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Bipolar Neutrosophic Frank Aggregation Operator and its application in Multi Criteria Decision Making Problem

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Abstract: Aggregation operators can be used to combine and synthesise a finite number of numerical values into a single numerical value. Many areas, including decision-making, expert systems, risk analysis, and image processing, rely heavily on aggregating functions. In real-world situations, the neutrosophic set can manage the uncertainties associated with information from any decision-making challenge, whereas the fuzzy set and intuitionistic set cannot. The term "bipolarity" refers to the propensity of the human mind to weigh pros and drawbacks when thinking through decisions. Triangular norms are aggregation operators in a variety of fields, including fuzzy set theory, probability and statistics, and decision sciences. Thus, the individual assessments in this paper's study of and approach to multi-criteria decision-making (MCDM) problems that use bipolar neutrosophic numbers as the individual evaluations. Frank operational laws of bipolar neutrosophic numbers, bipolar neutrosophic Frank weighted geometric aggregation (BNFWGA) and the bipolar neutrosophic frank ordered weighted geometric aggregation (BNFOWGA) operators have been developed with its desirable properties. Additionally, the suggested aggregation operators have been used in the selection of bridges. The outcomes demonstrate the applicability and validity of the suggested approach. Comparative analysis has been performed using the current approach.

Abbreviation:

NS	Neutrosophic Set
IFS	Intuitionistic fuzzy set
IVIFS	Interval-valued intuitionistic fuzzy set
INS	Interval neutrosophic set
SVNS	Single valued neutrosophic set
MCDM	Multi criteria decision making
BNS	Bipolar neutrosophic set
MAGDM	Multi attribute group decision making
BNNs	Bipolar neutrosophic numbers
BNFWGA	Bipolar neutrosophic frank aggregation weighted geometric aggregation
BNFOWGA	Bipolar neutrosophic ordered weighted geometric aggregation

Keywords: Bipolar neutrosophic set, Frank Triangular Norms, Operational Laws, Aggregation Operators, Decision Making

1. Introduction

A newly established model typically fixes the flaws of prior models in fuzzy theory. Because ambiguity and uncertainty present challenges in many real-world situations, routine mathematics is not always available. Many methods, including statistical hypothesis, probability, and fuzzy set hypothesis, have been presented as alternatives to traditional models and to guard against weaknesses in order to handle such difficulties. The majority of these mathematical alternatives, regrettably, have drawbacks and shortcomings of their own. Most words are in fact ambiguous and cannot be quantified, for instance, authentic and best-known. The authors of [1] started thinking about the chance based on the participation function that awards a membership grade in [0, 1] to handle such muddled and ambiguous information. Fuzzy sets are unable to handle the difficult issue since they only have one membership degree. Presented the intuitionistic fuzzy set (IFS) concept in [2]. IFS is used to provide an extremely flexible description of uncertain information. IFS offers degrees that are both membership- and non-membership-based. Introduced the idea of an intuitive fuzzy set with interval values in [3]. [4] developed the idea of a neutrosophic set (NS). NS includes membership, non-membership, and indeterminacy membership functions to define incompletes, inconsistent, and uncertain information. In order to adapt NS to real-world decision-making scenarios, [5, 6] introduced the interval Neutrosophic set (INS) and single valued Neutrosophic set (SVNS) concepts. Bipolar fuzzy sets are a generalization of fuzzy sets that were created by [7, 8]. The bipolar fuzzy relations study, in which each tuple is connected to a pair of satisfaction degrees, was first presented in [9]. Two applications for bipolar fuzzy sets in groups called the bipolar fuzzy groups and the norm have been introduced in [10].

A popular area of research in fuzzy theory of decision analysis is the study of fuzzy multi-attribute group decision-making. A sequence of judgments made in a fuzzy environment, which is usually ambiguous or uncertain, give decision information in the process of choosing the best possible options in terms of several criteria. Gaining a comprehensive understanding of the data is crucial for information fusion, particularly when making difficult decisions. Aggregation functions are one of the most efficient and simple methods for obtaining the aggregated result, although there are other methods as well. An n-tuple of data can be condensed into a single output using an aggregate function, which uses non-decreasing functions and keeps the output in the same set as the input. [11] In situations where all attribute values were defined as intuitionistic fuzzy numbers or interval-valued intuitionistic fuzzy numbers, aggregation functions were employed to handle dynamic multi-attribute decision-making in [11].

On the premise that the decision makers' criteria or preferences are unrelated and that the aggregating operators—defined by the independence axiom—are linear operators based on additive measures, multiple aggregation processes have been implemented in [12,13]. According to [14], real decision-making issues show the occurrence of unique dependencies or interactions between criteria. Decision-makers are typically invited from the same or related fields for a choice dilemma. They have a comparable social status, a similar level of knowledge, and similar tastes. Their arbitrary preferences can be demonstrated to exhibit nonlinearity. As a result, both the mutually preferred independence of these criteria and the independence of decision-makers are compromised. Advanced neutrosophic planar graph concepts and their applications were introduced in [15]. In [16], it was suggested to utilize a neutrosophic graph to predict linkages in social networks. A novel method of link prediction using the rsm index was developed by the authors of [17]. Radio fuzzy graphs and used radio k-colouring graphs to assign frequencies in radio stations introduced in [18]. [19] investigated the edge colouring of fuzzy graphs; chromatic index and the strong chromatic index have been proposed with its related properties in [19]. The colouring of directed fuzzy graphs based on the influence of relationship was proposed in [20]. Bipolar Neutrosophic TOPSIS was introduced in [21] as a method for resolving Multi Attribute Decision-Making (MADM) issues in a bipolar Neutrosophic fuzzy environment. In [22], methods based on Frank Choquet Bonferroni Mean Operators were developed to address MADM difficulties in a bipolar Neutrosophic fuzzy environment. [23, 24] discussed a few aggregation operators on different models.

The lattice of closed interval-valued fuzzy sets has been extended using Frank t-norms-based extension operations, which were proposed in [25]. These operations have been given the necessary and sufficient conditions to form a complete algebraic structure. [26] developed an analytical hierarchy process method for multi-attribute decision-making issues based on a logarithmic regression function and presented the idea of a triangular interval type-2 fuzzy set. [27] used arithmetic procedures like union and intersection between interval fuzzy linguistic numbers and Multi Attribute Group Decision Making problems to create a probabilistic linguistic framework. [28] a new signed distances-based linear assignment technique for MAGDM issues with fuzzy set information was created, and it was then applied to locate a landfill. [29] suggested a MAGDM and the idea of a trapezoidal interval type-2 fuzzy soft set. According to the aforementioned data, the research contribution based on bipolarity is relatively minimal, which is why we applied the concept in our research. MATLAB can be used to lessen the time complexity. The rest of the essay is organized as follows. Section 2 presents the fundamental antecedents. Section 3 established the Frank Aggregation operators' operating laws for bipolar neutrosophic numbers (BNNs). We go into great detail on BNFWGA and BNFOWGA, as well as their attributes, in Section 4. On the basis of bipolar neutrosophic Frank aggregation operations, we propose a number of comprehensive MCDM techniques in Section 5. The presented principles are applied to extend comprehensive techniques in Section 6. The proposed aggregation operators are used in section 7 to resolve the decision-making problem for choosing the best bridge. The comparative analysis and current methods have been discussed in Section 8. Section 9 provides the conclusion of the current study along with future directions.

2. Basic Concepts

For a better understanding, basic definitions pertaining to the current work are provided in this section.

Definition 2.1: Bipolar Neutrosophic Set (BNS) [30]

Let U be a fixed set. Then BNS can be defined as follows.

$$N(u) =< \chi_N^+(u), \xi_N^+(u), \Lambda_N^+(u), \chi_N^-(u), \xi_N^-(u), \Lambda_N^-(u) > / u \in U\},$$

Where $\chi_N^+(u), \xi_N^+(u), \Lambda_N^+(u) : U \to [0,1]$ and $\chi_N^-(u), \xi_N^-(u), \Lambda_N^-(u) : U \to [-1,0]$. The positive membership degrees $\chi_N^+(u), \xi_N^+(u), \Lambda_N^+(u)$ are the truth membership, indeterminacy membership degree and falsity membership degree of an element $u \in U$ corresponding to BNSN and the negative membership degrees $\chi_N^-(u), \xi_N^-(u), \Lambda_N^-(u)$ denote the truth membership degree, indeterminacy membership degree and falsity membership degree of an element $u \in U$ to some implicit counter property corresponding to a BNSN.

In particular, if 'U' has only one element, then

$N(u) =< \chi_N^+(u), \xi_N^+(u), \Lambda_N^+(u), \chi_N^-(u), \xi_N^-(u), \Lambda_N^-(u) >\},$ is called bipolar neutrosophic numbers (BNN).

Definition 2.2 : Algebraic operations of BNNs [30]

Let $N_1 =< \chi_1^+, \xi_1^+, \Lambda_1^+, \chi_1^-, \xi_1^-, \Lambda_1^- >$ and $N_2 =< \chi_2^+, \xi_2^+, \Lambda_2^+, \chi_2^-, \xi_2^-, \Lambda_2^- >$ be two BNNs. Then algebraic operations are defined as follows:

$$(1) N_1 \oplus N_2 =< \chi_1^+ + \chi_2^+ - \chi_1^+\chi_2^+, \xi_1^+\xi_2^+, \Lambda_1^+\Lambda_2^-, -\chi_1^+\chi_2^+, -(-\xi_1^- - \xi_2^- - \xi_1^-\xi_2^-), -(-\Lambda_1^- - \Lambda_2^- - \Lambda_1^-\Lambda_2^-) >$$

$$(2) N_1 \otimes N_2 =< \chi_1^+\chi_2^+, \xi_1^+ + \xi_2^+ - \xi_1^+\xi_2^+, \Lambda_1^+ + \Lambda_2^+ - \Lambda_1^+\Lambda_2^+, -(-\chi_1^- - \chi_2^- - \chi_1^-\chi_2^-), -\xi_1^-\xi_2^-, -\Lambda_1^-\Lambda_2^-) >$$

$$(3) l.N_1 =< 1-(1-\chi_1^+)^l, \left(\xi_1^+\right)^l, \left(\Lambda_1^+\right)^l, \left(-\chi_1^-\right)^l, -\left(1-\left(1-\left(-\xi_1^-\right)^l\right)\right), -\left(1-\left(1-\left(-\Lambda_1^-\right)^l\right)\right) > (l>0),$$

$$(4) \mathrm{N}_1^l =< \left(\chi_1^+\right)^l, 1-\left(1-\xi_1^+\right)^l, 1-\left(1-\Lambda_1^+\right)^l, -\left(1-\left(1-\left(-\chi_1^-\right)^l\right)\right), -\left(-\xi_1^-\right)^l, -\left(-\Lambda_1^-\right)^l > (l>0)$$

