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Classification of Ordered Semigroups Through Neutrosophic Generalized bi-ideals with Applications

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Abstract. An icebreaking theory known as neutrosophic theory opened a new direction for researchers of philosophy, logics, set theory and probability/statistics. Neutrosophy put the point base for a entire household of new mathematical speculations that summarized classical and fuzzy correspondence theories. In this article, we introduced the conception of neutrosophic fuzzy ideal theory of ordered semigroups based on belongs to relation and quasi-coincident with relation. Particularly, neutrosophic fuzzy generalized bi-ideal (resp. bi-ideal) of type $(\in, \in \vee q)$ have been developed and detail symposium on multi-dimension of the neutrosophic said ideals in ordered semigroup has given. Further, a verity of depictions of ordered semigroups in expression of $(\in, \in \vee q)$-fuzzy generalized bi-ideals have been constructed and several related examples have been formulated. Finally, the lower parts of neutrosophic $(\in, \in \vee q)$-fuzzy generalized bi-ideals were proposed and ordered semigroups have been discussed by the properties of these newly developed neutrosophic fuzzy generalized bi-ideals.

Keywords: Ordered Semigroup; Neutrosophic set; Neutrosophic $(\in, \in \vee q)$-fuzzy bi-ideal; Neutrosophic $(\in, \in \vee q)$-fuzzy generalized bi-ideal; lower Parts of the Neutrosophic generalized bi-ideals.)

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1. **Introduction**

In the modern times, economics and technological progress play a pivotal part in the evolution of at all particular country. Caused by high-quality analysis in the new field such as computer science, control system engineering, analyses of the data , economics, error-correction coding, answerable, prediction and automated, most realms have fallen back. These new realms spend a large scrap of their budgets in these areas. From another point of view, the above-mentioned meadows face several complex issues calling for uncertainty. These completed issues cannot be solved by traditional techniques. There are definite types of speculations, such as theoretical probability, fuzzy set theory, rough set theory, and soft set theory, which can be use for the above problems. However, all these theories have their importance and inherent limitations. One of the main problems accepted by these speculations is their incompatibility with parametric implements. In order to control such labours, in 1965, Zadeh [1] introduce the ice breaking conception of fuzzy subset, which could handling imprecision and uncertainties of these king of problems. So, here we specify some terms which is used throughout my thesis, FG is used for fuzzy group, OSG is used for ordered semigroup, OSGs used for ordered semigroups, SUBG used for subgroup, for fuzzy left (resp. right) ideal used FL(resp. right)I, for fuzzy generalized bi-ideal FGB-I is used, for quasi-ideal Q-I is used, for bi-ideal B-I is used, for fuzzy subgroup FSUBG is used, for regular RG is used, for completely regular CRG is used, for intra-regular intra-RG is used, for semigroup SG is used, for prime P is used, for semiprime SP is used, for simple SMP is used, for left simple LSMP is used, for quasi-prim Q-P is used, for weakly quasi-prim Q-P is used, for fuzzy left ideal FLI is used, $(\in, \in \vee q)$-fuzzy left ideal $(\in, \in \vee q)$-FLI is used, for subsemigroup SUBSG is used, for interior ideal II is used, for fuzzy set FS is used, for $(\in, \in \vee q)$-fuzzy generalized bi-ideal $(\in, \in \vee q)$-FGB-I is used, for $(\in, \in \vee q)$-fuzzy bi-ideal $(\in, \in \vee q)$-FB-I is used, for right simple RSMP is used, for fuzzy quasi-prime ideal FQ-PI is used, for strongly regular SRG is used, for fuzzy quasi-ideal FQ-I is used, and for plural only small "s" is added at the end e.g fuzzy ideals FIs, for weakly prime fuzzy ideal WPFI is used, for completely prime fuzzy ideal CPFI is used, for completely semiprime fuzzy ideal CSPFI is used, for fuzzy point FP is used, for quaasi Q is used, for subgroup SUBG is used, for $(\in, \in \vee q)$-fuzzy left (resp. right) ideal $(\in, \in \vee q)$-FL(resp. right)I is used, for intutionistic fuzzy set IFS is used, for neutrosophic set NS is used, for neutrosophic$(\in, \in \vee q)$-fuzzy generalized bi-ideal neutrosophic $(\in, \in \vee q)$-FGB-I is used, for intutionistic set IS is used, for paraconsistent set PS is used, for strongly simple SSMP is used, for characteristic function CF is used, for lower part LP used, and other also expressed on the same way else stated. Further Zadeh [4-7] elaborated the conception of fuzzy set to a large extent. After that 1971, Rosenfeld [2] proposed the new conception of fuzzy group(FG) it opened a new direction for the scientists to assessment different conceptions and consequence from the principality of algebra in the larger flow of fuzzy surroundings. Possess the inspiration considering, Das [3] suggested the conception of level subgroup of the FG. Further, Kruoki [12-16] described the notions of fuzzy left (resp. right, bi-, quasi-, generalised bi-) ideals of SGs and thereby identified various classes (regular, intra-regular, completely regular, semiprime, left simple) of semigroups in terms of these conceptions. The renowned research group of Kehayopulu [17-21] studied fuzzy left (right, bi-, interior and quasi-) ideals in OSGs to a greater extent. Moreover, The conception of (α, β)-FSUBG by utilizing the "belongs to" relation $(\in)$ and "quasi-coincident with "relation (q) of fuzzy point(FP) with fuzzy set(FS) by studied by Bakat and Das [9, 10] and Bakat [11]. Further the conception of the sort of (α, β)-FIIs, and new conception of sort an $(\in, \in \vee q_k)$-FII of an OSG of S is determined by khan et al. [32], here k is an testimonial component of $[0, 1)$. Otherwise expressed. Further demonstrated that in a regular(resp. semisimple) of OSG, the ideas of $(\in, \in \vee q_k)$-FI and $(\in, \in \vee q_k)$-FIIs matched. Similarly, the conception of (α, β)-FL(resp. right)Is of an OSG of S and the new kind of FL(resp. right)Is of the type $(\in, \in \vee q_k)$-F (resp. right)Is, here $k \in [0,\ 1)$. Special in this paper, reported the relation between ordinary FI and $(\in, \in \vee q_k)$-FIs of an OSG was initiated by Khan et al. [33]. After this, the conception of IFSs, which is a extension of FSs and provably equivalent to interval valued FSs are initiated by Atanassov [22], in 1986.

Further, in 1998 Smarandche [23] generalized the ice breaking conception of IFS, PS, and IS to the NS, and initiated the new conception of the NS.

After this big achievement Maji [24] reported the conception of neutrosophic soft set. Moreover, using the conception neutrosophic solution to make MCDM standard decisions. In addition to studying some interesting mathematical properties of the method, the algorithm neut-MCDM is also proposed. This work also provides a concise basis for the MCDM community with the first introduction of the NS this work proposes a multi approach which was investigated by Kharal [25]. However Salama et al. [26] investigate the notion of "neutosophic crisp neighborhoods system for the neutrosophic crisp point". In addition, to introduced and investigated the notion of the local function of the neutrosophic crispness, and constructed a new type of neutrosophic crisp topological space through the ideals of the neutrosophic crispness. It involves the possible application of GIS topology rules. Further the notion of rough NS was Studied by Broumi et al. [27], in this article they developed a hybrid structure said to be " Rough Neutrosophic Sets(RNSs)" and also, investigate their possessions. Therefore, both the NS theory and rough set theory are becoming a powerful tools for managing uncertainty, incompleteness and imprecision information. Mover the operation on the interval NS were investigated by Broumi and Smarandache [28], in this paper they further defines three new operation on interval NSs whic is based on arthmetic mean, geometric mean and harmonic mean. So the interval NS is an example of NS, which can be used in actual science and engineering

2. Some Basic Definitions and Results

Definition 2.1. If $(S, .)$ is semigroup, then the structure $(S, ., \leq)$ is called an OSG, $(S, \leq)$ is a partially ordered set (poset) i.e $\alpha \leq \alpha$ (reflexive), $\alpha \leq \beta, \beta \leq \gamma \Rightarrow \alpha = \beta$ (anti-symmetric), $\alpha \leq \beta, \beta \leq \gamma \Rightarrow \alpha \leq \gamma$,(transitive) $\forall \alpha,\ \beta,\ \gamma\ \in S$ and $x \leq y \Rightarrow x\alpha \leq y\beta$ and $xa \leq xb$ $\forall\ a, b, x\ \in S$.

Definition 2.2. Let X be an OSG S. Then interpret the subset (X] of S as.

$(X] = \{y \in S | y \leq x$ for some $x \in X\}$.
If $X = \{x\}$, then the notion (x] is used of $(\{x\}]$. For any subset X and Y of S, $XY = \{xy | x \in X$ and $y \in Y\}$ so throughout my thesis S is an OSG unless otherwise indicated.
The definition of SUBSG and left (right) ideal are discussed as follow.

Lemma 2.3. *If S be an OSG, then the understated condition are equivalently:*
(1) S is left WRG.
(2) $\Gamma \cap \Omega \subseteq (\Gamma\Omega], \ \forall$ *ideal* Γ *and GB-I* Ω *of S.*
(3) $\Gamma(c) \cap \Omega(c) \subseteq (\Gamma(c)\Omega(c)], \ \forall \ c \in S$.

