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On Continuity in Minimal Structure Neutrosophic Topological Space

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Abstract:

In this paper, we introduce the notion of continuity via neutrosophic minimal structure space. Besides, we introduce the notion of product minimal space in neutrosophic topological space. Further, we investigate some basic properties of N_m–continuity in neutrosophic minimal structure space, such as composition of N_m–continuous functions, product of N_m–continuous functions in product neutrosophic minimal structure space.

Keywords: *Neutrosophic Set*; *Neutrosophic Topology*; *Minimal Structure*; *Neutrosophic Pre-Open Set*; *Neutrosophic Semi-Open Set*; m_N-*Continuity*; *Neutrosophic Product Space*.

1. Introduction:

The existing theory of Cantor's crisp set theory was not sufficient to handle most of the problems in the real life situation. Uncertainty plays an important role in our everyday life problems. Then, L.A. Zadeh introduced notion of fuzzy set in the year 1965 to overcome the uncertainty situation on considering the membership of truthiness. This is considered as an important generalization of the two valued logic. Still the introduction of fuzzy sets was not sufficient to control the uncertainty. K. Atanaosv in the year 1986 considered non-membership value together with the membership value. He introduced the notion of intuitionistic set. Smarandache [24] realised that the existing tools are not sufficient to find solutions to all types of problems on uncertainty. He then considered the elements with truth membership, false membership and indeterministic membership values, and introduced the notion of neutrosophic set. The concept of neutrosophic set has been applied in many branches of science and technology. Das et al. [5] have studied algebraic operations neutrosophic fuzzy matrices. Das and Tripathy [6] have investigated different properties of neutrosophic multiset topological space. Das et al. [4] have applied the concept of neutrosophic sets for the solution of decision making problems.

The notion of neutrosophic topological space was introduced by Salama and Alblowi [21]. Salama and Alblowi [22] further studied the notion of generalized neutrosophic set and generalized neutrosophic topological space. Later on, Iswaraya and Bageerathi [10], Arokiarani et al. [2], Parimala et al. [17], Parimala et al. [18], Rao and Srinivasa [19], Salama et al. [23], Das and Tripathy [9] and others introduced different notions of open sets in neutrosophic topological space. Recently, Tripathy and Das [27] introduced the notion of b-locally open sets in neutrosophic topological space. The notion of b-locally open sets in bitopological space was introduced and investigated by Tripathy and Sarma [32]. In 2013, Tripathy and Sarma [33] studied the notion of weakly b-continuous functions in bitopological space. In 2020, Das and Tripathy [7] introduced the concept of pairwise neutrosophic b-open set via neutrosophic bitopological space. Later on, Tripathy and Das [26] defined pairwise neutrosophic b-continuous functions via neutrosophic bitopological space.

The notion of minimal structure in topological space was introduced by Maki et al. [12]. Thereafter, it was investigated by many others from different aspects. The notion of minimal structure in a fuzzy topological space was introduce by Alimohammady and Roohi [1], and further investigated by Tripathy and Debnath [28] and others.

Continuity on topological spaces is a very fundamental concept. It plays an important role and has successfully been applied in different areas of research in science and technology. Different types of continuity on topological spaces and fuzzy topological spaces has been investigated by Ray and Tripathy [20], Tripathy and Ray [29-31], Tripathy and Sarma [33] and others.

In this article we introduce the notion of continuity in minimal structure spaces in neutrosophic topological space and investigate its different properties.

The rest of the paper is divided into following sections:

Section 2 is on the preliminaries and definitions. All the existing definitions have been procured in this section those are very useful for the preparation of the main results of this article. Section 3 introduces mappings between neutrosophic sets, and some basic results have been proved. Section 4 is on ccontinuity in neutrosophic minimal structure spaces. Finally, in section 5, we conclude the work done in this article.

2. Materials and Methods (proposed work with more details):

In this section we procure some basic definitions and notations those will be used throughout this article.

Definition 2.1.[24] Let X be a universal set. A neutrosophic set A in X is a set contains triplet having truthness, falseness and indeterminacy membership values that can be characterized independently, denoted by T_A, F_A, I_A in [0,1]. The neutrosophic set A is denoted as follows:

A = {(x, $T_A(x)$, $I_A(x)$, $F_A(x)$): $x \in X$, and $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$}. Since, no restriction on the values of $T_A(x)$, $I_A(x)$ and $F_A(x)$ is imposed, so we have $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Example 2.1. Let X={n_1, n_2} be a non-empty fixed set. Clearly, W={(n_1,0.4,0.7,0.8), (n_2,0.2,0.7,0.8)} is a neutrosophic set over X.