Definition 2.3: Score and accuracy function of BNNs [26]

Let $N_1 =< \chi_1^+, \xi_1^+, \Lambda_1^+, \chi_1^-, \xi_1^-, \Lambda_1^- >$ be BNN. Then the score function s(aleph1),accuracy function $a\left(N_1\right)$ and certainty function

c(aleph1) are defined as:

$$s\left(N_1\right) = \frac{\chi_1^+ + 1 - \xi_1^+ + 1 - \Lambda_1^+ + 1 + \chi_1^- - \xi_1^- - \Lambda_1^-}{6} \quad \text{..................(1)}$$

$$a\left(N_1\right) = \left(\chi_1^+ - \Lambda_1^+\right) + \left(\chi_1^- - \Lambda_1^-\right) \quad \text{..................(2)}$$

$$c\left(N_1\right) = \chi_1^+ - \Lambda_1^- \quad \text{.......................(3)}$$

Definition 2.4: Properties on Bipolar Neutrosophic Sets [30]

Let $N_1 =< \chi_1^+, \xi_1^+, \Lambda_1^+, \chi_1^-, \xi_1^-, \Lambda_1^- >$

And $N_2 =< \chi_2^+, \xi_2^+, \Lambda_2^+, \chi_2^-, \xi_2^-, \Lambda_2^- >$

Be two BNNs therefore

(1) If $s\left(N_1\right) > s\left(N_2\right), then N_1 > N_2$.

(2) If $s\left(N_1\right)=s\left(N_2\right)\&a\left(N_1\right)=a\left(N_2\right), then N_1 > N_2$.

(3) If $s\left(N_1\right)=s\left(N_2\right)\&a\left(N_1\right)=a\left(N_2\right)\&c\left(N_1\right)=c\left(N_2\right), then N_1 > N_2$

(4) If $s\left(N_1\right)=s\left(N_2\right)\&a\left(N_1\right)=a\left(N_2\right)\&c\left(N_1\right)=c\left(N_2\right), then N_1 \sim N_2$

Definition 2.5: Frank Triangular Norms [27]

The sum and product of Frank triangular norms are defined as follows.

$$\mathrm{N}_1 \oplus_F \mathrm{N}_2 = 1-\log_\lambda\left(1+\frac{\left(\lambda^{1-\mathrm{N}_1}-1\right)\left(\lambda^{1-\mathrm{N}_2}-1\right)}{\lambda-1}\right)\lambda>1\forall\left(\mathrm{N}_1,\mathrm{N}_2\right)\in\left[0,1\right]^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

$$\mathrm{N}_1 \otimes_F \mathrm{N}_2 = \log_\lambda\left(1+\frac{\left(\lambda^{\mathrm{N}_1}-1\right)\left(\lambda^{\mathrm{N}_2}-1\right)}{\lambda-1}\right)\lambda>1\forall\left(\mathrm{N}_1,\mathrm{N}_2\right)\in\left[0,1\right]^2 \qquad \ldots\ldots\ldots\ldots\ldots(5)$$

3. Operational Laws of Frank Triangular Norms for Bipolar Neutrosophic Numbers:

In this section, Operational laws are proposed using Frank triangular norms for BNNs.

Definition 3.1: Let $\mathrm{N}_1=\left\langle\chi_1^+,\xi_1^+,\Lambda_1^+,\chi_1^-,\xi_1^-,\Lambda_1^-\right\rangle$ and $\mathrm{N}_2=\left\langle\chi_2^+,\xi_2^+,\Lambda_2^+,\chi_2^-,\xi_2^-,\Lambda_2^-\right\rangle$ be two BNNs and $\lambda\geq1$. Then the operational laws are as follows.

(i). Addition:

$$
\mathrm{N}_1 \oplus_F \mathrm{N}_2 = \begin{cases} <1-\log_\lambda\left(1+\dfrac{\left(\lambda^{1-\chi_1^+}-1\right)\left(\lambda^{1-\chi_2^+}-1\right)}{\lambda-1}\right)-\log_\lambda\left(1+\dfrac{\left(\lambda^{\chi_1^+}-1\right)\left(\lambda^{\chi_2^+}-1\right)}{\lambda-1}\right), \log_\lambda\left(1+\dfrac{\left(\lambda^{\xi_1^+}-1\right)\left(\lambda^{\xi_2^+}-1\right)}{\lambda-1}\right), \\ \log_\lambda\left(1+\dfrac{\left(\lambda^{\Lambda_1^+}-1\right)\left(\lambda^{\Lambda_1^-}-1\right)}{\lambda-1}\right), -\log_\lambda\left(1+\dfrac{\left(\lambda^{\chi_1^+}-1\right)\left(\lambda^{\chi_2^+}-1\right)}{\lambda-1}\right), \\ \log_\lambda\left(1+\dfrac{\left(\lambda^{1+\xi_1^-}-1\right)\left(\lambda^{1+\xi_2^-}-1\right)}{\lambda-1}\right)-1+\log_\lambda\left(1+\dfrac{\left(\lambda^{-\xi_1^-}-1\right)\left(\lambda^{-\xi_2^-}-1\right)}{\lambda-1}\right), \\ \log_\lambda\left(1+\dfrac{\left(\lambda^{1+\Lambda_1^-}-1\right)\left(\lambda^{1+\Lambda_2^-}-1\right)}{\lambda-1}\right)-1+\log_\lambda\left(1+\dfrac{\left(\lambda^{-\Lambda_1^-}-1\right)\left(\lambda^{-\Lambda_2^-}-1\right)}{\lambda-1}\right)> \end{cases}
$$

(ii). Multiplication:

$$
\mathrm{N}_1 \otimes_F \mathrm{N}_2 = \begin{cases} <\log_\lambda\left(1+\dfrac{\left(\lambda^{\chi_1^+}-1\right)\left(\lambda^{\chi_2^+}-1\right)}{\lambda-1}\right), \\ 1-\log_\lambda\left(\dfrac{\lambda+\lambda^{2-\xi_1^+-\xi_2^+}-\lambda^{1-\xi_1^+}-\lambda^{1-\xi_2^+}}{\lambda-1}\right)-\log_\lambda\left(\dfrac{\lambda+\lambda^{\xi_1^++\xi_2^+}-\lambda^{\xi_1^+}-\lambda^{\xi_2^+}}{\lambda-1}\right), \\ 1-\log_\lambda\left(\dfrac{\lambda+\lambda^{2-\Lambda_1^+-\Lambda_2^+}-\lambda^{1-\Lambda_1^+}-\lambda^{1-\Lambda_2^+}}{\lambda-1}\right)-\log_\lambda\left(\dfrac{\lambda+\lambda^{\Lambda_1^++\Lambda_2^+}-\lambda^{\Lambda_1^+}-\lambda^{\Lambda_2^+}}{\lambda-1}\right). \\ \log_\lambda\left(1-\dfrac{\left(\lambda^{1+\chi_1^-}-1\right)\left(\lambda^{1+\chi_2^-}-1\right)}{\left(\lambda^{-\chi_1^-}-1\right)\left(\lambda^{-\chi_2^-}-1\right)}\right)-1, -\log_\lambda\left(1+\dfrac{\left(\lambda^{\xi_1^-}-1\right)\left(\lambda^{\xi_2^-}-1\right)}{\lambda-1}\right), \\ -\log_\lambda\left(1+\dfrac{\left(\lambda^{\Lambda_1^-}-1\right)\left(\lambda^{\Lambda_2^-}-1\right)}{\lambda-1}\right)> \end{cases}
$$

(iii). Multiplication by an ordinary number:

$$
l._F \mathrm{N}_1 = \begin{cases} <1-\log_\lambda\left(1+\dfrac{\left(\lambda^{1-\chi_1^+}-1\right)^k}{\left(\lambda-1\right)^{k-1}}\right), \log_\lambda\left(1+\dfrac{\left(\lambda^{\xi_1^+}-1\right)^k}{\left(\lambda-1\right)^{k-1}}\right) \\ \log_\lambda\left(1+\dfrac{\left(\lambda^{\Omega_1^+}-1\right)^k}{\left(\lambda-1\right)^{k-1}}\right), -\log_\lambda\left(1+\dfrac{\left(\lambda^{\Lambda_1^-}-1\right)^k}{\left(\lambda-1\right)^{k-1}}\right), \\ \log_\lambda\left(1+\dfrac{\left(\lambda^{1-\xi_1^-}-1\right)^k}{\left(\lambda-1\right)^{k-1}}\right)-1, \log_\lambda\left(1+\dfrac{\left(\lambda^{1-\Lambda_1^-}-1\right)^k}{\left(\lambda-1\right)^{k-1}}\right)-1> \end{cases}
$$

(iv) Power Operation:

$$N_1^{\wedge_F^k} = \begin{cases} < \log_\lambda\left(1+\frac{\left(\lambda^{\chi_1^+}-1\right)^k}{(\lambda-1)^{k-1}}\right), 1-\log_\lambda\left(1+\frac{\left(\lambda^{1-\xi_1^+}-1\right)^k}{(\lambda-1)^{k-1}}\right) \\ 1-\log_\lambda\left(1+\frac{\left(\lambda^{1-\Lambda_1^+}-1\right)^k}{(\lambda-1)^{k-1}}\right), \log_\lambda\left(1+\frac{\left(\lambda^{1-\chi_1^-}-1\right)^k}{(\lambda-1)^{k-1}}\right)-1, \\ -\log_\lambda\left(1+\frac{\left(\lambda^{\xi_1^-}-1\right)^k}{(\lambda-1)^{k-1}}\right), -\log_\lambda\left(1+\frac{\left(\lambda^{\Lambda_1^-}-1\right)^k}{(\lambda-1)^{k-1}}\right) > \end{cases}$$

