ell_5, \ell_6))\right) = \gamma\left(\{r_1\}\right)$$
$$= \{\eta_1\}.$$

Then, $\Psi_{\gamma\delta\lambda}((g, \widehat{g}, \Lambda))((\acute{\ell_3}, \acute{\ell_5}, \acute{\ell_6})) = ((\acute{\ell_3}, \acute{\ell_5}, \acute{\ell_6}), \{\eta_3\}, \{\eta_1\})$.
Hence, $\Psi_{\gamma\delta\lambda}((g, \widehat{g}, \Lambda)) = \{((\acute{\ell_1}, \acute{\ell_5}, \acute{\ell_6}), \{\eta_1, \eta_3\}, \{\eta_2\}), ((\acute{\ell_2}, \acute{\ell_5}, \acute{\ell_6}), \phi, \aleph), ((\acute{\ell_3}, \acute{\ell_5}, \acute{\ell_6}), \{\eta_3\}, \{\eta_1\}),$ $((\acute{\ell_4}, \acute{\ell_5}, \acute{\ell_6}), \phi, \aleph)\}$.

Remark 3.3. In the next example, we illustrate the reason for choosing the mapping $\gamma : \Re \to \aleph$ in Definition 3.1 to be injective .

Example 3.4. Suppose $\Psi_{\gamma\delta\lambda}$ and $(g, \widehat{g}, \Lambda)$ be the same as in Example 3.2 but $\gamma(r_2) = \eta_1$ instead of $\gamma(r_2) = \eta_2$, then $\Psi_{\gamma\delta\lambda}((g, \widehat{g}, \Lambda))((\acute{\ell_1}, \acute{\ell_5}, \acute{\ell_6})) = ((\acute{\ell_1}, \acute{\ell_5}, \acute{\ell_6}), \{\eta_1, \eta_3\}, \{\eta_1\})$ which contradicts the definition of bipolar hypersoft set since $\Psi_{\gamma\delta\lambda}(g)((\acute{\ell_1}, \acute{\ell_5}, \acute{\ell_6})) \cap \Psi_{\gamma\delta\lambda}(\widehat{g})(\neg(\acute{\ell_1}, \acute{\ell_5}, \acute{\ell_6}))$ $\neq \phi$.

Definition 3.5. Suppose that $\Psi_{\gamma\delta\lambda} : \Omega_{(\Re,\Sigma)} \to \Omega_{(\aleph,\acute{\Sigma})}$ is a bipolar hypersoft mapping and $(g, \widehat{g}, \Lambda)$, $(f, \widehat{f}, \Delta) \in \Omega_{(\Re,\Sigma)}$. Then:

(1) The union of bipolar hypersoft image of $(g, \widehat{g}, \Lambda)$, $(f, \widehat{f}, \Delta) \in \Omega_{(\Re,\Sigma)}$ is defined as, for all $\acute{\ell} \in \acute{\Sigma}$,
$$\left(\Psi_{\gamma\delta\lambda}((g, \widehat{g}, \Lambda)) \tilde{\tilde{\sqcup}} \Psi_{\gamma\delta\lambda}((f, \widehat{f}, \Delta))\right)(\acute{\ell}) =$$
$$\left(\acute{\ell}, \Psi_{\gamma\delta\lambda}(g)(\acute{\ell}) \cup \Psi_{\gamma\delta\lambda}(f)(\acute{\ell}), \Psi_{\gamma\delta\lambda}(\widehat{g})(\neg\acute{\ell}) \cap \Psi_{\gamma\delta\lambda}(\widehat{f})(\neg\acute{\ell})\right).$$

(2) The intersection of bipolar hypersoft image of $(g, \widehat{g}, \Lambda)$, $(f, \widehat{f}, \Delta) \in \Omega_{(\Re,\Sigma)}$ is defined as, for all $\acute{\ell} \in \acute{\Sigma}$,
$$\left(\Psi_{\gamma\delta\lambda}((g, \widehat{g}, \Lambda)) \tilde{\tilde{\sqcap}} \Psi_{\gamma\delta\lambda}((f, \widehat{f}, \Delta))\right)(\acute{\ell}) =$$
$$\left(\acute{\ell}, \Psi_{\gamma\delta\lambda}(g)(\acute{\ell}) \cap \Psi_{\gamma\delta\lambda}(f)(\acute{\ell}), \Psi_{\gamma\delta\lambda}(\widehat{g})(\neg\acute{\ell}) \cup \Psi_{\gamma\delta\lambda}(\widehat{f})(\neg\acute{\ell})\right).$$

Definition 3.6. Suppose that $\Psi_{\gamma\delta\lambda}:\Omega_{(\Re,\Sigma)}\to\Omega_{(\aleph,\acute{\Sigma})}$ is a bipolar hypersoft mapping, where $\gamma:\Re\to\aleph$ is an injective mapping, $\delta:\Sigma\to\acute{\Sigma}$ and $\lambda:\neg\Sigma\to\neg\acute{\Sigma}$ are two mappings such that $\lambda(\neg\ell)=\neg\delta(\ell)$ for all $\neg\ell\in\neg\Sigma$. Then a bipolar hypersoft mapping $\Psi_{\gamma\delta\lambda}$ is called:

(1) A bipolar hypersoft surjective mapping if γ and δ are surjective mappings.
(2) A bipolar hypersoft injective mapping if γ and δ are injective mappings. (Provided that any bipolar hypersoft sets in $\Omega_{(\Re,\Sigma)}$ must have the same sets of parameters.)
(3) A bipolar hypersoft bijective mapping if γ and δ are bijective mappings.

Proposition 3.7. *Suppose that $\Psi_{\gamma\delta\lambda}:\Omega_{(\Re,\Sigma)}\to\Omega_{(\aleph,\acute{\Sigma})}$ is a bipolar hypersoft mapping, where $\gamma:\Re\to\aleph$ is an injective mapping, $\delta:\Sigma\to\acute{\Sigma}$ and $\lambda:\neg\Sigma\to\neg\acute{\Sigma}$ are two mappings such that $\lambda(\neg\ell)=\neg\delta(\ell)$ for all $\neg\ell\in\neg\Sigma$. If $(g,\widehat{g},\Lambda)$, $(f,\widehat{f},\Delta)\in\Omega_{(\Re,\Sigma)}$ then:*

(1) $\Psi_{\gamma\delta\lambda}((\Phi,\widehat{\Re},\Sigma))\ \tilde{\tilde{\sqsupseteq}}\ ((\Phi,\widehat{\aleph},\acute{\Sigma}))$. The equality holds if γ is a surjective mapping.
(2) $\Psi_{\gamma\delta\lambda}((\widehat{\Re},\Phi,\Sigma))\ \tilde{\tilde{\sqsubseteq}}\ ((\widehat{\aleph},\Phi,\acute{\Sigma}))$.
(3) If $(g,\widehat{g},\Lambda)\ \tilde{\tilde{\sqsubseteq}}\ (f,\widehat{f},\Delta)$, then $\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda))\ \tilde{\tilde{\sqsubseteq}}\ \Psi_{\gamma\delta\lambda}((f,\widehat{f},\Delta))$.
(4) $\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda)\tilde{\tilde{\sqcup}}(f,\widehat{f},\Delta))=\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda))\ \tilde{\tilde{\sqcup}}\ \Psi_{\gamma\delta\lambda}((f,\widehat{f},\Delta))$.
(5) $\Psi_{\gamma\delta\lambda}\Big((g,\widehat{g},\Lambda)\tilde{\tilde{\sqcap}}(f,\widehat{f},\Delta)=(h,\widehat{h},\Lambda\cap\Delta)\Big)\ \tilde{\tilde{\sqsubseteq}}\ \Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda))\ \tilde{\tilde{\sqcap}}\ \Psi_{\gamma\delta\lambda}((f,\widehat{f},\Delta))$. The equality holds if $\Psi_{\gamma\delta\lambda}$ is a bipolar hypersoft injective mapping.

Proof. 1. and 2. are straightforward.

