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Homomorphism and Isomorphism of Neutrosophic Over Topologized Graphs

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Abstract: In this paper introduce the homomorphism, isomorphism, weak isomorphism and co-weak isomorphism of Neutrosophic over topologized graphs. Some properties of isomorphism are introduced. The isomorphism of Neutrosophic over topologized graphs equivalnce relation, weak isomorphism of Neutrosophic over topologized graphs partial order relation and complement of Neutrosophic over topologized graphs also derived here.

Keywords: Neutrosophic over topologized graphs, homomorphsim,isomorphism, weak isomorphism and co-weak isomorphism

1 Introduction

In 1965 Zadeh [12] was invent the idea of a fuzzy set as a mathematical frame work for representing vagueness and imprecise information. Rosenfield (1975) introduced the notion of fuzzy graph [10]. Fuzzy graphs have numerous applications in diverse parts of science and engineering like broad cost communications producing, social network. Attanassov introduced the concept of intuitionstic fuzzy set as a gerealization of fuzzy sets [2]. Many researchers established and studied about fuzzy graphs and Intuitionstic fuzzy graphs in[1].Neutrosophic set proposed by Smarandache [11,13,14] is a powerful tool for dealing incomplete , inconsistance, imprecision, uncertain, false and inderminate problems in the real world whenever the fuzzy and intuitionstic fuzzy approches fail in such type of situation. Also he extended the neutrosophic set respectively to Neutrosophic Overset when some neutrosophic component is > 1, to Neutrosophic Underset when some neutrosophic component is < 0, and to Neutrosophic Offset when some neutrosophic components are off the interval [0, 1], i.e. some neutrosophic component $>$ 1 and other neutrosophic component < 0. since our real-world has numerous examples and applications of over-/under-/off-neutrosophic components[4,5,6]. Later, Narmada Devi [15,16,17,18,19,20,21,22,23] worked on new type of Neutrosophic over, Neutrosophic off graph and minimal domination via Neutrosophic over graph and Neutrosophic over topologized graph. In this paper, we introduce the notion of homomorphism and isomorphism between Neutrosophic over topologized graphs.

2 Preliminiaries

Definition 2.1. **[3]** A topologized graph is a topological space $\mathscr{X}$ such that

(i) every singleton is open or closed

(ii) $\forall x \in \mathscr{X}$, $|\eth(x)| \leq 2$, since $\eth(x)$ is denoted by the boundary of a point x.

Definition 2.2. **[16]** A single-valued *neutrosophic over set* A is defined as $A = (X, \langle \mathscr{T}(x), \mathscr{I}(x), \mathscr{F}(x)\rangle), x \in X$ such that there exist some elements in A that have atleast one neutrosophic component that is > 1 and no element has neutrosophic components that are < 0 and $\mathscr{T}(x), \mathscr{I}(x), \mathscr{F}(x) \in [0, \Omega]$, where Ω is called overlimit such that $0 < 1 < \Omega$.

Definition 2.3. **[16]** A *Neutrosophic over graph* $G = (P, Q)$ on a crisp graph G^* where P is an *neutrosophic vertex over set* on V and Q is a *neutrosophic edge over set* on E respectively such that

(i) $\mathscr{T}_Q\,(mn) \leq [\mathscr{T}_P(m) \wedge \mathscr{T}_P(n)]]$

(ii) $\mathscr{I}_Q\,(mn) \leq [\mathscr{I}_P(m) \wedge \mathscr{I}_P(n)]]$

(iii) $\mathscr{F}_Q\,(mn) \geq [\mathscr{F}_P(m) \vee \mathscr{F}_P(n)]]$ for every $mn \in E \subseteq V \times V$.

3 Homomorphism of Neutrosophic over Topologized Graphs

Definition 3.1. Let $G = (A, B)$ be a Neutrosophic over topologized graph [In short Neutrosophic over top graph] . The order of G denoted by $O(G)$ is defined as $O(G) = (O_T(G), O_I(G), O_F(G))$, where
$O_T(G) = \sum\limits_{v \in V} T_A(v)$ denotes the $T-$order of G,
$O_I(G) = \sum\limits_{v \in V} I_A(v)$ denotes the $I-$order of G,
$O_F(G) = \sum\limits_{v \in V} F_A(v)$ denotes the $F-$order of G.