Theorem 3.2:

Let $N_1 = \langle \chi_1^+, \xi_1^+, \Lambda_1^+, \chi_1^-, \xi_1^-, \Lambda_1^- \rangle$ and $N_2 = \langle \chi_2^+, \xi_2^+, \Lambda_2^+, \chi_2^-, \xi_2^-, \Lambda_2^- \rangle$ be two BNNs and $l, l_1, l_2 > 0$. Then the following properties can be proven easily

(a) $N_1 \oplus_F N_2 = N_2 \oplus_F N_1$

(b) $N_1 \otimes_F N_2 = N_2 \otimes_F N_1$

(c) $l \cdot_F \left(N_1 \oplus_F N_2\right) = l \cdot_F N_2 \oplus_F l \cdot_F N_1$

(d) $\left(N_1 \oplus_F N_2\right)^{\wedge_F^l} = N_1^{\wedge_F^l} \oplus_F N_2^{\wedge_F^l}$

(e) $(l_1 + l_2) \cdot_F N_1 = l_1 \cdot_F N_2 \oplus_F l_2 \cdot_F N_1$

(f) $N_1^{\wedge_F^{(l_1+l_2)}} = N_2^{\wedge_F^{l_1}} \otimes N_2^{\wedge_F^{l_2}}$

4. Bipolar Neutrosophic Frank Weighted Geometric Aggregation Operator

In this section, we proposed bipolar neutrosophic Frank weighted geometric aggregation (BNFWGA) and the bipolar neutrosophic Frank ordered weighted geometric aggregation (BNFOWGA) operators and discussed different properties.

Definition 4.1:

Let $N_j = \langle \chi_j^+, \xi_j^+, \Lambda_j^+, \chi_j^-, \xi_j^-, \Lambda_j^- \rangle \left(j = 1, 2, 3......n\right)$ be a family of BNN's.

A mapping BNFWGA: $U^{'} = U$ is called BNFWGA operator, if it satisfies

$$BNFWGA\left(N_1, N_2, N_3.........N_n\right) = \underset{j=1}{\overset{n}{\otimes}} N_n^{\wedge_F^{\omega_n}}$$

$$= N_1^{\wedge_F^{\omega_1}} \otimes_F N_2^{\wedge_F^{\omega_2}} \otimes_F ....\otimes_F N_n^{\wedge_F^{\omega_n}}$$

and $\omega = \left(\omega_1, \omega_2, \omega_3..........\omega_n\right)^T$ be the weight vector of $\mathrm{N}_j, \omega_j \in [0,1] \& \sum_{j=1} \omega_j = 1.$

Theorem 4.1 :

Let $\mathrm{N}_j = \left\langle \chi_j^+, \xi_j^+, \Lambda_j^+, \chi_j^-, \xi_j^-, \Lambda_j^- \right\rangle (j = 1, 2, 3......n)$ be a family of BNN's and $\omega = \left(\omega_1, \omega_2, \omega_3..........\omega_n\right)^T$ be the weight vector of $\mathrm{N}_j, \omega_j \in [0,1] \& \sum_{j=1} \omega_j = 1.$ Then, the value aggregated using BNFWGA operator is still a BNN i.e).

$$BNFWGA\left(N_1, N_2, N_3.........N_n\right) = N_1^{\wedge_F^{\omega_j}} \otimes_F N_2^{\wedge_F^{\omega_j}} \otimes_F ....\otimes_F N_n^{\wedge_F^{\omega_j}}$$

$$= \begin{cases} < \log_\lambda \left(1 + \prod_{j=1}^{n} \left(\lambda^{\chi_j^+} - 1\right)^{\omega_j}\right), 1 - \log\left(1 + \prod_{j=1}^{n}\left(\lambda^{1-\xi_j^+} - 1\right)^{\omega_j}\right) \\ 1 - \log \prod_{j=1}^{n} \left(\lambda^{1-\Lambda_j^+} - 1\right)^{\omega_j}, \log_\lambda\left(1 + \prod_{j=1}^{n}\left(\lambda^{1-(-\chi_j^-)} - 1\right)^{\omega_j}\right) - 1, \\ -\log_\lambda \left(1 + \prod_{j=1}^{n}\left(\lambda^{-\xi_j^-} - 1\right)^{\omega_j}\right), -\log_\lambda\left(1 + \prod_{j=1}^{n}\left(\lambda^{-\Lambda_j^-} - 1\right)^{\omega_j}\right), > (\omega > 0) \end{cases} \quad ....(6)$$

Proof:

By mathematical induction we prove the result.

Case (1): When n=2

Based on the Definition 4.1, the following result can be obtained

$$BNFWGA\left(N_1, N_2\right) = N_1^{\wedge_F^{\omega_1}} \otimes_F N_2^{\wedge_F^{\omega_2}}$$

$$N_1^{\wedge_F^{\omega}} = \begin{cases} < \log_\lambda\left(1 + \left(\lambda^{\chi_1^+} - 1\right)^{\omega}\right), 1 - \log_\lambda\left(1 + \left(\lambda^{1-\xi_1^+} - 1\right)^{\omega}\right), \\ 1 - \log_\lambda\left(1 + \left(\lambda^{1-\Lambda_1^+} - 1\right)^{\omega}\right), \log_\lambda\left(1 + \left(\lambda^{1-(-\chi_1^-)} - 1\right)^{\omega}\right) - 1, \\ -\log_\lambda\left(1 + \left(\lambda^{-\xi_1^-} - 1\right)^{\omega}\right), -\log_\lambda\left(1 + \left(\lambda^{-\Lambda_1^-} - 1\right)^{\omega}\right) > \end{cases}$$

$$N_2^{\wedge_F^{\omega_1}} = \begin{cases} < \log_\lambda\left(1+\left(\lambda^{\chi_2^+}-1\right)^{\omega_2}\right), 1-\log_\lambda\left(1+\left(\lambda^{1-\xi_2^+}-1\right)^{\omega_2}\right), \\ 1-\log_\lambda\left(1+\left(\lambda^{1-\Lambda_2^+}-1\right)^{\omega_2}\right), \log_\lambda\left(1+\left(\lambda^{1-(-\chi_2^-)}-1\right)^{\omega_2}\right)-1, \\ -\log_\lambda\left(1+\left(\lambda^{-\xi_2^-}-1\right)^{\omega_2}\right), -\log_\lambda\left(1+\left(\lambda^{-\Lambda_2^-}-1\right)^{\omega_2}\right) > \end{cases}$$

$$N_1^{\wedge_F^{\omega_1}} \otimes_F N_2^{\wedge_F^{\omega_2}} == \begin{cases} < \log_\lambda\left(1+\prod_{j=1}^{2}\left(\lambda^{\chi_j^+}-1\right)^{\omega_j}\right), 1-\log\prod_{j=1}^{2}\left(\lambda^{1-\xi_j^+}-1\right)^{\omega_j}, \\ 1-\log\prod_{j=1}^{2}\left(\lambda^{1-\Lambda_j^+}-1\right)^{\omega_j}, \log_\lambda\left(1+\prod_{j=1}^{2}\left(\lambda^{1-(-\chi_j^-)}-1\right)^{\omega_j}\right)-1, \\ -\log_\lambda\left(1+\prod_{j=1}^{2}\left(\lambda^{-\xi_j^-}-1\right)^{\omega_j}\right), -\log_\lambda\left(1+\prod_{j=1}^{2}\left(\lambda^{-\Lambda_j^-}-1\right)^{\omega_j}\right), > (\omega > 0) \end{cases}$$

Case (ii): When n=s

Using equation (6), the following result can be obtained.

$$BNFWGA\left(N_1, N_2.....N_s\right) = \otimes_{j=1}^{s} N_j^{\wedge^{\omega_j}}$$

$$= \begin{cases} < \log_\lambda\left(1+\prod_{j=1}^{s}\left(\lambda^{\chi_j^+}-1\right)^{\omega_j}\right), 1-\log\left(1+\prod_{j=1}^{s}\left(\lambda^{1-\xi_j^+}-1\right)^{\omega_j}\right), \\ 1-\log\prod_{j=1}^{s}\left(\lambda^{1-\Lambda_j^+}-1\right)^{\omega_j}, \log_\lambda\left(1+\prod_{j=1}^{s}\left(\lambda^{1-(-\chi_j^-)}-1\right)^{\omega_j}\right)-1, \\ -\log_\lambda\left(1+\prod_{j=1}^{s}\left(\lambda^{-\xi_j^-}-1\right)^{\omega_j}\right), -\log_\lambda\left(1+\prod_{j=1}^{s}\left(\lambda^{-\Lambda_j^-}-1\right)^{\omega_j}\right), > (\omega > 0) \end{cases}$$

Case (iii) When n=s+1 then following result can be obtained:

$$BNFWGA\left(N_1, N_2.....N_s, N_{s+1}\right) = \otimes_{j=1}^{s} N_j^{\wedge^{\omega_j}} \otimes N_j^{\wedge^{\omega_{s+1}}}$$

$$= \begin{cases} < \log_\lambda\left(1+\prod_{j=1}^{s}\left(\lambda^{\chi_j^+}-1\right)^{\omega_j}\right), 1-\log\left(1+\prod_{j=1}^{s}\left(\lambda^{1-\xi_j^+}-1\right)^{\omega_j}\right), \\ 1-\log\left(1+\prod_{j=1}^{s}\left(\lambda^{1-\Lambda_j^+}-1\right)^{\omega_j}\right), \log_\lambda\left(1+\prod_{j=1}^{s}\left(\lambda^{1-(-\chi_j^-)}-1\right)^{\omega_j}\right)-1, \\ -\log_\lambda\left(1+\prod_{j=1}^{s}\left(\lambda^{-\xi_j^-}-1\right)^{\omega_j}\right), -\log_\lambda\left(1+\prod_{j=1}^{s}\left(\lambda^{-\Lambda_j^-}-1\right)^{\omega_j}\right), > (\omega > 0) \end{cases}$$

$$\otimes_F \begin{cases} < \log_\lambda\left(1+\prod_{j=s+1}\left(\lambda^{\chi_j^+}-1\right)^{\omega_j}\right), 1-\log\left(1+\prod_{j=s+1}\left(\lambda^{1-\xi_j^+}-1\right)^{\omega_j}\right), \\ 1-\log\left(1+\prod_{j=s+1}\left(\lambda^{1-\Lambda_j^+}-1\right)^{\omega_j}\right), \log_\lambda\left(1+\prod_{j=s+1}\left(\lambda^{1-(-\chi_j^-)}-1\right)^{\omega_j}\right)-1, \\ -\log_\lambda\left(1+\prod_{j=s+1}\left(\lambda^{-\xi_j^-}-1\right)^{\omega_j}\right), -\log_\lambda\left(1+\prod_{j=s+1}\left(\lambda^{-\Lambda_j^-}-1\right)^{\omega_j}\right), > (\omega > 0) \end{cases}$$

