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Abstract:

In this paper a property which can be used to Baire spaces in fy. neutrosophic top. Spaces (simply as fy. – fuzzy, top. – topological) are introduced and studied. For this purpose, introduced fy. neutrosophic F_σ – set, fy. neutrosophic G_δ – set, fy. neutrosophic dense, fy. neutrosophic nowh. (Simply as nowh. - nowhere) dense, fy. neutrosophic one (one denotes first) category, fy. neutrosophic two (two denotes second) category and fy. neutrosophic re. (Simply as re. – residual) set are defined. Also, some characterizations about these concepts are obtained.

Keywords:

Fy. neutrosophic dense set, Fy. neutrosophic nowh. dense set, Fy. neutrosophic re. set, Fy. neutrosophic Baire spaces, Fy. neutrosophic one and two category.

AMS subject classification: 54A40, 03E72

1. Introduction:

The concept of fy. sets were introduced by L.A. Zadeh in 1965 [10]. Then the fy. set theory is extension by many researchers. The important concept of fuzzy topological space was offered by C.L. Chang [3] and from that point forward different ideas in topology have been reached out to fuzzy topological space. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a morden theory of fuzzy topology has been developed. The concept of fuzzy $\sigma -$ Baire spaces was introduced and studied by G. Thangaraj and E. Poongothai in [7]. The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In 2017, Veereswari [9] introduced fy. neutrosophic top. spaces. This concept is the solution and representation of the problems with various fields.

In this paper, we define new concepts of fy. neutrosophic $F_\sigma -$ set, fy. neutrosophic $G_\delta -$ set, fy. neutrosophic dense, fy. neutrosophic nowh. dense, fy. neutrosophic one and two category sets, fy. neutrosophic re. set, fy. neutrosophic Baire spaces, fy. neutrosophic one and two category spaces in fy. neutrosophic top. spaces, and we also discussed some new properties and examples based of this defined concept.

2. Preliminaries:

Definition 2.1 [2]:

A fy. neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x) \rangle$, $x \in X$ where $T, I, F : X \to [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

With the condition $0 \leq T_{A^*}(x) + \mathrm{I}_{A^*}(x) + F_{A^*}(x) \leq 2$.

Definition 2.2 [2]:

A fy. neutrosophic set A is a subset of a fy. neutrosophic set B (i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$, $F_A(x) \geq F_B(x)$.

Definition 2.3 [2]:

Let X be a non-empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x) \rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x) \rangle$ be two fy. neutrosophic sets. Then

$A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x)) \rangle$

$A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x)) \rangle$

Definition 2.4 [2]:

The difference between two fy. neutrosophic sets A and B is defined as

$A \backslash B(x) = \langle x, \min(T_A(x), F_B(x)), \min(I_A(x), 1 - I_B(x)), \max(F_A(x), T_B(x)) \rangle$

Definition 2.5 [2]:

A fy. neutrosophic set A over the universe X is said to be null or empty fy. neutrosophic set if $T_A(x) = 0,\ I_A(x) = 0,\ F_A(x) = 1\ for\ all\ x \in X$. It is denoted by 0_N.

Definition 2.6 [2]:

A fy. neutrosophic set A over the universe X is said to be absolute (universe) fy. neutrosophic set if $T_A(x) = 1,\ I_A(x) = 1,\ F_A(x) = 0\ for\ all\ x \in X$. It is denoted by 1_N.

Definition 2.7 [2]:

The complement of a fy. neutrosophic set A is denoted by A^C and is defined as

$A^C = \langle x, T_{A^C}(x), I_{A^C}(x), F_{A^C}(x) \rangle$ where

$T_{A^C}(x) = F_A(x), I_{A^C}(x) = 1 - I_A(x), F_{A^C}(x) = T_A(x)$

The complement of fy. neutrosophic set A can also be defined as $A^C = 1_N - A$.

