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Similarity Measures on Interval-Complex Neutrosophic Soft Sets with Applications to Decision Making and Medical Diagnosis under Uncertainty

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Abstract. The idea of an interval complex neutrosophic soft set (I-CNSS) emerges from the interval neutrosophic soft set (I-NSS) by the extension of its three membership functions (T, I, F) from real space to complex space (unit disc) to better handle uncertainties, vagueness, indeterminacy, and imprecision of information in the periodic nature. The novelty of I-CNSS lies in its more significant range of activity compared to CNS. Measures of similarity and distance are important tools that can be used to solve many problems in real life. Hence, this paper presents some interval complex neutrosophic soft similarities based on Hamming and Euclidean distances of I-CNSSs to deal with real-life problems that include uncertain information such as decision-making issues and medical diagnosis stats. Firstly, this paper reviews the definition of an interval complex neutrosophic soft set. Secondly, we defined distance Hamming measures and distance Euclidean measures on I-CNSSs . Next, the axiomatic definition of similarity measures based on Hamming and Euclidean distances of I-CNSSs is proposed. Moreover, a numerical example is given and relations between these measures are introduced and verified. Meanwhile, some applications are given to show how similarity can be used to help the user in making decisions and making medical diagnoses. Finally, a comparison of some current approaches is used to back up this study.

Keywords: similarity measure; decision making; interval complex neutrosophic soft set; distance measure.

1. Introduction

The idea of a complex fuzzy set (CFS) was established by Ramot [1] as a generalization of traditional fuzzy set theory [2] from real numbers to complex plane (unit disc) to represent the uncertain information that exists in two dimensions. The enormous success of this idea

has brought about the building up of many extensions of CFSs, such as complex intuitionistic fuzzy sets (CIFs) [3], complex multi-fuzzy sets (CMFSs) [4], complex vague sets (CVSs) [5], complex hesitant fuzzy sets (CHFSSs) [6] and interval-valued complex intuitionistic fuzzy sets (I-VCIFs) [7] which have been introduced and examined in several fields in our life, such as decision-making, image restoration and medical diagnosis. Ali and Smarandache [8] introduced a new model knowing a complex neutrosophic set where the three membership functions T, I, F instead of being real-valued functions with a range of $[0,1]$ is replaced by a complex-valued functions of the form $T = t_A(x).e^{j\mu_A(x)}, I = i_A(x).e^{j\beta_A(x)}$ and $F = f_A(x).e^{j\aleph_A(x)}$ with $(j = \sqrt{-1})$ and $\mu_A(x), \beta_A(x), \aleph_A(x)$ are periodic functions. To make the CNS more flexible and adaptable to cover vague periodic information in real-life problems, Ali and Smarandache again generalize the CNS idea to interval complex neutrosophic sets (I-CNSs) [9], which are characterized by the degrees of three complex membership functions that are characterized by interval values. Researchers utilized CNS and ICNS in different application areas and introduce new contributions, such as Broumi et al. [10] presented some of the potential properties and theorems with a multi-criteria decision-making process on bipolar complex neutrosophic sets. Quek et al. [11] introduced a new approach to neutrosophic graphs named complex neutrosophic graphs. Al-Quran et al. [12] investigated the relation between CNSs depending on the cartesian product of CNSs. Dat et al. [13] provided the connotation of a linguistic ICNS number, which is categorised independently by three membership functions linguistic variables for multiple attribute group decision-making (MGDM). All the existing mentioned theories work in several fields of life without considering the parameterization factor during the analysis. Consequently, these models lack parametrization tools to handle uncertainties and ambiguous issues in parameterized form. To cope with such challenges, Molodtsov (1999) [14] benchmarked and identified the theory of soft sets (SS) to handle uncertainty and vagueness by providing a general mathematical tool that can represent the problem parameters in a more wide and more complete form. A soft set is a set-valued map that provides a rough description of the things under consideration depending on several parameters. Since its inception, soft set has been studied and extended by researchers to several different hybrid models. First of all, Maji et al. employed the soft set theory with FSs and IFs and introduced new notions of the fuzzy soft set (FSS) [15] and intuitionistic fuzzy soft set (IFSS) [16]. Thus, the hybridization process of soft set theory with the other uncertainty concepts created many hybrid fuzzy structures, such as interval fuzzy soft set [17], interval-valued intuitionistic fuzzy soft set [18], neutrosophic soft set [19], interval neutrosophic soft set [20] Q-neutrosophic soft set [21, 22], etc. are introduced. Later, in order to incorporate the advantages of complex numbers into the concept of soft sets, fuzzy sets, and their generalizations, Thirunavukarasu et al. [23] developed the complex fuzzy soft set and tested it in decision making problems. Selvachandran and

Singh [24] extended the CFSSs theory and established a new vision which is known as interval-valued complex fuzzy soft sets (I-VCFSs). Kumar and Bajaj [25] introduced the concept of complex intuitionistic fuzzy soft sets, while Selvachandran et al. [26] proposed the concept of complex vague soft sets and defined several distance measures between these sets. Broumi et al. [27] presented the notion of CNSSs with their essential properties. Following that, these uncertainty sets have been actively used to address uncertainty in a variety of decision-making problems [28]- [35]. Recently, Al-Sharqi et al. [36] developed a hybrid model of complex neutrosophic sets with soft sets in an interval setting called the interval complex neutrosophic soft set (ICNSS). An ICNSS is defined by a complex interval-valued truth membership function which represents uncertainty with periodicity, complex interval-valued indeterminacy membership function which represents indeterminacy with periodicity, and a complex interval-valued falsity membership function which represents falsity with periodicity. This notion handles the neutrosophic environment data in a periodic manner, while the interval neutrosophic soft set provides a parameterization tool to handle the neutrosophic environment data. In addition, Al-Sharqi et al. also kept building on the idea of ICNSS by combining it with other mathematical techniques to solve problems with uncertainty more efficiently [37], [38].

Similarity measures are an important tool in fuzzy set theory and its hybrid structures. This tool indicates the degree of similarity between two objects in a fuzzy environment. Various similarity measures of fuzzy sets and their extensions have been offered and they have been successfully applied in solving real-world problems such as decision making [39], [40], medical diagnosis [41], [42], and pattern recognition [43], [44]. In neutrosophic environment, Broumi and Smarandache [45] introduced the concept of similarity of NSs. Jun Ye [46] proposed the concept of similarity measures between interval neutrosophic sets. Following that, Mukherjee and Sarkar [47] studied several similarity measures on interval neutrosophic soft sets with an application in pattern recognition. Abu Qamar and Hassan [48] applied similarity and entropy tools to Q-NSS and they examined these tools in decision-making problems and a medical diagnosis. The development of the uncertainty sets under the similarity measures environment mentioned above is not restricted to the real field but developed in the complex field. Recently, researchers [49]- [51] made noteworthy additions to the literature on similarity measures environments by using hybridized models to handle the uncertainty of periodic data, where time plays a vital role in describing it. Following this trend and to make our concept (ICNSS) more useful in modeling some problems in real life, in this article the Hamming and Euclidean distances between two interval complex neutrosophic soft sets (ICNS sets) are defined and similarity measures between two ICNS sets based on these distances are presented. Similarity measures between two ICNS sets based on a set theoretic approach are also proposed in this article. An application in decision-making and medical diagnosis methods is established based

on the proposed similarity measures. The rest of this paper is organized in the following way: In Section 2, we recall the fundamental concepts related to interval complex neutrosophic soft sets. In Section 3, we develop some similarity measures of interval complex neutrosophic soft sets based on the distance measures: Hamming distance and Euclidean distance. In Section 4, we apply these similarity measures to a decision-making problem and a medical diagnosis with interval-valued complex neutrosophic soft information. A detailed comparison between the tools used in this work and other existing tools in section 5. Finally, the conclusions are offered in Section 6.

2. Preliminaries

Now, in this current section, we recapitulate the idea of soft set (SS) [14], neutrosophic set (NS) [52, 53], interval neutrosophic set (INS) [54], complex neutrosophic set [8] and show an overview of the I-CNSS model [36].

2.1. Neutrosophic Set (NS)

Definition 2.1. [52, 53] A N is a neutrosophic set (NS) on universe of a non-empty universe V and defined as $N = \{\langle v, T_N(v), I_N(v), F_N(v) \rangle\}$, where $T_N(v), I_N(v), F_N(v)$ are truth, indeterminacy and falsity memberships respectively, such that $0 \leq T_N(v) + I_N(v) + F_N(v) \leq 3$.

