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Semi-Separation Axioms and Semi-Regularity Axioms in Linguistic Neutrosophic Topological Spaces

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Abstract. Several new linguistic neutrosophic semi-separation axioms and semi-regularity axioms are presented in this article and example cases are also given for non justifiable results. Additionally, a new class of spaces called linguistic neutrosophic semi-D_0, linguistic neutrosophic semi-D_1 and linguistic neutrosophic semi-D_2 is described and the inter relationships are analyzed with appropriate illustrations.

Keywords: Linguistic Neutrosophic semi-T_i spaces(i=0,1,2); Linguistic Neutrosophic semi-R_p spaces(p=0,1); Linguistic Neutrosophic semi-D_k spaces(k=0,1,2);

1. Introduction

It is known as separation axioms in topology and related fields of mathematics that one often makes several restrictions on the kinds of topological spaces that are to be considered. Maheswari and Prasad [9] generalized T_0, T_1 and T_2 spaces to semi-T_0, semi-T_1 and semi-T_2 respectively. The separation axioms R_0 and R_1 are introduced in topological spaces by Shanin [14] in 1943. Several intriguing results have been obtained by Murdashwar and Naimpally [11] studying the properties of R_0 topological spaces. Also, they proposed a second concept, R_1, was introduced which is independent of T_0 and T_1, but stronger than T_2.

As a continuation of fuzzy sets [17] and eventually intuitionistic fuzzy sets [1], Smarandache [16] introduced the idea of neutrosophic sets. Chang [2], Coker [3] and Salama, Alblowi [14] are the topologists who have instigated the notion of fuzzy topology, intuitionistic fuzzy topology and neutrosophic topology respectively. Meanwhile, Fang [5] found linguistic neutrosophic number which has led to the concept of linguistic neutrosophic topology introduced in 2021 by Gayathri and Helen [6]. In this article, linguistic neutrosophic semi-$T_k, (k = 0, 1, 2)$ spaces and linguistic neutrosophic semi-R_p spaces($p = 0, 1,$) are discussed. Aside from that, the new spaces called linguistic neutrosophic semi-D_k spaces($k = 0, 1, 2$) are introduced and their properties analyzed and numerous relationships are discussed.

Throughout this article, (S_{LN}, τ_{LN}) denotes the linguistic neutrosophic topological space

2. Preliminaries

Definition 2.1. [16] Let S be a space of points (objects), with a generic element in x denoted by S. A neutrosophic set A in S is characterized by a truth-membership function T_A, an indeterminacy membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is $T_A : S \to]0^-, 1^+[, I_A : S \to]0^-, 1^+[, F_A : S \to]0^-, 1^+[$. There is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2.2. [16] Let S be a space of points (objects), with a generic element in x denoted by S. A single valued neutrosophic set (SVNS) A in S is characterized by truth-membership function T_A, indeterminacy-membership function I_A and falsity-membership function F_A. For each point S in S, $T_A(x), I_A(x), F_A(x) \in [0, 1]$. When S is continuous, a SVNS A can be written as $A = \int \langle T(x), I(x), F(x)\rangle / x \in S$. When S is discrete, a SVNS A can be written as $A = \sum \langle T(x_i), I(x_i), F(x_i)\rangle / x_i \in S$.

Definition 2.3. [5] Let $S = \{s_\theta | \theta = 0, 1, 2, ..., \tau\}$ be a finite and totally ordered discrete term set, where τ is the even value and s_θ represents a possible value for a linguistic variable.

Definition 2.4. [5] Let $Q = \{s_0, s_1, s_2, ..., s_t\}$ be a linguistic term set (LTS) with odd cardinality $t+1$ and $\overline{Q} = \{s_h / s_0 \leq s_h \leq s_t, h \in [0, t]\}$. Then, a linguistic single valued neutrosophic set A is defined by,
$A = \{\langle x, s_\theta(x), s_\psi(x), s_\sigma(x)\rangle | x \in S\}$, where $s_\theta(x), s_\psi(x), s_\sigma(x) \in \overline{Q}$ represent the linguistic truth, linguistic indeterminacy and linguistic falsity degrees of S to A, respectively, with condition $0 \leq \theta + \psi + \sigma \leq 3t$. This triplet $(s_\theta, s_\psi, s_\sigma)$ is called a linguistic single valued neutrosophic number.

Definition 2.5. [6]Let $\alpha = (l_\theta, l_\psi, l_\sigma)$ be a LSVNN. The set of all labels is, $L = \{l_0, l_1, l_2, ..., l_t\}$.
Then the unit linguistic neutrosophic(LN in short) set (1_{LN}) is defined as $1_{LN} = (l_t, l_0, l_0)$, which is the truth membership,and the zero linguistic neutrosophic set (0_{LN}) is defined as $0_{LN} = (l_0, l_t, l_t)$, which is the falsehood membership.

Definition 2.6. [6] For a linguistic neutrosophic topology τ_{LN}, the collection of linguistic neutrosophic(LN in short) sets should obey,

(1) $0_{LN}, 1_{LN} \in \tau_{LN}$
(2) $K_1 \bigcap K_2 \in \tau_{LN}$ for any $K_1, K_2 \in \tau_{LN}$
(3) $\bigcup K_i \in \tau_{LN}, \forall \{K_i : i \in J\} \subseteq \tau_{LN}$

We call, the pair (S_{LN}, τ_{LN}), a linguistic neutrosophic topological space.

Definition 2.7. Let (S_{LN}, τ_{LN}) be a LNTS. Then, the LN semi-closure, for a LN subset E_{LN} is defined as the intersection of all LNSCSs in S_{LN}, that are contained in E_{LN}, (i.e) $LNSCl(E_{LN}) = \cap\{K_{LN} : K_{LN}$ is a LNSCS in S_{LN} and $K_{LN} \supseteq E_{LN}\}$.

Definition 2.8. A topological space (S_{LN}, τ_{LN}) is said to be

(1) semi-T_0 [9](semi-Kolomogorov) if for each pair of distinct points in X, there exists a semi-open set containing one but not the other.
(2) semi-T_1 [9](semi-Frechet) if for each pair of distinct points x and y in X, there exist semi-open sets U and V containing x and y such that $x \in U, y \notin V$ and $x \notin U, y \in V$.
(3) semi-T_2 [9](semi-Hausdorff) if every two points can be separated by disjoint semi-open sets.
(4) R_0 [11] if for each open set G, $x \in G \Rightarrow cl(\{x\}) \subseteq G$.
(5) R_1 [11] if for each $x, y \in X$ with $cl(\{x\} \neq cl(\{y\})$, there exist two disjoint open sets U and V such that $cl(\{x\} \subseteq U$ and $cl(\{y\} \in V$.

Definition 2.9. Let S_{LN} be a non-void set and $K_{LN} = \{\langle s, [T_{K_{LN}}, I_{K_{LN}}, F_{K_{LN}}]\rangle\}$ and $H_{LN} = \{\langle s, [T_{H_{LN}}, I_{H_{LN}}, F_{H_{LN}}]\rangle\}$ are LN sets in LNTS.