Definition 3.2. Let $G = (A, B)$ be a Neutrosophic over top graph. The size of G denoted by $S(G)$ is defined as $S(G) = (S_T(G), S_I(G), S_F(G))$, where
$S_T(G) = \sum\limits_{v_i \neq v_j} T_B(v_i, v_j)$ denotes the $T-$size of G,
$S_I(G) = \sum\limits_{v_i \neq v_j} I_B(v_i, v_j)$ denotes the $I-$size of G,
$S_F(G) = \sum\limits_{v_i \neq v_j} F_B(v_i, v_j)$ denotes the $F-$size of G,

Definition 3.3. Let G_1 and G_2 be the Neutrosophic over top graphs. A homomorphism $f : G_1 \rightarrow G_2$ is a map $f : V_1 \rightarrow V_2$ which satisfies the following conditions
(a) $T_{A_1}(x_1) = T_{A_2}(f(x_1)), I_{A_1}(x_1) = I_{A_2}(f(x_1)), F_{A_1}(x_1) = F_{A_2}(f(x_1))$
(b) $T_{B_1}(x_1y_1) = T_{B_2}(f(x_1)f(y_1)), I_{B_1}(x_1y_1) = I_{B_2}(f(x_1)f(y_1)), F_{B_1}(x_1y_1) = F_{B_2}(f(x_1)f(y_1))$,
$\forall x_1 \in V_1, x_1y_1 \in E_1$

Definition 3.4. Let G_1 and G_2 be the Neutrosophic over top graphs. Isomorphism $f : G_1 \rightarrow G_2$ is a map $f : V_1 \rightarrow V_2$ which is a bijective mapping that satisfies the following conditions
(i) $T_{A_1}(x_1) = T_{A_2}(f(x_1)), I_{A_1}(x_1) = I_{A_2}(f(x_1)), F_{A_1}(x_1) = F_{A_2}(f(x_1))$,
(ii) $T_{B_1}(x_1y_1) = T_{B_2}(f(x_1)f(y_1)), I_{B_1}(x_1y_1) = I_{B_2}(f(x_1)f(y_1)), F_{B_1}(x_1y_1) = F_{B_2}(f(x_1)f(y_1))$,
$\forall x_1 \in V_1, x_1y_1 \in E_1$

Definition 3.5. Let G_1 and G_2 be the Neutrosophic over top graphs. Then a weak isomorphism $f : G_1 \to G_2$ is bijective mapping $f : V_1 \to V_2$ which satisfies the following conditions

(1) f is homomorphism

(2) $T_{A_1}(x_1) = T_{A_2}(f(x_1))$, $I_{A_1}(x_1) = I_{A_2}(f(x_1))$ and $F_{A_1}(x_1) = F_{A_2}(f(x_1)), \forall x_1 \in V_1$.

Thus, a weak isomorphism preserves the weights of the vertex but not necessarily the weight of the edges.

Theorem 3.1. For any two isomorphic Neutrosophic over top graphs their order and size are same.

Proof: If $f : G_1 \to G_2$ is an isomorphism between the Neutrosophic over top graphs G_1 and G_2 with the underlying sets V_1 and V_2 respectively.

(i) $T_{A_1}(x_1) = T_{A_2}(f(x_1)), I_{A_1}(x_1) = I_{A_2}(f(x_1))$ and $F_{A_1}(x_1) = F_{A_2}(f(x_1)), \forall x_1 \in V_1$

(ii) $T_{B_1}(x_1y_1) = T_{B_2}(f(x_1)f(y_1)), I_{B_1}(x_1y_1) \ = I_{B_2}(f(x_1)f(y_1)), F_{B_1}(x_1y_1) = F_{B_2}(f(x_1)f(y_1)), \ \ \forall x_1y_1 \in E_1$

