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Abstract. In this paper, the priority disciplined queuing models are investigated under neutrosophic environment. It develops and optimizes a model with non-preemptive priorities system, denoted by $NM/NM/1$. It is a queuing model where the arrivals follow a Poisson process, service times are exponentially distributed and there is only one server whose arrival rate and service rate are represented in terms of single valued trapezoidal Neutrosophic number (SVTNN). Using (α, β, γ)-cut approach and Zadehs extension principle, the Neutrosophic queuing model is reduced to a crisp model and results are discussed. An illustrative example is provided to understand the analytical procedure developed in this paper.

Keywords: Neutrosophic set; single value trapezoidal Neutrosophic number; Neutrosophic Markov chain; priority queue.

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1. **Introduction**

Basic queueing systems involve organized queues where the arrival rate of customers is in an order and waiting discipline is ensured. But in real life situations most of the queuing models require priority discipline as most urgent work has to be given preference. Priority queueing models are useful in a variety of different applications. In priority queues customers are served based on their service priorities. The high-priority customers with high urgency are served first and the lower priority customers are served with less urgency. In communication engineering, priority queues are used to study networks with differentiated levels of quality of service. Steady state distribution of single server priority queue was developed by Miller [1]. Prade [2] dealt with fuzzy service time and fuzzy service rule in a queuing problem with application. Li et al. [3] investigated two fuzzy queues denoted by $M/F/1$ and $FM/FM/1$ whose interarrival time and service rate are fuzzified. Negi et al. [4] discussed analytical and simulation results of fuzzy and probability approaches of traditional queuing models. Maria et al. [5] developed two fuzzy queueing models with priority-discipline both with non-pre-emptive priorities system and pre-emptive priorities system. Varadharajan et al. [6] analysed fuzzy priority discipline queue models using a parametric programming approach.

Kalpana et al. [7] investigated the performance measures for non-pre-emptive priority fuzzy queues. Usha et al. [8] made an interpretation of a non-pre-emptive priority queueing system in fuzzy environment with asymmetrical service rates. Aarthi et al. [9] analyzed the performance of a non-pre-emptive intuitionistic fuzzy queuing model. Khudr Al-Kridi et al. [10] discussed the performance measures of $FM/FM/1$ queueing model where both arrival and departure rates are fuzzy numbers Kumuthavalli et al. [11] focused on developing a neutrosophic probability for solving queue operation in the real standard domain.

Fariborz Jolai et al. [12] presented a new formulation for the problem of fuzzy priority assignment and buffer control. Mohamed Bisher Zeina [13] provided Neutrosophic Littles Formulas which is a main tool in queueing systems problems under neutrosophic environment. Also he [14, 15] discussed about Erlang service queueing model under neutrosophic environment. Heba Rashad et al. [16] discussed the performance measures of $NM/NM/1, NM/NM/s$, and $NM/NM/1/b$ queueing models. Zhivko Tomov et al. [17] proposed generalized net models of different queueing disciplines in queueing systems. Buckley [18, 19] dealt fuzzy queue model using possibility theory. Many researchers [20, 21], have shown light over Intuitionistic fuzzy queueing models.

Florentin Smarandache [22] introduced Neutrosophic set as an generalization of Intutionistic fuzzy set developed by Atanassov [23] which is a powerful tool to deal with ambiguity compared to fuzzy set proposed by Zadeh [24] as it considers membership, indeterminacy and non-membership degree of an object simultaneously. Also Florentin Smarandache [25, 26] has explored various concepts such as Neutrosophic measure, Neutrosophic logic, Neutrosophic probability etc.,. Wang et al. [27] discussed about operations and properties of single valued Neutrosophic set (SVNS). Later applications involving SVNS are considered by many researchers [28, 29]. This paper aims at investigating a single server queuing model with priority discipline involving SVNS. A comparison table 1 of existing queueing model under uncertainty is discussed below.

Table 1. Comparison with the existing queueing model

Author	Queueing model	Uncertainty used	Methodology
Prade, H. M (1980)	General queuing model	Fuzzy sets	Zadehs extension principle
Li, R. J. et al. (1989)	General queuing model	Fuzzy sets	Zadehs extension principle
Negi, D. S. et al. (1992)	General queuing model	Fuzzy sets	-cut approach
Khudr Al-Kridi et al. (2018)	General queuing model	Fuzzy sets	Zadehs extension principle
Zhivko Tomov (2019)	General queuing model	Intuitionistic fuzzy set	Generalized Net models
Kumuthavalli et al. (2017)	General queuing model	Neutrosophic sets	Zadehs Exclusion Principle
Mohamed Bisher Zeina (2020)	General queuing model	Neutrosophic sets	Neutrosophic Littles Formulas
Mohamed Bisher Zeina (2020)	Erlang service queueing model	Neutrosophic sets	Neutrosophic statistical interval method
Maria Jose Pardoa et al. (2007)	Priority queues	Fuzzy sets	Zadehs extension principle
Varadharajan et al. (2018)	Priority queues	Fuzzy sets	α-cut approach
Kalpana et al. (2018)	Priority queues	Fuzzy sets	LR method
Usha Prameela et al.(2021)	Priority queue	Fuzzy sets	α-cut approach
Aarthi et al. (2022)	Priority queue	Intuitionistic Fuzzy sets	Ranking method
Fariborz Jolai et al. (2016)	Multi objective priority queue	Fuzzy sets	Fuzzy Data Envelopment Analysis
Heba Rashad et al. (2021)	General queueing model	Neutrosophic sets	Neutrosophic Littles Formulas
Proposed	Priority model	Neutrosophic sets	(α, β, γ)-cut

In this paper, we explored the neutrosophic queueing model and its application. To the best of the authors knowledge, none of the previous works has addressed the neutrosophic decision-making regarding prioritization and queue selection of service-needing people in disaster aftermath. The main contributions of the study include:

(1) The innovative concept of priority queuing model under neutrosophic sets is introduced.

(2) Formulation of $NM/NM/1$ queue with priority model is proposed.

(3) Also, a numerical example is discussed to show the effectiveness of the proposed queueing model.

(4) To make the decision maker understand the solution graphical representation are provided.

In Section 2, we discusses the Neutrosophic preliminaries. Section 3 briefly discussed the neutrosophic queueing model. In section 4, numerical illustration are solved for showing performance measures of neutrosophic in queueing model and Section 5 presents the conclusion, and future work.

2. Preliminaries

Definition 2.1. [26] A neutrosophic set N is given as

$$N = \{(s, \mathcal{T}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(s))/s \in s\}$$

where $\mathcal{T}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(s) : s \to]0^-, 1^+[$ are the degree of truth, ondeterminancy and falsity such that $0^- \leq \sup \mathcal{T}_{\mathcal{A}}(s) + \sup \mathcal{I}_{\mathcal{A}}(s) + \sup \mathcal{F}_{\mathcal{A}}(s) \leq 3^+$.

Definition 2.2. [26] A single valued neutrosophic set (SVNS) N in s is stated as

$$N = \{(s, \mathcal{T}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(s))/s \in s\}$$

where $\mathcal{T}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(s) \in [0,1]$ and $0 \leq \sup \mathcal{T}_{\mathcal{A}}(s) + \sup \mathcal{I}_{\mathcal{A}}(s) + \sup \mathcal{F}_{\mathcal{A}}(s) \leq 3$.