3. Let $(g,\widehat{g},\Lambda)\ \tilde{\tilde{\sqsubseteq}}\ (f,\widehat{f},\Delta)$, then we want to show that, for all $\acute{\ell}\in\acute{\Sigma}$, $\Psi_{\gamma\delta\lambda}(g)(\acute{\ell})\subseteq\Psi_{\gamma\delta\lambda}(f)(\acute{\ell})$ and, for all $\neg\acute{\ell}\in\neg\acute{\Sigma}$, $\Psi_{\gamma\delta\lambda}(\widehat{f})(\neg\acute{\ell})\subseteq\Psi_{\gamma\delta\lambda}(\widehat{g})(\neg\acute{\ell})$. Let $\acute{\ell}\in\delta(\Lambda)\subseteq\delta(\Delta)\subseteq\acute{\Sigma}$ (if $\acute{\ell}\notin\delta(\Lambda)$, then $\Psi_{\gamma\delta\lambda}(g)(\acute{\ell})=\phi\subseteq\Psi_{\gamma\delta\lambda}(f)(\acute{\ell})$), then

$$\begin{aligned}\Psi_{\gamma\delta\lambda}(g)(\acute{\ell})&=\gamma\left(\bigcup_{\ell\in\delta^{-1}(\acute{\ell})\cap\Lambda}g(\ell)\right)\\&\subseteq\gamma\left(\bigcup_{\ell\in\delta^{-1}(\acute{\ell})\cap\Delta}f(\ell)\right),since\,g(\ell)\subseteq f(\ell)\,for\,all\,\ell\in\Lambda\\&=\Psi_{\gamma\delta\lambda}(f)(\acute{\ell}).\end{aligned}$$

Now, for $\neg\acute{\ell}\in\lambda(\neg\Lambda)\subseteq\lambda(\neg\Delta)\subseteq\neg\acute{\Sigma}$ (if $\neg\acute{\ell}\notin\lambda(\neg\Lambda)$, then $\Psi_{\gamma\delta\lambda}(\widehat{g})(\neg\acute{\ell})=\aleph\supseteq\Psi_{\gamma\delta\lambda}(\widehat{f})(\neg\acute{\ell})$), we have

$$\begin{aligned}\Psi_{\gamma\delta\lambda}(\widehat{f})(\neg\acute{\ell})&=\gamma\left(\bigcap_{\neg\ell\in\lambda^{-1}(\neg\acute{\ell})\cap\neg\Delta}\widehat{f}(\neg\ell)\right)\\&\subseteq\gamma\left(\bigcap_{\neg\ell\in\lambda^{-1}(\neg\acute{\ell})\cap\neg\Lambda}\widehat{g}(\neg\ell)\right),since\,\widehat{f}(\neg\ell)\subseteq\widehat{g}(\ell)\,for\,all\,\neg\ell\in\neg\Lambda\\&=\Psi_{\gamma\delta\lambda}(\widehat{g})(\acute{\neg\ell}).\end{aligned}$$

Hence, $\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda)) \,\widetilde{\widetilde{\sqsubseteq}}\, \Psi_{\gamma\delta\lambda}((f,\widehat{f},\Delta))$.

4. To keep things simple, let

$$\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda))\widetilde{\widetilde{\sqcup}}\Psi_{\gamma\delta\lambda}((f,\widehat{f},\Delta)) = (h,\widehat{h},\acute{\Sigma})$$

$$\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda)\widetilde{\widetilde{\sqcup}}(f,\widehat{f},\Delta)) = \Psi_{\gamma\delta\lambda}((I,\widehat{I},\Lambda\cup\Delta)) = (\mathcal{J},\widehat{\mathcal{J}},\acute{\Sigma}).$$

We want to prove that , for all $\acute{\ell}\in\acute{\Sigma}$, $\mathcal{J}(\acute{\ell}) = h(\acute{\ell})$ and, for all $\neg\acute{\ell}\in\acute{\Sigma}$, $\widehat{\mathcal{J}}(\neg\acute{\ell}) = \widehat{h}(\neg\acute{\ell})$. For non-trivial case, let $\acute{\ell}\in\delta(\Lambda\cup\Delta) = \delta(\Lambda)\cup\delta(\Lambda) = \acute{\Lambda}\cup\acute{\Delta}$, then

$$\mathcal{J}(\acute{\ell}) = \Psi_{\gamma\delta\lambda}(I)(\acute{\ell}) = \gamma\left(\bigcup_{\ell\in\delta^{-1}(\acute{\ell})\cap(\Lambda\cup\Delta)} I(\ell)\right)$$

$$= \begin{cases} \gamma\left(\bigcup_{\ell\in\delta^{-1}(\acute{\ell})\cap(\Lambda\setminus\Delta)} g(\ell)\right), & \text{if } \acute{\ell}\in\acute{\Lambda}\setminus\acute{\Delta} \\ \gamma\left(\bigcup_{\ell\in\delta^{-1}(\acute{\ell})\cap(\Delta\setminus\Lambda)} f(\ell)\right), & \text{if } \acute{\ell}\in\acute{\Delta}\setminus\acute{\Lambda} \\ \gamma\left(\bigcup_{\ell\in\delta^{-1}(\acute{\ell})\cap\Lambda} g(\ell)\right)\cup\gamma\left(\bigcup_{\ell\in\delta^{-1}(\acute{\ell})\cap\Delta} f(\ell)\right), & \text{if } \acute{\ell}\in\acute{\Lambda}\cap\acute{\Delta} \end{cases}$$

$$= \begin{cases} \Psi_{\gamma\delta\lambda}(g)(\acute{\ell}), & \text{if } \acute{\ell}\in\acute{\Lambda}\setminus\acute{\Delta} \\ \Psi_{\gamma\delta\lambda}(f)(\acute{\ell}), & \text{if } \acute{\ell}\in\acute{\Delta}\setminus\acute{\Lambda} \\ \Psi_{\gamma\delta\lambda}(g)(\acute{\ell})\cup\Psi_{\gamma\delta\lambda}(f)(\acute{\ell}), & \text{if } \acute{\ell}\in\acute{\Lambda}\cap\acute{\Delta} \end{cases}$$

Since $\Psi_{\gamma\delta\lambda}(f)(\acute{\ell}) = \phi$ for $\acute{\ell}\in\acute{\Lambda}\setminus\acute{\Delta}$ and $\Psi_{\gamma\delta\lambda}(g)(\acute{\ell}) = \phi$ for $\acute{\ell}\in\acute{\Delta}\setminus\acute{\Lambda}$, then for all $\acute{\ell}\in\acute{\Sigma}$, we have

$$\mathcal{J}(\acute{\ell})) = \Psi_{\gamma\delta\lambda}(g)(\acute{\ell})\cup\Psi_{\gamma\delta\lambda}(f)(\acute{\ell})$$

$$= h(\acute{\ell})), \text{ by Definition 3.5 (1.).}$$

Also, for non-trivial case, let $\neg\acute{\ell}\in\neg(\acute{\Lambda}\cup\acute{\Delta}) = \neg\acute{\Lambda}\cup\neg\acute{\Delta}$, then

$$\widehat{\mathcal{J}}(\neg\acute{\ell}) = \Psi_{\gamma\delta\lambda}(\widehat{I})(\neg\acute{\ell}) = \gamma\left(\bigcap_{\neg\ell\in\lambda^{-1}(\neg\acute{\ell})\cap(\neg\Lambda\cup\neg\Delta)} \widehat{I}(\neg\ell)\right)$$

$$= \begin{cases} \gamma\left(\bigcap_{\neg\ell\in\lambda^{-1}(\neg\acute{\ell})\cap(\neg\Lambda\setminus\neg\Delta)} \widehat{g}(\neg\ell)\right), & \text{if } \neg\acute{\ell}\in\neg\acute{\Lambda}\setminus\neg\acute{\Delta} \\ \gamma\left(\bigcap_{\neg\ell\in\lambda^{-1}(\neg\acute{\ell})\cap(\neg\Delta\setminus\neg\Lambda)} \widehat{f}(\neg\ell)\right), & \text{if } \neg\acute{\ell}\in\neg\acute{\Delta}\setminus\neg\acute{\Lambda} \\ \gamma\left(\bigcap_{\neg\ell\in\lambda^{-1}(\neg\acute{\ell})\cap\neg\Lambda} \widehat{g}(\neg\ell)\right)\cap\gamma\left(\bigcap_{\neg\ell\in\lambda^{-1}(\neg\acute{\ell})\cap\neg\Delta} \widehat{f}(\neg\ell)\right), & \text{if } \neg\acute{\ell}\in\neg\acute{\Lambda}\cap\neg\acute{\Delta} \end{cases}$$

$$= \begin{cases} \Psi_{\gamma\delta\lambda}(\widehat{g})(\neg\acute{\ell}), & \text{if } \neg\acute{\ell}\in\neg\acute{\Lambda}\setminus\neg\acute{\Delta} \\ \Psi_{\gamma\delta\lambda}(\widehat{f})(\neg\acute{\ell}), & \text{if } \neg\acute{\ell}\in\neg\acute{\Delta}\setminus\neg\acute{\Lambda} \\ \Psi_{\gamma\delta\lambda}(\widehat{g})(\neg\acute{\ell})\cap\Psi_{\gamma\delta\lambda}(\widehat{f})(\neg\acute{\ell}), & \text{if } \neg\acute{\ell}\in\neg\acute{\Lambda}\cap\neg\acute{\Delta} \end{cases}$$

Since $\Psi_{\gamma\delta\lambda}(\widehat{f})(\neg\acute{\ell}) = \aleph$ for $\neg\acute{\ell}\in\neg\acute{\Lambda}\setminus\neg\acute{\Delta}$ and $\Psi_{\gamma\delta\lambda}(\widehat{g})(\neg\acute{\ell}) = \aleph$ for $\neg\acute{\ell}\in\neg\acute{\Delta}\setminus\neg\acute{\Lambda}$, then for all $\neg\acute{\ell}\in\neg\acute{\Sigma}$, we have

$$\widehat{\mathcal{J}}(\neg\acute{\ell})) = \Psi_{\gamma\delta\lambda}(\widehat{g})(\neg\acute{\ell})\cap\Psi_{\gamma\delta\lambda}(\widehat{f})(\neg\acute{\ell})$$

$$= \widehat{\hbar}(\neg\acute{\ell})), \text{ by Definition 3.5 (2.).}$$

Hence, $\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda)\widetilde{\widetilde{\sqcup}}(f,\widehat{f},\Delta)) = \Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda))\ \widetilde{\widetilde{\sqcup}}\ \Psi_{\gamma\delta\lambda}((f,\widehat{f},\Delta))$.