2.2. Interval-Neutrosophic Set (INS)

Definition 2.2. [54]. An INS A defined on V is given by:

$A = \left\{ \left(v, \tilde{T}_A(v), \tilde{I}_A(v), \tilde{F}_A(v) \right) : v \in V \right\}$ where,
 $\tilde{T}_A(v) = [\tilde{p}_A^L(v), \tilde{p}_A^U(v)] \subseteq [0, 1], \tilde{I}_A(v) = [\tilde{q}_A^L(v), \tilde{q}_A^U(v)] \subseteq [0, 1]$ and $\tilde{F}_A(v) = [\tilde{r}_A^L(v), \tilde{r}_A^U(v)] \subseteq [0, 1]$ represent the interval truth membership, interval indeterminacy membership, and interval non-membership degrees such that $0^- \leq \tilde{T}_A(v) + \tilde{I}_A(v) + \tilde{F}_A(v) \leq 3^+$ for all $v \in V$.

2.3. Complex Neutrosophic Set (CNS)

Definition 2.3. [8] Let V be an initial universe, E be a set of parameters and $A \subset E$. Let $P(V)$ denote the complex neutrosophic power set of V . Then, a pair (\tilde{S}, A) is dubbed a complex neutrosophic set (CNS) on V , where \tilde{S} defined as a mapping $\tilde{F} : A \rightarrow P(V)$ such that

$$\tilde{S}(v) = \left\{ \alpha, \left(\tilde{T}_{\tilde{S}(\alpha)}(v), \tilde{I}_{\tilde{S}(\alpha)}(v), \tilde{F}_{\tilde{S}(\alpha)}(v) \right) : \alpha \in A \subset E, v \in V \right\},$$

where $\widetilde{T}_{\check{S}(\alpha)}(v), \widetilde{I}_{\check{S}(\alpha)}(v)$ and $\widetilde{F}_{\check{S}(\alpha)}(v)$ are complex truth-membership function, complex indeterminacy-membership function and complex false-membership function and defined as bellow:

$$\widetilde{T}_{\check{S}(\alpha)}(v) = \widetilde{p}_{\check{S}(\alpha)}(v) \cdot e^{j2\pi\widetilde{\mu}_{\check{S}(\alpha)}(v)},$$

$$\widetilde{I}_{\check{S}(\alpha)}(v) = \widetilde{q}_{\check{S}(\alpha)}(v) \cdot e^{j2\pi\widetilde{\omega}_{\check{S}(\alpha)}(v)}$$

and

$$\widetilde{F}_{\check{S}(\alpha)}(v) = \widetilde{r}_{\check{S}(\alpha)}(v) \cdot e^{j2\pi\widetilde{\Phi}_{\check{S}(\alpha)}(v)}.$$

Where,

$\widetilde{p}_{\check{S}(\alpha)}(v), \widetilde{q}_{\check{S}(\alpha)}(v)$ and $\widetilde{r}_{\check{S}(\alpha)}(v)$ indicate to amplitude terms and $e^{j2\pi\widetilde{\mu}_{\check{S}(\alpha)}(v)}, e^{j2\pi\widetilde{\omega}_{\check{S}(\alpha)}(v)}$ and $e^{j2\pi\widetilde{\Phi}_{\check{S}(\alpha)}(v)}$ indicate to phase terms.

2.4. Soft Set (SS)

Definition 2.4. [14] Let V be an initial universe and E be a set of parameters. Then, a pair (\widetilde{F}, E) is dubbed a soft set on V , where \widetilde{F} defined as a mapping $\widetilde{F} : E \rightarrow P(V)$ such that $P(V)$ denotes the power of parameters set in V .

2.5. Interval-Complex Neutrosophic Soft Set (I-CNSS)

Faisal et al. [36] introduced the idea of I-CNSS by combined both concepts SS and INS under complex setting to address two-dimensional indeterminate and incompatible data in periodic nature.

Definition 2.5. [36]. An I-CNSS (\widehat{G}, A) defined on V is a set given by:

$$(\widehat{G}, A) = \left\{ \left\langle \alpha, \widehat{T}_{\widehat{G}(\alpha)}(v), \widehat{I}_{\widehat{G}(\alpha)}(v), \widehat{F}_{\widehat{G}(\alpha)}(v) \right\rangle : \alpha \in A \subseteq E, v \in V \right\}.$$

Where,

$$\widehat{T}_{\widehat{G}(\alpha)}(v) = p_{\widehat{G}(\alpha)}(v) \cdot e^{j\mu_{\widehat{G}(\alpha)}(v)}, \widehat{I}_{\widehat{G}(\alpha)}(v) = q_{\widehat{G}(\alpha)}(v) \cdot e^{j\omega_{\widehat{G}(\alpha)}(v)},$$

$$\widehat{F}_{\widehat{G}(\alpha)}(v) = r_{\widehat{G}(\alpha)}(v) \cdot e^{j\Phi_{\widehat{G}(\alpha)}(v)}.$$

And the amplitude interval terms $p_{\widehat{G}(\alpha)}(v), q_{\widehat{G}(\alpha)}(v), r_{\widehat{G}(\alpha)}(v)$ can be write as

$$p_{\widehat{G}(\alpha)}(v) = \left[p_{\widehat{G}(\alpha)}^L(v), p_{\widehat{G}(\alpha)}^U(v) \right]$$

$$q_{\widehat{G}(\alpha)}(v) = \left[q_{\widehat{G}(\alpha)}^L(v), q_{\widehat{G}(\alpha)}^U(v) \right]$$

$$r_{\widehat{G}(\alpha)}(v) = \left[r_{\widehat{G}(\alpha)}^L(v), r_{\widehat{G}(\alpha)}^U(v) \right]$$

and for the phases interval terms $\mu_{\widehat{G}(\alpha)}(v), \omega_{\widehat{G}(\alpha)}(v), \Phi_{\widehat{G}(\alpha)}(v)$ can be write as

$$\mu_{\widehat{G}(\alpha)}(v) = \left[\mu_{\widehat{G}(\alpha)}^L(v), \mu_{\widehat{G}(\alpha)}^U(v) \right]$$

$$\omega_{\widehat{G}(\alpha)}(v) = \left[\omega_{\widehat{G}(\alpha)}^L(v), \omega_{\widehat{G}(\alpha)}^U(v) \right]$$

$$\Phi_{\widehat{G}(\alpha)}(v) = \left[\Phi_{\widehat{G}(\alpha)}^L(v), \Phi_{\widehat{G}(\alpha)}^U(v) \right]$$

where $p_{\hat{G}(\alpha)}^L(v), q_{\hat{G}(\alpha)}^L(v), r_{\hat{G}(\alpha)}^L(v), p_{\hat{G}(\alpha)}^U(v), q_{\hat{G}(\alpha)}^U(v), r_{\hat{G}(\alpha)}^U(v), \mu_{\hat{G}(\alpha)}^L(v), \omega_{\hat{G}(\alpha)}^L(v), \Phi_{\hat{G}(\alpha)}^L(v), \mu_{\hat{G}(\alpha)}^U(v), \omega_{\hat{G}(\alpha)}^U(v),$ and $\Phi_{\hat{G}(\alpha)}^U(v)$ represent the lower and upper bound of amplitude interval terms and phases interval terms respectively.

