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PCTLHS-Matrix, Time-based Level Cuts, Operators, and unified time-layer health state Model.

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Abstract: This article aims to introduce a unique hypersoft time-based matrix model that organizes and classifies higher-dimensional information scattered in numerous forms and vague appearances varying on specific time levels. Classical matrices as rank-2 tensors single-handedly relate equations and variables across rows and columns are a limited approach to organizing higher-dimensional information. This Plithogenic Crisp Time Leveled Hypersoft Matrix (PCTLHS-Matrix) model is designed to sort the higher dimensional information flowing in parallel time layers as a combined view of events. This matrix has several parallel layers of time. The time-based level cuts as time layers are introduced to present an explicit view of information on certain required time levels as a separate reality. The sub-layers are formulated as sub-level cuts that represent a partial view of the event or reality. Further subdividing these sub-levels creates sub-sub-level cuts, which are the smallest focused partial view of the event, serving the purpose of zooming. These Level cuts are utilized to construct local aggregation operators for PCTLHS-Matrix. And the concept of timelessness is introduced by unifying the time levels of the universe. This means all attributes that exist in various time levels are merged to exist in a unified time called the unified time layer. In this way, the attributes are focused and the layers of time are merged as if there is no time. The particular types of time layers are unified by local operators to introduce the concept of timelessness that is obtained by unifying time levels. Finally, for a precise description of the model, a numerical example is constructed by assuming a classification of various health states with COVID-19 patients in a hospital.
Intuitionistic Fuzzy / Neutrosophic / and other fuzzy-extension IndetermSoft Set & IndetermHyperSoft Set are presented together with their applications.

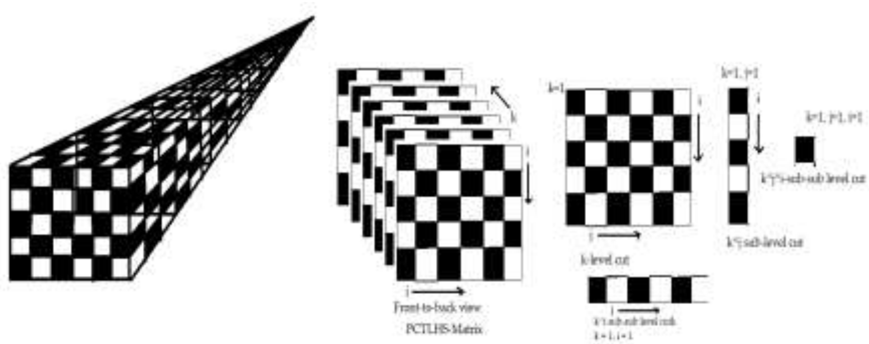
Keywords: : PCTLHS-Matrix; Time-Layers, Level-Cuts; Sub-Level Cuts; Sub-Sub- Level Cuts; Combined Event-View, Separate event-view; Partial event view; Aggregation-Operators.

1. Introduction

The discipline of modeling and decision-making in an uncertain and vague environment consisting of higher-dimensional information is an incredible task. To enhance the field modeling and decision-making in an uncertain and vague universe, the field of fuzzy theory was developed by Zadeh [1] in 1965. Later, in 1986, K. Atanassov [2] further expanded this state of vagueness by introducing intuition or hesitation into decision-making structures called intuitionistic fuzzy set theory (IFS). The three states of the human mind were represented by the level of membership, the level of non-membership, and the level of hesitation in IFS theory. In addition, K. Atanassov [3] introduced an interval-valued fuzzy set (IVFS) in 1999, which is another form of IFS (memberships and nonmemberships packed in unit intervals). Later, F. Smarandache [4-6] introduced neutrosophy by extending hesitation as an independent indeterminate neutral factor. Molodtsove formulated a soft set in 1999 [7-10], he expanded the theory of wage by considering multiple attributes parameterized by subjects. In 2018 [11] Smarandache introduced the Hypersoft set and the Plithogenic Hypersoft set earlier, giving the plithogenicy theory [12]. In these sets, he extended the attributes to the values of the attributes called sub-attributes and parametrized the subjects by several attributes and sub-attributes. By introducing the Hypersoft set and the Plithogenic Hypersoft set, he opened some problems in exploring these sets such as constructing aggregation operators and decision-making models.

Shazia et al [13], explored and extended these sets and addressed the problems opened by Smarandache. In addition, they introduced the plithogenic Fuzzy Whole Hypersoft Set (PFWHSS) and formulated a matrix representation form named Plithogenic Fuzzy Whole Hypersoft-Matrix. They developed some local aggregation operators for the plithogenic Fuzzy Hypersoft set (PFHSS). This matrix was developed for a specific combination of attributes and sub-attributes. The application of this matrix has been provided in the form of a decision-making model referred to as the Plithogenic Frequency Matrix Multi-Attribute Decision-Making technique. Later, Shazia et al [14] extended the Plithogenic Whole Hypersoft Matrix to a generalized form of the Matrix called the Plithogenic Subjective Hyper-Super-Soft Matrix.

It was a superior matrix to its previously developed matrix which has a greater capacity for expressing the variations of certain connected attributive levels. These attribute levels are presented as matrix layers. The application of this matrix is provided in the form of a new ranking model called the Plithogenic Subjective Local-Global Universal Ranking Model.



This model has provided a physical classification of the Universe (a combination of subjects with attributes). Later, in this research field, the hypersoft set expanded, and some MADM techniques and operators are developed. Saqlain, Saeed, et al [15] discussed the Generalization of TOPSIS for Neutrosophic Hypersoft set using Accuracy Function and its Application. Saqlain, et al [16] constructed some Aggregate Operators of Neutrosophic Hypersoft Set. Quek, et al [17] introduced Entropy Measures for Plithogenic Sets and Applications in Multi-Attribute Decision Making. Saqlain Smarandache [18] formulated octagonal neutrosophic numbers and discussed their different representations, properties, graphs, and de-eutrophication with the application of personnel selection. International Journal of Neutrosophic Science. Rahman et al, [19] developed a multi-attribute decision-support system that is based on aggregations of Interval-Valued Complex Neutrosophic Hypersoft Set. Saeed, Muhammad, and Atiqe Ur Rahman [20] constructed an optimal supplier selection model via a decision-making algorithmic technique that is based on a single-valued neutrosophic fuzzy hypersoft Set. Ihsan, Muhammad, Atiqe Ur Rahman, and Muhammad Saeed [21] discussed a single-valued neutrosophic hypersoft expert set with application in decision making. Saeed, Muhammad, et al. [22] formulated the model of the prognosis of allergy-based diseases using Pythagorean fuzzy hypersoft mapping structures and recommended medication. Rahman, Atiqe Ur, et al. [23] developed decision-making algorithmic techniques based on aggregation operations and similarity measures of possibility intuitionistic fuzzy hypersoft sets.

