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EOQ model with price, marketing, service and green dependent neutrosophic demand under uncertain resource constraint: A geometric programming approach

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Abstract. In the competitive market, a customer's choice for an item depends on several factors like management's marketing strategy and service, the item's price and greenness. Demand increases with the marketing strategy, service and item's greenness, but it is inversely related to the item's price. These relations are non-linear and imprecise. Recently, neutrosophic set has been introduced to represent impreciseness more realistically. Moreover, resources (capital, storage space, etc.) are generally uncertain (random or imprecise). Considering the above business scenarios, profit maximization EOQ models with price, marketing, service, and green dependent neutrosophic demand and order quantity dependent unit production cost are developed under different uncertain resource constraints. Models' parameters are pentagonal neutrosophic (PN) numbers. The proposed models are first made deterministic and then solved using the geometric programming technique. The PN parameters are made crisp using the score function. The random, fuzzy, rough and trapezoidal neutrosophic resource constraints in different models are converted to crisp using possibility measure, chance-constrained technique, trust measure and (α, β, γ)-cut with weighted mean, respectively. These processes reduce the objective function and constraints to signomial forms, and the reduced problems are solved by geometric programming technique with the degree of difficulty 2. Numerical experiments and sensitivity analyses are performed to illustrate the models.

Keywords: Inventory; Pentagonal neutrosophic number; Possibility; Chance constrained programming; Trust measure;

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1. Introduction

Nowadays, integration of the effects of marketing cost, service cost, green cost, etc., into demand in an EOQ model is a realistic production and business strategy. Marketing costs are generally the total expenditure of a manufacturing company on marketing activities. This cost includes advertisement of the products, campaigning, promotional events, market research, etc. Now these activities, and hence marketing costs, directly affect the demand of items. Again, some manufacturing companies spend incentives on their sales representatives for better performance. Sometimes incentives are given to the delivery agents to have perfect timing in delivering the items. These types of expenditure are termed service cost and this cost also directly affects total demand. The demands for green goods are always very high in any market. Green costs include the extra expenditure to produce green goods. Thus the demand of an item increases with its greenness. Moreover, it is well known that an item's demand is inversely related to its selling price, i.e., demand continuously decreases with the price. In practice, the relations mentioned above are not linear and deterministic. Demand is always related to the marketing effort, service provided, greenness and price non-linearly in an imprecise sense, i.e., fuzzy. Recently, neutrosophic set more realistically represents the impreciseness. Following these real-life facts, demand is taken as a non-linear function of marketing and service costs, item's greenness and price in a neutrosophic fuzzy sense. This presentation of demand is new in the literature. Lee and Kim [1] first identified the idea of marketing planning into a classical inventory problem. They formulated the model with price and marketing cost dependent demand and solved using the geometric programming (GP) method. Later, Lee [2, 3] investigated profit maximization problems with optimal selling price and order quantity as decision variables along with some constraints. A multi-objective marketing planning EOQ problem was studied by Islam [4]. Later, marketing cost, selling price and service cost dependent demand was considered by Samadi et al. [5]. They solved the model under a fuzzy environment. Recently, Aggarwal et al. [6] developed an inventory model with price and advertising expenditure dependent demand.

In reality, an inventory model is formulated along with one or more restrictions like a limitation on storage space, order, production cost, etc. Among these restrictions, storage space constraint is very common. A manufacturing company builds or hires a warehouse to store its products at the beginning of production or business. These warehouses bear certain dimension that limits total storage space. In practical situations, this space may not be adequate all the time. Hence, space may be augmented if necessary. This augmentation is usually uncertain, i.e., the total available area may be considered as imprecise, random, rough, etc., in nature. Roy and Maiti [7] investigated a fuzzy EOQ problem under space constraints where demand depends on unit cost. Later, multi-objective inventory problems were considered for deteriorating items with space constraints under fuzzy (cf. [8]) and intuitionistic fuzzy (cf. [9]) environments. Again, Mandal and Islam [10], Panda and Maiti [11] solved an EOQ model with space constraint having fuzzy coefficients by applying the GP method. Recently, Kar et al. [12, 13] proposed neutrosophic GP technique to solve inventory problems with space constraints under neutrosophic environment. Das et al. [14] investigated a multi-item production inventory model with limited storage area under fuzzy environment. Moreover, Karimi and Sadjadi [15] developed a deteriorating multi-item EOQ model under capacity constraint and solved by a dynamic programming approach.

Chance constrained programming is introduced in an optimization problem when the chances of satisfying a certain constraint are above a certain level. In other words, when any constraint involves one or more random parameters, it is called a chance constraint. Charnes and Cooper [16] first developed a chance-constrained programming technique to solve stochastic optimization. Later, it has been extended in various directions [17–19]. An EOQ model for stochastically imperfect products was investigated and solved using chance-constrained programming by Panda et al. [20]. In the recent era, Widyan [21] developed a multi-criteria inventory model with random constraints. Furthermore, Hajiagha et al. [22] solved a multi-criteria fuzzy inventory model using chance-constrained and probabilistic programming based hybrid algorithm.

In the real world scenario, the demand for items in the manufacturing companies changes frequently. We can only note down the sales data of an item for one period and suggest an estimated demand of that item for the next period based on the previous one. Depending on this estimate, which may be fuzzy, rough, random, etc., the amount of resources, such as budget, warehouse space, etc., are determined. In such situations, the rough set theory, developed by Pawlak [23] and others [24], is used to deal with imprecise, inconsistent, incomplete information and knowledge. Later, many researchers have studied the rough set theory in various working fields [25, 26]. Also, Xu and Zhao [27] solved a fuzzy rough multi-objective decision making problem. De et al. [28] investigated an imperfect economic production model over different time horizons. Xu and Yao [29] proposed randomness and roughness simultaneously and established that some parameters follow random distribution with rough expected value. Recently, Bera and Mandal [30] and Midya and Roy [31] investigated multi-objective transportation problems under rough environment.

The theory of impreciseness has been used in various fields in the recent era. The concept of fuzzy set was first came up with Zadeh [32], and after that, many researchers [33–39] extended it and applied it in different problems. Later, Atanassov [40] successfully introduced the generalization of fuzzy set, called intuitionistic fuzzy set (IFS). In IFS, there are two-degree functions: membership and non-membership. Many researchers [41–43] have applied IFS in various fields. Nowadays, to deal with indeterminate/inconsistent information, Smarandache [44–46] developed neutrosophic set (NS). Unlike IFS, NS has three independent components: membership, indeterminacy and falsity. These independent degrees lie within $]0^-, 1^+[$. Later, Wang et al. [47] developed single valued neutrosophic set, and Peng et al. [48] proposed simplified NS. In recent era, Chakraborty et al. [49–51] applied the idea of pentagonal neutrosophic number on different problems. Moreover, Khalid et al. [52–54] and Pramanik et al. [55] introduced neutrosophic GP technique in several fields.

Generally, the GP technique is a very effective method for solving a class of non-linear optimization problems. The GP technique's most remarkable advantage is that this converts a complicated non-linear optimization problem involving highly non-linear constraints into an equivalent linear optimization problem with only linear constraints. The basic concept of GP was initially introduced by Duffin [56]. GP has contributed several applications in various areas such as inventory systems, circuit design, system design, project management, etc. Kochenberger [57] first tackled the inventory problem by GP technique. Later, several researchers [58–61] efficiently applied the GP technique for solving various non-linear problems in different fields.

In spite of the above developments in GP and its application in inventory control problems under uncertain environments, very few researchers have used GP in EOQ with neutrosophic uncertainty (cf. Kar et al. [12]). Only a few analytically expressed the limited resource amounts using neutrosophic numbers. Moreover, there are very few inventory control problems for green products using GP. We have tried to fill up the above lacunas in the present investigation.

In this paper, single item profit maximization inventory models are formulated with selling price, marketing, service and green dependent neutrosophic demand. The models are developed with different uncertain storage space constraints. Parameters of all models' objective functions are considered as PN numbers to formulate the models more realistically. Again, the resource constraint is taken in various environments as fuzzy, random, rough and trapezoidal neutrosophic (TN) numbers to derive particular models. At first, all the models are transformed into equivalent crisp forms using score function, possibility measure, chance-constrained programming, trust measure and (α, β, γ)-cut for PN, fuzzy, random, rough and TN environments respectively. These processes lead both objective function and constraint expression to signomial forms, which are solved using the GP technique. Solution procedures for all models are numerically illustrated. Sensitivity analysis is presented to observe the changes in optimum results against various parameters.

Thus, the main contributions of this investigation are

- For the first time, marketing, service and green expenditures, along with the item's selling price, is integrated into the item's demand to develop the model more realistically. Relations of these parameters with the demand are imprecise, expressed by PN numbers.
- Realistically, in this investigation, uncertain resource capacities are considered as fuzzy, random, rough and neutrosophic.
- Unit production cost is taken as a non-linear function of the order quantity.
- Models are appropriately solved by the GP technique to get the exact values/expressions of the decision variable.
- Due to the presence of neutrosophic parameters in the model, the concept of score function is introduced to convert the model into a crisp maximization problem.
- For converting particular models from fuzzy, random and rough environments to an equivalent crisp form, possibility measure, chance-constrained programming and trust measure

are respectively applied. In the case of TN resource constraint, (α, β, γ)-cuts and weighted arithmetic mean are used.

The remaining part of the present investigation is arranged as follows: Section 2 represents the formulation of the proposed models. Section 3 derives some particular cases. The solution procedures for all the models are explained in Section 4. In Section 5, numerical experiments are performed and the optimum results are described. Section 6 represents a sensitivity analysis. Conclusion and future extensions are presented in Section 7. All required preliminaries are explained in Appendix.

2. Formulation of the proposed models

The proposed models are established using the following notations and assumptions:

Notations:

Inventory related parameters:

Symbol	Explanation
D:	demand rate per unit time
C:	production cost per unit item
A:	set-up cost per period
H:	holding cost per item per unit time
T:	period of each cycle
w:	available total storage capacity area
w_0:	capacity area to store per unit quantity
I(t):	inventory level at any time t (≥ 0)
a:	selling price elasticity to demand
b:	marketing expenditure elasticity to demand
c:	service expenditure elasticity to demand
d:	green expenditure elasticity to demand
θ:	lot size elasticity to unit production cost
Decision variables:	
P:	selling price per unit quantity
Q:	number of order quantity
M:	marketing expenditure per unit item
R:	service expenditure per unit item
G:	green expenditure per unit item

Assumptions:

(a) Replenishment rate is instantaneous.

(b) Shortages are not allowed.

(c) Lead time is negligible.

(d) The inventory system allows a single item.

(e) In real life, it is always seen that when the items are ordered in a lot, the per item production cost reduces with the size of ordered units. Therefore, the unit production cost is inversely related to order quantity. So it is taken as $C = rQ^{-\theta}$, where r is the scaling factor and $0 < \theta < 1$.