Definition 2.2.[24] The null and full neutrosophic set over a nonempty set X are denoted by 0_N and 1_N, given by

(i) 0_N={(x, 0, 1, 1) : $x \in X$};

(ii) 1_N={(x, 1, 0, 0) : $x \in X$}.

There are also other representations of 0_N and 1_N. One may refer to the references cited in the article. Clearly, $0_N \subseteq 1_N$. We have, for any neutrosophic set A, $0_N \subseteq A \subseteq 1_N$.

Definition 2.3.[24] Let A = {(x, $T_A(x)$, $I_A(x)$, $F_A(x)$): $x \in X$} be a neutrosophic set over X, then the complement of A is defined by A^c={(x, 1-$T_A(x)$, 1-$I_A(x)$, 1-$F_A(x)$) : $x \in X$}.

Example 2.2. Let X={n_1, n_2} be a non-empty set. Let W={(n_1,0.5,0.7,0.5), (n_2,0.5,0.8,0.7)} be a neutrosophic set over X. Then, the complement of W is W^c={(n_1,0.5,0.3,0.5), (n_2,0.5,0.2,0.3)}.

Definition 2.4.[24] A neutrosophic set A = {(x, $T_A(x)$, $I_A(x)$, $F_A(x)$): $x \in X$} is contained in the other neutrosophic set B = {(x, $T_B(x)$, $I_B(x)$, $F_B(x)$): $x \in X$} (i.e., $A \subseteq B$) if and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$, for each $x \in X$.

Example 2.3. Let X={n_1, n_2} be a non-empty set. Let W={(n_1,0.5,0.7,0.5), (n_2,0.5,0.8,0.7)} and M={(n_1,0.7,0.5,0.2), (n_2,0.9,0.3,0.5)} be two *neutrosophic sets* over X. Then, $W \subseteq M$.

Definition 2.5.[24] If A = {(x, $T_A(x)$, $I_A(x)$, $F_A(x)$): $x \in X$} and B = {(x, $T_B(x)$, $I_B(x)$, $F_B(x)$): $x \in X$} are any two neutrosophic sets over X, then $A \cup B$ and $A \cap B$ is defined by

$$A \cup B = \{(x, T_A(x) \vee T_B(x), I_A(x) \wedge I_B(x), F_A(x) \wedge F_B(x)): x \in X\},$$

$$\text{and } A \cap B = \{(x, T_A(x) \wedge T_B(x), I_A(x) \vee I_B(x), F_A(x) \vee F_B(x)): x \in X\}.$$

Example 2.4. Let X={n_1, n_2} be a non-empty set. Let W={(n_1,0.5,0.7,0.5), (n_2,0.5,0.8,0.7)} and M= {(n_1,0.7,0.5,0.2), (n_2,0.9,0.3,0.5)} be two neutrosophic sets over X. Then, $W \cup M$={(n_1,0.7,0.5,0.2), (n_2,0.9,0.3,0.5)} and $W \cap M$={(n_1,0.5,0.7,0.5), (n_2,0.5,0.8,0.7)}.

The notion of neutrosophic topological space is defined as follows:

Definition 2.6.[21] Let X be a non-empty set and τ be the collection of neutrosophic subsets of X then τ is said to be a neutrosophic topology (in short NT) on X if the following properties holds:

(i) 0_N, $1_N \in \tau$,

(ii) U_1, $U_2 \in \tau \Rightarrow U_1 \cap U_2 \in \tau$,

(iii) $\cup_{i \in \Delta} u_i \in \tau$, for every {$u_i$: $i \in \Delta$} $\subseteq \tau$.

Then, (X, τ) is called a neutrosophic topological space (in short NTS) over X. The members of τ are called neutrosophic open sets (in short NOS). A neutrosophic set D is called neutrosophic closed set (in short NCS) if and only if D^c is a neutrosophic open set.