This current article provides a further upgraded plithogenic Model. In this model, a new time-level variation has been introduced. This model is programmed with a magnified angle of vision to cope with scattered time-dependent information of the plithogenic Universe in a crisp environment. First, a three-dimensional expanded view of the PCTLHS matrix is presented to show the Plithogenic Crisp Time Lined Hypersoft Set. This PCTLHS The matrix is a third-rank tensor representing three types of variation; it consists of several matrix layers, each layer being a second-rank tensor expressed in the Crisp environment. Furthermore, this PCTLHS matrix represents multiple parallel universes or parallel realities. By using this connected-matrix expression, one can grasp and categorize all the information at a glimpse, i.e., the information from a crowd of people assigned to a combination of attributes and observed at different time levels. Therefore, it is obvious that the matrix expression is the most appropriate expression to represent the multidimensional data compared to the classical set expression. This new model would help to enhance and broaden the field of decision-making and artificial intelligence. After a detailed description, specific types of level cuts are constructed on the variation indices. These level cuts are named K-level cuts obtained by splitting the PCTLHS-Matrix at one of the three given variation indices. Further, these K-level cuts would provide a structure for viewing each event or reality separately and serve as the projection of higher-dimensional events in the lower-dimensional universe. Additionally, these K-level cuts are further broken down into sub-level cuts by splitting the matrix layer at either of the two remaining variation indices, i.e: J, I. While these sub-level cuts offer the projection of the previous lower dimension into a further lower dimension and provide an interior view of the expanded universe, this view may be called an implicit expanded universal view.

At a later stage from these sub-level cuts, sub-sub-level cuts are constructed by dividing the sub-level cuts (row or column of a given layer of the PCTLHS matrix) at the second variation index of the matrix and then further at the third variation index of the matrix. After applying all splits at indices the outcomes would be reflected as points. It is obvious that these level cuts, sublevel cuts, and sub-sub-level cuts serve to program zoom-in functions to look at an inside view of the event. This can be considered as a contraction of the expanded higher-dimensional universe. However, the sub-sub level cuts provide a contracted picture of the smallest part of single or multiple universes. In this way, the expanded universe of matrix layers could possibly be contracted at a single point. Similarly, by reversing the process, one can extend the same point to other higher dimensions of rows, columns, matrix, matrix layers, and clusters of matrix layers. In the final phase, plithogenic aggregation operators are developed and used to elaborate the activity of these different types of level cuts based on variation indices. These operators are plithogenic disjunction operators, plithogenic conjunction operators, plithogenic average operators, and vice versa. For a more precise and lucid explanation of the model, a numerical example related to the classification of COVID-19 patients and their health states in a hospital at two different time levels.

2. Prelimneries

This section summarizes some basic definitions of Soft Sets, Hypersoft Sets, Crisp Hypersoft Sets, Plithogenic Hypersoft Sets, Plithogenic Crisp Hypersoft Sets. These definitions would help expand the theory of plithogecy.

Definition 2.1 [7] (Soft Set)

Let $\boldsymbol{U}$ be the initial Universe of discourse, and $\boldsymbol{E}$ be a set of parameters or attributes with respect to $\boldsymbol{U}$ let $\boldsymbol{P}(\boldsymbol{U})$ denote the power set of $\boldsymbol{U}$, and $\boldsymbol{A} \subseteq \boldsymbol{E}$ is a set of attributes. Then pair $(\boldsymbol{F}, \boldsymbol{A})$, where $\boldsymbol{F}: \boldsymbol{A} \longrightarrow \boldsymbol{P}(\boldsymbol{U})$ is called Soft Set over $\boldsymbol{U}$. , In other words, a soft set $(\boldsymbol{F}, \boldsymbol{A})$ over $\boldsymbol{U}$ is a parameterized family of subsets of $\boldsymbol{U}$. Fore $\boldsymbol{e} \in \boldsymbol{A}$, $\boldsymbol{F}(\boldsymbol{e})$ may be considered as a set of $\boldsymbol{e}$ elements or $\boldsymbol{e}$ approximate elements

$$(\boldsymbol{F}, \boldsymbol{A}) = \{(\boldsymbol{F}(\boldsymbol{e}) \in \boldsymbol{P}(\boldsymbol{U}): \boldsymbol{e} \in \boldsymbol{E}, \boldsymbol{F}(\boldsymbol{e}) = \boldsymbol{\varphi} \,\mathbf{if}\, \boldsymbol{e} \notin \boldsymbol{A}\} \tag{2.1}$$

Definition 2.2 [11] (Hypersoft set)

Let $\boldsymbol{U}$ be the initial Universe of discourse $\boldsymbol{P}(\boldsymbol{U})$ the power set of $\boldsymbol{U}$.

let $\boldsymbol{a_1}, \boldsymbol{a_2}, \ldots, \boldsymbol{a_n}$ for $\boldsymbol{n} \geq \mathbf{1}$ be $\boldsymbol{n}$ distinct attributes, whose corresponding attributes values are respectively the sets $\boldsymbol{A_1}, \boldsymbol{A_2}, \ldots, \boldsymbol{A_n}$ with $\boldsymbol{A_i} \cap \boldsymbol{A_j} = \boldsymbol{\varphi}$ for $\boldsymbol{i} \neq \boldsymbol{j}$ and $\boldsymbol{i}, \boldsymbol{j} \in \{\mathbf{1}, \mathbf{2}, \ldots, \boldsymbol{n}\}$.

Then the pair $(\boldsymbol{F}, \boldsymbol{A_1} \times \boldsymbol{A} \times \ldots \times \boldsymbol{A_n})$ where,

$$\boldsymbol{F}: \boldsymbol{A_1} \times \boldsymbol{A} \times \ldots \times \boldsymbol{A_n} \rightarrow \boldsymbol{P}(\boldsymbol{U}), \tag{2.2}$$

is called a Hypersoft set over $\boldsymbol{U}$

Definition 2.3 [11] (plithogenic Crisp Hypersoft set)

Let $\boldsymbol{U_c}$ be the initial Crisp Universe of discourse $\boldsymbol{P}(\boldsymbol{U_c})$ the power set of $\boldsymbol{U}$. Let $\boldsymbol{a_1}, \boldsymbol{a_2}, \ldots, \boldsymbol{a_n}$ for $\boldsymbol{n} \geq \mathbf{1}$ be $\boldsymbol{n}$ distinct attributes, whose corresponding attributes values are respectively the sets $\boldsymbol{A_1}, \boldsymbol{A_2}, \ldots, \boldsymbol{A_n}$ with $\boldsymbol{A_i} \cap \boldsymbol{A_j} = \boldsymbol{\varphi}$ for $\boldsymbol{i} \neq \boldsymbol{j}$ and $\boldsymbol{i}, \boldsymbol{j} \in \{\mathbf{1}, \mathbf{2}, \ldots, \boldsymbol{n}\}$. Then

$\{\boldsymbol{F_c}, \boldsymbol{A_1} \times \boldsymbol{A_2} \times \ldots \times \boldsymbol{A_n}\}$ is called plithogenic Crisp Hypersoft set over $\boldsymbol{U_c}$ where, $\boldsymbol{F_c}: \boldsymbol{A_1} \times \boldsymbol{A} \times \ldots \times \boldsymbol{A_n} \rightarrow \boldsymbol{P}(\boldsymbol{U_c})$, *Definition 2.4 [24][25] [26] (super-matrices)*

A square or rectangular arrangements of numbers in rows and columns are matrices we shall call them as simple matrices while a super-matrix is one whose elements are themselves matrices with elements that can be either scalars or other matrices.