(f) It is universally accepted that the demand of an item is negatively influenced by its price - either inversely or linearly. Marketing effort in the form of advertisement in electronic media, displayed hoardings on roadsides, etc., always uplifts the demand. Similarly, the service sector, in the form of timely dispatch, availability, good handling of customers, etc., plays an important role in increasing the sale/demand of an item (cf. Samadi et al. [5]). Nowadays, due to increased environmental consciousness, the demand for green products gradually increases day by day. Thus, greenness also plays a role in increased demand. Hence, demand can be expressed as $D = kP^{-a}M^bR^cG^d$, where k is the scaling factor/market size and $a > 1, 0 < b, c, d < 1$. Normally, the market size for a customized product is considered to be very high. So, the market size parameter, k, is estimated by an uncertain high number (cf. Samadi et al. [5]).

2.1. *Mathematical model*

In the present investigation, the inventory level continuously decreases to satisfy the demand (See Figure 1). If I(t) be the inventory level at time t, then the governing differential equation over the time (0,T) is given by

$$D^{'}I(t) = -D,\ 0 \leqslant t \leqslant T\ , \qquad \left(D^{'} \equiv \frac{d}{dt}\right) \tag{1}$$

where $I(0) = Q$ and $I(T) = 0$.

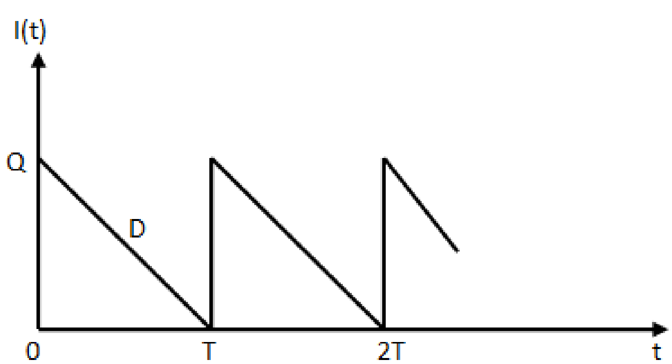

FIGURE 1. Crisp inventory model

Solving the above differential equation, we get $I(t) = Q - Dt$ and $T = \frac{Q}{D}$

Now, the average profit in the system involves the following:

Average sales revenue $= \frac{PQ}{T}$, average production cost $= \frac{CQ}{T}$, average marketing cost $= \frac{MQ}{T}$, average service cost $= \frac{RQ}{T}$, average green cost $= \frac{GQ}{T}$, average set-up cost $= \frac{A}{T}$ and average holding cost $= \int_0^T HI(t)\ dt = \frac{HQ}{2}$.

Hence total profit per unit time is

$$f = \textit{Sales revenue - Production cost - Merketing cost - Service cost - Green cost}$$
$$\textit{- Set-up cost - Holding cost}$$
$$= kP^{1-a}M^bR^cG^d - krP^{-a}M^bR^cG^dQ^{-\theta} - kP^{-a}M^{1+b}R^cG^d - kP^{-a}M^bR^{1+c}G^d$$
$$- kP^{-a}M^bR^cG^{1+d} - kAP^{-a}M^bR^cG^dQ^{-1} - 0.5HQ$$

Here, the space constraint is expressed as $w_0Q \leq w$.

2.2. *Model-1: Model in neutrosophic environment*

In reality, all data in an inventory model may not be found accurately. There may arise the case when some of the data are uncertain, incomplete and/or indeterminant. To deal with such a situation, the model's parameters are expressed in an imprecise environment considering fuzzy sets, intuitionistic sets, neutrosophic sets, rough sets, etc. The present inventory model considers all the elasticity parameters, scaling factors, holding cost, set-up cost, total available space, and per unit quantity area in the neutrosophic environment using PN numbers. Let us assume,

$$\left.\begin{array}{l}
\tilde{a}^n = \langle (a_1,a_2,a_3,a_4,a_5)\,;\mu_{\tilde{a}^n},\sigma_{\tilde{a}^n},\nu_{\tilde{a}^n}\rangle,\ \tilde{b}^n = \langle (b_1,b_2,b_3,b_4,b_5)\,;\mu_{\tilde{b}^n},\sigma_{\tilde{b}^n},\nu_{\tilde{b}^n}\rangle \\
\tilde{c}^n = \langle (c_1,c_2,c_3,c_4,c_5)\,;\mu_{\tilde{c}^n},\sigma_{\tilde{c}^n},\nu_{\tilde{c}^n}\rangle,\ \tilde{d}^n = \langle (d_1,d_2,d_3,d_4,d_5)\,;\mu_{\tilde{d}^n},\sigma_{\tilde{d}^n},\nu_{\tilde{d}^n}\rangle \\
\tilde{\theta}^n = \langle (\theta_1,\theta_2,\theta_3,\theta_4,\theta_5)\,;\mu_{\tilde{\theta}^n},\sigma_{\tilde{\theta}^n},\nu_{\tilde{\theta}^n}\rangle,\ \tilde{k}^n = \langle (k_1,k_2,k_3,k_4,k_5)\,;\mu_{\tilde{k}^n},\sigma_{\tilde{k}^n},\nu_{\tilde{k}^n}\rangle \\
\tilde{r}^n = \langle (r_1,r_2,r_3,r_4,r_5)\,;\mu_{\tilde{r}^n},\sigma_{\tilde{r}^n},\nu_{\tilde{r}^n}\rangle,\ \tilde{H}^n = \langle (H_1,H_2,H_3,H_4,H_5)\,;\mu_{\tilde{H}^n},\sigma_{\tilde{H}^n},\nu_{\tilde{H}^n}\rangle \\
\tilde{A}^n = \langle (A_1,A_2,A_3,A_4,A_5)\,;\mu_{\tilde{A}^n},\sigma_{\tilde{A}^n},\nu_{\tilde{A}^n}\rangle,\ \tilde{w}^n = \langle (w_1,w_2,w_3,w_4,w_5)\,;\mu_{\tilde{w}^n},\sigma_{\tilde{w}^n},\nu_{\tilde{w}^n}\rangle \\
\tilde{w_0}^n = \langle (w_{01},w_{02},w_{03},w_{04},w_{05})\,;\mu_{\tilde{w_0}^n},\sigma_{\tilde{w_0}^n},\nu_{\tilde{w_0}^n}\rangle
\end{array}\right\} \quad (2)$$

Thus, the inventory model in neutrosophic environment is formulated as

$$\text{Max } f = \tilde{k}^nP^{1-\tilde{a}^n}M^{\tilde{b}^n}R^{\tilde{c}^n}G^{\tilde{d}^n} - \tilde{k}^n\tilde{r}^nP^{-\tilde{a}^n}M^{\tilde{b}^n}R^{\tilde{c}^n}G^{\tilde{d}^n}Q^{-\tilde{\theta}^n} - \tilde{k}^nP^{-\tilde{a}^n}M^{1+\tilde{b}^n}R^{\tilde{c}^n}G^{\tilde{d}^n}$$
$$-\tilde{k}^nP^{-\tilde{a}^n}M^{\tilde{b}^n}R^{1+\tilde{c}^n}G^{\tilde{d}^n} - \tilde{k}^nP^{-\tilde{a}^n}M^{\tilde{b}^n}R^{\tilde{c}^n}G^{1+\tilde{d}^n} - \tilde{k}^n\tilde{A}^nP^{-\tilde{a}^n}M^{\tilde{b}^n}R^{\tilde{c}^n}G^{\tilde{d}^n}Q^{-1} - 0.5\tilde{H}^nQ \quad (3)$$

$$\text{subject to} \quad \tilde{w_0}^n\,Q \ \leq \tilde{w}^n \quad (4)$$

$$P, M, R, G, Q > 0 \quad (5)$$

3. **Particular cases**

3.1. *Model-1.1: (Model with fuzzy space constraint)*

In this consideration, the inventory problem remains similar as formulated previously, except that the space constraint is taken under fuzzy environment. Practically, the total available space in a production source point may not be predicted precisely. Keeping this fact in mind, it is assumed that the total storage space is imprecise in nature, and it is expressed by triangular fuzzy number $\tilde{w} = (w_1, w_2, w_3)$. Thus, for Model-1.1, the profit function (Max $f_{1.1}$, say) is same as in expression (3) subject to $w_0\ Q\ \leq \tilde{w}$ and positivity condition (5).

3.2. *Model-1.2: (Model with random space constraint)*

In this case, total available space $\bar{w}$ is assumed to be random in nature and all the other terms in the model are left same as in Model-1. Hence, the Model-1.2 is formulated with the profit expression (Max $f_{1.2}$, say) same as in equation (3) and the constraints are given by $w_0\ Q\ \leq \bar{w}$ and positivity condition (5).

3.3. *Model-1.3: (Model with rough space constraint)*

When the available total storage space $\hat{w}$ is a rough variable, the constraint reduces to the rough environment and is expressed as $w_0 Q \leq \hat{w}$, where $\hat{w} = ([w_1, w_2][w_3, w_4])\,,\ 0 \leq w_3 \leq w_1 \leq w_2 \leq w_4$ is a rough variable. Therefore, the Model-1.3 is described by the same profit function (say, $f_{1.3}$) as in equation (3) with the restriction $w_0\ Q\ \leq \hat{w}$ and condition (5).

3.4. *Model-1.4: (Model with neutrosophic space constraint)*

In this case, we express the total available storage area capacity using a TN number $\breve{w} = \langle (w_1, w_2, w_3, w_4); \mu_{\breve{w}}, \sigma_{\breve{w}}, \nu_{\breve{w}} \rangle$. The corresponding neutrosophic inventory model becomes maximize profit (say, $f_{1.4}$) as represented in (3) subject to the constraints $w_0\ Q\ \leq \breve{w}$ and positivity restriction (5).

3.5. *Model-1.5: (Model without space constraint)*

In this particular case, the inventory model is considered as an unconstrained profit maximization problem by omitting the space constraint from Model-1. Here, the expression for optimal profit (say, $f_{1.5}$) remains same as in equation (3) along with the positivity constraint (5) only.

4. **Solution procedure**

4.1. *Solution procedure for Model-1*

In this Section, a solution procedure is described to find a solution space for the neutrosophic inventory Model-1. Firstly, the model is converted into a crisp one from the neutrosophic one by applying the definition of the score function for PN numbers. It is then transformed into a posynomial problem. After that GP technique is used to get an ideal solution space.