3. Similarity measures based on distance measures between I-CNSSs

Now, first we present several distances in the interval-complex neutrosophic soft sets (I-CNSSs) case and support it with some numerical examples. Second, based on the proposed distance measures between I-CNSSs, we give the following definition of similarity measures:

Definition 3.1. Assume that $\xi = (\hat{G}, A), \bar{\xi} = (\hat{K}, A)$ and $\bar{\xi} = (\hat{L}, A)$ are an interval-complex neutrosophic soft sets (I-CNSSs) on universe of discourse V . A function $d : I - CNSS(V) \times I - CNSS(V) \rightarrow [0, 1]$ is called distance measure I-CNSS(V) if d fulfill the following three axioms:

- (1) $d(\xi, \bar{\xi}) \geq 0$ and $d(\xi, \bar{\xi}) = 0$ if and only if $\xi = \bar{\xi}$.
- (2) $d(\xi, \bar{\xi}) = d(\bar{\xi}, \xi)$.
- (3) $d(\xi, \bar{\xi}) \leq d(\xi, \xi) + d(\bar{\xi}, \bar{\xi})$. (triangle inequality)

Definition 3.2. If $V = \{v_1, v_2, \dots, v_n\}$ is a nonempty universal set, $A = \{\alpha_1, \alpha_2, \dots, \alpha_m\}$ being a parameters set. Then for all $\xi = (\hat{G}, A), \bar{\xi} = (\hat{K}, B)$ are a I-CNSSs(V) and d a distance measure between I-CNSSs for all $\alpha_i \in E$. Then, the diverse distances between two I-CNSSs ξ and $\bar{\xi}$ are defined as follows:

- (1) The Hamming distance measure:

$$d_{I-CNSS}^H(\xi, \bar{\xi}) = \frac{1}{12} \sum_{j=1}^n \sum_{i=1}^m \left\{ \left| p_{\hat{G}(\alpha_i)}^L(v_j) - p_{\hat{K}(\alpha_i)}^L(v_j) \right| + \left| p_{\hat{G}(\alpha_i)}^U(v_j) - p_{\hat{K}(\alpha_i)}^U(v_j) \right| + \left| q_{\hat{G}(\alpha_i)}^L(v_j) - q_{\hat{K}(\alpha_i)}^L(v_j) \right| + \left| q_{\hat{G}(\alpha_i)}^U(v_j) - q_{\hat{K}(\alpha_i)}^U(v_j) \right| + \left| r_{\hat{G}(\alpha_i)}^L(v_j) - r_{\hat{K}(\alpha_i)}^L(v_j) \right| + \left| r_{\hat{G}(\alpha_i)}^U(v_j) - r_{\hat{K}(\alpha_i)}^U(v_j) \right| + \left| \mu_{\hat{G}(\alpha_i)}^L(v_j) - \mu_{\hat{K}(\alpha_i)}^L(v_j) \right| + \left| \mu_{\hat{G}(\alpha_i)}^U(v_j) - \mu_{\hat{K}(\alpha_i)}^U(v_j) \right| + \left| \varphi_{\hat{G}(\alpha_i)}^L(v_j) - \varphi_{\hat{K}(\alpha_i)}^L(v_j) \right| + \left| \varphi_{\hat{G}(\alpha_i)}^U(v_j) - \varphi_{\hat{K}(\alpha_i)}^U(v_j) \right| + \left| \omega_{\hat{G}(\alpha_i)}^L(v_j) - \omega_{\hat{K}(\alpha_i)}^L(v_j) \right| + \left| \omega_{\hat{G}(\alpha_i)}^U(v_j) - \omega_{\hat{K}(\alpha_i)}^U(v_j) \right| \right\}$$

- (2) The normalized Hamming distance measure:

$$d_{I-CNSS}^{NH}(\xi, \bar{\xi}) = \frac{d_{I-CNSS}^H(\xi, \bar{\xi})}{mn}$$

- (3) The Euclidean distance measure:

$$d_{I-CNSS}^E(\xi, \bar{\xi}) = \left\{ \frac{1}{12} \sum_{j=1}^n \sum_{i=1}^m \left[\left| p_{\hat{G}(\alpha_i)}^L(v_j) - p_{\hat{K}(\alpha_i)}^L(v_j) \right|^2 + \left| p_{\hat{G}(\alpha_i)}^U(v_j) - p_{\hat{K}(\alpha_i)}^U(v_j) \right|^2 + \left| q_{\hat{G}(\alpha_i)}^L(v_j) - q_{\hat{K}(\alpha_i)}^L(v_j) \right|^2 + \left| q_{\hat{G}(\alpha_i)}^U(v_j) - q_{\hat{K}(\alpha_i)}^U(v_j) \right|^2 + \left| r_{\hat{G}(\alpha_i)}^L(v_j) - r_{\hat{K}(\alpha_i)}^L(v_j) \right|^2 + \left| r_{\hat{G}(\alpha_i)}^U(v_j) - r_{\hat{K}(\alpha_i)}^U(v_j) \right|^2 + \left| \mu_{\hat{G}(\alpha_i)}^L(v_j) - \mu_{\hat{K}(\alpha_i)}^L(v_j) \right|^2 + \left| \mu_{\hat{G}(\alpha_i)}^U(v_j) - \mu_{\hat{K}(\alpha_i)}^U(v_j) \right|^2 + \left| \varphi_{\hat{G}(\alpha_i)}^L(v_j) - \varphi_{\hat{K}(\alpha_i)}^L(v_j) \right|^2 + \left| \varphi_{\hat{G}(\alpha_i)}^U(v_j) - \varphi_{\hat{K}(\alpha_i)}^U(v_j) \right|^2 + \left| \omega_{\hat{G}(\alpha_i)}^L(v_j) - \omega_{\hat{K}(\alpha_i)}^L(v_j) \right|^2 + \left| \omega_{\hat{G}(\alpha_i)}^U(v_j) - \omega_{\hat{K}(\alpha_i)}^U(v_j) \right|^2 \right] \right\}$$

$$\left. \begin{aligned} & \left| q_{G(\alpha_i)}^U(v_j) - q_{K(\alpha_i)}^U(v_j) \right|^2 + \left| r_{G(\alpha_i)}^L(v_j) - r_{K(\alpha_i)}^L(v_j) \right|^2 + \left| r_{G(\alpha_i)}^U(v_j) - r_{K(\alpha_i)}^U(v_j) \right|^2 + \\ & \frac{1}{(2\pi)^2} \left(\left| \mu_{G(\alpha_i)}^L(v_j) - \mu_{K(\alpha_i)}^L(v_j) \right|^2 + \left| \mu_{G(\alpha_i)}^U(v_j) - \mu_{K(\alpha_i)}^U(v_j) \right|^2 + \left| \varphi_{G(\alpha_i)}^L(v_j) - \varphi_{K(\alpha_i)}^L(v_j) \right|^2 + \right. \\ & \left. \left| \varphi_{G(\alpha_i)}^U(v_j) - \varphi_{K(\alpha_i)}^U(v_j) \right|^2 + \left| \omega_{G(\alpha_i)}^L(v_j) - \omega_{K(\alpha_i)}^L(v_j) \right|^2 + \left| \omega_{G(\alpha_i)}^U(v_j) - \omega_{K(\alpha_i)}^U(v_j) \right|^2 \right) \right\}^{\frac{1}{2}} \end{aligned}$$

(4) The normalized Euclidean distance measure:

$$d_{I-CNSS}^{NE}(\xi, \bar{\xi}) = \frac{d_{I-CNSS}^E(\xi, \bar{\xi})}{\sqrt{mn}}.$$

Based on these distance measures, the following properties clearly hold:

- (a) $0 \leq d_{I-CNSS}^H(\xi, \bar{\xi}) \leq mn.$
- (b) $0 \leq d_{I-CNSS}^{NH}(\xi, \bar{\xi}) \leq 1.$
- (c) $0 \leq d_{I-CNSS}^E(\xi, \bar{\xi}) \leq \sqrt{mn}.$
- (d) $0 \leq d_{I-CNSS}^{NE}(\xi, \bar{\xi}) \leq 1.$

Theorem 3.3. All the distance measures on I-CNSSs which are given in Definition 3.2 are distance functions of I-CNSSs.

Proof. It is clear that all the distance measures presented in Definition 3.2 fulfil the mentioned conditions in Definition 3.1. Thus, tracking brevity, we only go to prove condition (3) (triangle inequality) for the Hamming distance measure.