$\boldsymbol{a} = \begin{bmatrix} \boldsymbol{a_{11}} & \boldsymbol{a_{12}} \\ \boldsymbol{a_{21}} & \boldsymbol{a_{22}} \end{bmatrix}$, where

$\boldsymbol{a_{11}} = \begin{bmatrix} \mathbf{2} & \mathbf{-4} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$, $\boldsymbol{a_{12}} = \begin{bmatrix} \mathbf{0} & \mathbf{40} \\ \mathbf{21} & \mathbf{-12} \end{bmatrix}$,

$\boldsymbol{a_{21}} = \begin{bmatrix} \mathbf{3} & \mathbf{-1} \\ \mathbf{5} & \mathbf{7} \\ \mathbf{-2} & \mathbf{9} \end{bmatrix}$, $\boldsymbol{a_{22}} = \begin{bmatrix} \mathbf{4} & \mathbf{12} \\ \mathbf{-17} & \mathbf{6} \\ \mathbf{3} & \mathbf{7} \end{bmatrix}$ $\boldsymbol{a}$ is a super-matrix.

Note: The elements of super-matrices are called sub-matrices i.e. $\boldsymbol{a_{11}}, \boldsymbol{a_{12}}, \boldsymbol{a_{21}}, \boldsymbol{a_{22}}$ are submatrices of the super-matrix $\boldsymbol{a}$.

in this example, the order of super-matrix $\boldsymbol{a}$ is $\mathbf{2} \times \mathbf{2}$ and order of sub-matrices $\boldsymbol{a_{11}}$ is $\mathbf{2} \times \mathbf{2}$, $\boldsymbol{a_{12}}$ is $\mathbf{2} \times \mathbf{2}$ $\boldsymbol{a_{21}}$ is $\mathbf{3} \times \mathbf{2}$ and order of sub-matrix $\boldsymbol{a_{22}}$ is $\mathbf{3} \times \mathbf{2}$, we can see that the order of super-matrix doesn't tell us about the order of its sub-matrices.

Definition 2.5 [27] (Hypermatrices)

For $\boldsymbol{n_1}, \ldots, \boldsymbol{n_d} \in \boldsymbol{N}$, a function $\boldsymbol{f}: (\boldsymbol{n_1}) \times \ldots \times (\boldsymbol{n_d}) \rightarrow \boldsymbol{F}$ is a hypermatrix, also called an order-d hypermatrix or d-hypermatrix. We often just write $\boldsymbol{a_{k_1 \ldots k_d}}$ to denote the value $\boldsymbol{f}(\boldsymbol{k_1} \ldots \boldsymbol{k_d})$ of $\boldsymbol{f}$ at $(\boldsymbol{k_1} \ldots \boldsymbol{k_d})$ and think of $\boldsymbol{f}$ (renamed as $\boldsymbol{A}$) as specified by a d-dimensional table of values, writing $\boldsymbol{A} = [\boldsymbol{a_{k_1 \ldots k_d}}]_{\boldsymbol{k_1 \ldots k_d}}^{\boldsymbol{n_1, \ldots, n_d}}$

A 3-hypermatrix may be conveniently written down on a (2-dimensional) piece of paper as a list of usual matrices, called slices. For example

$$\boldsymbol{A} = [\boldsymbol{a_{ijk}}] = \begin{bmatrix} \boldsymbol{a_{111}} & \boldsymbol{a_{121}} & \boldsymbol{a_{131}} & \cdot & \boldsymbol{a_{112}} & \boldsymbol{a_{122}} & \boldsymbol{a_{132}} \\ \boldsymbol{a_{211}} & \boldsymbol{a_{221}} & \boldsymbol{a_{231}} & \cdot & \boldsymbol{a_{212}} & \boldsymbol{a_{222}} & \boldsymbol{a_{232}} \\ \boldsymbol{a_{311}} & \boldsymbol{a_{321}} & \boldsymbol{a_{331}} & \cdot & \boldsymbol{a_{312}} & \boldsymbol{a_{322}} & \boldsymbol{a_{332}} \end{bmatrix}$$

3. Plithogenic Crisp Time-Leveled Hypersoft Matrix

Definition 3.1 (Plithogenic Crisp Time leveled Hypersoft Matrix):

Let $\boldsymbol{U_C}(\boldsymbol{X})$ be the Crisp universe of discourse, $\boldsymbol{P}(\boldsymbol{U_C})$ be the power set of $\boldsymbol{U_C}$, $\boldsymbol{A_j^k}$ is a combination of attributes sub-attributes for some $j = 1,2,3, \ldots, N$ attributes, $k = 1,2,3, \ldots, L$ time-leveled-attributes and x_i $i = 1,2,3, \ldots, M$ subjects under consideration then Plithogenic Crisp Time-Leveled Hypersoft-Matrix (PCTLHS-Matrix), is a mapping C from the cross product of attributes / time-leveled-attributes on the power set of universe $P(U_C)$ represented in matrix form. This mapping C and its matrix form in the plithogenic crisp environment is described below in Eq.3.1 and Eq.3.2 respectively,

$$F: A_1^k \times A_2^k \times A_3^k \times \ldots \times A_N^k \rightarrow P(\boldsymbol{U_C}) \tag{3.1}$$

$$F = \left[\mu_{A_j^k}(x_i)\right] \tag{3.2}$$

s.t $\mu_{A_j^k}(x_i) \in \{0.1\}$, are crisp states as memberships either "0" or "1",

$\mu_{A_j^k}(x_i)$ are crisp memberships for given x_i subjects regarding each given A_j^k attributes / time-leveled-attributes where, A_j^k is a combination of attributes / time leveled-attributes for some $j = 1,2,3,\ldots,N$ attributes, $k = 1,2,3,\ldots,L$ time levels associated to x_i $i = 1,2,3,\ldots, M$ subjects under consideration.

Or in simple words a Plithogenic Crisp Hypersoft Set, represented in the matrix form is called Plithogenic Crisp Time Lined Hypersoft Matrix, (PCTLHS-Matrix).

This matrix has three possible expanded forms or views described in Crisp environments.

3.2 PCTLHS-Matrix as a Tensor:

As we know, all matrices in the real vector space are rank-2 tensors, similar to how the PCTLHS matrix with its three variation indices is a rank 3 tensor. The PCTLHS matrix contains layers of ordinary matrices called matrix layers or Level cuts (plane slices).

