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Abstract: The purpose of this article is to study the equations of neutrosophic straight line and neutrosophic circle, where the neutrosophic point, general neutrosophic equation of a line, the equation of a neutrosophic straight line passing through two neutrosophic points and parallel and perpendicular neutrosophic lines are defined, in addition, four forms of the equations of neutrosophic circle were discussed. Where detailed examples are given to clarify each case.

Keywords: neutrosophic straight line; neutrosophic circle; the equations; polar; radius; center.

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1. Introduction

As an alternative to the existing logics, Smarandache proposed the Neutrosophic Logic to represent a mathematical model of uncertainty, vagueness, ambiguity, imprecsion, undefined, unknown, incompleteness, inconsistency, redundancy, contradiction, where the concept of neutrosophy is a new branch of philosophy introduced by Smarandache [3-13]. He presented the definition of the standard form of neutrosophic real number and conditions for the division of two neutrosophic real numbers to exist, he defined the standard form of neutrosophic complex number, and found root index $n \geq 2$ of a neutrosophic real and complex number [2-4], studying the concept of the Neutrosophic probability [3-5], the Neutrosophic statistics [4][6], and professor Smarandache entered the concept of preliminary calculus of the differential and integral calculus, where he introduced for the first time the notions of neutrosophic mereo-limit, mereo-continuity, mereoderivative, and mereo-integral [1-8]. Madeleine Al- Taha presented results on single valued neutrosophic (weak) polygroups [9]. Edalatpanah proposed a new direct algorithm to solve the neutrosophic linear programming where the variables and right-hand side represented with triangular neutrosophic numbers [10]. Chakraborty used pentagonal neutrosophic number in networking problem, and Shortest Path Problem [11-12]. Y.Alhasan studied the concepts of neutrosophic complex numbers, the general exponential form of a neutrosophic complex, and the neutrosophic integrals and integration methods [7-14-18]. On the other hand, M.Abdel-Basset presented study in the science of neutrosophic about an approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number [15]. Also, neutrosophic sets played an important role in applied science such as health care, industry, and optimization [16-17]. Giorgio Nordo, Arif Mehmood and Said Broumi studied single valued neutrosophic filters [19].

The paper is organized as follows. Section 1, provides an introduction, in which neutrosophic science review has given. Neutrosophic real number are discussed in Section 2. Section 3 frames the equations of neutrosophic straight line. the equations of neutrosophic circle were discussed in section 4. Finally, In Section 5 a conclusion to the paper is given.

2. Preliminaries

2.1. Neutrosophic Real Number [4]

Suppose that w is a neutrosophic number, then it takes the following standard form: $w = a + bI$ where a , b are real coefficients, and I represent indeterminacy, such $0.I = 0$ and $I^n = I$, for all positive integers n.

2.2. Division of neutrosophic real numbers [4]

Suppose that w_1, w_2 are two neutrosophic numbers, where:

$$w_1 = a_1 + b_1 I \ , \qquad w_2 = a_2 + b_2 I$$

To find $(a_1 + b_1 I) \div (a_2 + b_2 I)$, we can write:

$$\frac{a_1 + b_1 I}{a_2 + b_2 I} \equiv x + yI$$

where x and y are real unknowns.

$$a_1 + b_1 I \equiv (a_2 + b_2 I)(x + yI)$$

$$a_1 + b_1 I \equiv a_2 x + (b_2 x + a_2 y + b_2 y)I$$

by identifying the coefficients, we get:

$$a_1 = a_2 x$$

$$b_1 = b_2 x + (a_2 + b_{2)} y$$

We obtain unique one solution only, provided that:

$$\begin{vmatrix} a_2 & 0 \\ b_2 & a_2 + b_2 \end{vmatrix} \neq 0 \quad \Rightarrow \quad a_2(a_2 + b_2) \neq 0$$

Hence: $a_2 \neq 0$ $\quad and \ a_2 \neq -b_2$ are the conditions for the division of two neutrosophic real numbers to exist.