Thus the following result can be obtained.

$$BNFWGA\left(N_1, N_2, N_3.........N_n\right) = N_1^{\wedge_F^{\omega_j}} \otimes_F N_2^{\wedge_F^{\omega_j}} \otimes_F ....\otimes_F N_n^{\wedge_F^{\omega_j}}$$

$$= \begin{cases} < \log_\lambda\left(1+\prod_{j=1}^{n}\left(\lambda^{\chi_j^+}-1\right)^{\omega_j}\right), 1-\log\left(1+\prod_{j=1}^{n}\left(\lambda^{1-\xi_j^+}-1\right)^{\omega_j}\right), \\ 1-\log\prod_{j=1}^{n}\left(\lambda^{1-\Lambda_j^+}-1\right)^{\omega_j}, \log_\lambda\left(1+\prod_{j=1}^{n}\left(\lambda^{1-(-\chi_j^-)}-1\right)^{\omega_j}\right)-1, \\ -\log_\lambda\left(1+\prod_{j=1}^{n}\left(\lambda^{-\xi_j^-}-1\right)^{\omega_j}\right), -\log_\lambda\left(1+\prod_{j=1}^{n}\left(\lambda^{-\Lambda_j^-}-1\right)^{\omega_j}\right), > (\omega > 0) \end{cases}$$

Therefore, the theorem is true for n=s+1.

Hence the theorem.

Example 4.1: Let
$$N_1 =< 0.5, 0.7, 0.3, -0.6, -0.2, -0.6 >$$
$$N_2 =< 0.2, 0.5, 0.5, -0.8, -0.4, -0.3 >$$
$$N_3 =< 0.3, 0.6, 0.4, -0.7 - 0.3, -0.1 >$$

be three BNNs and let weight vector of BNNs $N_j \ \left(j = 1, 2, 3\right)$ be

$\omega = \left(\frac{1}{8}, \frac{3}{8}, \frac{1}{2}\right)^T, \omega_1 = \frac{1}{8}, \omega_2 = \frac{3}{8}, \omega_3 = \frac{1}{2}$ are the weight of $N_j \ \left(j = 1, 2, 3\right)$ such that $\prod_{j=1} \omega_j = 1$

Then by above theorem

$$BNFWGA\left(N_1,N_2,N_3\right)=\begin{cases}\log_3\left(1+\left(3^{0.5}-1\right)^{\frac{1}{8}}+\left(3^{0.2}-1\right)^{\frac{3}{8}}+\left(3^{0.3}-1\right)^{\frac{1}{2}}\right),\\ 1-\log_3\left(1+\left(3^{1-0.7}-1\right)^{\frac{1}{8}}+\left(3^{1-0.5}-1\right)^{\frac{3}{8}}+\left(3^{1-0.6}-1\right)^{\frac{1}{2}}\right),\\ 1-\log_3\left(1+\left(3^{1-0.3}-1\right)^{\frac{1}{8}}+\left(3^{1-0.5}-1\right)^{\frac{3}{8}}+\left(3^{1-0.4}-1\right)^{\frac{1}{2}}\right),\\ \log_3\left(1+\left(3^{1-0.6}-1\right)^{\frac{1}{8}}+\left(3^{1-0.8}-1\right)^{\frac{3}{8}}+\left(3^{1-0.7}-1\right)^{\frac{1}{2}}\right)-1,\\ \log_3\left(1+\left(3^{0.2}-1\right)^{\frac{1}{8}}+\left(3^{0.4}-1\right)^{\frac{3}{8}}+\left(3^{0.3}-1\right)^{\frac{1}{2}}\right),\\ l\,og_3\left(1+\left(3^{0.6}-1\right)^{\frac{1}{8}}+\left(3^{0.3}-1\right)^{\frac{3}{8}}+\left(3^{0.1}-1\right)^{\frac{1}{2}}\right)\end{cases}$$

$$BNFWGA\left(N_1,N_2,N_3\right)=\left(1.0,0.0391,0.2474,-0.3072,-1.0,-1.0\right)$$

The BNFWGA operator has the following properties:

(1) Idomopotency:

Let all the BNN's be $N_j=<\chi_j^+,\xi_j^+,\Lambda_j^+,\chi_j^-,\xi_j^-,\Lambda_j^->=N\left(j=1,2,3....n\right)$ where $\omega=\left(\omega_1,\omega_2,\omega_3..........\omega_n\right)^T$ be the weight vector of $N_j.\omega_j\in\left[0,1\right]$ and $\sum_{j=1}^{3}\omega_j=1$ then

$$BNFWGA\left(N_1,N_2,N_3.........N_j\right)=N$$

(2) Monotonicity: Let $N_j\ \left(j=1,2,3....n\right)$ and $N_j^{'}\ \left(j=1,2,3....n\right)$ be two families of BNNs, where $\omega=\left(\omega_1,\omega_2,\omega_3..........\omega_n\right)^T$ be the weight vector of N_j and $N_j^{'},\omega_j\in\left[0,1\right]$ and $\prod_{j=1}\omega_j=1$. For all 'j' if $N_j\geq N_j^{'}$ then

$$BNFWGA\left(N_1,N_2,N_3.........N_j\right)\geq BNFWGA\left(N_1^{'},N_2^{'},N_3^{'}.........N_j^{'}\right)$$

(3) Boundedness: Let $N_j =< \chi_j^+, \xi_j^+, \Lambda_j^+, \chi_j^-, \xi_j^-, \Lambda_j^- >= N\left(j=1,2,3....n\right)$ be a family of BNNs. Where $\omega = \left(\omega_1, \omega_2, \omega_3.........\omega_n\right)^T$ be the weight vector of N_j , $\omega_j \in [0,1]$ and $\sum_{j=1}^{3} \omega_j = 1$, Therefore we have

$$BNFWGA\left(N^-, N^-, ...N^-\right) \le BNFWGA\left(N_1, N_2, N_3........N_j\right) \le BNFWGA\left(N^+, N^+, ...N^+\right)$$

where $N^- =< \chi_{N^-}^+, \xi_{N^-}^+, \Lambda_{N^-}^+, \chi_{N^-}^-, \xi_{N^-}^-, \Lambda_{N^-}^- >$

$$= \begin{cases} \min\left(\chi_1^+, \chi_2^+, \chi_3^+, ...\chi_n^+\right), \max\left(\xi_1^+, \xi_2^+, \xi_3^+, ...\xi_n^+\right), \\ \max\left(\Lambda_1^+, \Lambda_2^+, \Lambda_3^+, ...\Lambda_n^+\right), \max\left(\chi_1^-, \chi_2^-, \chi_3^-, ...\chi_n^-\right) \\ \min\left(\xi_1^-, \xi_2^-, \xi_3^-, ...\xi_n^-\right), \min\left(\Lambda_1^-, \Lambda_2^-, \Lambda_3^-, ...\Lambda_n^-\right) > \end{cases}$$

And $N^+ =< \chi_{N^+}^+, \xi_{N^+}^+, \Lambda_{N^+}^+, \chi_{N^+}^-, \xi_{N^+}^-, \Lambda_{N^+}^- >$

$$= \begin{cases} \max\left(\chi_1^+, \chi_2^+, \chi_3^+, ...\chi_n^+\right), \min\left(\xi_1^+, \xi_2^+, \xi_3^+, ...\xi_n^+\right), \\ \min\left(\Lambda_1^+, \Lambda_2^+, \Lambda_3^+, ...\Lambda_n^+\right), \min\left(\chi_1^-, \chi_2^-, \chi_3^-, ...\chi_n^-\right) \\ \max\left(\xi_1^-, \xi_2^-, \xi_3^-, ...\xi_n^-\right), \max\left(\Lambda_1^-, \Lambda_2^-, \Lambda_3^-, ...\Lambda_n^-\right) > \end{cases}$$

Proof: Since $N_j =< \chi_j^+, \xi_j^+, \Lambda_j^+, \chi_j^-, \xi_j^-, \Lambda_j^- >= N\left(j=1,2,3....n\right)$ then, the following result can be obtained by using Equation (6). The following result can be obtained

$$BNFWGA\left(N_1, N_2, N_3........N_n\right) = \begin{cases} < \log_\lambda\left(1+\prod_{j=1}^{n}\left(\lambda^{\chi_j^+}-1\right)^{\omega_j}\right), 1-\log\prod_{j=1}^{n}\left(\lambda^{1-\xi_j^+}-1\right)^{\omega_j}, \\ 1-\log\prod_{j=1}^{n}\left(\lambda^{1-\Lambda_j^+}-1\right)^{\omega_j}, \log_\lambda\left(1+\prod_{j=1}^{n}\left(\lambda^{1-(-\chi_j^-)}-1\right)^{\omega_j}\right)-1, \\ -\log_\lambda\left(1+\prod_{j=1}^{n}\left(\lambda^{-\xi_j^-}-1\right)^{\omega_j}\right), -\log_\lambda\left(1+\prod_{j=1}^{n}\left(\lambda^{-\Lambda_j^-}-1\right)^{\omega_j}\right), > (\omega > 0) \end{cases}$$

$$= \begin{cases} < \log_\lambda\left(1+\left(\lambda^{\chi^+}-1\right)\right), 1-\log_\lambda\left(1+\left(\lambda^{1-\xi^+}-1\right)\right), \\ 1-\log_\lambda\left(1+\left(\lambda^{1-\Lambda^+}-1\right)\right), \log_\lambda\left(1+\left(\lambda^{1-(-\chi^-)}-1\right)\right)-1, \\ -\log_\lambda\left(1+\left(\lambda^{-\xi^-}-1\right)\right), -\log_\lambda\left(1+\left(\lambda^{-\Lambda^-}-1\right)\right) > \end{cases}$$

$=< \chi^+, \xi^+, \Lambda^+, \chi^-, \xi^-, \Lambda^- >= N$ holds

(1) The property is obvious based on the equation (6).