Definition 2.3. [25] Let $(\nu\Omega, NF, NP)$ be a neutrosophic probability space, where $\nu\Omega$ is a neutrosophic sample space, NF is a neutrosophic event space, and NP is a neutrosophic probability measure.The following neutrosophic probability axioms are as follows
(i) The neutrosophic probability of an event A
$NP(A) = \left(ch(A), ch(indeterm_A), ch(\bar{A})\right)$,
where $ch(A) \geq 0, ch(indeterm_A) \geq 0, ch(\bar{A}) \geq 0$, for any $A \in NF$; with the notations that $indeterm(A)$ means indeterminacy related to event A and $\bar{A}$ is the complement event of A (the $antiA$ event).
(ii) The neutrosophic probability of the sample space is between -0 and 3^+.
$NP(\nu\Omega) = \left(\sum_{x\in\nu\Omega} ch(x), ch(indeterm_{\nu\Omega}), ch(anti\nu\Omega)\right)$,
where $-0 \leq \sum_{x\in\nu\Omega} ch(x), ch(indeterm_{\nu\Omega}), ch(anti\nu\Omega) \leq 3^+$,
with the notation $indeterm_{\nu\Omega}$ means total indeterminacy that may occur in the neutrosophic sample space. For the classical complete (normalized) sample space, $ch(anti\nu\Omega) = 0$, but for incomplete sample space $ch(anti\nu\Omega) > 0$.
(iii) The neutrosophic σ-additivity is defined as

$NP\left(A_1 \cup A_2 \cup \ldots\right) = \left(\sum_{j=1}^{\infty} ch(A_j), ch(indeterm_{A_1 \cup A_2 \cup \ldots}), ch(\overline{A_1 \cup A_2 \cup \ldots})\right),$
where $A_1, A_2, \ldots$ is a countable sequence of disjoint neutrosophic events.

Definition 2.4. [25] A random variable (r.v) which have an indeterminate outcome is said to be neutrosophic r.v.

A neutrosophic stochastic process is a collection of neutrosophic r.v which represents the evolution over time of some neutrosophic random values.

Definition 2.5. [25] A neutrosophic stochastic process $\{X(n) : n \in \mathbb{N}\}$ is said to be a neutrosophic Markov chain if it satisfies the Markov property:
$P\left(X_{n+1} = j/X_n = i, X_{n-1} = k, \ldots X_0 = m\right) = P\left(X_{n+1} = j/X_n = i\right)$
where i, j, k establish the state space S of the process.
Here $\widetilde{P}_{ij} = P\left(X_{n+1} = j/X_n = i\right)$ are called the neutrosophic probabilities of moving from state i to state j in one step. Hence $\widetilde{P}_{ij} = \left(\mathcal{T}_{\widetilde{P}_{ij}}, \mathcal{I}_{\widetilde{P}_{ij}}, \mathcal{F}_{\widetilde{P}_{ij}}\right)$, where $\mathcal{T}_{\widetilde{P}_{ij}}(\mathcal{I}_{\widetilde{P}_{ij}}, \mathcal{F}_{\widetilde{P}_{ij}})$ is the truth (indeterminate, falsity) membership of the transition from state i to state j. The matrix $P = \widetilde{P}_{ij}$ is called the neutrosophic transition probability matrix.

Definition 2.6. [30] A single valued trapezoidal neutrosophic number (SVTNN) $\mathcal{A}$ is defined as follows

$$\mathcal{T}_{\mathcal{A}}(s) = \begin{cases} \dfrac{s^{\mathcal{T}} - t_1^{\mathcal{T}}}{t_2^{\mathcal{T}} - t_1^{\mathcal{T}}} & \text{for } t_1^{\mathcal{T}} \leq s^{\mathcal{T}} \leq t_2^{\mathcal{T}} \\ 1 & \text{for } t_2^{\mathcal{T}} \leq s^{\mathcal{T}} \leq t_3^{\mathcal{T}} \\ \dfrac{t_4^{\mathcal{T}} - s^{\mathcal{T}}}{t_4^{\mathcal{T}} - t_3^{\mathcal{T}}} & \text{for } t_3^{\mathcal{T}} \leq s^{\mathcal{T}} \leq t_4^{\mathcal{T}} \\ 0 & \text{otherwise} \end{cases}$$

where $t_1^{\mathcal{T}} \leq t_2^{\mathcal{T}} \leq t_3^{\mathcal{T}} \leq t_4^{\mathcal{T}}$.

$$\mathcal{I}_{\mathcal{A}}(s) = \begin{cases} \dfrac{t_2^{\mathcal{I}} - s^{\mathcal{I}}}{t_2^{\mathcal{I}} - t_1^{\mathcal{I}}} & \text{for } t_1^{\mathcal{I}} \leq s^{\mathcal{I}} \leq t_2^{\mathcal{I}} \\ 1 & \text{for } t_2^{\mathcal{I}} \leq s^{\mathcal{I}} \leq t_3^{\mathcal{I}} \\ \dfrac{t_4^{\mathcal{I}} - s^{\mathcal{I}}}{t_4^{\mathcal{I}} - t_3^{\mathcal{I}}} & \text{for } t_3^{\mathcal{I}} \leq s^{\mathcal{I}} \leq t_4^{\mathcal{I}} \\ 1 & \text{otherwise} \end{cases}$$

where $t_1^{\mathcal{I}} \leq t_2^{\mathcal{I}} \leq t_3^{\mathcal{I}} \leq t_4^{\mathcal{I}}$.

$$\mathcal{F}_{\mathcal{A}}(s) = \begin{cases} \dfrac{t_2^{\mathcal{F}} - s^{\mathcal{F}}}{t_2^{\mathcal{F}} - t_1^{\mathcal{F}}} & \text{for } t_1^{\mathcal{F}} \leq s^{\mathcal{F}} \leq t_2^{\mathcal{F}} \\ 1 & \text{for } t_2^{\mathcal{F}} \leq s^{\mathcal{F}} \leq t_3^{\mathcal{F}} \\ \dfrac{t_4^{\mathcal{F}} - s^{\mathcal{F}}}{t_4^{\mathcal{F}} - t_3^{\mathcal{F}}} & \text{for } t_3^{\mathcal{F}} \leq s^{\mathcal{F}} \leq t_4^{\mathcal{F}} \\ 1 & \text{otherwise} \end{cases}$$

where $t_1^{\mathcal{F}} \leq t_2^{\mathcal{F}} \leq t_3^{\mathcal{F}} \leq t_4^{\mathcal{F}}$.

