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Application of Neutrosophic Interval valued Goal Programming to a Supply Chain Inventory Model for Deteriorating Items with Time Dependent Demand

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Abstract. A single deteriorating product's EOQ model has been examined in the literature, where it is considered that the product deteriorate continuously but has a maximum lifespan. It has also been assumed that market demand is linearly related to time. Additionally, the credit-risk is required for the retailer to pay the purchase price is offered by the supplier. The total annual relevant cost has been demonstrated to be convex, suggesting that not only does the ideal replenishment cycle time exist, but that it is also singular. We identify the system's ideal replenishment strategy, which reduces the overall cost per unit of time. To generalize the model we used Neutrosophic triangular numbers for the parameters. Finally, an numerical example is given to illustrate the theoretical results of this model.

Keywords:EOQ model; deterioration; time dependent demand; Neutrosophic interval valued goal programming

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1. **Introduction**

In real life, certain type of products either deteriorate or become obsolete and can not serve the need of the customer for an extended period of time. For example, in the food industry items deteriorate continuously. Also, how items are stored also has an impact on the lifespan of the products. For example in the durg industry items become obsolete after a fixed period. So, deterioration impacts how well can a customer be served. A lot of literature explored inventory model with deterioration. Some recent papers including including [8–12] explored deterioration inventory models. A supply chain environment to determine retailer's optimal credit period and cycle time was consided by Mahata [29]. A two-warehouse inventory model for decaying goods having imperfect quality was considered by [28] . Mahata [30] considered supply chain inventory model for deteriorating items with maximum lifetime and partial trade credit to credit-risk customers. In the article, they showed that the total annual relevant cost is convex. An EOQ inventory model for non-instantaneous deteriorating products with advertisement and price sensitive demand under order quantity dependent trade credit was discussed in [31]. Some modified mathematical derivations of the annual total relevant cost of the inventory model with two levels of trade credit in the supply chain system was analyzed in [32]. Liao et. al. [27] studied Lot-sizing decisions for deteriorating items with two warehouses under an order-size-dependent trade credit. Abdel et. al. [2] discussed a hybrid approach of neutrosophic sets and DEMATEL method for the development in supplier selection criteria. [3] has done a case study using the integrated neutrosophic ANP and VIKOR technique to achieve sustainable supplier selection. [4] provided a Comprehensive Framework to evaluate sustainable green building indicators under an uncertain environment. [5] provided a bipolar neutrosophic multi criteria decision making technique for making a professional selection. [6] developed a hybrid multi-criteria decision making technique to evaluate thesustainable photovoltaic farms locations. [7] introduced an approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number.

In most of the articles above, the market demand is taken to be constant. Furthermore all the articles are formulated with the assumption all the data available in hand are precise. But, in real life problems the data available may not be exact. Using the arguments presented above, this paper investigate two important elements. Firstly, the market demand is taken to be a linear function of time. And secondly, the parameters are taken to be neutrosopic triangular numbers to consider the fuzziness in the data. Zadeh [26] developed the idea of fuzzy set. Bellman [21] explained the decision making in fuzzy systems. Zimmermann [22] implemented this concept for solving linear programming problem with several objective functions. Atanassov [19] introduced the concept of intuitionistic fuzzy set, where the sum of the membership degree and non-membership degreeis less than equal to one. Smarandache [25] developed the concept of neutrosophic by adding another independent membership function called as indeterminacy membership along with truth and falsity membership functions. Smarandache [14,18] introduced the idea of Neutrosophic interval valued number. Some basic properties as have been established in that paper. Ye [13] explained some basic properties and developed a linear programming method. Banerjee [17] dealt with a single objective linear goal programming model with neutrosophic numbers. [1] developed a EOQ model with trade credit model with deterioration with constant demand.

In this paper, we have developed a EOQ model where the said item deteriorates continuously with time and the demand is linearly dependent on time. Also to generalize the model we have taken demand as the triangular neutrosophic number. The rest of the manuscript is organized as follows; 2 provides some basic definitions. 3 presents the model. 4 forms the fuzzy problem, where demand is taken to be Neutrosophic triangular number. 5 provides an illustrative example. 6 gives the conclusion.