Example 2.5. Let W, E and Z be three neutrosophic sets over a fixed set X={p, q, r} such that:

W={(p,0.7,0.7,0.5), (q,0.5,0.5,0.1), (r,0.9,0.6,0.7): p, q, $r \in X$};

E={(p,0.6,0.8,0.9), (q,0.5,0.8,0.3), (r,0.4,0.7,0.8): $p, q, r\in X$};

Z={(p,0.5,0.8,1.0), (q,0.4,0.9,0.4), (r,0.3,0.7,1.0): $p, q, r\in X$}.

Then, the collection τ={$0_N, 1_N, W, E, Z$} forms a neutrosophic topology on X. Here, $0_N, 1_N, W, E, Z$ are NOSs in (X, τ), and their complements $1_N, 0_N$, W^c={(p,0.3,0.3,0.5), (q,0.5,0.5,0.9), (r,0.1,0.4,0.3): $p, q, r\in X$}, E^c={(p,0.4,0.2,0.1), (q,0.5,0.2,0.7), (r,0.6,0.3,0.2): $p, q, r\in X$} and Z={(p,0.5,0.2,0.0), (q,0.6,0.1,0.6), (r,0.7,0.3,0.0): $p, q, r\in X$} are NCSs in (X, τ).

The notion of neutrosophic interior and neutrosophic closure of a neutrosophic set is defined as follows:

Definition 2.7.[21] Let (X, τ) be a *NTS* and U be a *NS* in X. Then the neutrosophic interior (in short N_{int}) and neutrosophic closure (in short N_{cl}) of U are defined by

$$N_{int}(U) = \cup\{E : E \text{ is a } NOS \text{ in } X \text{ and } E\subseteq U\},$$

$$\text{and } N_{cl}(U) = \cap\{F : F \text{ is a } NCS \text{ in } X \text{ and } U\subseteq F\}.$$

Example 2.6. Let us consider a neutrosophic topological space as shown in **Example 2.5.** Let U={(p,0.5,0.7,0.5), (q,0.5,0.8,0.7), (r,0.3,0.7,1.0)} be a *neutrosophic set* over X. Then, $N_{int}(U)=0_N$ and $N_{cl}(U)$={(p,0.5,0.2,0.0), (q,0.6,0.1,0.6), (r,0.7,0.3,0.0)}.

Remark 2.1.[21] Clearly $N_{int}(U)$ is the largest neutrosophic open set over X which is contained in U and $N_{cl}(U)$ is the smallest neutrosophic closed set over X which contains U.

Definition 2.8.[2] Let (X, τ) be a neutrosophic topological space and G be a neutrosophic set over X. Then G is called,

(i) Neutrosophic semi-open (in short *NSO*) set if and only if $G\subseteq N_{cl}(N_{int}(G))$;

(ii) Neutrosophic pre-open (in short *NPO*) set if and only if $G\subseteq N_{int}(N_{cl}(G))$.

The collection of all NSO sets and NPO sets in (X, τ) are denoted by NSO(τ) and NPO(τ).

Example 2.7. Let X={a, b} be a non-empty set. Clearly, (X, τ) is a neutrosophic topological space, where τ={0_N, 1_N, {(a,0.3,0.4,0.3), (b,0.4,0.3,0.4): $a, b\in X$}, {(a,0.4,0.4,0.1), (b,0.5,0.1,0.3): $a, b\in X$}}. Then, the neutrosophic set Q={(a,0.6,0.4,0.1), (b,0.9,0.1,0.2): $a, b\in X$} is a NSO set and P={(a,0.3,0.9,0.2), (b,0.3,0.4,0.3): $a, b\in X$} is a NPO set in (X, τ).

Definition 2.9.[2] A neutrosophic set G is called a neutrosophic b-open set in a NTS (X, τ) if and only if $G\subseteq N_{int}(N_{cl}(G))\cup N_{cl}(N_{int}(G))$. A neutrosophic set H is said to be neutrosophic b-closed set if its complement H^c is a neutrosophic b-open. The collection of all neutrosophic b-open sets in (X, τ) is denoted by N-b-O(τ).

Example 2.8. Let (X, τ) be a neutrosophic topological space as shown in **Example 2.7**. Then, the neutrosophic set P={(a,0.3,0.9, 0.2), (b,0.3,0.4,0.3): $a, b\in X$} is a neutrosophic b-open set in (X, τ).

Definition 2.10.[27] Let (X, τ) be a neutrosophic topological space. A neutrosophic set G is said to be a neutrosophic locally open (in short *NLO*) set if $G= H\cup K$, where H is a neutrosophic open set and K is a neutrosophic closed set in X.