Definition 2.7. [30]
(α,β,γ)-cut of a TSVNN is defined as follows:
$\mathcal{A}_{\alpha,\beta,\gamma} = [A_1(\alpha), A_2(\alpha)]\,; \left[A_1^{'}(\beta), A_2^{'}(\beta)\right]; \left[A_1^{''}(\gamma), A_2^{''}(\gamma)\right], 0 \leq \alpha+\beta+\gamma \leq 3$, where

$$[A_1(\alpha), A_2(\alpha)] = \left[\left(t_1^{\mathcal{T}} + \alpha(t_2^{\mathcal{T}} - t_1^{\mathcal{T}})\right), \left(t_4^{\mathcal{T}} - \alpha(t_4^{\mathcal{T}} - t_3^{\mathcal{T}})\right)\right],$$
$$\left[A_1^{'}(\beta), A_2^{'}(\beta)\right] = \left[\left(t_2^{\mathcal{I}} - \beta(t_2^{\mathcal{I}} - t_1^{\mathcal{I}})\right), \left(t_3^{\mathcal{I}} + \beta(t_4^{\mathcal{I}} - t_3^{\mathcal{I}})\right)\right],$$
$$\left[A_1^{''}(\gamma), A_2^{''}(\gamma)\right] = \left[\left(t_2^{\mathcal{F}} - \gamma(t_2^{\mathcal{F}} - t_1^{\mathcal{F}})\right), \left(t_3^{\mathcal{F}} + \gamma(t_4^{\mathcal{F}} - t_3^{\mathcal{F}})\right)\right].$$

Definition 2.8. [32] Let $[r_1, r_2]$ and $[r_3, r_4]$ be two closed and bounded real intervals. If $*$ denotes addition, substraction, multiplication or division, then $[r_1, r_2] * [r_3, r_4] = [\alpha, \beta]$.
For division, it is assumed that $0 \notin [r_3, r_4]$. With basic operations, is developed as follows :

i . $[r_1, r_2] + [r_3, r_4] = [r_1 + r_3, r_2 + r_4]$
ii . $[r_1, r_2] - [r_3, r_4] = [r_1 - r_4, r_2 - r_3]$
iii . $[r_1, r_2]\,.\,[r_3, r_4] = [min\{r_1r_3, r_1, r_4, r_2r_3, r_2r_4\}, max\{r_1r_3, r_1, r_4, r_2r_3, r_2r_4\}]$
iv . $\dfrac{[r_1, r_2]}{[r_3, r_4]} = \left[min\left\{\dfrac{r_1}{r_3}, \dfrac{r_1}{r_4}, \dfrac{r_2}{r_3}, \dfrac{r_2}{r_4}\right\}, max\left\{\dfrac{r_1}{r_3}, \dfrac{r_1}{r_4}, \dfrac{r_2}{r_3}, \dfrac{r_2}{r_4}\right\}\right]$

3. The Neutrosophic Queueing Model

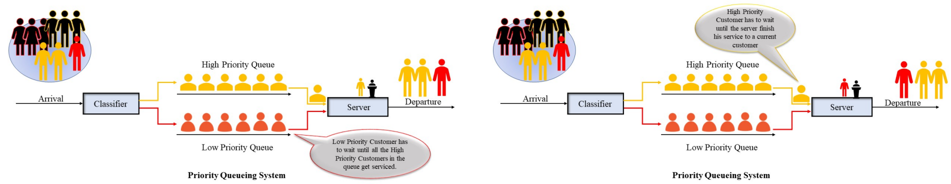
In this section, we analyze a single server queue with priority in neutrosophic environment.

3.1. Classical $M/M/1$ queue with priority queue

We consider a single server queue with priority. Assume that there are two arrival stream of customers called higher priority and low priority customers and they follow different Poisson process with parameters λ_1 and λ_2 respectively. A single server provides service to these customers and the service time follows exponential distribution with rate μ. The higher priority customers have the right to be served ahead of the others without preemption. The system capacity is infinite and within the priority group the first come first served rule is applied. Some system performance are

- Average queue length of higher priority: $L_{q_1} = \dfrac{\rho.\lambda_1}{\mu - \lambda_1}$
- Average queue length of low priority: $L_{q_2} = \dfrac{\rho.\lambda_2}{(1-\rho)(\mu - \lambda_1)}$
- Average waiting time of higher priority queue: $W_{q_1} = \dfrac{\rho}{\mu - \lambda_1}$
- Average waiting time of low priority queue: $W_{q_2} = \dfrac{\rho}{(\mu - \lambda)(\mu - \lambda_1)}$

where $\lambda = \lambda_1 + \lambda_2$ and traffic intensity $(\rho)= \dfrac{\lambda}{\mu}$.
An $M/M/1$ priority queue with infinite capacity as depicted in figure 1.



(A) Higher priority customers in service (B) Low priority customers in service

FIGURE 1. $M/M/1$ queue with priority queue

3.2. Formulation of $NM/NM/1$ queue with priority model

Consider a single server $NM/NM/1$ queueing system with priority. The neutrosophic interarrival times $\widetilde{A}_i, i = 1, 2$ of units in the first and second priority, neutrosophic service time $\widetilde{S}$ are approximately known and are represented by the follows

$$\widetilde{A_i} = \left\{ \left(a, \mathcal{T}_{\widetilde{A}_i}(a), \mathcal{I}_{\widetilde{A}_i}(a), \mathcal{F}_{\widetilde{A}_i}(a) \right) / a \in X \right\};\ i = 1, 2$$

$$\widetilde{S} = \left\{ \left(s, \mathcal{T}_{\widetilde{S}}(s), \mathcal{I}_{\widetilde{S}}(s), \mathcal{F}_{\widetilde{S}}(s) \right) / s \in Y \right\}$$

where X and Y are crisp universal sets of the neutrosophic interarrival times and neutrosophic service time and $\mu_{\widetilde{A}_i}(a); i = 1, 2, \mathcal{T}_{\widetilde{S}}(s)$ are the corresponding membership functions. The (α, β, γ)-cut of $\widetilde{A}_i; i = 1, 2$ and $\widetilde{S}$ are

$$A_i(\alpha, \beta, \gamma) = \left\{ a \in X / \mathcal{T}_{\widetilde{A}_i}(a) \geq \alpha, \mathcal{I}_{\widetilde{A}_i}(a) \leq \beta, \mathcal{F}_{\widetilde{A}_i}(a) \leq \gamma \right\}; i = 1, 2$$

$$S(\alpha, \beta, \gamma) = \left\{ s \in Y / \mathcal{T}_{\widetilde{S}}(s) \geq \alpha, \mathcal{I}_{\widetilde{S}}(s) \leq \beta, \mathcal{F}_{\widetilde{S}}(s) \leq \beta \right\}$$

where the $A_i(\alpha, \beta, \gamma)$ and $S(\alpha, \beta, \gamma)$ are the crisp subsets of X and Y respectively. Using (α, β, γ)-cuts, the Neutrosophic interarrival times and Neutrosophic service time can be represented by different levels of confidence intervals. Consequently, a Neutrosophic queue can be reduced to a family of crisp queues with different (α, β, γ)-cuts $\{A_i(\alpha, \beta, \gamma) : 0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1\}$ and
$\{S(\alpha, \beta, \gamma) : 0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1\}$.

In this paper, we proposed queueing model with both interarrival times $\widetilde{A}_i, i = 1, 2$ and service time $\widetilde{S}$ are represented as SVTNN. Denote confidence intervals of $\widetilde{A}_i$ and $\widetilde{S}$ by $[l_{\widetilde{A}_i(\alpha,\beta,\gamma)}, u_{\widetilde{A}_i(\alpha,\beta,\gamma)}]$ and $[l_{\widetilde{S}(\alpha,\beta,\gamma)}, u_{\widetilde{S}(\alpha,\beta,\gamma)}]$.
Let us denote the performance measure by $p(\widetilde{A_i}, \widetilde{S})$ and the truth membership function, the indeterminacy membership function and the falsity membership function of $p(\widetilde{A_i}, \widetilde{S})$ can be defined using Zadeh's extension principle [31, 32], as:

$$\mathcal{T}_{p(\widetilde{A_i}, \widetilde{S})}(z) = \sup \left\{ min_{a \in X, a' \in Y} (\mu_{\widetilde{A}_i(a)}, \mathcal{T}_{\widetilde{S}(a')}) : z = p(a, a') \right\}$$

$$\mathcal{I}_{p(\widetilde{A_i},\widetilde{S})}(z) = \inf\left\{min_{a\in X,a'\in Y}(\mu_{\widetilde{A_i(a)}}, \mathcal{T}_{\widetilde{S}(a')}) : z = p(a,a^{'})\right\}$$

and

$$\mathcal{F}_{p(\widetilde{A_i},\widetilde{S})}(z) = \inf\left\{min_{a\in X,a'\in Y}(\mu_{\widetilde{A_i(a)}}, \mathcal{T}_{\widetilde{S}(a')}) : z = p(a,a^{'})\right\}$$