2. **Preliminaries**

2.1. *Some Definitions*

Definition 2.1 (Fuzzy Sets)**.** According to [26], a fuzzy set $\widetilde{A}$ in a universe of discourse X is defined as the ordered pairs $\widetilde{A} = \{(x, M_{\widetilde{A}}(x)) : x \in X\}$ where $M_{\widetilde{A}} : X \to [0,1]$ is a function known as the membership function of the set $\widetilde{A}$. $M_{\widetilde{A}}(x)$ is the degree of membership of x $\in X$ in the fuzzy set $\widetilde{A}$. Higher value of $M_{\widetilde{A}}(x)$ indicates a higher degree of membership in $\widetilde{A}$.

Definition 2.2 (Neutrosophic sets)**.** According to [25], let X be a universe of discourse and let $x \in X$. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity- membership function $F_A(x)$, where $T_A(x), I_A(x), F_A(x) \in (0,1), \forall x \in X$ and $0^+ \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3^-$.

Definition 2.3 (Single valued neutrosophic sets)**.** According to [24], if X is a universe of discourse and if $x \in X$, a single valued neutrosophic set A is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity- membership function $F_A(x)$, where $T_A(x), I_A(x), F_A(x) \in [0,1], \forall x \in X$ and $0 \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3$.

Definition 2.4 (Interval valued Neutrosophic number)**.** As in [18], A neutrosophic number $\alpha = a + bI$ where a is the determinate part, b is the indeterminate part and I is the indeterminacy. Here $a, b \in \Re$ and I is an real interval.

$\alpha = a + bI$, where $I = [I^l, I^u] \implies \alpha = [a + bI^l, a + bI^u] = \{x \in \Re | a + bI^l < x < a + bI^u\} = [\alpha^l, \alpha^u]$(say).

Example: Let $\alpha = 1 + 2[0.1, 0.2]$ where 1 is the determinate part and 2 is the indeterminate part. Assume that $I \in [0.1, 0.2]$, then α becomes an interval $\alpha = [1.2, 1.4]$.

2.2. *Neutrosophic interval valued linear programming*

In this section we briefly discuss neutrosophic interval valued linear programming as in [13, 17].

$$\text{Minimize } Z_n = \sum_{i=1}^{n} [c_{ni}^l, c_{ni}^u] x_i \tag{1}$$

Subject to,

$$\sum_{k=1}^{K}[a^l_{mk}, a^u_{mk}]x_k \leq [b^l_m, b^u_m] \qquad m = 1, 2, ..., M \tag{2}$$

$$x_k \geq 0 \qquad k = 1, 2, ..., K \tag{3}$$

where Z_n for n=1,2,3,...,N are objective functions, $[c^l_{ni}, c^u_{ni}]$ are the interval coefficients for the p^{th} objective function, $[a^l_{mk}, a^u_{mk}]$, $[b^l_m, b^u_m]$ are the interval coefficients of the constraints.

Accordingly in [15, 16], the constraints in 2 can be transformed into two following inequalities,