Then:

$$\frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} . I$$

2.3. Root index $n \geq 2$ of a neutrosophic real number [4]

1) Case: $n = 2$

Let $w = a + \mathrm{bI}$ be a neutrosophic real number, then:

$$\sqrt{a + \mathrm{bI}} = x + y.I$$

$$a + \mathrm{bI} \equiv (x + y.I)^2$$

$$a + bI \equiv x^2 + 2xy.I + y^2 I$$

by identifying the coefficients, we get:

$$x^2 = a$$

$$y^2 + 2xy = b$$

Hence $x = \pm\sqrt{a}$

$$y^2 \pm 2\sqrt{a}y - b = 0$$

By solving the second equation in respect to y we find:

$$y = \frac{\mp 2\sqrt{a} \pm \sqrt{4a + 4\mathrm{b}}}{2} = \mp\sqrt{a} \pm \sqrt{a + \mathrm{b}}$$

Then we fined four solutions of $\sqrt{a + \mathrm{bI}}$:

$$\sqrt{a + \mathrm{bI}} = \sqrt{a} + \left(-\sqrt{a} + \sqrt{a + \mathrm{b}}\right).I$$

Or: $$\sqrt{a + \mathrm{bI}} = \sqrt{a} - \left(-\sqrt{a} + \sqrt{a + \mathrm{b}}\right).I$$

Or: $$\sqrt{a + \mathrm{bI}} = -\sqrt{a} + \left(\sqrt{a} + \sqrt{a + \mathrm{b}}\right).I$$

Or: $$\sqrt{a + \mathrm{bI}} = -\sqrt{a} + \left(\sqrt{a} - \sqrt{a + \mathrm{b}}\right).I$$

particular case: $\sqrt{I} = \pm I$

2) Case: $n > 2$

$$\sqrt[n]{a + \mathrm{bI}} = x + y.I$$

$$a + \mathrm{bI} \equiv (x + y.I)^n$$

$$a + \mathrm{bI} \equiv x^n + \left(\sum_{k=0}^{n-1} C_n^k\, y^{n-k}\, x^k\right).I$$

$$x^n = a \;\Rightarrow\; \qquad x = \begin{cases} \sqrt[n]{a} & ;\ n\ odd \\ \pm\sqrt[n]{a} & ;\ n\ even \end{cases}$$

$$\sum_{k=0}^{n-1} C_n^k\, y^{n-k}\, a^{\frac{k}{n}} = b$$

Solve it in respect to y, we can distinguish two cases:
When the x and y solutions are real, we get neutrosophic real solutions,

When x and y solutions are complex, we get neutrosophic complex solutions.

3. The equations of neutrosophic straight line

3.1. Neutrosophic point

Definition3.1.1:

Let $x_A = x_a + x_b I$ and $y_A = y_a + y_b I$, where x_a, x_b, y_a, y_b are real numbers, while $I =$ Indeterminacy, then $A(x_A, y_A)$ represent the neutrosophic point.

3.2. General neutrosophic equation of a line

Definition3.2.1:

The general equation of a neutrosophic straight line is given by the following formula:

$$(a_0 + a_1 I\,)x + (b_0 + b_1 I)y + c_0 + c_1 I = 0$$

Where $a_0, b_0 \neq 0$ and $a_0, a_1, b_0, b_1, c_0, c_1$ are real numbers, while $I =$ indeterminacy.

Definition3.2.2:

Slope-intercept form of the equation of a neutrosophic straight line is given by the following formula:

$$y = (m_a + m_b I)x + p_a + p_b I$$

where m_a, m_b, p_a, p_b are real numbers, while I =indeterminacy.

Example3.1.1:

$$y = (3 + 2I)x + 2 - 4I$$

Definition3.2.3:

Equation of a neutrosophic straight line passing through two neutrosophic points, $A(x_1 + x_2 I, y_1 + y_2 I)$ *and* $B(\acute{x}_1 + \acute{x}_2 I, \acute{y}_1 + \acute{y}_2 I)$, is given by the following formula:

$$\frac{y - y_1 - y_2 I}{x - x_1 - x_2 I} = \frac{\acute{y}_1 + \acute{y}_2 I - y_1 - y_2 I}{\acute{x}_1 + \acute{x}_2 I - x_1 - x_2 I}$$

where $x_1, x_2, y_1, y_2, \acute{x}_1, \acute{x}_2, \acute{y}_1, \acute{y}_2$ are real numbers ($\acute{x}_1 \neq x_1$ *and not zero*), while $I =$ indeterminacy.