Lemma 2.4. *If S be an OSG, then the understated axioms are equivalently:*
(1) S is LWRG.
(2) $\Gamma \cap \Psi \subseteq (\Gamma\Psi], \ \forall$ *ideal* Γ *and left ideal* Ψ *of S.*
(3) $\Gamma(x) \cap \Psi(x) \subseteq (\Gamma(x)\Psi(\alpha)], \ \forall \ x \in S$.

Lemma 2.5. *If S be an OSG, then the understated axioms are equivalently:*
(1) S is RG.
(2) $\Gamma \cap \Omega \cap \Psi \subseteq (\Gamma\Omega\Psi], \ \forall$ *right ideal* Γ*, GB-I* Ω *and left ideal* Ψ *of S.*
(3) $\Gamma(z) \cap \Omega(z) \cap \Psi(z) \subseteq (\Gamma(z)\Omega(z)\Psi(z)], \ \forall \ z \in S$.

Lemma 2.6. *If S be an OSG, then the understated axioms are equivalently:*
(1) S is RG.
(2) $\Omega \cap \Gamma \subseteq (\Omega\Gamma\Omega], \ \forall \ \Omega$ *GB-I and* Γ *ideal of S.*
(3) $(\Omega(k) \bigcap \Psi(k) \subseteq (\Omega(k)\Gamma(k)], \ \forall \ k \in S$.

Lemma 2.7. *An OSG S is completely regular* $\Leftrightarrow \forall \ X \subseteq S$*, we have,* $X \subseteq (X^2SX^2]$.

Lemma 2.8. *An OSG S is L(resp. right)SMP if,* $\forall$ *(Sx]=S, (resp.(xS]=S for all* $x \in S$.

Proposition

If χ and ψ are any subsets of an OSG S, then

(1) $\chi \subseteq (\chi]$.
(2) $(\chi](\psi] \subseteq (\chi\psi]$.
(3) $((\chi]] = (\chi]$.
(4) $((\chi](\psi] = (\chi\psi]$.

Proposition

Let X and $Y \neq \phi$ subsets of S, then we have the understated condition holds:

(1) $X \subseteq Y$ iff $A_X \preceq B_Y$.
(2) $A_X \wedge A_Y = A_{X \cap Y}$.
(3) $A_X \circ A_Y = A_{(XY]}$.

3. Neutrosophic sets (Basic Operation)

In the past two decennaries, the utilizes of soft set theory has made one more climacteric in mathematics. In mathematics, some mathematical enigmas contain indeterminacy in different paddock, such as answerable, automaton1 theory, coding theory, economics and memorandums of understanding, while other mathematical problems cannot be solved by ordinary mathematics. Due to the influence of parameterizations, tools (such as fuzzy set theory, probability theory,etc), the newest investigation on in this managment and the new investigation on theory of soft are fruitful due to the diversified uses of soft sets in the above-mentioned fields [27, 28]. It is worth noting that Sezgum and Atagum [29] studied various new actions on theory of soft and explained soft sets the following way:

Definition 3.1. If X is non-empty set, then structure λ in X is of the structure $\lambda = \{\langle a; \lambda_T(a), \lambda_I(a),$
$\lambda_F(a)\rangle | a \in X\}$ is called NS, where $\lambda_T : X \to [0,1]$ is a truth membership function, $\lambda_I : X \to [0,1]$ is an indeterminate membership function and $\lambda_F : X \to [0,1]$ is false membership function. Generalizing the notion of an ordered FP, we introduce a new concept called neutrosophic ordered points(NOPs) as follows:

Suppose S is an OSG, t $\in$ S *and* $u, v, w \in [0,1]$. By a NOPs, we mean $t_{\tilde{p}}(x) = \langle t_u(x), t_v(x), t_w(x)\rangle$ where $\tilde{p} = (u, v, w)$ and

$$t_u(x) = \begin{cases} u, \ if \ x \ \in \ (t], \\ \quad 1, \ ifnot. \end{cases}$$

$$t_v(x) = \begin{cases} v, \ if \ x \ \in \ (t], \\ \quad 1, \ ifnot. \end{cases}$$

$$t_w(x) = \begin{cases} w, \ if \ x \ \in \ (t], \\ \quad 1, \ ifnot. \end{cases}$$

Let $\lambda = \{\langle a; \ \lambda_T(a), \ \lambda_I(a), \ \lambda_F(a)\rangle\}$ be a NS and $t_{\tilde{p}}$ be a NOP, we define

$$i \rightarrow t_{\tilde{p}} \ \in \ \lambda \ if \begin{cases} \lambda_T(t) \leq \ u, \\ \lambda_I(t) \leq \ v, \\ \lambda_F(t) \geq \ w, \end{cases}$$

$$ii \rightarrow t_{\tilde{p}} \ q \ \lambda \ if \begin{cases} \lambda_T(t) + u \ < 1, \\ \lambda_I(t) + v \ < 1, \\ \lambda_F(t) + w \ > 1, \end{cases}$$

$$iii \rightarrow t_{\tilde{p}} \ \in \ \vee q \ \lambda \Rightarrow t_{\tilde{p}} \ \in \ \lambda \ or \ t_{\tilde{p}} q \lambda.$$

$$iv \rightarrow t_{\tilde{p}} \ \in \ \wedge q \ \lambda \Rightarrow t_{\tilde{p}} \ \in \ \lambda \ and \ t_{\tilde{p}} q \lambda.$$

$v \rightarrow t_{\tilde{p}} \ \overline{\in \wedge q} \ \lambda \Rightarrow t_{\tilde{p}} \ \in \ \wedge q \ \lambda$ does not hold.

If $\lambda = \langle x; \lambda_T(x), \ \lambda_I(x), \ \lambda_F(x) \rangle$ and $\eta = \langle x; \ \eta_T(x), \ \eta_I(x), \ \eta_F(x) \rangle$ be neutrosophic sets, then $\lambda \otimes \eta, \ \lambda \tilde{\cap} \eta$ and $\lambda \tilde{\cup} \eta$ are defined as follow:

$\lambda \tilde{\cap} \eta = \{ \langle x; \ ((\lambda_T \ o \ \eta_T), \ (\lambda_I \ o \ \eta_I), \ (\lambda_F \ o \ \eta_F)) \ (x) \rangle \}$
where $\lambda_T \ o \ \eta_T, \ \lambda_I \ o \ \eta_I \ and \ \lambda_F \ o \ \eta_F$ are defined as

$$(\lambda_T \ o \ \eta_T)(x) = \begin{cases} \wedge_{(y,z) \in A_x} [\lambda_T(y) \vee \eta_T(z)] \ if \ A_x \neq \phi, \\ 1 \ if \ A_x = \phi. \end{cases}$$

$$(\lambda_I \ o \ \eta_I)(x) = \begin{cases} \wedge_{(y,z) \in A_x} [\lambda_I(y) \vee \eta_I(z)] \ if \ A_x \neq \phi, \\ 1 \ if \ A_x = \phi. \end{cases}$$

$$(\lambda_F \ o \ \eta_F)(x) = \begin{cases} \vee_{(y,z) \in A_x} [\lambda_F(y) \wedge \eta_F(z)] \ if \ A_x \neq \phi, \\ 1 \ if \ A_x = \phi. \end{cases}$$

$\lambda \tilde{\cap} \eta = \{ \langle x; \ ((\lambda_T \cap \eta_T), \ (\lambda_I \cap \eta_I), \ (\lambda_F \cap \eta_F))(x) \rangle \},$

where

$(\lambda_T \cap \eta_T)(x) = \max\{\lambda_T(x), \ \eta_T(x), \ 0.5\}$

$(\lambda_I \cap \eta_I)(x) = \max\{\lambda_I(x), \ \eta_I(x), \ 0.5\}$

$(\lambda_F \cap \eta_F)(x) = \min\{\lambda_F(x), \eta_F(x), \ 0.5\}$

and

$\lambda\tilde{\cup}\eta = \{\langle x;\ ((\lambda_T \cup \eta),\ (\lambda_I \cup \eta_I),\ (\lambda_F \cup \eta_F))(x)\rangle\},$

where

$(\lambda_T \cup \eta_T)(x)$=min$\{\lambda_T(x),\ \eta_T(x),\ 0.5\}$

$(\lambda_I \cup \eta_I)(x)$=min$\{\lambda_I(x),\ \eta_I(x),\ 0.5\}$

$(\lambda_F \cup \eta_F)(x)$=max$\{\lambda_F(x),\ \eta_F(x),\ 0.5\}$

Note that if NOP (S) is the group of all NOPs in OSG S,

then

$t_{\tilde{p}}.s_{\tilde{q}} = (ts)\begin{pmatrix} u_1 \vee u_2 \\ v_1 \vee v_2 \\ w_1 \wedge w_2 \end{pmatrix} \in NOP\ (S)$ where$\tilde{p} = (u_1, v_1, w_1)$ *and* $\tilde{q} = (u_2, v_2, w_2)$.