Let $\xi, \bar{\xi}$ and $\bar{\bar{\xi}}$ be three I-CNSSs and for Hamming distance measure, we have:

$$\begin{aligned} & d_{I-CNSS}^H(\xi, \bar{\xi}) + d_{I-CNSS}^H(\bar{\xi}, \bar{\bar{\xi}}) = \\ & \frac{1}{12} \sum_{j=1}^n \sum_{i=1}^m \left\{ \left| p_{\hat{G}(\alpha_i)}^L(v_j) - p_{\hat{K}(\alpha_i)}^L(v_j) \right| + \left| p_{\hat{G}(\alpha_i)}^U(v_j) - p_{\hat{K}(\alpha_i)}^U(v_j) \right| + \left| q_{\hat{G}(\alpha_i)}^L(v_j) - q_{\hat{K}(\alpha_i)}^L(v_j) \right| + \right. \\ & \left. \left| q_{\hat{G}(\alpha_i)}^U(v_j) - q_{\hat{K}(\alpha_i)}^U(v_j) \right| + \left| r_{\hat{G}(\alpha_i)}^L(v_j) - r_{\hat{K}(\alpha_i)}^L(v_j) \right| + \left| r_{\hat{G}(\alpha_i)}^U(v_j) - r_{\hat{K}(\alpha_i)}^U(v_j) \right| + \right. \\ & \left. \frac{1}{2\pi} \left(\left| \mu_{\hat{G}(\alpha_i)}^L(v_j) - \mu_{\hat{K}(\alpha_i)}^L(v_j) \right| + \left| \mu_{\hat{G}(\alpha_i)}^U(v_j) - \mu_{\hat{K}(\alpha_i)}^U(v_j) \right| + \left| \varphi_{\hat{G}(\alpha_i)}^L(v_j) - \varphi_{\hat{K}(\alpha_i)}^L(v_j) \right| \right. \right. \\ & \left. \left. + \left| \varphi_{\hat{G}(\alpha_i)}^U(v_j) - \varphi_{\hat{K}(\alpha_i)}^U(v_j) \right| + \left| \omega_{\hat{G}(\alpha_i)}^L(v_j) - \omega_{\hat{K}(\alpha_i)}^L(v_j) \right| + \left| \omega_{\hat{G}(\alpha_i)}^U(v_j) - \omega_{\hat{K}(\alpha_i)}^U(v_j) \right| \right) \right\} + \\ & \frac{1}{12} \sum_{j=1}^n \sum_{i=1}^m \left\{ \left| p_{\hat{K}(\alpha_i)}^L(v_j) - p_{\hat{L}(\alpha_i)}^L(v_j) \right| + \left| p_{\hat{K}(\alpha_i)}^U(v_j) - p_{\hat{L}(\alpha_i)}^U(v_j) \right| + \left| q_{\hat{K}(\alpha_i)}^L(v_j) - q_{\hat{L}(\alpha_i)}^L(v_j) \right| + \right. \\ & \left. \left| q_{\hat{K}(\alpha_i)}^U(v_j) - q_{\hat{L}(\alpha_i)}^U(v_j) \right| + \left| r_{\hat{K}(\alpha_i)}^L(v_j) - r_{\hat{L}(\alpha_i)}^L(v_j) \right| + \left| r_{\hat{K}(\alpha_i)}^U(v_j) - r_{\hat{L}(\alpha_i)}^U(v_j) \right| + \right. \\ & \left. \frac{1}{2\pi} \left(\left| \mu_{\hat{K}(\alpha_i)}^L(v_j) - \mu_{\hat{L}(\alpha_i)}^L(v_j) \right| + \left| \mu_{\hat{K}(\alpha_i)}^U(v_j) - \mu_{\hat{L}(\alpha_i)}^U(v_j) \right| + \left| \varphi_{\hat{K}(\alpha_i)}^L(v_j) - \varphi_{\hat{L}(\alpha_i)}^L(v_j) \right| + \right. \right. \\ & \left. \left. \left| \varphi_{\hat{K}(\alpha_i)}^U(v_j) - \varphi_{\hat{L}(\alpha_i)}^U(v_j) \right| + \left| \omega_{\hat{K}(\alpha_i)}^L(v_j) - \omega_{\hat{L}(\alpha_i)}^L(v_j) \right| + \left| \omega_{\hat{K}(\alpha_i)}^U(v_j) - \omega_{\hat{L}(\alpha_i)}^U(v_j) \right| \right) \right\} \\ & = \frac{1}{12} \sum_{j=1}^n \sum_{i=1}^m \left\{ \left| p_{\hat{G}(\alpha_i)}^L(v_j) - p_{\hat{K}(\alpha_i)}^L(v_j) \right| + \left| p_{\hat{K}(\alpha_i)}^L(v_j) - p_{\hat{L}(\alpha_i)}^L(v_j) \right| + \dots + \left| r_{\hat{G}(\alpha_i)}^L(v_j) - r_{\hat{K}(\alpha_i)}^L(v_j) \right| \right. \\ & \left. + \left| r_{\hat{K}(\alpha_i)}^U(v_j) - r_{\hat{L}(\alpha_i)}^U(v_j) \right| + \frac{1}{2\pi} \left(\left| \mu_{\hat{G}(\alpha_i)}^L(v_j) - \mu_{\hat{K}(\alpha_i)}^L(v_j) \right| + \left| \mu_{\hat{K}(\alpha_i)}^L(v_j) - \mu_{\hat{L}(\alpha_i)}^L(v_j) \right| + \dots + \right. \right. \\ & \left. \left. \left| \omega_{\hat{G}(\alpha_i)}^L(v_j) - \omega_{\hat{K}(\alpha_i)}^L(v_j) \right| + \left| \omega_{\hat{K}(\alpha_i)}^U(v_j) - \omega_{\hat{L}(\alpha_i)}^U(v_j) \right| \right) \right\} \geq \frac{1}{12} \sum_{j=1}^n \sum_{i=1}^m \left\{ \left| p_{\hat{G}(\alpha_i)}^L(v_j) - p_{\hat{L}(\alpha_i)}^L(v_j) \right| + \right. \end{aligned}$$

$$\begin{aligned}
 & \left| p_{\hat{G}(\alpha_i)}^U(v_j) - p_{\hat{L}(\alpha_i)}^U(v_j) \right| + \left| q_{\hat{G}(\alpha_i)}^L(v_j) - q_{\hat{L}(\alpha_i)}^L(v_j) \right| + \left| q_{\hat{G}(\alpha_i)}^U(v_j) - q_{\hat{L}(\alpha_i)}^U(v_j) \right| + \\
 & \left| r_{\hat{G}(\alpha_i)}^L(v_j) - r_{\hat{L}(\alpha_i)}^L(v_j) \right| + \left| r_{\hat{G}(\alpha_i)}^U(v_j) - r_{\hat{L}(\alpha_i)}^U(v_j) \right| \\
 & + \frac{1}{2\pi} \left(\left| \mu_{\hat{G}(\alpha_i)}^L(v_j) - \mu_{\hat{L}(\alpha_i)}^L(v_j) \right| + \left| \mu_{\hat{G}(\alpha_i)}^U(v_j) - \mu_{\hat{L}(\alpha_i)}^U(v_j) \right| + \left| \varphi_{\hat{G}(\alpha_i)}^L(v_j) - \varphi_{\hat{L}(\alpha_i)}^L(v_j) \right| \right. \\
 & \left. + \left| \varphi_{\hat{G}(\alpha_i)}^U(v_j) - \varphi_{\hat{L}(\alpha_i)}^U(v_j) \right| + \left| \omega_{\hat{G}(\alpha_i)}^L(v_j) - \omega_{\hat{L}(\alpha_i)}^L(v_j) \right| + \left| \omega_{\hat{G}(\alpha_i)}^U(v_j) - \omega_{\hat{L}(\alpha_i)}^U(v_j) \right| \right) \} \\
 & = d_{I-CNSS}^H(\xi, \tilde{\xi})
 \end{aligned}$$

□

Now, we will introduce the concept of a similarity measure between I-CNSSs. We can look into ambiguous data in interval-complex neutrosophic soft sets by using these measures.

Definition 3.4. Assume that $\xi = (\hat{G}, A)$, $\tilde{\xi} = (\hat{K}, A)$ and $\tilde{\xi} = (\hat{L}, A)$ are an interval-complex neutrosophic soft sets (I-CNSSs) on universe of discourse V . A function $S : I-CNSS(V) \times I-CNSS(V) \rightarrow [0, 1]$ is called similarity measure between I-CNSSs if S fulfill the following axioms:

- (S1) $0 \leq S(\xi, \tilde{\xi}) \leq 1$,
- (S2) $S(\xi, \tilde{\xi}) = 1$ if and only if $\xi = \tilde{\xi}$,
- (S3) $S(\xi, \tilde{\xi}) = S(\tilde{\xi}, \xi)$,
- (S4) If $\xi \subseteq \tilde{\xi} \subseteq \xi$, then $S(\xi, \tilde{\xi}) \leq \min \{ S(\xi, \tilde{\xi}), S(\tilde{\xi}, \xi) \}$.

Distance and similarity measures are related concepts in mathematics. Thus, we can use the proposed distances in definition 3.2 to describe similarity measures between I-CNSSs. As a result, we can provide several measures of similarity between I-CNSSs, as shown below.