(I) $K_{LN} \cup H_{LN}$ can be defined as
  (a) $K_{LN} \cup H_{LN} = \{\langle s, [T_{K_{LN}} \wedge T_{H_{LN}}, I_{K_{LN}} \wedge I_{H_{LN}}, F_{K_{LN}} \vee F_{H_{LN}}]\rangle\}$
(II) $K_{LN} \cap H_{LN}$ can be defined as
  (a) $K_{LN} \cap H_{LN} = \{\langle s, [T_{K_{LN}} \wedge T_{H_{LN}}, I_{K_{LN}} \wedge I_{H_{LN}}, F_{K_{LN}} \vee F_{H_{LN}}]\rangle\}$
(III) The complement of $K_{LN} = \{\langle s, [T_{K_{LN}}, I_{K_{LN}}, F_{K_{LN}}]\rangle\}$ is defined as,
  (a) $(K_{LN})^c = \{\langle s, [F_{K_{LN}}, I_{K_{LN}}, T_{K_{LN}}]\rangle\}$
  (b) $((K_{LN})^c)^c = K_{LN}$
  (c) $(K_{LN} \cap H_{LN})^c = (K_{LN})^c \cup (H_{LN})^c$
  (d) $(K_{LN} \cup H_{LN})^c = (K_{LN})^c \cap (H_{LN})^c$

3. Linguistic Neutrosophic Separation Axioms

Definition 3.1. A LNS $P_{LN} = \{\langle s_1, T_{P_{LN}}(s_1), I_{P_{LN}}(s_1), F_{P_{LN}}(s_1)\rangle : s_1 \in S_{LN}\}$ is called a linguistic neutrosophic point(LNP in short) if and only if for any element $s_2 \in S_{LN}$,

$$\begin{cases} T_{P_{LN}}(s_1) = l_p, I_{P_{LN}}(s_1) = l_q, F_{P_{LN}}(s_1) = l_r, & \text{for} \quad s_2 = s_1, \\ T_{P_{LN}}(s_1) = 0, I_{P_{LN}}(s_1) = 0, F_{P_{LN}}(s_1) = 1, & \text{for} \quad s_2 \neq s_1. \end{cases}$$

where $0 < \mathrm{p} \leq t, 0 \leq q < t, 0 \leq r < t.$

Definition 3.2. A LNP $P_{LN} = \{\langle s, T_{P_{LN}}(s), I_{P_{LN}}(s), F_{P_{LN}}(s)\rangle : s \in S_{LN}\}$ will be denoted by ${P_{LN}}^{s}{}_{\langle l_p,l_q,l_r\rangle}$ or $P_{LN}\,\langle s, l_p, l_q, l_r\rangle$ or simply by $s_{\langle l_p,l_q,l_r\rangle}$.
The complement of the LNP ${P_{LN}}^{s}{}_{\langle l_p,l_q,l_r\rangle}$ will be denoted by $({P_{LN}}^{s}{}_{\langle l_p,l_q,l_r\rangle})^{c}$ or ${s^{c}}_{\langle l_p,l_q,l_r\rangle}$.

Definition 3.3. A LNTS (S_{LN}, τ_{LN}) is LN semi-T_0 space if for a couple of distinct points in S_{LN}, there lies a LNSO set containing one point not the other.

Example 3.4. Let the universe of discourse be $U = \{a, b, c\}$. The set of all linguistic term is, $L = \{$ very salt(l_0), salt(l_1), very sour(l_2), sour(l_3), bitter(l_4), sweet(l_5), very sweet(l_6)$\}$. Let $S_{LN} = \{c\}$. Let $s_{1\langle a,l_0,l_2,l_6\rangle}, s_{2\langle a,l_1,l_0,l_6\rangle}$ be any two distinct LN points in S_{LN}. Then $A_{LN} = \langle a, (l_0, l_4, l_6)\rangle$ and $B_{LN} = \langle a, (l_2, l_0, l_6)\rangle$ the LNSOSs that contains the points s_1 and s_2 respectively such that $s_2 \notin A_{LN}$ and $s_1 \notin B_{LN}$.

Theorem 3.5. *A LNTS* (S_{LN}, τ_{LN}) *is semi-*T_0 *iff each couple of points* s_1, s_2 *of* $S_{LN}, LNSCl(\{s_1\}) \neq LNSCl(\{s_2\})$.

Proof:
Necessity Part: Let the space (S_{LN}, τ_{LN}) is LN semi-T_0 and $s_1 \neq s_2$ where $s_1, s_2 \in S_{LN}$. Then there lies a LNSO set V_{LN} with $s_1 \in V_{LN}$ and $s_2 \notin V_{LN}$. So, $S_{LN}\backslash V_{LN}$ is a LNSC set containing s_2 but not s_1. Also, $s_2 \in LNSCl(\{s_2\}) \subseteq S_{LN}\backslash V_{LN}$ but $s_1 \notin LNSCl(\{s_2\})$.
Sufficiency Part: Let $s_1, s_2 \in S_{LN}$ with $s_1 \neq s_2$ where $LNSCl(\{s_1\}) \neq LNSCl(\{s_2\})$. Then, there lies an element $r \in S_{LN}$ with $r \in LNSCl(\{s_1\})$ and $r \notin LNSCl(\{s_2\})$. If $s_1 \in LNSCl(\{s_2\})$, then $LNSCl(\{s_1\}) \subseteq LNSCl(\{s_2\})$.(i.e) $r \in LNSCl(\{s_2\})$ which is a contradiction. Therefore, $s_1 \notin LNSCl(\{s_2\})$. Also, $s_2 \notin S_{LN}\backslash LNSCl(\{s_2\})$ where the set $S_{LN}\backslash LNSCl(\{s_2\})$ is LNSO.

Definition 3.6. Let A_{LN} be a LN subset of (S_{LN}, τ_{LN}). Then LN semi-kernel of A_{LN} is defined by, $LNSKer(A_{LN}) = \cap\{K_{LN} \subseteq S_{LN} | A_{LN} \subseteq K_{LN}$ and $K_{LN} \in LNSO(S_{LN}, \tau_{LN})\}$.

Theorem 3.7. *A LN topological space* (S_{LN}, τ_{LN}) *is semi-*T_0 *iff for any couple of points* s_1, s_2 *of* $S_{LN}, LNSKer(\{s_1\}) \neq LNSKer(\{s_2\})$.

Proof: Necessity Part: Suppose S_{LN} is a LN semi-T_0 space, then $LNSCl(\{s_1\}) \neq LNSCl(\{s_2\})$. Ergo, $LNSKer(\{s_1\}) \neq LNSKer(\{s_2\})$.
Sufficiency Part: Let $s_1 \neq s_2$ where $s_1, s_2 \in S_{LN}$ and $LNSKer(\{s_1\}) \neq LNSKer(\{s_2\})$. Then, $LNSCl(\{s_1\}) \neq LNSCl(\{s_2\})$.

Definition 3.8. A LNTS (S_{LN}, τ_{LN}) is LN semi-T_1 space if for every couple of distinct points in S_{LN}, there lies LNSO sets E_{LN} and F_{LN} containing two points respectively with $E_{LN} \cap F_{LN} = \phi$, (i.e) the intersection must be an empty set rather than zero element.