Example3.1.2:

Find the equation of a neutrosophic straight line passing through two neutrosophic points: $A(3 + 2I, 3I)$ *and* $B(7 - 3I, 5 + I)$

Solution:

$$\frac{y - 3I}{x - 3 - 2I} = \frac{5 + I - 3I}{7 - 3I - 3 - 2I}$$

$$\frac{y - 3I}{x - 3 - 2I} = \frac{5 - 2I}{4 - 5I}$$

$$\frac{y - 3I}{x - 3 - 2I} = \frac{5}{4} - \frac{17}{4}I$$

$$y - 3I = (x - 3 - 2I)\left(\frac{5}{4} - \frac{17}{4}I\right)$$

$$y - 3I = \left(\frac{5}{4} - \frac{17}{4}I\right)x - \frac{8}{4} + \frac{75}{4}I$$

$$y = \left(\frac{5}{4} - \frac{17}{4}I\right)x - \frac{8}{4} + \frac{75}{4}I + 3I$$

$$\Rightarrow \qquad y = \left(\frac{5}{4} - \frac{17}{4}I\right)x - \frac{8}{4} + \frac{87}{4}I$$

Definition3.2.4:

Let $d_1, and\ d_2$ are two straight lines, we say $d_1, and\ d_2$ are parallel if their slopes are equal, and we say that they are perpendicular if the product of their slopes is -1 .

Example3.1.3:

$$d_1: \ y = (2 - 3I)x + 4 + 3I$$

$$d_2: \ y = (2 - 3I)x + 5 + 4I$$

$$m_{d_1} = m_{d_2} = 2 - 3I \quad \Rightarrow d_1 \,//\, d_2$$

We can illustrate this by giving different values of I:

- $I = 0$, then:

$$d_1: \ y = 2x + 4$$

$$d_2: \ y = 2x + 5$$

- $I = 5$, then:

$$d_1: \ y = -7x + 13$$

$$d_2: \ y = -7x + 17$$

Example3.1.4:

$$d_1: \ y = (3 + 5I)x - 3 + I$$

$$d_2: \ y = \left(\frac{1}{-3-5I}\right)x + 7 - 2I$$

$$m_{d_1}.m_{d_2} = (3 + 5I)\left(\frac{1}{-3 - 5I}\right) = -1 \quad \Rightarrow d_1 \perp d_2$$

4. The equations of neutrosophic circle

Definition4.1:

The standard equation of a neutrosophic circle: for point $p(x, y)$ to lie on a circle with center $c(h + h_1I, k + k_1I)$ and radius $r + r_1I > 0$, the distance pc must be equal to radius $r + r_1I$. Then, using the distance formula we get:

$$\overline{pc} = \sqrt{(x - h - h_1I)^2 + (y - k - k_1I)^2}$$

$$\overline{pc} = r + r_1I$$

$$\sqrt{(x-h-h_1I)^2+(y-k-k_1I)^2}=r+r_1I$$

$$\left(\sqrt{(x-h-h_1I)^2+(y-k-k_1I)^2}\right)^2=(r+r_1I)^2$$

$$(x-h-h_1I)^2+(y-k-k_1I)^2=(r+r_1I)^2 \quad (1)$$

Example4.1:

$$(x-3-2I)^2+(y+4-3I)^2=4-3I$$

The center is $c(3+2I,-4+3I)$, we can find the radius as the following:

$$(r+r_1I)^2=4-3I$$

$$r+r_1I=\sqrt{4-3I}$$

Let's find $\sqrt{4-3I}$

$$\sqrt{4-3I}=r+r_1I$$

$$4-3I=r^2+2rr_1I+{r_1}^2I$$

$$4-3I=r^2+(2rr_1+{r_1}^2)I$$

then:

$$\begin{cases} r^2=4 \\ 2rr_1+{r_1}^2=-3 \end{cases}$$

$$\begin{cases} r=\pm 2 \\ r^2+2rr_1+3=0 \end{cases}$$

Find r_1:

