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Abstract.

This paper comes side by side with the complement paper (Original Methods for Obtaining the Initial Solution in Neutrosophic Transportation Problems), these two papers regarded as twins, they are both dedicated to sounding the transportation problems from the perspective of neutrosophic theory, having kinds of indeterminacy in three aspects are:

1- The entries of the payment matrix are neutrosophic values (i.e. $Nc_{ij} = [c_{ij} \pm \varepsilon]$); the indexes i&j have their usual meaning representing the transportation cost of one unit from the production center i to the consumption center j. Assume the indeterminate $\varepsilon = [\lambda_1, \lambda_2]$.

2- The available and the required quantities are both having neutrosophic values represented by $Na_i = a_i \pm \varepsilon_i$, $Nb_j = b_j \pm \delta_j$ respectively, where $\varepsilon_i = [\lambda_{i1}, \lambda_{i2}]$, $\delta_j = [\mu_{j1}, \mu_{j2}]$.

3- This kind of neutrosophic transportation problem is represented gathered from the above two cases.

Keywords: Linear Programming; Neutrosophic Transportation Problem (NTP); Neutrosophic Production Quantities; Neutrosophic Consumption Quantities.

Introduction

The operation research specifically mathematical programming is used in the daily recurrent problems that appear each time when we need to transfer materials from the production centers to the consumption centers.

After the transportation problems have been formulated, the yielded linear models will be solved by simplex method and its modifications. [1-5]. This manuscript contains a modeling study transportation problems using neutrosophic logic that first adopted by F Smarandache (1995) [6-9] is took recently a huge solicitude in effectively addressing the potential uncertainties in the real world, neutrosophic logic comes as a replacement to the fuzzy logic presented by L. Zadeh (1965) [10], intuitionistic fuzzy logic presented by K. Atanassov (1983) [11].

1. Discussion and the General Formulation of the NTP

There is no doubt of the TP importance in any Inc., because of the high costs paid by institutions and companies to secure their needs of raw materials or through the marketing of their products or even the process of transferring their administrative and functional members, so it was necessary to present a study that keeps with the frontiers of modern science in which this article studies transportation issues using the neutrosophic logic that takes into account all the changes that can occur during work, and provides companies with a safe working environment.

Assume that one material may be transferred from the production center $A_i, i = 1,2, \dots, m$ to the consumption center $B_j, j = 1,2, \dots, n$,where $a_1, a_2, \dots, a_m$ are available quantities, $b_1, b_2, \dots, b_n$ are required quantities, C_{ij} is the transfer cost of one unit from the production center i to the consumption center j, and are represent the entries of the payment matrix $C = [c_{ij}]$. To construct the mathematical model, x_{ij} denotes the transferred amount of material from the production center i to the consumption center j. The following tableau contains the basic symbols of the any transportation problems, so any later table will be read out of this table:

PC \ **CC**	B_1	B_2	B_3	…	B_n	**AQ**
A_1	c_{11} x_{11}	c_{12} x_{12}	c_{13} x_{13}	…	c_{1n} x_{1n}	a_1
A_2	c_{21} x_{21}	c_{22} x_{22}	c_{23} x_{23}	…	c_{2n} x_{2n}	a_2
A_3	c_{31} x_{31}	c_{32} x_{32}	c_{33} x_{33}	…	c_{3n} x_{3n}	a_3
. .	…	…	…	…	…	. .
A_m	c_{m1} x_{m1}	c_{m2} x_{m2}	c_{m3} x_{m3}	…	c_{mn} x_{mn}	a_m
RQ	b_1	b_2	b_3	…	b_n	

In any TP, there are two cases:

1- Balanced model satisfying $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

2- Unbalanced model at $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$.

For the purposes of generalizing any mathematical symbol in the above-mentioned tableau from its classical model to its neutrosophic meaning, it is enough to add a prefix N to indicate that this symbol will hold some indeterminacy as follow:

a. The matrix of transferring cost of one unit from the production center i to the consumption center j in its neutrosophic value is $Nc_{ij} = [c_{ij} \pm \varepsilon]$, where $\varepsilon = [\lambda_1, \lambda_2]$ represents the indeterminate value.

b. The symbol Nx_{ij} refers to the materials' amount that transported from the production center i to the consumption center j, so the matrix of the unknowns is written as $NX = [Nx_{ij}]$.

c. The neutrosophic meaning of the available quantities and the required quantities are $Na_i = a_i \pm \varepsilon_i$, $Nb_j = b_j \pm \delta_j$ respectively, where $\varepsilon_i = [\lambda_{i1}, \lambda_{i2}]$, $\delta_j = [\mu_{j1}, \mu_{j2}]$.

d. The neutrosophic encoding of the objective function in the linear programming is $NZ = \sum_{i=1}^{m} \sum_{j=1}^{n} Nc_{ij}\, x_{ij}$, or $NZ = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, Nx_{ij}$, or $NZ = \sum_{i=1}^{m} \sum_{j=1}^{n} Nc_{ij}\, Nx_{ij}$.