Let us find the score values for the neutrosophic parameters as follows: (See Appendix)

$$\left.\begin{aligned}
S(\tilde{a}^n) &= \frac{1}{15}\{(a_1+a_2+a_3+a_4+a_5)\times(2+\mu_{\tilde{a}^n}-\sigma_{\tilde{a}^n}-\nu_{\tilde{a}^n})\}\\
S(\tilde{b}^n) &= \frac{1}{15}\{(b_1+b_2+b_3+b_4+b_5)\times(2+\mu_{\tilde{b}^n}-\sigma_{\tilde{b}^n}-\nu_{\tilde{b}^n})\}\\
S(\tilde{c}^n) &= \frac{1}{15}\{(c_1+c_2+c_3+c_4+c_5)\times(2+\mu_{\tilde{c}^n}-\sigma_{\tilde{c}^n}-\nu_{\tilde{c}^n})\}\\
S(\tilde{d}^n) &= \frac{1}{15}\{(d_1+d_2+d_3+d_4+d_5)\times(2+\mu_{\tilde{d}^n}-\sigma_{\tilde{d}^n}-\nu_{\tilde{d}^n})\}\\
S(\tilde{\theta}^n) &= \frac{1}{15}\{(\theta_1+\theta_2+\theta_3+\theta_4+\theta_5)\times(2+\mu_{\tilde{\theta}^n}-\sigma_{\tilde{\theta}^n}-\nu_{\tilde{\theta}^n})\}\\
S(\tilde{k}^n) &= \frac{1}{15}\{(k_1+k_2+k_3+k_4+k_5)\times(2+\mu_{\tilde{k}^n}-\sigma_{\tilde{k}^n}-\nu_{\tilde{k}^n})\}\\
S(\tilde{r}^n) &= \frac{1}{15}\{(r_1+r_2+r_3+r_4+r_5)\times(2+\mu_{\tilde{r}^n}-\sigma_{\tilde{r}^n}-\nu_{\tilde{r}^n})\}\\
S(\tilde{H}^n) &= \frac{1}{15}\{(H_1+H_2+H_3+H_4+H_5)\times(2+\mu_{\tilde{H}^n}-\sigma_{\tilde{H}^n}-\nu_{\tilde{H}^n})\}\\
S(\tilde{A}^n) &= \frac{1}{15}\{(A_1+A_2+A_3+A_4+A_5)\times(2+\mu_{\tilde{A}^n}-\sigma_{\tilde{A}^n}-\nu_{\tilde{A}^n})\}\\
S(\tilde{w}^n) &= \frac{1}{15}\{(w_1+w_2+w_3+w_4+w_5)\times(2+\mu_{\tilde{w}^n}-\sigma_{\tilde{w}^n}-\nu_{\tilde{w}^n})\}\\
S(\tilde{w_0}^n) &= \frac{1}{15}\{(w_{01}+w_{02}+w_{03}+w_{04}+w_{05})\times(2+\mu_{\tilde{w_0}^n}-\sigma_{\tilde{w_0}^n}-\nu_{\tilde{w_0}^n})\}
\end{aligned}\right\} \tag{6}$$

Using these values in Model-1, the converted crisp model can be formulated as follows:

$$\begin{aligned}
\text{Max } f = {} & S(\tilde{k}^n)P^{1-S(\tilde{a}^n)}M^{S(\tilde{b}^n)}R^{S(\tilde{c}^n)}G^{S(\tilde{d}^n)} - S(\tilde{k}^n)S(\tilde{r}^n)P^{-S(\tilde{a}^n)}M^{S(\tilde{b}^n)}R^{S(\tilde{c}^n)}G^{S(\tilde{d}^n)}Q^{-S(\tilde{\theta}^n)}\\
& -S(\tilde{k}^n)P^{-S(\tilde{a}^n)}M^{1+S(\tilde{b}^n)}R^{S(\tilde{c}^n)}G^{S(\tilde{d}^n)} - S(\tilde{k}^n)P^{-S(\tilde{a}^n)}M^{S(\tilde{b}^n)}R^{1+S(\tilde{c}^n)}G^{S(\tilde{d}^n)}\\
& -S(\tilde{k}^n)P^{-S(\tilde{a}^n)}M^{S(\tilde{b}^n)}R^{S(\tilde{c}^n)}G^{1+S(\tilde{d}^n)} - S(\tilde{k}^n)S(\tilde{A}^n)P^{-S(\tilde{a}^n)}M^{S(\tilde{b}^n)}R^{S(\tilde{c}^n)}G^{S(\tilde{d}^n)}Q^{-1}\\
& -0.5S(\tilde{H}^n)Q
\end{aligned} \tag{7}$$

$$\text{subject to} \quad S(\tilde{w_0}^n)\ Q\ \le S(\tilde{w}^n) \tag{8}$$

$$P, M, R, G, Q > 0$$

This transformed form of the inventory problem represents by a signomial GP problem, and its degree of difficulty is 2. Some necessary modifications are needed to convert the model into a posynomial GP problem. For the conversion, an appropriate lower bound is considered for the objective function with the aim that the maximization of that lower bound will be equivalent to the maximization of the objective function of the model. In this way, the signomial model is converted into the following equivalent form by introducing another additional auxiliary variable and a constraint:

$$\begin{aligned}
&\text{Max} \quad f_0\\
&\text{subject to } S(\tilde{k}^n)P^{1-S(\tilde{a}^n)}M^{S(\tilde{b}^n)}R^{S(\tilde{c}^n)}G^{S(\tilde{d}^n)} - S(\tilde{k}^n)S(\tilde{r}^n)P^{-S(\tilde{a}^n)}M^{S(\tilde{b}^n)}R^{S(\tilde{c}^n)}G^{S(\tilde{d}^n)}Q^{-S(\tilde{\theta}^n)}\\
&-S(\tilde{k}^n)P^{-S(\tilde{a}^n)}M^{1+S(\tilde{b}^n)}R^{S(\tilde{c}^n)}G^{S(\tilde{d}^n)} - S(\tilde{k}^n)P^{-S(\tilde{a}^n)}M^{S(\tilde{b}^n)}R^{1+S(\tilde{c}^n)}G^{S(\tilde{d}^n)}\\
&-S(\tilde{k}^n)P^{-S(\tilde{a}^n)}M^{S(\tilde{b}^n)}R^{S(\tilde{c}^n)}G^{1+S(\tilde{d}^n)} - S(\tilde{k}^n)S(\tilde{A}^n)P^{-S(\tilde{a}^n)}M^{S(\tilde{b}^n)}R^{S(\tilde{c}^n)}G^{S(\tilde{d}^n)}Q^{-1}\\
&-0.5S(\tilde{H}^n)Q \leq f_0 \qquad (9)\\
&S(\tilde{w_0}^n)\ Q \ \leq S(\tilde{w}^n) \qquad (10)\\
&P, M, R, G, Q > 0 \qquad (11)
\end{aligned}$$

Again, this reduced model is equivalent to the following minimization problem:

$$\begin{aligned}
&\text{Min} \quad F = {f_0}^{-1}\\
&\text{subject to } f_0 S(\tilde{k^n})^{-1}P^{S(\tilde{a}^n)-1}M^{-S(\tilde{b}^n)}R^{-S(\tilde{c}^n)}G^{-S(\tilde{d}^n)} + S(\tilde{r}^n)P^{-1}Q^{-S(\tilde{\theta}^n)} + P^{-1}M + P^{-1}R\\
&+P^{-1}G + S(\tilde{A}^n)P^{-1}Q^{-1} + 0.5S(\tilde{H}^n)S(\tilde{k^n})^{-1}P^{S(\tilde{a}^n)-1}M^{-S(\tilde{b}^n)}R^{-S(\tilde{c}^n)}G^{-S(\tilde{d}^n)}Q \leq 1\\
&S(\tilde{w_0}^n)\ Q \ \leq S(\tilde{w}^n)\\
&P, M, R, G, Q > 0 \qquad (12)
\end{aligned}$$

The derived inventory model (12) is a posynomial GP problem whose degree of difficulty is 2. To solve this problem the dual geometric programming problem is expressed as follows:

$$\begin{aligned}
\text{Max } d(w) = &\left(\frac{1}{w_{00}}\right)^{w_{00}} \left(\frac{S(\tilde{k^n})^{-1}}{w_{01}}\right)^{w_{01}} \left(\frac{S(\tilde{r^n})}{w_{02}}\right)^{w_{02}} \left(\frac{1}{w_{03}}\right)^{w_{03}} \left(\frac{1}{w_{04}}\right)^{w_{04}} \left(\frac{1}{w_{05}}\right)^{w_{05}}\\
&\left(\frac{S(\tilde{A^n})}{w_{06}}\right)^{w_{06}} \left(\frac{0.5S(\tilde{H^n})}{w_{07}}\right)^{w_{07}} \left(\sum_{i=1}^{7} w_{0i}\right)^{\sum_{i=1}^{7} w_{0i}} \left(\frac{S(\tilde{w_0}^n)}{S(\tilde{w}^n)}\right)^{w_{11}} \qquad (13)
\end{aligned}$$

subject to the following conditions:

Normality condition: $w_{00} = 1$

Orthogonality conditions: $-w_{00} + w_{01} = 0$

$$\begin{aligned}
&(S(\tilde{a}^n) - 1)w_{01} - w_{02} - w_{03} - w_{04} - w_{05} - w_{06} + (S(\tilde{a}^n) - 1)w_{07} = 0\\
&-S(\tilde{b}^n)w_{01} + w_{03} - S(\tilde{b}^n)w_{07} = 0\\
&-S(\tilde{c}^n)w_{01} + w_{04} - S(\tilde{c}^n)w_{07} = 0\\
&-S(\tilde{d}^n)w_{01} + w_{05} - S(\tilde{d}^n)w_{07} = 0\\
&-S(\tilde{\theta}^n)w_{02} - w_{06} + w_{07} + w_{11} = 0
\end{aligned}$$

Positivity condition: $w_{00}, w_{01}, w_{02}, w_{03}, w_{04}, w_{05}, w_{06}, w_{07}, w_{11} > 0$

where $\dot{w} = (w_{00}, w_{01}, w_{02}, w_{03}, w_{04}, w_{05}, w_{06}, w_{07}, w_{11})^T$ and $w_{0i}(i = 0, 1, ..., 7), w_{11}$ are the dual variables against the primal variables P, M, R, G and Q for the problem defined by equation (12).
As the model has 2 degree of difficulty, we cannot calculate all the dual variables directly from the conditions. Therefore, solving the above constraints and expressing the dual variables $w_{0i},\ i = 2, 3, ..., 6$ in terms of w_{07} and w_{11} we get

$$\left.\begin{aligned}
w_{00} &= 1 = w_{01}\\
w_{02} &= \frac{S(\tilde{a}^n) - S(\tilde{b}^n) - S(\tilde{c}^n) - S(\tilde{d}^n) - 2}{1 - S(\tilde{\theta}^n)} w_{07} - \frac{1}{1 - S(\tilde{\theta}^n)} w_{11}\\
&\quad + \frac{S(\tilde{a}^n) - S(\tilde{b}^n) - S(\tilde{c}^n) - S(\tilde{d}^n) - 1}{1 - S(\tilde{\theta}^n)}\\
w_{03} &= S(\tilde{b}^n)(1 + w_{07})\\
w_{04} &= S(\tilde{c}^n)(1 + w_{07})\\
w_{05} &= S(\tilde{d}^n)(1 + w_{07})\\
w_{06} &= \frac{1 - S(\tilde{\theta}^n)\left(S(\tilde{a}^n) - S(\tilde{b}^n) - S(\tilde{c}^n) - S(\tilde{d}^n) - 1\right)}{1 - S(\tilde{\theta}^n)} w_{07} + \frac{1}{1 - S(\tilde{\theta}^n)} w_{11}\\
&\quad - \frac{S(\tilde{\theta}^n)\left(S(\tilde{a}^n) - S(\tilde{b}^n) - S(\tilde{c}^n) - S(\tilde{d}^n) - 1\right)}{1 - S(\tilde{\theta}^n)}
\end{aligned}\right\} \tag{14}$$