Definition 2.8 [1]:

A fy. neutrosophic topology on a non-empty set X is a τ of fy. neutrosophic sets in X

$(i)\ \ 0_N, 1_N \in \tau$

$(ii)\ \ A_1 \cap A_2 \in \tau\ \ for\ any\ A_1, A_2 \in \tau$

$(iii)\ \cup A_i \in \tau\ for\ any\ arbitrary\ family\ \{A_i : i \in J\}\ \in \tau$

Satisfying the following axioms.

In this case the pair (X, τ) is called fy. neutrosophic top. space and any Fy. neutrosophic set in τ is known as fy. neutrosophic open set in X.

Definition 2.9 [1]:

The complement A^C of a fy. neutrosophic set A in a fy. neutrosophic top. space (X, τ) is called fy. neutrosophic closed set in X.

Definition 2.10 [1]:

Let (X, τ_N) be a fy. neutrosophic top. space and $A = \langle x, T_A(x), I_A(x), F_A(x) \rangle$ be a fy. neutrosophic set in X. Then the closure and interior of A are defined by

$int(A) = \cup\{G : G\ is\ a\ fuzzy\ neutrosophic\ open\ set\ in\ X\ and\ G \subseteq A\}$

$cl(A) = \cap\{G : G\ is\ a\ fuzzy\ neutrosophic\ closed\ set\ in\ X\ and\ A \subseteq G\}$

3. On Fuzzy Neutrosophic Nowhere Dense Sets

Throughout the present paper, P denote the fy. neutrosophic top. spaces. Let A_N be a fy. neutrosophic set on P. The fy. neutrosophic interior and closure of A_N is denoted by $fn(A_N)^+$, $fn(A_N)^-$ respectively. A fy. neutrosophic set A_N is defined to be fy.

neutrosophic open set $(fnOS)$ if $A_N \leq fn(((A_N)^-)^+)^-$. The complement of a fy. neutrosophic open set is called fy. neutrosophic closed set $(fnCS)$.

Definition 3.1:

A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic $F_\sigma -$ set if $A_N = \vee_{i=1}^{\infty} A_{N_i}$, where $\overline{A_{N_i}} \in \tau_N$ for $i \in I$.

Definition 3.2:

A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic $G_\delta -$ set in (P, τ_N) if $A_N = \wedge_{i=1}^{\infty} A_{N_i}$, where $A_{N_i} \in \tau_N$ for $i \in I$.

Definition 3.3:

A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic semi-open if $A_N \leq fn(((A_N)^+)^-)$. The complement of A_N in (P, τ_N) is called a fy. neutrosophic semi-closed set in (P, τ_N).

Definition 3.4:

A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic dense if there exist no $fnCS$ B_N in (P, τ_N) s.t $A_N \subset B_N \subset 1_X$. That is, $fn(A_N)^- = 1_N$.

Definition 3.5:

A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic nowh. dense set if there exist no non-zero $fnOS$ B_N in (P, τ_N) s.t $B_N \subset fn(A_N)^-$. That is, $fn(((A_N)^-)^+) = 0_N$.

Example 3.1:

Let $P = \{a, b, c\}$ and consider the family $\tau_N = \{0_N, 1_N, A_N, B_N, C_N\}$ where

$A_N = \{\langle a, 0.3,0.3,0.5\rangle, \langle b, 0.6,0.6,0.5\rangle, \langle c, 0.6,0.6,0.5\rangle\}$

$B_N = \{\langle a, 0.6,0.6,0.6\rangle, \langle b, 0.3,0.3,0.3\rangle, \langle c, 0.6,0.6,0.6\rangle\}$

$C_N = \{\langle a, 0.3,0.3,0.4\rangle, \langle b, 0.7,0.7,0.4\rangle, \langle c, 0.3,0.3,0.4\rangle\}$

Thus (P, τ_N) is a fy. neutrosophic top. spaces.