We can find the lower and upper bounds of the (α,β,γ) cuts of $\widetilde{A_i},\widetilde{S}$ as follows:

$$l_{p(\alpha,\beta,\gamma)} = \min p(a,a^{'}) \text{ such that } l_{\widetilde{A}_i(\alpha,\beta,\gamma)} \le a \le u_{\widetilde{A}_i(\alpha,\beta,\gamma)}, l_{\widetilde{S}(\alpha,\beta,\gamma)} \le a^{'} \le u_{\widetilde{S}(\alpha,\beta,\gamma)} \quad (1)$$

$$u_{p(\alpha,\beta,\gamma)} = \max p(a,a^{'}) \text{ such that } l_{\widetilde{A}_i(\alpha,\beta,\gamma)} \le a \le u_{\widetilde{A}_i(\alpha,\beta,\gamma)}, l_{\widetilde{S}(\alpha,\beta,\gamma)} \le a^{'} \le u_{\widetilde{S}(\alpha,\beta,\gamma)} \quad (2)$$

provided $a \in \widetilde{A}_i(\alpha,\beta,\gamma)$ and $a^{'} \in \widetilde{S}(\alpha,\beta,\gamma)$.

If both $l_{p(\alpha,\beta,\gamma)}$ and $u_{p(\alpha,\beta,\gamma)}$ are invertible with respect to (α,β,γ) then the left shape function $L_{\mathcal{T}}(z) = \left(l_{p(\alpha,\beta,\gamma)}\right)^{-1}$ and the right shape function $R_{\mathcal{T}}(z) = \left(u_{p(\alpha,\beta,\gamma)}\right)^{-1}$ can be obtained, from which the truth membership function $\mu_{p(\widetilde{A_i},\widetilde{S})}(z)$ is given by

$$\mathcal{T}_{p(\widetilde{A_i},\widetilde{S})}(z) = \begin{cases} L_{\mathcal{T}}(z); & z_1^{\mathcal{T}} \le z \le z_2^{\mathcal{T}} \\ R_{\mathcal{T}}(z); & z_3^{\mathcal{T}} \le z \le z_4^{\mathcal{T}} \\ 0; & \text{otherwise} \end{cases}$$

where $z_1^{\mathcal{T}} \le z \le z_4^{\mathcal{T}}$ and $L_{\mathcal{T}}(z_1^{\mathcal{T}}) = R_{\mathcal{T}}(z_4^{\mathcal{T}}) = 0$ for the SVTNN.

Similarly the indeterminacy membership function $\eta_{p(\widetilde{A_i},\widetilde{S})}(z)$ and the falsity membership function $\nu_{p(\widetilde{A_i},\widetilde{S})}(z)$,are derived as follows

$$\mathcal{I}_{p(\widetilde{A_i},\widetilde{S})}(z) = \begin{cases} L_{\mathcal{I}}(z); & z_1^{\mathcal{I}} \le z \le z_2 v \\ R_{\mathcal{I}}(z); & z_3^{\mathcal{I}} \le z \le z_4^{\mathcal{I}} \\ 0; & \text{otherwise} \end{cases}$$

where $z_1^{\mathcal{I}} \le z \le z_4^{\mathcal{I}}$ and $L_{\mathcal{I}}(z_1^{\mathcal{I}}) = R_{\mathcal{I}}(z_4^{\mathcal{I}}) = 0$ for the SVTNN.

$$\mathcal{F}_{p(\widetilde{A_i},\widetilde{S})}(z) = \begin{cases} L_{\mathcal{F}}(z); & z_1^{\mathcal{F}} \le z \le z_2^{\mathcal{F}} \\ R_{\mathcal{F}}(z); & z_3^{\mathcal{F}} \le z \le z_4^{\mathcal{F}} \\ 0; & \text{otherwise} \end{cases}$$

where $z_1^{\mathcal{F}} \le z \le z_4^{\mathcal{F}}$ and $L_{\mathcal{F}}(z_1^{\mathcal{F}}) = R_{\mathcal{F}}(z_4^{\mathcal{F}}) = 0$ for the SVTNN.

The proposed $NM/NM/1$ queue with priority can be reduced it to classical $M/M/1$ queue with priority by using the concept of (α,β,γ)-cut approach.

4. Numerical Illustration

In this section, we present a numerical example to explain the proposed $NM/NM/1$ queueing model with priority.

Let the arrival rates of first and second priority with the same service rate are represented by SVTNN $\widetilde{A}_1 = \langle(3,4,5,6)\ (2,5,8,11)\ (2,4,6,8)\rangle$

$\widetilde{A}_2 = \langle(4,5,6,7)\ (3,4,5,6)\ (6,6,7,8)\rangle$ and

$\widetilde{S} = \langle(16,17,18,19)\ (18,20,22,24)\ (17,19,21,23)\rangle$ per hour respectively.

The (α, β, γ)-cut of $\widetilde{A}_i, i = 1, 2; \widetilde{S}$ are
$\widetilde{A}_1 = \langle [3 + \alpha, 6 - \alpha], [5 - 3\beta, 8 + 3\beta], [4 - 2\gamma, 6 + 2\gamma] \rangle$,
$\widetilde{A}_2 = \langle [4 + \alpha, 7 - \alpha], [4 - \beta, 5 + \beta], [6 - \gamma, 7 + \gamma] \rangle$ and
$\widetilde{S} = \langle [16 + \alpha, 19 - \alpha], [20 - 2\beta, 22 - 2\beta], [19 - 2\gamma, 21 + 2\gamma] \rangle$
From equations (1) and (2) the parametric programming problems are formulated to derive the membership function $\overline{L}_{q_1}, \overline{L}_{q_2}, \overline{W}_{q_1}$ and $\overline{W}_{q_2}$. They are calculated as follows.
The performance functions of (i) $\overline{L}_{q_1}$- average queue length of higher priority (ii) $\overline{L}_{q_2}$- average queue length of low priority (iii) $\overline{W}_{q_1}$ -average waiting time in higher priority queue (iv) $\overline{W}_{q_2}$ - average waiting time in low priority queue are derived from the respective parametric programs. These differ only in their objective functions and are listed below.

$$\begin{aligned} l_{L_{q_1}(\alpha)} &= \min \left\{ \frac{e_1(e_1+e_2)}{e_3(e_3-e_1)} \right\}, u_{L_{q_1}(\alpha)} = \max \left\{ \frac{e_1(e_1+e_2)}{e_3(e_3-e_1)} \right\} \\ \text{such that } & 3+\alpha < e_1 < 6-\alpha \\ & 4+\alpha < e_2 < 7-\alpha \\ & 16+\alpha < e_3 < 19-\alpha \end{aligned} \tag{3}$$

where $0 < \alpha \leq 1$. $l_{L_{q_1}(\alpha)}$ is found when e_1 and e_2 approach their lower bounds (l.b) and e_3 approaches its upper bound (u.b) and also $u_{L_{q_1}(\alpha)}$ is found when e_1 and e_2 approach their u.b's and e_3 approaches its l.b. Consequently the optimal solution for (3) are

$$l_{L_{q_1}(\alpha)} = \frac{21 + 13\alpha + 2\alpha^2}{304 - 54\alpha + 2\alpha^2} \text{ and } u_{L_{q_1}(\alpha)} = \frac{78 - 25\alpha + 2\alpha^2}{160 + 42\alpha + 2\alpha^2}$$