$$\sum_{k=1}^{K} a^l_{mk}x_k \leq b^u_m \qquad m = 1, 2, ..., M \tag{4}$$

$$\sum_{k=1}^{K} a^u_{mk}x_k \leq b^l_m \qquad m = 1, 2, ..., M \tag{5}$$

Therefore then minimization problem stated above can be written as,

$$\text{Minimize } Z_n = \sum_{i=1}^{n}[c^l_{ni}, c^u_{ni}]x_i \tag{6}$$

Subject to,

$$\sum_{k=1}^{K} a^l_{mk}x_k \leq b^u_m \qquad m = 1, 2, ..., M \tag{7}$$

$$\sum_{k=1}^{K} a^u_{mk}x_k \leq b^l_m \qquad m = 1, 2, ..., M \tag{8}$$

$$x_k \geq 0 \qquad k = 1, 2, ..., K \tag{9}$$

For the best possible solution, we solve the problem

$$\text{Minimize } Z_n = \sum_{i=1}^{n} c^l_{ni}x_i = Z^l(\text{say}) \tag{10}$$

Subject to,

$$\sum_{k=1}^{K} a^l_{mk}x_k \leq b^u_m \qquad m = 1, 2, ..., M \tag{11}$$

$$x_k \geq 0 \qquad k = 1, 2, ..., K \tag{12}$$

And for the worst possible solution, we solve the problem

$$\text{Minimize } Z_n = \sum_{i=1}^{n} c^u_{ni}x_i = Z^u(\text{say}) \tag{13}$$

Subject to,

$$\sum_{k=1}^{K} a_{mk}^{u} x_k \leq b_m^l \qquad m = 1, 2, ..., M \tag{14}$$

$$x_k \geq 0 \qquad k = 1, 2, ..., K \tag{15}$$

Let the best and worst possible solution respectively be $Z_n^b(x_n^b)$ and $Z_n^w(x_n^w)$. So the optimal solution lies in the interval $[Z_n^b(x_n^b), Z_n^w(x_n^w)]$. So, for the decision maker the objective function Z lies in $[Z_n^b(x_n^b), Z_n^w(x_n^w)]$. If $d^l, d^u \geq 0$ be devotional variables, then the goal achievement functions can be written as,

$$-Z^u + d^u = -Z_n^b(x_n^b) \text{ and } Z^l + d^l = Z_n^w(x_n^w) \tag{16}$$

So, the goal programming problem according to [17],

$$\text{Minimize}(d^u + d^l) \tag{17}$$

subject to,

$$-Z^u + d^u = -Z_n^b(x_n^b) \tag{18}$$

$$Z^l + d^l = Z_n^w(x_n^w) \tag{19}$$

$$\sum_{k=1}^{K} a_{mk}^{l} x_k \leq b_m^u \qquad m = 1, 2, ..., M \tag{20}$$

$$\sum_{k=1}^{K} a_{mk}^{u} x_k \leq b_m^l \qquad m = 1, 2, ..., M \tag{21}$$

$$d^l \geq 0 \tag{22}$$

$$d^u \geq 0 \tag{23}$$

$$x_k \geq 0 \qquad k = 1, 2, ..., K \tag{24}$$

(25)

3. Mathematical Model

In this manuscript, we study a supply chain system where a supplier supplies retailers with deteriorating products. Also, the give credit to pay the credit-risk of the retailer's accounts. For this we have the following notations and assumptions.

3.1. *Notations*

o per unit order cost
c per unit cost of purchasing
p per unit selling price
h per unit annual holding costs excluding interest costs

I_e interest that the retailer earns each year
I_p interest accrued annually
I(t) inventory level at any time t
$\theta(t)$ non-decreasing deterioration rate at any time t
m maximum lifetime of the products in years
M trade credit term in years set by the supplier
D time dependent demand rate per year
Q order amount in units per replenishment cycle
T the number of years in the replenishing cycle
Z(t) the total relevant yearly cost
T^* the optimal duration between replenishment cycles

3.2. *Assumptions*

(1) The item deteriorate continuously. Also, the item expires after the maximum lifetime. The deterioration rate is assumed to be closed to 1 when time approaches to the expiration date m. The deterioration rate is assumed to be same as that in [23] as follows:

$$\theta(t) = \frac{1}{1+m-t} \qquad (0 \leq t \leq T \leq m) \tag{26}$$

Clearly, $0 \leq \theta(t) \leq 1$, $\theta(m) = 1$ and $\theta^{'}(t) \geq 1$
(2) The market demand is a linear function of time as follows: $D(t) = a + bt$
(3) Shortages are not allowed.
(4) There is no delay in replenishment and also the lead time is zero.
(5) Time horizon is assumed to be infinite.
(6) The trade credit agreement is assumed to be as follow:
- The retailer initially borrows money to pay the supplier's procurement costs, after which interest charges are incurred during the time interval (0,M].
- In the event that the retailer does not settle the balance by time M, the supplier asks the retailer to pay the unpaid balance plus interest with interest rate I_p. The retailer then uses the earnings from the sale to settle the supplier's outstanding debt. Once all accounts have been settled, the retailer keeps the profit and uses sales income to earn interest for the course of the replenishment cycle (T).