Letting $k_1 = \frac{S(\tilde{a}^n) - S(\tilde{b}^n) - S(\tilde{c}^n) - S(\tilde{d}^n) - 1}{1 - S(\tilde{\theta}^n)}$ and $k_2 = \frac{1}{1 - S(\tilde{\theta}^n)}$ we get,

$$w_{02} = (k_1 - k_2)\, w_{07} - k_2 w_{11} + k_1 \quad \text{and} \quad w_{06} = \left(k_2 - S(\tilde{\theta}^n) k_1\right) w_{07} + k_2 w_{11} - S(\tilde{\theta}^n) k_1$$

Substituting these dual variables in equation (13), the dual objective function is expressed in terms of w_{07} and w_{11} as given below:

$$\begin{aligned}
d(w_{07}, w_{11}) = S(\tilde{k^n})^{-1} &\times \left(\frac{S(\tilde{r^n})}{(k_1 - k_2)\, w_{07} - k_2 w_{11} + k_1}\right)^{(k_1 - k_2) w_{07} - k_2 w_{11} + k_1}\\
&\times \left(\frac{1}{S(\tilde{b}^n)(1 + w_{07})}\right)^{S(\tilde{b}^n)(1 + w_{07})} \times \left(\frac{1}{S(\tilde{c}^n)(1 + w_{07})}\right)^{S(\tilde{c}^n)(1 + w_{07})}\\
&\times \left(\frac{1}{S(\tilde{d}^n)(1 + w_{07})}\right)^{S(\tilde{d}^n)(1 + w_{07})} \times \left(\frac{0.5 S(\tilde{H}^n)}{S(\tilde{k^n}) w_{07}}\right)^{w_{07}}\\
&\times \left(\frac{S(\tilde{A}^n)}{(k_2 - S(\tilde{\theta}^n) k_1) w_{07} + k_2 w_{11} - S(\tilde{\theta}^n) k_1}\right)^{\left(k_2 - S(\tilde{\theta}^n) k_1\right) w_{07} + k_2 w_{11} - S(\tilde{\theta}^n) k_1}\\
&\times (S(\tilde{a}^n)(1 + w_{07}))^{S(\tilde{a}^n)(1 + w_{07})} \times \left(\frac{S(\tilde{w_0}^n)}{S(\tilde{w}^n)}\right)^{w_{11}}
\end{aligned} \tag{15}$$

To evaluate the optimum dual variables w_{07}^* and w_{11}^* that optimize the dual objective $d(w_{07}, w_{11})$, we first take logarithm of equation (15) and get the following expression:

$$\begin{aligned}\log\, d(w_{07},w_{11}) = &\log S(\tilde{k^n})^{-1} + ((k_1-k_2)w_{07} - k_2w_{11} + k_1)\log S(\tilde{r^n})\\ &-((k_1-k_2)\,w_{07} - k_2w_{11} + k_1)\log((k_1-k_2)\,w_{07} - k_2w_{11} + k_1)\\ &-(S(\tilde{b}^n)(1+w_{07}))\log(S(\tilde{b}^n)(1+w_{07})) - (S(\tilde{c}^n)(1+w_{07}))\log(S(\tilde{c}^n)(1+w_{07}))\\ &-(S(\tilde{d}^n)(1+w_{07}))\log(S(\tilde{d}^n)(1+w_{07})) + w_{07}\log\left(\frac{0.5S(\tilde{H}^n)}{S(\tilde{k^n})}\right) - w_{07}\log w_{07}\\ &+((k_2 - S(\tilde{\theta}^n)k_1)w_{07} + k_2w_{11} - S(\tilde{\theta}^n)k_1)\log S(\tilde{A}^n)\\ &-((k_2 - S(\tilde{\theta}^n)k_1)w_{07} + k_2w_{11} - S(\tilde{\theta}^n)k_1)\log((k_2 - S(\tilde{\theta}^n)k_1)w_{07} + k_2w_{11} - S(\tilde{\theta}^n)k_1)\\ &+(S(\tilde{a}^n)(1+w_{07}))\log(S(\tilde{a}^n)(1+w_{07})) + w_{11}\log\left(\frac{S(\tilde{w_0}^n)}{S(\tilde{w}^n)}\right) \qquad (16)\end{aligned}$$

Since there are two variables w_{07} and w_{11} in the above logarithmic expression (16), to derive these optimal dual variables, we set the first order partial derivatives of log d(w_{07}, w_{11}) with respect to w_{07} and w_{11}, respectively, to zero and get the followings:

$$\begin{aligned}\frac{\partial \log\, d(w_{07},w_{11})}{\partial w_{07}} = &(k_1-k_2)\log S(\tilde{r^n}) - (k_1-k_2)\log((k_1-k_2)w_{07} - k_2w_{11} + k_1)\\ &-S(\tilde{b}^n)\log(S(\tilde{b}^n)(1+w_{07})) - S(\tilde{c}^n)\log(S(\tilde{c}^n)(1+w_{07}))\\ &-S(\tilde{d}^n)\log(S(\tilde{d}^n)(1+w_{07})) + (k_2 - S(\tilde{\theta}^n)k_1)\log S(\tilde{A}^n)\\ &-(k_2 - S(\tilde{\theta}^n)k_1)\log((k_2 - S(\tilde{\theta}^n)k_1)w_{07} + k_2w_{11} - S(\tilde{\theta}^n)k_1)\\ &+\log\left(\frac{0.5S(\tilde{H}^n)}{S(\tilde{k^n})}\right) + \log w_{07} + S(\tilde{a^n})\log(S(\tilde{a}^n)(1+w_{07}))\\ &= 0 \qquad (17)\end{aligned}$$

$$\begin{aligned}\frac{\partial \log\, d(w_{07},w_{11})}{\partial w_{11}} = &-k_2\log S(\tilde{r^n}) + k_2\log((k_1-k_2)w_{07} - k_2w_{11} + k_1) + k_2\log S(\tilde{A}^n)\\ &-k_2\log((k_2 - S(\tilde{\theta}^n)k_1)w_{07} + k_2w_{11} - S(\tilde{\theta}^n)k_1) + \log\left(\frac{S(\tilde{w_0}^n)}{S(\tilde{w}^n)}\right)\\ &= 0 \qquad (18)\end{aligned}$$

By using any search method or any software the above equations (17) and (18) can be solved to get the optimal dual variables w^*_{07} and w^*_{11}. With the help of these optimal values, other optimal dual variables can easily be evaluated from equation (14). Consequently, the optimum dual objective function $d^*(\dot{w}^*)$ can be calculated. Now, the primal-dual relations of the problem (12) are derived as:

$$\lambda = \sum_{i=1}^{7} w^*_{0i},\ S(\tilde{k^n})^{-1}P^{*S(\tilde{a}^n)-1}M^{*-S(\tilde{b}^n)}R^{*-S(\tilde{c}^n)}G^{*-S(\tilde{d}^n)} = \frac{w^*_{01}}{\lambda},\ S(\tilde{r}^n)P^{*-1}Q^{*-S(\tilde{\theta}^n)} = \frac{w^*_{02}}{\lambda}$$

$$P^{*-1}M^* = \frac{w^*_{03}}{\lambda},\ P^{*-1}R^* = \frac{w^*_{04}}{\lambda},\ P^{*-1}G^* = \frac{w^*_{05}}{\lambda},\ S(\tilde{A}^n)P^{*-1}Q^{*-1} = \frac{w^*_{06}}{\lambda},$$

$$0.5S(\tilde{H}^n)S(\tilde{k^n})^{-1}P^{*S(\tilde{a}^n)-1}M^{*-S(\tilde{b}^n)}R^{*-S(\tilde{c}^n)}G^{*-S(\tilde{d}^n)}Q^* = \frac{w_{07}^*}{\lambda},\ \left(\frac{S(\tilde{w_0}^n)}{S(\tilde{w}^n)}\right)Q^* = \frac{w_{11}^*}{w_{11}^*}$$

With the help of these equations, we obtain the following expressions for the optimal primal variables:

$$Q^* = \frac{S(\tilde{w}^n)}{S(\tilde{w_0}^n)},\ P^* = S(\tilde{A}^n)\frac{S(\tilde{w_0}^n)}{S(\tilde{w}^n)}\frac{\sum_{i=1}^7 w_{0i}^*}{w_{06}^*},\ M^* = S(\tilde{A}^n)\frac{S(\tilde{w_0}^n)}{S(\tilde{w}^n)}\frac{w_{03}^*}{w_{06}^*},$$
$$R^* = S(\tilde{A}^n)\frac{S(\tilde{w_0}^n)}{S(\tilde{w}^n)}\frac{w_{04}^*}{w_{06}^*},\ G^* = S(\tilde{A}^n)\frac{S(\tilde{w_0}^n)}{S(\tilde{w}^n)}\frac{w_{05}^*}{w_{06}^*}$$

Substituting the above obtained optimal variables in neutrosophic inventory model i.e. Model-1, the optimal profit becomes

$$f^* = \left(\frac{S(\tilde{A}^n)}{w_{06}^*}\frac{S(\tilde{w_0}^n)}{S(\tilde{w}^n)}\right)^{1-S(\tilde{a}^n)+S(\tilde{b}^n)+S(\tilde{c}^n)+S(\tilde{d}^n)} \sum_{i=1}^{7} w_{0i}^{*\ -S(\tilde{a}^n)} {w_{03}^*}^{S(\tilde{b}^n)} {w_{04}^*}^{S(\tilde{c}^n)} {w_{05}^*}^{S(\tilde{d}^n)} \times \left[\sum_{i=1}^{7} w_{0i}^*\right.$$
$$\left. -S(\tilde{r}^n)\left(\frac{S(\tilde{A}^n)}{w_{06}^*}\frac{S(\tilde{w_0}^n)}{S(\tilde{w}^n)}\right)^{S(\tilde{\theta}^n)-1} - w_{03}^* - w_{04}^* - w_{05}^* - S(\tilde{A}^n)\right] S(\tilde{k}^n) - 0.5S(\tilde{H}^n)\frac{S(\tilde{w}^n)}{S(\tilde{w_0}^n)}$$

4.2. *Solution procedure for Model-1.1*

In this case, the constraint is formulated under fuzzy environment. To deal with such type of constraint in inventory model, we follow the possibility theory. After defuzzification the space constraint reduces to $Pos\,(w_0Q \leq \tilde{w}) \geq \eta$, η represents the degree of fuzziness and $\tilde{w} = (w_1, w_2, w_3)$. Following Lemma 1, the crisp formulation of the constraint becomes :

$$w_0Q \leq \eta w_2 + (1-\eta)w_3 \tag{19}$$

Here the above obtained constraint is under crisp environment. Using the score function formula, the neutrosophic objective function is converted into a crisp one, and consequently, the equivalent crisp model becomes:
Maximize profit $f_{1.1}$ as given in equation (7) subject to conditions (19) and (5).