Example 3.9. Let the universe of discourse be $\mathcal{U} = \{a, b, c\}$ and the LTS be as in the example 3.4. Let $S_{LN} = \{c\}$. Let $s_{1\langle a,l_0,l_2,l_6\rangle}, s_{2\langle a,l_1,l_0,l_6\rangle}$ be any two distinct LN points in S_{LN}. Then $A_{LN} = \langle a,(l_0, l_4, l_6)\rangle$ and $B_{LN} = \langle a,(l_2, l_0, l_6)\rangle$ the LNSOSs that contains the points s_1 and s_2 respectively such that A_{LN} and B_{LN}.

Theorem 3.10. *The upcoming characterizations of a LN semi-T_1 space imply each other.*

(1) *The space S_{LN} is LN semi-T_1 space.*
(2) $\{s\} = LNSCl(\{s\})$ *for every* $s \in S_{LN}$.
(3) *For each $s \in S_{LN}$, the intersection of all LNSO sets containing s is $\{s\}$.*

Proof: (1) ⇒ (2): There lies a LNSO set V_{LN} in S_{LN} with $s_1 \in V_{LN}$ and $s_2 \notin V_{LN}$. If $s_1 \in LNSCl(\{s_2\})$, then s_1 is a LN semi-cluster point of $\{s_2\}$. So, U_{LN} is a LNSO set containing s_1 and $\{s_2\} \cap V_{LN} \neq \phi$, which arrives at a contradiction. Thus, $s_1 \notin LNSCl(\{s_2\})$.
(2) ⇒ (3): Suppose $\{s_1\} = LNSCl(\{s_1\})$. Then, $\{s_1\} \subseteq LNSCl(\{s_1\})$. If $s_2 \in LNSCl(\{s_1\})$, then $s_1 \in LNSKer(\{s_2\})$. Thus, $LNSKer(\{s_1\}) \subseteq \{s_1\}$. Thus, $\{s_1\} = LNSKer(\{s_1\})$. Also, $\{s_1\} = \cap\{V_{LN} : V_{LN} \in LNSO(S_{LN}, \tau_{LN})$ and $s_1 \in U_{LN}\}$.
(3) ⇒ (1): Let the intersections of all LNSO sets containing s is $\{s\}$. And let $s_1 \neq s_2$, where $s_1, s_2 \in S_{LN}$. By the hypothesis, $\{s_1\} = \cap\{V_{LN} : V_{LN} \in LNSO(S_{LN}, \tau_{LN})$ and $s_1 \in U_{LN}\}$. Thus, we can find a LNSO set V_{LN} containing s_1 but not s_2. Therefore, S_{LN} is LN semi-T_1 space.

Theorem 3.11. *The space (S_{LN}, τ_{LN}) is LN semi-T_1 iff the singleton sets are LNSC.*

Proof: Necessity Part: For every singleton set, $\{s\} = LNSCl(\{s\})$.
Sufficiency Part: Let $\{s\}$ is LNSC, $\{s\} = LNSCl(\{s\})$. Then the LNTS S_{LN} is LN semi-T_1.

Definition 3.12. A LNTS (S_{LN}, τ_{LN}) is LN semi-T_2 space if two distinct points s_1 and s_2 can be separated by disjoint LNSO sets U_{LN} and V_{LN} respectively.

Example 3.13. In example 3.9, A_{LN} and B_{LN} are disjoint LNSOSs.

Theorem 3.14. *The underneath characterizations of a LN semi-T_2 space imply each other.*

(1) *The space S_{LN} is LN semi-T_2 space.*
(2) *For every $s_2 \neq s_1$, there is a LNSO set U_{LN} containing s_1 with $s_2 \notin LNSCl(U_{LN})$.*

Proof: (1) ⇒ (2): For each $s_2 \neq s_1$, there lie LNSO sets K_{LN} and H_{LN} with $s_1 \in K_{LN}$ and $s_2 \in H_{LN}$ with $K_{LN} \cap H_{LN} = \phi$. Also, $K_{LN} \subseteq S_{LN}\backslash H_{LN}$ and $s_2 \notin S_{LN}\backslash H_{LN}$, which shows that $s_2 \notin \cap\{S_{LN}\backslash H_{LN} : S_{LN}\backslash H_{LN}$ is LNSC and $K_{LN} \subseteq S_{LN}\backslash H_{LN}\}$.
(2) ⇒ (1): Let $s_2 \neq s_1$, then there lies a LNSO set U_{LN} containing s_1 with $s_2 \notin LNSCl(U_{LN})$. Now, $s_1 \in U_{LN} \subseteq LNSCl(U_{LN})$ and $S_{LN}\backslash LNSCl(U_{LN})$ is LNSO which is evident that $U_{LN} \cap (S_{LN}\backslash LNSCl(U_{LN})) = \phi$.

4. **Linguistic Neutrosophic Regulation Axioms**

Definition 4.1. A LNTS (S_{LN}, τ_{LN}) is semi-R_0 if for each LNSO set K_{LN}, $s \in K_{LN} \Rightarrow LNSCl(\{s\}) \subseteq K_{LN}$.

Example 4.2. Let the universe of discourse be $U = \{x, y, z, w\}$ and let $S_{LN} = \{x, y\}$. The set of all LTS be L= {very strongly disagree(l_0), strongly disagree(l_1), disagree(l_2), mostly disagree(l_3), slightly disagree(l_4), neither disagree nor agree(l_5), slightly agree(l_6), mostly agree(l_7), agree(l_8), strongly agree(l_9), very strongly agree(l_{10})}. Let $F_{LN} = \{(x, \langle l_{10}, l_9, l_2\rangle), (y, \langle l_5, l_7, l_0\rangle)\}$ be an LNSOS. Let $s_{\{(x,\langle l_3,l_6,l_5\rangle),(y,\langle l_7,l_2,l_6\rangle)\}}$ be a LNP. Now $LNSCl(\{s\}) = (E_{LN})^c \subseteq F_{LN}$, where E_{LN} is a LNSOS.

Theorem 4.3. *A LNTS (S_{LN}, τ_{LN}) is LN semi-R_0 iff each LN subset of S_{LN} is the union of LNSC sets.*

Proof: Necessity Part: Let S_{LN} be a LN semi-R_0 space and $A_{LN} \subseteq S_{LN}$. Then for any $s \in A_{LN}, LNSCl(\{s\}) \subseteq A_{LN}$. Also, $\cup\{LNSCl(\{s\}) : s \in A_{LN}\} \subseteq A_{LN}$. Thus, $A_{LN} = LNSCl(\{s\}) = \cup\{LNSCl(\{s\}) : s \in A_{LN}\}$.
Sufficiency Part: Let $s \in A_{LN}$ where A_{LN} is LNSO. Then, there lie LNSC sets U_i with $A_{LN} = \cup\{U_i : i \in I\}$. Since $s \in A_{LN} \Rightarrow s \in U_i : i \in I$. Ergo, $s \in LNSCl(\{s\}) \subseteq U_i \subseteq A_{LN}$.

Remark 4.4. Every LN semi-T_1 space is LN semi-R_0 but not the reverse implication holds true.

Example 4.5. In example 4.2, the space is semi-R_0 but not semi-T_1 as $E_{LN} \cap F_{LN} \neq 0_{LN}$.