When $r=2 \quad \Rightarrow {r_1}^2+2r_1+3=0$

$$(r_1+3)(r_1+1)=0 \quad \Rightarrow r_1=-3\,, r_1=-1$$

$$(2,-3),(2,-1)$$

When $r=-2 \quad \Rightarrow {r_1}^2-4r_1+3=0$

$$(r_1-3)(r_1-1)=0 \quad \Rightarrow r_1=3\,, r_1=1$$

$$(-2,3),(-2,1)$$

Hence:

$$r+r_1I=2-3I \quad ; I<\frac{2}{3}$$

Or $\quad r+r_1I=2-I \quad ; I<2$

Or $\quad r+r_1I=-2+3I \quad ; I>2/3$

Or $\quad r+r_1I=-2+I \quad ; I>2$

Definition4.2:

Equation of a neutrosophic circle when the centre is origin $O(0,0)$, it given by formula:

$$x^2 + y^2 = (r + r_1 I)^2$$

Example4.2:

$$x^2 + y^2 = 16 - 15I$$

The center is $O(0,0)$, we can find the radius as the following:

$$(r + r_1 I)^2 = 16 - 15I$$

$$r + r_1 I = \sqrt{16 - 15I}$$

Let's find $\sqrt{16 - 15I}$

$$\sqrt{16 - 15I} = r + r_1 I$$

$$16 - 15I = r^2 + 2rr_1 I + {r_1}^2 I$$

$$16 - 15I = r^2 + (2rr_1 + {r_1}^2)I$$

then:

$$\begin{cases} r^2 = 16 \\ 2rr_1 + {r_1}^2 = -15 \end{cases}$$

$$\begin{cases} r = \pm 4 \\ r^2 + 2rr_1 + 15 = 0 \end{cases}$$

Find r_1:

When $r = 4 \quad \Rightarrow {r_1}^2 + 8r_1 + 15 = 0$

$$(r_1 + 3)(r_1 + 5) = 0 \quad \Rightarrow r_1 = -3 , r_1 = -5$$

$$(4, -3), (4, -5)$$

When $r = -4 \quad \Rightarrow {r_1}^2 - 8r_1 + 15 = 0$

$$(r_1 - 3)(r_1 - 5) = 0 \quad \Rightarrow r_1 = 3 , r_1 = 5$$

$$(-4,3), (-4,5)$$

Hence:

$$r + r_1 I = 4 - 3I \quad ; I < \frac{4}{3}$$

Or $\quad r + r_1 I = 4 - 5I \quad ; I < 4/5$

Or $\quad r + r_1 I = -4 + 3I \quad ; I > 4/3$

Or $\quad r + r_1 I = -4 + 5I \quad ; I > 4/5$

Definition4.3:

The general equation of a neutrosophic circle given by formula:

$$x^2 + y^2 + (a + a_1 I)x + (b + b_1 I)y + c + c_1 I = 0$$

Adding $(a + a_1 I)^2 + (b + b_1 I)^2$ on both sides of the equation gives, we get:

$$x^2 + y^2 + (a + a_1I)x + (b + b_1I)y + (a + a_1I)^2 + (b + b_1I)^2 = (a + a_1I)^2 + (b + b_1I)^2 - c - c_1I$$

$$x^2 + (a + a_1I)x + (a + a_1I)^2 + y^2 + (b + b_1I)y + (b + b_1I)^2 = (a + a_1I)^2 + (b + b_1I)^2 - c - c_1I$$

$$\left(x + \frac{a + a_1I}{2}\right)^2 + \left(y + \frac{b + b_1I}{2}\right)^2 = \left(\frac{a + a_1I}{2}\right)^2 + \left(\frac{b + b_1I}{2}\right)^2 - c - c_1I \quad (2)$$

Comparing (2) with (1), we find:

$$h + h_1I = -\left(\frac{a + a_1I}{2}\right)$$

$$k + k_1I = -\left(\frac{b + b_1I}{2}\right)$$

$$(r + r_1I)^2 = \left(\frac{a + a_1I}{2}\right)^2 + \left(\frac{b + b_1I}{2}\right)^2 - c - c_1I$$

$$\Rightarrow \quad r + r_1I = \sqrt{\left(\frac{a + a_1I}{2}\right)^2 + \left(\frac{b + b_1I}{2}\right)^2 - c - c_1I} \quad > 0$$