3.3. *Model formulation*

At any time $t \in [0, T]$, the inventory level is depleting from the demand and deterioration. The inventory level is described by the following differential equation:

$$\begin{aligned} \frac{\mathrm{d}I(t)}{\mathrm{d}t} &= -D - \theta(t)I(t) \\ &= -(a+bt) - \frac{1}{1+m-t}I(t) \qquad (0 \leq t \leq T \leq m) \end{aligned} \tag{27}$$

With the boundary condition I(T)=0.

Solving the differential eq. 27, we get,

$$I(t) = (1+m-t)[(a+(1+m)b)\ln\left(\frac{1+m-t}{1+m-T}\right) + b(t-T)] \tag{28}$$

Furthermore, by assumption 6, the retailer takes loan to pay off the supplier. So the amount of loan the retailer has to take $\int_0^T cDdt = c[aT + b\frac{T^2}{2}]$. Then the amount of interest charged $cMI_p[aT + \frac{bT^2}{2}]$ during the time interval (0, M]. Additionally, the retailer keeps the payments and receives interest during the same time period.,i.e. ,

$$p\int_0^M (a+bt)dt + pI_e\int_0^M (a+bt)tdt = p(aM + \frac{bM^2}{2}) + pI_e(\frac{aM^2}{2} + \frac{bM^3}{3})$$

Now, if $p(aM + \frac{bM^2}{2}) + pI_e(\frac{aM^2}{2} + \frac{bM^3}{3}) \geq c[aT + b\frac{T^2}{2}]$ then the retailer succeeds in paying off the loan and keeps earning interest on the remaining balance given by,

$$p(aM + \frac{bM^2}{2}) + pI_e(\frac{aM^2}{2} + \frac{bM^3}{3}) > -c[aT + b\frac{T^2}{2}]$$

If, $p(aM + \frac{bM^2}{2}) + pI_e(\frac{aM^2}{2} + \frac{bM^3}{3}) < c[aT + b\frac{T^2}{2}]$ then the retailer fails to pay off the loan and he/she has to reduce the loan amount from sales revenue.

Additionally, the retailer encounters the following costs,

(1)

$$\text{Annual ordering cost} = \frac{o}{T} \tag{29}$$

(2)

$$\text{Annual procurement cost} = \frac{cQ}{T} = \frac{cI(0)}{T} = \frac{c(1+m)[(a+(1+m)b)\ln\left(\frac{1+m}{1+m-T}\right) + b(-T)]}{T} \tag{30}$$

(3)

Annual holding cost excluding interest charge,

$$\begin{aligned} &= \frac{h}{T}\int_0^T I(t)dt \\ &= \frac{h(a+b(1+m))}{T}[\frac{(1+m)^2}{2}\ln\left(\frac{1+m}{1+m-T}\right) + \frac{T^2}{4} - \frac{(1+m)T}{2} - \frac{bTh}{2}] \end{aligned} \tag{31}$$

In this connection two cases arise. The first case is for when the replenishment cycle is less than trade credit period and the second one is for when replenishment cycle is greater than trade credit period.

Case 1: In this case we examine the case where the retailer pays off the loan in time t=M and $T \leq M$ Here, the retailer keeps the profit and sells revenue and earns profit on it untill the the replenishment cycle time T.

The annual interest payable is given by,

$$I_p c(aM + \frac{cM^2}{2}) \tag{32}$$

Interest earned by the retailer from t=0 to t=T with an interest rate I_e is given by,

$$I_e p \int_0^T (a + bt)tdt = I_e p[\frac{aT^2}{2} + \frac{bT^3}{3}]. \tag{33}$$

Additionally, the interest earned starting from the time t=T to t=M is given by,

$$I_e[p(aT + \frac{bT^2}{2}) + pI_e(\frac{aT^2}{2} + \frac{bT^3}{3})](M - T) \tag{34}$$