Example

General example of neutrosophic set. The premise " Today is Sunny" or "Today will be Sunny" it does not convey a fixed rate constituent structure; this may assume to 50% true, 45% indeterminate and 40% false at time t_n where $n \geq 0$,but at the time t_{n+1} may be alter at 55% true, 46% indeterminate, and 28% false,(as stated to the new conformation source) and today at utter t_{n+60} the same premise may be 100% true, 0% indeterminate and 0% false (if today indeed sunny) this structure is dynamic; so the truth value change from time to time, another point of view, the truth value of the premise may be change from place to place
e.g;
the premise "It is sunny" in Islamabad, 100% true, 0% uncertain, and 0% false, but on the move to another site the city of Karachi the truth rate will be altered and may be 0% true, 0% indeterminate and 100% false It is also alter w.r.t viewer(subject to the parameter of the function T, I, F)
e.g;
"Simith is longer" $(.42\%, .64\%, .56\%)$ as stated to his mother, but $(0.86\%$
$0.23\%, 0.7\%)$ as stated to his personal Secretary, or $(0.48\%, 0.21\%, 0.31\%)$ as stated to his Boss.

4. Neutrosophic generalized bi-ideals of ordered semigroups

In this part, we initiate the conceptions of NS, neutrosophic $(\in, \in \vee q)$-SUBSG, neutrosophic $(\in, \in \vee q)$-FGB-I, neutrosophic $(\in, \in \vee q)$-FB-I, neutrosophic $(\in, \in \vee q)$-FL(resp. right)Is, neutrosophic level subset ,regular, weakly regular, related examples, theorems and propositions in detail.

For simplicity throughout the paper λ will be denoted for NS instead of$\lambda = \langle a;\ \lambda_T(a),\ \lambda_I(a),\ \lambda_F(a)\rangle$ unless otherwise stated.

Definition 4.1. A NS λ of an OSG S is called a neutrosophic $(\in, \in \vee q)$-SUBSG of S if the understated axiom is satisfied:
$(\forall t,\ s \in S)\ (\tilde{p} = (u_1,\ v_1,\ w_1),\ \tilde{q} = (u_2,\ v_2,\ w_2) \in [0, 1])$

$$\left(t_{\tilde{p}} \in \lambda;\ s_{\tilde{q}} \in \lambda \Rightarrow (ts)_{\begin{pmatrix} u_1 \vee u_2 \\ v_1 \vee v_2 \\ w_1 \wedge w_2 \end{pmatrix}} \in \vee q\lambda \right).$$

Definition 4.2. A neutrosophic set λ of an OSG S is said to be a neutrosophic $(\in, \in \vee q)$-FL(resp. right)I of S if the understated axioms are contented:

$(i)(\forall t,\ s \in S\ with\ t \leq s)(\tilde{p} = (u_1,\ v_1,\ w_1) \in [0,\ 1])(s_{\tilde{p}} \in \lambda \Rightarrow t_{\tilde{p}} \in \vee q\lambda).$

$(ii)(\forall t,\ s \in S)(\tilde{p} = (u_1,\ v_1,\ w_1) \in [0,\ 1])(s_{\tilde{p}} \in \lambda \Rightarrow (ts)_{\tilde{p}} \in \vee q\lambda(resp.(st)$
$_{\tilde{p}} \in \vee q\lambda)).$
Note that aneutrosophic set λ of S is a neutrosophic $(\in, \in \vee q)$-FI of S if it is both neutrosophic $(\in, \in \vee q)$-FLI and neutrosophic $(\in, \in \vee q)$-FRI of S.

Definition 4.3. A neutrosophic set(NS) λ of an OSG S is said to be a neutrosophic $(\in, \in \vee q)$-FGB-I of S if the understating axioms are contented:

$(i)(\forall t,\ s \in S\ with\ t \leq s)(\tilde{p} = (u_1,\ v_1,\ w_1) \in [0,\ 1])(s_{\tilde{p}} \in \lambda \Rightarrow t_{\tilde{p}} \in \vee q\lambda).$

$$(ii)(\forall t,a,s \in S\ t \leq s)(\tilde{p}=(u_1,v_1,w_1),\tilde{q}=(u_2,v_2,w_2)\in[0,1])$$
$$\left(t_{\tilde{p}} \in \lambda; s_{\tilde{q}} \in \lambda \Rightarrow (tas)_{\begin{pmatrix} u_1 \vee u_2 \\ v_1 \vee v_2 \\ w_1 \wedge w_2 \end{pmatrix}} \in \vee q\lambda \right).$$

Definition 4.4. A neutrosophic set λ of an ordered semigroup S is said to be a neutrosophic $(\in,\in\vee q)$-FB-I of S if it is both a neutrosophic $(\in,\in\vee q)$-FGB-I and neutrosophic$(\in,\in\vee q)$-SUBSG of S.

Theorem 4.5. *Suppose that G is a GB-I of an OSG S and λ is a neutrosophic subset of S such that:*

$$\lambda_T(x)=\begin{cases} 1\ if\ x\ \in S-G, \\ \leq 0.5\ if\ x\in G. \end{cases}$$
$$\lambda_I(x)=\begin{cases} 1\ if\ x\ \in S-G, \\ \leq 0.5\ if\ x\in G. \end{cases}$$
$$\lambda_F(x)=\begin{cases} 1\ if\ x\ \in S-G, \\ \geq 0.5\ if\ x\in G. \end{cases}.$$

Then,

(i) λ is a neutrosophic$(q,\in\vee q)$-FGB-I of S.

(ii) λ is a neutrosophic $(\in,\in\vee q)$-FGB-I of S.

Proof. (i) Let $t,s\in S$ *and* $u_1,v_1,w_1\ \in[0,\ 1]$ with $t\leq s$ be such that
$s_{\tilde{p}}q\lambda$ where $\tilde{p}=(u_1,\ v_1,\ w_1)$.

Then $s_{\tilde{p}}q\lambda$ implies that $\begin{cases} \lambda_T(s)+u_1<1, \\ \lambda_I(s)+v_1<1, \\ \lambda_F(s)+w_1>1. \end{cases}.$

Thus, $s\in G$ but G is FGB-I.
Therefore, $t\in G$ which implies that
$\lambda_T(t)\leq 0.5,\ \lambda_I(t)\leq 0.5$ and
$\lambda_F(t)\geq 0.5$. Now, if $u_1\geq 0.5, v_1\geq 0.5$ and $w_1\leq 0.5$,then $\lambda_T(t)\leq 0.5\leq u_1, \lambda_I(tas)\leq 0.5\leq v_1$ and $\lambda_F(tas)\geq 0.5\geq w_1$.Hence $t_{\tilde{p}}\in\lambda$.
If $u_1<0.5, v_1<0.5, w_1>0.5$, then $\lambda_T(t)+u_1<0.5+0.5=1$.
$\lambda_I(t)+v_1<0.5+0.5=1, \lambda_F(t)+w_1>0.5+0.5=1$,

Therefore, $t_{\tilde{p}}q\lambda$.

Thus, $t_{\tilde{p}} \in \vee q\lambda$.

Now assume $t,\ a,\ s \in S$ and $u_1,\ u_2,\ v_1,\ v_2,\ w_1,\ w_2 \in [0,\ 1]$ be such that $t_{\tilde{p}}q\lambda$ and $s_{\tilde{q}}q\lambda$ where $\tilde{p} = (u_1,\ v_1,\ w_1)$ and $\tilde{q} = (u_2, v_2, w_2)$.

Then $t_{\tilde{p}}q\lambda$ implies $\begin{cases} \lambda_T(t) + u_1 < 1, \\ \lambda_I(t) + v_1 < 1, \\ \lambda_F(t) + w_1 > 1. \end{cases}$

And $s_{\tilde{q}}q\lambda$ implies $\begin{cases} \lambda_T(s) + u_1 < 1, \\ \lambda_I(s) + v_1 < 1, \\ \lambda_F(s) + w_1 > 1. \end{cases}$.

Hence, $t, s \in G$ but G is generalized bi-ideal. Therefore, $tas \in G$ which implies that $\lambda_T(tas) \leq 0.5$, $\lambda_T(tas) \leq 0.5$ and $\lambda_F(tas) \geq 0.5$. Now, if $u_1 \geq 0.5,\ v_1 \geq 0.5$ and $w_1 \leq 0.5$, then $\lambda_T(tas) \leq 0.5 \leq u_1 \leq u_1 \vee u_1$, $\lambda_I(tas) \leq 0.5 \leq v_1 \leq v_1 \vee v_2$ and $\lambda_F(tas) \geq 0.5 \geq w_1 \geq w_1 \wedge w_2$.

Hence, $(tas)_{\begin{pmatrix} u_1 \vee u_2 \\ v_1 \vee v_2 \\ w_1 \wedge w_2 \end{pmatrix}} \in \lambda$

The similar case is also hold if $u_2 \geq 0.5,\ v_2 \geq 0.5$ and $w_2 \leq 0.5$.

If $u_1 < 0.5,\ v_1 < 0.5,\ w_1 > 0.5,\ u_2 < 0.5,\ v_2 < 0.5$ and $w_2 > 0.5$, then $\lambda_T(tas) + u_1 < 0.5 + 0.5 = 1, \lambda_I(tas) + v_1 < 0.5 + 0.5 = 1, \lambda_F(tas) + w_1 > 0.5 + 0.5 = 1,\ \lambda_T(tas) + u_2 < 0.5 + 0.5 = 1,\ \lambda_I(tas) + v_2 < 0.5 + 0.5 = 1$ and $\lambda_F(tas) + w_2 > 0.5 + 0.5 = 1$.