(2) Let $N^{-} =< \chi^{+}_{N^{-}}, \xi^{+}_{N^{-}}, \Lambda^{+}_{N^{-}}, \chi^{-}_{N^{-}}, \xi^{-}_{N^{-}}, \Lambda^{-}_{N^{-}} > and\, N^{+} =< \chi^{+}_{N^{+}}, \xi^{+}_{N^{+}}, \Lambda^{+}_{N^{+}}, \chi^{-}_{N^{+}}, \xi^{-}_{N^{+}}, \Lambda^{-}_{N^{+}} >$

There are following inequalities:

$$\begin{cases} < \log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{\chi^{+}_{N^{-}}}-1\right)\right) \le \log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{\chi^{+}_{j}}-1\right)\right) \le \log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{\chi^{+}_{N^{+}}}-1\right)\right), \\ 1-\log\left(1+\left(\prod_{j=1}^{n}\left(\lambda^{1-\xi^{+}_{N^{-}}}-1\right)\right)\right) \le 1-\log\left(1+\left(\prod_{j=1}^{n}\left(\lambda^{1-\xi^{+}_{N^{-}}}-1\right)\right)\right) \le 1-\left(1+\left(\prod_{j=1}^{n}\left(\lambda^{1-\xi^{+}_{N^{-}}}-1\right)\right)\right) \\ 1-\log\prod_{j=1}^{n}\left(\lambda^{1-\Lambda^{+}_{N^{-}}}-1\right) \le 1-\log\prod_{j=1}^{n}\left(\lambda^{1-\Lambda^{+}_{j}}-1\right) \le 1-\log\prod_{j=1}^{n}\left(\lambda^{1-\Lambda^{+}_{N^{+}}}-1\right), \\ \log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{1-(-\chi^{-}_{N})}-1\right)\right)-1 \le \log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{1-(-\chi^{-}_{N})}-1\right)\right)-1 \le \log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{1-(-\chi^{-}_{N})}-1\right)\right)-1 \\ -\log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{-\xi^{-}_{N^{-}}}-1\right)\right) \le -\log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{-\xi^{-}_{j}}-1\right)\right) \le -\log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{-\xi^{-}_{N^{+}}}-1\right)\right), \\ -\log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{-\Lambda^{-}_{N^{-}}}-1\right)\right), \le -\log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{-\Lambda^{-}_{j}}-1\right)\right) -\log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{-\Lambda^{-}_{N^{+}}}-1\right)\right) > \end{cases}$$

Hence

$$BNFWGA\left(N^{-}, N^{-}, ...N^{-}\right) \le BNFWGA\left(N_1, N_2, N_3........N_j\right) \le BNFWGA\left(N^{+}, N^{+}, ...N^{+}\right)$$

holds.

5. Bipolar Neutrosophic Frank Ordered Weighted Geometric Aggregation (BNFOWGA) Operator

This section proposes the BNFOWGA operator and describes its properties in detail.

Definition 5.1:

Let $\mathrm{N}_j = \left\langle \chi^{+}_{j}, \xi^{+}_{j}, \Lambda^{+}_{j}, \chi^{-}_{j}, \xi^{-}_{j}, \Lambda^{-}_{j} \right\rangle \left(j = 1, 2, 3......n\right)$ be a family of BNNs.

A mapping BNFOWGA: $U^{'} = U$ is called BNDWGA operator, if it satisfies,

$$BNFWGA\left(N_1, N_2, N_3........N_n\right) = \overset{n}{\underset{k=1}{\otimes}} N_{\rho(k)}^{\wedge_F^{\omega_n}}$$

$$= N_{\rho(1)}^{\wedge_F^{\omega_1}} \otimes_F N_{\rho(2)}^{\wedge_F^{\omega_2}} \otimes_F .... \otimes_F N_{\rho(n)}^{\wedge_F^{\omega_n}}$$

Where ρ is permutation that orders the elements

$N_{\rho(1)} \geq N_{\rho(2)} \geq N_{\rho(3)} \geq ....N_{\rho(n)} \geq$

and $\omega = (\omega_1, \omega_2, \omega_3..........\omega_n)^T$ be the weight vector of $N_j, \omega_j \in [0,1] \,\&\, \sum_{j=1} \omega_j = 1.$

Theorem 5.1:

Let $N_j = \langle \chi_j^+, \xi_j^+, \Lambda_j^+, \chi_j^-, \xi_j^-, \Lambda_j^- \rangle (j = 1,2,3......n)$ be a family of BNN's and $\omega = (\omega_1, \omega_2, \omega_3..........\omega_n)^T$ be the weight vector of $N_j, \omega_j \in [0,1] \,\&\, \sum_{j=1} \omega_j = 1.$ Then, the value aggregated by using bipolar neutrosophic Frank ordered weighted geometric average operator is still a BNN

$$BNFWGA(N_1, N_2, N_3.........N_n) = \overset{n}{\underset{k=1}{\otimes}} N_{\rho(k)}^{\wedge_F^{\omega_n}}$$

$$= N_{\rho(1)}^{\wedge_F^{\omega_1}} \otimes_F N_{\rho(2)}^{\wedge_F^{\omega_2}} \otimes_F ....\otimes_F N_{\rho(n)}^{\wedge_F^{\omega_n}}$$

$$= \begin{cases} < \log_\lambda \left(1 + \prod_{k=1}^{n} \left(\lambda^{\chi_j^+} - 1\right)^{\omega_k}\right), 1 - \left(\log\left(1 + \prod_{k=1}^{n}\left(\lambda^{1-\xi_j^+} - 1\right)^{\omega_k}\right)\right) \\ 1 - \log \prod_{k=1}^{n} \left(\lambda^{1-\Lambda_j^+} - 1\right)^{\omega_k}, \log_\lambda \left(1 + \prod_{k=1}^{n}\left(\lambda^{1-(-\chi_j^-)} - 1\right)^{\omega_k}\right) - 1, \\ -\log_\lambda \left(1 + \prod_{k=1}^{n}\left(\lambda^{-\xi_j^-} - 1\right)^{\omega_k}\right), -\log_\lambda\left(1 + \prod_{k=1}^{n}\left(\lambda^{-\Lambda_j^-} - 1\right)^{\omega_k}\right), > (\omega > 0) \end{cases} \quad .... (1)$$

Proof:

If n=2, using Frank operations for bipolar neutrosophic numbers, the following result can be obtained $BNFWGA(N_1, N_2) = N_{\rho(1)}^{\wedge_F^{\omega_1}} \otimes_F N_{\rho(2)}^{\wedge_F^{\omega_2}}$

$$N_1^{\wedge_F^{\omega_1}} = \begin{cases} < \log_\lambda\left(1 + \left(\lambda^{\chi_1^+} - 1\right)^{\omega_1}\right), 1 - \log_\lambda\left(1 + \left(\lambda^{1-\xi_1^+} - 1\right)^{\omega_1}\right), \\ 1 - \log_\lambda\left(1 + \left(\lambda^{1-\Lambda_1^+} - 1\right)^{\omega_1}\right), \log_\lambda\left(1 + \left(\lambda^{1-(-\chi_1^-)} - 1\right)^{\omega_1}\right) - 1, \\ -\log_\lambda\left(1 + \left(\lambda^{-\xi_1^-} - 1\right)^{\omega_1}\right), -\log_\lambda\left(1 + \left(\lambda^{-\Lambda_1^-} - 1\right)^{\omega_1}\right) > \end{cases}$$

$$N_2^{\hat{\omega}_1 F} = \begin{cases} < \log_\lambda \left(1+\left(\lambda^{\chi_2^+}-1\right)^{\omega_2}\right), 1-\log_\lambda \left(1+\left(\lambda^{1-\xi_2^+}-1\right)^{\omega_2}\right), \\ 1-\log_\lambda \left(1+\left(\lambda^{1-\Lambda_2^+}-1\right)^{\omega_2}\right), \log_\lambda \left(1+\left(\lambda^{1-(-\chi_2^-)}-1\right)^{\omega_2}\right)-1, \\ -\log_\lambda \left(1+\left(\lambda^{-\xi_2^-}-1\right)^{\omega_2}\right), -\log_\lambda \left(1+\left(\lambda^{-\Lambda_2^-}-1\right)^{\omega_2}\right) > \end{cases}$$

$$N_1^{\hat{\omega}_1 F} \otimes_F N_2^{\hat{\omega}_2 F} == \begin{cases} < \log_\lambda \left(1+\prod_{j=1}^{2}\left(\lambda^{\chi_j^+}-1\right)^{\omega_j}\right), 1-\log\left(1+\prod_{j=1}^{2}\left(\lambda^{1-\xi_j^+}-1\right)^{\omega_j}\right), \\ 1-\log \prod_{j=1}^{2}\left(\lambda^{1-\Lambda_j^+}-1\right)^{\omega_j}, \log_\lambda \left(1+\prod_{j=1}^{2}\left(\lambda^{1-(-\chi_j^-)}-1\right)^{\omega_j}\right)-1, \\ -\log_\lambda \left(1+\prod_{j=1}^{2}\left(\lambda^{-\xi_j^-}-1\right)^{\omega_j}\right), -\log_\lambda \left(1+\prod_{j=1}^{2}\left(\lambda^{-\Lambda_j^-}-1\right)^{\omega_j}\right), > (\omega > 0) \end{cases}$$

If k=s then,

$$BNFWGA\left(N_1, N_2 \ldots\ldots N_s\right) = \otimes_{k=1}^{s} N_{\rho(k)}^{\hat{\omega}_k}$$

$$= \begin{cases} < \log_\lambda \left(1+\prod_{k=1}^{s}\left(\lambda^{\chi_k^+}-1\right)^{\omega_k}\right), 1-\log\left(1+\prod_{j=1}^{s}\left(\lambda^{1-\xi_k^+}-1\right)^{\omega_k}\right), \\ 1-\log \prod_{j=1}^{s}\left(\lambda^{1-\Lambda_k^+}-1\right)^{\omega_k}, \log_\lambda \left(1+\prod_{k=1}^{s}\left(\lambda^{1-(-\chi_k^-)}-1\right)^{\omega_k}\right)-1, \\ -\log_\lambda \left(1+\prod_{k=1}^{s}\left(\lambda^{-\xi_k^-}-1\right)^{\omega_k}\right), -\log_\lambda \left(1+\prod_{k=1}^{s}\left(\lambda^{-\Lambda_k^-}-1\right)^{\omega_k}\right), > (\omega > 0) \end{cases}$$