Theorem 4.6. *For any LNTS (S_{LN}, τ_{LN}) the upcoming statements imply each other.*

(1) *(S_{LN}, τ_{LN}) is LN semi-R_0.*
(2) *For any LNSC set V_{LN} and for $s \notin V_{LN}$, there lies a $U_{LN} \in LNSO(S_{LN}, \tau_{LN})$ with $s \notin U_{LN}$ and $V_{LN} \subseteq U_{LN}$.*
(3) *$LNSCl(\{s\}) \cap V_{LN} = \phi$, where V_{LN} is a LNSC set and $s \notin V_{LN}$.*

Proof: (1) $\Rightarrow$ (2): Let K_{LN} be a LNSC set with $s \notin K_{LN}$. By the definition, $LNSCl(\{s\}) \subseteq S_{LN} \backslash K_{LN}$ and so $K_{LN} \subseteq S_{LN} \backslash LNSCl(\{s\})$. Then, $S_{LN} \backslash LNSCl(\{s\})$ is the required LNSC set containing K_{LN} and $s \notin S_{LN} \backslash LNSCl(\{s\})$.
(2) $\Rightarrow$ (3): Let K_{LN} be a LNSC set with $s \notin K_{LN}$. By hypothesis, we can find a $U_{LN} \in LNSO(S_{LN}, \tau_{LN})$ with $s \notin U_{LN}$ and $K_{LN} \subseteq U_{LN}$. Suppose $LNSCl(\{s\}) \cap U_{LN} \neq \phi$, then there exists $r \in S_{LN}$ with $r \in U_{LN}$ and $r \in LNSCl(\{s\})$. Now, $H_{LN} \cap \{s\} \neq \phi$,(i.e)$s \in H_{LN}$. The result is that U_{LN} is a LNSO set that contains r and $s \in U_{LN}$, which arrives at a contradiction.
(3) $\Rightarrow$ (1): If H_{LN} is a LNSO set and $s \in H_{LN}$, then $S_{LN} \backslash H_{LN}$ is LNSC and $s \notin S_{LN} \backslash H_{LN}$. By the assumption, $LNSCl(\{s\}) \cap (S_{LN} \backslash H_{LN}) = \phi$.

Theorem 4.7. *In a LNTS* (S_{LN}, τ_{LN}) *for any two points* s, r*, the result* $LNSCl(\{s\}) \neq LNSCl(\{r\}) \Rightarrow LNSCl(\{s\}) \cap LNSCl(\{r\}) = \phi$ *holds iff the LNTS is LN semi-*R_0*.*

Proof: Necessity Part: Let $s, r \in S_{LN}$ with $LNSCl(\{s\}) \neq LNSCl(\{r\})$. Now, suppose we can find an element in $x \in S_{LN}$ with $x \in LNSCl(\{s\})$ and $x \in LNSCl(\{r\})$. There lies a LNSO set U_{LN} containing x with $\{r\} \cap U_{LN} = \phi$ so that $r \notin U_{LN}$. As $x \in LNSCl(\{s\})$, for each LNSO set H_{LN} containing x so that $H_{LN} \cap \{s\} = \phi$, which results in $s \in U_{LN}$. Now, $LNSCl(\{s\}) \subseteq S_{LN} \backslash LNSCl(\{t\})$.
Sufficiency Part: Let $LNSCl(\{s\}) \neq LNSCl(\{r\})$ implies $LNSCl(\{s\}) \cap LNSCl(\{t\}) = \phi$. Let H_{LN} be a LNSO set with $s \in H_{LN}$. If $t \notin H_{LN}$ and so $s \notin LNSCl(\{r\})$. By assumption, $LNSCl(\{s\}) \cap LNSCl(\{t\}) = \phi$ and $t \notin LNSCl(\{s\})$.

Theorem 4.8. *In a space* (S_{LN}, τ_{LN}) *for any two points* s, r*, the result* $LNSKer(\{s\}) \neq LNSKer(\{r\})$ *implies* $LNSKer(\{s\}) \cap LNSKer(\{r\}) = \phi$ *holds iff the LNTS is LN semi-*R_0*.*

Proof: Necessity Part: Let $s, t \in S_{LN}$ with $LNSKer(\{s\}) \neq LNSKer(\{r\})$ and let $x \in LNSKer(\{s\}) \cap LNSKer(\{r\})$. Then $x \in LNSKer(\{s\})$ and $x \in LNSKer(\{t\})$. Then, $s \in LNSCl(\{x\})$ and $t \in LNSCl(\{x\})$ and also $LNSCl(\{s\}) \cap LNSCl(\{x\}) \neq \phi$ and $LNSCl(\{r\}) \cap LNSKer(\{x\}) \neq \phi$. Now, $LNSCl(\{s\}) = LNSCl(\{r\})$. Then, $LNSKer(\{s\}) = LNSKer(\{r\})$ which arrives at a contradiction.
Sufficiency Part: Suppose $LNSKer(\{s\}) \neq LNSKer(\{r\})$ implies $LNSKer(\{s\}) \cap LNSKer(\{r\}) = \phi$. Let $s, t \in S_{LN}$ with $LNSCl(\{s\}) \neq LNSCl(\{r\})$ and let $x \in LNSCl(\{s\}) \cap LNSCl(\{r\})$, then $x \in LNSCl(\{s\})$ and $x \in LNSCl(\{t\})$. Then, $s \in LNSKer(\{x\})$ and $t \in LNSKer(\{x\})$. Now, $LNSKer(\{s\}) = LNSKer(\{x\})$ and $LNSKer(\{t\}) = LNSKer(\{x\})$, also $LNSKer(\{s\}) = LNSKer(\{t\})$. Then, $LNSCl(\{s\}) = LNSCl(\{r\})$ which is a contradiction.

Theorem 4.9. *For any LNTS* (S_{LN}, τ_{LN}) *the following imply each other.*

(1) *The space* (S_{LN}, τ_{LN}) *is LN semi-*R_0*.*
(2) *For a non-zero LN set*A_{LN} *and for a LNSO set* K_{LN}*,* $A_{LN} \cap K_{LN} \neq \phi$*, we can find a* $U_{LN} \in LNSC(S_{LN}, \tau_{LN})$ *with* $A_{LN} \cap U_{LN} \neq \phi$ *and* $U_{LN} \subseteq K_{LN}$*.*
(3) *For any* $H_{LN} \in LNSO(S_{LN}, \tau_{LN}), H_{LN} = \cup\{U_{LN} : U_{LN} \in LNSC(S_{LN}, \tau_{LN})$ *and* $U_{LN} \subseteq H_{LN}\}$*.*
(4) *For any* $K_{LN} \in LNSC(S_{LN}, \tau_{LN}), K_{LN} = \cap\{H_{LN} : H_{LN} \in LNSO(S_{LN}, \tau_{LN})\}$ *and* $U_{LN} \subseteq H_{LN}\}$*.*
(5) *For any* $s \in S_{LN}, LNSCl(\{s\}) \subseteq LNSKer(\{s\})$*.*
(6) *For any* $s, t \in S_{LN}$*,* $t \in LNSCl(\{s\}) \Leftrightarrow s \in LNSCl(\{t\})$*.*

Proof: (1) $\Rightarrow$ (2): Let $(\phi \neq A_{LN}) \subseteq S_{LN}$ and $K_{LN} \in LNSO(S_{LN}, \tau_{LN})$ with $A_{LN} \cap K_{LN} \neq \phi$ and let $s \in A_{LN} \cap K_{LN}$. Thus, $LNSCl(\{s\}) \cap A_{LN} \neq \phi$.