Consequently, $\lambda_T(tas) + u_1 \vee u_2 < 0.5 + 0.5 = 1$,

$\lambda_I(tas) + v_1 \vee v_2 < 0.5 + 0.5 = 1,\ \lambda_F(tas) + w_1 \wedge w_2 > 0.5 + 0.5 = 1$.

Resultantly, (tas)$_{\begin{pmatrix} u_1 \vee u_2 \\ v_1 \vee v_2 \\ w_1 \wedge w_2 \end{pmatrix}} q\lambda$.

Thus, $(tas)_{\begin{pmatrix} u_1 \vee u_2 \\ v_1 \vee v_2 \\ w_1 \wedge w_2 \end{pmatrix}} \in \vee q\lambda$.

Consequently, λ is a neutrosophic $(q, \in \vee q)$-FGB-I of S.

(ii) Assume that $t, s \in S$ and $u,\ v,\ w \in [0, 1]$ with $t \leq s$ be such that $s_{\tilde{p}} \in \lambda$ where $\tilde{p} = (u,\ v,\ w)$.

Then $s_{\tilde{q}} \in \lambda$ implies that $\begin{cases} \lambda_T(t) \leq u, \\ \lambda_I(t) \leq v, \\ \lambda_F(t) \geq w. \end{cases}$

Thus, $s \in G$ but G is FGB-I.

Therefore, $t \in G$ which implies that $\lambda_T(t) \leq 0.5,\ \lambda_I(t) \leq 0.5$ and $\lambda_F(t) \geq 0.5$. Now, if $u \geq 0.5,\ v \geq 0.5$ and $w \leq 0.5$, then $\lambda_T(t) \leq 0.5 \leq u,\ \lambda_I(tas) \leq 0.5 \leq v$ and $\lambda_F(tas) \geq 0.5 \geq w$. Hence $t_{\tilde{p}} \in \lambda$.

If $u < 0.5,\ v < 0.5,\ w > 0.5$, then $\lambda_T(t) + u < 0.5 + 0.5 = 1, \lambda_I(t) + v < 0.5 + 0.5 = 1,\ \lambda_F(t) + w > 0.5 + 0.5 = 1$, therefore, $t_{\tilde{p}} q \lambda$.

Thus, $t_{\tilde{p}} \in \vee q \lambda$.

Now suppose that $t,\ a,\ s \in S$ and $u_1, u_2,\ v_1,\ v_2,\ w_1, w_2 \in [0,1]$ be such that $t_{\tilde{p}} \in \lambda$ and $s_{\tilde{q}} \in \lambda$ where $\tilde{p} = (u_1, v_1, w_1)$ and $\tilde{q} = (u_2, v_2, w_2)$.

Then $t_{\tilde{p}} \in \lambda$. Implies $\begin{cases} \lambda_T(t) \leq u_1, \\ \lambda_I(t) \leq v_1, \\ \lambda_F(t) \geq w_1. \end{cases}$

And $s_{\tilde{q}} \in \lambda$ implies $\begin{cases} \lambda_T(s) \leq u_2, \\ \lambda_I(s) \leq v_2, \\ \lambda_F(s) \geq w_2. \end{cases}$.

Thus, $t,\ s \in G$ but G is FGB-I.

So, $tas \in G$ which implies that $\lambda_T(tas) \leq 0.5,\ \lambda_I(tas) \leq 0.5$ and $\lambda_F(tas) \geq 0.5$.

Now, if $u_1 \geq 0.5,\ v_1 \geq 0.5$ and $w_1 \leq 0.5$, then $\lambda_T(tas) \leq 0.5 \leq u_1 \leq u_1 \vee u_2, \lambda_I(tas) \leq 0.5 \leq v_1 \leq v_1 \vee v_2$ and $\lambda_F(tas) \geq 0.5 \geq w_1 \geq w_1 \wedge w_2$.

Hence, $(tas)_{\begin{pmatrix} u_1 \vee u_2 \\ v_1 \vee v_2 \\ w_1 \wedge w_2 \end{pmatrix}} \in \lambda$.

The similar case is also hold if $u_2 \geq 0.5,\ v_2 \geq 0.5$ and $w_2 \leq 0.5$.

If $u_1 < 0.5, v_1 < 0.5,\ w_1 > 0.5, u_2 < 0.5, v_2 < 0.5$ and $w_2 > 0.5$, then $\lambda_T(tas) + u_1 < 0.5 + 0.5 = 1$,

$\lambda_I(tas) + v_1 < 0.5 + 0.5 = 1, \lambda_T(tas) + w_1 > 0.5 + 9.5 = 1, \lambda_T(tas) + u_2 < 0.5 + 0.5 = 1, \lambda_I(tas) + v_2 < 0.5 + 0.5 = 1$ and $\lambda_F(tas) + w_2 > 0.5 + 0.5 = 1$.

Consequently, $\lambda_T(tas) + u_1 \vee u_2 < 0.5 + 0.5 = 1$,

$\lambda_I(tas) + v)_1 \vee v_2 < 0.5 + 0.5 = 1$,

$\lambda_F(tas) + w_1 \wedge w_2 > 0.5 + 0.5 = 1$.

Resultantly, $(tas)_{\begin{pmatrix} u_1 \vee u_2 \\ v_1 \vee v_2 \\ w_1 \wedge w_2 \end{pmatrix}} q\lambda.$

Thus, $(tas)_{\begin{pmatrix} u_1 \vee u_2 \\ v_1 \vee v_2 \\ w_1 \bigwedge w_2 \end{pmatrix}} \in \vee q\lambda.$

Therefore, λ is a neutrosophic $(\in, \in \vee q)$-FGB-I of S.□

Theorem 4.6. *If λ be a neutrosophic subset of an OSG S, then show that the understated condition are equivalently:*

(I) λ is a neutrosophic $(\in, \in \vee q)$-FGB-I of S.

$$(II)\ (i)\ (\forall s, t \in S \text{ such that } s \leq t) \left(\begin{cases} \lambda_T(s) \leq max\{\lambda_T(t), 0.5\}, \\ \lambda_T(s) \leq max\{\lambda_T(t), 0.5\}, \\ \lambda_T(s) \geq min\{\lambda_T(t), 0.5\}. \end{cases} \right)$$

$$(ii)\ (\forall t, a, s \in S) \left(\begin{cases} \lambda_T(tas) \leq max\{\lambda_T(tas), \lambda_T(t), 0.5\}, \\ \lambda_T(tas) \leq max\{\lambda_I(tas), \lambda_T(t), 0.5\}, \\ \lambda_T(tas) \geq min\{\lambda_F(tas), \lambda_T(t), 0.5\}. \end{cases} \right).$$

Proof. $(I) \Rightarrow (II)$: Let λ be a n neutrosophic $(\in, \in \vee q)$-FGB-I of S and assume on contrary bases that $\lambda_T(s) >$ max $\{\lambda_T(t),\ 0.5\}, \lambda_I(sat) >$ max $\{\lambda_I(t),\ 0.5\}$ and $\lambda_F(s) <$ min $\{\lambda_T(t),\ 0.5\}$, then $\exists\ u,\ v,\ w \in [0,\ 1] \ni \lambda_T(s) > u \geq$ max $\{\lambda_T(t),\ 0.5\},\ \lambda_I(tas) > v \geq$ max $\{\lambda_I(t),\ 0.5\}$ and $\lambda_F(s) \leq w <$ min $\{\lambda_T(t),\ 0.5\}$. It is clear that $\lambda_T(t) \leq u,\ \lambda_I(sat) \leq v$ and $\lambda_F(t) \leq w$ shows that $t_{\tilde{p}} \in \lambda$ but $s_{\tilde{q}} \overline{\in \vee q} \lambda$ which is a contradicts to the fact λ is a neutrosophic $(\in, \in \vee q)$-FGB-I.
Hence (i) hold. By similar argument we can also show that (ii) hold.
Thus, $(I) \Rightarrow (II)$.

$(II) \Rightarrow (I)$. Suppose (i) and (ii) hold, we need to manifest that λ is a neutrosophic $(\in, \in \vee q)$-FGB-I. For this let $s,\ t \in S$ such that $s \leq t,\ u,\ v,\ w \in [0,\ 1]$ and $t_{\tilde{p}} \in \lambda$ where $\tilde{p} = (u,\ v,\ w)$. Therefore,

$$t_{\tilde{p}} \in \lambda \text{ implies that } \begin{cases} \lambda_T(t) \leq u, \\ \lambda_I(t) \leq v, \\ \lambda_F(t) \geq w. \end{cases}.$$

Since by (i) $\begin{cases} \lambda_T(s) \leq \{\lambda_T(t),\ 0.5\} \leq u, \\ \lambda_I(s) \leq \{\lambda_T(t),\ 0.5\} \leq v, \\ \lambda_F(s) \geq \{\lambda_T(t), 0.5\} \geq w. \end{cases}$

which shows that $\begin{cases} \lambda_T(s) \leq u, \\ \lambda_I(s) \leq v, \\ \lambda_F(s) \geq w. \end{cases}$.