Example 2.9. Let (X, τ) be a neutrosophic topological space as shown in **Example 2.5**. Then, the *neutrosophic set* R={(p,0.7,0.2,0.0), (q,0.6,0.1,0.1), (r,0.9,0.3,0.0): p, q, $r \in X$} is a neutrosophic locally open set in (X, τ).

3. Mappings Between Neutrosophic Sets:

In this section, we prove some results on mappings between neurosophic subsets.

Proposition 3.1. Let $f:X\to Y$ be a mapping, and $\{U_i : i\in\Delta\}$ be a family of neurosophic subsets of Y, then we have

(i) $f^{-1}(\cup_{i\in\Delta} U_i) = \cup_{i\in\Delta} f^{-1}(U_i)$.

(ii) $f^{-1}(\cap_{i\in\Delta} U_i) = \cap_{i\in\Delta} f^{-1}(U_i)$.

Proof. The proofs are so easy, so omitted.

Theorem 3.2. If f_i: $X_i\to Y_i$ and U_i be neutrosophic sets of Y for i = 1, 2, then

$(f_1\text{x}f_2)^{-1}(U_1\text{x}U_2) = f_1^{-1}(U_1) \text{ x } f_2^{-1}(U_2)$.

Proof. Let f_i: $X_i\to Y_i$ be mappings for i = 1, 2. Let $U_1 = \{(T_1, F_1, I_1)(x_1) : x_1 \in X_1\}$ and U_2 $\{(T_2, F_2, I_2)(x_2) : x_2 \in X_2\}$ be neutrosophic subsets in Y_1 and Y_2 respectively. Then we have for (x_1, x_2) in $X_1\text{x}X_2$, we have

$(f_1\text{x}f_2)^{-1}(T_1\text{x}T_2)\ (x_1, x_2) = (T_1\text{x}T_2)\ (f_1(x_1), f_2(x_2))$

$= \min \{T_1f_1(x_1),\ T_2\, f_2(x_2)\}$

$= \min \{f_1^{-1}(T_1(x_1)),\ f_2^{-1}(T_2(x_2))\}$

$= (f_1^{-1}(T_1), f_2^{-1}(T_2))\ (x_1, x_2)$.

Following the above argument, we can show that

$$(f_1\text{x}f_2)^{-1}(F_1\text{x}F_2)\ (x_1, x_2) = (f_1^{-1}(F_1), f_2^{-1}(F_2))\ (x_1, x_2)$$

and $(f_1\text{x}f_2)^{-1}(I_1\text{x}I_2)\ (x_1, x_2) = (f_1^{-1}(I_1), f_2^{-1}(I_2))\ (x_1, x_2)$.

4. Continuity in Minimal Structure Neutrosophic Topological Space:

In this section we introduce the notion of continuous maps between minimal structures in neutrosophic topological spaces. We procure the following definitions on neutrosophic minimal structre spaces from the article by Pal et al. [15].

Definition 4.1. A family M of neutrosophic subsets of X if M $\subset P(X)$, where $P(X)$ denotes the power set of X is said to be a neutrosophic minimal structure on X if 0_N and 1_N belong to M. Then, the pair (X, M) is called a nutrosophic minimal space.

Example 4.1. Let W, E and Z be three neutrosophic sets over a fixed set X={p, q, r} such that:

W={(p,0.7,0.7,0.5), (q,0.5,0.5,0.1), (r,0.9,0.6,0.7): p, q, $r\in X$};

E={(p,0.6,0.8,0.9), (q,0.5,0.8,0.3), (r,0.4,0.7,0.8): p, q, $r\in X$};

Z={(p,0.5,0.8,1.0), (q,0.4,0.9,0.4), (r,0.3,0.7,1.0): p, q, $r\in X$}.

Clearly, the collection M={0_N, 1_N, W, E, Z} forms a neutrosophic minimal structure on X, and the pair (X, M) is a neutrosophic minimal structure space.

Remark 4.1. Eevery NTS is a nutrosophic minimal structure space. But every nutrosophic minimal structure space may not be a NTS in general. This follows from the following example.