The truth membership function is

$$\mathcal{T}_{\overline{L}_{q_1}}(z) = \begin{cases} L_{\mathcal{T}}(z); & \left[l_{L_{q_1}(\alpha)}\right]_{\alpha=0} \leq z \leq \left[l_{L_{q_1}(\alpha)}\right]_{\alpha=1} \\ R_{\mathcal{T}}(z); & \left[u_{L_{q_1}(\alpha)}\right]_{\alpha=1} \leq z \leq \left[u_{L_{q_1}(\alpha)}\right]_{\alpha=0} \\ 0; & \text{otherwise} \end{cases}$$

which is estimated as

$$\mathcal{T}_{\overline{L}_{q_1}}(z) = \begin{cases} \dfrac{(54z+13) - (484z^2 + 4004z - 1)^{\frac{1}{2}}}{2(2z-2)}; & 0.07 \leq z \leq 0.14 \\ \dfrac{-(42z+25) + (484z^2 + 4004z - 1)^{\frac{1}{2}}}{2(2z-2)}; & 0.27 \leq z \leq 0.49 \\ 0; & \text{otherwise} \end{cases}$$

$$\begin{aligned} l_{L_{q_1}(\beta)} &= \min \left\{ \frac{e_1(e_1+e_2)}{e_3(e_3-e_1)} \right\}, u_{L_{q_1}(\beta)} = \max \left\{ \frac{e_1(e_1+e_2)}{e_3(e_3-e_1)} \right\} \\ \text{such that } & 5-3\beta < e_1 < 8+3\beta \\ & 4-\beta < e_2 < 5+\beta \\ & 20-2\beta < e_3 < 22+2\beta \end{aligned} \tag{4}$$

where $0 < \beta \leq 1$. $l_{L_{q_1}(\beta)}$ is found when e_1 and e_2 approach their l.b's and e_3 approaches its u.b. and also $u_{L_{q_1}(\beta)}$ is found when e_1 and e_2 approach their u.b's and e_3 approaches its l.b. Consequently the optimal solution for (4) is

$$l_{L_{q_1}(\beta)} = \frac{45 - 47\beta + 12\beta^2}{374 + 144\beta + 10\beta^2} \text{ and } u_{L_{q_1}(\beta)} = \frac{104 + 71\beta + 12\beta^2}{240 - 124\beta + 10\beta^2}$$

The indeterminacy membership function is

$$\mathcal{I}_{\overline{L}_{q_1}}(z) = \begin{cases} L_{\mathcal{I}}(z); & \left[l_{L_{q_1}(\beta)}\right]_{\beta=1} \leq z \leq \left[l_{L_{q_1}(\beta)}\right]_{\beta=0} \\ R_{\mathcal{I}}(z); & \left[u_{L_{q_1}(\beta)}\right]_{\beta=0} \leq z \leq \left[u_{L_{q_1}(\beta)}\right]_{\beta=1} \\ 0; & \text{otherwise} \end{cases}$$

which is estimated as

$$\mathcal{I}_{\overline{L}_{q_1}}(z) = \begin{cases} \dfrac{-(144z + 47) + (5776z^2 + 33288z + 49)^{\frac{1}{2}}}{2(10z - 12)}; & 0.02 \leq z \leq 0.12 \\ \dfrac{(124z + 71) - (5776z^2 + 33288z + 49)^{\frac{1}{2}}}{2(10z - 12)}; & 0.43 \leq z \leq 1.48 \\ 0; & \text{otherwise} \end{cases}$$

$$\begin{aligned} & l_{L_{q_1}(\gamma)} = \min\left\{\frac{e_1(e_1 + e_2)}{e_3(e_3 - e_1)}\right\}, u_{L_{q_1}(\gamma)} = \max\left\{\frac{e_1(e_1 + e_2)}{e_3(e_3 - e_1)}\right\} \\ & \text{such that } 4 - 2\gamma < e_1 < 6 + 2\gamma \\ & \qquad 6 - \gamma < e_2 < 7 + \gamma \\ & \qquad 19 - 2\gamma < e_3 < 21 + 2\gamma \end{aligned} \tag{5}$$

where $0 < \gamma \leq 1$. $l_{L_{q_1}(\gamma)}$ is found when e_1 and e_2 approach their l.b's and e_3 approaches its u.b. and also $u_{L_{q_1}(\gamma)}$ is found when e_1 and e_2 approach their u.b's and e_3 approaches its l.b. Consequently the optimal solution for (5) is

$$l_{L_{q_1}(\gamma)} = \frac{40 - 32\gamma + 6\gamma^2}{357 + 118\gamma + 8\gamma^2} \text{ and } u_{L_{q_1}(\gamma)} = \frac{78 + 44\gamma + 6\gamma^2}{247 - 102\gamma + 8\gamma^2} \tag{6}$$

The falsity membership function is

$$\mathcal{F}_{\overline{L}_{q_1}}(z) = \begin{cases} L_{\mathcal{F}}(z); & \left[l_{L_{q_1}(\gamma)}\right]_{\gamma=1} \leq z \leq \left[l_{L_{q_1}(\gamma)}\right]_{\gamma=0} \\ R_{\mathcal{F}}(z); & \left[u_{L_{q_1}(\gamma)}\right]_{\gamma=0} \leq z \leq \left[u_{L_{q_1}(\gamma)}\right]_{\gamma=1} \\ 0; & \text{otherwise} \end{cases}$$

which is estimated as

$$\mathcal{F}_{\overline{L}_{q_1}}(z) = \begin{cases} \dfrac{-(118z+32)+(2500z^2+17400z+64)^{\frac{1}{2}}}{2(8z-6)}; & 0.03 \leq z \leq 0.11 \\ \dfrac{(102z+44)-(2500z^2+17400z+64)^{\frac{1}{2}}}{2(8z-6)}; & 0.32 \leq z \leq 0.83 \\ 0; & \text{otherwise} \end{cases}$$

For different values of $\alpha, \beta, \gamma \in [0, 1]$, the average queue length of higher priority $\overline{L}_{q_1}$ is calculated and given in table 2. Also a graphical interpolation of truth, Indeterminacy and falsity of average queue length of higher priority is shown in figure 2.