- $S_{I-CNSS}^H(\xi, \tilde{\xi}) = \frac{1}{1+d_{I-CNSS}^H(\xi, \tilde{\xi})}$.
- $S_{I-CNSS}^{NH}(\xi, \tilde{\xi}) = \frac{1}{1+d_{I-CNSS}^{NH}(\xi, \tilde{\xi})}$.
- $S_{I-CNSS}^E(\xi, \tilde{\xi}) = \frac{1}{1+d_{I-CNSS}^E(\xi, \tilde{\xi})}$.
- $S_{I-CNSS}^{NE}(\xi, \tilde{\xi}) = \frac{1}{1+d_{I-CNSS}^{NE}(\xi, \tilde{\xi})}$.

Example 3.5. Let $\xi = (\hat{G}, A)$ and $\tilde{\xi} = (\hat{K}, A)$ be two I-CNSSs on $V = \{v_1, v_2\}$ and defined as follows:

$$\xi = \left\{ \left\{ \alpha_1, \left(\frac{([0.4,0.6].e^{j2\pi[0.5,0.6]}, [0.1,0.7].e^{j2\pi[0.1,0.3]}, [0.3,0.5].e^{j2\pi[0.8,0.9]})}{v_1} \right), \right. \right. \\
 \left. \left. \left(\frac{([0.2,0.4].e^{j2\pi[0.3,0.6]}, [0.1,0.1].e^{j2\pi[0.7,0.9]}, [0.5,0.9].e^{j2\pi[0.2,0.5]})}{v_2} \right) \right\} \right\}$$

and,

$$\tilde{\xi} = \left\{ \left\{ \alpha_1, \left(\frac{\langle [0.2,0.7].e^{j2\pi[0.7,0.8]}, [0.4,0.9].e^{j2\pi[0.3,0.5]}, [0.6,0.8].e^{j2\pi[0.5,0.6]} \rangle}{v_1} \right) \right\}, \left(\frac{[0.15,0.52].e^{j2\pi[0.1,0.3]}, [0,0.5].e^{j2\pi[0.6,0.8]}, [0.3,0.3].e^{j2\pi[0.6,0.7]}}{v_2} \right) \right\}$$

Now, by Definition 3.2, we have the following results:

$$\begin{aligned} d_{I-CNSS}^H(\xi, \tilde{\xi}) &= \\ & \frac{1}{12} (|0.4 - 0.2| + |0.6 - 0.7| + |0.1 - 0.4| + |0.7 - 0.9| + |0.3 - 0.6| + |0.5 - 0.8| + \\ & \frac{1}{2\pi} (|0.5\pi - 0.7\pi| + |0.6\pi - 0.8\pi| + |0.1\pi - 0.3\pi| + |0.3\pi - 0.5\pi| + |0.8\pi - 0.5\pi| + |0.9\pi - 0.6\pi|)) \\ & + \frac{1}{12} (|0.2 - 0.15| + |0.4 - 0.52| + |0.1 - 0| + |0.1 - 0.5| + |0.5 - 0.3| + |0.9 - 0.3| + \\ & \frac{1}{2\pi} (|0.3\pi - 0.1\pi| + |0.6\pi - 0.3\pi| + |0.7\pi - 0.6\pi| + |0.9\pi - 0.8\pi| + |0.2\pi - 0.6\pi| + |0.5\pi - 0.7\pi|)) \\ & = 0.350 \end{aligned}$$

and

$$d_{I-CNSS}^{NH}(\xi, \tilde{\xi}) = \frac{0.350}{4} = 0.0875$$

$$\begin{aligned} d_{I-CNSS}^E(\xi, \tilde{\xi}) &= \\ & \frac{1}{12} \left((|0.4 - 0.2|^2 + |0.6 - 0.7|^2 + |0.1 - 0.4|^2 + |0.7 - 0.9|^2 + |0.3 - 0.6|^2 + |0.5 - 0.8|^2 + \right. \\ & \left. \frac{1}{4\pi^2} (|0.5\pi - 0.7\pi|^2 + |0.6\pi - 0.8\pi|^2 + |0.1\pi - 0.3\pi|^2 + |0.3\pi - 0.5\pi|^2 + |0.8\pi - 0.5\pi|^2 + |0.9\pi - 0.6\pi|^2) \right) \\ & + \frac{1}{12} (|0.2 - 0.15|^2 + |0.4 - 0.52|^2 + |0.1 - 0|^2 + |0.1 - 0.5|^2 + |0.5 - 0.3|^2 + |0.9 - 0.3|^2 + \\ & \left. \frac{1}{4\pi^2} (|0.3\pi - 0.1\pi|^2 + |0.6\pi - 0.3\pi|^2 + |0.7\pi - 0.6\pi|^2 + |0.9\pi - 0.8\pi|^2 + |0.2\pi - 0.6\pi|^2 + |0.5\pi - 0.7\pi|^2) \right) \Big)^{\frac{1}{2}} \\ & = 0.209 \end{aligned}$$

. and

$$d_{I-CNSS}^{NE}(\xi, \tilde{\xi}) = \frac{0.209}{2} = 0.1045$$

Now, by Equations in definition 3.4, respectively, we get the similarity between two I-CNSSs as following:

$$\begin{aligned} S_{I-CNSS}^H(\xi, \tilde{\xi}) &= \frac{1}{1+0.350} = 0.741, \quad S_{I-CNSS}^{NH}(\xi, \tilde{\xi}) = \frac{1}{1+0.0875} = 0.919. \\ S_{I-CNSS}^E(\xi, \tilde{\xi}) &= \frac{1}{1+0.209} = 0.827, \quad S_{I-CNSS}^{NE}(\xi, \tilde{\xi}) = \frac{1}{1+0.1045} = 0.905. \end{aligned}$$

Nonetheless, in practice, a different weight may have been assigned to the different sets. i.e, $\exists w_i \geq 0, i = 1, 2, \dots, m$, and $\sum_{i=1}^m w_i = 1$, for each element $v_i \in V$. Therefore, in this work, we will propose the weighted Hamming and Euclidean distance measures for I-CNSSs.

- The weighted Hamming distance measure:

$$d_{I-CNSS}^{wH}(\xi, \tilde{\xi}) = \frac{1}{12} \sum_{j=1}^n \sum_{i=1}^m w_i \left\{ |p_{G(\alpha_i)}^L(v_j) - p_{K(\alpha_i)}^L(v_j)| + |p_{G(\alpha_i)}^U(v_j) - p_{K(\alpha_i)}^U(v_j)| + |q_{G(\alpha_i)}^L(v_j) - q_{K(\alpha_i)}^L(v_j)| + |q_{G(\alpha_i)}^U(v_j) - q_{K(\alpha_i)}^U(v_j)| + |r_{G(\alpha_i)}^L(v_j) - r_{K(\alpha_i)}^L(v_j)| + |r_{G(\alpha_i)}^U(v_j) - r_{K(\alpha_i)}^U(v_j)| + \frac{1}{2\pi} \left(| \mu_{G(\alpha_i)}^L(v_j) - \mu_{K(\alpha_i)}^L(v_j) | + | \mu_{G(\alpha_i)}^U(v_j) - \mu_{K(\alpha_i)}^U(v_j) | + | \varphi_{G(\alpha_i)}^L(v_j) - \varphi_{K(\alpha_i)}^L(v_j) | + | \varphi_{G(\alpha_i)}^U(v_j) - \varphi_{K(\alpha_i)}^U(v_j) | + | \omega_{G(\alpha_i)}^L(v_j) - \omega_{K(\alpha_i)}^L(v_j) | + | \omega_{G(\alpha_i)}^U(v_j) - \omega_{K(\alpha_i)}^U(v_j) | \right) \right\}$$

and

- The normalized weighted Hamming distance measure:

$$d_{I-CNSS}^{nwH}(\xi, \tilde{\xi}) = \frac{d_{I-CNSS}^{wH}(\xi, \tilde{\xi})}{mn}.$$