Now $fn(((\overline{A_N})^-)^+) = 0_N, fn(((\overline{B_N})^-)^+) = 0_N$ and $fn(((\overline{C_N})^-)^+) = 0_N$. This gives that $\overline{A_N}, \overline{B_N}$ and $\overline{C_N}$ are fy. neutrosophic nowh. dense sets in (P, τ_N).

Definition 3.6:

Let (P, τ_N) be a fy. neutrosophic top. space. A fy. neutrosophic set A_N in (P, τ_N) is called fy. neutrosophic one category set if $A_N = \vee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fy. neutrosophic nowh. dense sets in (P, τ_N). Any other fy. neutrosophic set in (P, τ_N) is said to be of fy. neutrosophic two category.

Definition 3.7:

A fy. neutrosophic top. space (P, τ_N) is called fy. neutrosophic one category space if the fy. neutrosophic set 1_X is a fy. neutrosophic one category set in (P, τ_N). That is $1_X = \vee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fy. neutrosophic nowh. dense sets in (P, τ_N). Otherwise (P, τ_N) will be called a fy. neutrosophic two category space.

Definition 3.8:

Let A_N be a fy. neutrosophic one category set in (P, τ_N). Then $\overline{A_N}$ is called fy. neutrosophic re. set in (P, τ_N).

Proposition 3.1:

If A_N is a $fnCS$ in (P,τ_N) with $fn(A_N)^+ = 0_N$ then A_N is a Fy. neutrosophic nowh. dense set in (P,τ_N).

Proof:

Let A_N is a $fnCS$ (P,τ_N). Then $fn(A_N)^- = A_N$.

Now $fn(((A_N)^-)^+) = fn(A_N)^+ = 0_N$. and hence A_N is a fy. neutrosophic nowh. dense set in (P,τ_N).

Proposition 3.2:

If A_N is a fy. neutrosophic nowh. dense set in (P,τ_N) then $fn(A_N)^+ = 0_N$.

Proof:

Let A_N be a fy. neutrosophic nowh. dense set in (P,τ_N). Now $A_N \le fn(A_N)^-$ gives that $fn(A_N)^+ \le fn(((A_N)^-)^+) = 0_N$. Hence, we have $fn(A_N)^+ = 0_N$.

Remark 3.1:

The complement of a fy. neutrosophic nowh. dense set need not be a fy. neutrosophic nowh. dense set. For, consider the following example.

Example 3.2:

Let $P = \{a,b,c\}$ and consider the family $\tau_N = \{0_{N,}1_N, A_N, B_N, C_N\}$ where

$A_N = \{\langle a, 0.5,0.5,0.4\rangle, \langle b, 0.5,0.3,0.5\rangle, \langle c, 0.5,0.5,0.4\rangle\}$

$B_N = \{\langle a, 0.5,0.5,0.3\rangle, \langle b, 0.4,0.2,0.5\rangle, \langle c, 0.5,0.4,0.3\rangle\}$

$C_N = \{\langle a, 0.5,0.4,0.5\rangle, \langle b, 0.5,0.3,0.2\rangle, \langle c, 0.5,0.6,0.3\rangle\}$

Now $fn(((B_N)^-)^+) = 0_N$ is a fy. neutrosophic nowh. dense sets in (P,τ_N).

But $fn(((\overline{B_N})^-)^+) \neq 0_N$. Therefore $\overline{B_N}$ is not a fy. neutrosophic nowh. dense sets in (P, τ_N).

Proposition 3.3:

If A_N is a fy. neutrosophic dense, $fnOS$ in (P, τ_N) s.t $B_N \leq \overline{A_N}$, then B_N is a fy. neutrosophic nowh. dense set in (P, τ_N).

Proof:

Let A_N be a $fnOS$ in (P, τ_N) s.t $fn(A_N)^- = 1$. Now $B_N \leq \overline{A_N}$ gives that $fn(B_N)^- \leq fn(\overline{A_N})^- = (1 - A_N)$ [$(\overline{A_N})$ is a $fnCS$ in (P, τ_N)] Then we have $fn(((B_N)^-)^+) \leq fn(\overline{A_N})^+ = \overline{(fn(A_N))^-} = 1 - 1 = 0_N$. and hence $fn(((B_N)^-)^+) = 0_N$. Therefore B_N is a fy. neutrosophic nowh. dense set in (P, τ_N).