$\boldsymbol{A} = [\boldsymbol{A_{ijk}}]$ is an example of PCTLHS-Matrix. Index i refers to variations in rows used to represent subjects under consideration j specifies a variation in columns used to represent attributes of subjects, and k provides variations of layers of rows and columns used to represent the attributes on specific time levels (varying matrix layers as clusters of rows and columns). Similarly $[\boldsymbol{A_{jki}}]$ is interpreted as the index j referred to variation of rows k gives a variation of columns, and i offers a variation of clusters of rows and columns.

3.3 Level Cuts, Sub-Level Cuts, and Sub-Sub-Level Cuts of PCTLHS-Matrix

We may define Level Cuts, Sub-Level Cuts, and Sub-Sub-Level Cuts by specifying the variation indices i, j, k for their positive integer values.

3.3.1 Level Cuts**:** Level Cuts are sub-matrices (first level splits) of PCTLHS-Matrix that can be further described as parallel matrix layers. The PCTLHS-Matrix is generated by uniting these matrix layers. These level cuts of PCTLHS-Matrix are obtained by assigning a specific integer value to one of the three variation indexes at a time.

The level cuts are of three types according to three types of views of the PCTLHS-Matrix i.e i-Level Cuts, j-Level Cuts, k-Level Cuts the detailed mathematical description of k-Level Cuts is described in section 4.

3.3.2 Sub-Level Cuts: Sub-Level Cuts are Level Cuts of Level Cuts (second splits applied over first splits) of PCTLHS-Matrix that are columns or rows of the Sub-Matrix or a Layer-Matrix. The Sub-Level Cuts are obtained by assigning a specific integer value to one of the two variation indices of a parallel layer (Sub-Matrix) of PCHS-Matrix. The detailed description and construction of sub-level cuts are presented in sec 4.

3.3.3 Sub-Sub-Level Cuts: The Sub-Sub-Level Cuts are obtained by assigning a specific Integer value to a variation index of sub-Level Cut (the third level splits over second splits). The Sub-Sub-Level Cut is one Specific element (point) of the Sub-Level Cut (Column or Row). These Level Cuts, Sub-Level Cuts, and Sub-Sub-Level Cuts are images of the higher dimensional Universe in the lower dimensional Universe and can be used as tools for getting images and transformations.

The detailed organization of these level cuts, sub-level cuts, and sub-sub-level cuts are described in the next section with the specific time based view of the PCTLHS-Matrix.

The utilization of these Cuts is that one can contract the expanded dimension of PCTLHS-Matrix to a Matrix, then to a row or column matrix, and then further to a single point. However, the reverse procedure would provide an expansion of the Universe in a similar manner. In PCHs-Matrix three types of variation indices are introduced on Crisp memberships $\boldsymbol{\mu_{A_j^k}(x_i)}$. For example, in the Universe of subjects attributes and sub-attributes, one may consider the first variation on the index $\boldsymbol{i}$ that is referred to as subjects representing $\boldsymbol{M}$ rows of an $\boldsymbol{M \times N}$ Sub-Matrix of an $\boldsymbol{M \times N \times L}$ PCHS-Matrix. The second variation on $\boldsymbol{j}$ is used to specify attributes representing $\boldsymbol{N}$ columns of a Sub-Matrix of an $\boldsymbol{M \times N \times L}$ Hs-Matrix. A third variation on $\boldsymbol{k}$ is introduced to express attributive levels and is represented in the form of $\boldsymbol{L}$ layers or $\boldsymbol{L}$ level-Cuts of $\boldsymbol{M} \times \boldsymbol{N} \times \boldsymbol{L}$ PCHS-Matrix.

4. Three-dimensional view and a front-to-back view of PCTLHS-Matrix

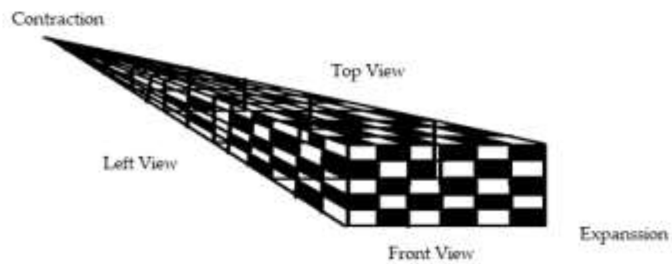


Figure 1 (Three-dimensional view of PCTLHS-Matrix)

The three-dimensional indexed-based PCTLHS-Matrix and level cuts, sub-level cuts, and Sub-Sub-Level Cuts are described below:

$$A = \begin{bmatrix} \mu_{A_1^K}(x_1) & \mu_{A_2^K}(x_1) & . & . & . & \mu_{A_N^K}(x_1) \\ \mu_{A_1^K}(x_2) & \mu_{A_2^K}(x_2) & . & . & . & \mu_{A_N^k}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_N^k}(x_M) & \mu_{A_N^k}(x_M) & . & . & . & \mu_{A_N^K}(x_M) \end{bmatrix} \quad (4.1)a$$

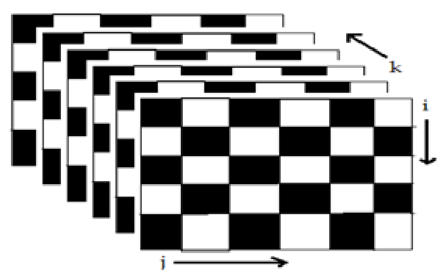


Figure 2 (front-to-back view of the matrix)

front-to-back view of PCTLHS-Matrix is described in Eq. (4.1)a figure 2

Front to back view of PCTLHS-Matrix in a more expanded form is described in Eq. (4.1)b as,

$$A=\begin{bmatrix}\begin{bmatrix}\mu_{A_1^1}(x_1) & \mu_{A_2^1}(x_1) & . & . & . & \mu_{A_N^1}(x_1)\\ \mu_{A_1^1}(x_2) & \mu_{A_2^1}(x_2) & . & . & . & \mu_{A_N^1}(x_2)\\ . & . & . & . & . & .\\ . & . & . & . & . & .\\ . & . & . & . & . & .\\ \mu_{A_1^1}(x_M) & \mu_{A_2^1}(x_M) & . & . & . & \mu_{A_N^1}(x_M)\end{bmatrix}\\ \begin{bmatrix}\mu_{A_1^2}(x_1) & \mu_{A_2^2}(x_1) & . & . & . & \mu_{A_N^2}(x_1)\\ \mu_{A_1^2}(x_2) & \mu_{A_2^2}(x_2) & . & . & . & \mu_{A_N^2}(x_2)\\ . & . & . & . & . & .\\ . & . & . & . & . & .\\ . & . & . & . & . & .\\ \mu_{A_1^2}(x_M) & \mu_{A_2^2}(x_M) & . & . & . & \mu_{A_N^2}(x_M)\end{bmatrix}\\ .\\ .\\ .\\ \begin{bmatrix}\mu_{A_1^L}(x_1) & \mu_{A_2^L}(x_1) & . & . & . & \mu_{A_N^k}(x_1)\\ \mu_{A_1^L}(x_2) & \mu_{A_2^L}(x_2) & . & . & . & \mu_{A_N^L}(x_2)\\ . & . & . & . & . & .\\ . & . & . & . & . & .\\ . & . & . & . & . & .\\ \mu_{A_1^L}(x_M) & \mu_{A^L}(x_M) & . & . & . & \mu_{A_N^L}(x_M)\end{bmatrix}\end{bmatrix} \quad (4.1)b$$