Example4.3:

To find the standard equation, the center and radius of the following neutrosophic circle:

$$x^2 - 6x + y^2 - 6Iy + 2I = 0$$

we follow these steps:

$$h + h_1I = -\left(\frac{a + a_1I}{2}\right) = \frac{6}{2} = 3$$

$$k + k_1I = -\left(\frac{b + b_1I}{2}\right) = \frac{6I}{2} = 3I$$

$$(r + r_1I)^2 = \left(\frac{a + a_1I}{2}\right)^2 + \left(\frac{b + b_1I}{2}\right)^2 - c - c_1I$$

$$= 9 + 9I - 2I \ = 9 + 7I$$

hence:

$$(x - 3)^2 + (y - 3I)^2 = 9 + 7I$$

The center is $c(3,3I)$, we can find the radius as the following:

$$(r + r_1I)^2 = 9 + 7I$$

$$r + r_1I = \sqrt{9 + 7I}$$

Let's find $\sqrt{9 + 7I}$

$$\sqrt{9 + 7I} = r + r_1I$$

$$9 + 7I = r^2 + 2rr_1I + {r_1}^2I$$

$$9+7I=r^2+(2rr_1+r_1{}^2)I$$

then:

$$\begin{cases} r^2=9 \\ 2rr_1+r_1{}^2=7 \end{cases}$$

$$\begin{cases} r=\pm 3 \\ r^2+2rr_1-7=0 \end{cases}$$

find r_1:

when $r=3 \quad \Rightarrow r_1{}^2+6r_1-7=0$

$$(r_1+7)(r_1-1)=0 \quad \Rightarrow r_1=-7\,, r_1=1$$

$$(3,-7),(3,1)$$

when $r=-3 \quad \Rightarrow r_1{}^2-6r_1-7=0$

$$(r_1-7)(r_1+1)=0 \quad \Rightarrow r_1=7\,, r_1=-1$$

$$(-3,7),(-3,-1)$$

hence:

$$r+r_1I=3-7I \quad ; I<\frac{3}{7}$$

Or $\quad r+r_1I=3+I \quad ; I>-3$

Or $\quad r+r_1I=-3+7I \quad ; I>3/7$

Or $\quad r+r_1I=-3-I \quad ; I<-3$

4.1. Polar equation of a neutrosophic circle

The polar form of equation of a neutrosophic circle, with a center $S(\acute{r}+\acute{r}_1I,\varphi+\varphi_1I)$ and radius $R+R_1I$, using the low of cosine:

$$(r+r_1I)^2+(\acute{r}+\acute{r}_1I)^2-2(r+r_1I)(\acute{r}+\acute{r}_1I)\cos(\theta+\theta_1I-\varphi-\varphi_1I)=(R+R_1I)^2$$

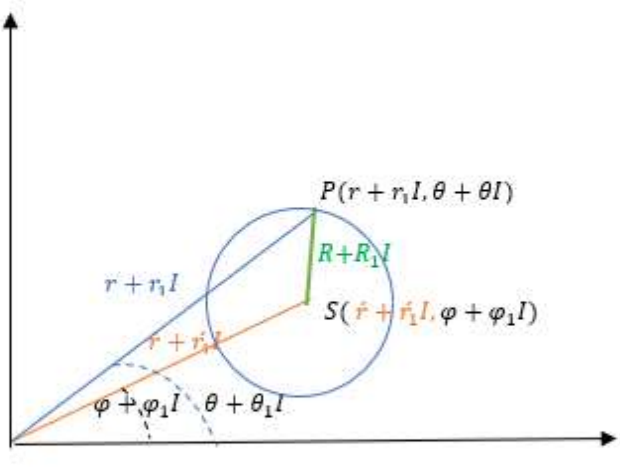

Figure 1

Note:

The polar equation of a neutrosophic circle, with radius $R+R_1I$ and a center on the polar axis running through the pole O (origin):