Proposition 3.4:

If A_N is a non-zero fy. neutrosophic nowh. dense set in (P, τ_N), is a fy. neutrosophic nowh. dense set then A_N is fy. neutrosophic semi-closed set in (P, τ_N).

Proof:

Let A_N be a fy. neutrosophic nowh. dense set in (P, τ_N). Then $fn(((A_N)^-)^+) = 0_N$. and therefore $fn(((A_N)^-)^+) \leq A_N$. Hence, A_N is fy. neutrosophic semi-closed set in (P, τ_N).

Proposition 3.5:

If a $fnCS$ A_N is a fy. neutrosophic nowh. dense set in (P, τ_N) if and only if $fn(A_N)^+ = 0_N$.

Proof:

Let A_N be a $fnCS$ in (P, τ_N) with $fn(A_N)^+ = 0_N$. Then by proposition 3.1, A_N is a fy. neutrosophic nowh. dense set in (P, τ_N). Conversely, let A_N is a fy. neutrosophic nowh. dense set in (P, τ_N), then $fn(((A_N)^-)^+) = 0_N$, which gives that $fn(A_N)^+ = 0_N$, [since A_N is $fnCS$ in $fn(A_N)^- = A_N$].

Proposition 3.6:

If A_N is a fy. neutrosophic nowh. dense set in (P, τ_N), then $\overline{A_N}$ is a fy. neutrosophic dense set in (P, τ_N).

Proof:

Let A_N be a fy. neutrosophic nowh. dense set in (P, τ_N).

Then by proposition 3.2, we have, $fn(A_N)^+ = 0_N$. Now $fn(\overline{A_N})^- = \overline{(fn(\overline{A_N}))^+} = 1 - 0_N = 1_N$. Therefore $\overline{A_N}$ is a fy. neutrosophic dense set in (P, τ_N).

Proposition 3.7:

If A_N is a fy. neutrosophic nowh. dense set and $fnOS$ in (P, τ_N), then $\overline{A_N}$ is a fy. neutrosophic nowh. dense set in (P, τ_N).

Proof:

Let A_N be a $fnOS$ in (P, τ_N) s.t, $fn(A_N)^- = 1$. Now $fn(((\overline{A_N})^-)^+) = \overline{(fn((A_N)^+)^-)} = \overline{fn(A_N)^-} == 1 - 1 = 0_N$. Hence $\overline{A_N}$ is a fy. neutrosophic nowh. dense set in (P, τ_N).

Proposition 3.8:

If A_N is a fy. neutrosophic nowh. dense set in (P, τ_N), then $fn(A_N)^-$ is a fy. neutrosophic nowh. dense set in (P, τ_N).

Proof:

Let $fn(A_N)^- = B_N$, Now $fn(((B_N)^-)^+)$ $= fn((((A_N)^-)^-)^+) = fn(((A_N)^-)^+) = 0_N$.

Hence $B_N = fn(A_N)^-$ is a fy. neutrosophic nowh. dense set in (P, τ_N).

Proposition 3.9:

If A_N is a fy. neutrosophic nowh. dense set in (P, τ_N), then $\overline{fn(A_N)^-}$ is a fy. neutrosophic dense set in (P, τ_N).

Proof:

By proposition 3.8, we have $fn(A_N)^-$ is a fy. neutrosophic nowh. dense set in (P, τ_N). By proposition 3.7, we have $\overline{fn(A_N)^-}$ is a fy. neutrosophic nowh. dense set in (P, τ_N).

Proposition 3.10:

Let A_N be a fy. neutrosophic dense set in (P, τ_N).