4.1 k-Level Cuts $A^{[k]}$ of PCTLHS-Matrix:

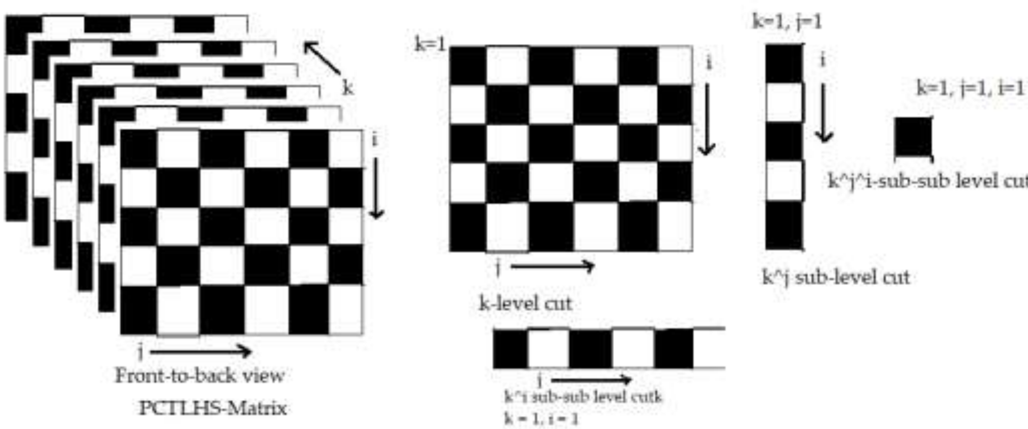


Figure 3 (level cuts sub-level cuts and sub-sub-level cuts)

k-Level Cuts of PCTLHS-Matrix are front-to-back k-splits of the matrix obtained by varying the index stepwise. These are the $\boldsymbol{L}$ number of front-to-back Matrix layers of $M \times N \times L$ PCTLHS-Matrix. Each layer is an $M \times N$ Matrix as described in Figure 3. L number of $k - level$ cuts ($A^{[k]}$) of PCTLHS-Matrix in the general contracted form are described in the matrix form by specifying $k = l$,

$$A^{[1]} = \left[\mu_{A_j^1}(x_i)\right],\ A^{[2]} = \left[\mu_{A_j^2}(x_i)\right], \dots, A^{[M]} = \left[\mu_{A_j^L}(x_i)\right]$$

$i = 1,2,\dots,M,\ j = 1,2,\dots,N$ and $k = 1,2,3,\dots,L$

k-Level Cuts of PHCTLHS-Matrix in the expanded form are described underneath in Eq. (4.2)

$$A^{[K]} = \begin{bmatrix} \mu_{A_1^k}(x_1) & \mu_{A_2^k}(x_1) & . & . & . & \mu_{A_N^k}(x_1) \\ \mu_{A_1^k}(x_2) & \mu_{A_2^k}(x_2) & . & . & . & \mu_{A_N^k}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_1^k}(x_M) & \mu_{A_2^k}(x_M) & . & . & . & \mu_{A_N^k}(x_M) \end{bmatrix} \tag{4.2}$$

k-Level Cuts $A^{[k]}$ of PCTLHS-Matrix in the expanded form are given below in EQs. (4.3), (4.4), and (4.5).

$$A^{[1]} = \begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_2^1}(x_1) & . & . & . & \mu_{A_N^1}(x_1) \\ \mu_{A_1^1}(x_2) & \mu_{A_2^1}(x_2) & . & . & . & \mu_{A_N^1}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_1^1}(x_M) & \mu_{A_2^1}(x_M) & . & . & . & \mu_{A_N^1}(x_M) \end{bmatrix} \tag{4.3}$$

$$A^{[2]} =, \begin{bmatrix} \mu_{A_1^2}(x_1) & \mu_{A_2^2}(x_1) & . & . & . & \mu_{A_N^2}(x_1) \\ \mu_{A_1^2}(x_2) & \mu_{A_2^2}(x_2) & . & . & . & \mu_{A_N^2}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_1^2}(x_M) & \mu_{A_2^2}(x_M) & . & . & . & \mu_{A_N^2}(x_M) \end{bmatrix} \tag{4.4}$$

$$A^{[L]} = \begin{bmatrix} \mu_{A_1^L}(x_1) & \mu_{A_2^L}(x_1) & . & . & . & \mu_{A_N^k}(x_1) \\ \mu_{A_1^L}(x_2) & \mu_{A_2^L}(x_2) & . & . & . & \mu_{A_N^L}(x_2) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \mu_{A_1^L}(x_M) & \mu_{A^L}(x_M) & . & . & . & \mu_{A_N^L}(x_M) \end{bmatrix} \tag{4.5}$$

4.2 k_i-Sub-Level Cuts: For k-Level Cuts further offer i-splits (row-wise splits) by specifying i and varying j k_i-Sub-Level Cuts that are obtained. These are rows of $M \times N$ Matrix Layer that are obtained by specifying $k = l, i = m,$ .and varying $j = 1,2 \dots N$

$$A^{[l_m]} = \begin{bmatrix} \mu_{A_1^l}(x_\mathrm{m}) & \mu_{A_2^l}(x_\mathrm{m}) & . & . & . & \mu_{A_N^l}(x_\mathrm{m}) \end{bmatrix} \tag{4.6}$$

4.3 k_j-Sub-Level Cuts: For k-Level Cuts further provide j-splits (column-wise splits) by specifying j and varying i k_j-Sub-Level Cuts are obtained. These are columns of $M \times N$ Matrix Layer that are obtained by specifying $k = l, j = n,$ and varying $i = 1,2 \dots M$

$$A^{[l_n]} = \begin{bmatrix} \mu_{A_n^l}(x_1) \\ \mu_{A_n^l}(x_2) \\ . \\ . \\ . \\ \mu_{A_n^l}(x_M) \end{bmatrix} \tag{4.7}$$

4.4 k_{i_j}-Sub-Sub-Level Cuts $A^{[k_{i_j}]}$: For Specific k_i-Sub-Level Cuts further specifying j $A^{[k_{i_j}]}$ Sub-Sub-Level Cuts are obtained. These are elements of rows of $M \times N$ Matrix Layer that are obtained by specifying $k = l, i = m,$ .and varying $\boldsymbol{j} = 1,2 \dots N$.

$$A^{[n_{m_l}]} = [\mu_{A_n^l}(x_m)] \tag{4.8}$$

It observed that k_{i_j} -Sub-Sub-Level Cuts $A^{[k_{i_j}]}$ and k_{j_i} -Sub-Sub-Level Cuts $A^{[k_{j_i}]}$ are identical i.e

$$A^{[n_{m_l}]} = [\mu_{A_n^l}(x_m)]$$

Note: It is obvious that by following the division procedure mentioned above, one would zoom into the given PCTLHS matrix. The first split would provide a zoom into the layer of the matrix, then zoom into the column or row, then next zoom into the element of the column or row. And the reverse process can serve as a zoom-out function. In this way one can approach the smallest unit of the extended universe.