Example 4.2. Let W, E and Z be three neutrosophic sets over a fixed set X={p, q, r} such that:

W={(p,0.5,0.7,0.5), (q,0.5,0.9,0.1), (r,0.9,0.6,0.7): p, q, $r\in X$};

E={(p,0.6,0.6,0.9), (q,0.5,0.8,0.3), (r,0.4,0.7,0.8): p, q, $r\in X$};

Z={(p,0.5,0.5,1.0), (q,0.4,0.7,0.4), (r,1.0,0.7,1.0): p, q, $r\in X$}.

Clearly, the collection M={0_N, 1_N, W, E, Z} forms a neutrosophic minimal structure on X, and the pair (X, M) is a neutrosophic minimal structure space. But (X, M) is not a NTS.

Definition 4.2. Let (X, M) be a neutrosophic minimal structure space. If $E\in$M, then E is called a neutrosophic m-open set, and its complement is called a neutrosophic m-closed set in (X, M).

Example 4.3. Let us consider a neutrosophic minimal structure space (X, M) as shown in **Example 4.1.** Clearly, 0_N, 1_N, W, E, Z are neutrosophic m-open sets in (X, M), and their complements 1_N, 0_N, W^c={(p,0.3,0.3,0.5), (q,0.5,0.5,0.9), (r,0.1,0.4,0.3): p, q, $r\in X$}, E^c={(p,0.4,0.2,0.1), (q,0.5,0.2,0.7), (r,0.6,0.3,0.2): p, q, $r\in X$} and Z={(p,0.5,0.2,0.0), (q,0.6,0.1,0.6), (r,0.7,0.3,0.0): p, q, $r\in X$} are neutrosophic m-closed sets in (X, M).

The notion of neutrosophic minimal interior and neutrosophic minimal closure of a neutrosophic set in a neutrosophic minimal structure space is defined as follows:

Definition 4.3. Let (X, M) be a neutrosophic minimal structure space, and U be a neutrosophic set over X. Then, the neutrosophic minimal interior (in short $N_{m\text{-}int}$) and neutrosophic minimal closure (in short $N_{m\text{-}cl}$) of U are defined as follows:

$$N_{m\text{-}int}(U) = \cup\{E : E \text{ is a neutrosophic } m\text{-open set in } X \text{ and } E\subseteq U\},$$

$$\text{and } N_{m\text{-}cl}(U) = \cap\{F : F \text{ is a neutrosophic } m\text{-closed set in } X \text{ and } U\subseteq F\}.$$

Example 4.4. Let (X, M) be a neutrosophic minimal structure space as defined in **Example 4.1.** Then, the neutrosophic minimal interior and neutrosophic minimal closure of U={(p,0.2,0.4,0.6), (q,0.4,0.7, 0.9), (r,0.0,0.5,0.4)} are $N_{m\text{-}int}(U)$={(p,0,1,1), (q,0,1,1), (r,0,1,1)} and $N_{m\text{-}cl}(U)$={(p,0.3,0.3,0.5), (q,0.5,0.5,0.9), (r,0.1,0.4,0.3)} respectively.

Exzample 4.5. From the above definitions, it is clear that every neutrosophic pre-open sets, neutrosophic semi-open sets, neutrosophic b-open sets are neutrosophic m-open sets.

Example 4.6. Let W, E and Z be three neutrosophic sets over a non-empty set X={p, q} such that:

W={(p,0.7,0.7,0.5), (q,0.5,0.5,0.1) : p, $q\in X$};

E={(p,0.6,0.8,0.9), (q,0.5,0.8,0.3) : p, $q\in X$};

Z={(p,0.5,0.8,1.0), (q,0.4,0.9,0.4) : p, $q\in X$}.

Here, the family τ={0_N, 1_N, W, E, Z} forms a neutrosophic topology on X, and so (X, τ) is a neutrosophic topological space. Suppose M = $\tau\cup$NPO(τ)$\cup$NSO(τ)$\cup$N-b-O(τ), then (X, M) is a neutrosophic minimal structure. Now, from the above it is clear that, every neutrosophic pre-open sets, neutrosophic semi-open sets, neutrosophic *b*-open sets in (X, τ) are neutrosophic *m*-open sets in (X, M). Further, it is also seen that, every neutrosophic *m*-open set in (X, M) is also a neutrosophic pre-open set, neutrosophic semi-open set, neutrosophic *b*-open set in (X, τ).

Definition 4.4. The function f: (X,M_1)⟶ (Y,M_2) is said to be minimal continuous (in short m_N continuous) if $f^1(U)$ is an m-open set, where U is any m-open set in M_2 .