- The weighted Euclidean distance measure:

$$d_{I-CNSS}^{wE}(\xi, \tilde{\xi}) = \left\{ \frac{1}{12} \sum_{j=1}^n \sum_{i=1}^m w_i \left[|p_{G(\alpha_i)}^L(v_j) - p_{K(\alpha_i)}^L(v_j)|^2 + |p_{G(\alpha_i)}^U(v_j) - p_{K(\alpha_i)}^U(v_j)|^2 + |q_{G(\alpha_i)}^L(v_j) - q_{K(\alpha_i)}^L(v_j)|^2 + |q_{G(\alpha_i)}^U(v_j) - q_{K(\alpha_i)}^U(v_j)|^2 + |r_{G(\alpha_i)}^L(v_j) - r_{K(\alpha_i)}^L(v_j)|^2 + |r_{G(\alpha_i)}^U(v_j) - r_{K(\alpha_i)}^U(v_j)|^2 + \frac{1}{(2\pi)^2} \left(| \mu_{G(\alpha_i)}^L(v_j) - \mu_{K(\alpha_i)}^L(v_j) |^2 + | \mu_{G(\alpha_i)}^U(v_j) - \mu_{K(\alpha_i)}^U(v_j) |^2 + | \varphi_{G(\alpha_i)}^L(v_j) - \varphi_{K(\alpha_i)}^L(v_j) |^2 + | \varphi_{G(\alpha_i)}^U(v_j) - \varphi_{K(\alpha_i)}^U(v_j) |^2 + | \omega_{G(\alpha_i)}^L(v_j) - \omega_{K(\alpha_i)}^L(v_j) |^2 + | \omega_{G(\alpha_i)}^U(v_j) - \omega_{K(\alpha_i)}^U(v_j) |^2 \right) \right] \right\}^{\frac{1}{2}}$$

- The normalized weighted Euclidean distance measure:

$$d_{I-CNSS}^{nwE}(\xi, \tilde{\xi}) = \frac{d_{I-CNSS}^{wE}(\xi, \tilde{\xi})}{mn}.$$

Also, the similarity measure on weighted Hamming and Euclidean distance measures for I-CNSSs defined as following:

- $S_{I-CNSS}^{wH}(\xi, \tilde{\xi}) = \frac{1}{1+d_{I-CNSS}^{wH}(\xi, \tilde{\xi})}.$
- $S_{I-CNSS}^{nwH}(\xi, \tilde{\xi}) = \frac{1}{1+d_{I-CNSS}^{nwH}(\xi, \tilde{\xi})}.$

- $S_{I-CNSS}^{wE}(\xi, \tilde{\xi}) = \frac{1}{1+d_{I-CNSS}^{wE}(\xi, \tilde{\xi})}$.
- $S_{I-CNSS}^{wnE}(\xi, \tilde{\xi}) = \frac{1}{1+d_{I-CNSS}^{wnE}(\xi, \tilde{\xi})}$.

Theorem 3.6. *If S be the similarity measure between two I-CNSSs $\xi, \tilde{\xi}$. Then,*

1. $S(\xi, \tilde{\xi}) = S(\tilde{\xi}, \xi)$.
2. $0 \leq S(\xi, \tilde{\xi}) \leq 1$.
3. $0 \leq S(\xi, \tilde{\xi}) = 0$ iff $\xi = \tilde{\xi}$.

Proof. Immediately follows from definitions 3.4. \square

4. Applications

I-CNSS is a hybrid tool for modeling two-dimensional information of a periodic nature in our everyday lives. In this section, we introduce some practical examples of I-CNSSs to show that the proposed similarity measures play an important role in solving some real-life problems such as decision-making problems and medical diagnosis.

4.1. Similarity Measures of I-CNSSs Applied to Medical Diagnosis

In this current subsection, we developed an algorithm based on the Hamming similarity measure of two I-CNS sets to evaluate the possibility that a sick person having related symptoms is suffering from typhoid. This algorithm depends on data described by two I-CNSS models, and it is built with the assistance of a medicinal master person. where the first I-CNSS indicates illness, and the second I-CNSS indicates the ill person. Then we find the similarity measure of these two I-CNS sets. We also think that the person may have typhoid if the similarity measure between these two I-CNS sets is greater than or equal to 0.6, which can be fixed with the help of a medical professional.

4.1.1. Algorithm

Step 1: Construct a model I-CNSS over the universe V for illness, which may be developed with the aid of a medical expert person.

Step 2: Build I-CNSS over the universe V for the patient person by helping a medical expert person.

Step 3: Compute Hamming distance between the model I-CNSS for illness and the I-CNSS for the patient person.

Step 4: Compute similarity Hamming measure between the I-CNSS for illness and the I-CNSS for the patient person.

Step 5: If the similarity Hamming measure between two I-CNSSs is greater than or equal to 0.6, then the person may possibly be suffering from typhoid, while if the similarity Hamming measure between two I-CNSSs is less than 0.6, then the person may not possibly be suffering from typhoid.

Now, we give a numerical example that shows a way out of these diagnosis problems from a scientific point of view. This is done to show how the above-proposed algorithm can be used to tell a patient if they have typhoid or not.

Example 4.1. Let $V = \{v_1 = \text{typhoid}, v_2 = \text{not typhoid}\}$, and $E = \{\alpha_1 = \text{flu}, \alpha_2 = \text{headache}, \alpha_3 = \text{body pain}\}$ be set of parameters which consist of symptoms of typhoid disease.

Step 1: Construct the model I-CNSS for typhoid:

$$\xi = \left\{ \left\{ \alpha_1, \left(\frac{\langle [0.3,0.4].e^{j2\pi[0.4,0.5]}, [0.4,0.5].e^{j2\pi[0.2,0.3]}, [0.6,0.7].e^{j2\pi[0.1,0.2]} \rangle}{v_1} \right), \left(\frac{\langle [0.6,0.7].e^{j2\pi[0.2,0.3]}, [0.2,0.3].e^{j2\pi[0.6,0.7]}, [0.1,0.2].e^{j2\pi[0.6,0.8]} \rangle}{v_2} \right) \right\}, \left\{ \alpha_2, \left(\frac{\langle [0.3,0.4].e^{j2\pi[0.7,0.8]}, [0.0,0.3].e^{j2\pi[0.4,0.6]}, [0.3,0.4].e^{j2\pi[0.4,0.5]} \rangle}{v_1} \right), \left(\frac{\langle [0.5,0.6].e^{j2\pi[0.2,0.3]}, [0.5,0.6].e^{j2\pi[0.1,0.2]}, [0.0,0.1].e^{j2\pi[0.2,0.4]} \rangle}{v_2} \right) \right\}, \left\{ \alpha_3, \left(\frac{\langle [0.4,0.6].e^{j2\pi[0.5,0.6]}, [0.1,0.7].e^{j2\pi[0.1,0.3]}, [0.3,0.5].e^{j2\pi[0.8,0.9]} \rangle}{v_1} \right), \left(\frac{\langle [0.6,0.7].e^{j2\pi[0.4,0.6]}, [0.1,0.1].e^{j2\pi[0.7,0.9]}, [0.5,0.9].e^{j2\pi[0.2,0.5]} \rangle}{v_2} \right) \right\} \right\}$$

Step 2: Create two models of I-CNSS for patients X and Y, respectively, as:

$$\tilde{\xi} = \left\{ \left\{ \alpha_1, \left(\frac{\langle [0.3,0.45].e^{j2\pi[0.35,0.5]}, [0.3,0.5].e^{j2\pi[0.25,0.35]}, [0.5,0.7].e^{j2\pi[0.1,0.2]} \rangle}{v_1} \right), \left(\frac{\langle [0.2,0.4].e^{j2\pi[0.6,0.7]}, [0.05,0.3].e^{j2\pi[0.5,0.7]}, [0.3,0.4].e^{j2\pi[0.6,0.65]} \rangle}{v_2} \right) \right\}, \left\{ \alpha_2, \left(\frac{\langle [0.05,0.2].e^{j2\pi[0.4,0.5]}, [0.1,0.2].e^{j2\pi[0.5,0.7]}, [0.1,0.3].e^{j2\pi[0.3,0.3]} \rangle}{v_1} \right), \left(\frac{\langle [0.25,0.4].e^{j2\pi[0.5,0.6]}, [0.6,0.6].e^{j2\pi[0.2,0.25]}, [0.25,0.3].e^{j2\pi[0.4,0.8]} \rangle}{v_2} \right) \right\}, \left\{ \alpha_3, \left(\frac{\langle [0.2,0.5].e^{j2\pi[0.4,0.5]}, [0,0.3].e^{j2\pi[0.14,0.2]}, [0.1,0.3].e^{j2\pi[0.5,0.7]} \rangle}{v_1} \right), \left(\frac{\langle [0.3,0.5].e^{j2\pi[0.2,0.4]}, [0.2,0.2].e^{j2\pi[0.3,0.5]}, [0.6,0.7].e^{j2\pi[0.3,0.6]} \rangle}{v_2} \right) \right\} \right\}$$