5. COVID-19 Patients unified time-based health state Model

The mathematical modeling of the organization and analysis of information and observations of some patients with COVID-19 symptoms is described in the given example.

Example 5.1

Considering $U_{PC} = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ representing six patients (vaccinated) who presented to the hospital with symptoms of COVID-19. Description of an investigation case of a doctor who examined three of them for four symptoms (attributes) These symptoms are observed and organized as doctor visits at two specific times, considered as symptoms observed at two distinct time levels (time leveled attributes). Patients under observation are recognized as subjects (material bodies). The observations of the visits are expressed in the Crisp environment and analyzed using Plithogenic Crisp-Time-Leveled Hyper-Soft-Matrix.

$let\ T = \{x_1, x_2, x_3\} \subset U_C$ be the set of these three patients considered by a doctor for examination.

Let the attributes be A_j^k; $j = 1,2,3,4$ with Time Leveled-attributes k = 1, 2 are described as,

A_1^k = Fever with numeric values, $k = 1\ ,2$ representing first-time level and second-time level states of attributes

A_1^1 = State of fever at the first visit, A_1^2 = State of fever at the second visit

A_2^k = Dry cough, with numeric values, $k = 1{,}2$

A_2^1 = Condition of cough at the first visit, A_2^2 = Condition of cough at the second visit

A_3^k = Breathing difficulty with numeric values, $k = 1{,}2$

A_3^1 = Breathing difficulty level at first visit,

A_3^2 = Breathing difficulty level at first visit,

A_4^k = Sickness record, with numeric values $k = 1{,}2$

A_4^1 = Sickness state at first visit,) A_4^2 = Sickness state at second visit.

In the next two subsections, this information now consisted of symptoms (attributes) of patients (subjects) observed at two stages, as two levels of time are organized and presented in two ways. One as a set, i.e. PCTLHS set and the other as a connected matrix of two matrices, that is the PCTLHS matrix.

5.1 Plithogenic Crisp Time-Leveled Hypersoft set (PCTLHS-Set) representation:

Let the Function A is reflecting given attributes/Time leveled-attributes as described below,

$$\boldsymbol{A}: A_1^k \times A_2^k \times A_3^k \times A_4^k \to P(U_C)$$

$$\text{S.t}\quad \boldsymbol{A}(A_1^1, A_2^1, A_3^1, A_4^1) = \{x_1, x_2, x_3\} \tag{5.1}$$

be a Time-leveled hypersoft set. Consider $A_1^1, A_2^1, A_3^1, A_4^1$ a combination of attributes at the first visit level (α combination)

$$\boldsymbol{A}(A_1^2, A_2^2, A_3^2, A_4^2) = \{x_1, x_2, x_3\} \tag{5.2}$$

$A_1^2, A_2^2, A_3^2, A_4^2$ a combination of attributes at the second visit level (β combination)

The Individual Crisp memberships are assigned to $\boldsymbol{A}$ = $\{x_1, x_2, x_3\}$ according to the doctor's opinion and then represented in PCTLHS-Set A=$\{x_1, x_2, x_3\}$. the opinion of the physician represented in the PCTLHS-Set as Crisp memberships i.e if the given symptom is present membership is one if not present membership is zero.

$\boldsymbol{A} = \{x_1\ (\mu_{A_j^1}(x_1)), x_2\ (\mu_{A_j^1}(x_2)), x_3(\mu_{A_j^1}(x_3))\}$ as $\mu_{A_j^k}(x_i)$ for $i = 1,2,3$ and $j = 1,2,3,4$ in $\boldsymbol{A}$ (these plithogenic crisp memberships reflect whether the A_j^k attribute is present ($\mu_{A_j^k}(x_i) = 1$) in x_i subject or not present ($\mu_{A_j^k}(x_i) = 0$) associated to time leveled α-combination of attributes.

$$A(\alpha) = A(A_1^1, A_2^1, A_3^1, A_4^1) = \begin{Bmatrix} x_1(1,0,1,1), \\ x_2(1,1,1,1), \\ x_3(1,0,0,1) \end{Bmatrix} \tag{5.3}$$

The first visit information is organized as PCTLHS-Set would produce the first level of the PCTLHS Matrix.

Regarding the second visit (second level of time) of patients for β-combination of attributes, the information is now presented as a PCTLHS Set

$$A(\beta) = A(A_1^1, A_2^1, A_3^1, A_4^1) = \begin{Bmatrix} x_1(0,0,0,0), \\ x_2(0,1,0,0), \\ x_3(1,0,1,0) \end{Bmatrix} \tag{5.4}$$

5.2 PCTLHS -Matrix representation: Let A be the representation matrix for PCTLHS-Set. The rows of the matrix represent x_1, x_2, x_3 (physical bodies or subjects) and columns represent (the non-Physical aspect of subjects) $A_1^k, A_2^k, A_3^k, A_4^k$ Attributes.

This information is organized in the form of PCTLHS-Matrix A as,

$$F = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{bmatrix} \tag{5.5}$$

This PCTLHS-Matrix consists of two layers for the first layer, it is interpreted that patient x_1 has a fever without a dry cough at the first visit but feels suffocation and nausea. Patient x_2 suffers from a fever with a dry cough, fits of suffocation, and nausea. Patient x_3 has a fever with a dry cough, no difficulty breathing, but nausea. During the second visit, while the patient x_1 has all symptoms resolved, patient x_2 only feels difficulty in breathing, and patient x_3 suffers from fever and difficulty breathing. One can see clearly by using this connected matrix expression, we can see and classify all the information at a glance, i.e. the information from a group of patients assigned to a combination of attributes and observed at different time levels. Therefore, it is obvious that the matrix expression is the more appropriate expression to represent the multidimensional data as compared to the classical Set expression.