(i) order of $G = (O_T(G_1), O_I(G_1), O_F(G_1))$

$$= \left(\sum_{x\in V} T_A(x), \sum_{x\in V} I_A(x), \sum_{x\in V} F_A(x)\right)$$

$$= \left(\sum_{x,y\in E} T_A(f(x)), \sum_{x,y\in E} I_A(f(x)), \sum_{x,y\in E} F_A(f(x))\right)$$

$$= (O_T(G_2), O_I(G_2), O_F(G_2))$$

$$= O(G_2)$$

(ii) $S = (S_T(G_1), S_I(G_1), S_F(G_1))$

$$= \left(\sum_{x_1\in V_1} T_{B_1}(x_1y_1), \sum_{x_1\in V_1} I_{B_1}(x_1y_1), \sum_{x_1\in V_1} F_{B_1}(x_1y_1)\right)$$

$$= \left(\sum_{x_1,y_1\in E_1} T_{B_2}(f(x_1), f(y_1)), \sum_{x_1,y_1\in E_1} I_{B_2}(f(x_1), f(y_1)), \sum_{x_1,y_1\in E_1} F_{B_2}(f(x_1), f(y_1))\right)$$

$$= (S_T(G_2), S_I(G_2), S_F(G_2))$$

$$= S(G_2)$$

Hence the theorem.

Theorem 3.2. Isomorphism between Neutrosophic over top graphs is an equivalence relation.

Proof: Let $G = (A, B)$, $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be Neutrosophic over top graphs with underlying sets V, V_1 and V_2 respectively.

(i) Reflexive:
Consider the identity map $f : V \to V$ such that $f(v) = v, \forall v \in V$.

This f is a bijective map satisfying $T_A(v_i) = T_A(f(v_i)), I_A(v_i) \ = I_A(f(v_i)), F_A(v_i) = F_A(f(v_i)), \forall v_i \in V$
$T_B(v_i, v_j) = T_A(f(v_i), f(v_j)), I_A(v_i, v_j) \ = I_A(f(v_i), f(v_j)), F_A(v_i, v_j) = F_A(f(v_i), f(v_j)), \forall v_i, v_j \in V$
Hence f is an isomorphism of the Neutrosophic over top graph to itself. $\therefore$ It satisfies reflexive relation.

(ii) Symmetric:
Let $f : V \to V_1$ be an isomorphism of G onto G_1, then f is a bijective map such that $f(v) = v_1, v \in V$ satisfying

$$\begin{aligned} T_A(v) &= T_A(f(v)) \\ I_A(v) &= I_A(f(v)) \\ F_A(v) &= F_A(f(v)), \forall v \in V \\ T_B(v_i, v_j) &= T_{B_1}(f(v_i), f(v_j)) \\ I_B(v_i, v_j) &= I_{B_1}(f(v_i), f(v_j)) \\ F_B(v_i, v_j) &= F_{B_1}(f(v_i), f(v_j)), \forall v_i, v_j \in V \end{aligned} \tag{3.1}$$

As f is bijective, by equation (3.1)

$$f^1(v_1) = v, \forall v_1 \in V_1 \tag{3.2}$$

using (3.2) in (3.1), we get

$$\begin{aligned} T_A(f^{-1}(v_1)) &= T_{A_1}(v_1) \\ I_A(f^{-1}(v_1)) &= I_{A_1}(v_1) \\ F_A(f^{-1}(v_1)) &= F_{A_1}(v_1), \forall v_1 \in V_1 \\ T_B(f^{-1}(v_i), f^{-1}(v_j)) &= T_{B_1}(v_{i1}), f(v_j 1)) \\ I_B(f^{-1}(v_i), f^{-1}(v_j)) &= I_{B_1}(v_{i1}), f(v_j 1)) \\ F_B(f^{-1}(v_i), f^{-1}(v_j)) &= F_{B_1}(v_{i1}), f(v_j 1)), \forall v_{i1}, v_{j1} \in V_1 \end{aligned} \tag{3.3}$$

Hence we get a 1-1, onto map $f^{-1} : V_1 \in V$, which is an isomorphism from G_1 to G

$$\text{i.e., } G \cong G^{'} \implies G^{'} \cong G$$

$\therefore$ It satisfies symmetric property.