Hence, the total annual interest earned is given by,

$$\frac{1}{T}[I_e p(\frac{aT^2}{2} + \frac{bT^3}{3}) + I_e(M - T)\{p(aT + \frac{bT^2}{2}) + pI_e(\frac{aT^2}{2} + \frac{bT^3}{3})\}] \tag{35}$$

So, the annual opportunity cost of capital is given by,

$$Z_1 = \frac{o}{T} + \frac{c(1+m)[(a + (1+m)b)\ln\left(\frac{1+m}{1+m-T}\right) + b(-T)]}{T} + \frac{h(a + b(1+m))}{T}[\frac{(1+m)^2}{2}\ln\left(\frac{1+m}{1+m-T}\right) + \frac{T^2}{4} - \frac{(1+m)T}{2} - \frac{bTh}{2}] + I_p c(aM + \frac{bM^2}{2}) - \frac{1}{T}[I_e p(\frac{aT^2}{2} + \frac{bT^3}{3}) + I_e(M - T)\{p(aT + \frac{bT^2}{2}) + pI_e(\frac{aT^2}{2} + \frac{bT^3}{3})\}] \tag{36}$$

With the inventory constant, $I(0) \leq I$

Case 2: In this case, we study the case when the replenishment cycle is greater than trade credit i.e. $M \leq T$. Similar to the previous case, the retailer has to pay the annual interest,

$$I_p c(aM + \frac{bM^2}{2}) \tag{37}$$

The retailer earns interest on sales revenue from t=0 to t=M and it is given by,

$$I_e p \int_0^M (a + bt)tdt = I_e p[\frac{aM^2}{2} + \frac{bM^3}{3}]. \tag{38}$$

After paying the loan interest, the retailer uses the remaining revenue to earn more interest. Since the retailer pays off in time t=M, he earns interest on the net revenue from time t=M to t=T on every replenishment cycle. So, the annual interest earned is given by,

$$I_e[(p(aM + \frac{bM^2}{2})) + pI_e(\frac{aM^2}{2} + \frac{bM^3}{3}) - c(aT + \frac{bT^2}{2})](T - M) + pI_e(\frac{a(T - m)^2}{2} + \frac{b(T - M)^3}{3}) \tag{39}$$

So, the annual opportunity cost of capital is given by,

$$Z_2 = \frac{o}{T} + \frac{c(1+m)[(a+(1+m)b)\ln\left(\frac{1+m}{1+m-T}\right) + b(-T)]}{T} + \frac{h(a+b(1+m))}{T}[\frac{(1+m)^2}{2}\ln\left(\frac{1+m}{1+m-T}\right) + \frac{T^2}{4} - \frac{(1+m)T}{2} - \frac{bTh}{2}] + I_p c(aM + \frac{bM^2}{2}) - \frac{1}{T}[I_e p\left(\frac{aM^2}{2} + \frac{bM^3}{3}\right) + I_e[(p(aM + \frac{bM^2}{2})) + pI_e(\frac{aM^2}{2} + \frac{bM^3}{3}) - c(aT + \frac{bT^2}{2})](T-M) + pI_e(\frac{a(T-m)^2}{2} + \frac{b(T-M)^3}{3})] \tag{40}$$

So we have,

$$Z(T) = \begin{cases} Z_1(T) & \text{if} \quad T \leq M \\ Z_2(T) & \text{if} \quad T \geq M \end{cases} \tag{41}$$

4. **Fuzzy Model formulation**

Sometimes it is hard to predict the market demand precisely. The approximate demand within a range may be predicted. So for generalization we form the same problem with the help of neutrosophic triangular number. We take the market demand as $D = [a^l, a^u] + [b^l, b^u]t$, where $[a^l, a^u], [b^l, b^u]$ are interval coefficients of fuzzy demand function. Here again the inventory level is described by the following differential equation:

$$\begin{aligned} \frac{\mathrm{d}I(t)}{\mathrm{d}t} &= -D - \theta(t)I(t) \\ &= -([a^l, a^u] + [b^l, b^u]t) - \frac{1}{1+m-t}I(t) \qquad (0 \leq t \leq T \leq m) \end{aligned} \tag{42}$$