Definition 4.5. Let (X, M) be a minimal structure on the neutrosophic set X. Let (X, τ) and (Y, σ) be neutrosophic topological spaces. Then the mapping f: (X, τ)→(Y, σ) is said to be weakly m-continuous if for each neutrosophic point x_0 and each neutrosophic open set V with $f(x_0)\in V$, there exists a neutrosophic open set U such that $x_0\in U$ and $f(U) \subset N_{cl}(V)$.

We state the following result without proof, in view of the above definition:

Proposition 4.1. Let f: (X,M_1)⟶ (Y,M_2) and g: (X,M_2)⟶ (Y,M_3) be N_m–continuous functions. Then the composition function gof : (X,M_1)⟶ (Y,M_3) is N_m–continuous.

Theorem 4.1. Let (Y,M_2) be a minimal space and f: X ⟶(Y,M_2) be a function. Then there is a weaker minimal structure M_1 on X for which f is N_m– continuous.

Proof: Let X and Y be non-empty sets and M_2 be a minimal structure on Y. Let f: X ⟶(Y,M_2) be a function. Let $M_1 \subseteq$ P (X) be defined by $M_1 =\{f^{-1}(V) : V\in M_2 \}$. Hence ($X$, M_1) is a minimal structure on X. From the definition of N_m–continuity and construction of M_1, it follows that f: (X,M_1)⟶ (Y,M2) is N_m–continuous. Further by the definition of weaker minimal structure and construction of M_1, it follows that M_1 is a weaker minimal structure on X.

We state the following result without proof.

Proposition 4.2. Let (X,M) be a minimal space and $Y\subseteq X$, then (Y,M⋂Y) is a minimal structure on Y. Further for (X, M⋂Y) there is a weaker minimal structure space.

Theorem 4.2. Let minimal (X,M_1) be a minimal structure and $Y\subseteq X$. Then there is a weakest minimal structure on Y say M_2 such the map i_f : (Y,M_2) ⟶(X,M_1⋂Y) is N_m–continuous.

Proof: In view of the above Theorem 4.2 on considering the identity map we can have the map i_f to be N_m– continuous.

Remark 4.2. The above result is true for the inclusion map i : (Y,M_2) ⟶(X,M_1). In this case M_2 is called as the induced minimal structure on Y.

Theorem 4.3. Let $Y\subseteq X$ and f:(X,M_1)⟶ (Z,N_1) be N_m–continuous. Then $f|_Y$:(Y,M2) ⟶(Z,N_1⋂$f(y)$) is m-continuous, where Y is endowed with M_2, induced minimal structure.

Proof: By the above remark and theorem 4.1, we have $f|_Y$ = foi (or foi_f) and hence $f|_Y$ is N_m–continuous.

Theorem 4.4. Let {(X_i, M_i) : $i\in\Delta$} be a family of minimal spaces, , where Δ being the index set and {f_i :X⟶(X_i, M_i) : $i\in\Delta$ } be a family of N_m–continuous functions. Then there is a weakest minimal structure M on X such that f_i's are N_m –continuous.

Proof: Let $\{(X_i, M_i) : i\in\Delta\}$, where Δ is the index set be a family of minimal spaces and $\{f_i : X\longrightarrow(X_i, M_i) : i\in\Delta\}$ be a family functions. Let $E_i = f^{-1}(M_i) = \{f^{-1}(V) : V\in M_i\}$ for $i\in\Delta$. Consider $M = \cup_{i\in\Delta}E_i$. Then (X, M) is a minimal structure on X by definition. Further from the construction of M, it is clear that $f_i : (X, M) \longrightarrow(X_i, M_i)$ are N_m–continuous. Since we have considered the union while considering the minimal structure M on X, so it will include all other minimal structures on X, so it is the weakest minimal structure on X.

Theorem 4.5. Let $\{f_i : X\longrightarrow(X_i, M_i) : i\in\Delta\}$ be a family of N_m–continuous functions, where, (X_i, M_i) are minimal spaces. Let the minimal structure M in X be generated by $\{f_i : i\in\Delta\}$. Then the function $f : (Y,N)\longrightarrow (X,M)$ is N_m–continuous if and only if $f_i o f$ is N_m–continuous function for all $i\in\Delta$.