Hence $s_{\tilde{p}} \in \lambda$. If $u, v < 0.5$ and $w > 0.5$,

then $\begin{cases} \lambda_T(t) \leq u < 0.5, \\ \lambda_I(t) \leq v < 0.5, \\ \lambda_F(t) \geq w > 0.5. \end{cases}$

implies that $\lambda_T(t) < 0.5,\ \lambda_I(t) < 0.5$ and $\lambda_F(t) > 0.5$.
Consequently, $\lambda_T(s) < 0.5,\ \lambda_I(s) < 0.5$ and $\lambda_F(s) > 0.5$.
Therefore, $\lambda_T(s) + u < 0.5 + 0.5 = 1,\ \lambda_I(s) + u < 0.5 + 0.5 = 1$ and $\lambda_F(s) + u > 0.5 + 0.5 = 1$.
Hence, $s_{\tilde{p}} q \lambda$, So $s_{\tilde{p}} \in \vee q \lambda$.
Similarly, for $s,\ a,\ t \in S$ such that $t_{\tilde{p}} \in \lambda,\ s_{\tilde{q}} \in \lambda$.
$\Rightarrow\ (tas)_{\begin{pmatrix} u_1 \vee u_2 \\ v_1 \vee v_2 \\ w_1 \wedge w_2 \end{pmatrix}} \in \vee q \lambda$.
Resultantly, λ is a neutrosophic$(\in, \in \vee q)$-FGB-I of S.
Since every neutrosophiuc $(\in, \in \vee q)$-FB-I of S is a neutrosophic $(\in, \in \vee q)$-FGB-I of S but the opposite statement is generally inncorrect.□

Definition 4.7. Let λ be a neutrosophic subset(NSUBS) of an OSG S, for any $u,\ v,\ w \in [0,\ 1]$ the set
$U(\lambda, \tilde{p}) = \left\{ x \in S | \begin{array}{c} \lambda_T(x) \leq u, \\ \lambda_I(x) \leq v, \\ \lambda_F(x) \geq w. \end{array} \right\}$ is SAID TO BE a neutrosophic level subset(NLSUBS) of λ.

Theorem 4.8. *Suppose that λ is a neutrosophic subset of an OSG S. Then show that λ is a neutrosophic $(\in, \in \vee q)$-FGB-I of S $\Leftrightarrow U(\lambda,\ \tilde{p})(\neq \phi)$ is FGB-I of S for $(u,\ v \in (0,\ 0.5],\ w \in (0,\ 0.5])$.*

Proof. Let λ is a neutrosophic $(\in, \in\ \vee q)$-FGB-I of S. Consider such that $s,\ t \in S$ and $t \in U(\lambda,\ \tilde{p})$.

$$\text{Then} \quad \begin{array}{l} \lambda_T(t) \leq u, \\ \lambda_I(t) \leq v, \\ \lambda_F(t) \geq w. \end{array} .$$

Since λ is a neutrosophic $(\in, \in \vee q)$-FGB-I. Therefore,

$$\text{by Theorem [4.2]}, \begin{cases} \lambda_T(s) \leq max\{\lambda_T(t),\ 0.5\}, \\ \lambda_I(s) \leq max\{\lambda_I(t),\ 0.5\}, \\ \lambda_F(s) \geq min\{\lambda_F(t),\ 0.5\}. \end{cases}$$

which implies that $\begin{cases} \lambda_T(s) \leq max\{\lambda_T(t),\ 0.5\} = u, \\ \lambda_I(s) \leq max\{\lambda_I(t),\ 0.5\} = v, \\ \lambda_F(s) \geq min\{\lambda_F(t),\ 0.5\} = w. \end{cases}$ because $(u,\ v \in (0.5,\ 1],\ w \in (0,\ 0.5])$.

Thus, $s \in U(\lambda,\ \tilde{p})$.

Similarly, for $s,\ a,\ t \in S$ such that $s,\ t \in U(\lambda,\ \tilde{p})$ implies $sat \in U(\lambda,\ \tilde{p})$.

Hence, $U(\lambda,\ \tilde{p})$ is a FGB-I of S.

$\Longleftarrow$, assume that $U(\lambda,\ \tilde{p})$ is FGB-I of S for $(u,\ v \in (0.5,\ 1],\ w \in (0,\ 0.5])$.

Let $s,\ t \in S \ni s \leq t$.

$$\text{Suppose by contradiction} \begin{cases} \lambda_T(s) > max\{\lambda_T(t),\ 0.5\}, \\ \lambda_I(s) > max\{\lambda_I(t),\ 0.5\}, \\ \lambda_F(s) < min\{\lambda_F(t),\ 0.5\}. \end{cases} .$$

Then for some $u,\ v \in (0.5,\ 1],\ w \in (0,\ 0.5]$,

$$\begin{cases} \lambda_T(s) > u \geq max\{\lambda_T(t),\ 0.5\}, \\ \lambda_I(s) > v \geq max\{\lambda_I(t),\ 0.5\}, \\ \lambda_F(s) < w \leq min\{\lambda_F(t),\ 0.5\}. \end{cases} .$$

Implies that $t \in U(\lambda,\ \tilde{p})$ but $s \overline{\in} U(\lambda,\ \tilde{p})$ which is a contradicts to the fact that $U(\lambda,\ \tilde{p})$ is FGB-I of S.

$$\text{Therefore,} \begin{cases} \lambda_T(s) \leq max\{\lambda_T(s),\ \lambda_T(t),\ 0.5\}, \\ \lambda_I(s) \leq max\{\lambda_I(s),\ \lambda_I(t),\ 0.5\}, \\ \lambda_F(s) \geq min\{\lambda_F(s),\ \lambda_F(t),\ 0.5\}. \end{cases} .$$

$$\text{Similarly, for } s,\ a,\ t \in S, \left(\begin{cases} \lambda_T(tas) \leq max\{\lambda_T(t), 0.5\}, \\ \lambda_T(tas) \leq max\{\lambda_T(t), 0.5\}, \\ \lambda_T(tas) \geq min\{\lambda_T(t), 0.5\}. \end{cases} \right) \text{ also hold.}$$

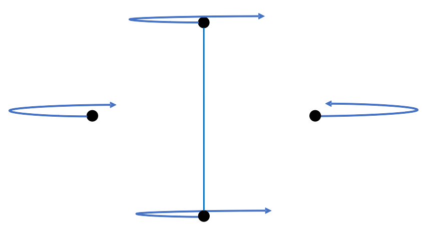
Thus, by Theorem [4.2], λ is a neutrosophic $(\in, \in \vee q)$-FGB-I of S. □

Example

Let $S = \{a, b, c, d\}$ be an OSG with understated multiplication table and ordered relation " $\leq$ " as follows: $\leq := \{(a, a), (b, b), (c, c), (d, d), (a, b)\}$

.	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	b	a
d	a	a	b	b

Table 1. Multiplicative table of Ordered Semigroup



S	$\lambda_T(x)$	$\lambda_I(x)$	$\lambda_F(x)$
a	0.18	0.15	0.30
b	0.20	0.19	0.28
c	0.17	0.16	0.33
d	0.20	0.20	0.32

TABLE 2. Example of Neutrosophic $(\in, \in \vee q)$-fuzzy generalized bi-ideals

Using definition (4.3) λ is neutrosophic $(\in, \in \vee q)$-FGB-I where $\tilde{p} = (0.25,\ 0.20,\ 0.22)$ and $\tilde{q} = (0.26,\ 0.30,\ 0.28)\ \in\ [0,\ 1]$

Definition 4.9. Let S be an OSG. The neutrosophic characteristic function $X_A = (X_{\lambda_T},\ X_{\lambda_I},\ X_{\lambda_F})$ of $A = \langle x, (\lambda_T,\ \lambda_I,\ \lambda_F)(x)\rangle$ is defined as

$$X_{\lambda_T}(x) = \begin{cases} 1\ if\ x \in A, \\ 0\ ifnot. \end{cases}$$

$$X_{\lambda_I}(x) = \begin{cases} 1\ if\ x \in A, \\ 0\ ifnot. \end{cases}$$

$$X_{\lambda_F}(x) = \begin{cases} 0\ if\ x \notin A, \\ 1\ ifnot. \end{cases}$$

Theorem 4.10. *A non-empty set B of an OSG S is a FGB-I of S* $\Leftrightarrow$ *the characteristic function* X_B *is a neutrosophic* $(\in, \in \vee q)$*-FGB-I of S.*

Proof. The proof is follows from theorem [4.3]. □

Theorem 4.11. *Suppose S is an OSG and L is a L(resp. right)I of S. If* λ *is defined by the neutrosophic subset of S*

$$\lambda_T(x) = \begin{cases} 1\ if\ x \in S - L, \\ \leq 0.5\ if\ x \in L. \end{cases}$$

$$\lambda_I(x) = \begin{cases} 1\ if\ x \in S - L, \\ \leq 0.5\ if\ x \in L. \end{cases}$$

$$\lambda_F(x) = \begin{cases} 1\ if\ x \in S - L, \\ \geq 0.5\ if\ x \in L. \end{cases}$$

Then

(i) λ *is a neutrosophic* $(q, \in \vee q)$*-FL(res. right)I of S.*

(ii) λ *is a neutrosophic* $(\in, \in \vee q)$*-FL(res. right)I of S.*

Proof. .□

Proved by theorem [4.1].