5. Simply, let

$$\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda)\widetilde{\widetilde{\sqcap}}(f,\widehat{f},\Delta)) = \Psi_{\gamma\delta\lambda}((\hbar,\widehat{\hbar},\Lambda\cap\Delta))$$

$$\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda))\widetilde{\widetilde{\sqcap}}\Psi_{\gamma\delta\lambda}((f,\widehat{f},\Delta)) = (I,\widehat{I},\acute{\Sigma}).$$

We want to show that, for all $\acute{\ell}\in\acute{\Sigma}$, $\Psi_{\gamma\delta\lambda}(\hbar)(\acute{\ell})\subseteq(I)(\acute{\ell})$ and, for all $\neg\acute{\ell}\in\neg\acute{\Sigma}$, $(\widehat{I})(\neg\acute{\ell})\subseteq\Psi_{\gamma\delta\lambda}(\widehat{\hbar})(\neg\acute{\ell})$. For a non-trivial case, let $\acute{\ell}\in\delta(\Lambda\cap\Delta)\subseteq\acute{\Sigma}$, then

$$\begin{aligned}
\Psi_{\gamma\delta\lambda}(\hbar)(\acute{\ell}) &= \gamma\left(\bigcup_{\ell\in\delta^{-1}(\acute{\ell})\cap(\Lambda\cap\Delta)}\hbar(\ell)\right)\\
&= \gamma\left(\bigcup_{\ell\in\delta^{-1}(\acute{\ell})\cap(\Lambda\cap\Delta)}g(\ell)\cap f(\ell)\right)\\
&= \gamma\left(\bigcup_{\ell\in\delta^{-1}(\acute{\ell})\cap(\Lambda\cap\Delta)}g(\ell)\right)\cap\gamma\left(\bigcup_{\ell\in\delta^{-1}(\acute{\ell})\cap(\Lambda\cap\Delta)}f(\ell)\right)\\
&\subseteq \gamma\left(\bigcup_{\ell\in\delta^{-1}(\acute{\ell})\cap\Lambda}g(\ell)\right)\cap\gamma\left(\bigcup_{\ell\in\delta^{-1}(\acute{\ell})\cap\Delta}f(\ell)\right)\\
&= \Psi_{\gamma\delta\lambda}(g)(\acute{\ell})\cap\Psi_{\gamma\delta\lambda}(f)(\acute{\ell})\\
&= I(\acute{\ell}).
\end{aligned}$$

Now, for a non-trivial case, let $\neg\acute{\ell}\in\lambda(\neg\Lambda\cap\neg\Delta)\subseteq\neg\acute{\Sigma}$, then

$$\begin{aligned}
\Psi_{\gamma\delta\lambda}(\widehat{\hbar})(\neg\acute{\ell}) &= \gamma\left(\bigcap_{\neg\ell\in\lambda^{-1}(\neg\acute{\ell})\cap(\neg\Lambda\cap\neg\Delta)}\widehat{\hbar}(\neg\ell)\right)\\
&= \gamma\left(\bigcap_{\neg\ell\in\lambda^{-1}(\neg\acute{\ell})\cap(\neg\Lambda\cap\neg\Delta)}\widehat{g}(\neg\ell)\cap\widehat{f}(\neg\ell)\right)\\
&\supseteq \gamma\left(\bigcap_{\neg\ell\in\lambda^{-1}(\neg\acute{\ell})\cap(\neg\Lambda\cap\neg\Delta)}\widehat{g}(\ell)\right)\cup\gamma\left(\bigcap_{\neg\ell\in\lambda^{-1}(\neg\acute{\ell})\cap(\neg\Lambda\cap\neg\Delta)}\widehat{f}(\neg\ell)\right)\\
&\supseteq \gamma\left(\bigcap_{\neg\ell\in\lambda^{-1}(\neg\acute{\ell})\cap\neg\Lambda}\widehat{g}(\neg\ell)\right)\cup\gamma\left(\bigcap_{\neg\ell\in\lambda^{-1}(\neg\acute{\ell})\cap\neg\Delta}\widehat{f}(\neg\ell)\right)\\
&= \Psi_{\gamma\delta\lambda}(\widehat{g})(\neg\acute{\ell})\cup\Psi_{\gamma\delta\lambda}(\widehat{f})(\neg\acute{\ell})\\
&= \widehat{I}(\neg\acute{\ell}).
\end{aligned}$$

Therefore, $\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda)\tilde{\tilde{\sqcap}}(f,\widehat{f},\Delta)) = \Psi_{\gamma\delta\lambda}((\hbar,\widehat{\hbar},\Lambda\cap\Delta)) \tilde{\tilde{\sqsubseteq}} \Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda)) \tilde{\tilde{\sqcap}} \Psi_{\gamma\delta\lambda}((f,\widehat{f},\Delta))$.

□

Remark 3.8. The reverse of Proposition 3.7 (5) is incorrect.

Example 3.9. Let $\Psi_{\gamma\delta\lambda}$ and $(g,\widehat{g},\Lambda)$ be the same as in Example 3.2. Let $(f,\widehat{f},\Delta) = \{((\ell_1,\ell_5,\ell_6),\{r_2\},\{r_1,r_3\}),((\ell_2,\ell_5,\ell_6),\{r_1\},\{r_2\}),((\ell_3,\ell_5,\ell_6),\Re,\phi)),((\ell_4,\ell_5,\ell_6),\{r_3\},\{r_1\}))\}$, then $\Psi_{\gamma\delta\lambda}((f,\widehat{f},\Delta)) = \{((\acute{\ell_1},\acute{\ell_5},\acute{\ell_6}),\{\eta_1,\eta_2\},\phi),\ ((\acute{\ell_2},\acute{\ell_5},\acute{\ell_6}),\phi,\aleph),\ ((\acute{\ell_3},\acute{\ell_5},\acute{\ell_6}),\ \{\eta_1,\eta_2,\eta_3\},\phi),\ ((\acute{\ell_4},\acute{\ell_5},\acute{\ell_6}),\{\eta_3\},\{\eta_1\})\}$. Now,

$\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda))\tilde{\tilde{\sqcap}}\Psi_{\gamma\delta\lambda}((f,\widehat{f},\Delta)) = \{((\acute{\ell_1},\acute{\ell_5},\acute{\ell_6}),\{\eta_1\},\{\eta_2\}),\ ((\acute{\ell_2},\acute{\ell_5},\acute{\ell_6}),\phi,\aleph),\ ((\acute{\ell_3},\acute{\ell_5},\acute{\ell_6}),\ \{\eta_3\},\{\eta_1\}),\ ((\acute{\ell_4},\acute{\ell_5},\acute{\ell_6}),\phi,\aleph)\}$.

On the other hand, $(g,\widehat{g},\Lambda)\tilde{\tilde{\sqcap}}(f,\widehat{f},\Delta) = \{((\ell_1,\ell_5,\ell_6),\phi,\{r_1,r_2,r_3\}),((\ell_2,\ell_5,\ell_6),\phi,\{r_1,r_2\}),\ ((\ell_3,\ell_5,\ell_6),\{r_3\},\{r_1\})\}$, then

$\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda)\tilde{\tilde{\sqcap}}(f,\widehat{f},\Delta)) = \{((\acute{\ell_1},\acute{\ell_5},\acute{\ell_6}),\phi,\{\eta_1,\eta_2\}),\ ((\acute{\ell_2},\acute{\ell_5},\acute{\ell_6}),\phi,\aleph),\ ((\acute{\ell_3},\acute{\ell_5},\acute{\ell_6}),\ \{\eta_3\},\{\eta_1\}),\ ((\acute{\ell_4},\acute{\ell_5},\acute{\ell_6}),\phi,\aleph)\}$.

Therefore, $\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda)) \tilde{\tilde{\sqcap}} \Psi_{\gamma\delta\lambda}((f,\widehat{f},\Delta)) \tilde{\tilde{\not\sqsubseteq}} \Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda)\tilde{\tilde{\sqcap}}(f,\widehat{f},\Delta))$.

Remark 3.10. In Proposition 3.7 (5.), $\Psi_{\gamma\delta\lambda}\left((g,\widehat{g},\Lambda)\tilde{\tilde{\sqcap}}(f,\widehat{f},\Delta) = (\hbar,\widehat{\hbar},\Lambda\cup\Delta)\right) \tilde{\tilde{\not\sqsubseteq}} \Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda)) \tilde{\tilde{\sqcap}} \Psi_{\gamma\delta\lambda}((f,\widehat{f},\Delta))$.