Solving the differential eq. we get,

$$I(t) = (1+m-t)[([a^l, a^u] + (1+m)[b^l, b^u])\ln\left(\frac{1+m-t}{1+m-T}\right) + [b^l, b^u](t-T) \tag{43}$$

Proceeding similar way, the loan amount will be $c[[a^l, a^u]T + [b^l, b^u]\frac{T^2}{2}]$ and the interest charged will be $cMI_p[[a^l, a^u]T + [b^l, b^u]\frac{T^2}{2}]$. Here the costs the retailer encounters are,

(1)

$$\text{Annual ordering cost} = \frac{o}{T} \tag{44}$$

(2)

$$\text{Annual procurement cost} = \frac{cQ}{T} = \frac{cI(0)}{T} = \frac{c(1+m)[([a^l, a^u] + (1+m)[b^l, b^u])\ln\left(\frac{1+m}{1+m-T}\right) + [b^l, b^u](-T)]}{T} \tag{45}$$

(3)

Annual holding cost excluding interest charge,

$$= \frac{h}{T}\int_0^T I(t)dt$$
$$= \frac{h([a^l,a^u]+[b^l,b^u](1+m))}{T}[\frac{(1+m)^2}{2}\ln\left(\frac{1+m}{1+m-T}\right)+\frac{T^2}{4}-\frac{(1+m)T}{2}-\frac{[b^l,b^u]Th}{2}] \tag{46}$$

Again two cases arise.

Case 1: Similarly as the crisp cases we get the following annual opportunity cost of capital,

$$\begin{aligned}\tilde{Z}_1 =& \frac{o}{T}+\frac{c(1+m)[([a^l,a^u]+(1+m)[b^l,b^u])\ln\left(\frac{1+m}{1+m-T}\right)+[b^l,b^u](-T)]}{T}\\ &+\frac{h([a^l,a^u]+[b^l,b^u](1+m))}{T}[\frac{(1+m)^2}{2}\ln\left(\frac{1+m}{1+m-T}\right)+\frac{T^2}{4}\\ &-\frac{(1+m)T}{2}-\frac{[b^l,b^u]Th}{2}]+I_pc([a^l,a^u]M+\frac{[b^l,b^u]M^2}{2})-\frac{1}{T}[I_ep(\frac{[a^l,a^u]T^2}{2}+\frac{[b^l,b^u]T^3}{3})+I_e(M-T)\{p([a^l,a^u]\\ &+\frac{[b^l,b^u]T^2}{2})+pI_e(\frac{[a^l,a^u]T^2}{2}+\frac{[b^l,b^u]T^3}{3})\}]\end{aligned} \tag{47}$$

With the inventory constant, $I(0) \leq I$

We solve the problem, using neutrosophic goal programming method.

Case 2: Here again, as before we get the following annual opportunity cost of capital,

$$\begin{aligned}\tilde{Z}_2 =& \frac{o}{T}+\frac{c(1+m)[([a^l,a^u]+(1+m)[b^l,b^u])\ln\left(\frac{1+m}{1+m-T}\right)+[b^l,b^u](-T)]}{T}\\ &+\frac{h([a^l,a^u]+[b^l,b^u](1+m))}{T}[\frac{(1+m)^2}{2}\ln\left(\frac{1+m}{1+m-T}\right)+\frac{T^2}{4}-\frac{(1+m)T}{2}-\frac{[b^l,b^u]Th}{2}]\\ &+I_pc([a^l,a^u]M+\frac{[b^l,b^u]M^2}{2})-\frac{1}{T}[I_ep\left(\frac{[a^l,a^u]M^2}{2}+\frac{[b^l,b^u]M^3}{3}\right)+I_e[(p([a^l,a^u]M+\frac{[b^l,b^u]M^2}{2}))\\ &+pI_e(\frac{[a^l,a^u]M^2}{2}+\frac{[b^l,b^u]M^3}{3})-c([a^l,a^u]T+\frac{[b^l,b^u]T^2}{2})](T-M)\\ &+pI_e(\frac{[a^l,a^u](T-m)^2}{2}+\frac{[b^l,b^u](T-M)^3}{3})]\end{aligned} \tag{48}$$

Similarly we have,

$$\tilde{Z}(T) = \begin{cases}\tilde{Z}_1(T) & \text{if} \quad T \leq M\\ \tilde{Z}_2(T) & \text{if} \quad T \geq M\end{cases} \tag{49}$$

5. Numerical example

In this section we discuss the numerical results in two cases. In the first case the crisp model is discussed. And in the second case the fuzzy model is discussed.