$$\tilde{\xi} = \left\{ \left\{ \alpha_1, \left(\frac{\langle [0.8,0.9].e^{j2\pi[0.3,0.6]}, [0.3,0.7].e^{j2\pi[0.4,0.6]}, [0.2,0.3].e^{j2\pi[0.4,0.5]} \rangle}{v_1} \right), \left(\frac{\langle [0.1,0.5].e^{j2\pi[0.2,0.4]}, [0,0.3].e^{j2\pi[0.3,0.3]}, [0.2,0.3].e^{j2\pi[0.2,0.5]} \rangle}{v_2} \right) \right\}, \left\{ \alpha_2, \left(\frac{\langle [0.5,0.6].e^{j2\pi[0.4,0.4]}, [0.1,0.2].e^{j2\pi[0.2,0.3]}, [0.2,0.4].e^{j2\pi[0.1,0.4]} \rangle}{v_1} \right), \left(\frac{\langle [0.1,0.3].e^{j2\pi[0.5,0.6]}, [0.4,0.4].e^{j2\pi[0.1,0.2]}, [0.3,0.5].e^{j2\pi[0.2,0.5]} \rangle}{v_2} \right) \right\}, \left\{ \alpha_3, \left(\frac{\langle [0.3,0.6].e^{j2\pi[0.2,0.3]}, [0.1,0.1].e^{j2\pi[0.3,0.4]}, [0.4,0.5].e^{j2\pi[0.4,0.6]} \rangle}{v_1} \right), \left(\frac{\langle [0.6,0.6].e^{j2\pi[0.3,0.2]}, [0.1,0.4].e^{j2\pi[0.2,0.4]}, [0.3,0.5].e^{j2\pi[0.2,0.6]} \rangle}{v_2} \right) \right\} \right\}$$

In $\bar{\xi}$ and $\tilde{\xi}$ above, which are based on the physician’s report, the amplitude term of the lower and upper bounds of the complex interval truth membership function denotes the strength and intensity of the symptoms that the patient suffers, and the phase term of the lower and upper bounds of the complex interval truth membership function denotes the period of the symptoms. At the same time, the amplitude term of the lower and upper bounds of the complex interval indeterminacy membership function means the inability to indeterminate these symptoms during the period mentioned in the phase term. The lower and upper bounds of the complex interval falsity membership function denote the absence of these symptoms during the period mentioned in the phase term of the lower and upper bounds of the complex interval falsity membership function. Here it is necessary to point out that the values of the amplitude term closer to 1 describe heavy symptoms, and values of the amplitude term closer to zero represent moderate symptoms. The period of the phase term is defined as weeks, so for values equal to 1, the period will be maximum.

Step 3:Based on Definition 3.2, the Hamming distance between ξ and $\tilde{\xi}$ is 0.725 while between ξ and $\bar{\xi}$ is 0.821.

Step 4:Based on Definition 3.4, the similarity Hamming measure between ξ and $\tilde{\xi}$ is 0.579 while between ξ and $\bar{\xi}$ is 0.549.

Step 4:Here $S_{I-CNSS}^H(\xi, \tilde{\xi}) < 0.6$ and $S_{I-CNSS}^H(\xi, \bar{\xi}) < 0.6$ that means both patients X and Y may possibly suffer from no typhoid.

Remark 4.2. In case no such conclusion can be drawn with the given information then we need to reassess all the symptoms with the help of expert and then repeat all the steps proposed in I-CNSSM-algorithm

4.2. Similarity Measures of I-CNSSs Applied to Multicriteria Decision Making

In this section, we developed an algorithm based on similarity measures of interval complex neutrosophic soft sets as defined in definition 3.4 for possible application in multicriteria decision making.

4.2.1. Algorithm

Step 1: Construct a model I-CNSS $\xi = (K, A)$ over the universe V depends on the opinion of one of the experts and the customer satisfaction rate.

Step 2: Compute the distance measured between the optimality choice $([1, 1] \cdot e^{2\pi[1,1]})$ and α_i (for $i = 1, 2, 3$) with the weighting vector w .

Step 3: Compute similarity measure for all the distances we got in **Step 2:**.

Step 4: Analyze the result and the decision is to select the alternative which has the highest similarity to the optimality of I-CNSS.

Example 4.3. Suppose a customer wishes to buy a new computer for his personal usage. There are four computers (four alternatives) that can be represented by $V = \{v_1, v_2, v_3, v_4\}$, a customer can pick out one of them. The customer considers three features (attributes) in his choice, namely performance, service, and price, which can be represented by $A = \{\alpha_1, \alpha_2, \alpha_3\}$ respectively with the weight vectors $w = \{0.3, 0.3, 0.4\}$. To help the customer choose the right apparatus, we will apply the proposed algorithm.

Step 1: Construct the model I-CNSS for the opinion of one of the experts and the customer satisfaction rate:

$$\begin{aligned} \xi = & \left\{ \left\{ \alpha_1, \left(\frac{\langle [0.8,0.9].e^{2\pi[0.7,0.8]}, [0.2,0.3].e^{2\pi[0.1,0.2]}, [0.2,0.4].e^{2\pi[0.2,0.3]} \rangle}{v_1} \right), \left(\frac{\langle [0.4,0.65].e^{j2\pi[0.55,0.7]}, [0.1,0.4].e^{j2\pi[0.15,0.4]}, [0.4,0.6].e^{j2\pi[0.2,0.25]} \rangle}{v_2} \right) \right. \right. \\ & \left. \left(\frac{\langle [0.3,0.7].e^{j2\pi[0.6,0.8]}, [0.4,0.5].e^{j2\pi[0.3,0.6]}, [0.5,0.8].e^{j2\pi[0.6,0.64]} \rangle}{v_3} \right) \left(\frac{\langle [0.2,0.4].e^{j2\pi[0.6,0.7]}, [0.05,0.3].e^{j2\pi[0.5,0.7]}, [0.3,0.4].e^{j2\pi[0.6,0.65]} \rangle}{v_4} \right) \right\}, \\ & \left\{ \alpha_2, \left(\frac{\langle [0.5,0.6].e^{j2\pi[0.4,0.5]}, [0.1,0.2].e^{j2\pi[0.5,0.7]}, [0.1,0.3].e^{j2\pi[0.3,0.3]} \rangle}{v_1} \right), \left(\frac{\langle [0.3,0.45].e^{j2\pi[0.35,0.5]}, [0.3,0.5].e^{j2\pi[0.25,0.35]}, [0.5,0.7].e^{j2\pi[0.1,0.2]} \rangle}{v_2} \right) \right. \\ & \left. \left(\frac{\langle [0.2,0.5].e^{j2\pi[0.25,0.3]}, [0.4,0.7].e^{j2\pi[0.45,0.5]}, [0.3,0.6].e^{j2\pi[0.2,0.35]} \rangle}{v_3} \right) \left(\frac{\langle [0.25,0.4].e^{j2\pi[0.5,0.6]}, [0.6,0.6].e^{j2\pi[0.2,0.25]}, [0.25,0.3].e^{j2\pi[0.4,0.8]} \rangle}{v_4} \right) \right\}, \\ & \left\{ \alpha_3, \left(\frac{\langle [0.2,0.5].e^{j2\pi[0.4,0.5]}, [0.0,3].e^{j2\pi[0.14,0.2]}, [0.1,0.3].e^{j2\pi[0.5,0.7]} \rangle}{v_1} \right), \left(\frac{\langle [0.6,0.6].e^{j2\pi[0.3,0.6]}, [0.33,0.4].e^{j2\pi[0.51,0.61]}, [0.4,0.8].e^{j2\pi[0.2,0.5]} \rangle}{v_2} \right) \right. \\ & \left. \left(\frac{\langle [0.3,0.45].e^{j2\pi[0.35,0.5]}, [0.4,0.5].e^{j2\pi[0.18,0.5]}, [0.6,0.9].e^{j2\pi[0.3,0.5]} \rangle}{v_3} \right), \left(\frac{\langle [0.3,0.5].e^{j2\pi[0.2,0.4]}, [0.2,0.2].e^{j2\pi[0.3,0.5]}, [0.6,0.7].e^{j2\pi[0.3,0.6]} \rangle}{v_4} \right) \right\} \end{aligned}$$

In I-CNSS model $\xi = (K, A)$ the interval complex neutrosophic soft values clarify the expert opinion about the attributes of all alternatives. For instance, in the interval-complex neutrosophic soft value of the alternative v_1 under α_1 attribute $[0.8, 0.9] \cdot e^{2\pi[0.7, 0.8]}, [0.2, 0.3] \cdot e^{2\pi[0.1, 0.2]}, [0.2, 0.4] \cdot e^{2\pi[0.2, 0.3]}$ the truth interval membership function $[0.8, 0.9] \cdot e^{2\pi[0.7, 0.8]}$ show that the expert here means that the performance of the computer v_1 is marked by an amplitude interval value of 0.8 to 0.9, and this percentage confirms that this computer is characterized by high performance, while the phase interval value of 0.7 to 0.8 indicate that the customer satisfied degree between 70% to 80% . While the indeterminacy interval membership function $[0.2, 0.3] \cdot e^{2\pi[0.1, 0.2]}$ reveal that the expert cannot determine if this computer has high performance or not by degree between 0.2 to 0.3, and the phase interval value indicates the degree of confusion of the customer for this device between 10% to 20%. Also, for the falsity interval membership function $[0.2, 0.4] \cdot e^{2\pi[0.2, 0.3]}$ show that the expert is unsatisfied with this device by a degree ranging from 0.2 to 0.4, and the customer is unsatisfied by a degree of 20% to 30%.