Example 3.11. Let $\Psi_{\gamma\delta\lambda}$ and $(g,\widehat{g},\Lambda)$ be the same as in Example 3.2. Let $(f,\widehat{f},\Delta) = \{((\ell_1,\ell_5,\ell_6),\{r_1,r_2\},\phi),((\ell_3,\ell_5,\ell_6),\{r_3\},\{r_1\}),((\ell_4,\ell_5,\ell_6),\Re,\phi)\}$, then $\Psi_{\gamma\delta\lambda}((f,\widehat{f},\Delta)) = \{((\acute{\ell_1},\acute{\ell_5},\acute{\ell_6}),\{\eta_1,\eta_2\},\phi),\ ((\acute{\ell_2},\acute{\ell_5},\acute{\ell_6}),\phi,\aleph),\ ((\acute{\ell_3},\acute{\ell_5},\acute{\ell_6}),\ \{\eta_3\},\{\eta_1\}),\ ((\acute{\ell_4},\acute{\ell_5},\acute{\ell_6}),\{\eta_1,\eta_2,\eta_3\},\phi)\}$. Now,

$\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda))\tilde{\tilde{\sqcap}}\Psi_{\gamma\delta\lambda}((f,\widehat{f},\Delta)) = \{((\acute{\ell_1},\acute{\ell_5},\acute{\ell_6}),\{\eta_1\},\{\eta_2\}),\ ((\acute{\ell_2},\acute{\ell_5},\acute{\ell_6}),\phi,\aleph),\ ((\acute{\ell_3},\acute{\ell_5},\acute{\ell_6}),\ \{\eta_3\},\{\eta_1\}),\ ((\acute{\ell_4},\acute{\ell_5},\acute{\ell_6}),\phi,\aleph)\}$.

On the other hand, $(g,\widehat{g},\Lambda)\tilde{\tilde{\sqcap}}(f,\widehat{f},\Delta) = \{((\ell_1,\ell_5,\ell_6),\{r_1\},\{r_2\}),((\ell_2,\ell_5,\ell_6),\{r_3\},\{r_1,r_2\}),\ ((\ell_3,\ell_5,\ell_6),\{r_3\},\{r_1\}),((\ell_4,\ell_5,\ell_6),\Re,\phi\})\}$, then

$\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda)\tilde{\tilde{\sqcap}}(f,\widehat{f},\Delta)) = \{((\acute{\ell_1},\acute{\ell_5},\acute{\ell_6}),\{\eta_1,\eta_3\},\{\eta_2\}),\ ((\acute{\ell_2},\acute{\ell_5},\acute{\ell_6}),\phi,\aleph),\ ((\acute{\ell_3},\acute{\ell_5},\acute{\ell_6}),\ \{\eta_3\},\{\eta_1\}),\ ((\acute{\ell_4},\acute{\ell_5},\acute{\ell_6}),\{\eta_1,\eta_2,\eta_3\},\phi)\}$.

Therefore, $\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda)\widetilde{\widetilde{\sqcap}}(f,\widehat{f},\Delta)) \widetilde{\widetilde{\not\supseteq}} \Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda)) \widetilde{\widetilde{\sqcap}} \Psi_{\gamma\delta\lambda}((f,\widehat{f},\Delta))$.

4. Bipolar Hypersoft Inverse Image

This section focuses on bipolar hypersoft inverse image and its relation to the bipolar hypersoft image on bipolar hypersoft sets.

Definition 4.1. Let $\gamma : \Re \to \aleph$ be an injective mapping. Let $\delta : \Sigma \to \acute{\Sigma}$ and $\lambda : \neg\Sigma \to \neg\acute{\Sigma}$ be two mappings such that $\lambda(\neg\ell) = \neg\delta(\ell)$ for all $\neg\ell \in \neg\Sigma$, and $\Psi_{\gamma\delta\lambda} : \Omega_{(\Re,\Sigma)} \to \Omega_{(\aleph,\acute{\Sigma})}$ be a bipolar hypersoft mapping. The inverse image of a bipolar hypersoft set $(g,\widehat{g},\acute{\Lambda})$ under $\Psi_{\gamma\delta\lambda}$, $\Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\Lambda)) = (\Psi^{-1}_{\gamma\delta\lambda}(g),\Psi^{-1}_{\gamma\delta\lambda}(\widehat{g}),\Sigma)$ is a bipolar hypersoft set in $\Omega_{(\Re,\Sigma)}$ given as, for all $\ell \in \Sigma$:

$$\Psi^{-1}_{\gamma\delta\lambda}(g)(\ell) = \begin{cases} \gamma^{-1}\left(g(\delta(\ell))\right), & \text{if } \delta(\ell) \in \acute{\Lambda} \\ \phi, & \text{if } \delta(\ell) \notin \acute{\Lambda} \end{cases}$$

$$\Psi^{-1}_{\gamma\delta\lambda}(\widehat{g})(\neg\ell) = \begin{cases} \gamma^{-1}(\widehat{g}(\lambda(\neg\ell))), & \text{if } \lambda(\neg\ell) \in \neg\acute{\Lambda} \\ \Re, & \text{if } \lambda(\neg\ell) \notin \neg\acute{\Lambda} \end{cases}$$

Example 4.2. Let $\Psi_{\gamma\delta\lambda}$ be the same as in Example 3.2. Let $(g,\widehat{g},\acute{\Lambda}) = \{((\acute{\ell_3},\acute{\ell_5},\acute{\ell_6}),\aleph,\phi),((\acute{\ell_4},\acute{\ell_5},\acute{\ell_6}),\{\eta_1,\eta_3\},\{\eta_2\})\}$.

Since $\delta^{-1}(\acute{\Lambda}) = \delta^{-1}(\{(\acute{\ell_3},\acute{\ell_5},\acute{\ell_6}),(\acute{\ell_4},\acute{\ell_5},\acute{\ell_6})\}) = \{(\ell_3,\ell_5,\ell_6),(\ell_4,\ell_5,\ell_6)\}$ and $\delta((\ell_3,\ell_5,\ell_6)) = (\acute{\ell_3},\acute{\ell_5},\acute{\ell_6}) \in \acute{\Lambda}$, then

$$\Psi^{-1}_{\gamma\delta\lambda}(g)(\ell_3,\ell_5,\ell_6) = \gamma^{-1}(g(\delta((\ell_3,\ell_5,\ell_6)))) = \gamma^{-1}(g((\acute{\ell_3},\acute{\ell_5},\acute{\ell_6}))) = \gamma^{-1}(\aleph) = \Re.$$

Also, $\lambda^{-1}(\neg\acute{\Lambda}) = \lambda^{-1}(\{(\neg\acute{\ell_3},\neg\acute{\ell_5},\neg\acute{\ell_6}),(\neg\acute{\ell_4},\neg\acute{\ell_5},\neg\acute{\ell_6})\}) = \{(\neg\ell_3,\neg\ell_5,\neg\ell_6),(\neg\ell_4,\neg\ell_5,\neg\ell_6)\}$ and $\lambda((\neg\ell_3,\neg\ell_5,\neg\ell_6)) = (\neg\acute{\ell_3},\neg\acute{\ell_5},\neg\acute{\ell_6}) \in \neg\acute{\Lambda}$, then

$$\Psi^{-1}_{\gamma\delta\lambda}(g)(\ell_3,\ell_5,\ell_6) = \gamma^{-1}(g(\delta((\ell_3,\ell_5,\ell_6)))) = \gamma^{-1}(\widehat{g}((\neg\acute{\ell_3},\neg\acute{\ell_5},\neg\acute{\ell_6}))) = \gamma^{-1}(\phi) = \phi.$$

Then, $\Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\acute{\Lambda}))((\ell_3,\ell_5,\ell_6)) = ((\ell_3,\ell_5,\ell_6),\Re,\phi)$.

Now, for (ℓ_4,ℓ_5,ℓ_6): $\delta((\ell_4,\ell_5,\ell_6)) = (\acute{\ell_4},\acute{\ell_5},\acute{\ell_6}) \in \acute{\Lambda}$, then

$$\Psi^{-1}_{\gamma\delta\lambda}(g)(\ell_4,\ell_5,\ell_6) = \gamma^{-1}(g(\delta((\ell_4,\ell_5,\ell_6)))) = \gamma^{-1}(g((\acute{\ell_4},\acute{\ell_5},\acute{\ell_6}))) = \gamma^{-1}(\{\eta_1,\eta_3\}) = \{r_1,r_3\}.$$

Also, $\lambda((\neg\ell_4,\neg\ell_5,\neg\ell_6)) = (\neg\acute{\ell}_4,\neg\acute{\ell}_5,\neg\acute{\ell}_6) \in \neg\acute{\Lambda}$, then

$\Psi^{-1}_{\gamma\delta\lambda}(g)(\ell_4,\ell_5,\ell_6) = \gamma^{-1}(g(\delta((\ell_4,\ell_5,\ell_6)))) = \gamma^{-1}(\widehat{g}((\neg\acute{\ell}_4,\neg\acute{\ell}_5,\neg\acute{\ell}_6))) = \gamma^{-1}(\{\eta_2\}) = \{r_2\}$.
Then, $\Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\acute{\Lambda}))((\ell_4,\ell_5,\ell_6)) = ((\ell_4,\ell_5,\ell_6),\{r_1,r_3\},\{r_2\})$.

Hence, $\Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\acute{\Lambda})) = \{((\ell_1,\ell_5,\ell_6),\phi,\Re), ((\ell_2,\ell_5,\ell_6),\phi,\Re), ((\ell_3,\ell_5,\ell_6),\Re,\phi), ((\ell_4,\ell_5,\ell_6),\{r_1,r_3\},\{r_2\})\}$.