(iii) Transitive:
Let $f : V \to V_1$ and $g : V_1 \to V_2$ be isomorphisms of the Neutrosophic over top graphs G onto G_1 and G_1 onto G_2 respectively.

Then $g \circ f$ is a 1-1 onto map from $V \to V_2$ where

$$(g \circ f)(v) = g(f(v)), \forall v \in V$$

As $f : V \to V_1$ is an isomorphism

$$f(v) = v_1, v \in V \tag{3.4}$$

$$\begin{aligned} T_A(v) &= T_{A_1}(f(v)) \\ I_A(v) &= I_{A_1}(f(v)) \\ F_A(v) &= F_{A_1}(f(v)), \forall v \in V \\ T_B(v_i, v_j) &= T_{B_1}(f(v_i), f(v_j)) \\ I_B(v_i, v_j) &= I_{B_1}(f(v_i), f(v_j)) \\ F_B(v_i, v_j) &= F_{B_1}(f(v_i), f(v_j)), \forall v_i, v_j \in V \end{aligned} \tag{3.5}$$

using equation (3.4) in equation (3.5), we have

$$\begin{aligned} T_A(v)) &= T_{A_1}(v_1) \\ I_A(v) &= I_{A_1}(v_1) \\ F_A(v)) &= F_{A_1}(v_1), \forall v \in V \end{aligned} \tag{3.6}$$

$$\begin{aligned} T_B(v_i, v_j) &= T_{B_1}(f(v_{1i}), f(v_{1j})) \\ I_B(v_i, v_j) &= I_{B_1}(f(v_{1i}), f(v_{1j})) \\ F_B(v_i, v_j) &= F_{B_1}(f(v_{1i}), f(v_{1j})), \forall v_i, v_j \in V \end{aligned} \tag{3.7}$$

As $g : V_1 \to V_2$ is an isomorphisms

$$g(v_1) = v_2, v_1 \in V_1 \tag{3.8}$$

$$\begin{aligned} T_{A1}(v_1)) &= T_{A_2}(g(v_1)) \\ I_{A1}(v_1)) &= I_{A_2}(g(v_1)) \\ F_{A1}(v_1)) &= F_{A_2}(g(v_1)), \forall v_1 \in V_1 \end{aligned} \tag{3.9}$$

$$\begin{aligned} T_{B_1}(v_{1i}, v_{1j}) &= T_{B_2}(g(v_{1i}), f(v_{1j})) \\ I_{B_1}(v_{1i}, v_{1j}) &= I_{B_2}(g(v_{1i}), f(v_{1j})) \\ F_{B_1}(v_{1i}, v_{1j}) &= F_{B_2}(g(v_{1i}), f(v_{1j})), \forall v_i, v_j \in V_1 \end{aligned} \tag{3.10}$$

Equations (3.5), (3.7) and (3.10) implies

$$\begin{aligned} T_A(v) &= T_{A_2}(g(v_1)) = T_{A_2}(g(f(v))) \\ I_A(v) &= I_{A_2}(g(f(v))) \\ F_A(v) &= F_{A_2}(g(v_1)) \end{aligned} \tag{3.11}$$

Equations (3.5), (3.8) and (3.11) implies

$$\begin{aligned} T_B(v_i, v_j) &= T_{B_2}(g(v_{1i}), g(v_{1j})) = T_{B_2}(g(f(v_i)), g(f(v_j))) \\ I_B(v_i, v_j) &= I_{B_2}(g(f(v_i)), g(f(v_j))) \\ F_B(v_i, v_j) &= F_{B_2}(g(f(v_i)), g(f(v_j))) \end{aligned} \tag{3.12}$$

Equations (3.12) and (3.13) implies

$g \circ f$ is an isomorphism between G and G'' is $G \cong G''$ i.e., isomorphism between Neutrosophic over top graphs is an equivalence relation.

Theorem 3.3. Weak isomorphism between Neutrosophic over top graphs satisfies the partial order relation.

Proof: Let $G = (A, B)$, $G' = (A', B')$, $G'' = (A'', B'')$ be Neutrosophic over top graphs with underlying sets V, V' and V'' respectively.

(i) Reflexive:

Consider the identity map $h : V \to V$ such that $h(v) = v, \forall v \in V$.