If k=s+1 then there is following result:

$$BNFOWGA\left(N_1, N_2 \ldots\ldots N_s, N_{s+1}\right) = \otimes_{k=1}^{s} N_{\rho(s)}^{\hat{\omega}_s} \otimes N_{\rho(s+1)}^{\hat{\omega}_{s+1}}$$

$$= \begin{cases} < \log_\lambda \left(1+\prod_{k=1}^{s}\left(\lambda^{\chi_k^+}-1\right)^{\omega_k}\right), 1-\log\left(1+\prod_{k=1}^{n}\left(\lambda^{1-\xi_k^+}-1\right)^{\omega_k}\right) \\ 1-\log\left(1+\prod_{k=1}^{s}\left(\lambda^{1-\Lambda_k^+}-1\right)^{\omega_k}\right), \log_\lambda \left(1+\prod_{k=1}^{s}\left(\lambda^{1-(-\chi_k^-)}-1\right)^{\omega_k}\right)-1, \\ -\log_\lambda \left(1+\prod_{k=1}^{s}\left(\lambda^{-\xi_k^-}-1\right)^{\omega_k}\right), -\log_\lambda \left(1+\prod_{k=1}^{s}\left(\lambda^{-\Lambda_k^-}-1\right)^{\omega_k}\right), > (\omega > 0), \end{cases}$$

Hence the theorem true for r=s+1. Thus the result

$$\begin{cases} < \log_{\lambda}\left(1+\prod_{k=1}^{n}\left(\lambda^{\chi_k^+}-1\right)^{\omega_k}\right), 1-\log\left(1+\prod_{k=1}^{n}\left(\lambda^{1-\xi_k^+}-1\right)^{\omega_k}\right) \\ 1-\log\prod_{k=1}^{n}\left(\lambda^{1-\Lambda_k^+}-1\right)^{\omega_k}, \log_{\lambda}\left(1+\prod_{k=1}^{n}\left(\lambda^{1-(-\chi_k^-)}-1\right)^{\omega_k}\right)-1, \\ -\log_{\lambda}\left(1+\prod_{k=1}^{n}\left(\lambda^{-\xi_k^-}-1\right)^{\omega_k}\right), -\log_{\lambda}\left(1+\prod_{k=1}^{n}\left(\lambda^{-\Lambda_k^-}-1\right)^{\omega_k}\right), > (\omega > 0) \end{cases}$$

holds for all n.

Example 5.1: Let $N_1 =< 0.5, 0.7, 0.3, -0.6, -0.2, -0.6 >$, $N_2 =< 0.2, 0.5, 0.5, -0.8, -0.4, -0.3 >$, $N_3 =< 0.3, 0.6, 0.4, -0.7-0.3, -0.1 >$ be three BNNs and let weight vector of BNNs N_k $(k=1,2,3)$ be $\omega = \left(\frac{1}{8}, \frac{3}{8}, \frac{1}{2}\right)^T, \omega_1 = \frac{1}{8}, \omega_2 = \frac{3}{8}, \omega_3 = \frac{1}{2}$ are the weight of N_k $(k=1,2,3)$ such that $\prod_{k=1} \omega_k = 1$. Then by above theorem

$$BNFOWGA(N_1, N_2, N_3) = \begin{cases} \log_3\left(1+\left(3^{0.3}-1\right)^{\frac{1}{8}}+\left(3^{0.2}-1\right)^{\frac{3}{8}}+\left(3^{0.5}-1\right)^{\frac{1}{2}}\right), \\ 1-\log_3\left(1+\left(3^{1-0.6}-1\right)^{\frac{1}{8}}+\left(3^{1-0.5}-1\right)^{\frac{3}{8}}+\left(3^{1-0.7}-1\right)^{\frac{1}{2}}\right), \\ 1-\log_3\left(1+\left(3^{1-0.4}-1\right)^{\frac{1}{8}}+\left(3^{1-0.5}-1\right)^{\frac{3}{8}}+\left(3^{1-0.3}-1\right)^{\frac{1}{2}}\right), \\ \log_3\left(1+\left(3^{1-0.7}-1\right)^{\frac{1}{8}}+\left(3^{1-0.8}-1\right)^{\frac{3}{8}}+\left(3^{1-0.6}-1\right)^{\frac{1}{2}}\right)-1, \\ \log_3\left(1+\left(3^{0.3}-1\right)^{\frac{1}{8}}+\left(3^{0.4}-1\right)^{\frac{3}{8}}+\left(3^{0.2}-1\right)^{\frac{1}{2}}\right), \\ l\log_3\left(1+\left(3^{0.1}-1\right)^{\frac{1}{8}}+\left(3^{0.3}-1\right)^{\frac{3}{8}}+\left(3^{0.6}-1\right)^{\frac{1}{2}}\right) \end{cases}$$

$$BNFOWGA(N_1, N_2, N_3) = (1.0, 0.125, 0.2520, -0.0652, -1.0, -1.0).$$

The BNFOWGA operator has the following properties:

(4) Idomopotency:

Let all the BNN's be $N_n =< \chi_k^+, \xi_k^+, \Lambda_k^+, \chi_k^-, \xi_k^-, \Lambda_k^- >= N(k=1,2,3....n)$ where $\omega = (\omega_1, \omega_2, \omega_3.........\omega_n)^T$ be the weight vector of $N_k.\omega_k \in [0,1]$ and $\prod_{k=1} \omega_k = 1$ then

$BNFWGA\left(N_1, N_2, N_3.........N_k\right) = N$.

(5) Monotonicity: Let $N_k\ \left(k = 1,2,3....n\right)$ and $N_j^{'}\ \left(j = 1,2,3....n\right)$ be two families of BNNs, where $\omega = \left(\omega_1, \omega_2, \omega_3..........\omega_n\right)^T$ be the weight vector of N_k and $N_k^{'}$, $\omega_k \in [0,1]$ and $\prod_{k=1} \omega_k = 1$. For all 'j' if $N_j \geq N_j^{'}$ then

$$BNFWGA\left(N_1, N_2, N_3.........N_k\right) \geq BNFWGA\left(N_1^{'}, N_2^{'}, N_3^{'}.........N_k^{'}\right).$$

(6) Boundedness: Let $N_k =< \chi_k^+, \xi_k^+, \Lambda_k^+, \chi_k^-, \xi_k^-, \Lambda_j^- >= N\left(k = 1,2,3....n\right)$ be a family of BNNs. Where $\omega = \left(\omega_1, \omega_2, \omega_3..........\omega_n\right)^T$ be the weight vector of N_k , $\omega_k \in [0,1]$ and $\sum_{k=1}^{3} \omega_k = 1$, Therefore we have

$$BNFOWGA\left(N^-, N^-, ...N^-\right) \leq BNFOWGA\left(N_1, N_2, N_3.........N_j\right) \leq BNFOWGA\left(N^+, N^+, ...N^+\right)$$

where $N^- =< \chi_{N^-}^+, \xi_{N^-}^+, \Lambda_{N^-}^+, \chi_{N^-}^-, \xi_{N^-}^-, \Lambda_{N^-}^- >$

$$= \begin{cases} \min\left(\chi_1^+, \chi_2^+, \chi_3^+, ...\chi_n^+\right), \max\left(\xi_1^+, \xi_2^+, \xi_3^+, ...\xi_n^+\right), \\ \max\left(\Lambda_1^+, \Lambda_2^+, \Lambda_3^+, ...\Lambda_n^+\right), \max\left(\chi_1^-, \chi_2^-, \chi_3^-, ...\chi_n^-\right) \\ \min\left(\xi_1^-, \xi_2^-, \xi_3^-, ...\xi_n^-\right), \min\left(\Lambda_1^-, \Lambda_2^-, \Lambda_3^-, ...\Lambda_n^-\right) > \end{cases} \text{And}$$

$$N^+ =< \chi_{N^+}^+, \xi_{N^+}^+, \Lambda_{N^+}^+, \chi_{N^+}^-, \xi_{N^+}^-, \Lambda_{N^+}^- >$$

$$= \begin{cases} \max\left(\chi_1^+, \chi_2^+, \chi_3^+, ...\chi_n^+\right), \min\left(\xi_1^+, \xi_2^+, \xi_3^+, ...\xi_n^+\right), \\ \min\left(\Lambda_1^+, \Lambda_2^+, \Lambda_3^+, ...\Lambda_n^+\right), \min\left(\chi_1^-, \chi_2^-, \chi_3^-, ...\chi_n^-\right) \\ \max\left(\xi_1^-, \xi_2^-, \xi_3^-, ...\xi_n^-\right), \max\left(\Lambda_1^-, \Lambda_2^-, \Lambda_3^-, ...\Lambda_n^-\right) > \end{cases}.$$

Proof:

(3) Since $N_j =< \chi_j^+, \xi_j^+, \Lambda_j^+, \chi_j^-, \xi_j^-, \Lambda_j^- >= N\left(j = 1,2,3....n\right)$ then, the following result can be obtained by using Equation (1)The following result can be obtained

$$BNFOWGA\left(N_1,N_2,N_3.........N_n\right)=\begin{cases}<\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{\chi_k^+}-1\right)^{\omega_k}\right),1-\log\prod_{k=1}^{n}\left(\lambda^{1-\xi_k^+}-1\right)^{\omega_k},\\ 1-\log\prod_{k=1}^{n}\left(\lambda^{1-\Lambda_k^+}-1\right)^{\omega_k},\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{1-(-\chi_k^-)}-1\right)^{\omega_k}\right)-1,\\ -\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{-\xi_k^-}-1\right)^{\omega_k}\right),-\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{-\Lambda_k^-}-1\right)^{\omega_k}\right),>(\omega>0)\end{cases}$$

$$=\begin{cases}<\log_\lambda\left(1+\left(\lambda^{\chi^+}-1\right)\right),1-\log_\lambda\left(1+\left(\lambda^{1-\xi^+}-1\right)\right),\\ 1-\log_\lambda\left(1+\left(\lambda^{1-\Lambda^+}-1\right)\right),\log_\lambda\left(1+\left(\lambda^{1-(-\chi^-)}-1\right)\right)-1,\\ -\log_\lambda\left(1+\left(\lambda^{-\xi^-}-1\right)\right),-\log_\lambda\left(1+\left(\lambda^{-\Lambda^-}-1\right)\right)>\end{cases}$$

$$=<\chi^+,\xi^+,\Lambda^+,\chi^-,\xi^-,\Lambda^->=N \text{ holds}$$

(4) The property is obvious based on the equation (6).