$A = \left[\mu_{A_j^k}(x_i)\right]$ $i = 1,2,3$ $j = 1,2,3,4$, and $k = 1,2$

This ($\boldsymbol{A}$) is a PCTLHS-Matrix of rank 3 and order $(i \times j \times k) = (3 \times 4 \times 2)$

The front-to-back view of this PCTLHS-Matrix consists of two parallel layers of ordinary 3×4 ordered matrices. These layers when separated are called k-level cuts described as underneath,

5.3 k-Level Cuts $A^{[k]}$ of PCTLHS-Matrix

$$A = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{bmatrix} \tag{5.6}$$

Eq. (5.6) represents a front-to-back view of PHS-Matrix in Crisp environment for $i = 1,2,3$ $j = 1,2,3,4$ and $k = 1,2$. This $3 \times 4 \times 2$ hyper matrix has two 3×4 Matrices as two front-to-back layers. These layers are separated to construct k-level cuts (time-wise level cuts).

The two k-Level Cuts of PCTLHS-Matrix (eq. (5.6)) are given below, which serves to focus the time levels first.

$A^{[k=1]} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ $A^{[k=2]} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

5.4 k_i-Sub-Level Cuts: Sub-layers of PCTLHS-Matrix obtained by specifying $k = 1, i = 1$, and varying $j = 1,2,3,4$ are given below as k_i-Sub-Level Cuts as rows of the matrix layers.

$A^{[1_1]} = [1 \quad 0 \quad 1 \quad 1], A^{[1_2]} = [1 \quad 1 \quad 1 \quad 1], A^{[1_3]} = [1 \quad 1 \quad 0 \quad 1]$

$A^{[2_1]} = [0 \quad 0 \quad 0 \quad 0], A^{[2_2]} = [0 \quad 1 \quad 0 \quad 0], A^{[2_3]} = [1 \quad 0 \quad 1 \quad 0]$

For example the description of $A^{[2_1]} = [0 \quad 0 \quad 0 \quad 0]$ is that by the second visit to the first patient, all four symptoms are clear.

5.5 k_j-Sub-Level Cuts: Sub-layers of PCTLHS-Matrix obtained by specifying $k = 1, j = 1$, respectively, and varying $i = 1,2,3$, are given below as k_j-Sub-Level Cuts as columns of the matrix layers.

$A^{[1_1]} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, A^{[1_2]} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, A^{[1_3]} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, A^{[1_4]} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, A^{[2_1]} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, A^{[2_2]} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, A^{[2_3]} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

For example the given $A^{[1_3]}$ describes that at the first time level state of the third symptom is described individually in all three patients.

5.6 k_{j_i} Sub-Sub-Level Cut: For a given k_j -Sub-Level Cuts after specifying the time and attribute respectively our final focus is the patient (subject) i.e specifying finally $i = m$ we get k_{j_i}Sub-Sub-Level Cut for a fixed k_j-Sub-Level Cuts. $A^{[l_{n_m}]}$ is obtained by specifying $k = l$, $j = n$ respectively, and finally $i = m$. This sub-sub level cut is the smallest unit of the matrix that is the single element as described below,

$A^{[k_{j_i}]}$: $A^{[1_{1_1}]} = [1]$ $A^{[1_{2_1}]} = [0]$ $A^{[2_{3_1}]} = [0]$

For example $A^{[2_{3_1}]} = [0]$ represents the condition of the first patient for the third symptom at the second time level.

Example 5.2

$$B = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{bmatrix} \tag{5.7}$$

Is a $3 \times 3 \times 2$ PCTLHS-Matrix with two attribute time levels.

$i = 1,2,3$ $j = 1,2,3$ and $k = 1,2$

A Front to back view of PCTLHS-Matrix with two k-level cuts, each level cut is a 3×3 Matrix is given below in Eq. (5.9)

$$B = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{bmatrix} \tag{5.9}$$

$k - level\ cuts\ of\ B$ are $B^{[k]}$

$$B^{[1]} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix},\ B^{[2]} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

6 Local Aggregation Operators of $k - Level\ Cuts$

Operations of PCTLHS-Matrix are the basic set laws of union, intersection, average, and compliment that is defined by using k-level cuts. After using these operations cumulative memberships $\Omega^{t}_{\mathbf{A}_j^k}(x_i)$ are obtained by combining corresponding memberships of k-level cuts (time-based level cuts). These local operators serve to unify the time levels of the universe. This means that all attributes that are present in different time levels are considered as present in a unified single time level being reflected from many entities of the universe. In this way, attributes are focused and time levels are merged as there is no time, therefore, these special types of k-level cuts and their local operators introduce the concept of no time that is obtained by the unification of time levels by using aggregation operators. Three local operators are formulated and described, $t = 1$ used for max -operator $t = 2$ used for min-operator, and $t = 3$ used for the $averaging$-operator. These three operators are described as under,

6.1 Union of $k - Level\ Cuts$: The union between front to back parallel layers is defined as

$$\cup_k \left[\mu_{A_j^k}(x_i)\right] = \underset{k}{Max}\left(\mu_{A_j^k}(x_i)\right) = \left[\Omega_{A_j}(x_i)\right] \tag{6.1}$$

$\left[\Omega_{A_j^k}(\mathrm{x})\right]$ is the cumulated layer of highest memberships considered as the Extreme front level layer.

Example 6.1: For Matrix **A** given in Ex-1 ∪ ($\boldsymbol{A}^{[k]}$)**) is**

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \cup \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

6.2 Intersection of $\boldsymbol{j}$ − Level Cuts: The intersection between front to back parallel layers is defined below,

$$\cap \left[\mu_{A_j^k}(x_i)\right] = \underset{k}{Min}\left(\mu_{A_j^k}(x_i)\right) = \left[\Omega_{A_j}(x_i)\right] \tag{6.2}$$

$\left[\Omega_{A_j^k}(\mathrm{x})\right]$ is the cumulated layer of lowest memberships considered as the Extreme back level layer.

Example 6.2: For Matrix **A** given in Ex-1 ∪ ($\boldsymbol{A}^{[k]}$)**) is**

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \cap \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

This lowest back level layer is reflecting the accumulated lowest state of symptoms throughout the two time-levels.

6.3 Average of $k - Level\ Cuts$: This is the average between front to back parallel layers is defined below,

$\Gamma\left[\mu_{A_j^k}(x_i)\right] = \left(\Omega_{A_j}(x_i)\right)$ such that

$$\left(\Omega_{A_j}(x_i)\right) = \begin{bmatrix} 1\ if\ \sum_{j=1}^{L} \frac{\left(\mu_{A_j^k}(x_i)\right)}{N} \geq 0.5 \\ 0\ if\ \sum_{j=1}^{L} \frac{\left(\mu_{A_j^k}(x_i)\right)}{N} < 0.5 \end{bmatrix} \tag{6.3}$$

$\left[\Omega_{A_j^k}(\mathrm{x})\right]$ is the cumulated layer of average memberships considered as the interior level layer.

Example 6.3: For Matrix **A** given in Ex-1 ∪ ($\boldsymbol{A}^{[k]}$)**) is**

$$\begin{bmatrix}1&0&1&1\\1&1&1&1\\1&1&0&1\end{bmatrix}\Gamma\begin{bmatrix}0&0&0&0\\0&1&0&0\\1&0&1&0\end{bmatrix}=\begin{bmatrix}1&0&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix}$$

This average middle-level layer is reflecting the accumulated average state of symptoms throughout the two distinct time levels.