TABLE 2. $\overline{L}_{q_1}$

α	$l_{L_{q_1(\alpha)}}$	$u_{L_{q_1(\alpha)}}$	β,γ	$l_{L_{q_1(\beta)}}$	$u_{L_{q_1(\beta)}}$	$l_{L_{q_1(\gamma)}}$	$u_{L_{q_1(\gamma)}}$
0.0	0.06908	0.48750	1.0	0.12032	0.43333	0.11204	0.31579
0.1	0.07474	0.45987	0.9	0.10404	0.48845	0.09992	0.34811
0.2	0.08074	0.43376	0.8	0.08948	0.55046	0.08884	0.38357
0.3	0.08709	0.40908	0.7	0.07649	0.62042	0.07870	0.42253
0.4	0.09380	0.38573	0.6	0.06491	0.69958	0.06945	0.46539
0.5	0.10090	0.36364	0.5	0.05463	0.78947	0.06100	0.51263
0.6	0.10840	0.34273	0.4	0.04552	0.89196	0.05331	0.56477
0.7	0.11633	0.32293	0.3	0.03748	1.00936	0.04631	0.62244
0.8	0.12469	0.30419	0.2	0.03043	1.14457	0.03995	0.68637
0.9	0.13353	0.28643	0.1	0.02427	1.30125	0.03419	0.75742
1.0	0.14286	0.26961	0.0	0.01894	1.48413	0.02899	0.83660

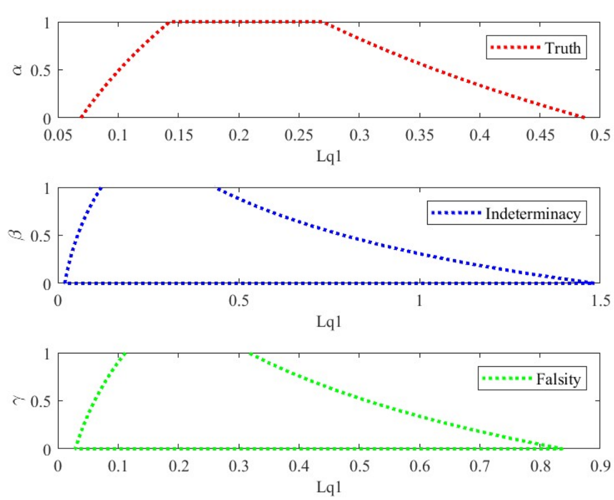


FIGURE 2. Average queue length of higher priority

Similarly the performance functions of $\overline{L}_{q2}$ is derived from the respective parametric programs. These differ only in their objective functions and are listed below.

$$l_{L_{q_2}(\alpha)} = \min\left\{\frac{e_2(e_1+e_2)}{[e_3-(e_1+e_2)](e_3-e_1)}\right\} \tag{7}$$

and

$$u_{L_{q_2}(\alpha)} = \max\left\{\frac{e_2(e_1+e_2)}{[e_3-(e_1+e_2)](e_3-e_1)}\right\} \tag{8}$$

The objective functions given through the equations (7) and (8) with the constraints given with the equation (3) yield the following results:

$$l_{L_{q_2}(\alpha)} = \frac{28+15\alpha+2\alpha^2}{192-72\alpha+6\alpha^2};\ u_{L_{q_2}(\alpha)} = \frac{91-27\alpha+2\alpha^2}{30+36\alpha+6\alpha^2}$$

$$\mathcal{T}_{\overline{L}_{q_2}}(z) = \begin{cases} \dfrac{(72z+15)-(576z^2+4368z+1)^{\frac{1}{2}}}{2(6z-2)}; & 0.14 \le z \le 0.36 \\ \dfrac{-(36z+27)+(576z^2+4368z+1)^{\frac{1}{2}}}{2(6z-2)}; & 0.92 \le z \le 3.03 \\ 0; & \text{otherwise} \end{cases}$$

Similarly the performance functions of $\overline{L}_{q2}$ is derived from the respective parametric programs. These differ only in their objective functions and are listed below.

$$l_{L_{q_2}(\beta)} = \min\left\{\frac{e_2(e_1+e_2)}{[e_3-(e_1+e_2)](e_3-e_1)}\right\} \tag{9}$$

and

$$u_{L_{q_2}(\beta)} = \max\left\{\frac{e_2(e_1+e_2)}{[e_3-(e_1+e_2)](e_3-e_1)}\right\} \tag{10}$$

The objective functions given through the equations (9) and (10) with the constraints given with the equations (4) yield the following results:

$$l_{L_{q_2}(\beta)} = \frac{36-25\beta+4\beta^2}{221+167\beta+30\beta^2};\ u_{L_{q_2}(\beta)} = \frac{65+33\beta+4\beta^2}{84-107\beta+30\beta^2}$$

$$\mathcal{I}_{\overline{L}_{q_2}}(z) = \begin{cases} \dfrac{-(167z+25)+(1369z^2+16206z+49)^{\frac{1}{2}}}{2(30z-4)}; & 0.04 \le z \le 0.16 \\ \dfrac{(107z+33)-(1369z^2+16206z+49)^{\frac{1}{2}}}{2(30z-4)}; & 0.77 \le z \le 14.57 \\ 0; & \text{otherwise} \end{cases}$$

Similarly the performance functions of $\overline{L}_{q2}$ is derived from the respective parametric programs. These differ only in their objective functions and are listed below.

$$l_{L_{q_2}(\gamma)} = \min\left\{\frac{e_2(e_1+e_2)}{[e_3-(e_1+e_2)](e_3-e_1)}\right\} \tag{11}$$

and

$$l_{L_{q2}(\gamma)} = \max\left\{\frac{e_2(e_1+e_2)}{[e_3-(e_1+e_2)](e_3-e_1)}\right\} \tag{12}$$

The objective functions given through the equations (11) and (12) with the constraints given with the equations (5) yield the following results:

$$l_{L_{q2}(\gamma)} = \frac{60-28\gamma+3\gamma^2}{187+129\gamma+20\gamma^2};\ u_{L_{q2}(\gamma)} = \frac{91+34\gamma+3\gamma^2}{78-89\gamma+20\gamma^2}$$

$$\mathcal{F}_{\overline{L}_{q2}}(z) = \begin{cases} \dfrac{-(129z+28)+(1681z^2+14268z+64)^{\frac{1}{2}}}{2(20z-3)}; & 0.1 \le z \le 0.32 \\ \dfrac{(89z+34)-(1681z^2+14268z+64)^{\frac{1}{2}}}{2(20z-3)}; & 1.17 \le z \le 14.22 \\ 0; & \text{otherwise} \end{cases}$$

For different values of $\alpha, \beta, \gamma \in [0,1]$, the average queue length of low priority $\overline{L}_{q2}$ is calculated and given in table 3. Also a graphical interpolation of truth, Indeterminacy and falsity of average queue length of low priority is shown in figure 3.

Table 3. $\overline{L}_{q2}$

α	$l_{L_{q2}(\alpha)}$	$u_{L_{q2}(\alpha)}$	β,γ	$l_{L_{q2}(\beta)}$	$u_{l_{q2}(\beta)}$	$l_{L_{q2}(\gamma)}$	$u_{l_{q2}(\gamma)}$
0.0	0.14583	3.03333	1.0	0.16290	0.77381	0.32086	1.16667
0.1	0.15969	2.62389	0.9	0.14092	0.92853	0.28601	1.36263
0.2	0.17476	2.28846	0.8	0.12191	1.12476	0.25524	1.60525
0.3	0.19118	2.00968	0.7	0.10541	1.37839	0.22800	1.91092
0.4	0.20906	1.77513	0.6	0.09105	1.71391	0.20380	2.30439
0.5	0.22857	1.57576	0.5	0.07853	2.17105	0.18226	2.82468
0.6	0.24987	1.40476	0.4	0.06759	2.81830	0.16303	3.53711
0.7	0.27314	1.25697	0.3	0.05803	3.78403	0.14584	4.55961
0.8	0.29861	1.12835	0.2	0.04965	5.33864	0.13043	6.12857
0.9	0.32652	1.01576	0.1	0.04232	8.16167	0.11660	8.79646
1.0	0.35714	0.91667	0.0	0.03589	14.57144	0.10417	14.22223