(5) Let $N^-=<\chi^+_{N^-},\xi^+_{N^-},\Lambda^+_{N^-},\chi^-_{N^-},\xi^-_{N^-},\Lambda^-_{N^-}>\ and\ N^+=<\chi^+_{N^+},\xi^+_{N^+},\Lambda^+_{N^+},\chi^-_{N^+},\xi^-_{N^+},\Lambda^-_{N^+}>$.

There are the following inequalities:

$$\begin{cases}<\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{\chi^+_{N^-}}-1\right)\right)\le\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{\chi_k^+}-1\right)\right)\le\log_\lambda\left(1+\prod_{j=1}^{n}\left(\lambda^{\chi^+_{N^+}}-1\right)\right),\\ 1-\log\prod_{k=1}^{n}\left(\lambda^{1-\xi^+_{N^-}}-1\right)\le1-\log\prod_{k=1}^{n}\left(\lambda^{1-\xi_k^+}-1\right)\le,1-\log\prod_{k=1}^{n}\left(\lambda^{1-\xi^+_{N^+}}-1\right),\\ 1-\log\prod_{k=1}^{n}\left(\lambda^{1-\Lambda^+_{N^-}}-1\right)\le1-\log\prod_{k=1}^{n}\left(\lambda^{1-\Lambda_k^+}-1\right)\le1-\log\prod_{j=1}^{n}\left(\lambda^{1-\Lambda^+_{N^+}}-1\right),\\ \log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{1-(-\chi^-_{N^-})}-1\right)\right)-1\le\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{1-(-\chi^-_{N})}-1\right)\right)-1\le\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{1-(-\chi^-_{N^+})}-1\right)\right)-1\\ -\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{-\xi^-_{N^-}}-1\right)\right)\le-\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{-\xi_k^-}-1\right)\right)\le-\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{-\xi^-_{N^+}}-1\right)\right),\\ -\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{-\Lambda^-_{N^-}}-1\right)\right),\le-\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{-\Lambda_k^-}-1\right)\right)-\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{-\Lambda^-_{N^+}}-1\right)\right)>\end{cases}$$

.

Hence

$$BNFOWGA\left(N^-,N^-,...........N^-\right)\le BNFOWGA\left(N_1,N_2,N_3.........N_j\right)\le BNFOWGA\left(N^+,N^+,...........N^+\right)$$

holds for all 'n'.

6. Model for MCDM Using Bipolar Neutrosophic Information

Using the BNFWGA and BNFOWGA operators that are suggested, two comprehensive MCDM approaches are expanded in this section.

For MCDM model with bipolar neutrosophic fuzzy information, Let $A=(A_1, A_2, A_3.......A_n)$ be the set of alternatives and $C=(C_1, C_2, C_3.......C_n)$ be a set of attributes.

Let $\omega=(\omega_1, \omega_2, \omega_3..........\omega_n)^T$ be the weight to attribute C_K.

Suppose that $N=\left(N_{jk}\right)_{s\times r}=\left(\chi_{jk}^{+}, \xi_{jk}^{+}, \Lambda_{jk}^{+}, \chi_{jk}^{-}, \xi_{jk}^{-}, \Lambda_{jk}^{-}\right)_{s\times r}(j=1,2,3.....s)(k=1,2,.....r)$ is BNN decision matrix, where $\chi_{jk}^{+}, \xi_{jk}^{+}, \Lambda_{jk}^{+}$ indicates the truth membership degree, indeterminacy membership degree and falsity membership degree of alternative A_j under attribute C_K with respect to positive preferences and $\chi_{jk}^{-}, \xi_{jk}^{-}, \Lambda_{jk}^{-}$ indicates the truth membership degree, indeterminacy membership degree and falsity membership degree of alternative A_j under attribute C_K with respect to negative preferences. We have conditions $\chi_{jk}^{+}, \xi_{jk}^{+}, \Lambda_{jk}^{+}, \chi_{jk}^{-}, \xi_{jk}^{-}, \&\Lambda_{jk}^{-}\in[0,1]$ such that $0\le\chi_{jk}^{+}, \xi_{jk}^{+}, \Lambda_{jk}^{+}, \chi_{jk}^{-}, \xi_{jk}^{-}, \Lambda_{jk}^{-}\le 6$ for $(j=1,2,3.....s)(k=1,2,.....r)$.

6.1 Proposed Algorithm using BNFOWGA operator to solve MCDM problem

Step 1 Collect information on the bipolar neutrosophic evaluation

Step 2 Calculate score and the accuracy values of collected information.

The score values $s\left(N_{jk}\right)$ and accuracy values $a\left(N_{jk}\right)$ of alternatives A_j

can be calculated by using Equations (1) and (2).

Step 3 The compression method in Definition 4.1 to reorder information on evaluation under each attribute. The comparison method is used to reorder $\left(N_{jk}\right)$.

Step 4 Derive the collective BNN $N_j\ (j=1,2,.....s)$ for the alternative $A_j\ (j=1,2,.....s)$

Method (1) .

Utilize BNFWGA operator to calculate the collective BNN for each alternative, then $N_j = BNFWGA\left(N_{j1}, N_{j2}, N_{j3}.........N_{jn}\right) = \overset{n}{\underset{k=1}{\otimes}} N_{jk}^{\wedge_F^{\omega_k}}$.

$$N_j = BNFWGA\left(N_1, N_2.....N_{jn}\right) = \otimes_{k=1}^{n} N_{\rho(jk)}^{\wedge^{\omega_k}}$$

$$= \begin{cases} < \log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{\chi_{jk}^{+}}-1\right)^{\omega_k}\right), \\ 1-\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{1-\xi_{jk}^{+}}-1\right)^{\omega_k}\right), \\ 1-\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{1-\Lambda_{jk}^{+}}-1\right)^{\omega_k}\right), \\ \log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{1-(-\chi_{jk}^{-})}-1\right)^{\omega_k}\right)-1, \\ -\log_\lambda\left(1+\prod_{k=1}^{s}\left(\lambda^{-\xi_{jk}^{-}}-1\right)^{\omega_k}\right), \\ -\log_\lambda\left(1+\prod_{k=1}^{s}\left(\lambda^{-\Lambda_{jk}^{-}}-1\right)^{\omega_k}\right), > (\omega > 0) \end{cases}$$

Where $\omega = \left(\omega_1, \omega_2, \omega_3..........\omega_n\right)^T$ is the weight vector such that $\omega_k \in [0,1]$ and $\sum_{k=1}^{n} \omega_k = 1$

Method (2).

Utilize BNFOWGA operator to calculate the collective BNN for each alternative, then

$$N_j = BNFOWGA\left(N_1, N_2.....N_{jl}\right) = \otimes_{k=1}^{n} N_{\rho(jl)}^{\wedge^{\omega_k}}$$

$$= \begin{cases} < \log_\lambda\left(1+\prod_{l=1}^{n}\left(\lambda^{\chi_{jl}^{+}}-1\right)^{\omega_l}\right), 1-\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{1-\xi_{jl}^{+}}-1\right)^{\omega_l}\right), \\ 1-\log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{1-\Lambda_{jl}^{+}}-1\right)^{\omega_l}\right), \log_\lambda\left(1+\prod_{k=1}^{n}\left(\lambda^{1-(-\chi_{jl}^{-})}-1\right)^{\omega_l}\right)-1, \\ -\log_\lambda\left(1+\prod_{k=1}^{s}\left(\lambda^{-\xi_{jl}^{-}}-1\right)^{\omega_l}\right), -\log_\lambda\left(1+\prod_{k=1}^{s}\left(\lambda^{-\Lambda_{jl}^{-}}-1\right)^{\omega_l}\right), > (\omega > 0) \end{cases}$$

Where ρ is permutation that orders the elements: $N_{\rho(j1)} \geq N_{\rho(j2)} \geq N_{\rho(j3)} \geq ....N_{\rho(jn)}$

Where $\omega = \left(\omega_1, \omega_2, \omega_3..........\omega_n\right)^T$ is the weight vector such that $\omega_l \in [0,1]$ and $\sum_{l=1}^{n} \omega_l = 1$.

Step 5: Calculate the score values $s\left(N_j\right)\left(j = 1, 2, 3........s\right)$ of BNNs $\left(N_j\right)\left(j = 1, 2, 3........s\right)$ to rank all the alternatives $A_j\left(j = 1, 2, 3........s\right)$ and then select favorable one. If score values of BNNs

N_j & N_k are equal, then we calculate accuracy values $a\left(N_j\right) \& a\left(N_k\right)$ of BNNs N_j & N_k respectively and then rank the alternatives A_j & A_l as accuracy values $a\left(N_j\right) \& a\left(N_k\right)$

Step 6 Rank all the alternatives $A_j\left(j=1,2,3........s\right)$ and select favorable one.

Step 7 End

7. Bridge Management to avoid traffic congestion using proposed Algorithm

The best solution to a bridge management problem from Amin Amini & Navid Nikraz [32] is discovered in this part in order to prevent traffic jams. To determine the optimum path that avoids traffic jams. Additionally, parametric analysis and comparison analysis are performed to confirm the adaptability and efficiency of the suggested algorithm to address the problem of decision-making.
People who reside in and travel through affected neighbourhoods, as well as on state routes, are greatly affected by traffic jams. The behaviour of road users, the safety of the road and bridge infrastructure conditions and characteristics, and vehicles interact continuously to form the traffic process. People that are delayed are late for key daily tasks including work, school, appointments, and other things. When clients and consumers have trouble contacting them, business suffers. When ambulances, rescue teams, and fire vehicles are unable to drive on their usual routes, routine incidents can quickly become life-threatening. Therefore, our programme was created to determine the optimum path while taking into consideration three factors: connectivity in a single lane, avoiding traffic incidents, and saving time for human resources. Consider three bridges A_1, A_2, A_3.