This h is a bijective map satisfying

$$\begin{aligned} T_A(v_i) &= T_A(h(v_i)) \\ I_A(v_i) &= I_A(h(v_i)) \\ F_A(v_i) &= F_A(h(v_i)), \forall v_i \in V \end{aligned}$$

$$\begin{aligned} T_B(v_i, v_j) &\leq T_B(h(v_i), h(v_j)) \\ I_B(v_i, v_j) &\leq I_B(h(v_i), h(v_j)) \\ F_B(v_i, v_j) &\leq F_B(h(v_i), h(v_j)), \forall v_i, v_j \in V \end{aligned}$$

Hence h is a weak isomorphism of the Neutrosophic over top graph to itself.

$\therefore$ it satisfies reflexive relation.

(ii) Anti symmetric:

Let h be a weak isomorphism between G and $G^{'}$ and g be a weak isomorphism between $G^{'}$ and G.

i.e., $h : V \to V^{'}$ is a bijective map such that $h(v) = v^{'}$, $v \in V$ satisfying

$$\begin{aligned} T_A(v) &= T_{A'}(h(v)) \\ I_A(v) &= I_{A'}(h(v)) \\ F_A(v) &= F_{A'}(h(v)), \forall v \in V \end{aligned}$$

$$\begin{aligned} T_B(v_i, v_j) &\leq T_{B'}(h(v_i), h(v_j)) \\ I_B(v_i, v_j) &\leq I_{B'}(h(v_i), h(v_j)) \\ F_B(v_i, v_j) &\leq F_{B'}(h(v_i), h(v_j)), \forall v_i, v_j \in V \end{aligned} \tag{3.13}$$

and $g : V^{'} \to V$ is a bijective map satisfying

$$\begin{aligned} T_{A'}(v^{'}) &= T_A(g(v^{'})) \\ I_{A'}(v^{'}) &= I_A(g(v^{'})) \\ F_{A'}(v^{'}) &= F_A(g(v^{'})), \forall v^{'} \in V^{'} \end{aligned}$$

$$\begin{aligned} T_{B'}(v_i^{'}, v_j^{'}) &\leq T_B(h(v_i^{'}), h(v_j^{'})) \\ I_{B'}(v_i^{'}, v_j^{'}) &\leq I_B(h(v_i^{'}), h(v_j^{'})) \\ F_{B'}(v_i^{'}, v_j^{'}) &\leq F_B(h(v_i^{'}), h(v_j^{'})), \forall v_i^{'}, v_j^{'} \in V^{'} \end{aligned} \tag{3.14}$$

The inequalities (3.13) and (3.14) hold good on the finite sets V $V^{'}$ only when G and $G^{'}$ have the same number of edges and the corresponding edges have same weights.

Hence G and $G^{'}$ are identical.

(iii) Transitive:

Let $h : V \to V^{'}$ and $g : V^{'} \to V^{''}$ be weak isomorphism of the Neutrosophic over top graphs G onto $G^{'}$ and $G^{'}$ onto $G^{''}$ respectively.

Then $g \circ h$ is a 1-1, onto map from $V \to V^{''}$ where $(g \circ h)(v) = g(h(v)), v \in V$.

As $h : V \to V^{'}$ is a weak isomorphism $h(v) = v^{'}, v \in V$.

$$\begin{aligned} T_A(v) &= T_{A'}(h(v)) \\ I_A(v) &= I_{A'}(h(v)) \\ F_A(v) &= F_{A'}(h(v)), \forall v \in V \end{aligned}$$

$$\begin{aligned} T_B(v_i, v_j) &\leq T_{B'}(h(v_i), h(v_j)) \\ I_B(v_i, v_j) &\leq I_{B'}(h(v_i), h(v_j)) \\ F_B(v_i, v_j) &\leq F_{B'}(h(v_i), h(v_j)), \forall v_i, v_j \in V \end{aligned} \tag{3.15}$$

As $g : V^{'} \to V^{''}$ is a weak isomorphism $g(v^{'}) = v^{''}, \forall v^{'} \in V^{'}$.