Definition 4.3. Suppose that $\Psi_{\gamma\delta\lambda} : \Omega_{(\Re,\Sigma)} \to \Omega_{(\aleph,\acute{\Sigma})}$ is a bipolar hypersoft mapping and $(g,\widehat{g},\acute{\Lambda})$, $(f,\widehat{f},\acute{\Delta}) \in \Omega_{(\aleph,\acute{\Sigma})}$. Then:

(1) The union of bipolar hypersoft inverse image of $(g,\widehat{g},\acute{\Lambda})$, $(f,\widehat{f},\acute{\Delta}) \in \Omega_{(\aleph,\acute{\Sigma})}$ is defined as, for all $\ell \in \Sigma$,

$$\left(\Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\acute{\Lambda}))\tilde{\tilde{\sqcup}}\Psi^{-1}_{\gamma\delta\lambda}((f,\widehat{f},\acute{\Delta}))\right)(\ell) =$$

$$\left(\ell, \Psi^{-1}_{\gamma\delta\lambda}(g)(\ell) \cup \Psi^{-1}_{\gamma\delta\lambda}(f)(\ell), \Psi^{-1}_{\gamma\delta\lambda}(\widehat{g})(\neg\ell) \cap \Psi^{-1}_{\gamma\delta\lambda}(\widehat{f})(\neg\ell)\right).$$

(2) The intersection of bipolar hypersoft inverse image of $(g,\widehat{g},\acute{\Lambda})$, $(f,\widehat{f},\acute{\Delta}) \in \Omega_{(\aleph,\acute{\Sigma})}$ is defined as, for all $\ell \in \Sigma$,

$$\left(\Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\acute{\Lambda}))\tilde{\tilde{\sqcap}}\Psi^{-1}_{\gamma\delta\lambda}((f,\widehat{f},\acute{\Delta}))\right)(\ell) =$$

$$\left(\ell, \Psi^{-1}_{\gamma\delta\lambda}(g)(\ell) \cap \Psi^{-1}_{\gamma\delta\lambda}(f)(\ell), \Psi^{-1}_{\gamma\delta\lambda}(\widehat{g})(\neg\ell) \cup \Psi^{-1}_{\gamma\delta\lambda}(\widehat{f})(\neg\ell)\right).$$

Proposition 4.4. *Suppose that $\Psi_{\gamma\delta\lambda} : \Omega_{(\Re,\Sigma)} \to \Omega_{(\aleph,\acute{\Sigma})}$ is a bipolar hypersoft mapping, where $\gamma : \Re \to \aleph$ is an injective mapping, $\delta : \Sigma \to \acute{\Sigma}$ and $\lambda : \neg\Sigma \to \neg\acute{\Sigma}$ are two mappings such that $\lambda(\neg\ell) = \neg\delta(\ell)$ for all $\neg\ell \in \neg\Sigma$. If $(g,\widehat{g},\acute{\Lambda})$, $(f,\widehat{f},\acute{\Delta}) \in \Omega_{(\aleph,\acute{\Sigma})}$ then:*

(1) $\Psi^{-1}_{\gamma\delta\lambda}((\Phi,\widehat{\aleph},\acute{\Sigma})) = (\Phi,\widehat{\Re},\Sigma)$.
(2) $\Psi^{-1}_{\gamma\delta\lambda}((\widehat{\aleph},\Phi,\acute{\Sigma})) = (\widehat{\Re},\Phi,\Sigma)$.
(3) If $(g,\widehat{g},\acute{\Lambda}) \tilde{\tilde{\sqsubseteq}} (f,\widehat{f},\acute{\Delta})$, *then* $\Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\acute{\Lambda})) \tilde{\tilde{\sqsubseteq}} \Psi^{-1}_{\gamma\delta\lambda}((f,\widehat{f},\acute{\Delta}))$.
(4) $\Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\acute{\Lambda})\tilde{\tilde{\sqcup}}(f,\widehat{f},\acute{\Delta})) = \Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\acute{\Lambda})) \tilde{\tilde{\sqcup}} \Psi^{-1}_{\gamma\delta\lambda}((f,\widehat{f},\acute{\Delta}))$.
(5) $\Psi^{-1}_{\gamma\delta\lambda}\left((g,\widehat{g},\acute{\Lambda})\tilde{\tilde{\sqcap}}(f,\widehat{f},\acute{\Delta}) = (h,\widehat{h},\acute{\Lambda}\cap\acute{\Delta})\right) = \Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\acute{\Lambda})) \tilde{\tilde{\sqcap}} \Psi^{-1}_{\gamma\delta\lambda}((f,\widehat{f},\acute{\Delta}))$.
(6) $\Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\acute{\Sigma})^c) = (\Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\acute{\Sigma})))^c$.

Proof. 1. and 2. are straightforward.
3. Let $(g,\widehat{g},\acute{\Lambda}) \tilde{\tilde{\sqsubseteq}} (f,\widehat{f},\acute{\Delta})$, then we want to show that, for all $\ell \in \Sigma$, $\Psi^{-1}_{\gamma\delta\lambda}(g)(\ell) \subseteq \Psi^{-1}_{\gamma\delta\lambda}(f)(\ell)$ and, for all $\neg\ell \in \neg\Sigma$, $\Psi^{-1}_{\gamma\delta\lambda}(\widehat{f})(\neg\ell) \subseteq \Psi^{-1}_{\gamma\delta\lambda}(\widehat{g})(\neg\ell)$. Let $\ell \in \Sigma$ where $\delta(\ell) \in \acute{\Lambda} \subseteq \acute{\Delta}$ (if $\delta(\ell) \notin \acute{\Lambda}$,

then $\Psi^{-1}_{\gamma\delta\lambda}(g)(\ell) = \phi \subseteq \Psi^{-1}_{\gamma\delta\lambda}(f)(\ell))$, then

$$\begin{aligned}\Psi^{-1}_{\gamma\delta\lambda}(g)(\ell) &= \gamma^{-1}\left(g(\delta(\ell))\right)\\ &\subseteq \gamma^{-1}(f(\delta(\ell))), since\, g(\acute{\ell}) \subseteq f(\acute{\ell})\, for\, all\, \acute{\ell} \in \acute{\Lambda}\\ &= \Psi^{-1}_{\gamma\delta\lambda}(f)(\ell).\end{aligned}$$

Now, for $\neg\ell \in \neg\Sigma$ where $\lambda(\neg\ell) \in \neg\acute{\Lambda} \subseteq \neg\acute{\Delta}$ (if $\lambda(\neg\ell) \notin \neg\acute{\Lambda}$), then $\Psi^{-1}_{\gamma\delta\lambda}(\widehat{f})(\neg\ell) \subseteq \Psi^{-1}_{\gamma\delta\lambda}(\widehat{g})(\neg\ell) = \Re$, we have

$$\begin{aligned}\Psi^{-1}_{\gamma\delta\lambda}(\widehat{f}(\neg\ell)) &= \gamma^{-1}(\widehat{f}(\lambda(\neg\ell)))\\ &\subseteq \gamma^{-1}(\widehat{g}(\lambda(\neg\ell))), since\, \widehat{f}(\neg\acute{\ell}) \subseteq \widehat{g}(\acute{\ell})\, for\, all\, \neg\acute{\ell} \in \neg\acute{\Lambda}\\ &= \Psi^{-1}_{\gamma\delta\lambda}(\widehat{g})(\neg\ell).\end{aligned}$$

Hence, $\Psi^{-1}_{\gamma\delta\lambda}((g, \widehat{g}, \acute{\Lambda})) \,\tilde{\tilde{\sqsubseteq}}\, \Psi^{-1}_{\gamma\delta\lambda}((f, \widehat{f}, \acute{\Delta}))$.

4. To keep things simple, let

$$\Psi^{-1}_{\gamma\delta\lambda}((g, \widehat{g}, \acute{\Lambda}) \,\tilde{\tilde{\sqcup}}\, (f, \widehat{f}, \acute{\Delta})) = \Psi^{-1}_{\gamma\delta\lambda}((I, \widehat{I}, \acute{\Lambda} \cup \acute{\Delta})) = (\mathcal{J}, \widehat{\mathcal{J}}, \Sigma)$$

$$\Psi^{-1}_{\gamma\delta\lambda}((g, \widehat{g}, \acute{\Lambda})) \,\tilde{\tilde{\sqcup}}\, \Psi^{-1}_{\gamma\delta\lambda}((f, \widehat{f}, \acute{\Delta})) = (\hbar, \widehat{\hbar}, \Sigma).$$

We want to prove that, for all $\ell \in \Sigma$, $\mathcal{J}(\ell) = \hbar(\ell)$ and, for all $\neg\ell \in \neg\Sigma$, $\widehat{\mathcal{J}}(\neg\ell) = \widehat{\hbar}(\neg\ell)$. For a non-trivial case, let $\ell \in \Sigma$ where $\delta(\ell) \in \acute{\Lambda} \cup \acute{\Delta}$, then

$$\mathcal{J}(\ell) = \Psi^{-1}_{\gamma\delta\lambda}(I)(\ell) = \gamma^{-1}(I(\delta(\ell)))$$

$$= \begin{cases} \gamma^{-1}(g(\delta(\ell))), & \text{if } \delta(\ell) \in \acute{\Lambda} \setminus \acute{\Delta}\\ \gamma^{-1}(f(\delta(\ell))), & \text{if } \delta(\ell) \in \acute{\Delta} \setminus \acute{\Lambda}\\ \gamma^{-1}(g(\delta(\ell)) \cup f(\delta(\ell))), & \text{if } \delta(\ell) \in \acute{\Lambda} \cap \acute{\Delta}\end{cases}$$

$$= \begin{cases} \gamma^{-1}(g(\delta(\ell))), & \text{if } \delta(\ell) \in \acute{\Lambda} \setminus \acute{\Delta}\\ \gamma^{-1}(f(\delta(\ell))), & \text{if } \delta(\ell) \in \acute{\Delta} \setminus \acute{\Lambda}\\ \gamma^{-1}(g(\delta(\ell))) \cup \gamma^{-1}(f(\delta(\ell))), & \text{if } \delta(\ell) \in \acute{\Lambda} \cap \acute{\Delta}\end{cases}$$

$$= \begin{cases} \Psi^{-1}_{\gamma\delta\lambda}(g(\ell)), & \text{if } \delta(\ell) \in \acute{\Lambda} \setminus \acute{\Delta}\\ \Psi^{-1}_{\gamma\delta\lambda}(f(\ell)), & \text{if } \delta(\ell) \in \acute{\Delta} \setminus \acute{\Lambda}\\ \Psi^{-1}(g(\ell)) \cup \Psi^{-1}(f(\ell)), & \text{if } \delta(\ell) \in \acute{\Lambda} \cap \acute{\Delta}\end{cases}$$