So, in this article, the authors assumed the representation of neutrosophic numbers as intervals such as $\boldsymbol{\varepsilon_i = [\lambda_{i1}, \lambda_{i2}]}$, $\boldsymbol{\delta_j = [\mu_{j1}, \mu_{j2}]}$. It is important to notice that the authors did not adopt (trapezoidal numbers, pentagonal numbers, or any other neutrosophic numbers which need to specify using the membership functions, this kind of neutrosophic numbers or parameters represented by intervals have been firstly introduced by Smarandache F. in his main published books [12-14]),

2. Types of Unbalanced Neutrosophic Transportation Problems

Without loss of generality, any solver can faces unbalanced NTP (i.e. $\sum_{i=1}^{m} Na_i \neq \sum_{j=1}^{n} Nb_j$) in which two types of problems can be extracted:

1. Overproduction problems occur when $\sum_{i=1}^{m} Na_i > \sum_{j=1}^{n} Nb_j$. To treat this case, the solver should balance this problem by adding an artificial consumption center B_{n+1} has need of

$Nb_{n+1} = \sum_{i=1}^{m} Na_i - \sum_{j=1}^{n} Nb_j$, where the transformation cost of one unit from the all production centers to this artificial consumption center equal to zero (i.e. $c_{i\,n+1} = 0; i = 1,2,\dots,m$), also, in the data table, the solver should add a new column concerning the new consumption center. The conditions of this linear programming problems that should be satisfied are:

$\sum_{j=1}^{n+1} Nx_{ij} = Na_i \quad ; \quad \sum_{i=1}^{m} Nx_{ij} = Nb_j \ ; \ Nx_{ij} \geq 0 \ ; \ j = 1,2,\dots n+1 \ ; i = 1,2,\dots,m$

2. production deficient case (type of problems having production shortfall) occur when $\sum_{i=1}^{m} Na_i < \sum_{j=1}^{n} Nb_j$. The same above strategy of balancing the problem has been applied by adding an artificial production center A_{m+1} has production power of $Na_{m+1} = \sum_{j=1}^{n} Nb_j - \sum_{i=1}^{m} Na_i$, where the transformation cost of one unit from this artificial production center to all consumption centers equal to zero (i.e. $c_{m+1\,j} = 0; j = 1,2,\dots,n$), also, in the data table, the solver should add a new row concerning the new production center. The conditions of this linear programming problems that should be satisfied are:

$\sum_{j=1}^{n} Nx_{ij} = Na_i \quad ; \quad \sum_{i=1}^{m+1} Nx_{ij} = Nb_j \ ; \ Nx_{ij} \geq 0 \ ; \ j = 1,2,\dots,n \ ; i = 1,2,\dots,m+1$

3. Miscellaneous NT Problems

In the upcoming subsections (3.1,3.2,3.3), the problem text will be: A quantity of fuel is intended to be shipped from three stations A_1, A_2, A_3 to four cities B_1, B_2, B_3, B_4. The available quantities at each station, and the demand quantities in each city, with the transportation costs in each direction are demonstrated in accompanied tables;

3.1 Balanced Neutrosophic Transportation Problem (NTP)

CC / PC	B_1	B_2	B_3	B_4	AQ
A_1	$7+\varepsilon$ Nx_{11}	$4+\varepsilon$ Nx_{12}	$15+\varepsilon$ Nx_{13}	$9+\varepsilon$ Nx_{14}	$120+\varepsilon_1$
A_2	$11+\varepsilon$ Nx_{21}	$2+\varepsilon$ Nx_{22}	$7+\varepsilon$ Nx_{23}	$3+\varepsilon$ Nx_{24}	$80+\varepsilon_2$

A_3	$4+\varepsilon$ Nx_{31}	$5+\varepsilon$ Nx_{32}	$2+\varepsilon$ Nx_{33}	$8+\varepsilon$ Nx_{34}	$100+\varepsilon_3$
RQ	$85+\delta_1$	$65+\delta_2$	$90+\delta_3$	$60+\delta_4$	