6.4 Compliment of $k-Level\ Cuts$: The complement of each membership of the k-Level Cut is defined in eq (5.3)d

$$\mathrm{C}\left(A^{[k]}\right)=\left[1-\mu_{A_j^k}(x_i)\right] \tag{6.4}$$

Example 6.4: Compliments of k-Level Cuts of A are,

$$\mathrm{C}\left(A^{[1]}\right)=\begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&1&0\end{bmatrix},\mathrm{C}\left(A^{[2]}\right)=\begin{bmatrix}1&1&1&1\\1&0&1&1\\0&1&0&1\end{bmatrix}$$

7. Conclusion & Analysis

7.1 Conclusions

1. We can portray an extensive indeterminable Plithogenic Universe by using PCTLHS-Matrix.
2. We can display Multiple-dimensional views of the Universe (subjects versus attributes and time lined-attributes) By considering all possible views of the PCTLHS-Matrix
3. We can classify and analyze the universe explicitly and implicitly through level cuts, sublevel cuts, and sub-sub-level cuts.
4. The PCTLHS -Matrix provides the broader Exterior and interior view of displaying all possible Events (realities) together.
5. The Level Cuts of PCTLHS-Matrix Portray an explicit event or reality at an instance
6. Choosing the level cut based on the Variation index (i, j, k) Provides a view of reality or events from multiple angles of vision.
7. We can analyze the Universe by choosing the best possible reality out of multiple possible realities with the help of level cuts and operators. This fact would be helpful in the development of artificial intelligence programs.
8. The disjunction operator, i.e., the $Max-operator$ Provides the optimist view of the reality.
9. The conjunction operator, i.e., the $Min-operator$ Provides the pessimist view of the reality
10. The Averaging operator Provides a neutral view of the reality.
11. The Complement operator depicts the inverted reflection of the event or reality.
12. These local aggregation operators that are designed for k-level cuts introduce the concept of timelessness by unifying the time levels of the universe. This means that all attributes that exist in different time levels are merged and considered to exist in a unified single time level. In this way, attributes are focused, and time layers are merged as if there is no time for them.

7.2 Comparisons of former fuzzy extensions and models:

This section describes a brief comparison of previous and recent fuzzy extensions.

The soft set is an improved and extended version of the fuzzy set since it handles numerous attributes at the same time regardless of the fuzzy set, which only holds one attribute at a time.

Hypersoft Set is a superior extension of Soft Set because it can adapt multidimensional information by handling various attributes and their values as sub-attributes simultaneously.

Plithogenic Hypersoft Set The Plithogenic Hypersoft Set manages multiple attributes and their values (sub-attributes) simultaneously and beyond by observing each attribute separately therefore it is a more inovative version as compared to the Hypersoft Set, Soft set, and Fuzzy Set. It manages detailed information in a single structure. The spectator can precieve the state of element x (subject) by observing each attribute separately.

Plithogenic Fuzzy Whole Hypersoft-Set/Matrix (PFWHS-Set/Matrix) is a more aplicable choice compared to the previously mentioned extensions as it manages the states of subjects (attributes/sub-attributes) at the isolated level for each attribute/sub-attribute (the case of Plithogenic Hypersoft Set) and also at the combined level for merged attributes as a whole (the case of hypersoft set). Therefore, it is an extended and a hybrid version of the hypersoft set and plithogenic hypersoft set. By using PFWHS-Set/Matrix, one can observe a more transparent inner perception (case of a single state representation) or outer view (case of a combined state representation) of the information/facts/events.

The Plithogenic Subjective Hyper-Super-Soft Matix (PSHSS-Matrix) It is a generalized and an advanced form of the PFWHS-Matrix, as it has a higher capability to manage numerous connected attributes/sub-attributes separately and as a whole by considering connected attribute / sub-attribute levels.

Plithogenic Time-Leveled Hypersoft-Matrix (PTLHS-Matrix) is a unique case of the previous mentioned form (PSHSS-Matrix). It can manage time-based connected attributes. It is more suitable than other extended fuzzy sets mentioned (Soft Set, Hypersoft, Plithogenic Hypersoft Set, PFWHS Set / Matrix PSHSS Matrix) for the subsequent valid reasons.

1. Most of the variations in this universe are time-dependent like weather graphs, stock exchange, website ratings, etc. Therefore, it is of great help if this PCTLHS-Matrix is used to manage the scattered time-varying piece of information.
2. It manages several attributes sub-attributes interiorly such that each attribute has many values varying in the flow of time called time-based attributes.
3. By using PCTLHS-Matrix one can organize and classify multidimensional information into the shape of connected matrix layers as hypermatrices.
4. The matrix expression is the most applicable expression to represent multidimensional information compared to the classic set expression.
5. The observer can see the information down to its innermost level through level cuts, sub-level cuts, and sub-level cuts of PCTLHS-Matrix.
6. PCTLHS-Matrix offers a broader view of multi-dimensional information by viewing the entire universe as a hypersoft time-leveled matrix. Therefore, the observer can see and analyze the whole universe externally at a single glance.
7. The level cuts offers the observer to focus on one required piece of information that is displayed as a single matrix layer of PTLHS-Matrix. Whereas the other information can vary in the flow of time being displayed as other matrix layers.
8. The sub-level cuts can focus on required information that is displayed as a single column or row of the given layer (sub-matrix) of PTLHS-Matrix.
9. The sub-sub-level cuts can focus on one required information that is displayed as a single element of the sub-matrix of PTLHS-Matrix.
10. The Sub-Level Cuts offer the representation of the previous lower dimension in the further lower dimension and enable us to sneak in an inside view of the expanded universe, i.e after explicitly focusing on a subject through an i-level cut (single level of the layered matrix) our next focus is on that subject's (patient's) attribute (a particular symptom) through the sub-level cut (row or column of one layer of the multi-layered matrix).

11. It also offers the unification of the information by applying the aggregation operators, in this way all the extended information of the universe that is represented as a matrix having multiple layers can be transformed into a single layer of the matrix.

Open problems:

Now, let us list some of the open problems that might be addressed in future research.

- In this article, we have portrayed the Plithognic Hypersoft Matrix in a Crisp Environment.
- The expression of this matrix in other environments like Fuzzy, intuitionistic, and neutrosophic, or any mixed or combined environment, i.e., containing several environments, would provide the variation of fuzziness levels of reflected events.
- One can extend this model in other environments like intuitionistic environment, Neutrosophic environment, or any other mixed environment according to required conditions.
- By introducing these Level Cuts, we have provided the concept of contracting the expanded dimension of PCTLHS-Matrix to a single point (serving as a zoom-in function.
- Moreover, some other kinds of the local operator can be provided for unification purposes according to the requirement of the concerned bodies.

The operations and properties of these hypersoft matrices need to be explored.

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