Case 1: For the following example, we have taken the ordering cost 10\$ per order. Per unit cost of purchasing and selling are taken respectively 10\$,15\$. We have taken holding cost per unit per year excluding interest charge to be 1\$. We assumed the retailer earns 0.12\$ per year and the retailer pays 0.15\$ per year interest. We have the following problem,

$$\text{Minimize } \tilde{Z}(T) \tag{50}$$

$$Z(T) = \begin{cases} Z_1(T) & \text{if} \quad T \leq M \\ Z_2(T) & \text{if} \quad T \geq M \end{cases} \tag{51}$$

We have used Lingo software for solving this optimization problem.

We have the following results,

Table 1

a	b	o	c	p	h	I_e	I_p	M	m	T^*	$Z^*(T^*)$
100	.1	10	10	15	10	.12	.15	1	1	0.1071494	653.5241
105	.11	10	10	15	10	.12	.15	1	1	0.1046234	628.8297
110	.12	10	10	15	10	.12	.15	1	1	0.1022692	599.0090
95	.1	10	10	15	10	.12	.15	1	1	0.1098596	625.4176

5.1. *Numerical example(Fuzzy)*

In this section we solve the problem using the neutrosophic interval valued linear programming to solve the problem as discussed in 2.2. Again for the following example, we have taken the ordering cost 10\$ per order. Per unit cost of purchasing and selling are taken respectively 10\$,15\$. We have taken holding cost per unit per year excluding interest charge to be 1\$. We assumed the retailer earns 0.12\$ per year and the retailer pays 0.15\$ per year interest.

The problem is,

$$\text{Minimize } \tilde{Z}(T) \tag{52}$$

where,

$$\tilde{Z}(T) = \begin{cases} \tilde{Z}_1(T) & \text{if} \quad T \leq M \\ \tilde{Z}_2(T) & \text{if} \quad T \geq M \end{cases} \tag{53}$$

Similarly using Lingo program we get the following results for different values of the market demand.

By similar arguments, for different values of the score functions we get the following results,

Table 2

$[a^l, a^u]$	$[b^l, b^u]$	o	c	p	h	I_e	I_p	M	m	T^*	$Z^*(T^*)$
[103,115]	[.11,.15]	10	10	15	10	.12	.25	1	1	0.3269101	[448.55,618.64]
[101,110]	[.10,.13]	10	10	15	10	.12	.25	1	1	0.3656319	[543.78,659.13]
[99,108]	[.12,.16]	10	10	15	10	.12	.25	1	1	0.2551095	[372.84,548.5184]
[105,125]	[.09,.12]	10	10	15	10	.12	.25	1	1	0.4338459	[667.10,734.18]

So, when we use the neutrosophic interval valued number we get an range of value for the objective function rather than getting a fixed value. So, the decision maker has more freedom in choosing the approximate demand.

6. **Conclusion**

In this paper we have developed a EOQ model for a single deteriorating product which deteriorate continuously. Also, the market demand is considered to be linearly dependent of time. The retailer is given trade credit with a fixed interest. The retailer assumed to be earning interest on the profit. Since the market demand cannot be predicted precisely, the model is further generalized using neutrosophic triangular numbers for the parameters. The final model is solved using neutrosophic interval valued goal programming method. Through an example we have shown that the retailer has more freedom in choosing the approximate demand for the later case.

The model has been formed assuming the demand function is a linear function of time. For future work, the demand function can be assumed to be more complex functions of time or other parameters. Furthermore, the model can be more generalized by considering the other parameters as neutrosophic triangular number.

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