$$\begin{aligned} T_{A'}(v^{'}) &= T_{A''}(h(v^{'})) \\ I_{A'}(v^{'}) &= I_{A''}(h(v^{'})) \\ F_{A'}(v^{'}) &= F_{A''}(h(v^{'})), \forall v^{'} \in V \end{aligned}$$

$$\begin{aligned} T_{B'}(v_i, v_j) &\leq T_{B''}(h(v_i^{'}), h(v_j^{'})) \\ I_{B'}(v_i, v_j) &\leq I_{B''}(h(v_i^{'}), h(v_j^{'})) \\ F_{B'}(v_i, v_j) &\leq F_{B''}(h(v_i^{'}), h(v_j^{'})), \forall v_i^{'}, v_j^{'} \in V^{'} \end{aligned} \tag{3.16}$$

Equation (3.15) and (3.16) implies

$$\begin{aligned} T_A(v) &= T_{A''}(g(v^{'})) = T_{A''}(g(h(v))) \\ I_A(v) &= I_{A''}(g(v^{'})) = I_{A''}(g(h(v))) \\ F_A(v) &= F_{A''}(g(v^{'})) = F_{A''}(g(h(v))), \forall v^{'} \in V \end{aligned} \tag{3.17}$$

$$\begin{aligned} T_B(v_i, v_j) &\leq T_{B''}(g(v_i^{'}), g(v_j^{'})) = T_{B''}(g(h(v_i)), g(h(v_j))) \\ I_B(v_i, v_j) &\leq I_{B''}(g(v_i^{'}), g(v_j^{'})) = I_{B''}(g(h(v_i)), g(h(v_j))) \\ F_B(v_i, v_j) &\leq F_{B''}(g(v_i^{'}), g(v_j^{'})) = F_{B''}(g(h(v_i)), g(h(v_j))) \end{aligned} \tag{3.18}$$

Equations (3.17) & (3.18) implies $g \circ h$ is a weak isomorphism between G & $G^{''}$.

i.e., weak isomorphism satisfies transitity.

(i), (ii) & (iii) implies weak isomorphism between Neutrosophic over top graphs is partial relation.

4 Isomorphic Neutrosophic Over topologized graphs and their complements

Definition 4.1. The complement of a Neutrosophic over top graph $G = (A, B)$ is a Neutrosophic over top graph $\overline{G} = (\overline{A}, \overline{B})$, where

(1) $\overline{V} = V$

(2) $\overline{T}_A(v_i) = T_A(v_i)$
$\overline{I}_A(v_i) = I_A(v_i)$
$\overline{F}_A(v_i) = F_A(v_i)$

(3) $\overline{T}_B(v_i, v_j) = \begin{cases} \min[T_A(v_i), T_A(v_j)], & \text{if } T_B(v_i, v_j) \\ \min[T_A(v_i), T_A(v_j)] - T_B(v_i, v_j), & \text{if } T_B(v_i, v_j) > 0 \end{cases}$

$\overline{I}_B(v_i, v_j) = \begin{cases} \min[I_A(v_i), I_A(v_j)], & \text{if } I_B(v_i, v_j) \\ \min[I_A(v_i), I_A(v_j)] - I_B(v_i, v_j), & \text{if } I_B(v_i, v_j) > 0 \end{cases}$

$\overline{F}_B(v_i, v_j) = \begin{cases} \max[F_A(v_i), F_A(v_j)], & \text{if } F_B(v_i, v_j) \\ \max[F(v_i), F_A(v_j)] - F_B(v_i, v_j), & \text{if } F_B(v_i, v_j) > 0 \end{cases}$

$\forall v_i, v_j \in V$

Example 4.1. Consider a Neutrosophic over top graph $G = (A, B)$ on the non-empty set $V = \{v_1, v_2, v_3, v_4\}$ $E = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$. Neutrosophic over top graph $G = (A, B)$ and complement Neutrosophic over top graph $\overline{G} = (\overline{A}, \overline{B})$.