By assuming $\varepsilon=[0,2], \varepsilon_1=[0,35], \varepsilon_2=[0,10], \varepsilon_3=[0,15], \delta_1=[0,7], \delta_2=[0,18], \delta_3=[0,25], \delta_4=[0,10]$, the above table can be rewritten as:

CC / PC	B_1	B_2	B_3	B_4	**AQ**
A_1	[7,9] Nx_{11}	[4,6] Nx_{12}	[15,17] Nx_{13}	[9,11] Nx_{14}	[120,155]
A_2	[11,13] Nx_{21}	[2,4] Nx_{22}	[7,9] Nx_{23}	[3,5] Nx_{24}	[80,90]
A_3	[4,6] Nx_{31}	[5,7] Nx_{32}	[2,4] Nx_{33}	[8,10] Nx_{34}	[100,115]
RQ	[85,92]	[65,83]	[90,115]	[60,70]	

Obviously, the problem is balanced cause $\sum_{i=1}^{3} Na_i = \sum_{j=1}^{4} Nb_j = [300,360]$

The model of the linear programming is

$$\text{Min } NZ = [7,9]Nx_{11} + [4,6]Nx_{12} + [15,17]Nx_{13} + [9,11]Nx_{14} + [11,13]Nx_{21} + [2,4]Nx_{22} + [7,9]Nx_{23} + [3,5]Nx_{24} + [4,6]Nx_{31} + [5,7]Nx_{32} + [2,4]Nx_{33} + [8,10]Nx_{34}$$

Subject to

$$\sum_{j=1}^{4} Nx_{ij} = a_i \quad ; \quad \sum_{i=1}^{3} Nx_{ij} = b_j \ ; \ Nx_{ij} \geq 0 \ ; \ j = 1,2,3,4 \ ; i = 1,2,3$$

3.2 Unbalanced Overproduction NTP Case Study

CC / PC	B_1	B_2	B_3	B_4	**AQ**
A_1	$7+\varepsilon$ Nx_{11}	$4+\varepsilon$ Nx_{12}	$15+\varepsilon$ Nx_{13}	$9+\varepsilon$ Nx_{14}	$120+\varepsilon_1$

A_2	$11+\varepsilon$ Nx_{21}	$2+\varepsilon$ Nx_{22}	$7+\varepsilon$ Nx_{23}	$3+\varepsilon$ Nx_{24}	$95+\varepsilon_2$
A_3	$4+\varepsilon$ Nx_{31}	$5+\varepsilon$ Nx_{32}	$2+\varepsilon$ Nx_{33}	$8+\varepsilon$ Nx_{34}	$100+\varepsilon_3$
RQ	$85+\delta_1$	$65+\delta_2$	$90+\delta_3$	$60+\delta_4$	

By assuming $\varepsilon=[0,2], \varepsilon_1=[0,35], \varepsilon_2=[0,10], \varepsilon_3=[0,15], \delta_1=[0,7], \delta_2=[0,18], \delta_3=[0,25], \delta_4=[0,10]$, the above table can be rewritten as:

PC \ CC	B_1	B_2	B_3	B_4	**AQ**
A_1	[7,9] Nx_{11}	[4,6] Nx_{12}	[15,17] Nx_{13}	[9,11] Nx_{14}	[120,155]
A_2	[11,13] Nx_{21}	[2,4] Nx_{22}	[7,9] Nx_{23}	[3,5] Nx_{24}	[95,105]
A_3	[4,6] Nx_{31}	[5,7] Nx_{32}	[2,4] Nx_{33}	[8,10] Nx_{34}	[100,115]
RQ	[85,92]	[65,83]	[90,115]	[60,70]	

It is obvious that $\sum_{i=1}^{3} Na_i=[315,375]>\sum_{j=1}^{4} Nb_j=[300,360]$; this lead to add an artificial consumption center b_5 has need of the value $b_5=\sum_{i=1}^{m} a_i-\sum_{j=1}^{n} b_j=[315,375]-[300,360]=[15,15]=15$