(2) $\Rightarrow$ (3): If $K_{LN} \in LNSO(S_{LN}, \tau_{LN})$ and $s \in K_{LN}$, then one can find a LNSC set U_{LN} with $\{s\} \cap U_{LN} \neq \phi$ and $U_{LN} \subseteq K_{LN}$. This implies $s \in U_{LN}$ and $U_{LN} \subseteq K_{LN}\}$ and so $K_{LN} \subseteq \cup\{U_{LN} : U_{LN} \in LNSC(S_{LN}, \tau_{LN})$ and $U_{LN} \subseteq K_{LN}\}$. Also, $\cup\{U_{LN} : U_{LN} \in LNSC(S_{LN}, \tau_{LN})$ and $U_{LN} \subseteq K_{LN}\} \subseteq K_{LN}$.

(3) $\Rightarrow$ (4): If $U_{LN} \in LNSC(S_{LN}, \tau_{LN}$, then $S_{LN} \backslash U_{LN} \in LNSO(S_{LN}, \tau_{LN})$. By hypothesis, $S_{LN} \backslash U_{LN} = \cup\{S_{LN} \backslash K_{LN} : S_{LN} \backslash K_{LN} \in LNSC(S_{LN}, \tau_{LN})$ and $S_{LN} \backslash K_{LN} \subseteq S_{LN} \backslash U_{LN}\}$. This implies that $U_{LN} = \cap\{K_{LN} : K_{LN} \in LNSO(S_{LN}, \tau_{LN})$ and $U_{LN} \subseteq K_{LN}\}$.

(4) $\Rightarrow$ (5): If $t \notin LNSKer(\{s\})$, then $s \notin LNSCl(\{t\})$. Then we can find a LNSO set V_{LN} containing s with $V_{LN} \cap \{t\} = \phi$, which implies that $LNSCl(\{t\}) \cap V_{LN} = \phi$. Then, $LNSCl(\{t\}) = \cap\{K_{LN} : K_{LN} \in LNSO(S_{LN}, \tau_{LN})$ and $LNSCl(\{t\}) \subseteq K_{LN}\}$. Since $s \in V_{LN}$, we have $s \notin LNSCl(\{t\})$ and so there exists $K_{LN} \in LNSO(S_{LN}, \tau_{LN})$ with $LNSCl(\{t\}) \subseteq K_{LN}$ and $s \notin K_{LN}$. This follows that $LNSCl(\{s\}) \cap K_{LN} = \phi$. Thus, $t \notin LNSCl(\{s\})$ and so $LNSCl(\{s\}) \subseteq LNSKer(\{s\})$.

(5) $\Rightarrow$ (6): If $t \in LNSCl(\{s\})$ then by hypothesis, $t \in LNSKer(\{s\})$ and $s \in LNSCl(\{t\})$. Similarly, if $s \in LNSCl(\{t\})$ and $s \in LNSKer(\{t\})$ then $t \in LNSKer(\{s\})$. This shows that $s \in LNSCl(\{y\}) \Leftrightarrow t \in LNSKer(\{s\})$.

(6) $\Rightarrow$ (1): Let K_{LN} be a LNSO set in (S_{LN}, τ_{LN}) and let $s \in K_{LN}$. If $t \notin K_{LN}$,then $t \in S_{LN} \backslash K_{LN}$. Since $LNSCl(\{t\})$ is the smallest LNSC set that contains t, we have $t \in LNSCl(\{t\}) \subseteq S_{LN} \backslash K_{LN}$. Then $LNSCl(\{t\}) \cap K_{LN} = \phi$, which results that $s \notin LNSCl(\{t\})$.

Theorem 4.10. *For any LNTS (S_{LN}, τ_{LN}) the following imply each other.*

(1) *(S_{LN}, τ_{LN}) is LN semi-R_0.*
(2) *If H_{LN} is LNSC, then $H_{LN} = LNSKer(H_{LN})$.*
(3) *If H_{LN} is LNSC and $s \in H_{LN}$, then $LNSKer(\{s\}) \subseteq H_{LN}$.*
(4) *If $s \in S_{LN}$, then $LNSKer(\{s\}) \subseteq LNSCl(\{s\})$.*

Proof: Proof is direct. (1) $\Rightarrow$ (2): Let H_{LN} be a LNSC and $s \notin H_{LN}$. Then $S_{LN} \backslash H_{LN}$ is a LNSO set containing s. Then by definition, $LNSCl(\{s\}) \subseteq S_{LN} \backslash H_{LN}$ and also $LNSCl(\{s\}) \cap H_{LN} = \phi$. Also, $s \notin LNSKer(H_{LN})$. This means that $LNSKer(H_{LN}) \subseteq H_{LN}$.

(2) $\Rightarrow$ (3): Proof is direct.

(3) $\Rightarrow$ (4): Let $s \in LNSCl(\{s\})$ and the set $LNSCl(\{s\})$ is LNSC. From the assumption, $LNSKer(\{s\}) \subseteq LNSCl(\{s\})$.

(4) $\Rightarrow$ (1): Let $s \in LNSCl(\{t\})$. Then $t \in LNSKer(\{s\})$ and by hypothesis $t \in LNSCl(\{s\})$. On the other hand, let $t \in LNSCl(\{s\})$. Then, $s \in LNSKer(\{t\})$ and $s \in LNSCl(\{t\})$. This reveals that $s \in LNSCl(\{t\})$ iff $t \in LNSCl(\{s\})$.

Theorem 4.11. *For any LNTS* (S_{LN}, τ_{LN}) *the following imply each other.*

(1) *The space* (S_{LN}, τ_{LN}) *is LN semi-*R_0.

(2) *For* $A_{LN} \neq \phi$ *and for a LNSO set* K_{LN}, $A_{LN} \cap K_{LN} \neq \phi$, *we can find a* $U_{LN} \in LNSC(S_{LN}, \tau_{LN})$ *with* $A_{LN} \cap U_{LN} \neq \phi$ *and* $U_{LN} \subseteq K_{LN}$.

(3) *For any* $H_{LN} \in LNSO(S_{LN}, \tau_{LN}), H_{LN} = \cup\{U_{LN} : U_{LN} \in LNSC(S_{LN}, \tau_{LN})$ *and* $U_{LN} \subseteq H_{LN}\}$.

(4) *For any* $K_{LN} \in LNSC(S_{LN}, \tau_{LN}), K_{LN} = \cap\{H_{LN} : H_{LN} \in LNSO(S_{LN}, \tau_{LN})\}$ *and* $U_{LN} \subseteq H_{LN}\}$.

(5) *For any* $s \in S_{LN}, LNSCl(\{s\}) \subseteq LNSKer(\{s\})$.

(6) *For any* $s, t \in S_{LN}$, $t \in LNSCl(\{s\}) \Leftrightarrow s \in LNSCl(\{t\})$.