Since $\Psi^{-1}_{\gamma\delta\lambda}(f)(\ell) = \phi$ for $\delta(\ell) \in \acute{\Lambda} \setminus \acute{\Delta}$ and $\Psi^{-1}_{\gamma\delta\lambda}(g)(\acute{\ell}) = \phi$ for $\delta(\ell) \in \acute{\Delta} \setminus \acute{\Lambda}$, then for all $\ell \in \Sigma$, we have

$$\begin{aligned}\mathcal{J}(\ell) &= \Psi^{-1}_{\gamma\delta\lambda}(g)(\ell) \cup \Psi^{-1}_{\gamma\delta\lambda}(f)(\ell)\\ &= \hbar(\ell), \text{ by Definition 4.3 (1.).}\end{aligned}$$

Also, for a non-trivial case, let $\neg\ell \in \neg\Sigma$ where $\lambda(\neg\ell) \in \neg\acute{\Lambda} \cup \neg\acute{\Delta}$, then

$$\mathcal{I}(\neg\ell) = \Psi^{-1}_{\gamma\delta\lambda}(\widehat{I})(\neg\ell) = \gamma^{-1}(\widehat{I}(\lambda(\neg\ell)))$$

$$= \begin{cases} \gamma^{-1}(\widehat{g}(\lambda(\neg\ell))), & \text{if } \lambda(\neg\ell) \in \neg\acute{\Lambda} \setminus \neg\acute{\Delta} \\ \gamma^{-1}(\widehat{f}(\lambda(\neg\ell))), & \text{if } \lambda(\neg\ell) \in \neg\acute{\Delta} \setminus \neg\acute{\Lambda} \\ \gamma^{-1}(\widehat{g}(\lambda(\neg\ell)) \cap \widehat{f}(\lambda(\neg\ell))), & \text{if } \lambda(\neg\ell) \in \neg\acute{\Lambda} \cap \neg\acute{\Delta} \end{cases}$$

$$= \begin{cases} \gamma^{-1}(\widehat{g}(\lambda(\neg\ell))), & \text{if } \lambda(\neg\ell) \in \neg\acute{\Lambda} \setminus \neg\acute{\Delta} \\ \gamma^{-1}(\widehat{f}(\lambda(\neg\ell))), & \text{if } \lambda(\neg\ell) \in \neg\acute{\Delta} \setminus \neg\acute{\Lambda} \\ \gamma^{-1}(\widehat{g}(\lambda(\neg\ell))) \cap \gamma^{-1}(\widehat{f}(\lambda(\neg\ell))), & \text{if } \lambda(\neg\ell) \in \neg\acute{\Lambda} \cap \neg\acute{\Delta} \end{cases}$$

$$= \begin{cases} \Psi^{-1}_{\gamma\delta\lambda}(\widehat{g}(\neg\ell)), & \text{if } \lambda(\neg\ell) \in \neg\acute{\Lambda} \setminus \neg\acute{\Delta} \\ \Psi^{-1}_{\gamma\delta\lambda}(\widehat{f}(\neg\ell)), & \text{if } \lambda(\neg\ell) \in \neg\acute{\Delta} \setminus \neg\acute{\Lambda} \\ \Psi^{-1}(\widehat{g}(\neg\ell)) \cap \Psi^{-1}(\widehat{f}(\neg\ell)), & \text{if } \lambda(\neg\ell) \in \neg\acute{\Lambda} \cap \neg\acute{\Delta} \end{cases}$$

Since $\Psi^{-1}_{\gamma\delta\lambda}(\widehat{f})(\neg\ell) = \Re$ for $\lambda(\neg\ell) \in \neg\acute{\Lambda} \setminus \neg\acute{\Delta}$ and $\Psi^{-1}_{\gamma\delta\lambda}(\widehat{g})(\neg\acute{\ell}) = \Re$ for $\lambda(\neg\ell) \in \neg\acute{\Delta} \setminus \neg\acute{\Lambda}$, then for all $\neg\ell \in \neg\Sigma$, we have

$$\widehat{\mathcal{I}}(\neg\ell) = \Psi^{-1}_{\gamma\delta\lambda}(\widehat{g})(\neg\ell) \cap \Psi^{-1}_{\gamma\delta\lambda}(\widehat{f})(\neg\ell)$$

$$= \widehat{h}(\neg\ell), \text{ by Definition 4.3 (2.).}$$

Hence, $\Psi^{-1}_{\gamma\delta\lambda}((g, \widehat{g}, \acute{\Lambda})\widetilde{\widetilde{\sqcup}}(f, \widehat{f}, \acute{\Delta})) = \Psi^{-1}_{\gamma\delta\lambda}((g, \widehat{g}, \acute{\Lambda}))\ \widetilde{\widetilde{\sqcup}}\ \Psi^{-1}_{\gamma\delta\lambda}((f, \widehat{f}, \acute{\Delta}))$.

5. Simply, let

$$\Psi^{-1}_{\gamma\delta\lambda}((g, \widehat{g}, \acute{\Lambda})\widetilde{\widetilde{\sqcap}}(f, \widehat{f}, \acute{\Delta})) = \Psi^{-1}_{\gamma\delta\lambda}((I, \widehat{I}, \acute{\Lambda} \cap \acute{\Delta})) = (\mathcal{I}, \widehat{\mathcal{I}}, \Sigma)$$

$$\Psi^{-1}_{\gamma\delta\lambda}((g, \widehat{g}, \acute{\Lambda}))\widetilde{\widetilde{\sqcap}}\Psi^{-1}_{\gamma\delta\lambda}((f, \widehat{f}, \acute{\Delta})) = (h, \widehat{h}, \Sigma).$$

We want to prove that, for all $\ell \in \Sigma$, $\mathcal{I}(\ell) = h(\ell)$ and, for all $\neg\ell \in \neg\Sigma$, $\widehat{\mathcal{I}}(\neg\ell) = \widehat{h}(\neg\ell)$. For a non-trivial case, let $\ell \in \delta^{-1}(\acute{\Lambda} \cap \acute{\Delta}) = \delta^{-1}(\acute{\Lambda}) \cap \delta^{-1}(\acute{\Delta})$, then

$$\begin{aligned} \mathcal{I}(\ell) &= \Psi^{-1}_{\gamma\delta\lambda}(I)(\ell) \\ &= \gamma^{-1}(g(\delta(\ell)) \cap f(\delta(\ell))) \\ &= \gamma^{-1}(g(\delta(\ell))) \cap \gamma^{-1}(f(\delta(\ell))) \\ &= \Psi^{-1}_{\gamma\delta\lambda}(g)(\ell) \cap \Psi^{-1}_{\gamma\delta\lambda}(f(\delta(\ell))) \\ &= h(\ell). \end{aligned}$$

Also, let $\neg\ell \in \lambda^{-1}(\neg\acute{\Lambda} \cap \neg\acute{\Delta}) = \lambda^{-1}(\neg\acute{\Lambda}) \cap \lambda^{-1}(\neg\acute{\Delta})$, then

$$\begin{aligned}\widehat{\mathcal{J}}(\neg\ell) &= \Psi^{-1}_{\gamma\delta\lambda}(\widehat{I})(\neg\ell)\\ &= \gamma^{-1}(\widehat{g}(\lambda(\neg\ell)) \cup \widehat{f}(\lambda(\neg\ell)))\\ &= \gamma^{-1}(\widehat{g}(\lambda(\neg\ell))) \cup \gamma^{-1}(\widehat{f}(\lambda(\neg\ell)))\\ &= \Psi^{-1}_{\gamma\delta\lambda}(\widehat{g})(\neg\ell) \cup \Psi^{-1}_{\gamma\delta\lambda}(\widehat{f}(\lambda(\neg\ell)))\\ &= \widehat{h}(\neg\ell).\end{aligned}$$

Hence, $\Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\acute{\Lambda})\widetilde{\widetilde{\sqcap}}(f,\widehat{f},\acute{\Delta})) = \Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\acute{\Lambda}))\ \widetilde{\widetilde{\sqcap}}\ \Psi^{-1}_{\gamma\delta\lambda}((f,\widehat{f},\acute{\Delta}))$.