Theorem 4.12. *Assume that S is an OSG and I is an ideal of S. If λ be a neutrosophic subset of S defined as in Theorem [4.5], then λ is both a neutrosophic $(q, \in \vee q)$-FI and a neutrosophic $(\in, \in \vee q)$-FI of s.*

Proof. the proof follow by combining Theorem [4.5] and Theorem [4.1].□

Theorem 4.13. *If λ be a NSUBS of an OSG S, then show that the understating condition are equivalently:*

(I) *λ is a neutrosophic $(\in, \in \vee q)$-FL(resp. right)I of S.*

(II) *(i)*$(\forall s,\ t \in S\ suchthat\ s \leq t) \left(\begin{cases} \lambda_T(s) \leq max\{\lambda_T(t),\ 0.5\}, \\ \lambda_I(s) \leq max\{\lambda_T(t),\ 0.5\}, \\ \lambda_F(s) \geq min\{\lambda_T(t),\ 0.5\}. \end{cases} \right).$

(ii) $(\forall s, t \in S) \left(\begin{cases} \lambda_T(st) \leq max\{\lambda_T(t),\ 0.5\},\ (resp.max\{\lambda_T(s),\ 0.5\}), \\ \lambda_I(st) \leq max\{\lambda_I(t),\ 0.5\},\ (resp.max\{\lambda_I(s), 0.5\}), \\ \lambda_F(st) \geq min\{\lambda_F(t),\ 0.5\}.\ (resp.min\{\lambda_F(s), 0.5\}) \end{cases} \right).$

Proof. Proved by theorem [4.2]. □

Theorem 4.14. *Suppose that λ is a neutrosophic subset of an OSG S. Then λ is a neutrosophis $(\in, \in \vee q)$-FL(resp. right)I of S $\Leftrightarrow$ $U(\lambda, \tilde{p})(\neq\ \phi) \left(x \in S |\ \begin{matrix} \lambda_T(x) \leq u, \\ \lambda_I(x) \leq v, \\ \lambda_F(x) \geq w. \end{matrix} \right\}$ is a L(resp. right)I of S for $(u,\ v \in (0,\ 0.5],\ w \in (0,\ 0.5])$.*

Proof. Proved by theorem [4.3]. □

Definition 4.15. If S is an OSG, then S is RG $\Leftrightarrow \forall\ x \in S\ \exists\ a \in S \ni x \leq xax$ or $A \subseteq (XSX]$ $\forall\ X \subseteq S$.

Definition 4.16. If S is an OSG, then S is left weakly RG $\Leftrightarrow x \in S\ \exists\ a, b \in S \ni x \leq axay$ or $X \subseteq ((SX)^2]\ \forall\ X \subseteq S$.

Proposition

Let λ be a neutrosophic subset of a regular OSG S. Then every neutrosophic $(\in, \in \vee q)$-FGB-I of S is a neutrosophic $(\in, \in \vee q)$-FB-I of S.

Proof. Assume that $s,\ t \in S$ and λ is a neutrosophic $(\in, \in \vee q)$-FGB-I of S. Since S is RG so $\exists\ x \in S \ni s \leq sxs. \implies \lambda_T(s) \leq \max\{\lambda_T(sxs), 0.5\}$.

Hence $\lambda_T(st) \leq \max\{\lambda_T(sxst),\ 0.5\}$
$= max\{\lambda_T(s(xs)t),\ 0.5\}$
$\leq \max\{\lambda_T(s),\ \lambda_T(t),\ 0.5\}$,
by similar argument
$\lambda_I(st) \leq \max\{\lambda_I(s),\ \lambda_I(t),\ 0.5\}$ and $\lambda_F(st) \geq \min\{\lambda_F(s),\ \lambda_F(t),\ 0.5\}$ also hold.
Hence λ is a neutrosophic $(\in, \in \vee q)$-FB-I of S.□

Proposition

Consider that λ is a neutrosophic subset of a left weakly regular OSG S.Then every neutrosophic $(\in, \in \vee q)$-FGB-I of S is a neutrosophic $(\in, \in \vee q)$-FB-I of S.

Proof. Proved by proposition [3]. □

5. **Lower Parts of Neutrosophic** $(\in, \in \vee q)$**-generalized bi-ideals**

In this section, we will start the fundamental operations of the lower parts of the neutrosophic subset, the neutrosophic characteristic function(CF) lower parts, left (resp. right)RG, left (resp. right)SMP, the related theorems and the lammas of the lower parts.

Definition 5.1. Let λ be a neutrosophic subset of an OSG S, we stated the LP as $\lambda^- = \langle x, \lambda_T^-, \lambda_I^-,\ \lambda_F^- \rangle$ of λ as follows;

$\lambda_T^-(x)$=$\max\{\lambda_T(x),\ 0.5\}$

$\lambda_I^-(x)$=$\max\{\lambda_I(x),\ 0.5\}$

$\lambda_F^-(x)$=$\min\{\lambda_F(x),\ 0.5\}$

For any subset $A \neq \phi$ and neutrosophic subsets(NSUBSs) λ of OSG S, then LP of neutrosophic CF $(X_A)^-$ will be denoted by X_A^-.

Definition 5.2. Let λ and η any tow NSUBSs of an OSG S, we stated $(\lambda\tilde{\cap}\eta)^{-}$, $(\lambda\tilde{\cup}\eta)^{-}$ and $(\lambda\otimes\eta)^{-}$ as follows:

$$(\lambda\tilde{\cap}\eta)^{-}(x)=\max\{\lambda\tilde{\cap}\eta(x),\ 0.5\}$$

$$(\lambda\tilde{\cup}\eta)^{-}(x)=\max\{\lambda\tilde{\cup}\eta(x),\ 0.5\}$$

$$(\lambda\otimes\eta)^{-}(x)=\max\{\lambda\otimes\eta(x),\ 0.5\}.$$

Lemma 5.3. *Let λ and η be any tow NSUBSs of an OSG S, then $(\lambda^{-})^{-}=\lambda^{-}$ where $\lambda^{-}=\langle x,\ \lambda_T^{-},\ \lambda_I^{-},\ \lambda_F^{-}\rangle$ is the LP of λ.*

Proof. Assume that λ_0^{-} is the lower part of λ, then by definition [5.2]□

$\lambda_T^{-}\ (x)=\max\{\lambda_T(x),0.5\}$
$(\lambda_T^{-})_T^{-}(x)=\max\{\{max\{\lambda_T(x),0.5\}\},0.5\}$
$=\max\{\lambda_T(x),0.5\}=\lambda_T^{-}(x)$
Similarly $(\lambda_I^{-})_I^{-}=\lambda_I^{-}$ and $(\lambda_F^{-})_F^{-}=\lambda_F^{-}$ also hold.
Thus, $(\lambda^{-})^{-}=\lambda^{-}$.

Lemma 5.4. *Let λ and η be any tow NSUBSs of an OSG S, then*

- $(\lambda\tilde{\cap}\eta)^{-}=\lambda^{-}\tilde{\cap}\eta^{-}$.
- $(\lambda\tilde{\cup}\eta)^{-}=\lambda^{-}\tilde{\cup}\eta^{-}$
- $(\lambda\otimes\eta)^{-}=\lambda^{-}\otimes\eta^{-}$.

Proof. Proof is straightforward.□

Definition 5.5. Let S be an OSG. Then the LP of the neutrosophic CF $X_A^{-}=(X_{\lambda_T}^{-},\ X_{\lambda_I}^{-},\ X_{\lambda_F}^{-})$ of $A=\langle x,\ (\lambda_T,\lambda_I,\ \lambda_F)(x)\rangle$ is defined as

$$X^{-}{}_{\lambda_T}(x)=\begin{cases}0.5\ if\ x\in A,\\ 1\ otherwise.\end{cases}$$

$$X^{-}{}_{\lambda_I}(x)=\begin{cases}0.5\ if\ x\in A,\\ 1\ otherwise.\end{cases}$$

$$X^{-}{}_{\lambda_F}(x)=\begin{cases}1\ if\ x\notin A,\\ 0.5\ otherwise.\end{cases}$$

Theorem 5.6. *Let $A = \langle x,\ (\lambda_T,\ \lambda_I,\ \lambda_F)(x)\rangle$ and $B = \langle x,\ (\eta_T, \eta_I,\ \lambda_F)(x)\rangle$ are any tow NSUBS of an OSG S, then*

(1) $(X_A \tilde{\cap} X_B)^{-} = X^{-}_{A\cap B}$

(2) $(X_A \tilde{\cup} X_B)^{-} = X^{-}_{A\cup B}$

(3) $(X_A \otimes X_B)^{-} = X^{-}(_{AB}]$.

Proof. The proof of (1) and (2) is simple here. So for the proof of (3), Suppose that $x \in (AB]$, then $X^{-}(_{AB}](x) = 0.5$. Since $x \in (AB]$, then $x \leq ab$ for some $a \in A$ and $b \in B$ implies that $(a, b) \in A_x$.Thus $A_x \neq \phi$. Therefore,

$$(X_A \otimes X_B)^{-}(x) = \{(X_A \otimes X_B)(x), 0.5\}$$

Since $a \in A$ and $b \in B$, therefore, $X_{\lambda_T}(x) = 0.5 = X_{\eta_T}(b)$. Hence,

$$(X_{\lambda_T} \circ X_{\eta_T})(x) = \left\{\ \wedge_{(a,\ b)\in A_x} [X_{\lambda_T}(a) \vee X_{\eta_T}(b)]\ \right\}$$

$$= \left\{\ \wedge_{(a,\ b)\in A_x} [0.5, 0.5]\ \right\} = 0.5.$$

Similarly,
$(X_{\lambda_T} \circ X_{\eta_T})(x) = 0.5$ and
$(X_{\lambda_F} \circ X_{\eta_F})(x) = 0.5$.