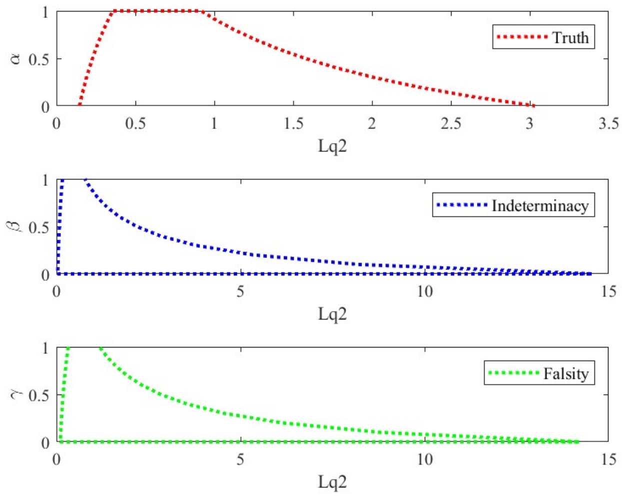


FIGURE 3. Average queue length of low priority

Similarly the performance functions of $\overline{W}_{q_1}$ is derived from the respective parametric programs. These differ only in their objective functions and are listed below.

$$u_{W_{q_1}(\alpha)} = \min\left\{\frac{(e_1+e_2)}{e_3(e_3-e_1)}\right\} \tag{13}$$

and

$$u_{W_{q_1}(\alpha)} = \max\left\{\frac{(e_1+e_2)}{e_3(e_3-e_1)}\right\} \tag{14}$$

The objective functions given through the equations (13) and (14) with the constraints given with the equations (3) yield the following results:

$$l_{W_{q_1}(\alpha)} = \frac{7+2\alpha}{304-54\alpha+2\alpha^2};\ u_{W_{q_1}(\alpha)} = \frac{13-2\alpha}{160+42\alpha+2\alpha^2}$$

$$\mathcal{T}_{\overline{W}_{q_1}}(z) = \begin{cases} \dfrac{(54z+2)-(484z^2+272z+4)^{\frac{1}{2}}}{4z}; & 0.02 \le z \le 0.04 \\ \dfrac{-(42z+2)+(484z^2+272z+4)^{\frac{1}{2}}}{4z}; & 0.05 \le z \le 0.08 \\ 0; & \text{otherwise} \end{cases}$$

Similarly the performance functions of $\overline{W}_{q_1}$ is derived from the respective parametric programs. These differ only in their objective functions and are listed below.

$$l_{W_{q_1}(\beta)} = \min\left\{\frac{(e_1+e_2)}{e_3(e_3-e_1)}\right\} \tag{15}$$

and

$$u_{W_{q_1}(\beta)} = \max\left\{\frac{(e_1+e_2)}{e_3(e_3-e_1)}\right\} \tag{16}$$

The objective functions given through the equations (15) and (16) with the constraints given with the equations (3) yield the following results:

$$l_{W_{q_1}(\beta)} = \frac{9-4\beta}{374+144\beta+10\beta^2};\ u_{W_{q_1}(\beta)} = \frac{13+4\beta}{240+124\beta+10\beta^2}$$

$$\mathcal{I}_{\overline{W}_{q_1}}(z) = \begin{cases} \dfrac{-(144z+4)+(5776z^2+1512z+16)^{\frac{1}{2}}}{20z}; & 0.009 \le z \le 0.02 \\ \dfrac{(124z+4)-(5776z^2+1512z+16)^{\frac{1}{2}}}{20z}; & 0.05 \le z \le 0.13 \\ 0; & \text{otherwise} \end{cases}$$

Similarly the performance functions of $\overline{W}_{q_1}$ is derived from the respective parametric programs. These differ only in their objective functions and are listed below.

$$l_{W_{q_1}(\gamma)} = \min\left\{\frac{(e_1+e_2)}{e_3(e_3-e_1)}\right\} \tag{17}$$

and

$$u_{W_{q_1}(\gamma)} = \max\left\{\frac{(e_1+e_2)}{e_3(e_3-e_1)}\right\} \tag{18}$$

The objective functions given through the equations (17) and (18) with the constraints given with the equations (5) yield the following results:

$$l_{W_{q_1}(\gamma)} = \frac{10-3\gamma}{357+118\gamma+8\gamma^2};\ u_{W_{q_1}(\gamma)} = \frac{13+3\gamma}{247-102\gamma+8\gamma^2}$$

$$\mathcal{F}_{\overline{W}_{q_1}}(z) = \begin{cases} \dfrac{-(118z+3)+(2500z^2+1028z+9)^{\frac{1}{2}}}{16z}; & 0.01 \le z \le 0.03 \\ \dfrac{(102z+3)-(2500z^2+1028z+9)^{\frac{1}{2}}}{16z}; & 0.05 \le z \le 0.11 \\ 0; & \text{otherwise} \end{cases}$$

For different values of $\alpha, \beta, \gamma \in [0,1]$, the average waiting time in the higher priority queue $\overline{W}_{q_1}$ is calculated and given in table 4. Also a graphical interpolation of truth, Indeterminacy and falsity of average waiting time in the higher priority queue is shown in figure 4.

TABLE 4. $\overline{W}_{q_1}$

α	$l_{W_{q_1(\alpha)}}$	$u_{W_{q_1(\alpha)}}$	β,γ	$l_{W_{q_1(\beta)}}$	$u_{W_{q_1(\beta)}}$	$l_{W_{q_1(\gamma)}}$	$u_{W_{q_1(\gamma)}}$
0.0	0.02303	0.08125	1.0	0.02406	0.05417	0.02801	0.05263
0.1	0.02411	0.07794	0.9	0.02214	0.05885	0.02630	0.05615
0.2	0.02523	0.07479	0.8	0.02034	0.06401	0.02468	0.05993
0.3	0.02639	0.07177	0.7	0.01866	0.06971	0.02315	0.06402
0.4	0.02759	0.06888	0.6	0.01708	0.07604	0.02170	0.06844
0.5	0.02883	0.06612	0.5	0.01561	0.08310	0.02033	0.07323
0.6	0.03011	0.06347	0.4	0.01422	0.09102	0.01904	0.07844
0.7	0.03144	0.06093	0.3	0.01292	0.09994	0.01781	0.08411
0.8	0.03281	0.05850	0.2	0.01170	0.11005	0.01665	0.09031
0.9	0.03424	0.05616	0.1	0.01055	0.12161	0.01554	0.09711
1.0	0.03571	0.05392	0.0	0.00947	0.13492	0.01449	0.10458

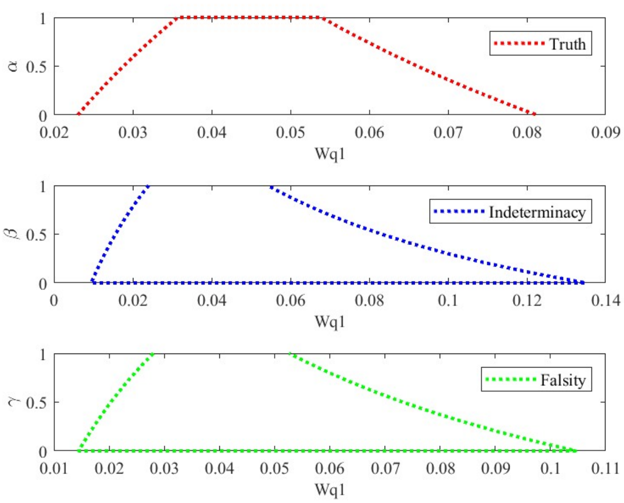


FIGURE 4. Average waiting time in the higher priority queue

Similarly the performance functions of $\overline{W}_{q_2}$ is derived from the respective parametric programs. These differ only in their objective functions and are listed below.