6. Simply, let $\Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\acute{\Sigma})) = (h,\widehat{h},\Sigma)$. Let $\ell \in \Sigma$, then

$$\begin{aligned}\left(\Psi^{-1}_{\gamma\delta\lambda}(g)(\ell)\right)^c &= \left(\gamma^{-1}(g(\delta(\ell)))\right)^c\\ &= \left(h(\ell)\right)^c\\ &= \widehat{h}(\neg\ell).\end{aligned}$$

Again, $(g,\widehat{g},\acute{\Sigma})^c = (g^c,\widehat{g}^c,\acute{\Sigma})$, then

$$\begin{aligned}\Psi^{-1}_{\gamma\delta\lambda}(g^c)(\ell) &= \gamma^{-1}(g^c(\delta(\ell)))\\ &= \gamma^{-1}(\widehat{g}(\neg\delta(\ell)))\\ &= \gamma^{-1}(\widehat{g}(\lambda(\neg\ell)))\\ &= \widehat{h}(\neg\ell).\end{aligned}$$

Hence, $\left(\Psi^{-1}_{\gamma\delta\lambda}(g)(\ell)\right)^c = \Psi^{-1}_{\gamma\delta\lambda}(g)^c(\ell)$. Using the same technique, we can show that $\left(\Psi^{-1}_{\gamma\delta\lambda}(\widehat{g})(\neg\ell)\right)^c = \Psi^{-1}_{\gamma\delta\lambda}(\widehat{g})^c(\neg\ell)$ for all $\neg\ell \in \neg\Sigma$.

Therefore, $\Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\acute{\Sigma})^c) = (\Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\acute{\Sigma})))^c$. □

Remark 4.5. In Proposition 4.4 (5.), $\Psi^{-1}_{\gamma\delta\lambda}\left((g,\widehat{g},\acute{\Lambda})\widetilde{\widetilde{\sqcap}}(f,\widehat{f},\acute{\Delta}) = (h,\widehat{h},\acute{\Lambda}\cup\acute{\Delta})\right) \neq \Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\acute{\Lambda}))\ \widetilde{\widetilde{\sqcap}}\ \Psi^{-1}_{\gamma\delta\lambda}((f,\widehat{f},\acute{\Delta}))$.

Example 4.6. Consider $\Psi_{\gamma\delta\lambda}$ in Example 3.2 and $(g,\widehat{g},\acute{\Lambda})$ in Example 4.2. Let $(f,\widehat{f},\acute{\Delta}) = \{((\acute{\ell}_1,\acute{\ell}_5,\acute{\ell}_6),\{\eta_4\},\{\eta_3\}),((\acute{\ell}_2,\acute{\ell}_5,\acute{\ell}_6),\{\eta_1\},\{\eta_2\}),((\acute{\ell}_3,\acute{\ell}_5,\acute{\ell}_6),\{\eta_1\},\{\eta_3,\eta_4\}),((\acute{\ell}_4,\acute{\ell}_5,\acute{\ell}_6),\phi,\aleph)\}$, then $\Psi^{-1}_{\gamma\delta\lambda}((f,\widehat{f},\acute{\Delta})) = \{((\ell_1,\ell_5,\ell_6),\phi,\{r_3\}),\ ((\ell_2,\ell_5,\ell_6),\phi,\{r_3\}),\ ((\ell_3,\ell_5,\ell_6),\ \{r_1\},\{r_3\}),((\ell_4,\ell_5,\ell_6),\phi,\Re)\}$. Now,

$\Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\acute{\Lambda}))\widetilde{\widetilde{\sqcap}}\Psi^{-1}_{\gamma\delta\lambda}((f,\widehat{f},\acute{\Delta})) = \{((\ell_1,\ell_5,\ell_6),\phi,\Re),\ ((\ell_2,\ell_5,\ell_6),\phi,\Re),\ ((\ell_3,\ell_5,\ell_6),\{r_1\},\{r_3\}),\ ((\ell_4,\ell_5,\ell_6),\phi,\Re)\}$.

On the other hand, $(g,\widehat{g},\acute{\Lambda})\ \widetilde{\widetilde{\sqcap}}\ (f,\widehat{f},\acute{\Delta}) = (f,\widehat{f},\acute{\Delta})$, then $\Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\Lambda)\widetilde{\widetilde{\sqcap}}(f,\widehat{f},\Delta)) = \Psi^{-1}_{\gamma\delta\lambda}((f,\widehat{f},\acute{\Delta}))$.

Therefore, $\Psi^{-1}_{\gamma\delta\lambda}\left((g,\widehat{g},\acute{\Lambda})\widetilde{\widetilde{\sqcap}}(f,\widehat{f},\acute{\Delta})\right) \neq \Psi^{-1}_{\gamma\delta\lambda}((g,\widehat{g},\acute{\Lambda}))\ \widetilde{\widetilde{\sqcap}}\ \Psi^{-1}_{\gamma\delta\lambda}((f,\widehat{f},\acute{\Delta}))$.

In what follows, the bipolar hypersoft image and the bipolar hypersoft inverse image of bipolar hypersoft sets are discussed.

Proposition 4.7. *Suppose that $\Psi_{\gamma\delta\lambda} : \Omega_{(\Re,\Sigma)} \to \Omega_{(\aleph,\acute{\Sigma})}$ is a bipolar hypersoft mapping, where $\gamma : \Re \to \aleph$ is an injective mapping, $\delta : \Sigma \to \acute{\Sigma}$ and $\lambda : \neg\Sigma \to \neg\acute{\Sigma}$ are two mappings such that $\lambda(\neg\ell) = \neg\delta(\ell)$ for all $\neg\ell \in \neg\Sigma$. If $(g,\widehat{g},\Lambda) \in \Omega_{(\Re,\Sigma)}$, then $(g,\widehat{g},\Lambda)\ \widetilde{\widetilde{\sqsubseteq}}\ \Psi^{-1}_{\gamma\delta\lambda}(\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda)))$. The equality holds if $\Lambda = \Sigma$ and $\Psi_{\gamma\delta\lambda}$ is a bipolar hypersoft injective mapping.*

Proof. Let $\Psi^{-1}_{\gamma\delta\lambda}(\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda))) = \Psi^{-1}_{\gamma\delta\lambda}((f,\widehat{f},\acute{\Sigma})) = (h,\widehat{h},\Sigma)$. We want to show that, for all $\ell \in \Lambda$, $\Psi_{\gamma\delta\lambda}(g)(\ell) \subseteq \Psi_{\gamma\delta\lambda}(h)(\ell)$ and, for all $\neg\ell \in \neg\Lambda$, $\Psi_{\gamma\delta\lambda}(\widehat{h})(\neg\ell) \subseteq \Psi_{\gamma\delta\lambda}(\widehat{g})(\neg\ell)$. Let $\ell \in \Lambda$, then

$$\begin{aligned}
h(\ell) &= \Psi^{-1}_{\gamma\delta\lambda}(f)(\ell)\\
&= \gamma^{-1}(f(\delta(\ell)))\\
&= \gamma^{-1}\left(\gamma(\bigcup_{\ell\in\delta^{-1}(\delta(\ell))\cap\Lambda} g(\ell))\right)\\
&= \bigcup_{\ell\in\delta^{-1}(\delta(\ell))\cap\Lambda} \gamma^{-1}(\gamma(g(\ell)))\\
&= \bigcup_{\ell\in\delta^{-1}(\delta(\ell))\cap\Lambda} g(\ell),\ since\,\gamma\,is\,injective\,mapping\\
&\supseteq g(\ell).
\end{aligned}$$

Also, for $\neg\ell \in \neg\Lambda$, then

$$\begin{aligned}
\widehat{h}(\neg\ell) &= \Psi^{-1}_{\gamma\delta\lambda}(\widehat{f})(\neg\ell)\\
&= \gamma^{-1}(\widehat{f}(\lambda(\neg\ell)))\\
&= \gamma^{-1}\left(\gamma(\bigcap_{\neg\ell\in\lambda^{-1}(\lambda(\neg\ell))\cap\neg\Lambda} \widehat{g}(\neg\ell))\right)\\
&= \bigcap_{\neg\ell\in\lambda^{-1}(\lambda(\neg\ell))\cap\neg\Lambda} \gamma^{-1}(\gamma(\widehat{g}(\neg\ell))),\ since\,\gamma\,is\,injective\,mapping\\
&= \bigcap_{\neg\ell\in\lambda^{-1}(\lambda(\neg\ell))\cap\neg\Lambda} \widehat{g}(\neg\ell),\ since\,\gamma\,is\,injective\,mapping\\
&\subseteq \widehat{g}(\neg\ell).
\end{aligned}$$

Hence, the proof is completed. □

Remark 4.8. The equality is false in Proposition 4.7.

Example 4.9. Consider $\Psi_{\gamma\delta\lambda}$, $(g,\widehat{g},\Lambda)$, and $\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda))$ in Example 3.2. Then $\Psi^{-1}_{\gamma\delta\lambda}(\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda))) = \{((\ell_1,\ell_5,\ell_6),\{r_1,r_3\},\{r_2\}),((\ell_2,\ell_5,\ell_6),\{r_1,r_3\},\{r_2\}),$ $((\ell_3,\ell_5,\ell_6),\{r_3\},\{r_1\}),((\ell_4,\ell_5,\ell_6),\phi,\Re)\}$. Hence, $(g,\widehat{g},\Lambda) \neq \Psi^{-1}_{\gamma\delta\lambda}(\Psi_{\gamma\delta\lambda}((g,\widehat{g},\Lambda)))$.