Now the model reduces to a signomial GP problem with degree of difficulty 2. Therefore, this problem can be solved by the GP technique. Following the similar approach as computed in section 4.1, the optimum results are evaluated as follows:

$$Q^* = \frac{\eta w_2 + (1-\eta)w_3}{w_0}, \qquad P^* = S(\tilde{A}^n)\frac{w_0}{\eta w_2 + (1-\eta)w_3}\frac{\sum_{i=1}^7 w_{0i}^*}{w_{06}^*},$$
$$M^* = S(\tilde{A}^n)\frac{w_0}{\eta w_2 + (1-\eta)w_3}\frac{w_{03}^*}{w_{06}^*}, \qquad R^* = S(\tilde{A}^n)\frac{w_0}{\eta w_2 + (1-\eta)w_3}\frac{w_{04}^*}{w_{06}^*},$$
$$G^* = S(\tilde{A}^n)\frac{w_0}{\eta w_2 + (1-\eta)w_3}\frac{w_{05}^*}{w_{06}^*}$$

and the optimum profit becomes

$$f_{1.1}^{*}=\left(\frac{S(\tilde{A}^{n})}{w_{06}^{*}}\frac{w_{0}}{\eta w_{2}+(1-\eta)w_{3}}\right)^{1-S(\tilde{a}^{n})+S(\tilde{b}^{n})+S(\tilde{c}^{n})+S(\tilde{d}^{n})}\sum_{i=1}^{7}w_{0i}^{*}{}^{-S(\tilde{a}^{n})}w_{03}^{*}{}^{S(\tilde{b}^{n})}w_{04}^{*}{}^{S(\tilde{c}^{n})}w_{05}^{*}{}^{S(\tilde{d}^{n})}$$
$$\times\left[\sum_{i=1}^{7}w_{0i}^{*}-S(\tilde{r}^{n})\left(\frac{S(\tilde{A}^{n})}{w_{06}^{*}}\frac{w_{0}}{\eta w_{2}+(1-\eta)w_{3}}\right)^{S(\tilde{\theta}^{n})-1}-w_{03}^{*}-w_{04}^{*}-w_{05}^{*}-S(\tilde{A}^{n})\right]S(\tilde{k}^{n})$$
$$-0.5S(\tilde{H}^{n})\frac{\eta w_{2}+(1-\eta)w_{3}}{w_{0}}$$

4.3. *Solution procedure for Model-1.2*

For this particular case, the space constraint is considered in a random sense. To deal with such types of constraints, we follow the chance-constrained programming approach and the corresponding constraint becomes:

$$Pr\left(w_{0}Q\leq\bar{w}\right)\geq p\ ,\ 0<p<1$$

where 'Pr' indicates probability and p represents the prescribed permissible probability.
Now, assume that $\bar{w}$ be normally distributed random variable with m_w and σ_w as mean and standard deviation respectively. Then, the constraint can be expressed as

$$Pr\left[\frac{w-m_{w}}{\sigma_{w}}\geq\frac{w_{0}Q-m_{w}}{\sigma_{w}}\right]\geq p$$

where $\frac{w-m_w}{\sigma_w}$ is a standard normal variate.
If we consider $\phi(p)$, such that $\int_{\phi(p)}^{\infty}\phi(t)\,dt$, where $\phi(t)$ is the standard normal density function, then we get

$$\frac{w-m_{w}}{\sigma_{w}}\leq\phi(p)$$

Thus, using chance-constrained programming the reduced crisp constraint can be written as

$$w_{0}Q\leq m_{w}+\sigma_{w}\phi(p)\tag{20}$$

Again, Definition 7 is used to transform the objective function into a crisp expression from a neutrosophic one. Thus, the corresponding crisp model can be expressed by the objective function given in equation (7) subject to the constraints (20) and (5). The above is again a signomial GP problem having degree of difficulty 2. Now we follow the GP approach as presented in section 4.1 to solve the model. Finally, the optimal solution is obtained in the following form:

$$Q^{*}=\frac{m_{w}+\sigma_{w}\phi(p)}{w_{0}},\ \ P^{*}=S(\tilde{A}^{n})\frac{w_{0}}{m_{w}+\sigma_{w}\phi(p)}\frac{\sum_{i=1}^{7}w_{0i}^{*}}{w_{06}^{*}},\ \ M^{*}=S(\tilde{A}^{n})\frac{w_{0}}{m_{w}+\sigma_{w}\phi(p)}\frac{w_{03}^{*}}{w_{06}^{*}},$$
$$R^{*}=S(\tilde{A}^{n})\frac{w_{0}}{m_{W}+\sigma_{W}\phi(p)}\frac{w_{04}^{*}}{w_{06}^{*}},\ \ G^{*}=S(\tilde{A}^{n})\frac{w_{0}}{m_{w}+\sigma_{w}\phi(p)}\frac{w_{05}^{*}}{w_{06}^{*}}$$

and corresponding optimum profit becomes

$$f_{1.2}^* = \left(\frac{S(\tilde{A}^n)}{w_{06}^*}\frac{w_0}{m_w+\sigma_w\phi(p)}\right)^{1-S(\tilde{a}^n)+S(\tilde{b}^n)+S(\tilde{c}^n)+S(\tilde{d}^n)} \sum_{i=1}^{7} w_{0i}^{*\;-S(\tilde{a}^n)} w_{03}^{*\;S(\tilde{b}^n)} w_{04}^{*\;S(\tilde{c}^n)} w_{05}^{*\;S(\tilde{d}^n)}$$
$$\times\left[\sum_{i=1}^{7} w_{0i}^* - S(\tilde{r}^n)\left(\frac{S(\tilde{A}^n)}{w_{06}^*}\frac{w_0}{m_w+\sigma_w\phi(p)}\right)^{S(\tilde{\theta}^n)-1} - w_{03}^* - w_{04}^* - w_{05}^* - S(\tilde{A}^n)\right] S(\tilde{k}^n)$$
$$-0.5S(\tilde{H}^n)\frac{m_w+\sigma_w\phi(p)}{w_0}$$

4.4. *Solution procedure for Model-1.3*

In this case, the space constraint is supposed to be in rough environment. Therefore, using Definition 12, the trust measure to the crisp conversion becomes

$$Tr\left(\hat{w}\geq w_0 Q\right)\geq \eta_1$$

where $\eta_1\in[0,1]$ is the trust level.

Now, following Theorem 2, the rough constraint can be written in equivalent crisp form as expressed below:

$$w_0Q \leq \begin{cases} w_4 - \frac{\eta_1(w_4-w_3)}{\zeta} & \text{if } w_2\leq w_0Q\leq w_4 \\ \frac{\zeta(w_2-w_1)+(1-\zeta)w_2(w_4-w_3)-\eta_1(w_4-w_3)(w_2-w_1)}{\zeta(w_2-w_1)+(1-\zeta)(w_4-w_3)} & \text{if } w_1\leq w_0Q\leq w_2 \\ w_4 + \frac{(1-\zeta-\eta_1)(w_4-w_3)}{\zeta} & \text{if } w_3\leq w_0Q\leq w_1 \\ w_3 \end{cases} \quad (21)$$

where $\zeta\in(0,1)$.

After reducing the neutrosophic objective function into its crisp form, the consequent model can be expressed by the profit function $f_{1.3}$ as given in expression (7) subject to restrictions (21) and(5). This obtained signomial GP problem bearing degree of difficulty 2 is then solved using GP technique as described in section 4.1. The optimal decision variables and profit are respectively obtained as:

$$Q^*=\frac{T}{w_0},\quad P^*=S(\tilde{A}^n)\frac{w_0}{T}\frac{\sum_{i=1}^{7} w_{0i}^*}{w_{06}^*},\quad M^*=S(\tilde{A}^n)\frac{w_0}{T}\frac{w_{03}^*}{w_{06}^*},\quad R^*=S(\tilde{A}^n)\frac{w_0}{T}\frac{w_{04}^*}{w_{06}^*},$$
$$G^*=S(\tilde{A}^n)\frac{w_0}{T}\frac{w_{05}^*}{w_{06}^*}\quad \text{and}$$
$$f_{1.3}^* = \left(\frac{S(\tilde{A}^n)}{w_{06}^*}\frac{w_0}{T}\right)^{1-S(\tilde{a}^n)+S(\tilde{b}^n)+S(\tilde{c}^n)+S(\tilde{d}^n)} \sum_{i=1}^{7} w_{0i}^{*\;-S(\tilde{a}^n)} w_{03}^{*\;S(\tilde{b}^n)} w_{04}^{*\;S(\tilde{c}^n)} w_{05}^{*\;S(\tilde{d}^n)}$$
$$\times\left[\sum_{i=1}^{7} w_{0i}^* - S(\tilde{r}^n)\left(\frac{S(\tilde{A}^n)}{w_{06}^*}\frac{w_0}{T}\right)^{S(\tilde{\theta}^n)-1} - w_{03}^* - w_{04}^* - w_{05}^* - S(\tilde{A}^n)\right] S(\tilde{k}^n) - 0.5S(\tilde{H}^n)\frac{T}{w_0}$$

where

$$T = \begin{cases} w_4 - \frac{\eta_1(w_4-w_3)}{\zeta} & \text{if } w_2 \leq w_0Q \leq w_4 \\ \frac{\zeta(w_2-w_1)+(1-\zeta)w_2(w_4-w_3)-\eta_1(w_4-w_3)(w_2-w_1)}{\zeta(w_2-w_1)+(1-\zeta)(w_4-w_3)} & \text{if } w_1 \leq w_0Q \leq w_2 \\ w_4 + \frac{(1-\zeta-\eta_1)(w_4-w_3)}{\zeta} & \text{if } w_3 \leq w_0Q \leq w_1 \\ w_3 & \end{cases}$$

and $\zeta \in (0,1)$.