Since:

$$\cos(\theta + \theta_1 I) = \frac{r + r_1 I}{2(R+R_1 I)}$$

then:

$$r + r_1 I = 2(R+R_1 I)\cos(\theta + \theta_1 I)$$

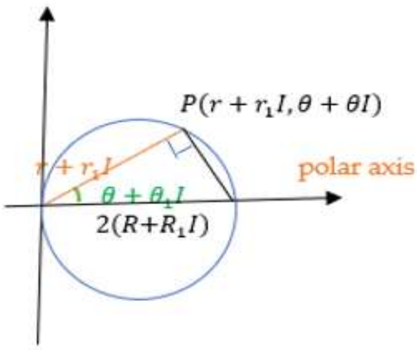

Figure 2

Example4.1.1:

Convert The polar equation of a neutrosophic circle:

$$r + r_1 I = (-4 + 6I)\cos(\theta + \theta_1 I)$$

into cartesian coordinatedes.

Solution:

$$r + r_1 I = (-4 + 6I)\cos(\theta + \theta_1 I)$$

$$(r + r_1 I)^2 = (-4 + 6I)(r + r_1 I)\cos(\theta + \theta_1 I)$$

by substitute in:

$$x^2 + y^2 = (r + r_1 I)^2$$

we get:

$$x^2 + y^2 = (-4 + 6I)(r + r_1 I)\cos(\theta + \theta_1 I)$$

we know:

$$x = (r + r_1 I)\cos(\theta + \theta_1 I)$$

then:

$$x^2 + y^2 = (-4 + 6I)x$$

$$x^2 - (-4 + 6I)x + y^2 = 0$$

$$h + h_1 I = -\left(\frac{a + a_1 I}{2}\right) = \frac{-4 + 6I}{2} = -2 + 3I$$

$$k + k_1 I = -\left(\frac{b + b_1 I}{2}\right) = \frac{0}{2} = 0 + 0I$$

$$(r + r_1 I)^2 = \left(\frac{a + a_1 I}{2}\right)^2 + \left(\frac{b + b_1 I}{2}\right)^2 - c - c_1 I$$

$$= (-2 + 3I\,)^2 + 0 + 0I - (0 + 0I) = 4 - 3I$$

hence:

$$(x + 2 - 3I)^2 + y^2 = 4 - 3I$$

The center is $c(-2 + 3I\,, 0 + 0I)$, we can find the radius as the following:

$$(r + r_1 I)^2 = 4 - 3I$$

$$r + r_1 I = \sqrt{4 - 3I}$$

let's find $\sqrt{4 - 3I}$

$$\sqrt{4 - 3I} = r + r_1 I$$

$$4 - 3I = r^2 + 2rr_1 I + {r_1}^2 I$$

$$4 - 3I = r^2 + (2rr_1 + {r_1}^2)I$$

then:

$$\begin{cases} r^2 = 4 \\ 2rr_1 + {r_1}^2 = -3 \end{cases}$$

$$\begin{cases} r = \pm 2 \\ r^2 + 2rr_1 + 3 = 0 \end{cases}$$

Find r_1:
When $r = 2 \quad \Rightarrow {r_1}^2 + 2r_1 + 3 = 0$

$$(r_1 + 3)(r_1 + 1) = 0 \quad \Rightarrow r_1 = -3\,, r_1 = -1$$

$$(2, -3), (2, -1)$$

When $r = -2 \quad \Rightarrow {r_1}^2 - 4r_1 + 3 = 0$

$$(r_1 - 3)(r_1 - 1) = 0 \quad \Rightarrow r_1 = 3\,, r_1 = 1$$

$$(-2,3), (-2,1)$$

Hence:

$$r + r_1 I = 2 - 3I \quad ; I < \frac{2}{3}$$

Or $\quad r + r_1 I = 2 - I \quad ; I < 2$

Or $\quad r + r_1 I = -2 + 3I \quad ; I > 2/3$

Or $\quad r + r_1 I = -2 + I \quad ; I > 2$

4. Conclusions

Geometry is important for many reasons. The world is overflowing with geometric shapes, and since geometric shapes surround us from every side, our understanding and appreciation of our world will be better if we learn something about geometry. This led us to introduce the concept of neutrosophic in geometry and to write this paper. The equations of the circle and the straight line in the neutrosophic field are defined. This paper is considered an introduction to the neutrosophic geometry.

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