Hence, the value of the objective function is

$$\begin{aligned}\text{Min } NZ = {} & [7,9]Nx_{11}+[4,6]Nx_{12}+[15,17]Nx_{13}+[9,11]Nx_{14}+0.Nx_{15}+[11,13]Nx_{21}+[2,4]Nx_{22} \\ & +[7,9]Nx_{23}+[3,5]Nx_{24}+0.Nx_{25}+[4,6]Nx_{31}+[5,7]Nx_{32}+[2,4]Nx_{33}+[8,10]Nx_{34} \\ & +0.Nx_{35}\end{aligned}$$

Subject to

$$\sum_{j=1}^{5} Nx_{ij}=a_i \quad ; \quad \sum_{i=1}^{3} Nx_{ij}=b_j \ ; \ Nx_{ij}\ge 0 \ ; \ j=1,2,3,4,5\ ; i=1,2,3$$

3.3 Unbalanced Production Deficient NTP Case Study

PC \ CC	B_1	B_2	B_3	B_4	**AQ**
A_1	$7+\varepsilon$ Nx_{11}	$4+\varepsilon$ Nx_{12}	$15+\varepsilon$ Nx_{13}	$9+\varepsilon$ Nx_{14}	$120+\varepsilon_1$
A_2	$11+\varepsilon$ Nx_{21}	$2+\varepsilon$ Nx_{22}	$7+\varepsilon$ Nx_{23}	$3+\varepsilon$ Nx_{24}	$80+\varepsilon_2$
A_3	$4+\varepsilon$ Nx_{31}	$5+\varepsilon$ Nx_{32}	$2+\varepsilon$ Nx_{33}	$8+\varepsilon$ Nx_{34}	$100+\varepsilon_3$
RQ	$85+\delta_1$	$100+\delta_2$	$90+\delta_3$	$60+\delta_4$	

Where, $\varepsilon=[0,2], \varepsilon_1=[0,35], \varepsilon_2=[0,10], \varepsilon_3=[0,15], \delta_1=[0,7], \delta_2=[0,18], \delta_3=[0,25], \delta_4=[0,10]$, the above table can be rewritten as:

PC \ CC	B_1	B_2	B_3	B_4	**AQ**
A_1	[7,9] Nx_{11}	[4,6] Nx_{12}	[15,17] Nx_{13}	[9,11] Nx_{14}	[120,155]
A_2	[11,13] Nx_{21}	[2,4] Nx_{22}	[7,9] Nx_{23}	[3,5] Nx_{24}	[80,90]
A_3	[4,6] Nx_{31}	[5,7] Nx_{32}	[2,4] Nx_{33}	[8,10] Nx_{34}	[100,115]
RQ	[85,92]	[100,118]	[90,115]	[60,70]	

It is worthy to mention that $\sum_{i=1}^{3} Na_i=[300,360]<\sum_{j=1}^{4} Nb_j=[335,395]$; this lead to add an artificial production center a_4 has production power equal to $a_4=\sum_{j=1}^{4} b_j-\sum_{i=1}^{3} a_i=[315,395]-[300,360]=[35,35]=35$

Hence, the model of the linear programming is

$$\text{Min } NZ = [7,9]Nx_{11} + [4,6]Nx_{12} + [15,17]Nx_{13} + [9,11]Nx_{14} + [11,13]Nx_{21} + [2,4]Nx_{22} + [7,9]Nx_{23} + [3,5]Nx_{24} + [4,6]Nx_{31} + [5,7]Nx_{32} + [2,4]Nx_{33} + [8,10]Nx_{34} + 0.Nx_{41} + 0.Nx_{42} + 0.Nx_{43} + 0.Nx_{44}$$

Subject to

$$\sum_{j=1}^{4} Nx_{ij} = a_i \quad ; \quad \sum_{i=1}^{4} Nx_{ij} = b_j \ ; \ Nx_{ij} \geq 0 \ ; \ j = 1,2,3,4 \ ; i = 1,2,3,4$$

4. Conclusion and Results

This manuscript presented new insights into remodeling transportation problems from classical models to the corresponding neutrosophic ones through three basic aspects containing some indeterminacies either in transformation costs, or in the available quantities existed in production centers side by side with required quantities existing in consumption centers, or in the general case study that represents the existence of indeterminacies in all previous states. The solutions to these neutrosophic transportation problems NTP give the solver a margin of freedom and reduce the loss resulting from transporting process through the availability of the indetermination notion in neutrosophic theory. This study was presented simultaneously with the paper entitled (Original Methods for Obtaining the Initial Solution in Neutrosophic Transportation Problems) regarded as complementary to this recent paper in which the authors discuss the initial solution for the NTP models and study the modifications of this initial solution aiming to get the optimal solutions.

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