$$l_{W_{q_2}(\alpha)} = \min\left\{\frac{(e_1+e_2)}{[e_3-(e_1+e_2)](e_3-e_1)}\right\} \tag{19}$$

and

$$u_{W_{q_2}(\alpha)} = \max\left\{\frac{(e_1+e_2)}{[e_3-(e_1+e_2)](e_3-e_1)}\right\} \tag{20}$$

The objective functions given through the equations (19) and (20) with the constraints given with the equations (3) yield the following results:

$$l_{W_{q_2}(\alpha)} = \frac{7+2\alpha}{192-72\alpha+6\alpha^2};\ u_{W_{q_2}(\alpha)} = \frac{13-2\alpha}{30+36\alpha+6\alpha^2}$$

$$\mathcal{T}_{\overline{W}_{q_2}}(z) = \begin{cases} \dfrac{(72z+2) - (576z^2+456z+4)^{\frac{1}{2}}}{12z}; & 0.04 \leq z \leq 0.07 \\ \dfrac{-(36z+2) + (576z^2+456z+4)^{\frac{1}{2}}}{12z}; & 0.15 \leq z \leq 0.43 \\ 0; & \text{otherwise} \end{cases}$$

Similarly the performance functions of $\overline{W}_{q_2}$ is derived from the respective parametric programs. These differ only in their objective functions and are listed below.

$$l_{W_{q_2}(\beta)} = \min\left\{\frac{(e_1+e_2)}{[e_3-(e_1+e_2)](e_3-e_1)}\right\} \tag{21}$$

and

$$u_{W_{q_2}(\beta)} = \max\left\{\frac{(e_1+e_2)}{[e_3-(e_1+e_2)](e_3-e_1)}\right\} \tag{22}$$

The objective functions given through the equations (21) and (22) with the constraints given with the equations (3) yield the following results:

$$l_{W_{q_2}(\beta)} = \frac{9-4\beta}{221+167\beta+30\beta^2};\ u_{W_{q_2}(\beta)} = \frac{13+4\beta}{84-107\beta+30\beta^2}$$

$$\mathcal{I}_{\overline{W}_{q_2}}(z) = \begin{cases} \dfrac{-(167z+4) + (1369z^2+2416z+16)^{\frac{1}{2}}}{60z}; & 0.01 \leq z \leq 0.04 \\ \dfrac{(107z+4) - (1369z^2+2416z+16)^{\frac{1}{2}}}{60z}; & 0.15 \leq z \leq 2.43 \\ 0; & \text{otherwise} \end{cases}$$

Similarly the performance functions of $\overline{W}_{q_2}$ is derived from the respective parametric programs. These differ only in their objective functions and are listed below.

$$l_{W_{q_2}(\gamma)} = \min\left\{\frac{(e_1+e_2)}{[e_3-(e_1+e_2)](e_3-e_1)}\right\} \tag{23}$$

and

$$u_{W_{q_2}(\gamma)} = \max\left\{\frac{(e_1+e_2)}{[e_3-(e_1+e_2)](e_3-e_1)}\right\} \tag{24}$$

The objective functions given through the equations (23) and (24) with the constraints given with the equations (5) yield the following results:

$$l_{W_{q_2}(\gamma)} = \frac{10-3\gamma}{187+129\gamma+20\gamma^2};\ u_{W_{q_2}(\gamma)} = \frac{13+3\gamma}{78-89\gamma+20\gamma^2}$$

$$\mathcal{F}_{\overline{W}_{q_2}}(z) = \begin{cases} \dfrac{-(129z+3) + (1681z^2+1574z+9)^{\frac{1}{2}}}{40z}; & 0.02 \leq z \leq 0.05 \\ \dfrac{(89z+3) - (1681z^2+1574z+9)^{\frac{1}{2}}}{40z}; & 0.17 \leq z \leq 1.78 \\ 0; & \text{otherwise} \end{cases}$$

For different values of $\alpha, \beta, \gamma \in [0,1]$, the average waiting time in the low priority queue $\overline{W}_{q2}$ is calculated and given in table 5. Also a graphical interpolation of truth, Indeterminacy and falsity of average waiting time in the low priority queue is shown in figure 5.

TABLE 5. $\overline{W}_{q2}$

α	$l_{W_{q2(\alpha)}}$	$u_{W_{q2(\alpha)}}$	β, γ	$l_{W_{q2(\beta)}}$	$u_{W_{q2(\beta)}}$	$l_{W_{q2(\gamma)}}$	$u_{W_{q2(\gamma)}}$
0.0	0.03646	0.43333	1.0	0.04072	0.15476	0.05348	0.16667
0.1	0.03895	0.38027	0.9	0.03613	0.18207	0.04848	0.19192
0.2	0.04161	0.33654	0.8	0.03208	0.21630	0.04401	0.22295
0.3	0.04446	0.29995	0.7	0.02849	0.26007	0.04000	0.26177
0.4	0.04751	0.26896	0.6	0.02529	0.31739	0.03639	0.31140
0.5	0.05079	0.24242	0.5	0.02244	0.39474	0.03314	0.37662
0.6	0.05432	0.21949	0.4	0.01988	0.50327	0.03019	0.46541
0.7	0.05812	0.19952	0.3	0.01758	0.663870	0.02752	0.59216
0.8	0.06221	0.18199	0.2	0.01552	0.92045	0.02508	0.78571
0.9	0.06664	0.16652	0.1	0.01365	1.38333	0.02286	1.11348
1.0	0.07143	0.15278	0.0	0.01196	2.42857	0.02083	1.77778

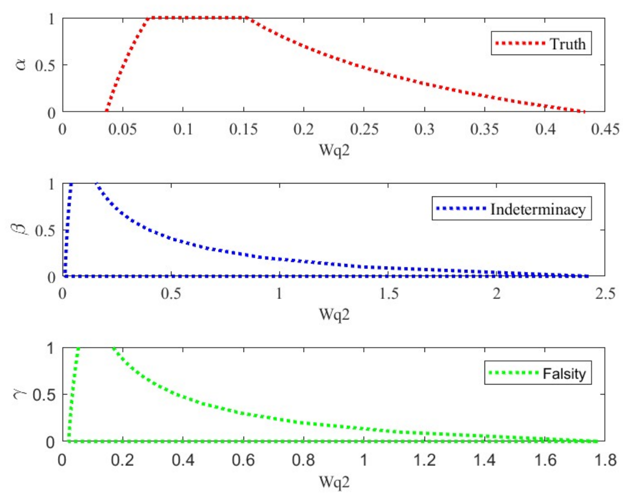


FIGURE 5. Average waiting time in the low priority queue

5. Conclusion

Priority queueing models are useful in real world problems such as emergency cases in hospital medical treatment, communication networks etc. The parameters for queueing decision models can be known imprecisely and hence the performance measurements of the system can be dealt in neutrosophic environment. This paper, proposes a single server queuing model with priority discipline and its characteristics. The service time and arrival time of proposed model are expressed in terms of single valued trapezoidal Neutrosophic number. An illustrative example is provided to show the performance measures of the proposed model which are constructed using truth, indeterminacy and falsity membership degree of SVTNN. In future, this queueing model can extended multi objective priority queuing model. The extensions of neutrosophic sets such as pythogorean and Fermatean neutrosophic sets can used in the proposed model to explore its new aspects.

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