If B_N is any fy. neutrosophic set in (P, τ_N), then B_N is a fy. neutrosophic nowh. dense set in (P, τ_N), if and only if $A_N \wedge B_N$ is a fy. neutrosophic nowh. dense set in (P, τ_N).

Proof:

Let B_N be a Fy. neutrosophic nowh. dense set in (P, τ_N).

Now,

$fn(((A_N \wedge B_N)^-)^+) = (fn(fn(A_N)^- \wedge fn\ (B_N)^-))^+ = (fn(1 \wedge fn(B_N)^-))^+$

$$= fn(((B_N)^-)^+) = 0_N.$$

Therefore $A_N \wedge B_N$ is a fy. neutrosophic nowh. dense set in (P, τ_N).

Conversely let $A_N \wedge B_N$ is a fy. neutrosophic nowh. dense set in (P, τ_N). Then $fn(((A_N \wedge B_N)^-)^+) = 0_N$ Gives that $(fn(fn(A_N)^- \wedge fn\ (B_N)^-))^+$.

Hence $(fn(1 \wedge fn(B_N)^-))^+ = 0_N$ and therefore $fn(((B_N)^-)^+) = 0_N$ which means that B_N is a fy. neutrosophic nowh. dense set in (P, τ_N).

Proposition 3.11:

Every fy. neutrosophic nowh. dense sets is a $fnCS$.

Proof:

Let A_N be any fy. neutrosophic nowh. dense set in a fy. neutrosophic top. space (P, τ_N). Therefore, we have $fn(((A_N)^-)^+) = 0_N$ and it means that there does not exist any $fnOS$ in between A_N and $(A_N)^-$. Also, let us suppose that $A_N \leq B_N$, where B_N is $fnOS$ and obviously $(A_N)^- \leq B_N$. Therefore B_N is a $fnCS$.

4. Fuzzy Neutrosophic Baire Space

Definition 4.1:

A fy. neutrosophic top. space (P, τ_N) is called fy. neutrosophic Baire space if $fn(\vee_{i=1}^{\infty}(A_{N_i}))^+ = 0_N$, where A_{N_i}'s are fy. neutrosophic nowh. dense sets in (P, τ_N).

Example 4.1:

Let $P = \{a, b, c\}$ and consider the family $\tau_N = \{0_N, 1_N, A_N, B_N, C_N, D_N\}$ where

$A_N = \{\langle a, 0.3, 0.3, 0.5\rangle, \langle b, 0.6, 0.6, 0.5\rangle, \langle c, 0.6, 0.6, 0.5\rangle\}$

$B_N = \{\langle a, 0.3, 0.3, 0.3\rangle, \langle b, 0.6, 0.6, 0.5\rangle, \langle c, 0.6, 0.6, 0.6\rangle\}$

$C_N = \{\langle a, 0.7,0.7,0.4\rangle, \langle b, 0.4,0.3,0.3\rangle, \langle c, 0.3,0.3,0.4\rangle\}$

$D_N = \{\langle a, 0.3,0.3,0.3\rangle, \langle b, 0.7,0.7,0.7\rangle, \langle c, 0.3,0.3,0.3\rangle\}$

Now $\overline{A_N}$, $\overline{B_N}$ and $\overline{C_N}$ are fy. neutrosophic nowh. dense sets in (P, τ_N). Also $fn(\overline{A_N} \vee \overline{B_N} \vee \overline{C_N})^+ = 0_N$. Hence (P, τ_N) be a fy. neutrosophic Baire Space.

Proposition 4.1:

Let (P, τ_N) be a fy. neutrosophic top. space. Then the following are equivalent.

i) (P, τ_N) is a fy. neutrosophic baire space.

ii) $fn\ (A_N)^+ = 0_N$, for every fy. neutrosophic one category set A_N in (P, τ_N).

iii) $fn\ (B_N)^+ = 1_N$, for every fy. neutrosophic re. set B_N in (P, τ_N).