Consequently, $(X_A \otimes X_B)(x) = 0.5$
$\Rightarrow$ $(X_A \otimes X_B)^{-}(x) = 0.5$.
Thus $(X_A \otimes X_B)^{-}(x) = X^{-}{}_{(AB]}$.
If $x \notin (Ab]$, then $X^{-}{}_{(AB](x)} = 1$. Let $(y, z) \in A_x$, then

$$(X_{\lambda_T} \circ X_{\eta_T})(x) = \left\{\ \wedge_{(y,z)\in A_x} [X_{\lambda_T}(y) \vee X_{\eta_T}(z)]\ \right\}$$

Since$(y,\ z) \in A_x$ then $x \leq yz$. If $y \in A$ and $z \in B$, then $yz \in AB$ implies that $x \in (AB]$ which goes to contradiction. Therefore, if $y \notin A$ and $z \in B$, then $X_{\lambda_T}(y) = 1,\ X_{\eta_T}(b) = 0.5$. Hence

$$(X_{\lambda_T} \circ X_{\eta_T})(x) = \left\{\ \wedge_{(y,z)\in A_x} [1 \vee 0.5]\ \right\} = 1.$$

The similar case hold if $y \in A$ and $z \notin B$. By similar way,
$(X_{\lambda_I} \circ X_{\eta_I})(x) = 1$ and $(X_{\lambda_F} \circ X_{\eta_F})(x) = 1$.

Consequently, $(X_A \otimes X_B)(x) = 1$.

Hence, $(X_A \otimes X_B)^{-} = X^{-}{}_{(AB]}$.□

Lemma 5.7. *The LP $X^{-}{}_A$ of the CF X_A of A is a neutrosophic $(\in, \in \vee q)$-FGB-I of an OSG $S \Leftrightarrow A$ is a GB-I of S.*

Proof. Let A is a GB-I of S, then by theorem [4.4], $X^{-}{}_A$ is a neutrosophic $(\in, \in \vee q)$-FGB-I of S.

$\Leftarrow$, suppose that $X^{-}{}_A$ is a neutrosophic $(\in, \in \vee q)$-FGB-I of S.

Let $x, y \in S \ni x \leq y$ and $y \in A$, then $X^{-}{}_A(y) = 0.5$.

Since $X^{-}{}_A$ is a neutrosophic $(\in, \in \vee q)$-FGB-I of S.

Thus $X^{-}{}_A(x) \leq max\{X^{-}{}_A(y),\ 0.5\} = 0.5$. Also $X^{-}{}_A \geq 0.5$ (always).

Therefore, $X^{-}{}_A(x) = 0.5$ shows that $x \in A$.

Similarly, for $x,\ y,\ z \in S$ and $x, z \in A$, then $X^{-}{}_A(y) = 0.5 = X^{-}{}_A(z)$.

Since $X^{-}{}_A$ is a neutrosophic $(\in, \in \vee q)$-FGB-I of S.

So $X^{-}{}_A(xyz) \leq max\{X^{-}{}_A(x),\ X^{-}{}_A(z),\ 0.5\} = 0.5$. also $X^{-}{}_A(xyz) \geq 0.5$ (always).

Therefore, $X^{-}{}_A(xyz) = 0.5$. Shows that $xyz \in A$.

Hence, A is a GB-I of S. □

Lemma 5.8. *The LP $X^{-}{}_A$ of the CF X_A of A is a neutrosophic $(\in, \in \vee q)$-FL(resp. right)I of $S \Leftrightarrow A$ is L(resp. right)I of S.*

Proof. Follows from the lemma [5.4].□

Definition 5.9. Let S be an OSG . Then S is L(resp. right)RG if $\forall\ a \in S, \exists\ x \in S \ni a \leq xa^2$ $(resp. a \leq a^2x)$ or $A \subseteq (SA^2]$ (resp. $A \subseteq (A^2S]$).

Definition 5.10. S is L(resp. right)SMP $\forall$ L(resp. right)I A of S, A=S. S is SMP if it is both left and right SMP , and is left, right and RG then S is CRG.

Lemma 5.11. *An OSG S is CRG $\Leftrightarrow \forall\ A \subseteq S$, we have, $A \subseteq (A^2SA^2]$.*

Lemma 5.12. *An OSG S is L(resp. right)SMP $\Leftrightarrow \forall$ (Sa]=S, (resp. (aS]=S $\forall\ a \in S$.*

Theorem 5.13. *If S is RG, left and right SMP, then $\lambda^{-}(a) = \lambda^{-}(b)\ \forall\ a, b \in S$ where λ is a neutrosophic $(\in, \in \vee q)$-FGB-I of S.*

Proof. Suppose that S is RG, left and right SMP and λ is a neutrosophic $(\in, \in \vee q)$-FGB-I of S. Let $E_s = \{s \in S | s \leq s^2\}$. Since S is RG, therefore $\forall\ a \in S, \exists\ x \in S \ni a \leq axa$ also $ax \leq axax = (ax)^2$. Thus $ax \in E_s$ implies that $E_s \neq \phi$. Now let $b,\ e \in S$, using Lemma

[3.3.7], $S=(Sb]$ and $S=(bS]$. Since $e\in S$ it implies that $e\in(Sb]$ and $e\in(bS]$, then $e\leq xb$, by for some $x,\ y\in S$.
Hence $e^2\leq(by)(xb)=b(yx)b$. Now as λ is a neutrosophic $(\in,\in\vee q)$-FGB-I of S.

$\lambda_T(e^2)\leq$max$\{\lambda_T(b(yx)b),\ 0.5\}$

$\leq\max\{\lambda_T(b),\ \lambda_T(b),0.5\}$

$=$max$\{\lambda_T(b),\ 0.5\}$

$\lambda_T(e^2)\vee 0.5\leq\max\{\lambda_T(b),\ 0.5\}\vee 0.5$

$=$max$\{\lambda_T(b),\ 0.5\}$

$\bar{\lambda}_T(e^2)\leq\bar{\lambda}_T(b)$

Since $e\in E_s$, so $e\leq e^2$ that is $\lambda_T(e)\leq\max\{\lambda_T(e^2),0.5\}$
implies that $\bar{\lambda}_T(e)\leq\bar{\lambda}_T(b)$.
By similar way $\bar{\lambda}_T(e)\leq\bar{\lambda}_I(b)$ and $\bar{\lambda}_F(e)\geq\bar{\lambda}_F(b)$. Therefore, $\bar{\lambda}$ is a constant on E_s. Now since S is RG so for $a\in S,\ ax,\ xa\in E_s$ follows that $\bar{\lambda}_T(ax)=\bar{\lambda}_T(b)=\bar{\lambda}_T(xa)$. Since $a\leq ax(axa)=(ax)a(xa)$. Therefore

$\lambda_T(a)\leq\max\ \{\lambda_T((ax)a(xa)),\ 0.5\}$

$\leq\max\ \{\lambda_T(ax),\ \lambda_T(xa),\ 0.5\}$

$=\max\ \{(\lambda_T(ax),\ 0.5),(\lambda_T(xa),\ 0.5)\}$

$\lambda_T(a)\vee 0.5=\max\ \{(\lambda_T(ax),\ 0.5),\ (\lambda_T(xa),\ 0.5)\}\bigvee 0.5$

$\bar{\lambda}_T(a)\leq\max\ \{\bar{\lambda}_T(ax),\ \bar{\lambda}_T(xa)\}=\bar{\lambda}_T(b)$.
By similar way, $\bar{\lambda}_I(a)\leq\bar{\lambda}_I(b),\ \bar{\lambda}_F(a)\geq\bar{\lambda}_F(b)$. Since $b\in(Sa],\ (aS]$, therefore, $b\leq sa,\ at$ for some $s,\ t\in S$. Thus

$\lambda_T(b^2)\leq\max\ \{\lambda_t(a(ts)a),\ 0.5\}$

$\leq \max\{\lambda_T(a),\ \lambda_T(a),\ 0.5\}$

$= \max\ \{\lambda_T(a),\ 0.5\}$

$\lambda_T(b^2) \leq \max\{\lambda_T(a),\ 0.5\} \vee 0.5$

$=\max\{\lambda_T(a),\ 0.5\}$

$\lambda^{-}{}_T(b^2) \leq \lambda^{-}{}_T(a)$

since $b \in E_s$ so $b \leq b^2$ that is $\lambda_T(b) \leq \max\ \{\lambda_T(b^2), 0.5\}$ implies that $\lambda^{-}{}_T(b) \leq \lambda^{-}{}_T(b^2)$.
Thus $\lambda^{-}{}_T(b) \leq \lambda^{-}{}_T(b^2) \leq \lambda^{-}{}_T(a)$.
By similar way $\lambda^{-}{}_I(b) \leq \lambda^{-}{}_I(a)$ and $\lambda^{-}{}_F(b) \geq \lambda^{-}{}_F(a)$.
Thus $\lambda^{-}{}_T(b) = \lambda^{-}{}_T(a), \lambda^{-}{}_I(b) = \lambda^{-}{}_I(a)$, and $\lambda^{-}{}_F(b) = \lambda^{-}{}_F(a)$. Resultantly, $\lambda^{-}(a) = \lambda^{-}(b)$. □

Theorem 5.14. *If S is an OSG, then it is RG* $\Leftrightarrow \forall$ *neutrosophic* $(\in, \in \vee q)$*-FGB-I of S,* $\lambda^{-}(a) = \lambda^{-}(a^2) \forall a \in S$.