4.5. *Solution procedure for Model-1.4*

To convert this neutrosophic model into a crisp representation, we first need to calculate the $\alpha - cut,\ \beta - cut$ and $\gamma - cut$ of the TN number. Using Definition 10 , we derive the following $\alpha - cut,\ \beta - cut$ and $\gamma - cut$ for the neutrosophic total space

$$\breve{w}_\alpha = [L_{\breve{w}}(\alpha), R_{\breve{w}}(\alpha)] = \left[\frac{(\mu_{\breve{w}} - \alpha)w_1 + \alpha w_2}{\mu_{\breve{w}}}, \frac{(\mu_{\breve{w}} - \alpha)w_4 + \alpha w_3}{\mu_{\breve{w}}}\right]$$

$$\breve{w}_\beta = [L^{'}_{\breve{w}}(\beta), R^{'}_{\breve{w}}(\beta)] = \left[\frac{(1-\beta)w_2 + (\beta - \sigma_{\breve{w}})w_1}{1-\sigma_{\breve{w}}}, \frac{(1-\beta)w_3 + (\beta - \sigma_{\breve{w}})w_4}{1-\sigma_{\breve{w}}}\right]$$

$$\breve{w}_\gamma = [L^{''}_{\breve{w}}(\gamma), R^{''}_{\breve{w}}(\gamma)] = \left[\frac{(1-\gamma)w_2 + (\gamma - \nu_{\breve{w}})w_1}{1-\nu_{\breve{w}}}, \frac{(1-\gamma)w_3 + (\gamma - \nu_{\breve{w}})w_4}{1-\nu_{\breve{w}}}\right]$$

After that, we transform the neutrosophic parameters in the constraint into a crisp interval number by using Theorem 1. Thus, we have the crisp constraint as $w_0Q \leq [L_{\breve{w}}, R_{\breve{w}}]$,
where $L_{\breve{w}} = max\left\{L_{\breve{w}}(\alpha),\ L^{'}_{\breve{w}}(\beta),\ L^{''}_{\breve{w}}(\gamma)\right\},\ R_{\breve{w}} = min\left\{R_{\breve{w}}(\alpha),\ R^{'}_{\breve{w}}(\beta),\ R^{''}_{\breve{w}}(\gamma)\right\}$
Now applying the weighted mean approach (cf. Lemma 2), convert the interval number into parametric function and get the crisp formulation of the constraint as follows:

$$w_0Q \leq L_{\breve{w}}(1-\rho) + R_{\breve{w}}\rho \tag{22}$$

After reducing the PN objective of the model-1.4 into a crisp one using the score function, the converted equivalent crisp formulation is expressed with the objective function same as in (7) along with the constraints given in (22) and (5). Now the reduced model is again a signomial GP problem having 2 degree of difficulty. Following the solution procedure, explained in Section 4.1, we get the following optimal solutions:

$$Q^* = \frac{L_{\breve{w}}(1-\rho) + R_{\breve{w}}\rho}{w_0}, \qquad P^* = S(\tilde{A}^n)\frac{w_0}{L_{\breve{w}}(1-\rho) + R_{\breve{w}}\rho}\frac{\sum_{i=1}^{7} w^*_{0i}}{w^*_{06}},$$

$$M^* = S(\tilde{A}^n)\frac{w_0}{L_{\breve{w}}(1-\rho) + R_{\breve{w}}\rho}\frac{w^*_{03}}{w^*_{06}}, \qquad R^* = S(\tilde{A}^n)\frac{w_0}{L_{\breve{w}}(1-\rho) + R_{\breve{w}}\rho}\frac{w^*_{04}}{w^*_{06}},$$

$$G^* = S(\tilde{A}^n)\frac{w_0}{L_{\breve{w}}(1-\rho) + R_{\breve{w}}\rho}\frac{w^*_{05}}{w^*_{06}}$$

along with the optimum profit given by

$$f_{1.4}^* = \left(\frac{S(\tilde{A}^n)}{w_{06}^*}\frac{w_0}{L_{\breve{w}}(1-\rho)+R_{\breve{w}}\rho}\right)^{1-S(\tilde{a}^n)+S(\tilde{b}^n)+S(\tilde{c}^n)+S(\tilde{d}^n)} \sum_{i=1}^{7} w_{0i}^{*\;-S(\tilde{a}^n)} w_{03}^{*\;S(\tilde{b}^n)} w_{04}^{*\;S(\tilde{c}^n)} w_{05}^{*\;S(\tilde{d}^n)}$$
$$\times\left[\sum_{i=1}^{7} w_{0i}^* - S(\tilde{r}^n)\left(\frac{S(\tilde{A}^n)}{w_{06}^*}\frac{w_0}{L_{\breve{w}}(1-\rho)+R_{\breve{w}}\rho}\right)^{S(\tilde{\theta}^n)-1} - w_{03}^* - w_{04}^* - w_{05}^* - S(\tilde{A}^n)\right] S(\tilde{k}^n)$$
$$-0.5S(\tilde{H}^n)\frac{L_{\breve{w}}(1-\rho)+R_{\breve{w}}\rho}{w_0}$$

4.6. *Solution procedure for Model-1.5*

The objective function of this particular model is transformed into crisp environment by using the definition of score function, and obtained crisp expression is given in the equation (7). Since there is only the positivity restriction in this model, the problem is a signomial GP problem with 1 degree of difficulty. Thus, the model can be solved by the GP technique, and the optimum solutions are as follows:

$$Q^* = \left(\frac{S(\tilde{A}^n)}{S(\tilde{r}^n)}\frac{w_{02}^*}{w_{06}^*}\right)^{\frac{1}{1-S(\tilde{\theta}^n)}},\ P^* = \left(\frac{S(\tilde{r}^n)}{w_{02}^*}\right)^{\frac{1}{1-S(\tilde{\theta}^n)}}\left(\frac{w_{06}^*}{S(\tilde{A}^n)}\right)^{\frac{S(\tilde{\theta}^n)}{1-S(\tilde{\theta}^n)}},$$
$$M^* = w_{03}^*\left(\frac{S(\tilde{r}^n)}{w_{02}^*}\right)^{\frac{1}{1-S(\tilde{\theta}^n)}}\left(\frac{w_{06}^*}{S(\tilde{A}^n)}\right)^{\frac{S(\tilde{\theta}^n)}{1-S(\tilde{\theta}^n)}},\ R^* = w_{04}^*\left(\frac{S(\tilde{r}^n)}{w_{02}^*}\right)^{\frac{1}{1-S(\tilde{\theta}^n)}}\left(\frac{w_{06}^*}{S(\tilde{A}^n)}\right)^{\frac{S(\tilde{\theta}^n)}{1-S(\tilde{\theta}^n)}},$$
$$G^* = w_{05}^*\left(\frac{S(\tilde{r}^n)}{w_{02}^*}\right)^{\frac{1}{1-S(\tilde{\theta}^n)}}\left(\frac{w_{06}^*}{S(\tilde{A}^n)}\right)^{\frac{S(\tilde{\theta}^n)}{1-S(\tilde{\theta}^n)}}.$$

Now using these optimum decision variables, optimal profit can be calculated easily.

5. **Numerical experiments**

Input data for Model-1:

The proposed models (Model-1,-1.1,-1.2,-1.3,-1.4 and -1.5) are illustrated with a theoretical example in this section. For this, the following inputs in appropriate units are considered:

$$\tilde{a}^n = \langle(2,3,7,8,10);0.6,0.5,0.6\rangle,\ \tilde{b}^n = \langle(0.2,0.32,0.45,0.52,0.61);0.9,0.1,0.3\rangle,$$
$$\tilde{c}^n = \langle(0.09,0.18,0.34,0.43,0.52);0.9,0.3,0.1\rangle,\tilde{d}^n = \langle(0.09,0.12,0.19,0.28,0.37);0.8,0.5,0.3\rangle,$$
$$\tilde{\theta}^n = \langle(0.01,0.02,0.03,0.04,0.05);0.8,0.4,0.4\rangle,\tilde{r}^n = \langle(2,5.5,8,10,12);0.6,0.2,0.4\rangle,$$
$$\tilde{k}^n = \langle(200000,400000,600000,750000,1050000);0.9,0.3,0.1\rangle,$$
$$\tilde{H}^n = \langle(0.9,1.2,2.5,3.8,4.1);0.9,0.8,0.6\rangle,\tilde{A}^n = \langle(36,58,86,110.5,122);0.7,0.4,0.3\rangle,$$
$$\tilde{w_0}^n = \langle(1,6,10,15,18);0.5,0.5,0.2\rangle,\tilde{w}^n = \langle(280,450,695,860,1390);0.7,0.4,0.3\rangle.$$

Input data for particular models:

For the particular models (Model-1.1,-1.2,1.3 and -1.4) the required inputs are presented in Table 1. All the other parameters for these models are similar as in Model-1.

TABLE 1. Other input data for particular models

Environments	Related data
Fuzzy	$\tilde{w} = (410, 490, 570)$ $\eta = 0.5$
Random	$(m_w, \sigma_w) = (480, 20)$ $p = 0.96$
Rough	$\hat{w} = ([460, 510][420, 580]),\ \zeta = 0.5,\ \eta_1 = 0.6$
TN	$\check{w} = \langle(420, 465, 500, 550); 0.7, 0.3, 0.3\rangle,\ \alpha = 0.1,\ \beta = 0.7,\ \gamma = 0.9$

Optimum results:

All models are formulated with the help of above considered parameters and optimized using the proposed solution approach. The optimum values of selling price P^*, number of order quantity Q^*, marketing expenditure M^*, service expenditure R^*, green expenditure G^* and profit f^* are evaluated for the inventory models with the constraint in different environments, and the results are indicated in Table 2.

TABLE 2. Optimal solutions for all models

Models	Environments		P^*	M^*	R^*	G^*	Q^*	f^*
	Objective	constraint						
1	PN	PN	12.600	1.470	1.092	0.588	81.67	1089.50
1.1	PN	Fuzzy	12.466	1.454	1.080	0.581	88.33	1101.27
1.2	PN	Random	12.790	1.492	1.108	0.597	74.17	1073.64
1.3	PN	Rough	12.610	1.471	1.093	0.588	81.43	1089.04
1.4	PN	TN	12.710	1.483	1.102	0.593	77.21	1080.44
1.5	PN	No constraint	11.463	1.337	0.993	0.535	197.87	1160.63

From Table 2, it is observed that the optimal profit for the model with PN numbers (Model-1) is 1089.50 \$. Again, the optimal selling price per unit item is 12.6 \$ for this model, and the total order quantity is 81.67 units. Moreover, the marketing, service and green expenditure per item are 1.47 \$, 1.092 \$ and 0.588 \$, respectively. Similarly, we note that the optimal profit for the particular cases, i.e., models with fuzzy constraint (Model-1.1), random constraint (Model-1.2), rough constraint (Model-1.3) and TN constraint (Model-1.4) are 1101.27 \$, 1073.64 \$, 1089.04 \$ and 1080.44 \$ respectively. The optimal values of these particular models' decision variables are presented in Table 2. Again, as per expectation, the unconstrained model (Model-1.5) gives the maximum profit among all models. Strictly speaking, as the environments under which the models are formulated are different, their optimum results can not be compared.

6. Sensitivity analysis

In this Section, we perform some sensitivity analysis to observe the changes in optimal profit with respect to the changes in parameters for the particular models. From Figure 2a, it is noted that whenever the degree of fuzziness (η) or the trust level (η_1) increases from 0.1 to 0.9, the optimal profit decreases continuously. But, with the increases of weights (ρ) of the weighted mean used in Model-1.4, optimal profit increases (cf. Figure 2a). Again, the decrease of optimal profit with respect to the considered probability (p) of Model-1.2 is plotted in Figure 2b. The changes in optimal profit of all the models along with the changes in total available space and set-up cost are figured in Figures 2c and 2d respectively.