Proof: (1) $\Rightarrow$ (2): Proof is direct. (2) $\Rightarrow$ (3): If $K_{LN} \in LNSO(S_{LN}, \tau_{LN})$ and $s \in K_{LN}$, then we can find a LNSC set U_{LN} in S_{LN} with $\{s\} \cap U_{LN} \neq \phi$ where $U_{LN} \subseteq K_{LN}$. This implies that $s \in U_{LN}$ and also $s \in \cup\{U_{LN} : U_{LN} \in LNSC(S_{LN}, \tau_{LN})$ and $U_{LN} \subseteq K_{LN}$ and so $K_{LN} \subseteq \cup\{U_{LN} : U_{LN} \in LNSC(S_{LN}, \tau_{LN})$. Also, $\cup\{U_{LN} : U_{LN} \in LNSC(S_{LN}, \tau_{LN})$ and $U_{LN} \subseteq K_{LN}\} \subseteq K_{LN}$.
(3) $\Rightarrow$ (4): If $U_{LN} \in LNSC(S_{LN}, \tau_{LN})$, then $S_{LN} \backslash U_{LN} \in LNSO(S_{LN}, \tau_{LN})$. By hypothesis, $S_{LN} \backslash U_{LN} = \cup\{S_{LN} \backslash K_{LN} : S_{LN} \backslash K_{LN} \in LNSC(S_{LN}, \tau_{LN})$ and $S_{LN} \backslash K_{LN} \subseteq S_{LN} \backslash U_{LN}\}$. Thus $U_{LN} = \cap\{K_{LN} : K_{LN} \in LNSO(S_{LN}, \tau_{LN})$ and $U_{LN} \subseteq K_{LN}\}$.
(4) $\Rightarrow$ (5): If $t \notin LNSKer(\{s\})$, then $s \notin LNSCl(\{t\})$. Then we can find a LNSO set V_{LN} containing s with $V_{LN} \cap \{t\} = \phi$, which implies that $LNSCl(\{t\}) \cap V_{LN} = \phi$. Then $LNSCl(\{t\}) = \cap\{K_{LN} : K_{LN} \in LNSO(S_{LN}, \tau_{LN})$ and $LNSCl(\{t\}) \subseteq K_{LN}\}$. Since $s \in V_{LN}$, $s \notin LNSCl(\{t\})$ and so there lies $K_{LN} \in LNSO(S_{LN}, \tau_{LN})$ with $LNSCl(\{t\}) \subseteq K_{LN}$ and $s \notin K_{LN}$.
Proof is direct for (5) $\Rightarrow$ (6) and (6) $\Rightarrow$ (1).

Theorem 4.12. *For any LNTS* (S_{LN}, τ_{LN}) *the following are equivalent.*

(1) (S_{LN}, τ_{LN}) *is LN semi-*R_0.

(2) *If* H_{LN} *is LNSC, then* $H_{LN} = LNSKer(H_{LN})$.

(3) *If* H_{LN} *is LNSC and* $s \in H_{LN}$, *then* $LNSKer(\{s\}) \subseteq H_{LN}$.

(4) *If* $s \in S_{LN}$, *then* $LNSKer(\{s\}) \subseteq LNSCl(\{s\})$.

Proof: (1) $\Rightarrow$ (2): Let H_{LN} be a LNSC and $s \notin H_{LN}$. Then $S_{LN} \backslash H_{LN}$ is a LNSO set containing s. Then, $LNSCl(\{s\}) \subseteq S_{LN} \backslash H_{LN}$ and also $LNSCl(\{s\}) \cap H_{LN} = \phi$. $s \notin LNSKer(H_{LN})$. This means that $LNSKer(H_{LN}) \subseteq H_{LN}$. Now, $H_{LN} \subseteq LNSKer(H_{LN})$.
(2) $\Rightarrow$ (3): Proof is direct.
(3) $\Rightarrow$ (4): Proof is direct.
(4) $\Rightarrow$ (1): Let $s \in LNSCl(\{t\})$. Then, $t \in LNSKer(\{s\})$ and $t \in LNSCl(\{s\})$. Let

$t \in LNSCl(\{s\})$. Then, $s \in LNSKer(\{t\})$ and $s \in LNSCl(\{t\})$. This results that $s \in LNSCl(\{t\})$ iff $t \in LNSCl(\{s\})$. Also , the LNTS is LN semi-R_0.

Definition 4.13. A LNTS (S_{LN}, τ_{LN}) is semi-R_1 if for any couple of points $s_1, s_2 \in S_{LN}$, with $LNSCl(\{s_1\}) \neq LNSCl(\{s_2\})$, there lie LNSO sets K_{LN} and H_{LN} with $LNSCl(\{s_1\}) \subseteq K_{LN}$ and $LNSCl(\{s_2\}) \subseteq H_{LN}$ where $K_{LN} \cap H_{LN} = \phi$.

Example 4.14. Let the universe of discourse be $U = \{x, y, z, w\}$ and let $S_{LN} = \{x, y, z\}$. The set of all LTS be L= { very poor (l_0), poor(l_1), very weak(l_2), weak(l_3), below average (l_4), average (l_5), above average (l_6), good (l_7), very good (l_8), excellent (l_9), outstanding (l_{10})}. Let $s_{1\{}(x, \langle l_1, l_2, L_5\rangle), (y, \langle l_0, l_4, L_6\rangle), (z, \langle l_4, l_4, l_9\rangle)\}$, $s_{2\{}(x, \langle l_2, l_2, l_4\rangle), (y, \langle l_1, l_4, l_5\rangle), (z, \langle l_7, l_4, l_9\rangle)\}$ be two distinct LNPs. Now, $LNSCl(\{s_1\}) = A_{LN} \subseteq K_{LN}$ and $LNSCl(\{s_2\}) = B_{LN} \subseteq H_{LN}$, where A_{LN}, B_{LN} are LNSCSs and $K_{LN}.H_{LN}$ are LNSOSs given by,
$A_{LN} = \{(x, \langle l_1, l_3, l_4\rangle), (y, \langle l_1, l_6, l_5\rangle), (z, \langle l_5, l_8, l_9\rangle)\}$
$B_{LN} = \{(x, \langle l_4, l_5, l_3\rangle), (y, \langle l_2, l_6, l_1\rangle), (z, \langle l_8, l_5, l_7\rangle)\}$
$K_{LN} = \{(x, \langle l_5, l_6, l_1\rangle), (y, \langle l_4, l_6, l_1\rangle), (z, \langle l_9, l_8, l_5\rangle)\}$
$H_{LN} = \{(x, \langle l_5, l_6, l_1\rangle), (y, \langle l_4, l_7, l_1\rangle), (z, \langle l_9, l_6, l_8\rangle)\}$

Theorem 4.15. *Every LN semi-R_1 space is LN semi-R_0 space.*

Proof: If $S_{LN}, \tau_{LN})$ is LN semi-R_1 and K_{LN} be a LNSO set in S_{LN}. Then for any $s_1 \in S_{LN}$ and $s_2 \in S_{LN} \backslash K_{LN}$, $s_1 \neq s_2$ which implies $LNSCl(\{s_1\}) \neq LNSCl(\{s_2\})$. We can find two disjoint LNSO sets K_{s_2} and H_{s_2} with $LNSCl(\{s_1\}) \subseteq K_{s_2}$ and $LNSCl(\{s_2\}) \subseteq H_{s_2}$. Let $H_{LN} = \cup\{H_{s_2}/s_2 \in S_{LN} \backslash K_{LN}\}$. If $s_1 \in LNSCl(\{s_1\}) \subseteq K_{s_2}$ and $K_{s_2} \cap H_{s_2} = \phi$ for every $s_2 \in S_{LN} \backslash K_{LN}$ and so $s_1 \notin H_{s_2}$ for each $s_2 \in S_{LN} \backslash K_{LN}$ which implies $s_1 \notin H_{LN}$. Now, $s_1 \in LNSCl(\{s_1\}) \subseteq S_{LN} \backslash H_{LN} K_{LN}$.