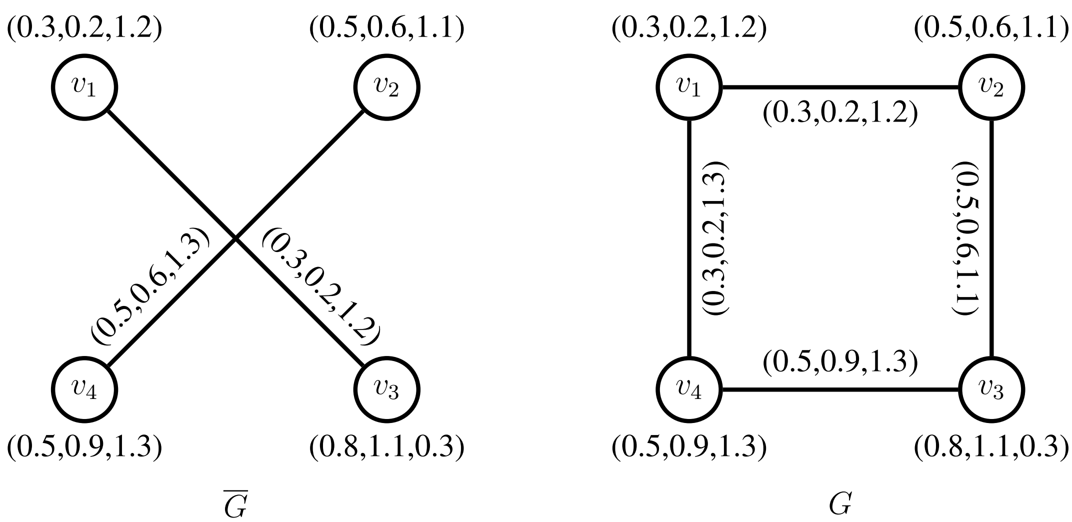


Figure 1: Neutrosophic over top graph G and its complement $\overline{G}$

Theorem 4.1. If two Neutrosophic over top graphs are isomorphic then their complements are isomorphic.

Proof: Let $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be the two Neutrosophic over top graphs. Assume $G_1 \cong G_2$. There exists a bijective map $h : V_1 \to V_2$ satisfying

$$T_{A_1}(v) = T_{A_2}(h(v))$$

$$\begin{aligned} I_{A_1}(v) &= I_{A_2}(h(v)) \\ F_{A_1}(v) &= F_{A_2}(h(v)), \forall v \in V \\ T_{B_1}(v_i, v_j) &= T_{B_2}(h(v_i), h(v_j)) \\ I_{B_1}(v_i, v_j) &= I_{B_2}(h(v_i), h(v_j)) \\ F_{B_1}(v_i, v_j) &= F_{B_2}(h(v_i), h(v_j)), \forall v_i, v_j \in V \end{aligned}$$

By definition

$$\begin{aligned} \overline{T}_{B_1}(v_i, v_j) &= \min[T_{A_1}(v_i), T_{A_1}(v_j)] - T_{B_1}(v_i, v_j) \\ &= \min[T_{A_2}(h(v_i)), T_{A_2}(h(v_j))] - T_{B_2}(h(v_i), h(v_j)) \\ &= \overline{T}_{B_2}(h(v_i), h(v_j)), \forall v_i, v_j \in V \\ \overline{I}_{B_1}(v_i, v_j) &= \overline{I}_{B_2}(h(v_i), h(v_j)), \forall v_i, v_j \in V \\ \overline{F}_{B_1}(v_i, v_j) &= \overline{F}_{B_2}(h(v_i), h(v_j)), \forall v_i, v_j \in V \end{aligned}$$

Hence $\overline{G}_1 \cong \overline{G}_2$.

Theorem 4.2. If $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ are weak isomorphic, then $\bar{G}_2 = (\bar{A}_2, \bar{B}_2)$ and $\bar{G}_1 = (\bar{A}_1, \bar{B}_1)$ are also weak isomorphic.