Proof: Let $\{f_i : X\longrightarrow(X_i, M_i) : i\in\Delta\}$ be a family of N_m–continuous functions and $f : (Y,N)\longrightarrow (X,M)$ be N_m–continuous then by proposition 4.1, $f_i o f$ is N_m– continuous.

Next, let $f_i o f$ be N_m–continuous functions for each $i\in\Delta$, but f is not N_m–continuous. Thus we have $B\in M$ such that $f^{-1}(B) \notin N$. Then we have the following possibilities:

(i) There exist $i_0\in\Delta$ and $B_{i_0} \in M_{i_0}$ such that $B = f_{i_0}^{-1}(B_{i_0})$.

(ii) For every $i\in\Delta$ and every $B_i\in M_i$, $B\neq f_{i_0}^{-1}(B_{i_0})$.

Consider case (i), we have $B = f_{i_0}^{-1}(B_{i_0})$, implies $f^{-1}(f_{i_0}^{-1}(B_{i_0})) = (f_{i_0}\circ f)^{-1}(B_{i_0})$. Thus, for $B_{i_0} \in M_{i_0}$, we have $(f_{i_0}\circ f)^{-1}(B_{i_0}) \notin N$, which shows that $f_{i_0}\circ f$ is not N_m–continuous. Hence we arrive at a contradiction. Thus our supposition is wrong.

Next, consider the case (ii), we have $f^{-1}(0_{N,X})=0_{N,Y}$ and $f^{-1}(1_{N,X}) = 1_{N,Y}$, which leads to $B(0_{N,Y}, 1_{N,Y})$. Hence $M \setminus\{B\}$ is a minimal structure on X. Thus for each $i\in\Delta$, $f_i:(X, M\setminus\{B\})\longrightarrow(X_i, M_i)$, we have $f_i o f : (Y, N)\longrightarrow(X_i, M_i)$ is m_N – continuous for each $i\in\Delta$,. This leads to a contradiction to the choice of the minimal structure M on X. Thus, f is N_m–continuous, whenever $f_i o f$ is N_m–continuous for each $i\in\Delta$.

Remark 4.3. Let $\{(X_i, M_i), i\in\Delta\}$ be a family of minimal structures, then the product space is defined by $\Pi_{i\in\Delta} X_i$. It can be easily verify that $(\Pi_{i\in\Delta} X_i, \Pi_{i\in\Delta} M_i)$ is a minimal structure on $\Pi_{i\in\Delta} X_i$. Further $M = \Pi_{i\in\Delta} M_i$ is the weakest minimal structure on $\Pi_{i\in\Delta} X_i$.

One can easily verify that for each $j\in\Delta$, the canonical projection $\pi_j : \Pi_{i\in\Delta} X_i\longrightarrow X_j$ is N_m–continuous.

In view of the above theorem and remark, we formulate the following results.

Proposition 4.3. Let $\{(X_i, M_i), i\in\Delta\}$ be a family of minimal spaces and $X = \Pi_{i\in\Delta} X_i$ exists.

Proposition 4.4. Let $\{(X_i, M_i), i\in\Delta\}$ be a family of minimal spaces and $X = \Pi_{i\in\Delta} X_i$. Let the minimal structure on X be generated by $\Pi_{i\in\Delta} X_i$. Let $f : (Y,N)\longrightarrow (X,M)$ be a function. Then f is N_m-continuous if and only if $\pi_i o f$ is N_m–continuous for all $i\in\Delta$.

The following result is a consequence of above results.

Corollary 4.1. Let f : (X,M)⟶ (Y,N) and g: (X,M)⟶(Z, Q) be N_m–continuous functions. Then the function f xg :(X,M) ⟶(YxZ, N x Q) defined by (fxg)(x)=(f(x), g(x)) is N_m-continuous.

5. Conclusion:

In this article, we have introduced the notion of continuity and product minimal space in neutrosophic minimal structure spaces. Besides, we have investigated some basic properties of N_m–continuity in neutrosophic minimal structure spaces, such as composition of N_m–continuous functions, product of N_m–continuous functions in product neutrosophic topological space etc. It is hoped that, these kind of notions can also be investigated in the field of Neutrosophic Multiset Topological Space [6], Neutrosophic Bitopological Space [7], Pentapartitioned Neutrosophic Topological Space [8], Neutrosophic Complex Topological Space [11], Generalized Neutrosophic Topological Space [22], etc.

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