Remark 4.16. The reverse implication of the above theorem need not be true unless one condition is satisfied.

Theorem 4.17. *A LN semi-R_0 space is LN semi-R_1, if for each couple of points s_1 and s_2 in S_{LN} satisfying $LNSCl(\{s_1\}) \neq LNSCl(\{s_2\})$ with we can find two disjoint LNSO sets K_{LN} and H_{LN} so that $s_1 \in K_{LN}$ and $s_2 \in H_{LN}$ respectively.*

Proof: Let the space S_{LN} be LN semi-R_0. Also for each couple of points s_1 and s_2 in S_{LN} satisfying $LNSCl(\{s_1\}) \neq LNSCl(\{s_2\})$, there lie LNSO sets K_{LN} and H_{LN} so that $s_1 \in K_{LN}$ and $s_2 \in H_{LN}$ with $K_{LN} \cap H_{LN} = \phi$. Then, $LNSCl(\{s_1\}) \subseteq K_{LN}$ and $LNSCl(\{s_1\}) \subseteq H_{LN}$.

Theorem 4.18. *A LNTS is semi-R_1 iff for every couple of points $s_1, s_2 \in S_{LN}$ with $LNSKer(\{s_1\}) \neq LNSKer(\{s_2\})$, there lie LNSO sets K_{LN} and H_{LN} in S_{LN} with $LNSCl(\{s_1\}) \subseteq K_{LN}$ and $LNSCl(\{s_2\}) \subseteq H_{LN}$ and $K_{LN} \cap H_{LN} = \phi$.*

Proof: Let s_1 and s_2 be any two points of $S_{LN}, \tau_{LN})$ with $LNSKer(\{s_1\}) \neq LNSKer(\{s_2\})$. Now we can find disjoint LNSO subsets K_{LN} and H_{LN} with $LNSCl(\{s_1\}) \subseteq K_{LN}$ and $LNSCl(\{s_2\}) \subseteq H_{LN}$, which reveals that S_{LN} is semi-R_0.
Conversely, suppose for each couple of points $s_1, s_2 \in S_{LN}$ with $LNSKer(\{s_1\}) \neq LNSKer(\{s_2\})$, there lie disjoint LNSO subsets K_{LN} and H_{LN} with $LNSCl(\{s_1\}) \subseteq K_{LN}$ and $LNSCl(\{s_2\}) \subseteq H_{LN}$. Assume that $LNSCl(\{s_1\}) \neq LNSCl(\{s_2\})$. Then, $LNSKer(\{s_1\}) \neq LNSKer(\{s_2\})$.

Theorem 4.19. *For the LNTS $S_{LN}, \tau_{LN})$, the following imply each other.*

(1) (S_{LN}, τ_{LN}) *is LN semi-T_2.*
(2) (S_{LN}, τ_{LN}) *is both LN semi-R_1 and LN semi-T_1.*
(3) (S_{LN}, τ_{LN}) *is both LN semi-R_1 and LN semi-T_0.*

Proof: (1) $\Rightarrow$ (2): Let the space be LN semi-T$_2$, then the LNTS is LN semi-T$_1$. If there exist two points $s_1, s_2 \in (S_{LN}, \tau_{LN})$ with $LNSCl(\{s_1\}) \neq LNSCl(\{s_2\})$, then $s_1 \neq s_2$. Since the points are distinct, there lie LNSO subsets U_{LN} and V_{LN} with $s_1 \in U_{LN}$, $s_2 \in V_{LN}$ and $U_{LN} \cap V_{LN} = \phi$. This implies $\{s_1\} = LNSCl(\{s_1\}) \subseteq U_{LN}$ and $\{s_2\} = LNSCl(\{s_2\}) \subseteq V_{LN}$.
(2) $\Rightarrow$ (3):Proof is direct.
(3) $\Rightarrow$ (1): Suppose the space is both LN semi-R$_1$ and LN semi-T$_0$. Then for any two distinct points $s_1, s_2 \in (S_{LN}, \tau_{LN})$, we have $LNSCl(\{s_1\}) \neq LNSCl(\{s_2\})$. Then we can find LNSO subsets U_{LN} and V_{LN} with $LNSCl(\{s_1\}) \subseteq U_{LN}$ and $LNSCl(\{s_2\}) \subseteq V_{LN}$.

Theorem 4.20. *A LNTS (S_{LN}, τ_{LN}) is semi-R_1 iff for each points $s_1, s_2 \in (S_{LN}, \tau_{LN})$ with $LNSCl(\{s_1\}) \neq LNSCl(\{s_2\})$, there lie LNSC subsets U_{LN} and V_{LN} with $s_1 \in U_{LN}, s_2 \notin U_{LN}, s_2 \in V_{LN}, s_1 \notin V_{LN}$ and $S_{LN} = U_{LN} \cup V_{LN}$.*

Proof: Let the space (S_{LN}, τ_{LN}) be LN semi-R$_1$ with $LNSCl(\{s_1\}) \neq LNSCl(\{s_2\})$. Then we can find LNSO subsets K_1 and K_2 with $LNSCl(\{s_1\}) \subseteq K_1$ and $LNSCl(\{s_2\}) \subseteq K_2$. Then $U_{LN} = S_{LN} \backslash K_2$ and $V_{LN} = S_{LN} \backslash K_1$ which are LNSC subsets with $s_1 \in U_{LN}, s_2 \notin U_{LN}, s_2 \in V_{LN}, s_1 \notin V_{LN}$ and $S_{LN} = U_{LN} \cup V_{LN}$.
Let $LNSCl(\{s_1\}) \neq LNSCl(\{s_2\})$ for any two disjoint points $s_1, s_2 \in S_{LN}$. Now by the hypothesis, there exists LNSC subsets U_{LN} and V_{LN} with $s_1 \in U_{LN}, s_2 \notin U_{LN}, s_2 \in V_{LN}, s_1 \notin V_{LN}$ and $S_{LN} = U_{LN} \cup V_{LN}$. Then $K_1 = S_{LN} \backslash V_{LN}$ and $K_2 = S_{LN} \backslash U_{LN}$, which are LNSO subsets with $s_1 \in KO1$ and $s_2 \in K_2$ and $K_1 \cap K_2 = \phi$. Ergo, the space is LN semi-T$_2$ and also LN semi-R$_1$.