Proof. The direct part of the theorem derived from Theorem [5.8].
$\Leftarrow$, suppose that $a \in S$, assume $B(a^2) = (a^2 \cup a^2Sa^2]$ is GB-I of S

$$X^{-}{}_{B(a^2)}(a) = \begin{cases} 0.5\ if\ a \in B(a^2) \\ 1\ otherwise \end{cases}$$

is a neutrosophic $(\in, \in \vee q)$-FGB-I S, $X^{-}{}_{B(a^2)}(a^2) = X^{-}{}_{B(a^2)}(a)$. Thus,$a \in B(a^2)$, hence $a \leq a^2$ or $a \leq a^2xa^2$. Now if $a \leq a^2$, then $a \leq a^2 = aa \leq a^2a^2 = aaa^2 \leq a^2aa^2 \in a^2Sa^2$ and $a \in (a^2Sa^2]$. If $a \leq a^2xa^2$, then $a \in (a^2Sa^2]$. $\therefore$, S is CRG.□

Lemma 5.15. *If S is an OSG, then the understating axioms are equivalently:*
(1) S is RG.
(2) $G \cap L \subseteq (GL]\ \forall$ *GB-I G and LI L of S.*
(3) $G(k) \cap L(k) \subseteq (G(k)L(k)]\ \forall\ k \in S$.

Theorem 5.16. *If S be an OSG, then the understating condition are equivalently:*
(I) S is RG.
(II) $(\lambda \otimes \eta)^{-} \preceq (\lambda \tilde{\cap} \eta)^{-}\ \forall$ *neutrosophic* $(\in, \in \vee q)$*-FGB-I of* λ *and neutrosophic* $(\in, \in \vee q)$*-FLI* η *of S.*

Proof. $(I) \Rightarrow (II)$:. Assume that S is RG, λ is a neutrosophic $(\in, \in \vee q)$-FGB-I and η is a neutrosophic $(\in, \in \vee q)$-FLI of S. Then $\exists\ x \in S \ni a \leq axa \leq (axa)(xa)$ implies that $(axa, xa) \in A_a$

which shows that $A_a \neq \phi$. Hence

$$(\lambda_T \circ \eta_T)^{-}(a) = \max\{(\lambda_T \circ \eta_T)(a),\ 0.5\}$$

$$= \max\Big\{\Big(\ \wedge_{(y,\ z)\in A_a} [\lambda_T(y) \vee \eta_T(z)].0.5\ \Big)\Big\}$$

$$\leq \max\ \{(\lambda_T(axa) \vee \eta_T(xa),\ 0.5)\}$$

Since λ is a neutrosophic $(\in, \in \bigvee q)$-FGB-I and η is a neutrosophic $(\in, \in \vee q)$-FLI of S. Then, $\lambda_T(axa) \leq \max\ \{\lambda_T(a),\ \lambda_T(a),\ 0.5\} = \max\{\lambda_T(a),\ 0.5\}$ and $\lambda_T(xa) \leq \max\ \{\lambda_T(a),\ 0.5\}$. Therefore, $(\lambda_T \circ \eta_T)^{-}(a) \leq (\lambda_T \cap \eta_T)^{-}(a)$.

Similarly, $(\lambda_I \circ \eta_I)^{-}(a) \leq (\lambda_I \cap \eta_I)^{-}(a)$ and $(\lambda_F \circ \eta_F)^{-}(a) \geq (\lambda_F \cap \eta_F)^{-}(a)$

Consequently, $(\lambda \otimes \eta)^{-} \preceq (\lambda \tilde{\cap} \eta)^{-}$

$\Leftarrow$, suppose that $(\lambda \otimes \eta)^{-} \preceq (\lambda \tilde{\cap} \eta)^{-}$. To demonstrate that S is RG, by Lemma [5.10], it is adequate to demonstrate that $G \cap L \subseteq (GL]$ for GB-I G and LI L of S. Let $x \in G \cap L$, then $x \in G$ and $x \in L$. Thus by Lemma [5.10], $X^{-}{}_G$ is a neutrosophic $(\in, \in \vee q)$-FGB-I and $X^{-}{}_L$ is a neutrosophic $(\in, \in \vee q)$-FLI of S. By supposition, $(X_G \otimes X_L)^{-}(x) \leq (X_G \tilde{\cap} X_L)^{-}(x) = \max\ \{(X_G \tilde{\cap} X_L)(x),\ 0.5\}$.

Since, $x \in G$ and $x \in L$, then $X_G(x) = 0.5$ or $X_G(x) \vee 0.5 = 0.5 = 0.5 \vee 0.5$ implies that $X^{-}{}_G(x) = 0.5$ similarly $X^{-}{}_L(x) = 0.5$ which show that $X^{-}{}_G \tilde{\cap} X^{-}{}_L = 0.5$. Follows that $(X_G \tilde{\cap} X_L)^{-}(x) = 0.5$. By Lemma [5.10], $(X_G \tilde{\cap} X_L)^{-}(x) = X^{-}{}_{(GL]} = 0.5$ therefore, $x \in (GL]$.

Hence,S is RG. □

Lemma 5.17. *If S be an OSG, then the understating axioms are equivalently:*

(1) S is RG.

(2) $G \cap T \subseteq (GT]$ $\forall$ *GB-I G and ideal T of S.*

(3) $G(k) \bigcap L(k) \subseteq (G(k)L(a)]$ $\forall$ $k \in S$.

Theorem 5.18. *If S be an OSG, then the understating condition are equivalently:*

(I) S is RG.

(II) $(\lambda \otimes \eta \otimes \lambda)^{-} \preceq (\lambda \tilde{\cap} \eta)^{-}$ $\forall$ *neutrosophic* $(\in, \in \vee q)$*-FGB-I* λ *and neutrosophic* $(\in, \in \vee q)$*-FLI* η *of S.*

Proof. The proof of the theorem can be obtained by following the same procedure as follows in the proof of Theorem [5.11].□

Lemma 5.19. *If S be an OSG, then the understating axioms are equivalently:*

(1) S is RG.

(2) $R \cap G \cap L \subseteq (RGL]$ $\forall$ *RI* R*, GB-I* G *and LI* L *of* S*.*

(3) $R(k) \cap G(k) \cap L(k) \subseteq (R(k)G(k)L(k)]$ $\forall$ $k \in S$*.*

Theorem 5.20. *If* S *be an OSG, then the understating condition are equivalent:*

(I) S *is RG.*

(II) $(\lambda \otimes \eta \otimes \xi)^{-} \preceq (\lambda \tilde{\cap} \eta \tilde{\cap} \xi)^{-}$ $\forall$ *neutrosophic* $(\in, \in \vee q)$*-FRI* λ*, neutrosophic* $(\in, \in \vee q)$*-FGB-I* η *and neutrosophic* $(\in, \in \vee q)$*-FLI* ξ *of* S*.*

Proof. Follows from the theorem [5.11]. □

Lemma 5.21. *If* S *be an OSG, then the understating axioms are equivalently:*

(1) S *is LWRG.*

(2) $T \cap L \subseteq (TL]$ $\forall$ *ideal* T *and LI* L *of* S*.*

(3) $T(k) \cap L(k) \subseteq (T(k)L(k)]$ $\forall$ $k \in S$*.*

Theorem 5.22. *If* S *be an OSG, then the understating condition are equivalently:*

(I) S *is LWRG.*

(II) $(\lambda \otimes \eta)^{-} \preceq (\lambda \tilde{\cap} \eta)^{-}$ $\forall$ *neutrosophic* $(\in, \in \vee q)$*-FI* λ *and neutrosophuic* $(\in, \in \vee q)$*-FLI* η *of* S*.*

Proof. Follows from the Theorem [5.11] and Lemma [5.16]. □

Lemma 5.23. *If* S *be an OSG, then the understating condition are equivalent:*

(1) S *is LWRG.*

(2) $T \cap G \subseteq (TG]$ $\forall$ *ideal* T *and GB-I* G *of* S*.*

(3) $T(k) \cap G(k) \subseteq (T(k)G(k)]$ $\forall$ $k \in S$*.*

Theorem 5.24. *If* S *be an OSG, then the understating condition are equivalently:*

(I) S *is LWRG.*

(II) $(\lambda \otimes \eta)^{-} \preceq (\lambda \tilde{\cap} \eta)^{-}$ $\forall$ *neutrosophic* $(\in, \in \vee q)$*-FI* λ *and neutrosophic* $(\in, \in \vee q)$*-FGB-I* η *of* S*.*

Proof. Follows from Theorem [5.11] and Lemma [5.18]. □

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