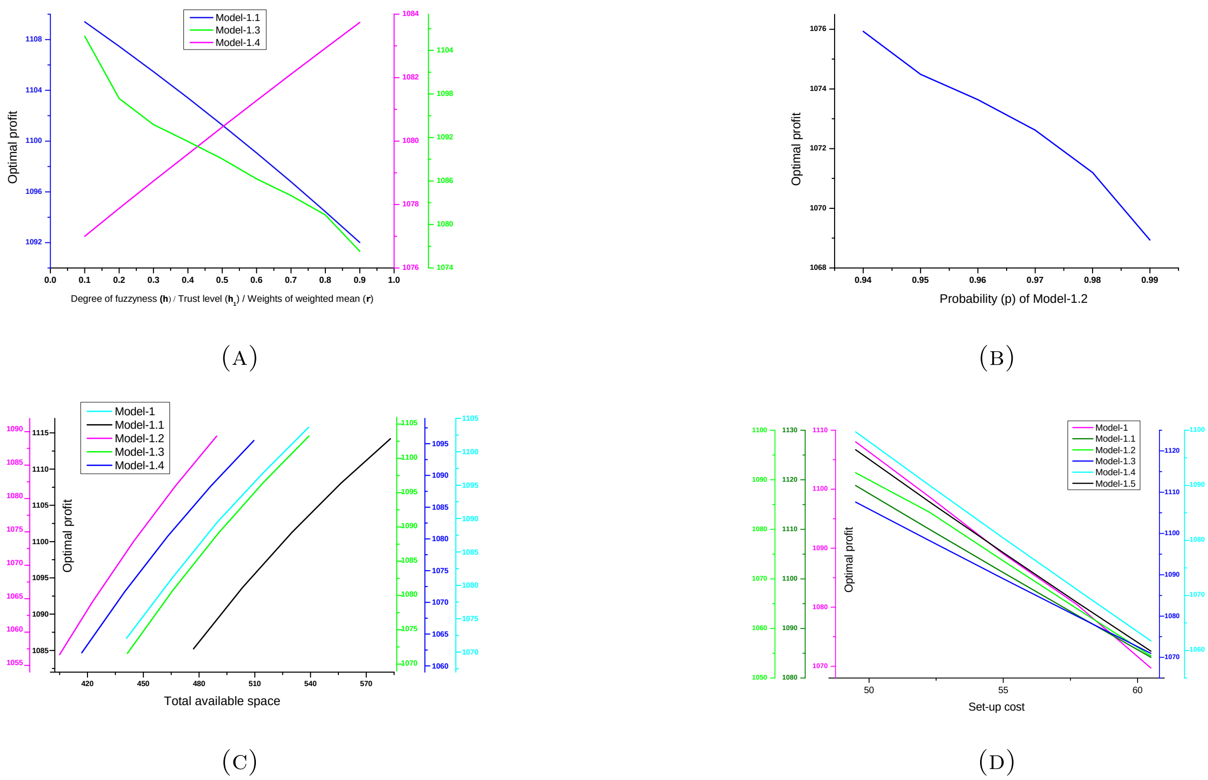

FIGURE 2. (A) Optimal profit vs. degree of fuzzyness (η)/ trust measure (η_1)/ weight of weighted mean (ρ); (B) Optimal profit vs. permissible probability (p) for Model-1.2; (C) Optimal profit of all models vs. total available space; (D) Optimal profit of all models vs. set-up cost

7. Conclusions

The main goal of the present study is to develop an economic order quantity model with space constraint having all its parameters as PN numbers. In reality, marketing expenditure and service quality play a significant role in the demand of any manufacturing company. Again, the demand for green items is always very high in the market. Therefore, the demand in this model is considered as a function of the selling price, marketing, service and green expenditure to be more realistic. Moreover, order quantity dependent unit production cost is assumed here. Again, the space constraint is considered in several environments like fuzzy, random, rough, and TN. Possibility measure and chance-constrained programming are used to deal with the fuzzy and random constraint goals, respectively. For all the models, a solution procedure is suggested. Finally, the GP technique is applied to solve the converted crisp models. Moreover, some numerical experiments and sensitivity analyses are done to illustrate the models.

In the future, the model can be developed more realistically by assuming ramp type demand, power demand, probabilistic demand, etc. Here, the model is considered for a single item, and hence it can be extended to a multi-item model. Also, shortages can be allowed in the problem. Furthermore, the items may be damageable, and preservation technology may be introduced. Moreover, different environments can be considered, such as intuitionistic, fuzzy-random, fuzzy-rough, type-2 fuzzy, etc. Again, the researchers may suggest different solution procedures and compare the optimum results with the present one.

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Appendix

Definition 1 (Fuzzy set). *[32] A fuzzy set $\tilde{B}$ in X (space of points/objects) is an object having the form $\tilde{B} = \{(x, \mu_{\tilde{B}}(x)) : x \in X\}$ where $\mu_{\tilde{B}} : X \to [0,1]$ is the membership function of the fuzzy set $\tilde{B}$.*

Definition 2 (Triangular fuzzy number). *[20] A triangular fuzzy number $\tilde{B} = (b_1, b_2, b_3)$ is a fuzzy number whose membership function $\mu_{\tilde{B}}(x)$ is defined as*

$$\mu_{\tilde{B}}(x) = \begin{cases} \frac{x-b_1}{b_2-b_1} & b_1 \leq x < b_2 \\ \frac{b_3-x}{b_3-b_2} & b_2 \leq x \leq b_3 \\ 0 & otherwise \end{cases}$$

where $[b_1, b_3]$ is the supporting interval and the point $(b_2, 1)$ is the peak.

Definition 3 (Possibility measure). *[20] Let $\tilde{m}$ and $\tilde{n}$ be two fuzzy numbers and the corresponding memberships functions be $\mu_{\tilde{m}}(x)$ and $\mu_{\tilde{n}}(x)$ respectively. Then the possibility measure of $\tilde{m}$ and $\tilde{n}$ is defined as:* $Pos(\tilde{m} * \tilde{n}) = Sup\,\{min(\mu_{\tilde{m}}(x), \mu_{\tilde{n}}(y)),\ x, y \in \mathbb{R},\ x * y\}$

Here, the abbreviation 'Pos' stands for possibility measure and '' represents any one of the relations $<, >, =, \leq, \geq$. Analogously if $\tilde{n}$ be a crisp number,say n, then we have*

$$Pos(\tilde{m} * n) = Sup\,\{min(\mu_{\tilde{m}}(x),\ x \in \mathbb{R},\ x * n\}$$

Lemma 1. *Let $\tilde{m} = (m_1, m_2, m_3)$ be any triangular fuzzy number and $0 < m_1$ and n be any crisp number. Then,*

$$Pos(\tilde{m} > n) \geq \alpha \; iff \; \frac{m_3 - n}{m_3 - m_2} \geq \alpha$$

Definition 4 (Neutrosophic set)**.** *[45] Let the set X be a space of points (objects) and $x \in X$. A neutrosophic set (NS) $\tilde{B}^n \subset X$ is represented by three independent functions: membership function $\mu_{\tilde{B}^n}(x)$, hesitation function $\sigma_{\tilde{B}^n}(x)$ and non-membership function $\nu_{\tilde{B}^n}(x)$ and expressed as*

$$\tilde{B}^n = \{(x, \mu_{\tilde{B}^n}(x), \sigma_{\tilde{B}^n}(x), \nu_{\tilde{B}^n}(x)) : x \in X\}.$$

where $\mu_{\tilde{B}^n}(x),\ \sigma_{\tilde{B}^n}(x),\ \nu_{\tilde{B}^n}(x) : X \to]0^-, 1^+[\ \forall\ x \in X$ are real standard or non-standard subset of $]0^-, 1^+[$. The sum of these three independent functions is related as follows:

$$0^- \leq Sup\ \mu_{\tilde{B}^n}(x) + Sup\ \sigma_{\tilde{B}^n}(x) + Sup\ \nu_{\tilde{B}^n}(x) \leq 3^+\ \forall\ x \in X.$$

Definition 5 (Single valued neutrosophic set)**.** *[45] Let the set X be a space of points (objects). A single valued neutrosophic set (SVNS) $\tilde{B} \subset X$ is expressed as*

$$\tilde{B}^n = \{(x, \mu_{\tilde{B}^n}(x), \sigma_{\tilde{B}^n}(x), \nu_{\tilde{B}^n}(x)) : x \in X\}$$

where $\mu_{\tilde{B}^n}, \sigma_{\tilde{B}^n}, \nu_{\tilde{B}^n} : X \to [0, 1]$ satisfy the condition $0 \leq \mu_{\tilde{B}^n}(x) + \sigma_{\tilde{B}^n}(x) + \nu_{\tilde{B}^n}(x) \leq 3\ \forall\ x \in X$. $\mu_{\tilde{B}^n}(x)$, $\sigma_{\tilde{B}^n}(x)$ and $\nu_{\tilde{B}^n}(x)$ denote the membership, hesitation and non-membership function respectively.

Definition 6 (Single valued pentagonal neutrosophic number)**.** *[51] A single valued pentagonal neutrosophic number (SVPN-number) $\tilde{B}^n$ having the form*

$$\tilde{B}^n = \langle [(b_1^{'}, b_2^{'}, b_3^{'}, b_4^{'}, b_5^{'}); w], [(b_1^{''}, b_2^{''}, b_3^{''}, b_4^{''}, b_5^{''}); u], [(b_1^{'''}, b_2^{'''}, b_3^{'''}, b_4^{'''}, b_5^{'''}); y] \rangle$$

where $w, u, y \in [0, 1]$. Here, membership function $\mu_{\tilde{B}^n}(x) : \mathbb{R} \to [0, w]$, hesitation function $\sigma_{\tilde{B}^n}(x) : \mathbb{R} \to [u, 1]$ and non-membership function $\nu_{\tilde{B}^n}(x) : \mathbb{R} \to [y, 1]$ are defined as follows:

$$\mu_{\tilde{B}^n}(x) = \begin{cases} \mu_{\widetilde{B^n l1}}(x) & b_1^{'} \leq x < b_2^{'} \\ \mu_{\widetilde{B^n l2}}(x) & b_2^{'} \leq x < b_3^{'} \\ w & x = b_3^{'} \\ \mu_{\widetilde{B^n r2}}(x) & b_3^{'} \leq x < b_4^{'} \\ \mu_{\widetilde{B^n r1}}(x) & b_4^{'} \leq x < b_5^{'} \\ 0 & otherwise \end{cases}, \quad \sigma_{\tilde{B}^n}(x) = \begin{cases} \sigma_{\widetilde{B^n l1}}(x) & b_1^{''} \leq x < b_2^{''} \\ \sigma_{\widetilde{B^n l2}}(x) & b_2^{''} \leq x < b_3^{''} \\ u & x = b_3^{''} \\ \sigma_{\widetilde{B^n r2}}(x) & b_3^{''} \leq x < b_4^{''} \\ \sigma_{\widetilde{B^n r1}}(x) & b_4^{''} \leq x < b_5^{''} \\ 0 & otherwise \end{cases}$$

and

$$\nu_{\tilde{B}^n}(x) = \begin{cases} \nu_{\widetilde{B^n l1}}(x) & b_1^{'''} \leq x < b_2^{'''} \\ \nu_{\widetilde{B^n l2}}(x) & b_2^{'''} \leq x < b_3^{'''} \\ y & x = b_3^{'''} \\ \nu_{\widetilde{B^n r2}}(x) & b_3^{'''} \leq x < b_4^{'''} \\ \nu_{\widetilde{B^n r1}}(x) & b_4^{'''} \leq x < b_5^{'''} \\ 0 & otherwise \end{cases}$$

respectively.