Proof:

$(i) \Rightarrow (ii)$

Let A_N be a fy. neutrosophic one category set in (P, τ_N). Then $A_N = (\vee_{i=1}^{\infty} A_{N_i})$, where A_{N_i}' s are fy. neutrosophic nowh. dense sets in (P, τ_N). Now $fn\ (A_N)^+ = fn\ (\vee_{i=1}^{\infty} A_{N_i})^+ = 0_N$. Since (P, τ_N) is a fy. neutrosophic Baire space. Therefore $fn\ (A_N)^+ = 0_N$.

$(ii) \Rightarrow (iii)$

Let B_N be a fy. neutrosophic re. set in (P, τ_N). Then $\overline{B_N}$ is a fy. neutrosophic one category set in (P, τ_N). By hypothesis, $fn(\overline{B_N})^+ = 0_N$ which gives that $\overline{fn(A_N)^-} = 0_N$. Hence $fn\ (A_N)^- = 1_N$.

$(iii) \Rightarrow (i)$

Let A_N be a fy. neutrosophic one category set in (P, τ_N). Then $A_N = (\vee_{i=1}^{\infty} A_{N_i})$, where A_{N_i}'s are fy. neutrosophic nowh. dense sets in (P, τ_N). Now A_N is a fy. neutrosophic one category set gives that $\overline{A_N}$ is a fy. neutrosophic re. set in (P, τ_N). By hypothesis, we have $fn(\overline{A_N})^- = 1_N$ which gives that $\overline{fn(A_N)^+} = 1_N$. Hence $fn\,(A_N)^+ = 0_N$. That is, $fn\,(\vee_{i=1}^{\infty} A_{N_i})^+ = 0_N$, where A_{N_i}'s are fy. neutrosophic nowh. dense sets in (P, τ_N). Hence (P, τ_N) is a fy. neutrosophic Baire space.

Proposition 4.2:

If A_N be a fy. neutrosophic one category set in (P, τ_N) then $\overline{A_N} = \wedge_{i=1}^{\infty} B_{N_i}$, where $fn\,(B_{N_i})^- = 1_N$.

Proof:

Let A_N be a fy. neutrosophic one category set in (P, τ_N).

Then $A_N = (\vee_{i=1}^{\infty} A_{N_i})$, where A_{N_i}'s are fy. neutrosophic nowh. dense sets in (P, τ_N). Now $\overline{A_N} = \overline{\vee_{i=1}^{\infty} A_{N_i}} = \wedge_{i=1}^{\infty} \bar{A}_{N_i}$ Now A_{N_i} is a fy. neutrosophic nowh. dense sets in (P, τ_N). Then by proposition [3.6] we have $\overline{A_{N_i}}$. is a fy. neutrosophic dense sets in (P, τ_N). Let us put $B_{N_i} = \overline{A_{N_i}}$. Then $\overline{A_N} = \wedge_{i=1}^{\infty} B_{N_i}$, where $fn\,(B_{N_i})^- = 1_N$.

Proposition 4.3:

If $fn\,(\vee_{i=1}^{\infty} A_{N_i})^+ = 0_N$ where $fn\,(A_{N_i})^+ = 0_N$ and $A_{N_i} \in \tau_N$, then (P, τ_N) is a fy. neutrosophic Baire space.

Proof:

Now $A_{N_i} \in \tau_N$ gives that A_{N_i} is a $fnOS$ in (P, τ_N). Since $fn\,(A_{N_i})^+ = 0_N$. By proposition (3.2), A_{N_i} is a fy. neutrosophic nowh. dense sets in (P, τ_N). Therefore

$fn\ (\vee_{i=1}^{\infty} A_{N_i})^{+} = 0_N$, where $A_{N_i}{}'s$ is a fy. neutrosophic nowh. dense sets in (P, τ_N). Hence (P, τ_N) is a fy. neutrosophic Baire space.