Proposition 4.10. *Suppose that $\Psi_{\gamma\delta\lambda} : \Omega_{(\Re,\Sigma)} \to \Omega_{(\aleph,\acute{\Sigma})}$ is a bipolar hypersoft mapping, where $\gamma : \Re \to \aleph$ is a bijective mapping, $\delta : \Sigma \to \acute{\Sigma}$ and $\lambda : \neg\Sigma \to \neg\acute{\Sigma}$ are two mappings such that $\lambda(\neg\ell) = \neg\delta(\ell)$ for all $\neg\ell \in \neg\Sigma$. If $(f,\widehat{f},\acute{\Sigma}) \in \Omega_{(\aleph,\acute{\Sigma})}$, then $\Psi_{\gamma\delta\lambda}(\Psi^{-1}_{\gamma\delta\lambda}((f,\widehat{f},\acute{\Sigma}))) \,\tilde{\tilde{\sqsubseteq}}\, (f,\widehat{f},\acute{\Sigma})$. The equality holds if $\Psi_{\gamma\delta\lambda}$ is a bipolar hypersoft surjective mapping.*

Proof. Let $\Psi_{\gamma\delta\lambda}(\Psi^{-1}_{\gamma\delta\lambda}((f,\widehat{f},\acute{\Sigma}))) = \Psi_{\gamma\delta\lambda}((g,\widehat{g},\Sigma)) = (h,\widehat{h},\acute{\Sigma})$. We want to show that, for all $\acute{\ell} \in \acute{\Sigma}$, $\Psi_{\gamma\delta\lambda}(h)(\acute{\ell}) \subseteq \Psi_{\gamma\delta\lambda}(f)(\acute{\ell})$ and, for all $\neg\acute{\ell} \in \neg\acute{\Sigma}$, $\Psi_{\gamma\delta\lambda}(\widehat{f})(\neg\acute{\ell}) \subseteq \Psi_{\gamma\delta\lambda}(\widehat{h})(\neg\acute{\ell})$. Let $\acute{\ell} \in \delta(\delta^{-1}(\acute{\Sigma})) \subseteq \acute{\Sigma}$ (if $\acute{\ell} \in \acute{\Sigma} \setminus \delta(\delta^{-1}(\acute{\Sigma}))$, then $h(\ell) = \phi \subseteq g(\ell)$), then

$$\begin{aligned} h(\acute{\ell}) &= \Psi_{\gamma\delta\lambda}(g)(\acute{\ell}) \\ &= \gamma\left(\bigcup_{\ell\in\delta^{-1}(\acute{\ell})\cap\Sigma} g(\ell)\right) \\ &= \gamma\left(\bigcup_{\ell\in\delta^{-1}(\acute{\ell})} \gamma^{-1}(f(\delta(\ell)))\right) \\ &= \gamma\left(\gamma^{-1}(\bigcup_{\ell\in\delta^{-1}(\acute{\ell})} f(\delta(\ell)))\right) \\ &= \gamma\left(\gamma^{-1}(f(\acute{\ell}))\right), since\, f(\delta(\ell)) = f(\acute{\ell})\, for\, all\, \ell \in \delta^{-1}(\acute{\ell}) \\ &= f(\acute{\ell}),\ since\, \gamma\, is\, surjective\, mapping. \end{aligned}$$

Also, for $\neg\acute{\ell} \in \lambda(\lambda^{-1}(\neg\acute{\Sigma})) \subseteq \neg\acute{\Sigma}$ (if $\neg\acute{\ell} \in \neg\acute{\Sigma} \setminus \lambda(\lambda^{-1}(\neg\acute{\Sigma}))$, then $\widehat{h}(\neg\ell) = \aleph \supseteq \widehat{f}(\neg\ell)$), then

$$\begin{aligned} \widehat{h}(\neg\acute{\ell}) &= \Psi_{\gamma\delta\lambda}(\widehat{g})(\neg\acute{\ell}) \\ &= \gamma\left(\bigcap_{\neg\ell\in\lambda^{-1}(\neg\acute{\ell})\cap\neg\Sigma} \widehat{g}(\neg\ell)\right) \\ &= \gamma\left(\bigcap_{\neg\ell\in\lambda^{-1}(\neg\acute{\ell})} \gamma^{-1}(\widehat{f}(\lambda(\neg\ell)))\right) \\ &= \gamma\left(\gamma^{-1}(\bigcap_{\neg\ell\in\lambda^{-1}(\neg\acute{\ell})} \widehat{f}(\lambda(\neg\ell)))\right) \\ &= \gamma\left(\gamma^{-1}(\widehat{f}(\neg\acute{\ell}))\right), since\, \widehat{f}(\lambda(\neg\ell)) = \widehat{f}(\neg\acute{\ell})\, for\, all\, \neg\ell \in \lambda^{-1}(\neg\acute{\ell}) \\ &= \widehat{f}(\neg\acute{\ell}),\ since\, \gamma\, is\, surjective\, mapping. \end{aligned}$$

Hence, the proof is completed. □

Remark 4.11. If γ is not surjective in Proposition 4.10, then the subset relation is not true in general.

Example 4.12. Consider $\Psi_{\gamma\delta\lambda}$ in Example 3.2 and $(f, \widehat{f}, \acute{\Lambda} = \acute{\Sigma})$, $\Psi^{-1}_{\gamma\delta\lambda}((f, \widehat{f}, \acute{\Sigma}))$ in Example 4.6. Then $\Psi_{\gamma\delta\lambda}(\Psi^{-1}_{\gamma\delta\lambda}((f, \widehat{f}, \acute{\Sigma}))) = \{((\acute{\ell}_1, \acute{\ell}_5, \acute{\ell}_6), \phi, \{\eta_3\}), ((\acute{\ell}_2, \acute{\ell}_5, \acute{\ell}_6), \phi, \aleph), ((\acute{\ell}_3, \acute{\ell}_5, \acute{\ell}_6), \{\eta_1\}, \{\eta_3\}), ((\acute{\ell}_4, \acute{\ell}_5, \acute{\ell}_6), \phi, \{\eta_1, \eta_2, \eta_3\})\}$. Hence, $\Psi_{\gamma\delta\lambda}(\Psi^{-1}_{\gamma\delta\lambda}((f, \widehat{f}, \acute{\Sigma}))) \tilde{\tilde{\not\subseteq}} (f, \widehat{f}, \acute{\Sigma})$.

Remark 4.13. The equality does not hold in Proposition 4.10.

Example 4.14. Consider $\Psi_{\gamma\delta\lambda}$ in Example 3.2 but if we take $\aleph = \{\eta_1, \eta_2, \eta_3\}$ instead of $\aleph = \{\eta_1, \eta_2, \eta_3, \eta_4\}$, then γ will be a bijective mapping. Let $(f, \widehat{f}, \acute{\Sigma}) = \{((\acute{\ell}_1, \acute{\ell}_5, \acute{\ell}_6), \{\eta_1, \eta_2\}, \phi), ((\acute{\ell}_2, \acute{\ell}_5, \acute{\ell}_6), \aleph, \phi), ((\acute{\ell}_3, \acute{\ell}_5, \acute{\ell}_6), \aleph, \phi), ((\acute{\ell}_4, \acute{\ell}_5, \acute{\ell}_6), \{\eta_3\}, \{\eta_1\})\}$, then $\Psi^{-1}_{\gamma\delta\lambda}((f, \widehat{f}, \acute{\Sigma})) = \{((\ell_1, \ell_5, \ell_6), \{r_1, r_2\}, \phi), ((\ell_2, \ell_5, \ell_6), \{r_1, r_2\}, \phi), ((\ell_3, \ell_5, \ell_6), \Re, \phi), ((\ell_4, \ell_5, \ell_6), \{r_3\}, \{r_1\})\}$. Therefore, $\Psi_{\gamma\delta\lambda}(\Psi^{-1}_{\gamma\delta\lambda}((f, \widehat{f}, \acute{\Sigma}))) = \{((\acute{\ell}_1, \acute{\ell}_5, \acute{\ell}_6), \{\eta_1, \eta_2\}, \phi), ((\acute{\ell}_2, \acute{\ell}_5, \acute{\ell}_6), \phi, \aleph), ((\acute{\ell}_3, \acute{\ell}_5, \acute{\ell}_6), \aleph, \phi), ((\acute{\ell}_4, \acute{\ell}_5, \acute{\ell}_6), \{\eta_3\}, \{\eta_1\})\}$. Hence, $\Psi_{\gamma\delta\lambda}(\Psi^{-1}_{\gamma\delta\lambda}((f, \widehat{f}, \acute{\Sigma}))) \neq (f, \widehat{f}, \acute{\Sigma})$.

5. Conclusions

Throughout this study, we have introduced bipolar hypersoft mapping as well as various associated concepts and properties. Also, the definition of the bipolar hypersoft inverse image along with some of the related results are then presented. We examined, on a bipolar hypersoft set, the relationship between bipolar hypersoft image and the bipolar hypersoft inverse image. In the future, we strongly recommend applying these results and suggestions to real-life problems in decision-making and medical diagnosis, as well as examining the behavior of specific topological and algebraic concepts.

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