Definition 7 (Score function of PN number). *[51] The score function is significantly used in the conversion of PN number into a crisp real number. The value of the score function utterly depends on the value of the membership, hesitation and non-membership degree of the PN number. Let* $\tilde{B}^n = \langle (b_1, b_2, b_3, b_4, b_5); \mu_{\tilde{B}^n}, \sigma_{\tilde{B}^n}, \nu_{\tilde{B}^n} \rangle$ *be any SVPN-number. Then the value of the score function is evaluated as*

$$S(\tilde{B}^n) = \frac{1}{15} \left\{ (b_1 + b_2 + b_3 + b_4 + b_5) \times (2 + \mu_{\tilde{B}^n} - \sigma_{\tilde{B}^n} - \nu_{\tilde{B}^n}) \right\}$$

Definition 8 (Single valued trapezoidal neutrosophic number). *[62] A single valued trapezoidal neutrosophic number (SVTN-number)* $\check{B}$ *is a special neutrosophic set on R (set of real number) having the form*

$$\check{B} = \langle (b_1, b_2, b_3, b_4); w_{\check{B}}, u_{\check{B}}, y_{\check{B}} \rangle$$

where $w_{\check{B}}, u_{\check{B}}, y_{\check{B}} \in [0, 1]$ *be any real numbers and* $b_1, b_2, b_3, b_4 \in \mathbb{R}$, $b_1 \leq b_2 \leq b_3 \leq b_4$ *are the values of the trapezoidal number. Here, membership function* $\mu_{\check{B}}(x)$*, hesitation function* $\sigma_{\check{B}}(x)$ *and non-membership function* $\nu_{\check{B}}(x)$ *are defined as follows:*

$$\mu_{\check{B}}(x) = \begin{cases} \frac{x-b_1}{b_2-b_1} w_{\check{B}} & if\ b_1 \leq x < b_2 \\ w_{\check{B}} & if\ b_2 \leq x \leq b_3 \\ \frac{b_4-x}{b_4-b_3} w_{\check{B}} & if\ b_3 < x \leq b_4 \\ 0 & otherwise \end{cases}, \quad \sigma_{\check{B}}(x) = \begin{cases} \frac{b_2-x+u_{\check{B}}(x-b_1)}{b_2-b_1} & if\ b_1 \leq x < b_2 \\ u_{\check{B}} & if\ b_2 \leq x \leq b_3 \\ \frac{x-b_3+u_{\check{B}}(b_4-x)}{b_4-b_3} & if\ b_3 < x \leq b_4 \\ 0 & otherwise \end{cases}$$

and

$$\nu_{\check{B}}(x) = \begin{cases} \frac{b_2-x+y_{\check{B}}(x-b_1)}{b_2-b_1} & if\ b_1 \leq x < b_2 \\ y_{\check{B}} & if\ b_2 \leq x \leq b_3 \\ \frac{x-b_3+y_{\check{B}}(b_4-x)}{b_4-b_3} & if\ b_3 < x \leq b_4 \\ 0 & otherwise \end{cases}$$

respectively.

Definition 9 ($\langle \alpha, \beta, \gamma \rangle$-cut set of SVTN-number). *[62] Let* $\check{B} = \langle (b_1, b_2, b_3, b_4); w_{\check{B}}, u_{\check{B}}, y_{\check{B}} \rangle$ *be a SVTN-number. Then,* $\langle \alpha, \beta, \gamma \rangle$*-cut set of the SVTN-number* $\check{B}$ *is denoted by* $\check{B}_{\langle \alpha, \beta, \gamma \rangle}$ *and defined as:*

$$\check{B}_{\langle\alpha,\beta,\gamma\rangle} = \{x|\mu_{\check{B}}(x) \geq \alpha, \sigma_{\check{B}}(x) \leq \beta, \nu_{\check{B}}(x) \leq \gamma, x \in R\}$$

which satisfies following the conditions:

$$0 \leq \alpha \leq w_{\check{B}},\ u_{\check{B}} \leq \beta \leq 1,\ y_{\check{B}} \leq \gamma \leq 1 \text{ and } 0 \leq \alpha+\beta+\gamma \leq 3$$

where $\mu_{\check{B}}$, $\sigma_{\check{B}}$ and $\nu_{\check{B}}$ represent membership, hesitation and non-membership function respectively.

Definition 10 (α-cut, β-cut, γ-cut of SVTN-number). *[62] Let $\check{B} = \langle(b_1, b_2, b_3, b_4); w_{\check{B}}, u_{\check{B}}, y_{\check{B}}\rangle$ be an arbitrary SVTN-number. Then*
1. α-cut of $\check{B}$ is defined by

$$\check{B}_\alpha = [L_{\check{B}}(\alpha), R_{\check{B}}(\alpha)] = \left[\frac{(w_{\check{B}}-\alpha)b_1+\alpha b_2}{w_{\check{B}}}, \frac{(w_{\check{B}}-\alpha)b_4+\alpha b_3}{w_{\check{B}}}\right]$$

where $\alpha \in [0, w_{\check{B}}]$.
2. β-cut of $\check{B}$ is defined by

$$\check{B}_\beta = [L'_{\check{B}}(\beta), R'_{\check{B}}(\beta)] = \left[\frac{(1-\beta)b_2+(\beta-u_{\check{B}})b_1}{1-u_{\check{B}}}, \frac{(1-\beta)b_3+(\beta-u_{\check{B}})b_4}{1-u_{\check{B}}}\right]$$

where $\beta \in [u_{\check{B}}, 1]$.
3. γ-cut of $\check{B}$ is defined by

$$\check{B}_\gamma = [L''_{\check{B}}(\gamma), R''_{\check{B}}(\gamma)] = \left[\frac{(1-\gamma)b_2+(\gamma-y_{\check{B}})b_1}{1-y_{\check{B}}}, \frac{(1-\gamma)b_3+(\gamma-y_{\check{B}})b_4}{1-y_{\check{B}}}\right]$$

where $\gamma \in [y_{\check{B}}, 1]$.

Theorem 1. *Let $\check{B} = \langle(b_1, b_2, b_3, b_4); w_{\check{B}}, u_{\check{B}}, y_{\check{B}}\rangle$ be an arbitrary SVTN-number. Then, $\check{B}_{\langle\alpha,\beta,\gamma\rangle} = \check{B}_\alpha \cap \check{B}_\beta \cap \check{B}_\gamma$ is hold for any $0 < \alpha < w_{\check{B}}$, $u_{\check{B}} < \beta < 1$ and $y_{\check{B}} < \gamma < 1$ where $0 \leq \alpha+\beta+\gamma \leq 3$.*

Proof: See [62]

Lemma 2. *Let $A = [a, b], a, b > 0$ be a closed interval with weights $w_1(> 0), w_2(> 0)$. Then the interval can be represented by a function using the weighted arithmetic mean*

$$WAM_A(\rho) = \frac{w_1 a + w_2 b}{w_1 + w_2} = a(1-\rho) + b\rho$$

where $\rho = \frac{w_2}{w_1+w_2}$, $\rho \in [0, 1]$.

Definition 11 (Rough variable). *[28] Let $(\Lambda, \delta, \mathcal{A}, \pi)$ be a rough space. A rough variable ξ is a measurable function from the rough space $(\Lambda, \delta, \mathcal{A}, \pi)$ to $\mathbb{R}$ (the set of real numbers). That is, for every Borel set B of $\mathbb{R}$, we have $\{\eta \in \Lambda : \xi(\eta) \in B\} \in \mathcal{A}$*
The upper and lower approximations of the rough variable ξ are denoted and defined by $\bar{\xi} = \{\xi(\eta) : \eta \in \Lambda\}$ and $\underline{\xi} = \{\xi(\eta) : \eta \in \delta\}$ respectively.

Definition 12 (Trust measure). *[28] Let $(\Lambda, \delta, \mathcal{A}, \pi)$ be a rough space. The trust measure of event A is denoted by $Tr\{A\}$ and defined by $Tr\{A\} = \frac{1}{2}(\underline{Tr}\{A\} + \overline{Tr}\{A\})$, where the lower and upper trust measure of event A are defined by $\underline{Tr}\{A\} = \frac{\pi\{A\cap\delta\}}{\pi\{\delta\}}$ and $\overline{Tr}\{A\} = \frac{\pi\{A\}}{\pi\{\Lambda\}}$ respectively.*

For a real life problem when sufficient information is given about the measurement of π*, it may be treated as Lebesgue measure. More generally, the trust measure can be considered in the form as* $Tr\{A\} = (1-\eta)\underline{Tr}\{A\} + \eta\overline{Tr}\{A\},\ 0 < \eta < 1$.

Let $\hat{\xi} = ([m,n][p,q]),\ p \le m \le n \le q$ *be a rough variable. Lebesgue measure is considered for the trust measure of the rough event associated with* $\hat{\xi} \ge r$*. Then the trust measure of the rough event* $\hat{\xi} \ge r$ *is defined by the following function curve*

$$Tr\left\{\hat{\xi} \ge r\right\} = \begin{cases} 0 & if\ q \le r \\ \frac{\eta(q-r)}{q-p} & if\ n \le r \le q \\ \frac{\eta(q-r)}{q-p} + \frac{(1-\eta)(n-r)}{n-m} & if\ m \le r \le n \\ \frac{\eta(q-r)}{q-p} + (1-\eta) & if\ p \le r \le m \\ 1 & if\ r \le p \end{cases}$$

Theorem 2. *[28] Let* $\hat{\xi} = ([m,n][p,q]),\ p \le m \le n \le q$ *be a rough variable and* $\hat{\xi} \ge r$ *be a rough event. Then* $Tr\left\{\hat{\xi} \ge r\right\} \ge \alpha$ *iff*

$$r \le \begin{cases} q - \frac{\alpha(q-p)}{\eta} & if\ n \le r \le q \\ \frac{\eta(n-m)+(1-\eta)n(q-p)-\alpha(q-p)(n-m)}{\eta(n-m)+(1-\eta)n(q-p)} & if\ m \le r \le n \\ q + \frac{(1-\eta-\alpha)(q-p)}{\eta} & if\ p \le r \le m \\ p \end{cases}$$

for any predetermined level $\alpha \in [0,1]$

Proof. For any $\alpha \in [0,1]$, we have

$$Tr\left\{\hat{\xi} \ge r\right\} \ge \alpha$$

$$\Leftrightarrow \quad \alpha \le Tr\left\{\hat{\xi} \ge r\right\}$$

$$\Leftrightarrow \quad \alpha \le \begin{cases} 0 & if\ q \le r \\ \frac{\eta(q-r)}{q-p} & if\ n \le r \le q \\ \frac{\eta(q-r)}{q-p} + \frac{(1-\eta)(n-r)}{n-m} & if\ m \le r \le n \\ \frac{\eta(q-r)}{q-p} + (1-\eta) & if\ p \le r \le m \\ 1 & if\ r \le p \end{cases}$$

[With the help of definition of trust measure of an event]

$$\Leftrightarrow r \le \begin{cases} q - \frac{\alpha(q-p)}{\eta} & if\ n \le r \le q \\ \frac{\eta(n-m)+(1-\eta)n(q-p)-\alpha(q-p)(n-m)}{\eta(n-m)+(1-\eta)n(q-p)} & if\ m \le r \le n \\ q + \frac{(1-\eta-\alpha)(q-p)}{\eta} & if\ p \le r \le m \\ p \end{cases}$$

Hence the proof is done.

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