Step 2: Compute the distance measured between the optimality choice $([1, 1] \cdot e^{2\pi[1, 1]})$ and α_i (for $i=1,2,3$) with the weighting vector $w = \{0.3, 0.3, 0.4\}$ and obtain the results as shown in Table 1.

TABLE 1. Distance Measures results

V_i	d^H	d^E	d^{nH}	d^{nE}	d^{nwH}	d^{nwE}
v_1	1.439	0.863	0.119	0.253	0.4848	0.253
v_2	1.268	0.760	0.105	0.223	0.418	0.228
v_3	1.166	0.699	0.097	0.205	0.388	0.203
v_4	1.357	0.814	0.113	0.239	0.453	0.236

Step 3: Compute similarity measure for all the distances we got in Table 1.

TABLE 2. Similarity Measures results

V_i	S^H	S^E	S^{nH}	S^{nE}	S^{nwH}	S^{nwE}
v_1	0.410	0.536	0.893	0.798	0.673	0.798
v_2	0.440	0.568	0.904	0.817	0.705	0.814
v_3	0.461	0.588	0.911	0.829	0.720	0.831
v_4	0.424	0.551	0.898	0.807	0.688	0.809

TABLE 3. Ordering of the given alternatives.

	Ordering.
S^H	$v_3 \succ v_2 \succ v_4 \succ v_1$
S^E	$v_3 \succ v_2 \succ v_4 \succ v_1$
S^{nH}	$v_3 \succ v_2 \succ v_4 \succ v_1$
S^{nE}	$v_3 \succ v_2 \succ v_4 \succ v_1$
S^{nwH}	$v_3 \succ v_2 \succ v_4 \succ v_1$
S^{nwE}	$v_3 \succ v_2 \succ v_4 \succ v_1$

Step 4: From Table 3, we analyse the similarity values in Table 2, which we got based on the distance measures given in Table 1. Clearly, the most useful alternative is v_3 , which is the one with the highest similarity to the ideal choice.

5. Comparison with Existing Models/Methods in Literature

This work is based on the similarity measures of interval-complex neutrosophic soft sets (I-CNSSs) and employs these measures to solve some real-life applications such as decision making and medical diagnosis under uncertainty. In this section, we compare the I-CNSS model to other similar models in the literature based on the characteristics in Table 4.

TABLE 4. Characteristic comparison of the I-CNSS with other variants.

Methods	Uncertainty	Three membership function	Parameterization	Interval form	Periodicity
[52]	T	T	F	F	F
[55]	T	F	T	T	F
[20]	T	T	T	T	F
[50]	T	F	T	F	T
Our model:I-CNSS	T	T	T	T	T

On the other hand, compared with INSSM [20], which used the INSS setting to describe the decision-making information, our suggested I-CNSS is a new approach created to conceptualize uncertainty issues that have two dimensions. From Example 4.1, it can be noted that the concept of INSS cannot cover the factors affecting the problem (symptoms severity and period of symptoms) in two stages simultaneously. Because it is not adapted to deal with two-dimensional issues, i.e., it doesn't have enough tools to do that. But the I-CNSS model can put the phase and amplitude terms together and can be used to represent these two variables together. So, we can say that the INSSM can't directly handle the problem given above with two-dimensional information in this way. Otherwise, we can say that the INSS model is a particular case of our model I-CNSS and can be conceptualized in the form of I-CNSS. In other words, the INSS is an I-CNSS with phase terms equal to zero. For example, the INSS.

other hand, we can say that the INSS model is a particular case of our model I-CNSS and can be conceptualized in the form of I-CNSS. In other words, the INSS is an I-CNSS with phase terms equal to zero. For example, the INSS $([0.3, 0.7], [0.1, 0.4], [0.5, 0.6])$ can be represented as $([0.3, 0.7] \cdot e^{j2\pi[0,0]}, [0.1, 0.4] \cdot e^{j2\pi[0,0]}, [0.5, 0.6] \cdot e^{j2\pi[0,0]})$ employing I-CNSS. Furthermore, our approach I-CNSS, is appropriate for other methods such as interval intuitionistic fuzzy soft problems, since interval intuitionistic fuzzy soft sets are a particular case of INSS and I-CNSS. For example, the interval intuitionistic fuzzy soft value $([0.3, 0.3], [0.5, 0.5])$ can be written as $([0.3, 0.3], [0.5, 0.5], [0.2, 0.2])$ employing INSS and hence can be written as $([0.3, 0.3] \cdot e^{j2\pi[0,0]}, [0.5, 0.5] \cdot e^{j2\pi[0,0]}, [0.2, 0.2] \cdot e^{j2\pi[0,0]})$ using I-CNSS, since the sum of the degrees of lower and upper interval true membership, lower and upper interval nonmembership, and lower and upper interval indeterminacy membership of an interval intuitionistic fuzzy value is equal to $([1, 1])$. Note that the lower and upper interval indeterminacy degree in an interval intuitionistic fuzzy set is provided by default. It cannot be defined alone, unlike the interval neutrosophic set, where the lower and upper interval indeterminacy membership are determined independently and quantified explicitly. As for the CVSS model [50], this model is based on a vague set model and also misses indeterminacy membership. Therefore, it is difficult for this model to deal with the data of the problem entirely like our proposed model.

5.1. *Pros of I-CNSS model*

Based on all of the above, our proposed method has particular advantages. Firstly, the main feature of I-CNSS is the presence of a phase and its membership in the form of an interval. Researchers realized that the time period is an important factor along with the membership value so that decision-makers can make the real decision, and it is more reliable and more acceptable than the other existing theories in which there is no scope for considering time-period. So, this new concept provides more scope for the decision-makers to make real decisions with more feasibility. Secondly, a practical formula is utilised to convert the I-CNSVs (complex stats) to the INSVs (real stats), which sustains the entirety of the original data without diminishing or distorting them. Thirdly, our technique allows for decision-making using a simple computational procedure that does not require the use of directed operations on complex numbers. Finally, the I-CNSS that is used in our approach has the capability to handle the imprecise, indeterminate, inconsistent, and incomplete information that is captured by the amplitude terms and phase terms simultaneously. As a result, the proposed method is capable of dealing with more uncertain data.

- Established on all that is mentioned in this comparison, we see that the similarity measures based on I-CNSS that are given in this work are more effective in dealing with uncertainty issues than other concepts mentioned in the literature.

6. Conclusion

Al-Sharqi et al. [36] established the idea of ICNSS as a substantial and essential generalization of the soft set to deal with the uncertain, inconsistent, and incomplete information in periodic data. In this paper, we have proposed several distance measures in the case of the interval complex neutrosophic soft sets, which are very helpful in dealing with the two-dimensional data in some real-world applications. Based on these distance measures, we introduced an axiomatic definition of similarity measure to measure the degree of fuzzy information in interval complex neutrosophic soft sets. Moreover, a numerical example is given, and relations between this similarity measure and these distances are introduced and verified. In addition, a proposed algorithm based on these measures has been built and applied in some daily life applications like decision-making problems and medical diagnoses. Finally, a comparison between the existing methods and I-CNSS was given, and some features of I-CNSS were revealed. In future possible research, we can extend from soft to hypersoft set [56] (by transforming the function F into a multi-attribute function). We also want to combine these measures with other kinds of algebraic structures, such as group structures [57]- [59], ring structures [60]- [62], and topological structures [63]- [65].

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