5. Linguistic Neutrosophic Semi-D$_0$, Semi-D$_1$, Semi-D$_2$ Spaces

Definition 5.1. Let $A_{LN} = \langle s, (T_{A_{LN}}, I_{A_{LN}}, F_{A_{LN}})\rangle$ and $B_{LN} = \langle s, (T_{B_{LN}}, I_{B_{LN}}, F_{B_{LN}})\rangle$ be LNS's, then $A_{LN}\backslash B_{LN}$ is defined by,
$A_{LN}\backslash B_{LN} = \langle s, min(T_{A_{LN}}, F_{B_{LN}}), min(I_{A_{LN}}.I_{B_{LN}}), max(F_{A_{LN}}, T_{B_{LN}})\rangle$.

Definition 5.2. Let A_{LN} be a LN subset of (S_{LN}, τ_{LN}) is a LN semi-difference set(LNSDS in short) if there exist LNSOSs U_{LN} and V_{LN} such that $U_{LN} \subset S_{LN}$ and $A_{LN} = U_{LN}\backslash V_{LN}$. The collection of all LNSDS's is denoted by $LNSD(S_{LN}, \tau_{LN})$.

Remark 5.3. Every LNSOS A_{LN} that is different from S_{LN} is a LNSDS if $U_{LN} = A_{LN}$ and $V_{LN} = \phi$. The reverse implication need not be true, which is given by a counter example.

Example 5.4. Let the universe of discourse $\mathcal{U}$ and LTS be as in example 4.2. The LN sets $E_{LN} = \langle(x, l_4, l_6, l_4), (y, l_8, l_3, l_4)\rangle$ and $F_{LN} = \langle(x, l_{10}, l_9, l_2), (y, l_5, l_7, l_0)\rangle$ are LNSOSs. Now, the LNDS K_{LN} is $E_{LN}\backslash F_{LN} = \langle(x, l_2, l_6, l_0), (y, l_0, l_3, l_5)\rangle$ which is not a LNSOS because $LNInt(LNCl(K_{LN})) = 1_{LN}$.

Definition 5.5. A LNTS (S_{LN}, π_{LN}) is

(1) LN semi-D_0 if for two distinct points $s_1, s_2 \in S_{LN}$ there lies LNSD set containing one of the point but not the other.
(2) LN semi-D_1 if for two distinct points $s_1, s_2 \in S_{LN}$ there lie LNSD sets U_{LN} and V_{LN} with $s_1 \in U_{LN}, s_2 \notin U_{LN}$ and $s_2 \in V_{LN}, s_1 \notin V_{LN}$.
(3) LN semi-D_2 if for two distinct points $s_1, s_2 \in S_{LN}$ there lie LNSD sets U_{LN} and V_{LN} with $s_1 \in U_{LN}, s_2 \in V_{LN}$ and $U_{LN} \cap V_{LN} = \phi$.

Theorem 5.6. *A LNTS (S_{LN}, τ_{LN}) is LN semi-T_0 iff it is LN semi-D_0.*

Proof: Let the space be LN semi-T_0 and let $S_1 \neq s_2$ for $s_1, s_2 \in S_{LN}$. Then by the definition of semi-T_0, there exists a LNSO set U_{LN} containing one of the points but not the other, (i.e) $s_1 \in U_{LN}$ but $s_2 \notin U_{LN}$. Then $U_{LN} \neq S_{LN}$. By remark above, U_{LN} is a LNSD set containing s_1 but not s_2. Hence, (S_{LN}, τ_{LN}) is LN semi-D_0.
Conversely, let the space be LN semi-D_0. Then by the definition of semi-D_0, there exists a LNSD set A_{LN} containing one of the point but not the other, (i.e)$s_1 \in A_{LN}, s_2 \notin A_{LN}$. Thus, there exists LNSO sets $U_{LN}$and V_{LN} with $U_{LN} \neq S_{LN}$ and $A_{LN} = U_{LN}\backslash V_{LN}$. As $s_1 \in A_{LN}, s_1 \in U_{LN}$ but $s_1 \notin V_{LN}$. For $s_2 \notin A_{LN}$, we have two cases.

(i) $s_2 \notin U_{LN}$ but $s_1 \in U_{LN}$.
(ii) $s_2 \in U_{LN}$ and $s_2 \in V_{LN}$. But $s_1 \notin V_{LN}$.

In both cases, the LNTS is LN semi-T_0.

Theorem 5.7. *A LNTS (S_{LN}, τ_{LN}) is LN semi-D_1 iff it is LN semi-D_2.*

Proof: Let the space be LN semi-D_1, then for each pair of distinct points $s_1, s_2 \in S_{LN}$, we can find LNSD sets K_{LN} and H_{LN} with $s_1 \in K_{LN}, s_2 \notin K_{LN}$ and $s_2 \in H_{LN}, s_2 \notin K_{LN}$. Thus, there lie LNSO sets $A_1, A_2, B_1, B_2 \in S_{LN}$ with $A_1 \neq S_{LN}, A_2 \neq S_{LN}, K_{LN} = A_1 B_1$ and $H_{LN} = A_2 \backslash B_2$. For $s_1 \notin H_{LN}$, the two cases are given below.

Case 1: $s_1 \notin B_2$. As $s_2 \notin K_{LN}$, either $s_2 \notin A_1$ or ($s_2 \in A_1$ and $s_2 \in B_1$). If $s_2 \notin A_1$, from $s_2 \in H_{LN} = A_2 \backslash B_2$, (i.e) $s_2 \in A_2 \backslash (B_2 \cup B_1)$ and $s_1 \notin A_2$. Now, we have $s_1 \in A_1 \backslash (B_1 \cup B_2)$ and $(A_2 \backslash (B_2 \cup B_1)) \cap (A_1 \backslash (B_1 \cup B_2)) = \phi$. If $s_2 \in A_1$ and $s_2 \in B_1$, then we have $s_1 \in K_{LN} = A_1 \backslash B_1$ and $\overline{(A_1 \backslash B_1)} \cap B_1 = \phi$.

Case 2: $s_1 \in A_2$ and $s_2 \in B_2$ Now, $s_2 \in H_{LN} = A_2 \backslash B_2, s_1 \in B_2$ and $(A_2 \backslash B_2) \cap B_2 = \phi$. Since $s_1 \notin B_1$ and $s_2 \notin B_2$, B_1 and B_2 are LNSO sets different from (S_{LN}, τ_{LN}). Then by remark(5.2), B_1 and B_2 are LNSD sets. Since $B_2 \cup A_1$ and $B_1 \cup A_2$ are LNSO sets, we have $A_2 \backslash (B_2 \cup A_1)$ and $A_1 \backslash (B_1 \cup A_2)$ are LNSD sets.

Conversely, suppose the space is LN semi-D_2 and let $s_1, s_2 \in S_{LN}$ with $s_1 \neq s_2$. There lie distinct LNSD sets K_{LN} and H_{LN} with $s_1 \in K_{LN}$ and $s_2 \in H_{LN}$, (i.e) $s_1 \in K_{LN}, s_2 \notin K_{LN}$ and $s_2 \in H_{LN}, s_1 \notin H_{LN}$.

6. Conclusion

Separation axioms and axioms of regularity are outlined in a new space referred to as linguistic neutrosophic topological spaces. A variety of concepts and ideas are explored with suitable examples. In addition, semi-different axioms are introduced and discussed through the use of linguistic neutrosophic semi-difference sets, and numerous intriguing results are obtained.

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