Proposition 4.4:

If $fn\ (\vee_{i=1}^{\infty} A_{N_i})^{+}$ where $fn\ (A_{N_i})^{+} = 0_N$ and A_{N_i}'s are $fnCS$ in fy. neutrosophic top. space in (P, τ_N) then (P, τ_N) is a fy. neutrosophic Baire space.

Proof:

Let $A_{N_i}{}'s$ be $fnCS$ in (P, τ_N). Since $fn\ (A_{N_i})^{+} = 0_N$, by proposition (3.2), $A_{N_i}{}'s$ are fy. neutrosophic nowh. dense sets in (P, τ_N). Thus $fn\ (\vee_{i=1}^{\infty} A_{N_i})^{+} = 0_N$, where $A_{N_i}{}'s$ are fy. neutrosophic nowh. dense sets in (P, τ_N). Hence (P, τ_N) is a fy. neutrosophic Baire space.

Proposition 4.5

If $fn\ (\wedge_{i=1}^{n} A_{N_i},)^{-} = 1_N$, where $A_{N_i}{}'s$ are fy. neutrosophic dense and $fnOS$ in fy. neutrosophic top. space (P, τ_N) if and only if (P, τ_N) is a fy. neutrosophic Baire space.

Proof:

Let $A_{N_i}{}'s$ be fy. neutrosophic dense sets in (P, τ_N). Then $fn\ (\wedge_{i=1}^{n} A_{N_i},)^{-} = 1_N$ which gives that $1 - fn\ (\wedge_{i=1}^{n} A_{N_i},)^{-} = 0_N$. That is $fn\ ((1 - \wedge_{i=1}^{n} A_{N_i})\ )^{+} = 0_N$ gives that $fn\ (1 - \vee_{i=1}^{\infty} A_{N_i})^{+} = 0_N$. Since $A_{N_i}{}'s$ be fy. neutrosophic dense, $fn\ (A_{N_i})^{-} = 1_N$. Hence $fn\ (1 - A_{N_i})^{+} = 1 - fn\ \left(A_{N_i}\right)^{-} = 0_N$. Consequently $fn\ (1 - \vee_{i=1}^{\infty} A_{N_i}\ )^{+} = 0_N$, where $fn\ (1 - A_{N_i})^{+} = 0_N$ and $A_{N_i}{}'s$ be $fnCS$ in (P, τ_N). By proposition 4.4, (P, τ_N) is a fy. neutrosophic Baire space.

Conversely, let A_{N_i}'s are fy. neutrosophic dense and $fnCS$ in (P,τ_N). By proposition (3.7), $1 - A_{N_i}$'s are fy. neutrosophic nowh. dense sets in (P,τ_N). Then $A_N = \vee_{i=1}^{\infty} 1 - A_{N_i}$ is a fy. neutrosophic one category set in (P,τ_N). Now $fn\,(A_N)^+ = fn\,(\vee_{i=1}^{\infty}(1 - A_{N_i}))^+ = fn\,(1 - \wedge_{i=1}^{\infty} A_{N_i})^+ = (1 - fn\,(\vee_{i=1}^{\infty} A_{N_i}))^-$.

Since (P,τ_N) is a fy. neutrosophic Baire space, by proposition 4.1, we set $fn\,(A_N)^+ = 0_N$. Then $(1 - fn\,(\wedge_{i=1}^{\infty} A_{N_i})^-) = 0_N$. This gives that $(fn\,(\wedge_{i=1}^{\infty} A_{N_i})^-) = 1_N$.

Conclusion:

In this paper, the concept of a new class of sets, spaces and called them fy. neutrosophic dense, fy. neutrosophic nowh. dense, fy. neutrosophic re. set, fy. neutrosophic one category set, fy. neutrosophic two category sets, fy. neutrosophic Baire spaces, fy. neutrosophic one category space, fy. neutrosophic two category space. Some of its characterizations of fy. neutrosophic Baire spaces are also studied. As fuzzy neutrosophic have many applications in many fields: information technology, information system, decision support system. In the future research presented some of the applications.

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