Proof: If h is a weak isomorphic between G_1 & G_2 then $h : V_1 \to V_2$ is a one-one-onto mapping and $h(v) = v_1, v \in V$

$$\begin{aligned} T_{A_1}(v) &= T_{A_2}(h(v)) \\ I_{A_1}(v) &= I_{A_2}(h(v)) \\ F_{A_1}(v) &= F_{A_2}(h(v)), \forall v \in V \end{aligned} \tag{4.1}$$

$$\begin{aligned} T_{B_1}(v_i, v_j) &= T_{B_2}(h(v_i), h(v_j)) \\ I_{B_1}(v_i, v_j) &= I_{B_2}(h(v_i), h(v_j)) \\ F_{B_1}(v_i, v_j) &= F_{B_2}(h(v_i), h(v_j)), \forall v_i, v_j \in V \end{aligned} \tag{4.2}$$

Since $h^{-1} : V_2 \to V_1$ is also one-one and onto for every v in V_2 there is a $v \in V_1$ such that $h^{-1}(v_1) = V$.
By equation number (4.2),
we have

$$\begin{aligned} T_{A_2}(v) &= T_{A_1}(h^{-1}(v)) \\ I_{A_2}(v) &= I_{A_1}(h^{-1}(v)) \\ F_{A_2}(v) &= F_{A_1}(h^{-1}(v)), \forall v \in V \end{aligned} \tag{4.3}$$

$$\begin{aligned} T_{B_1}(v_i, v_j) &= \min[T_{A_1}(v_i), T_{A_1}(v_j)] - T_{B_1}(v_i, v_j) \\ &= \min[T_{A_2}(h(v_i)), T_{A_2}(h(v_j))] - T_{B_2}(h(v_i), h(v_j)) \\ &= \bar{T}_{B_2}(h(v_i), h(v_j))_{,i}, v_j \in V \end{aligned}$$

Similarly

$$\begin{aligned} \bar{I}_{B_1}(v_i, v_j) &= \bar{I}_{B_2}(h(v_i), h(v_j))_{,i}, v_j \in V \\ \bar{F}_{B_1}(v_i, v_j) &= \bar{F}_{B_2}(h(v_i), h(v_j))_{,i}, v_j \in V \end{aligned}$$

Definition 4.2. Let G_1 and G_2 be Neutrosophic over top graphs. A co-weak isomorphism $f : G_1 \to G_2$ is bijective mapping $f : V_1 \to V_2$ which satisfies the following conditions

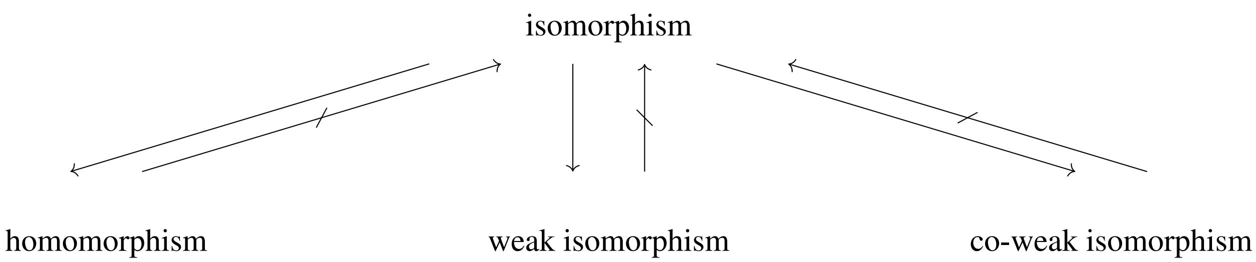
(i) f is homomorphism

(ii) $T_{B_1}(x_1y_1) = T_{B_2}(f(x_1), f(y_1))$
$I_{B_1}(x_1y_1) = I_{B_2}(f(x_1), f(y_1))$
$F_{B_1}(x_1y_1) = F_{B_2}(f(x_1), f(y_1))$

Thus, a co-weak isomorphism preserves the weight of the arcs but not necessarily the weights of the nodes.

Remark 4.1.
1. If $G_1 = G_2 = G$, then the homomorphism f over itself is called an endomorphism. An isomorphism f over G is called an automorphism.
2. If $G_1 = G_2$, then the weak and co-weak isomorphism actually become isomorphic.
3. If $V_1 \to V_2$ is a bijective map, then $f^{-1} : V_1 \to V_2$ is also a bijective map.

Remark 4.2. The interrelationship among Neutrosophic over top graphs as given below



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