Based on the recommendations of the experts group in terms of three criteria, the roads departments chose to construct the bridge in order to reduce traffic namely multiple road connection in single lane $\left(C_1\right)$, avoid road accidents $\left(C_2\right)$, time saver for human resources $\left(C_3\right)$.

Using the proposed algorithm bridge selection has been done as follows:
Step1: Collect information on bipolar neutrosophic evaluation
The information collected from expert discussion on evaluation is given in Table1

Table 1: Bipolar-neutrosophic evaluation information under

	C_1	C_2	C_3
A_1	(0.5,0.7,0.2,-0.7,-0.3,-0.6)	(0.4,0.4,0.5,-0.7,-0.8,-0.4)	(0.7,0.7,0.5,-0.8,-0.7-0.6)

A_2	(0.9,0.7,0.5,-0.7,-0.7,-0.1)	(0.7,0.6,0.8,-0.7,-0.5,-0.1)	(0.9,0.4,0.6,-0.1,-0.7,-0.5)
A_3	(0.3,0.4,0.2,-0.6,-0.3,-0.7)	(0.2,0.2,0.2,-0.4,-0.7,-0.4)	(0.9,0.5,0.5,-0.6,-0.5,-0.2)

Step 2: Calculate Score and accuracy values of collected information.

For each alternative A_j the attribute C_k ,the score values $s\left(N_{jk}\right)$ and the accuracy values $a\left(N_{jk}\right)$ can be calculated based on Equation (1) and Equation (2). The Score values $s\left(N_{jk}\right)$ and the accuracy values $a\left(N_{jk}\right)$ are shown in Tables 2 and 3 respectively.

Table 2: Score values $s\left(N_{jk}\right)$

	C_1	C_2	C_3
A_1	0.4067	0.5000	0.5000
A_2	0.4667	0.3667	0.6667
A_3	0.5167	0.5833	0.5000

Table 2: Accuracy Values $a\left(N_{jk}\right)$

	C_1	C_2	C_3
A_1	0.2000	-0.4000	0
A_2	-0.2000	-0.7000	0.7000
A_3	0.2000	0	0

Step 3: Reorder information on evaluation under each attribute

The comparison method in Definition 4.1 is used to reorder N_{jk}

Table 4 reordering bipolar neutrosophic evaluation information by using comparison method based on Definition 4.1

	C_1	C_2	C_3
A_1	(0.7,0.7,0.5,-0.8,-0.7-0.6)	(0.4,0.4,0.5,-0.7,-0.8,-0.4)	(0.5,0.7,0.2,-0.7,-0.3,-0.6)
A_2	(0.9,0.4,0.6,-0.1,-0.7,-0.5)	0.5,0.2,0.7,-0.5,-0.1,-0.9)	(0.9,0.7,0.5,-0.7,-0.7,-0.1)
A_3	(0.2,0.2,0.2,-0.4,-0.7,-0.4)	(0.3,0.4,0.2,-0.6,-0.3,-0.7)	(0.9,0.5,0.5,-0.6,-0.5,-0.2)

Table 5: Score values $s\left(N_{jk}\right)$

	C_1	C_2	C_3
A_1	0.5000	0.5000	0.4067
A_2	0.6667	0.5167	0.4667
A_3	0.5833	0.5167	0.5000

Table 6: Accuracy Values $a\left(N_{jk}\right)$

	C_1	C_2	C_3
A_1	0	-0.4000	0.2000
A_2	0.7000	0.2000	-0.2000
A_3	0	0.2000	0

Step 4: Derive the collective BNN $N_j\left(j=1,2,3...s\right)$ for the alternative $A_j\left(j=1,2,3...s\right)$ Method 1 BNFWGA operator using Eqn(8) and supporting $\lambda=7$ to calculate the collective BNN for each alternative, then

$$N_1=\langle 0.1301, 0.6857, 0.6345, -0.7517, -0.2494, -0.4480\rangle$$
$$N_2=\langle 0.5735, 0.6795, 0.7665, -0.6913, -0.1301, -0.1038\rangle$$
$$N_3=\langle 0.1301, 0.6857, 0.6345, -0.7517, -0.2494, -0.4480\rangle$$

The value of BNN by power operation when $\lambda=7$

$$\langle 0.7168, 0.2758, 0.2858, -0.2968, -0.6981, -0.6484\rangle$$

Method (2) Utilize BNFOWGA operator using Eq (9) and supporting $\lambda = 7$ to calculate the collective BNN for each alternative, then

$$N_1 = \langle 0.1301, 0.6857, 0.6345, -0.78414, -0.2512, -0.4480 \rangle$$
$$N_2 = \langle 0.5493, 0.6357, 0.7496, -0.6569, -0.1193, -0.1193 \rangle$$
$$N_3 = \langle 0.2160, 0.4527, 0.4264, -0.5661, -0.2506, -0.2335 \rangle$$

The value is $\langle 0.7100, 0.2758, 0.2668, -0.2739, -0.6343, -0.6529 \rangle$

Step 5: Calculate the score values $s(N_j)$ $A_j\ (j=1,2,3)$ of BNNs N_j $(j=1,2,3)$ for each alternatives $A_j\ (j=1,2,3)$ $A_k\ (k=1,2,3)$. The score values $s(N_{jk})$ are calculated using Equation (2)

Method 1 The following score values are obtained by using the BNFWGA operation

$s(N_1) = 0.2875 : s(N_2) = 0.2783 : s(N_3) = 0.3751$

Method 2 The following score values are obtained by using the BNFOWGA operation

$s(N_1) = 0.2875 : s(N_2) = 0.2910 : s(N_3) = 0.3758$

Step 6: Rank all the alternatives $A_j\ (j=1,2,3)$ and select favorable one.

The alternative can be ranked in descending order based on the comparison method, and favorable alternative can be selected.

Method 1 The following ranking order based on score values is obtained by using BNFWGA operator:

$A_3 > A_2 > A_1$ Thus A_3 is favorable.

Method 2 The following ranking order based on the score values is obtained by using BNFOWGA operator: $A_3 > A_2 > A_1$ Thus A_3 is favorable.

Step 7: End

8. Comparative Analysis

To demonstrate the soundness of the suggested work, this section contrasts and compares it to the current approaches.

Table: 7 Ranking Orders obtained by Different Methods.

Methods	Rankings

A_ω [22]	$A_3 > A_2 > A_1$
G_ω [22]	$A_3 > A_2 > A_1$
BNDGA Operator $\lambda = 7$ [33]	$A_3 > A_2 > A_1$
BNDOGA Operator $\lambda = 7$ [33]	$A_3 > A_2 > A_1$
FBNCGBM operator (s,t=1)[21]	$A_3 > A_2 > A_1$
FBNCOGBM operator (s,t=1)[21]	$A_3 > A_2 > A_1$
FATTT2FFAAOWA $\lambda = 7$ []	$A_2 > A_1 > A_3$
FATTT2FFAAFWA $\lambda = 7$ [33]	$A_2 > A_3 > A_1$
BNFWGA Operator $\lambda = 7$	$A_3 > A_2 > A_1$
BNFOWGA Operator $\lambda = 7$	$A_3 > A_2 > A_1$

Table: 8 Characteristic Comparison of different methods.

Methods	Flexible measure easier
A_ω [18]	No
G_ω [18]	No
BNDGA Operator $\lambda = 7$ [29]	No
BNDOGA Operator $\lambda = 7$ [29]	No
FBNCGBM operator (s,t=1)[22]	No
FBNCOGBM operator (s,t=1)[22]	No
FATTT2FFAAOWA $\lambda = 7$ [31]	Yes
FATTT2FFAAFWA $\lambda = 7$ [31]	Yes
BNFWGA Operator $\lambda = 7$	Yes
BNFOWGA Operator $\lambda = 7$	No

In G_{ω} , BNDGA and BNDOGA methods, the proposed method based on proposed BNFWGA operator does not consider the interaction and interrelation among attributes. In contrast to A_{ω}, G_{ω}, FATTT2FFAAOWA $(\lambda = 7)$ and FATTT2FFAAFWA $(\lambda = 7)$ methods, the proposed BNFWGA operator choose the appropriate parameters according to the preferences of Decision Makings. In similar to FBNCGBM operator (s,t=1) and FBNCOGBM operator (s,t=1), BNFWGA Operator for $\lambda = 7$, BNFOWGA Operator for $\lambda = 7$, the proposed BNFWGA operators can select the appropriate parameters according to preferences of the decision making.

In G_{ω} , BNDGA and BNDOGA methods, the proposed method based on proposed BNFOWGA operator does not consider the interaction and interrelation among attributes. In contrast to A_{ω}, G_{ω}, FATTT2FFAAOWA $(\lambda = 7)$ and FATTT2FFAAFWA $(\lambda = 7)$ methods, the proposed BNFOWGA operator selects the relevant settings in accordance with Decision Makings' preferences. In similar to FBNCGBM operator (s,t=1) and FBNCOGBM operator (s,t=1), BNFWGA Operator $\lambda = 7$, BNFOWGA Operator $\lambda = 7$ the proposed BNFWGA operators can select the appropriate parameters according to preferences of the decision makings. Therefore, based on the proposed BNFWGA operator, the proposed technique may be used for decision-making. As a result, the suggested operators are more dependable and adaptable. These suggested strategies can be used in practice MCDM situations for decision-making based on the suggested operators and their requirements.

9. Conclusion

The tendency of the human mind to think about both positive and negative impacts when making decisions is referred to as bipolarity. A generalization of fuzzy, intuitionistic, and neutrosophic sets, the bipolar neutrosophic set enables it to handle ambiguous information in the decision-making process with higher adaptability. As a result, the operational laws of the proposed aggregation operation for both BNFWGA and BNFOWGA have been investigated and given in this study utilising Frank triangular norms in a bipolar neutrosophic environment. The use of the BNFWGA and BNFOWGA operators' proposed method to the MCDM bridge selection problem demonstrated its viability and cogency, and the suggested principles were used to choose the best bridge for reducing traffic congestion. Additionally, a comparison of the new method with the old method has been conducted. In the future, aggregation operators may be created employing a variety of triangular norms in different neutrosophic contexts.

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